Self-production of Scalar Gravitons after Starobinsky Inflation

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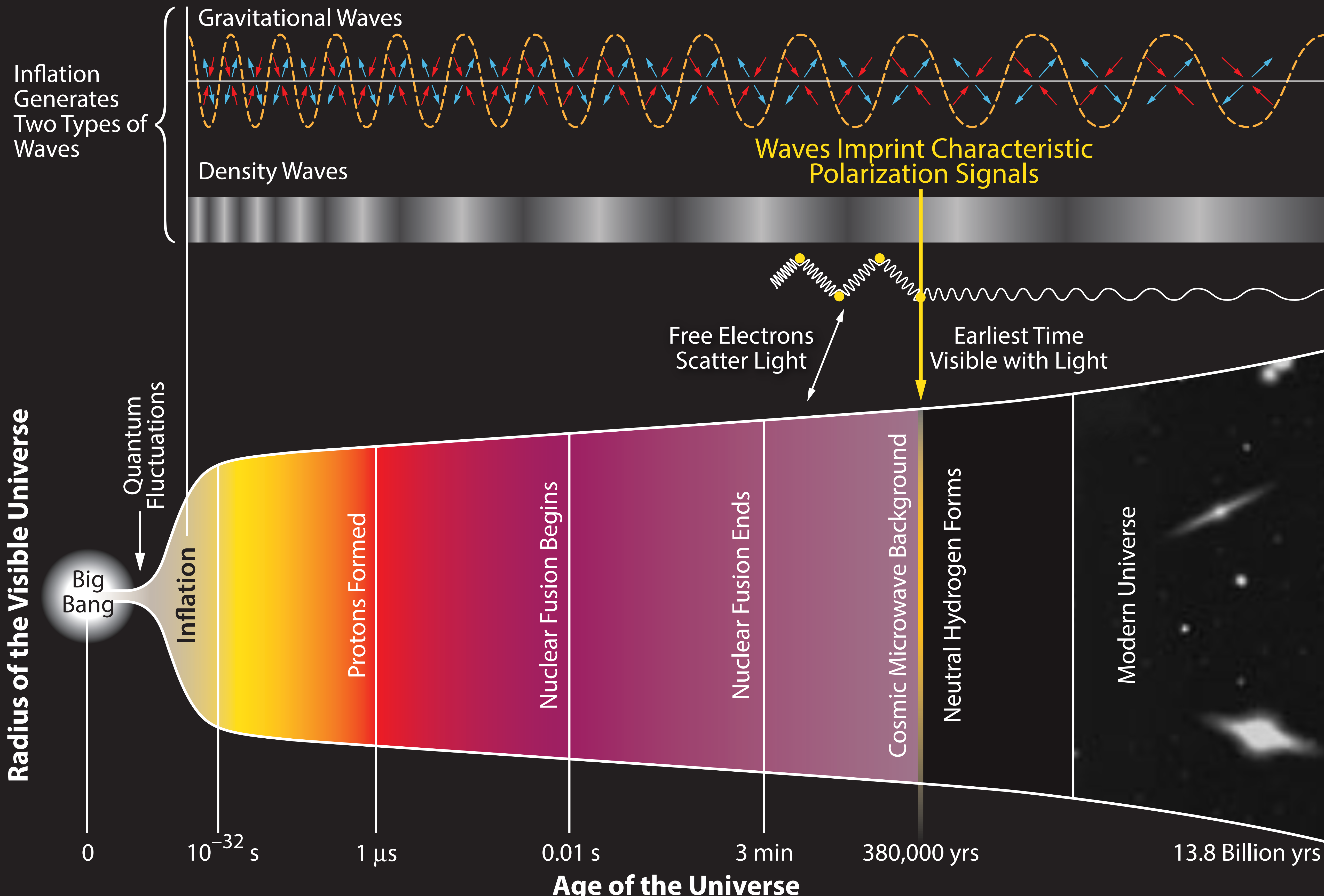
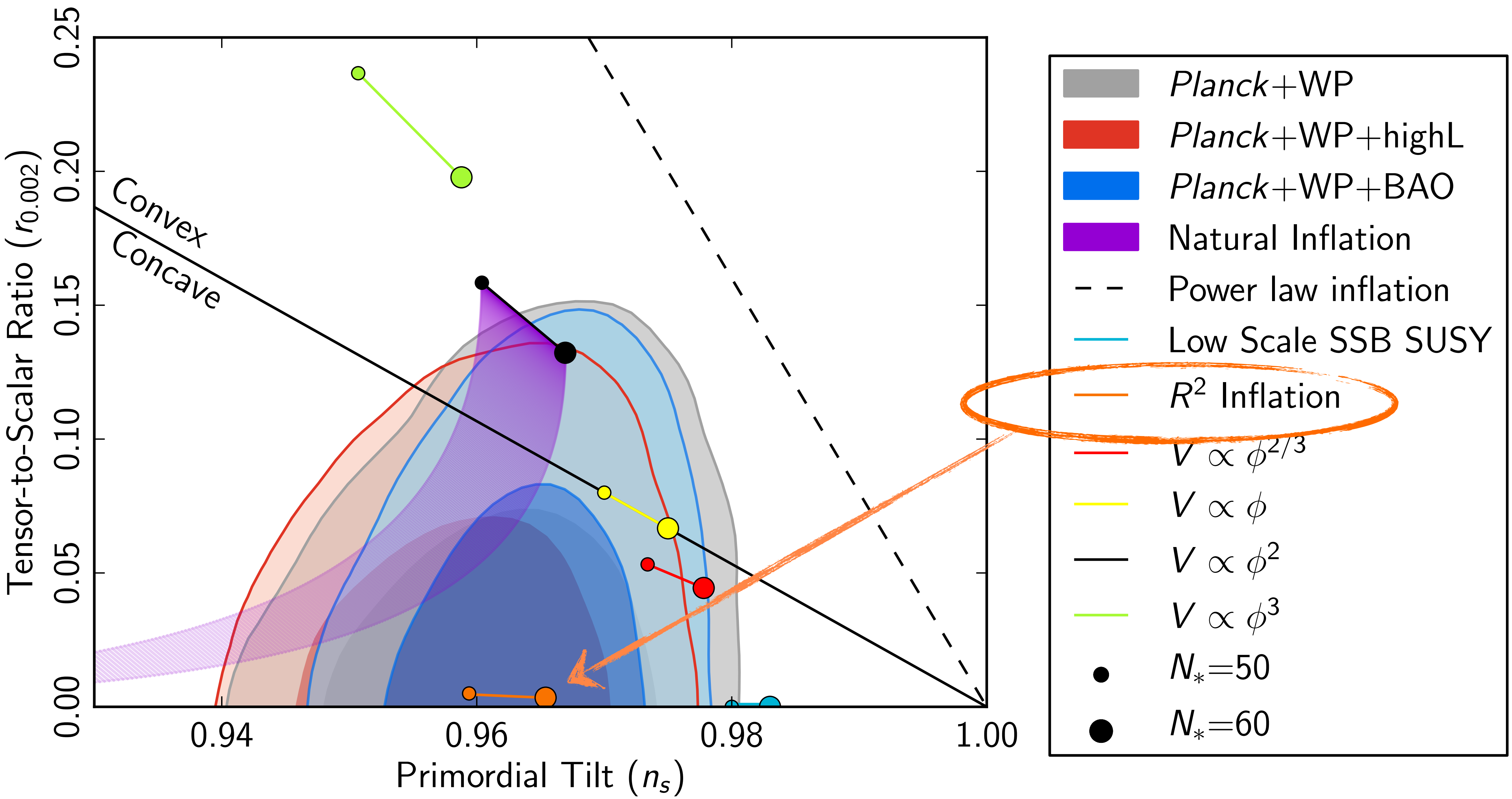
History of the Universe



