RESCEU Summer School

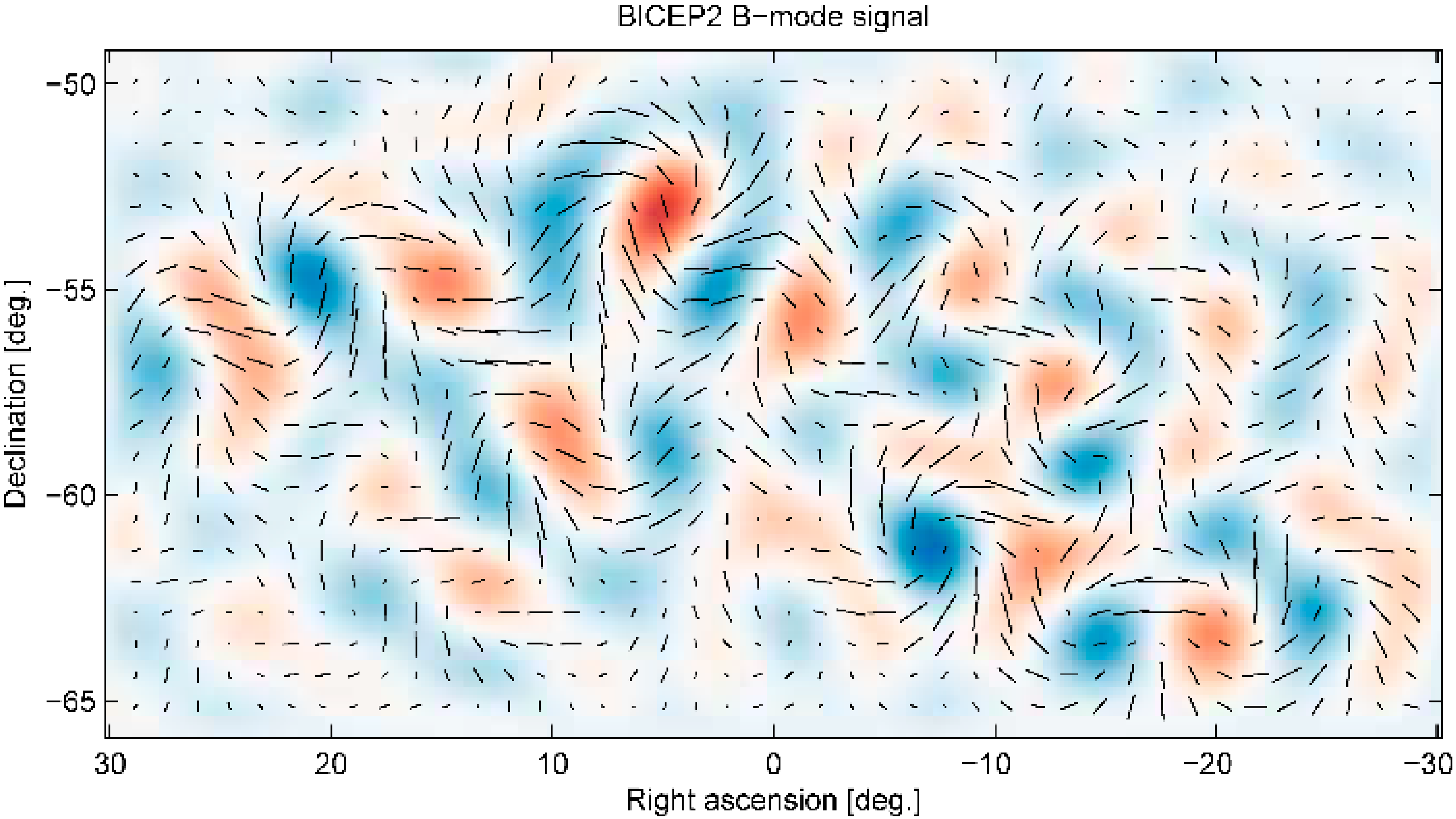
B-mode polarization of the CMB

Kiyotomo ICHIKI (KMI, Nagoya U.)

