Matter Power Spectrum in Viable $f(R)$ Models with Massive Neutrino

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Outline

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2. $f(R)$ Gravity and Perturbation
3. Numerical Results
4. Conclusion

Outline

Motivation

- The viable $f(R)$ models are suitable to explain Dark Energy Problem.
- Neutrino mass difference from oscillation experiments:

$$\Delta m_{21}^2 = (7.50 \pm 0.20) \times 10^{-5} eV^2 \,,$$
$$\Delta m_{32}^2 = \left(2.32^{+0.12}_{-0.08}\right) \times 10^{-3} eV^2 \,.$$

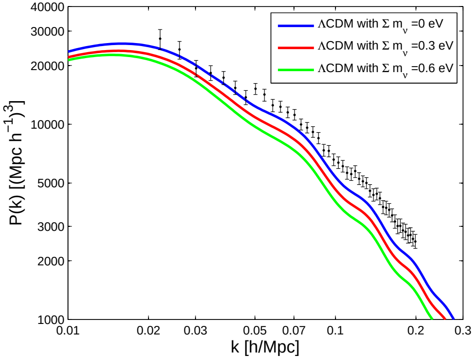
- Neutrono mass from cosmology:

$$\Sigma m_\nu < 0.23 eV \;\; (\text{best fit} : 0.001 eV) \,.$$
$$(95\%; \text{Planck} + \text{WMAP} + \text{highL} + \text{BAO})$$

Motivation

- Matter Power Spectrum $P(k)$ (data from SDSS DR7):

$$\langle \delta_m(\vec{k})\delta_m^*(\vec{k'})\rangle = (2\pi)^3 P(k)\delta^3(\vec{k}-\vec{k'})\,.$$



- The Starobinsky model allows 1 eV sterile neutrino[1] (WMAP7 and SDSS DR7).

[1] H. Motohashi, A. A. Starobinsky and J. Yokoyama, PRL **110**, 121302

Outline

$f(R)$ Gravity

- $f(R)$ gravity:
  One of the simplest modified gravity model, which extends Einstein-Hilbert action to higher order terms.
- The action of $f(R)$ gravity:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m \,,$$

where $f(R)$ is an arbitrary function.

Perturbation

- The field(modified Einstein) equation:

$$f_R R_{\mu\nu} - \frac{f}{2} g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu) f_R = \kappa^2 T_{\mu\nu} \, ,$$

where the subscript R denotes d/dR.

- The perturbed FLRW metric in Newtonian gauge:

$$ds^2 = a(\tau)^2 \left[-(1+2\Psi) d\tau^2 + (1-2\Phi) dx_i dx^i \right] \, .$$

- The perturbed energy-momentum tensor:

$$\begin{aligned} T^0_0 &= -\rho_m + \delta\rho_m \, , \\ T^0_i &= -\rho_m v_{m,i} \, , \\ T^i_j &= \delta P \delta^i_j + \pi^i_j \, . \end{aligned}$$

Perturbation

- In ΛCDM model:

$$\frac{k^2}{a^2}\Psi = -4\pi G\rho\delta_m\,, \qquad \Psi = \Phi\,,$$

where $\mathcal{H} = \frac{1}{a}\frac{da}{d\tau}$, $w = \frac{P}{\rho}$ and $\delta_m = \frac{\delta\rho}{\rho} + 3\frac{\mathcal{H}}{k}(1+w)$.

- In $f(R)$ gravity:

$$\frac{k^2}{a^2}\Psi = -4\pi G\mu(k,a)\rho\delta_m\,, \qquad \frac{\Phi}{\Psi} = \gamma(k,a)\,.$$

and

$$\mu(k,a) = \frac{1}{f_R}\frac{1+4\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}{1+3\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}\,, \qquad \gamma(k,a) = \frac{1+2\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}{1+4\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}\,,$$

which are used in MGCAMB[2]and CosmoMC.

- The evolution equation of matter perturbation is derived as

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\,\mu(k,a)\,\rho_m\delta_m = 0\,.$$

[2] A. Hojjati, G.B. Zhao, L. Pogosian and A. Silvestri, JCAP **1108** 005, http://www.sfu.ca/ aha25/MGCAMB.html

$f(R)$ Gravity
Viability Conditions

- $\frac{\partial f(R)}{\partial R} > 0$ for $R > R_0$.
- $\frac{\partial^2 f(R)}{\partial R^2} > 0$ for $R > R_0$.
- $f(R) \to R - 2\Lambda$ in the large curvature regime $(R \gg R_0)$.
- Having a stable late-time de-Sitter point.
- Passing local gravity constraints.