From BICEP2 website

Which model of inflation is correct?

- Observational constraints on inflation as of 3/2013 [Planck 2013]
- R2 inflation is in good shape! (n_s = 0.964, r = 3.9 x 10^{-3} for N = 55)
  Can we further confirm or falsify it with Planck 2014?



Starobinsky R^2 Inflation [Starobinsky 1980]

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R + \frac{R^2}{6M^2}\right) + S_m$$

$$S_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\nabla\sigma)^2 - V(\sigma)\right]$$ ← Higgs and other SM particles.

- One of the oldest models of Inflation, before models of Sato and Guth

- A single parameter **M** characterizes the model.

R^2 Inflation as scalar-tensor theory

[Whitt 1984; Maeda 1988]

$$S_J = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\hat{g}}\left(\hat{R} + \frac{\hat{R}^2}{6M^2}\right) + S_m$$

$$S_m = \int d^4x\sqrt{-\hat{g}}\left[-\frac{1}{2}(\hat{\nabla}\hat{\sigma})^2 - V(\hat{\sigma})\right]$$

Jordan frame $\hat{g}_{\mu\nu}$

↓

Einstein frame $g_{\mu\nu}$

$$g_{\mu\nu} = \hat{g}_{\mu\nu}\Omega^2$$

$$\Omega^2 = 2\kappa^2\left|\frac{\partial\mathcal{L}_J}{\partial\hat{R}}\right| = 1 + \frac{\hat{R}}{3M^2} \equiv e^{\sqrt{\frac{2}{3}}\kappa\varphi}$$

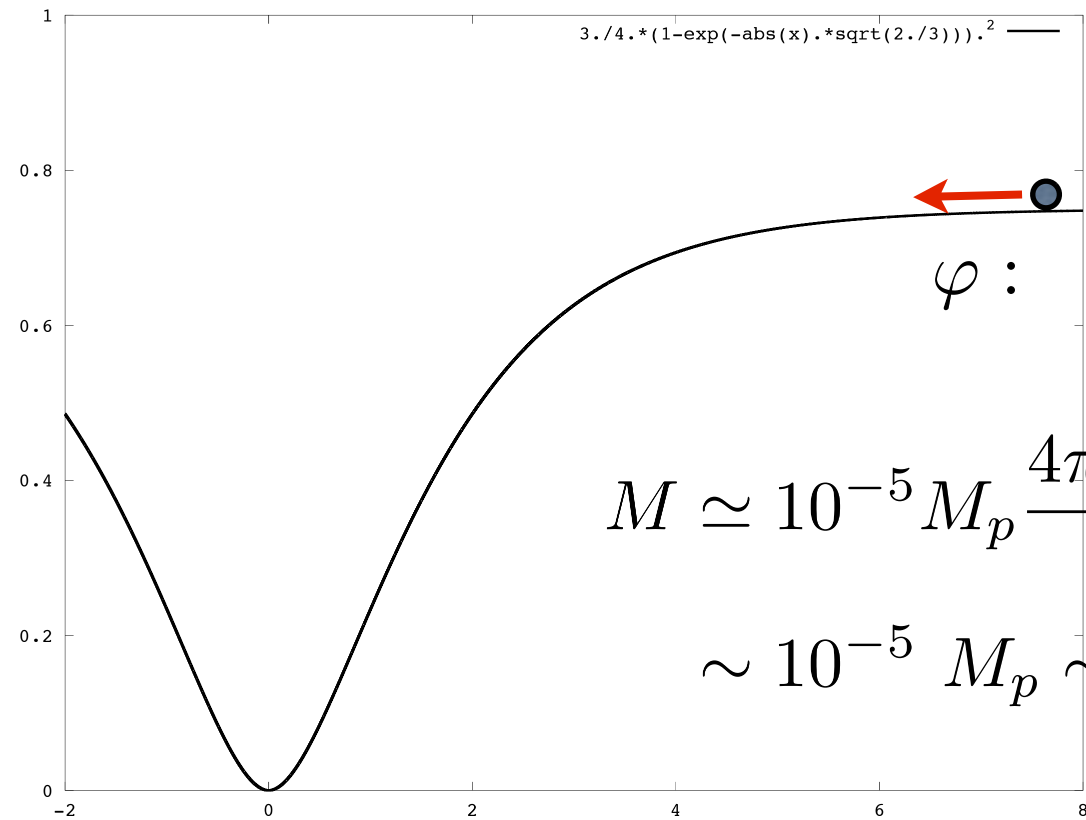
$$\hat{R} = \Omega^2[R + 3\Box(\ln\Omega^2) - \frac{3}{2}g^{\mu\nu}\partial_\mu(\ln\Omega^2)\partial_\nu(\ln\Omega^2)]$$

