QED effective action for background field in dS spacetime

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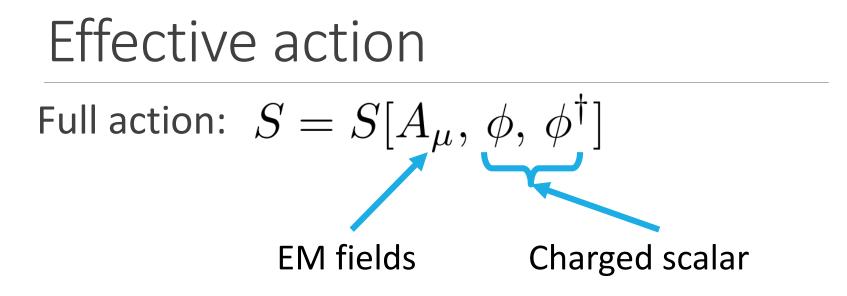
Motivation

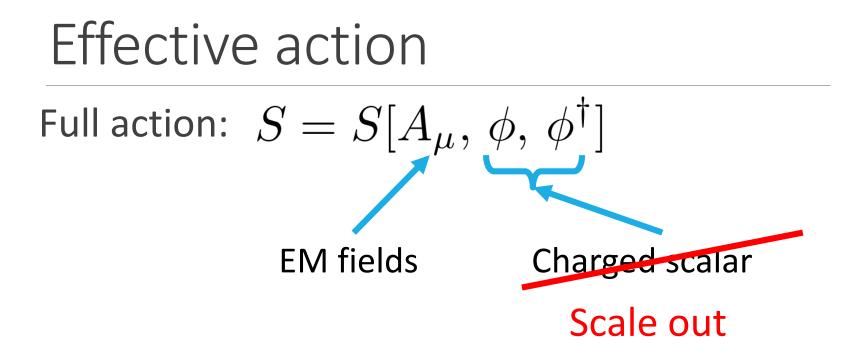
Origin of the Large scale magnetic field is still unknown

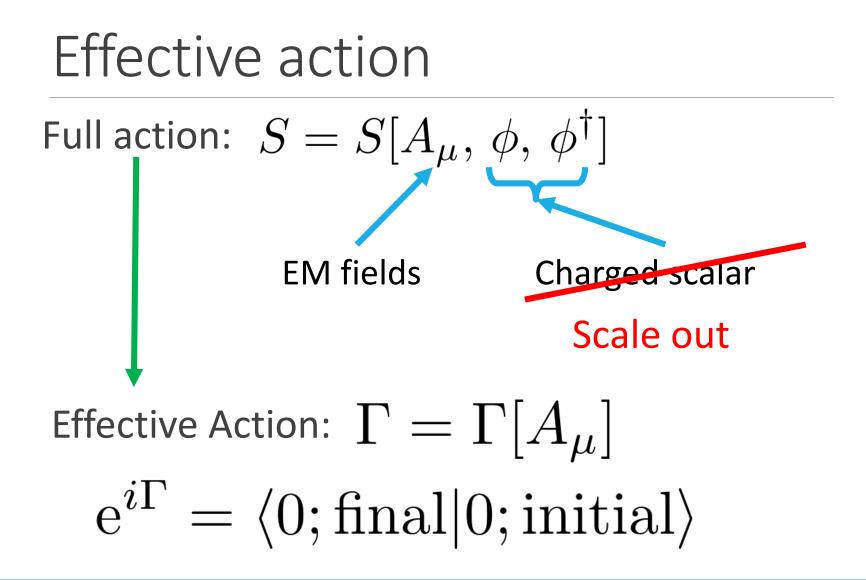
Inflationary magnetogenesis is a possible candidate, but...

Magnetogenesis theory often predict strong electric field

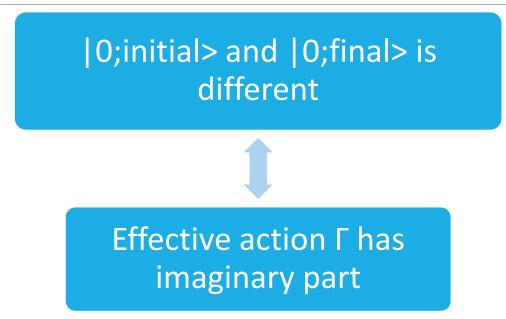
Can Schwinger effect remove this obstacle?







Effective action



Vacuum state for the initial time is not a vacuum for the final time.

Effective action

Schwinger's proper time method

$$\Gamma[A_{\mu}] = \int \mathrm{d}^4 x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) - i \int_0^\infty \frac{\mathrm{d}s}{s} \mathrm{e}^{-im^2 s} \Theta_{\hat{\mathcal{H}}}(is)$$

$$\Theta_{\hat{\mathcal{H}}}(is) \equiv \operatorname{Tr}(e^{-i\hat{\mathcal{H}}s}) \qquad \hat{\mathcal{H}} \equiv (\hat{p}_{\mu} + eA_{\mu})^2$$

$$\int_0^\infty \frac{\mathrm{d}s}{s} \mathrm{e}^{-m^2 s} \Theta_{\hat{\mathcal{H}}}(s) = -\ln\det(m^2) = -\ln(\prod_k (H_k + m^2))$$

In perturbation theory

We can always write down the effective action/Lagrangian $\mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e^4 (4 - \text{point interaction}) + \cdots$ Maxwell Lagrangian Self interaction term, Non linear