$f(R)$ Gravity

- Hu-Sawicki model:
  $f(R) = R - R_{\mathrm{HS}} \frac{c_1 (R/R_{\mathrm{HS}})^p}{c_2 (R/R_{\mathrm{HS}})^p + 1}$
- Starobinsky model:
  $f(R) = R - \lambda R_{\mathrm{S}} \left[1 - \left(1 + \frac{R^2}{R_{\mathrm{S}}^2} \right)^{-n} \right]$
- Tsujikawa model:
  $f(R) = R - \mu R_{\mathrm{T}} \tanh \left(\frac{R}{R_{\mathrm{T}}} \right)$
- Exponential gravity model:
  $f(R) = R - \beta R_{\mathrm{E}} \left(1 - e^{-R/R_{\mathrm{E}}} \right)$
- Appleby-Battye model:
  $(1 - g)R + g R_{AB} \ln \left[\frac{\cosh \left(R/R_{\mathrm{AB}} - b \right)}{\cosh\, b} \right]$

Outline

Numerical Results

- Starobinsky model:

$$f(R) = R - \lambda R_{\rm S}\left[1 - \left(1 + \frac{R^2}{R_{\rm S}^2}\right)^{-n}\right] .$$

- Exponential model:

$$f(R) = R - \beta R_s\left(1 - e^{-R/R_s}\right) .$$

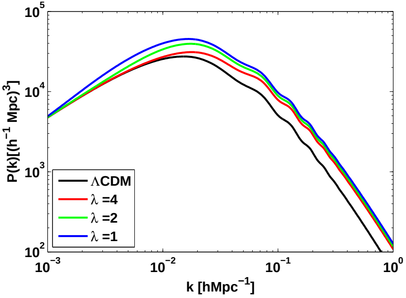
- Assumption:
  - Linear perturbation.
  - The ΛCDM background evolution.
  - One massive neutrinos and two massless neutrinos.

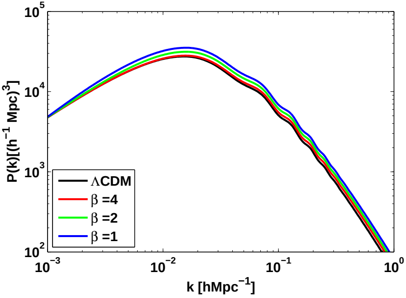
Numerical Results

- Matter density perturbation:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\,\mu(k,a)\,\rho_m\delta_m = 0, \text{ where } \mu(k,a) = \frac{1}{f_R}\frac{1+4\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}{1+3\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}.$$

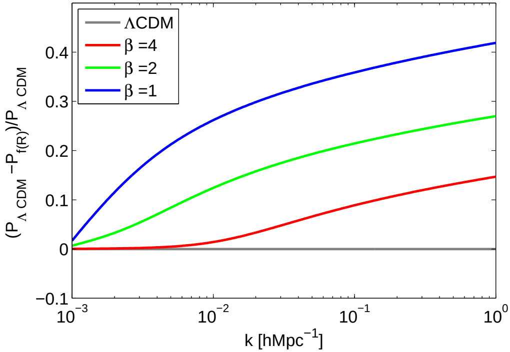
- Matter power spectrum $P(k)$ in Starobinsky (left) and Exponential gravity (right) model:





Numerical Results

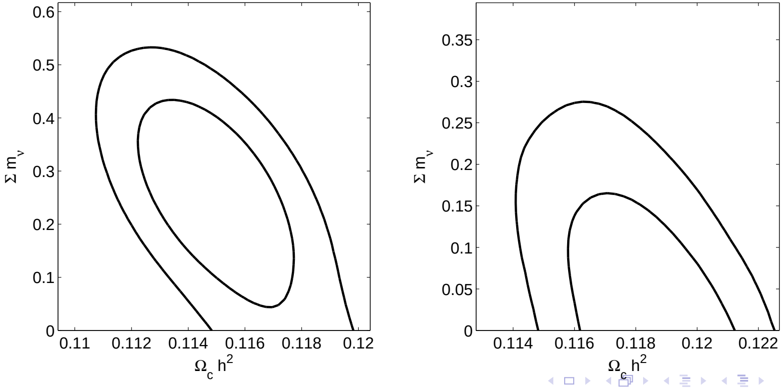
- $\frac{\mathrm{P}_{exp} - \mathrm{P}_{\Lambda CDM}}{\mathrm{P}_{\Lambda CDM}}$ in Exponential gravity model with $\beta = 1$ (red), 2 (green) and 4 (blue).



Numerical Results

- Fitting result from CMB (Planck and WMAP), BAO (SDSS DR11), SNIa (SNLS) and MPK (SDSS and WiggleZ)

Model	Σm_ν	$\Omega_c h^2$
Starobinsky (left)	$0.25^{+0.20}_{-0.23}$ eV	$0.114^{+0.004}_{-0.002}$
Exponential (right)	< 0.21 eV	0.118 ± 0.03
ΛCDM	< 0.20 eV	$0.117^{+0.004}_{-0.002}$



Outline

Conclusion

- In Starobinsky model, the neutrino mass best-fit locates at $\Sigma m_\nu = 0.25\ \mathrm{eV}$, which is consistent with the result from neutrino oscillation experiments.
- The Exponential gravity model might not be distinguishable from ΛCDM model.

Summary

Thank you for your attention!!

Numerical Results

- The probability of model parameter in Starobinsky (left) and Exponential (right) models.