名古屋大学
NAGOYA UNIVERSITY

素粒子宇宙起源研究機構

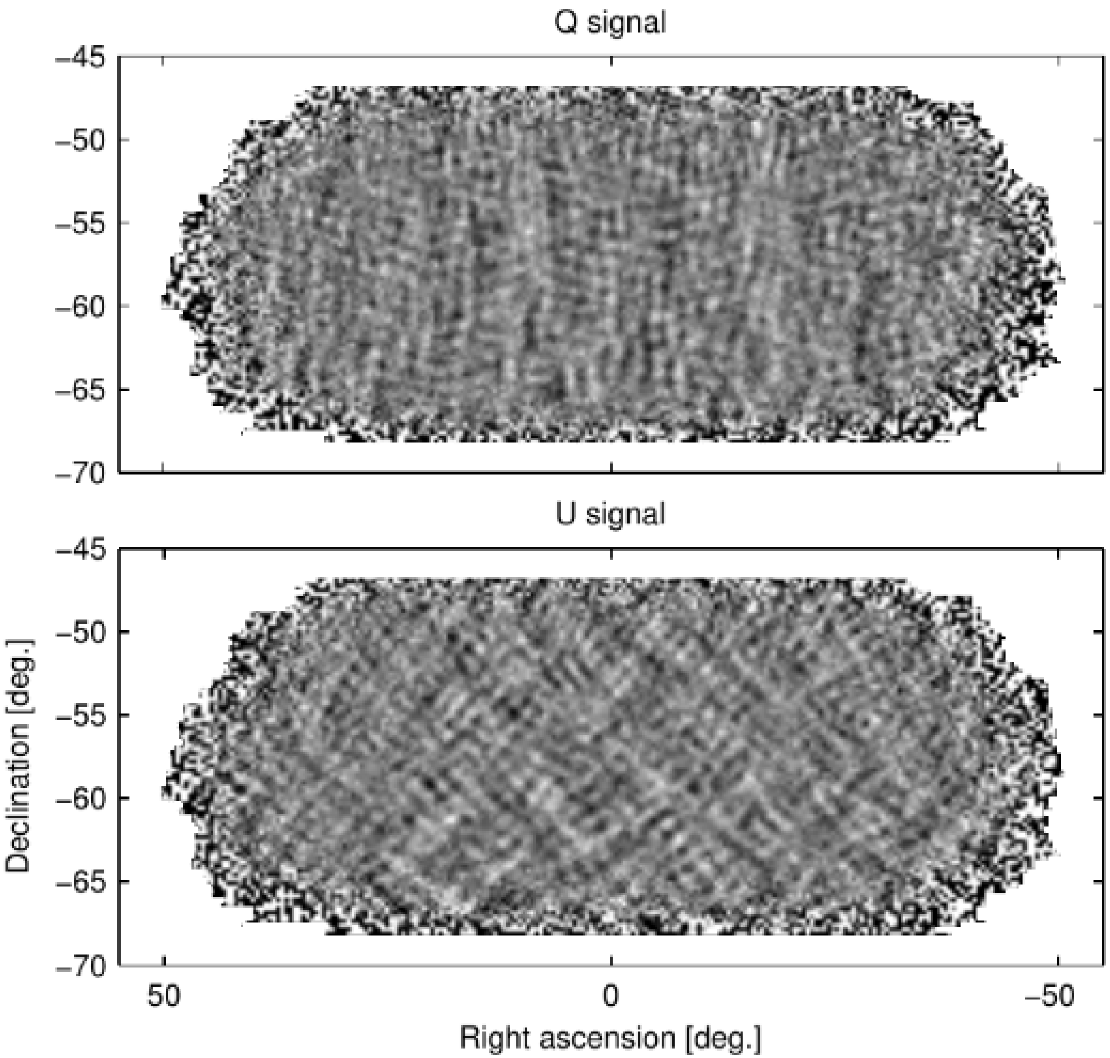
The Goal



Shown here is the actual B-mode pattern observed with the BICEP2 telescope, with the line segments showing the polarization from different spots on the sky. The red and blue shading shows the degree of clockwise and anti-clockwise twisting of this B-mode pattern.

The BICEP2 Collaboration, Phys. Rev. Lett. 112, 241101, 2014

The Goal



BICEP2 T, Q, U maps. The figure shows the basic signal maps with 0.25° pixelization as output by the reduction pipeline. …… Note that the structure seen in the Q and U signal maps is as expected for an E-mode dominated sky.

The BICEP2 Collaboration, Phys. Rev. Lett. 112, 241101, 2014

outline

- Introduction
  - Cosmic Microwave Background (CMB)

- CMB polarizations
  - Q&U stokes paramters and E & B modes

- The sources of B-mode polarization
  - Gravitatinal waves (BICEP2)
  - Gravitational lensing (POLARBEAR, SPTPol)

- Discussion & Summary

The standard model

The standard cosmological model is based on:
Big-Bang & Inflation

Demand for the Big-Bang

Hubble's law

light element abundance pattern

Evidence for the Big-Bang

Cosmic Microwave Background (CMB)

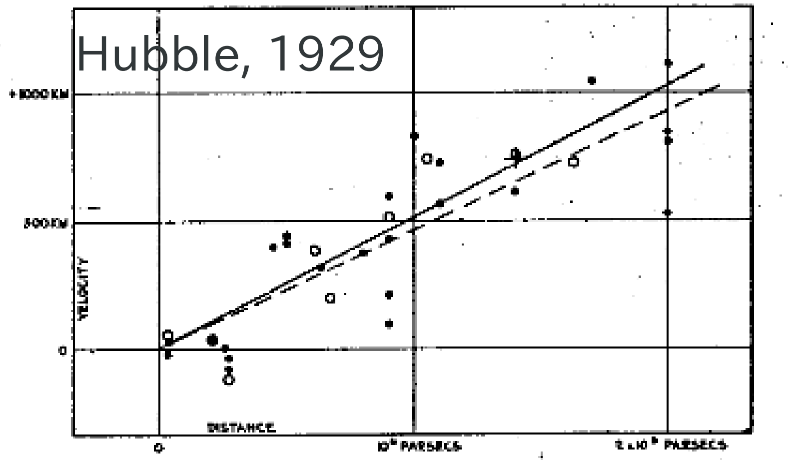
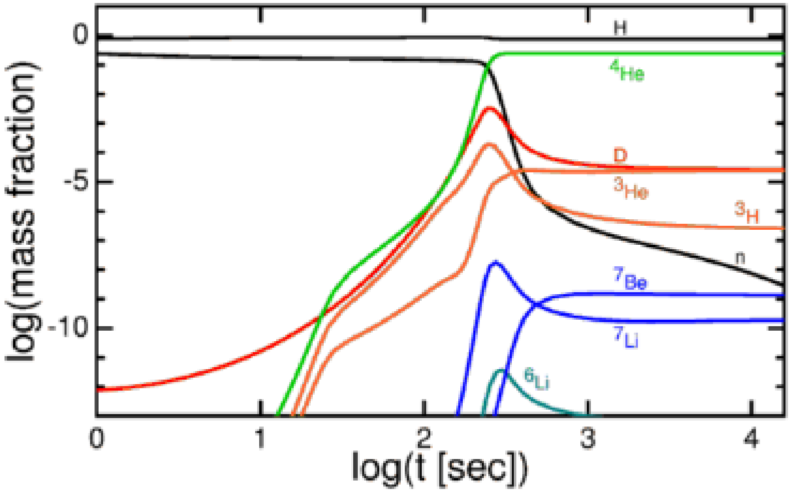
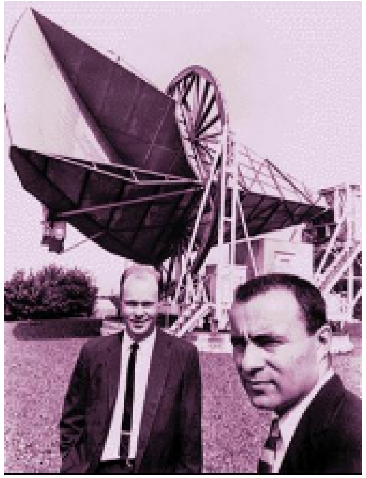

Hubble, 1929

FIGURE 1




Bell lab

The standard model

The standard cosmological model is based on:

Big-Bang & Inflation

Demand for the inflation

Cosmic homogeneity

Cosmic flatness

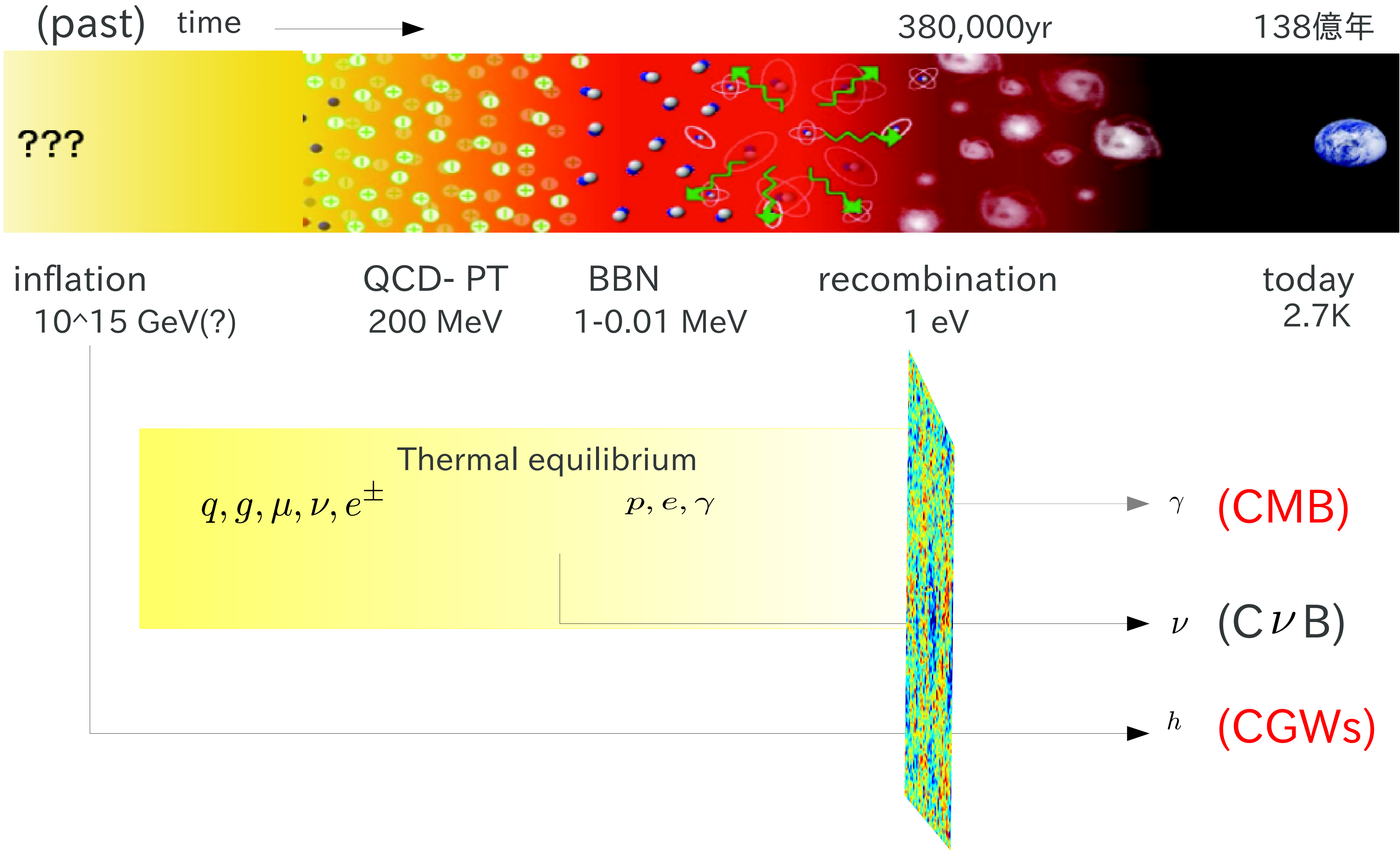
Monopole problem

Evidence for the inflation

Adiabatic density fluctuations

Small gravitational wave background

History of the expanding universe



(past) time

380,000yr

138億年

inflation
10^15 GeV(?)

QCD- PT
200 MeV

BBN
1-0.01 MeV

recombination
1 eV

today
2.7K

Thermal equilibrium

$q, g, \mu, \nu, e^{\pm}$

p, e, γ

γ (CMB)

ν (CνB)

h (CGWs)

THE CMB: stats

(1) Planck Spectrum with T=2.725

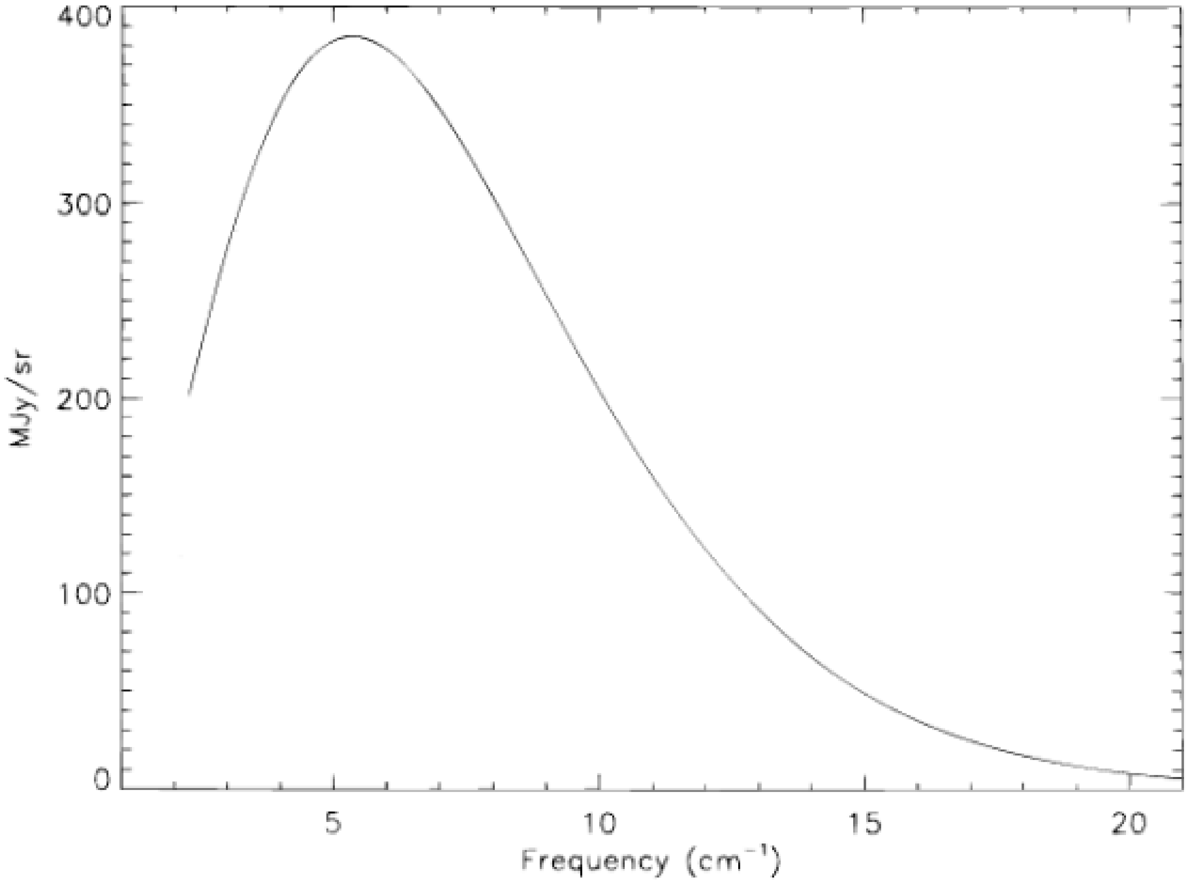
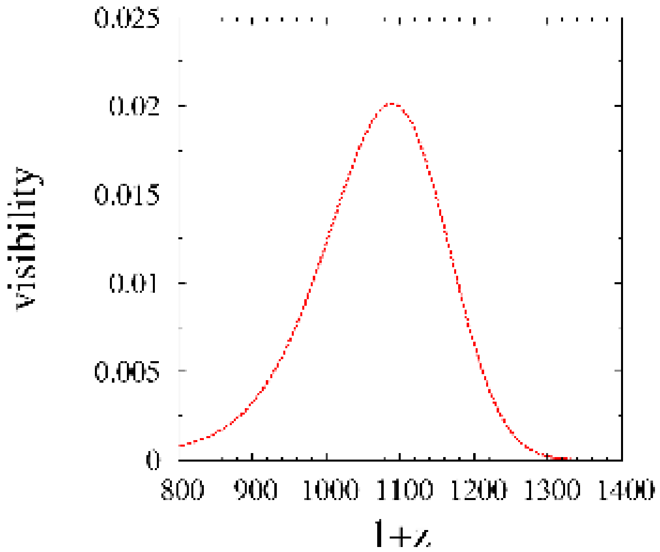