$$S_E = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\nabla\varphi)^2 - U(\varphi) - \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\kappa\varphi}(\nabla\hat{\sigma})^2 - e^{-\sqrt{\frac{8}{3}}\kappa\varphi}V(\hat{\sigma})\right]$$

$$U(\varphi) = \frac{3}{4}M^2M_p^2\left(1 - e^{-\sqrt{\frac{2}{3}}\kappa\varphi}\right)^2 = \begin{cases}\frac{3}{4}M^2M_p^2 & \text{for} \quad \varphi \gg \varphi_f \\ \frac{1}{2}M^2\varphi^2 & \text{for} \quad \varphi \ll \varphi_f\end{cases}$$

φ : Scalaron = Inflaton

R^2 Inflation [Starobinsky 1980]



φ : Scalaron = Inflaton

$$M \simeq 10^{-5} M_p \frac{4\pi\sqrt{30}}{N_*} \left(\frac{\mathcal{P}_\zeta(k_*)}{2 \times 10^{-9}} \right)^{1/2}$$

$$\sim 10^{-5}\ M_p \sim 10^{27}\mathrm{cm}^{-1} \sim 10^{51}\mathrm{Mpc}^{-1},$$

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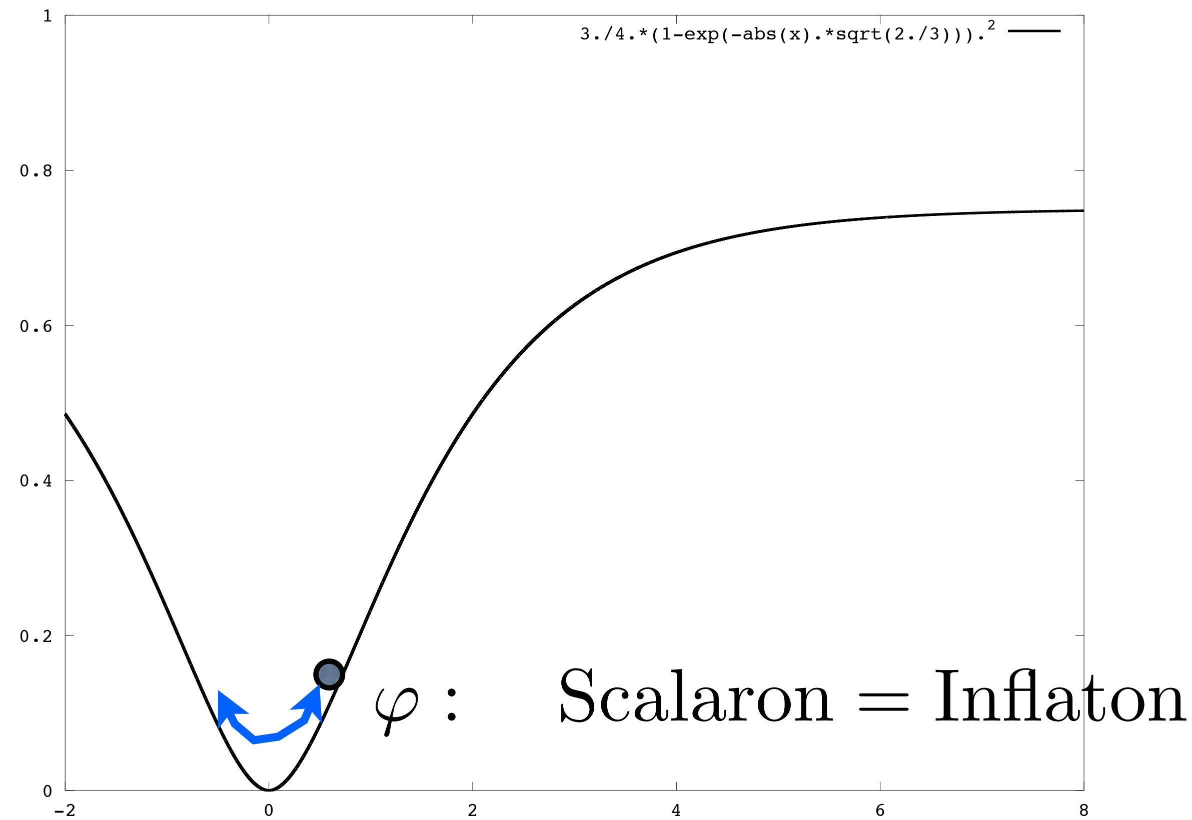
R^2 Inflation [Starobinsky 1980]



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Gravitational reheating by scalaron decay

[YW & Komatsu gr-qc/0612120; YW 1011.3348]