There is no imaginary part

Schwinger effect is a *non-perturbative* effect

Example: Constant electric field in Minkowski spacetime

$$\Im\Gamma\propto\mathrm{e}^{-\frac{\pi m^2}{eE}}$$

Schwinger's original discussion

Schwinger(1951)

Determine the effective Lagrangian from QED aciton

$$\mathcal{L} = -\mathcal{F} - \frac{1}{8\pi^2} \int_0^\infty \frac{\mathrm{d}s}{s^3} \mathrm{e}^{-m^2 s} \left[(es)^2 \frac{\Re \cosh(es\sqrt{2(\mathcal{F} + i\mathcal{G})})}{\Im \cosh(es\sqrt{2(\mathcal{F} + i\mathcal{G})})} - \frac{2}{3}(es)^2 \mathcal{F} - 1 \right]$$

$$\text{Minkowski, Constant EM background} \quad (\mathcal{F} \equiv \frac{1}{2}(\mathcal{B}^2 - \mathcal{E}^2), \mathcal{G} \equiv \mathcal{E} \cdot \mathcal{B})$$

Imaginary part comes from the poles on the integration path

$$\frac{1}{x+i\epsilon} = \mathcal{P}\frac{1}{x} - i\pi\delta(x)$$

View point

Imaginary part is totally determined by the analytic structure (singularities, poles, cuts...) of the kernel of the (Laplace-type) integral From spectrum problem to scattering problem

We assume $\hat{\mathcal{H}} = \partial_t^2 + V(t)$ Eigen value problem $\hat{\mathcal{H}}\psi = \lambda\psi$ has two WKB-type solutions

$$\psi_{\pm} = \frac{1}{\sqrt{\pi(t)}} e^{\pm i \int dt' \pi(t')}, \ (\pi(t) = +\sqrt{V-\lambda})$$

This is valid for $t \to -\infty, \ \lambda \to -\infty$

From spectrum problem to scattering problem

- There is only one (exact) solution which satisfies $\lim_{t \to -\infty} \psi/\psi_+ = 1$
- We define the Jost function as

$$a(\lambda) \equiv \lim_{t \to +\infty} \psi/\psi_+$$
 1-dim scattering matrix
f λ equals to the eigen value of $\hat{\mathcal{H}}$, $a(\lambda)=0$.
Then the Jost function is essentially same as
 $\det(\partial_t^2 + V(t) - \lambda)$

View point

Imaginary part is totally determined by the analytic structure (singularities, poles, cuts...) of the kernel of the (Laplace-type) integral

Calculation of the spectral determinant is reduced to the scattering problem

Equation of motion we have to consider $(\hbar^2 \partial_t^2 + \pi(t)^2) \psi = 0$

is rewritten as

$$(\partial_t^2 + \pi^2(t)\partial_s^2)\tilde{\psi} = 0$$

by the Laplace transformation

$$\psi(t,\hbar) = \hbar^{-1} \int \mathrm{d}s \mathrm{e}^{-s/\hbar} \tilde{\psi}(t,s)$$

Can we find the analytic structure of $\tilde{\psi}$?

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We have to use a new mathematics called "Resurgence" which is developed by J. Ecalle and A. Voros independently.

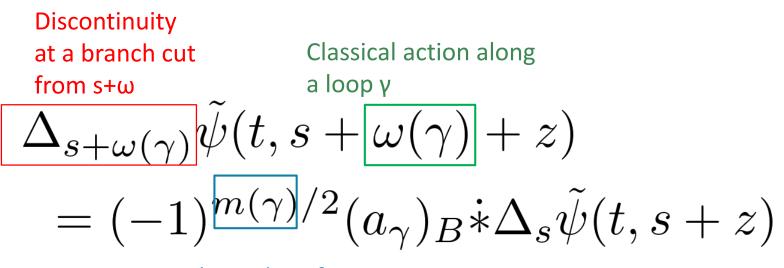
Bootstrap relation can be used to investigate analytic structure of $\tilde{\psi}$

$$\Delta_{s+\omega(\gamma)}\tilde{\psi}(t,s+\omega(\gamma)+z)$$

= $(-1)^{m(\gamma)/2}(a_{\gamma})_B \dot{*} \Delta_s \tilde{\psi}(t,s+z)$

A. Voros, J. Ecalle (80's)

Bootstrap relation can be used to investigate analytic structure of $\tilde{\psi}$



Maslov index of γ: winding number

Bootstrap relation:

$$\Delta_{s+\omega(\gamma)}\tilde{\psi}(t,s+\omega(\gamma)+z)$$

= $(-1)^{m(\gamma)/2}(a_{\gamma})_B \dot{*} \Delta_s \tilde{\psi}(t,s+z)$

The analytic singularity (discontinuity) is discribed by the discontinuity *itself* of other point.

View point

Imaginary part is totally determined by the analytic structure (singularities, poles, cuts...) of the kernel of the (Laplace-type) integral

Calculation of the spectral determinant is reduced to the scattering problem

All the analytic structure can be obtained by the resurgence technique even without any knowledge of the exact (analytic) solution of the e.o.m

Summary

Schwinger effect is a non-perturbative effect

We can catch this effect even in case of the unsolvable (=no known analytic solution) problem

I already checked this machinery works in the easiest case, but more interesting case is undone

 Extension to the QFT (multi/infinite-dimensional) problem