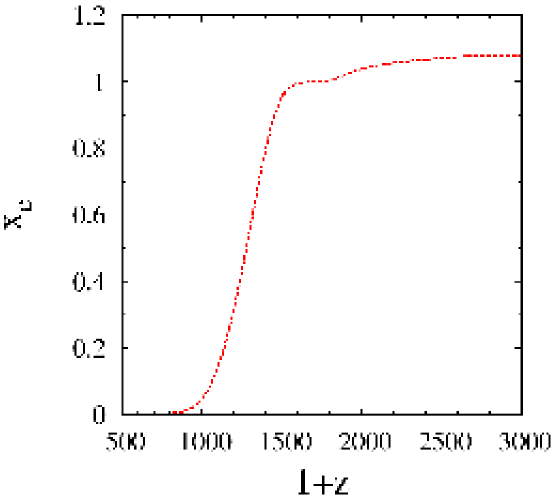
FIG. 4.—Uniform spectrum and fit to Planck blackbody (T). Uncertainties are a small fraction of the line thickness.

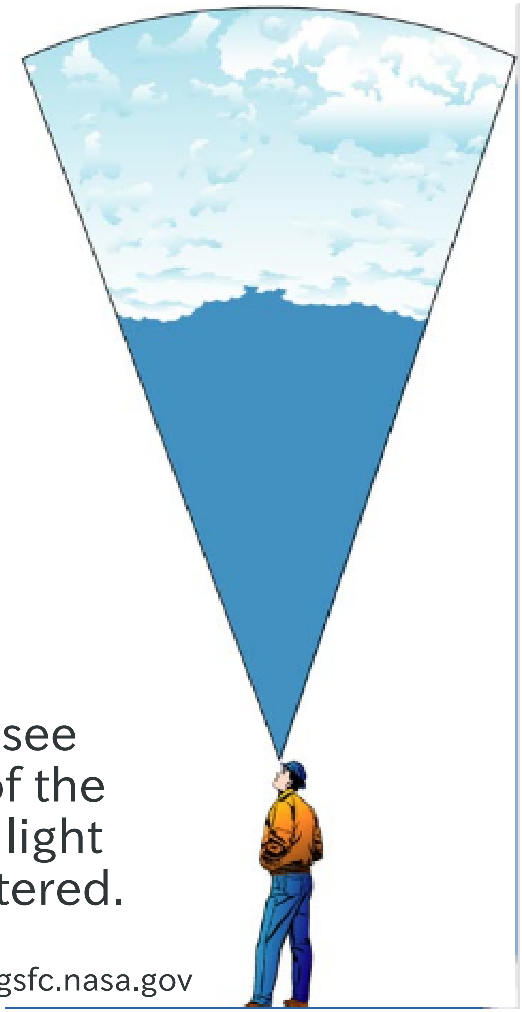
COBE FIRAS data (Fixsen et al., ApJ, 1996)

THE CMB: stats

(2) Coming from the Universe at z=1087





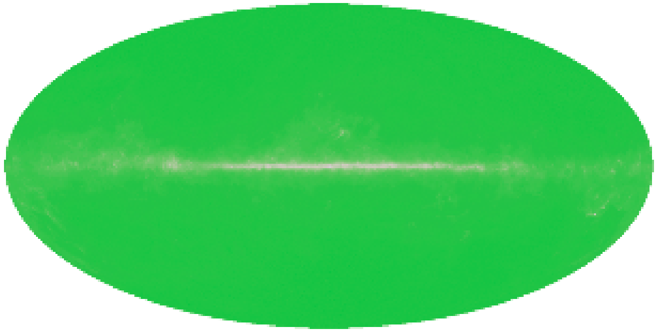


We can only see the surface of the cloud where light was last scattered.

from wmap.gsfc.nasa.gov

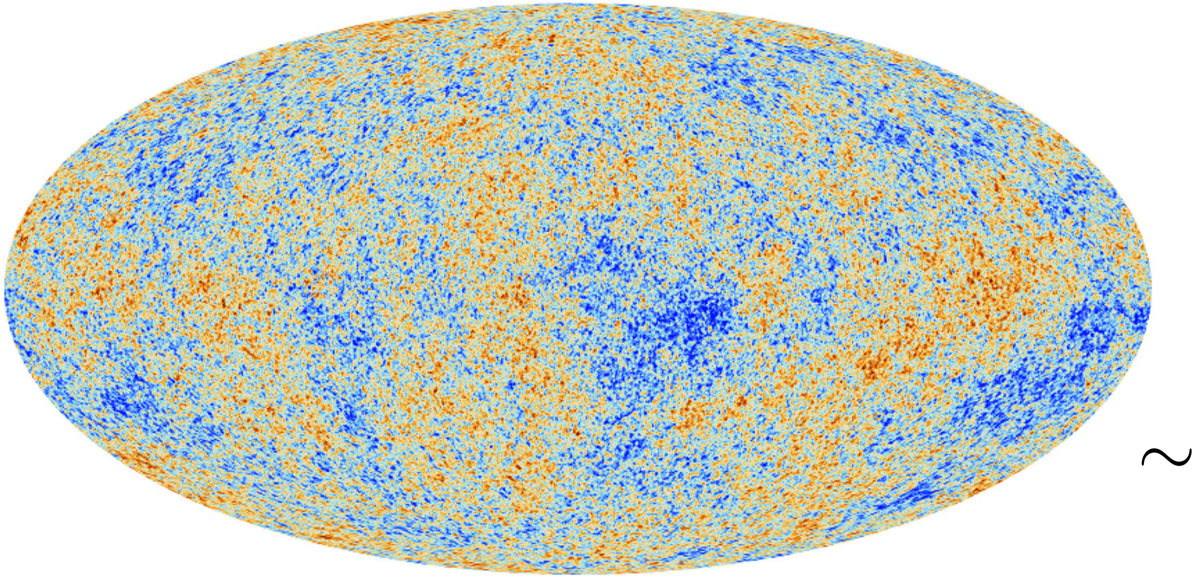
THE CMB: stats

(3) Having 10^{-5} level Temperature fluctuations



$\sim 3\mathrm{K}$

Penzias and Wilson (1965)