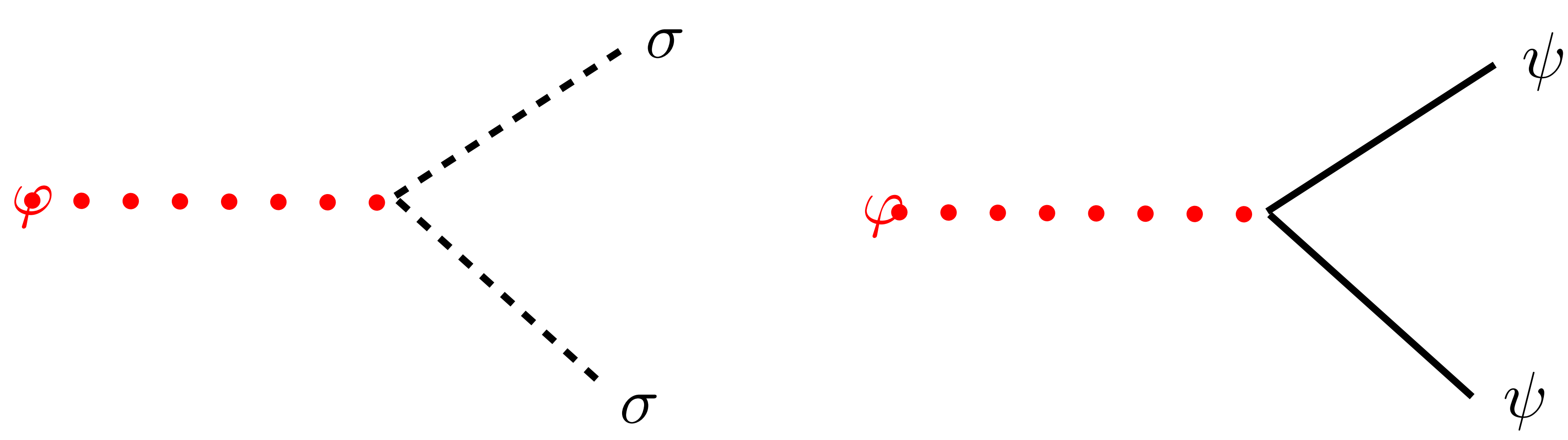
$$\sigma \equiv e^{-\frac{\kappa}{\sqrt{6}}\varphi}\hat{\sigma}$$

$$\mathcal{L}_{\rm scalar} = -\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{\kappa\sigma}{\sqrt{6}}\partial_\mu\sigma\partial^\mu\varphi - \frac{\kappa^2\sigma^2}{12}\partial_\mu\varphi\partial^\mu\varphi - \frac{m_\sigma^2}{2}e^{-\frac{2}{\sqrt{6}}\kappa\varphi}\sigma^2$$

$$\psi \equiv e^{-\frac{3\kappa}{2\sqrt{6}}\varphi}\hat{\psi}$$

$$\mathcal{L}_{\rm fermion} = -\bar{\psi}\not{D}\psi - e^{-\frac{1}{\sqrt{6}}\kappa\varphi}m_\psi\bar{\psi}\psi$$

$$\mathcal{L}_{\rm 3leg} = \frac{1}{\sqrt{6}M_{\rm Pl}}\varphi\partial^\mu\sigma\partial_\mu\sigma + \frac{2m_\sigma^2}{\sqrt{6}M_{\rm Pl}}\varphi\sigma^2 + \frac{m_\psi^2}{\sqrt{6}M_{\rm Pl}}\varphi\bar{\psi}\psi$$



Gravitational reheating by scalaron decay

[YW & Komatsu gr-qc/0612120; YW 1011.3348]

$$\Gamma(\varphi \to \sigma\sigma) = \frac{\mathcal{N}_\sigma (M^2 + 2m_\sigma^2)^2}{192\pi M_{\rm Pl}^2 M}$$

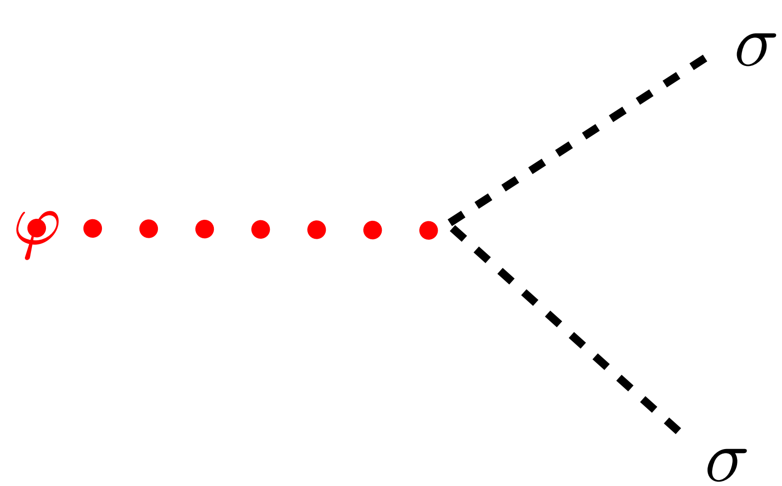
$$\simeq \frac{\mathcal{N}_\sigma M^3}{192\pi M_{\rm Pl}^2} + \frac{\mathcal{N}_\sigma m_\sigma^2 M}{48\pi M_{\rm Pl}^2}$$

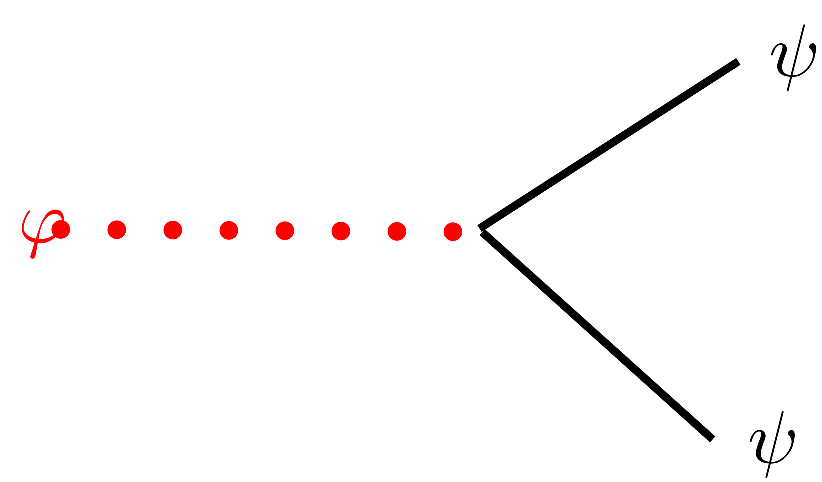
$$\Gamma(\varphi \to \bar{\psi}\psi) = \frac{\mathcal{N}_\psi m_\psi^2 M}{48\pi M_{\rm Pl}^2}$$

Leading term

$$H_{\rm rh} = \Gamma$$

$$T_{\rm rh} \simeq 0.1\sqrt{\Gamma_{\rm tot} M_p} \left(\frac{\mathcal{N}_{\rm tot}}{100}\right)^{-1/4}$$





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Leading term

$$\Gamma(\varphi \to \bar{\psi}\psi) = \frac{\mathcal{N}_\psi m_\psi^2 M}{48\pi M_{\rm Pl}^2}$$

$$T_{\rm rh} \simeq 0.1\sqrt{\Gamma_{\rm tot} M_p}\left(\frac{\mathcal{N}_{\rm tot}}{100}\right)^{-1/4} \sim 10^{-9} M_p,$$

$$N_* \simeq 54 + \frac{1}{3}\ln\left(\frac{T_{\rm rh}}{10^9\ {\rm GeV}}\right),$$

If we know the matter sector (e.g. SM minimally coupled to gravity), inflationary predictions can be made without uncertainty.

