QED effective action for background field in dS spacetime

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Motivation

- Origin of the Large scale magnetic field is still unknown
- Inflationary magnetogenesis is a possible candidate, but…
- Magnetogenesis theory often predict strong *electric* field
- Can Schwinger effect remove this obstacle?

Effective action

Full action: $S = S[A_\mu,\, \phi,\, \phi^\dagger]$

EM fields (A_μ)

Charged scalar ($\phi,\, \phi^\dagger$)

Effective action

Full action: $S = S[A_\mu, \phi, \phi^\dagger]$

EM fields

~~Charged scalar~~

Scale out

Effective action

Full action: $S = S[A_\mu, \phi, \phi^\dagger]$

EM fields

~~Charged scalar~~

Scale out

Effective Action: $\Gamma = \Gamma[A_\mu]$

$$\mathrm{e}^{i\Gamma} = \langle 0; \text{final} | 0; \text{initial} \rangle$$

Effective action

|0;initial> and |0;final> is different

↕

Effective action Γ has imaginary part

Vacuum state for the initial time is not a vacuum for the final time.

Effective action

Schwinger's proper time method

$$\Gamma[A_\mu] = \int \mathrm{d}^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) - i\int_0^\infty \frac{\mathrm{d}s}{s}\mathrm{e}^{-im^2 s}\Theta_{\hat{\mathcal{H}}}(is)$$

$$\Theta_{\hat{\mathcal{H}}}(is) \equiv \mathrm{Tr}(\mathrm{e}^{-i\hat{\mathcal{H}}s}) \qquad \hat{\mathcal{H}} \equiv (\hat{p}_\mu + eA_\mu)^2$$

$$\int_0^\infty \frac{\mathrm{d}s}{s}\mathrm{e}^{-m^2 s}\Theta_{\hat{\mathcal{H}}}(s) = -\ln\det(m^2) = -\ln(\prod_k (H_k + m^2))$$

In perturbation theory

- We can always write down the effective action/Lagrangian

$$\mathcal{L}_{\text{eff}} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Maxwell Lagrangian}} + \underbrace{e^4(4-\text{point interaction}) + \cdots}_{\text{Self interaction term, Non linear}}$$

Maxwell Lagrangian — Self interaction term, *Non linear*

  - There is no imaginary part

- Schwinger effect is a *non-perturbative* effect
  - Example: Constant electric field in Minkowski spacetime

$$\Im\Gamma \propto \mathrm{e}^{-\frac{\pi m^2}{eE}}$$

Schwinger's original discussion

- Schwinger(1951)
  - Determine the effective Lagrangian from QED aciton

$$\mathcal{L} = -\mathcal{F} - \frac{1}{8\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s^3}\mathrm{e}^{-m^2 s}\left[(es)^2 \frac{\Re\cosh(es\sqrt{2(\mathcal{F}+i\mathcal{G})})}{\Im\cosh(es\sqrt{2(\mathcal{F}+i\mathcal{G})})} - \frac{2}{3}(es)^2\mathcal{F} - 1\right]$$

Minkowski, Constant EM background $\left(\mathcal{F} \equiv \frac{1}{2}(\boldsymbol{B}^2 - \boldsymbol{E}^2),\ \mathcal{G} \equiv \boldsymbol{E}\cdot\boldsymbol{B} \right)$

- Imaginary part comes from the poles on the integration path

$$\frac{1}{x+i\epsilon} = \mathcal{P}\frac{1}{x} - \underline{i\pi\delta(x)}$$

View point

- Imaginary part is totally determined by the analytic structure (singularities, poles, cuts…) of the kernel of the (Laplace-type) integral

From spectrum problem to scattering problem

We assume $\hat{\mathcal{H}} = \partial_t^2 + V(t)$

Eigen value problem $\hat{\mathcal{H}}\psi = \lambda\psi$ has two

WKB-type solutions

$$\psi_\pm = \frac{1}{\sqrt{\pi(t)}} \mathrm{e}^{\pm i\int \mathrm{d}t' \pi(t')}, \ (\pi(t) = +\sqrt{V - \lambda})$$

This is valid for $t \to -\infty, \ \lambda \to -\infty$

From spectrum problem to scattering problem

There is only one (exact) solution which satisfies $\lim_{t\to-\infty} \psi/\psi_+ = 1$

We define the Jost function as

$$a(\lambda) \equiv \lim_{t\to+\infty} \psi/\psi_+$$ 1-dim scattering matrix

If λ equals to the eigen value of $\hat{\mathcal{H}}$, a(λ)=0.

Then the Jost function is essentially same as

$$\det(\partial_t^2 + V(t) - \lambda)$$

View point

- Imaginary part is totally determined by the analytic structure (singularities, poles, cuts…) of the kernel of the (Laplace-type) integral
- Calculation of the spectral determinant is reduced to the scattering problem

Bootstrap/Resurgence

Equation of motion we have to consider

$$(\hbar^2\partial_t^2 + \pi(t)^2)\psi = 0$$

is rewritten as

$$(\partial_t^2 + \pi^2(t)\partial_s^2)\tilde{\psi} = 0$$

by the Laplace transformation

$$\psi(t,\hbar) = \hbar^{-1}\int \mathrm{d}s \mathrm{e}^{-s/\hbar}\tilde{\psi}(t,s)$$

Bootstrap/Resurgence

Can we find the analytic structure of $\tilde{\psi}$?

Bootstrap/Resurgence

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Yes, we can!

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Yes, we can!

We have to use a new mathematics called "Resurgence" which is developed by J. Ecalle and A. Voros independently.

Bootstrap/Resurgence

Bootstrap relation can be used to investigate analytic structure of $\tilde{\psi}$

$$\Delta_{s+\omega(\gamma)}\tilde{\psi}(t, s+\omega(\gamma)+z)$$
$$= (-1)^{m(\gamma)/2}(a_\gamma)_B \dot{*} \Delta_s\tilde{\psi}(t, s+z)$$

A. Voros, J. Ecalle (80's)

Bootstrap/Resurgence

Bootstrap relation can be used to investigate analytic structure of $\tilde{\psi}$

Discontinuity at a branch cut from s+ω

Classical action along a loop γ

$$\Delta_{s+\omega(\gamma)}\tilde{\psi}(t, s+\omega(\gamma)+z)$$
$$= (-1)^{m(\gamma)/2}(a_\gamma)_B \dot{*} \Delta_s\tilde{\psi}(t, s+z)$$

Maslov index of γ: winding number

Bootstrap/Resurgence

Bootstrap relation:

$$\Delta_{s+\omega(\gamma)}\tilde{\psi}(t, s+\omega(\gamma)+z)$$

$$=(-1)^{m(\gamma)/2}(a_\gamma)_B\dot{*}\Delta_s\tilde{\psi}(t, s+z)$$

The analytic singularity (discontinuity) is discribed by the discontinuity *itself* of other point.

View point

- Imaginary part is totally determined by the analytic structure (singularities, poles, cuts…) of the kernel of the (Laplace-type) integral
- Calculation of the spectral determinant is reduced to the scattering problem
- All the analytic structure can be obtained by the resurgence technique even <u>without any knowledge of the exact (analytic) solution of the e.o.m</u>

Summary

- Schwinger effect is a non-perturbative effect
- We can catch this effect even in case of the unsolvable (=no known analytic solution) problem

- I already checked this machinery works in the easiest case, but more interesting case is undone
- Extension to the QFT (multi/infinite-dimensional) problem