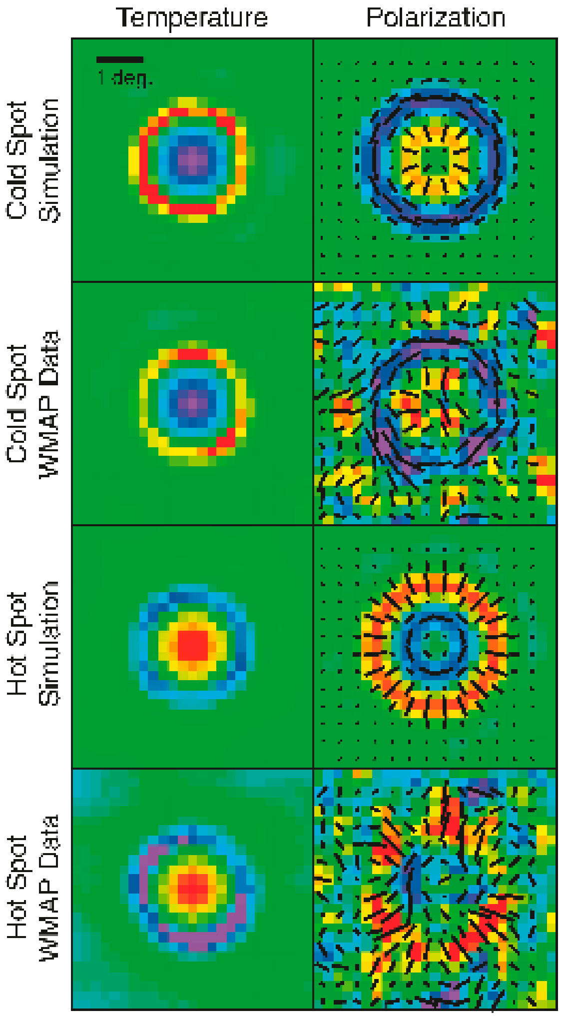


$\sim 30\mu\mathrm{K}$

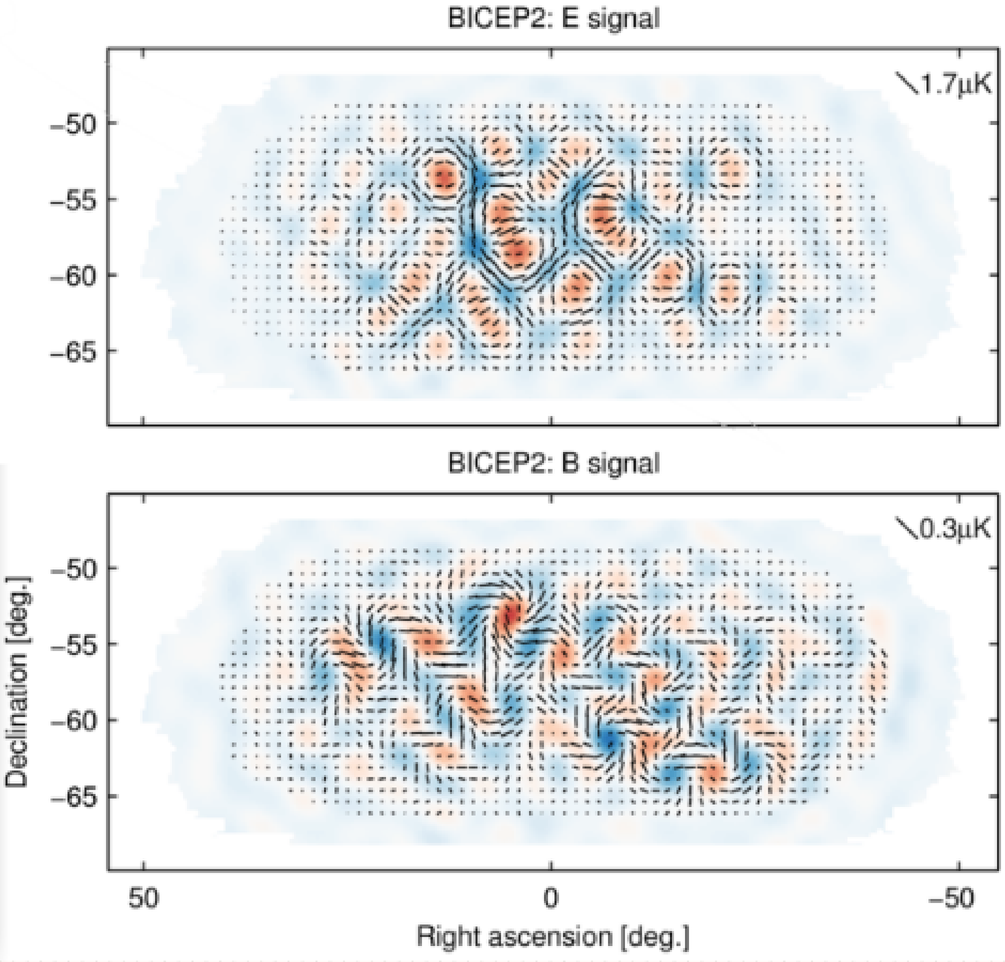
Planck collaboration (2013)