Predictions depend on reheating temperature

scalaron mass

$$M \simeq 10^{-5} M_p \frac{4\pi\sqrt{30}}{N_*} \left(\frac{\mathcal{P}_\zeta(k_*)}{2 \times 10^{-9}} \right)^{1/2}$$
$$\sim 10^{-5}\ M_p \sim 10^{27}\mathrm{cm}^{-1} \sim 10^{51}\mathrm{Mpc}^{-1},$$

e-folds of inflation

$$N_* \simeq 54 + \frac{1}{3} \ln \left(\frac{T_{\rm rh}}{10^9\ \mathrm{GeV}} \right),$$

grav. waves

$$r = \frac{\mathcal{P}_\gamma(k)}{\mathcal{P}_\zeta(k)} \simeq 16\epsilon \simeq \frac{12}{N_*^2}.$$

tilt and running of spectra

$$n_s - 1 = \frac{d \ln \mathcal{P}_\zeta(k)}{d \ln k} \simeq -6\epsilon_V + 2\eta_V \simeq -\frac{2}{N_*},$$
$$n_t = \frac{d \ln \mathcal{P}_\gamma(k)}{d \ln k} \simeq -2\epsilon_V \simeq -\frac{3}{2N_*^2},$$
$$\frac{dn_s}{d \ln k} \simeq 16\epsilon_V \eta_V - 24\epsilon_V^2 - 2\xi_V^2 \simeq -\frac{2}{N_*^2},$$
$$\frac{dn_t}{d \ln k} \simeq 4\epsilon_V \eta_V - 8\epsilon_V^2 \simeq -\frac{3}{N_*^3},$$

No ambiguity in reheating?

- During the oscillations of the inflaton, pamaretric resonance (preheating) may happen. [Kofman, Linde, Starobinsky 1994]
- If it happens, **long-living localized objects (oscillons/I-balls)** would be formed, making the inflaton decay non-perturbative. [Amin, Easther et al 2012]

$$V(\phi) = \frac{m^2 M^2}{2\alpha}\left[\left(1+\frac{\phi^2}{M^2}\right)^{\alpha} - 1\right]$$

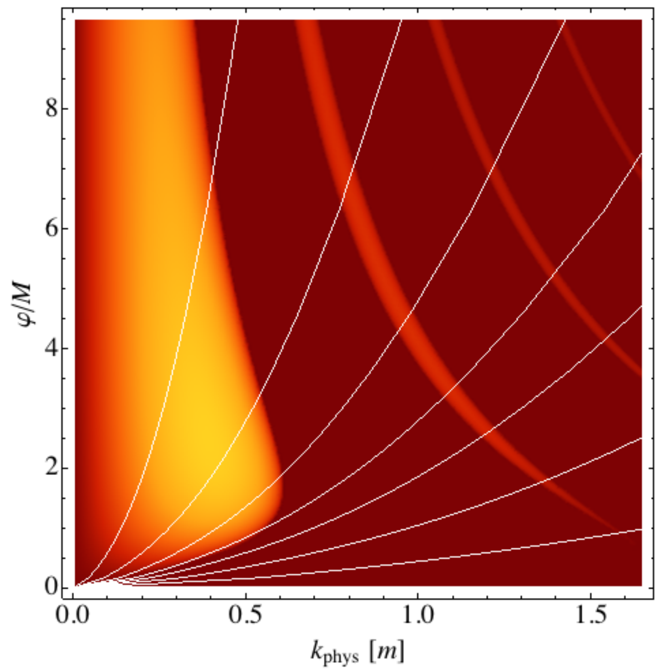


FIG. 1: Floquet diagram with $\alpha = 1/2$, $\beta = 100$. The stable regions are dark red. Within the unstable bands, lighter colors

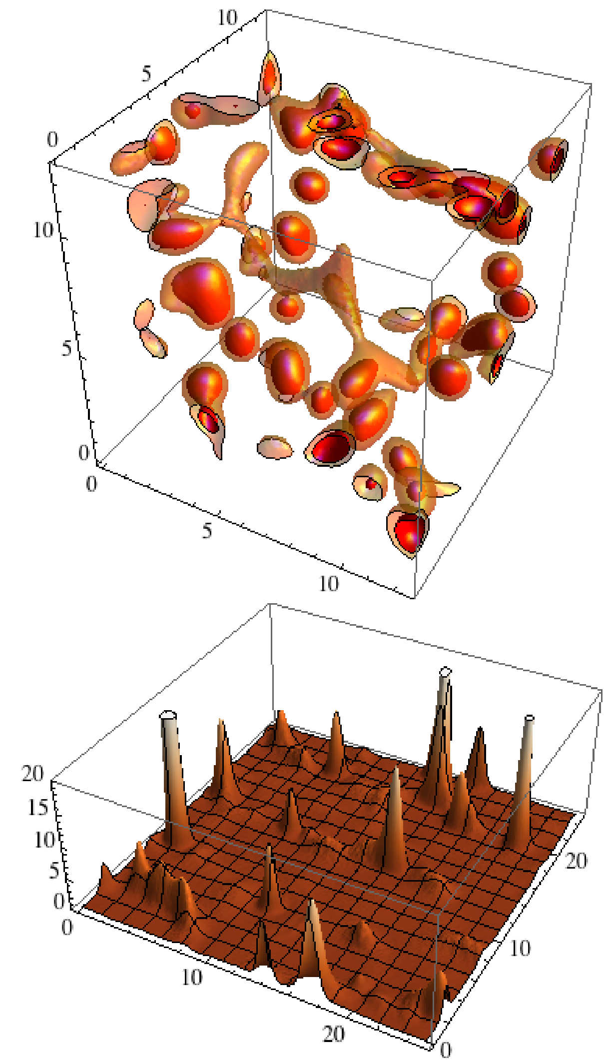
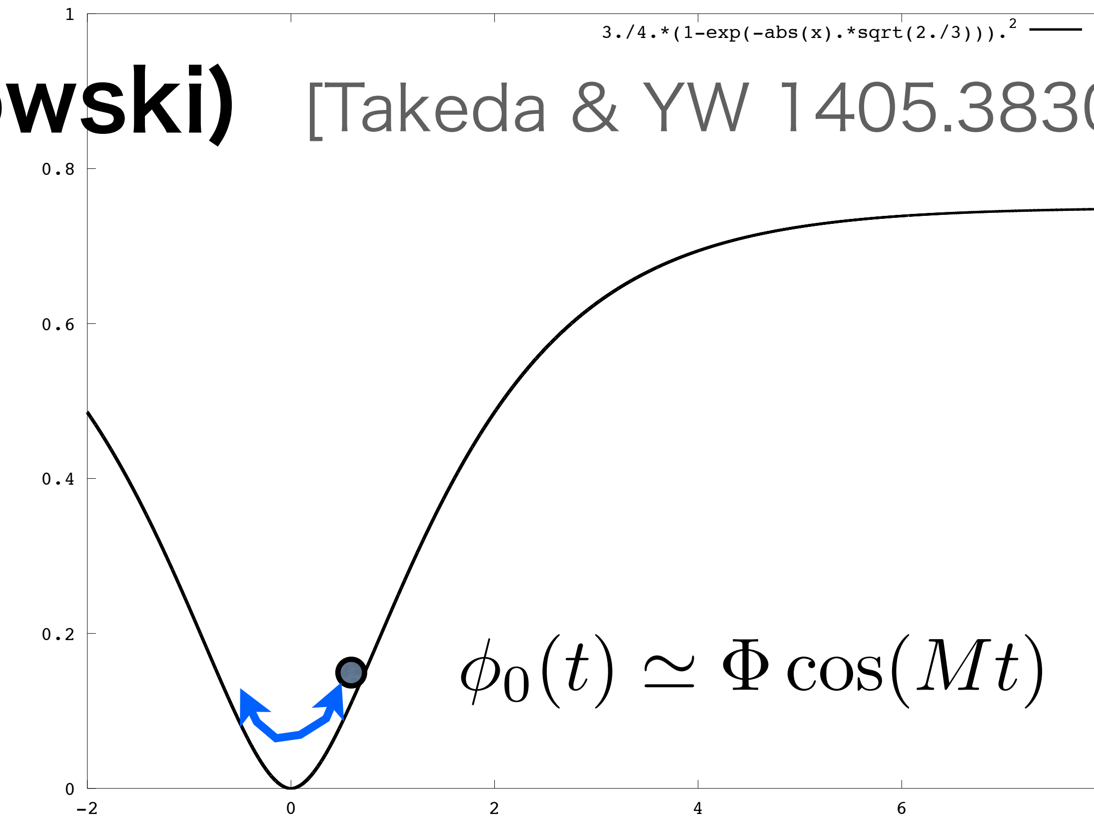


FIG. 2: Oscillon configuration with $\alpha = 1/2$ and $\beta = 50$. The top plot shows regions where $\rho/\langle\rho\rangle > 4$ (transparent) and 12 (solid), while the lower plot shows $\rho/\langle\rho\rangle$ on a two dimensional

Preheating in R^2 inflation (Minkowski) [Takeda & YW 1405.3830]



$$\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$$

$$\delta\ddot{\phi}_k + \omega_k^2 \delta\phi_k = 0$$

$$\omega_k^2 = k^2 + M^2 \left[1 + \frac{7}{6}\left(\frac{\Phi}{M_p}\right)^2\right] - \sqrt{6}M^2\frac{\Phi}{M_p}\cos(Mt) + \frac{7}{6}M^2\left(\frac{\Phi}{M_p}\right)^2 \cos(2Mt)$$

$$\delta\phi_k'' + \left[A_{1k} - 2q_1\cos(2\hat{T})\right]\delta\phi_k = 0$$

$$q_1 \equiv 2\sqrt{6}\frac{\Phi}{M_p},$$

$$A_{1k} \equiv 4 + 4\left(\frac{k}{M}\right)^2 + \frac{7}{36}q_1^2$$

2nd narrow resonance: $-\frac{q^2}{12} < A_k - 4 < \frac{5q^2}{12},$ $\quad \Phi < 0.2M_p$