THE CMB: stats

(4) Having 10^{-6} level polarization fluctuations



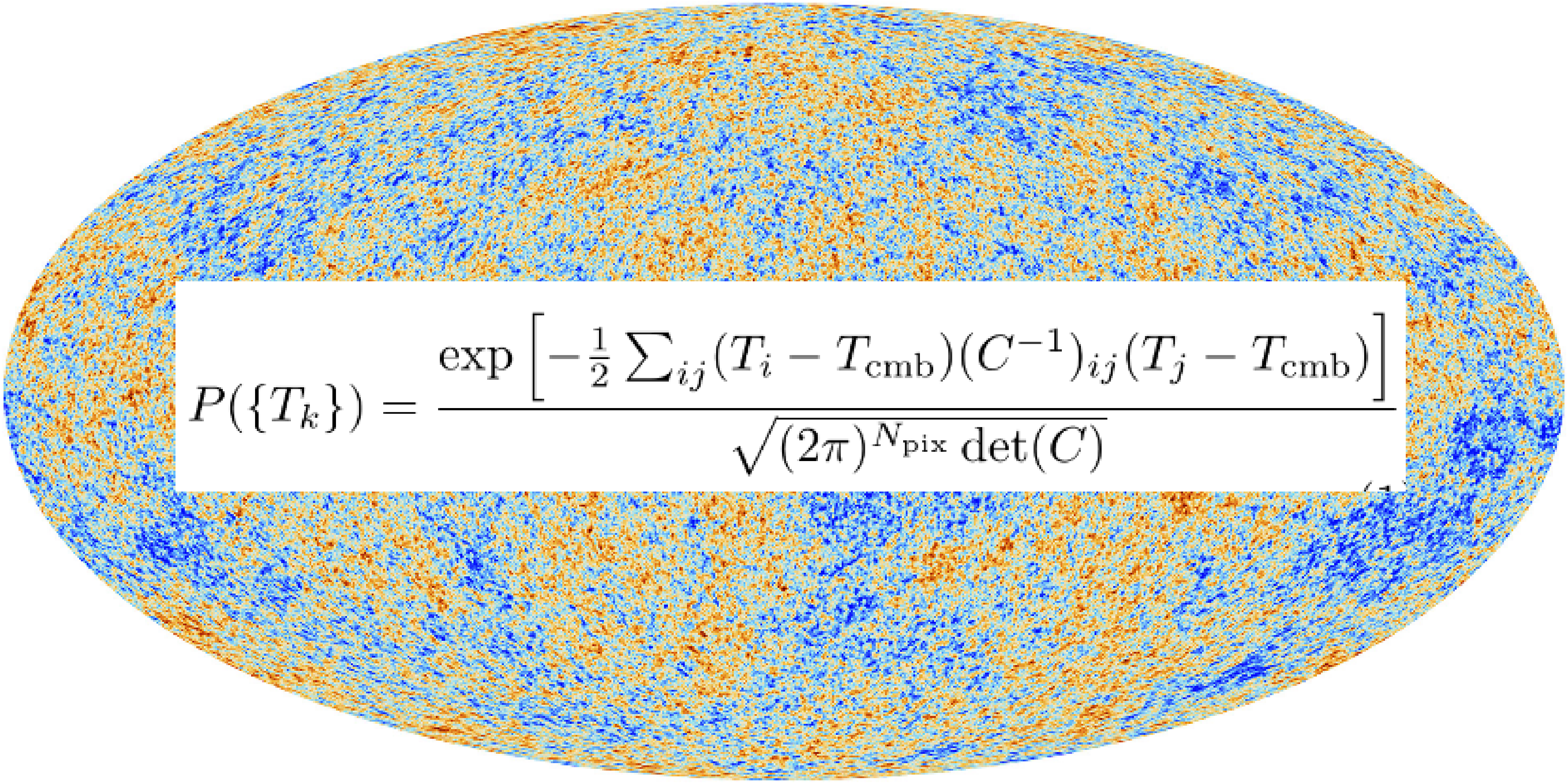
WMAP 7yr result



BICEP2 2014

THE CMB: stats

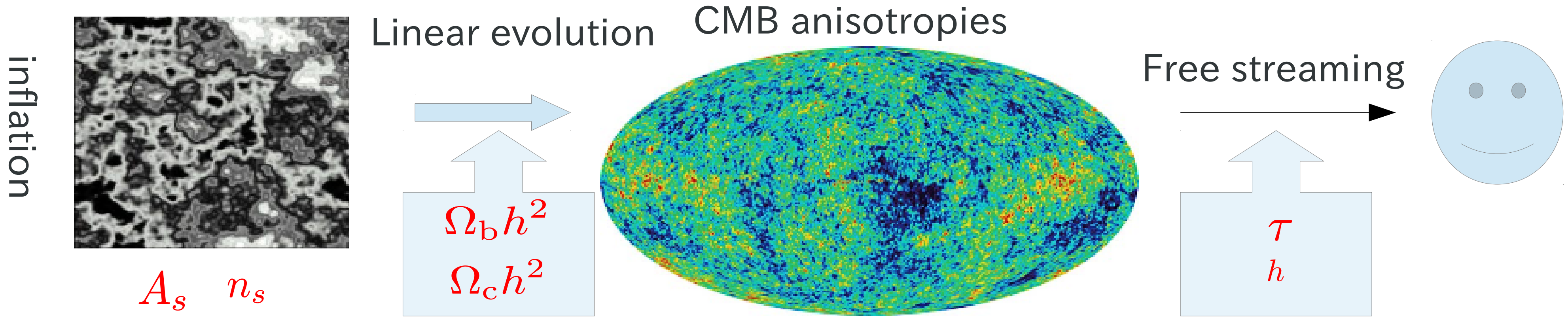
(5) Obeying Gaussian statistics



$$P(\{T_k\}) = \frac{\exp\left[-\frac{1}{2}\sum_{ij}(T_i - T_{\rm cmb})(C^{-1})_{ij}(T_j - T_{\rm cmb})\right]}{\sqrt{(2\pi)^{N_{\rm pix}}\det(C)}}$$

“The standard model”

- Geometry of the universe
  - h hubble parameter,
- Initial conditions
  - matter densities… $\Omega_{\rm b}h^2$ baryons, $\Omega_{\rm c}h^2$ cold dark matter
  - fluctuations… $P_{\mathcal{R}} = A_s(k/k_0)^{n_s-1}$ A_s amplitude, n_s spectral index
- astrophysics
  - τ optical depth to the last scattering surface



inflation

A_s n_s

Linear evolution

$\Omega_{\rm b}h^2$
$\Omega_{\rm c}h^2$

CMB anisotropies

Free streaming

τ
h

Cosmological Perturbations

(e.g., Kodama&Sasaki, PTP, 1984)

Metric (space-time)

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\underline{\delta_{ij}} + \underline{h_{ij}})dx^i dx^j\right]$$

Homogeneous& Isotropic

small perturbations
symmetric tensor (6 components)

scalar　　scalar　　vector　　tensor

$$h_{ij} = \hat{k}_i\hat{k}_j h + \left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)6\eta + 2i\hat{k}_{(i}h^V_{j)} + h^T_{ij}$$