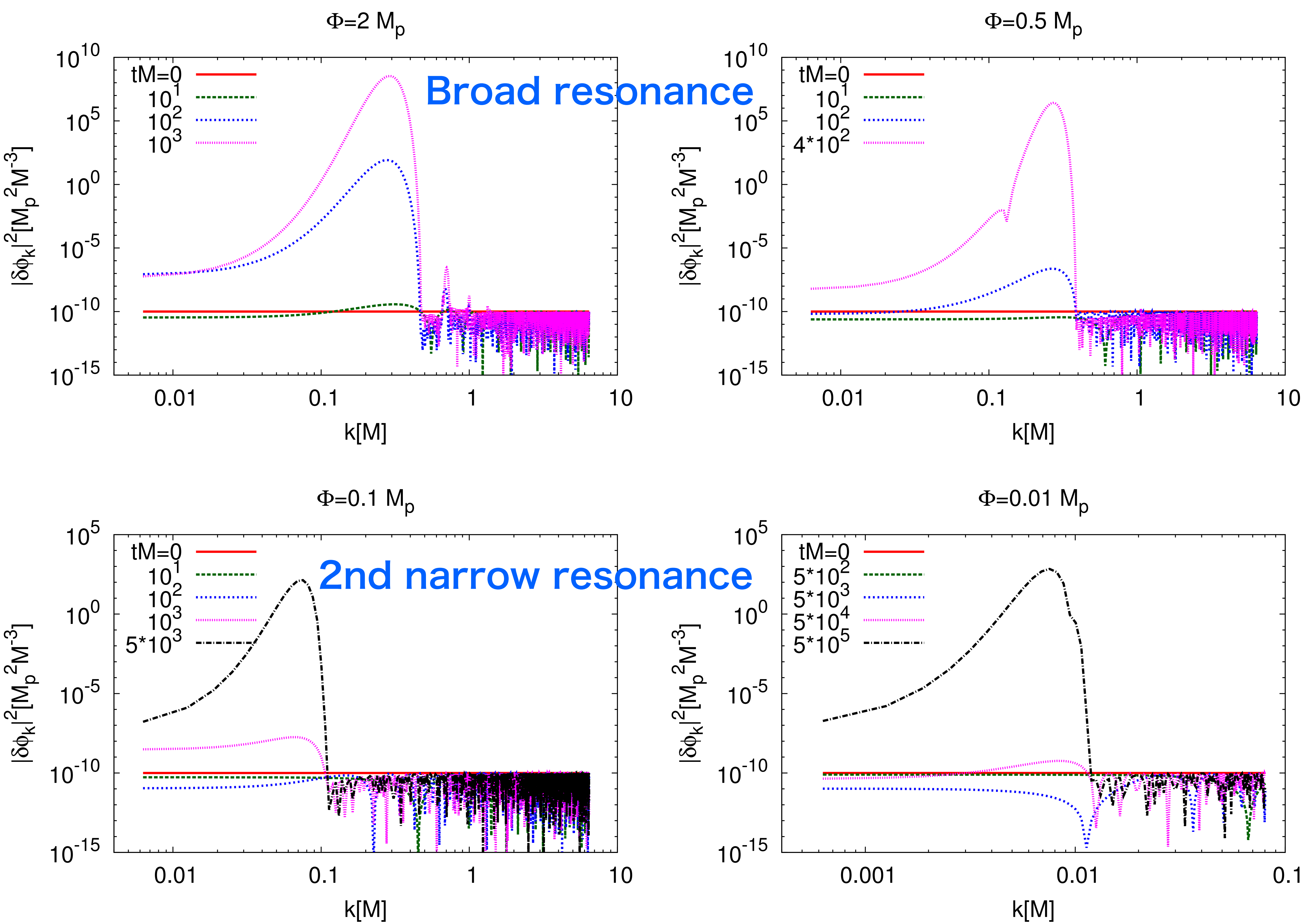
$$0 \le \frac{k}{M} < \frac{q_1}{3\sqrt{2}}$$

Broad resonance: $|d\omega/dt|/\omega^2 > 1$ $\quad 0.2M_p \lesssim \Phi \lesssim 2M_p$

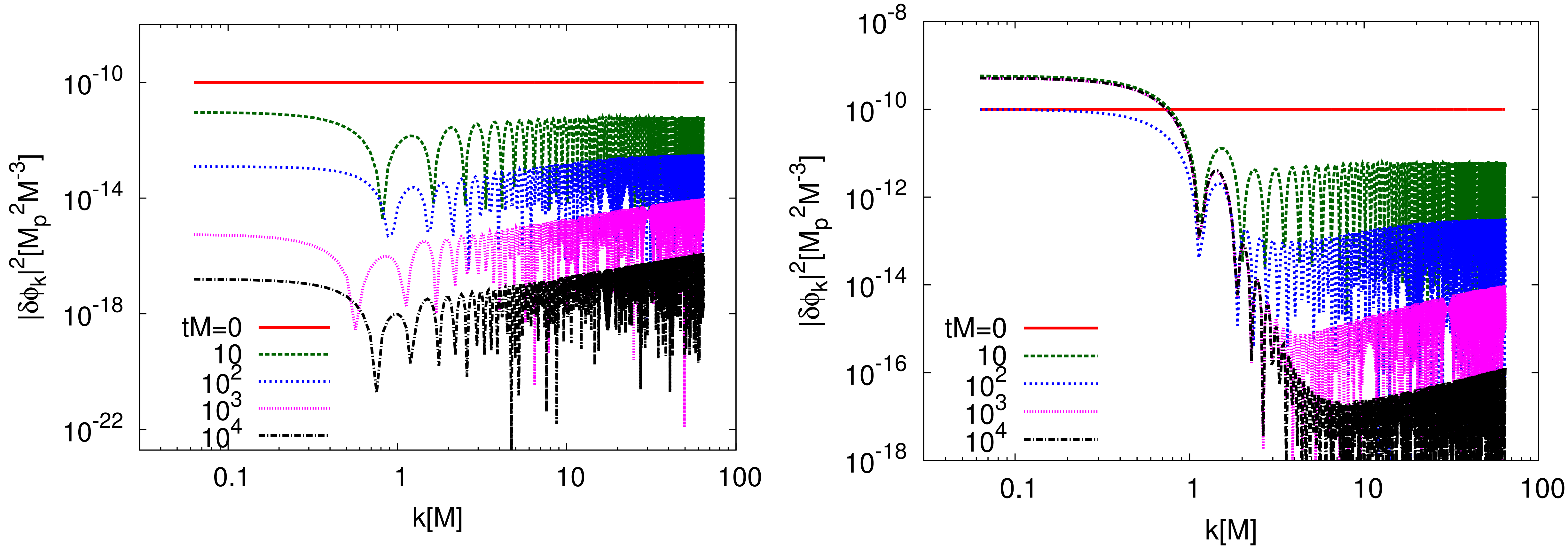
$$\left(\frac{k}{M}\right)^2 < -1 - \frac{7}{6}\left(\frac{\Phi}{M_p}\right)^2 + \sqrt{6}\frac{\Phi}{M_p}\cos(Mt) + \left(\frac{3}{2}\right)^{\frac{1}{3}}\left(\frac{\Phi}{M_p}\right)^{\frac{2}{3}}|\sin(Mt)|^{\frac{2}{3}},$$

Parametric resonant spectrum

[Takeda & YW 1405.3830]



Preheating in R^2 inflation (Friedmann) [Takeda & YW 1405.3830]



Without back-reaction from metric, Hubble damping wins over instabilities.