6 dof =　1 dof　　1 dof　　2 dof　　2 dof

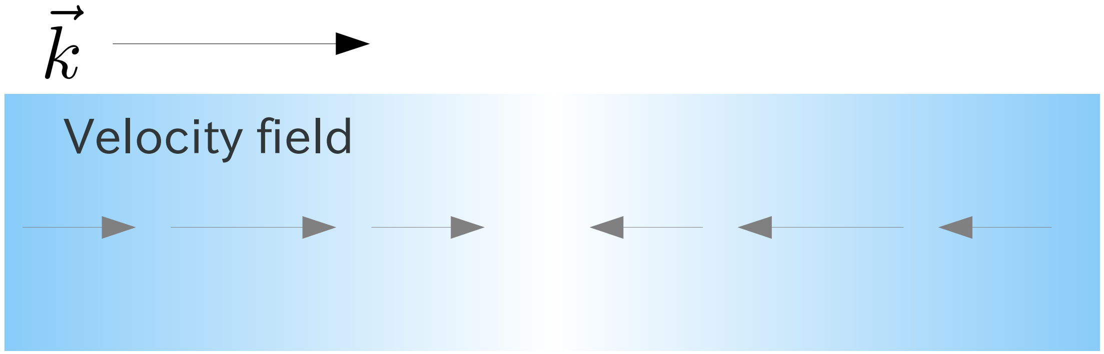
$$k^i h^V_i = 0$$

$$k^i h^T_{ij} = 0, \quad h^{Ti}{}_i = 0$$

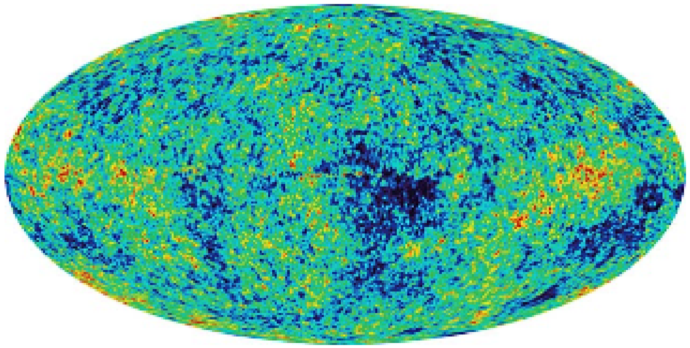
scalar mode

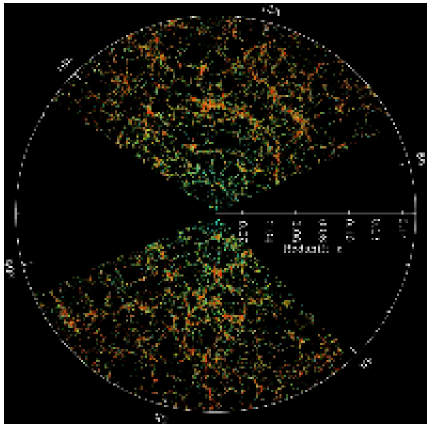
$$h_{ij} = \overset{\text{scalar}}{\hat{k}_i\hat{k}_j h} + \left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)\overset{\text{scalar}}{6\eta} + 2i\hat{k}_{(i}h^V_{j)} + h^T_{ij}$$

- gravitational potentials (density fluctuations)
- Velocity fields are written by gradient of velocity potential
- generated during inflation



One Fourier mode

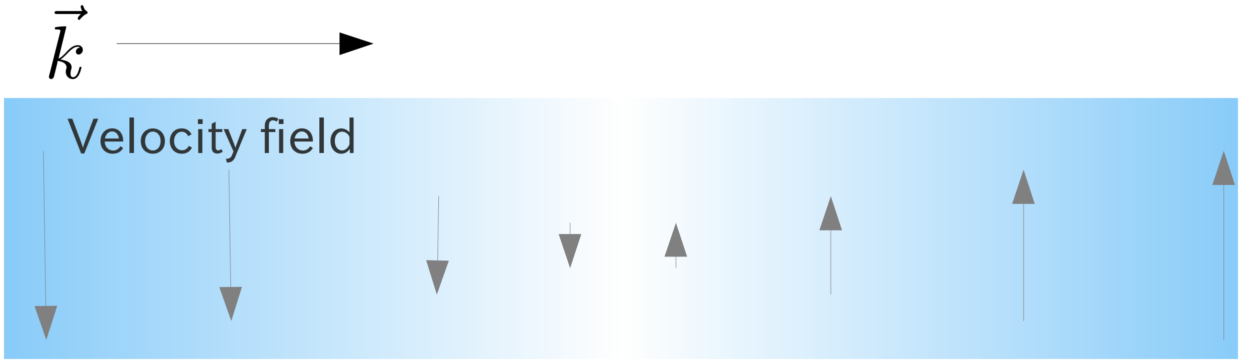




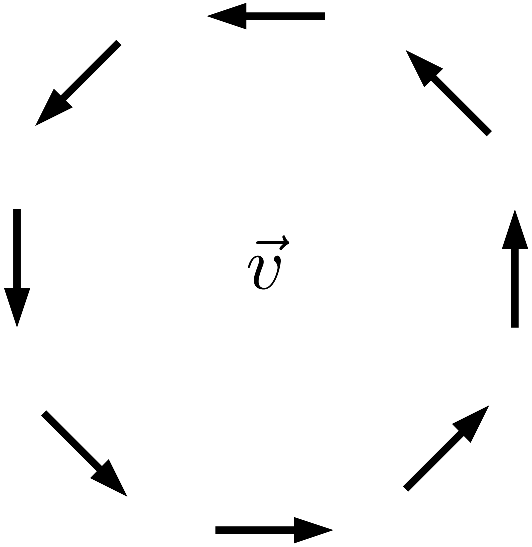
vector mode

$$h_{ij} = \hat{k}_i\hat{k}_j h + \left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right) 6\eta + 2i\hat{k}_{(i}\overset{\text{vector}}{h^{V}_{j)}} + h^{T}_{ij}$$

- divergenceless velocity field (vorticity)
- velocity fields perpendicular to $\vec{k}$
- negligible in the standard model (see, Saga-kun's Poster)