With back-reaction from metric, preheating is balanced with Hubble damping.

Mukhanov-Sasaki eqn.

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \underbrace{\left[\frac{k^2}{a^2} + V''(\phi_0) + \Delta F\right]}_{\omega_k^2} \delta\phi_k = 0,$$

Back-reaction from metric:

$$\Delta F \equiv \frac{2\dot{\phi}_0}{M_p^2 H} V'(\phi_0) + \frac{\dot{\phi}_0^2}{M_p^4 H^2} V(\phi_0).$$

Metric preheating in R² inflation [Takeda & YW 1405.3830]

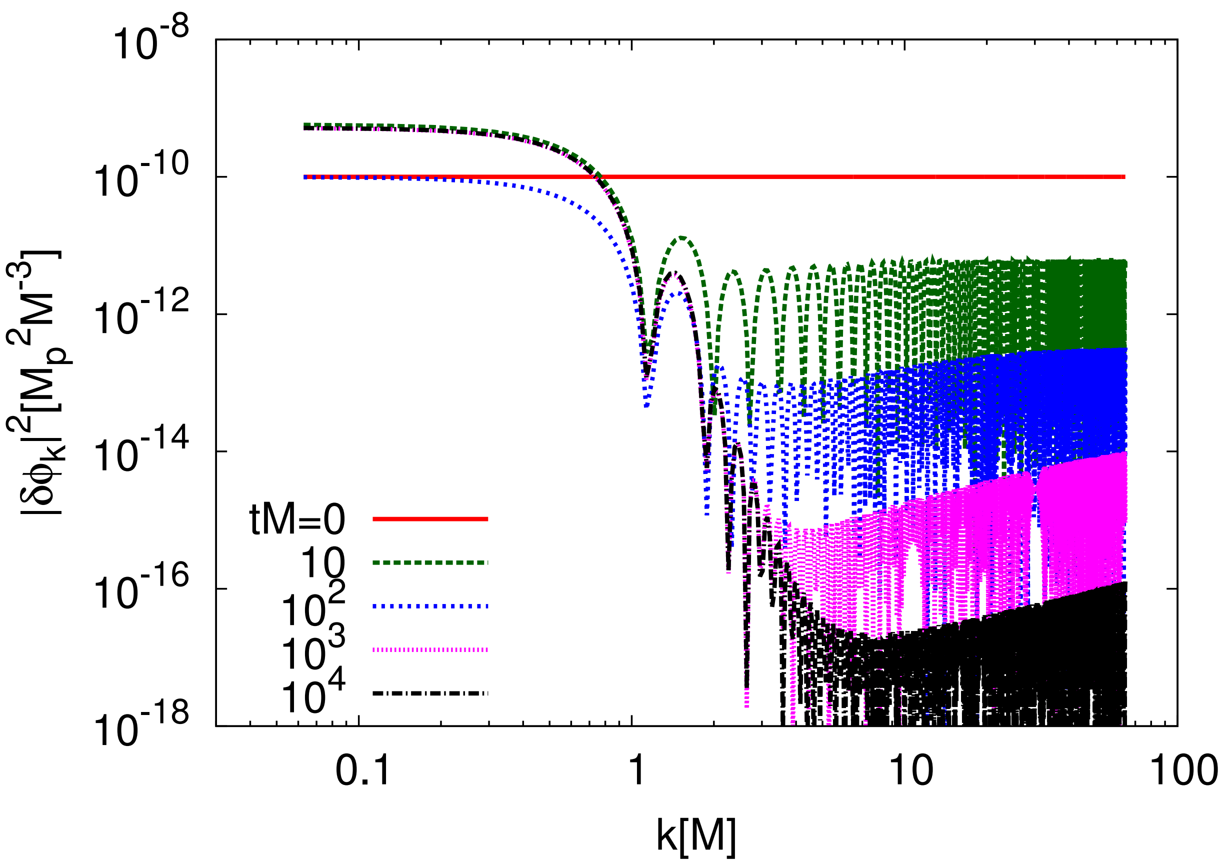
$$\phi_0(t) \simeq \phi_0(t_{\rm ini}) \left(\frac{a_{\rm ini}}{a}\right)^{\frac{3}{2}} \sin(Mt)$$

$$\omega_k^2 \simeq \frac{k^2}{a^2} + M^2 \left(1 - \sqrt{6}\frac{\phi_0}{M_p} + \frac{2\dot{\phi}_0\phi_0}{HM_p^2}\right)$$

$$\widetilde{\delta\phi}_k'' + \left[A_{3k} - 2q_3\cos(2\hat{T})\right]\widetilde{\delta\phi}_k = 0$$

$$q_3 \equiv \frac{a_{\rm ini}^3\phi_0^2(t_{\rm ini})M}{2a^3HM_p^2}, \quad A_{3k} \equiv 1 + \frac{k^2}{a^2M^2}.$$

$$\widetilde{\delta\phi}_k \equiv a^{3/2}\delta\phi_k$$



1st narrow resonance: $-q^2 < A_k - 1 < q^2,$

$$0 \leq \frac{k}{M} \lesssim a_{\rm ini}H_{\rm ini}\sqrt{\frac{3a_{\rm ini}}{aHM}} \propto a^{1/2}$$

Conclusion

- We are now in the golden age of cosmology. Data is getting more and more precise, and even a surprise is coming! The detection of inflationary gravitational waves by BICEP2 will be confirmed or falsified by Planck 10/2014.
- The physics of reheating affects precise predictions of inflationary models. Oscillons/I-balls do not form after Starobinsky inflation, thereby reheating proceeds through perturbative particle production of Higgs with $T_{\rm rh} \simeq 10^9 {\rm GeV}$ and $N_* \simeq 54.$
- Halos and PBHs ($M_{\rm nl} \simeq 4 \times 10^6 {\rm g}$) may be formed by metric preheating after Starobinsky inflation since $\delta\rho/\rho \propto a$ for ~13 e-folds. [cf. Jedamzik, Lemoine, Martin 2010]