One Fourier mode

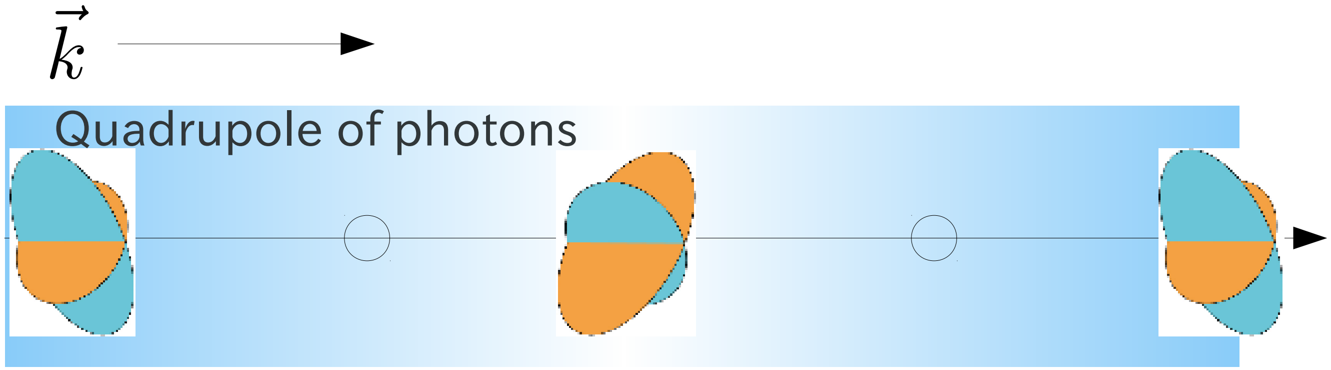


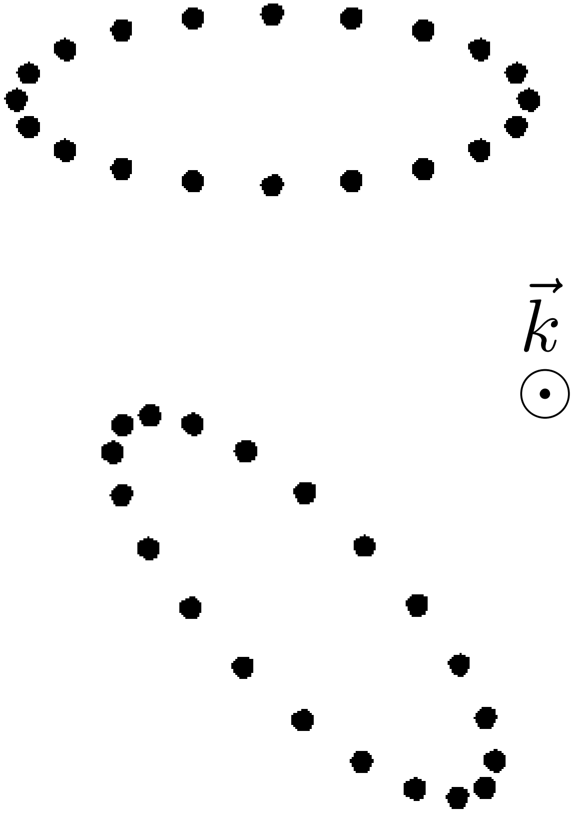
superposition of the modes

tensor mode

$$h_{ij} = \hat{k}_i\hat{k}_j h + \left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right) 6\eta + 2i\hat{k}_{(i}h^V_{j)} + \overset{\text{tensor}}{h^T_{ij}}$$

- gravitational waves (see Ando-san's talk)
- quadrupole pattern
- generated during inflation





GWs from Inflation

- GW = tensor-type（TT）perturbations in the metric:

$$h_{ij} = h^{+} e^{+}_{ij} + h^{\times} e^{\times}_{ij}$$

- EoM for the GWs (Einstein eq.)
  - takes the same form as that for massless scalar field

$$\ddot{h}_{+,\times} + 3H\dot{h}_{+,\times} + \frac{k^2}{a^2} h_{+,\times} = 0$$

$$\phi \equiv \frac{M_{\mathrm{pl}}}{\sqrt{2}} h_{+,\times}$$

$$\mathcal{L} = M_{\mathrm{pl}} R$$

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

- Solution in the De-Sitter spacetime

$$\phi = -\frac{1}{\sqrt{2k^3}} (i - k\eta) \frac{e^{-ik\eta}}{a\eta} \qquad \eta = -\frac{1}{aH}$$

$$P_\phi(k) = \frac{k^3}{2\pi^2} \left\langle |\phi|^2 \right\rangle = \left(\frac{H}{2\pi} \right)^2 \longleftarrow$$

Amplitude of GWs is determined by the energy scale of inflation

GWs from Inflation

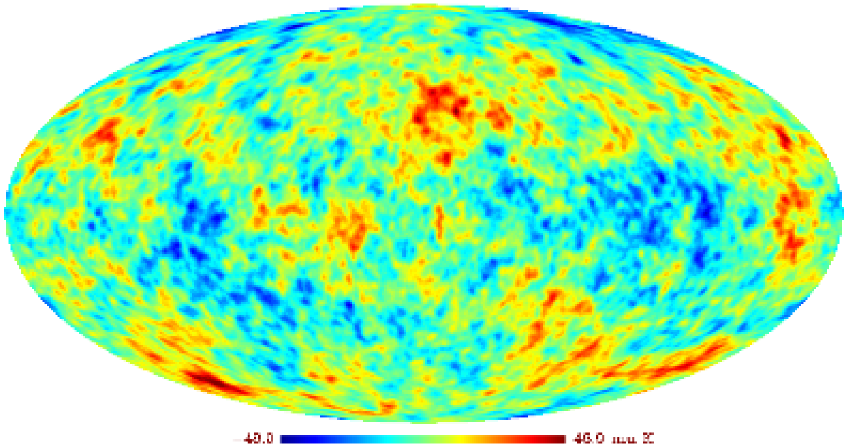
- How do we know the spectrum of the GWs?

$$P_\phi(k) = \frac{k^3}{2\pi^2}\left\langle|\phi|^2\right\rangle = \left(\frac{H}{2\pi}\right)^2 \qquad \phi \equiv \frac{M_{\rm pl}}{\sqrt{2}} h_{+,\times}$$

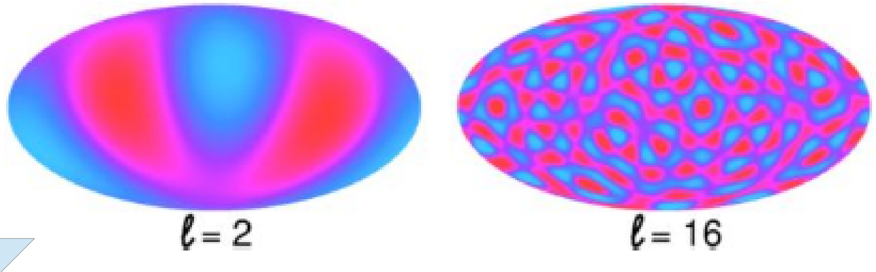
- Two possible signatures in the CMB
  - gravitational redshift (Sachs & Volfe, ApJ, 1967)
  - distinct polarization pattern (Polnarev, SvA 29, 1985)

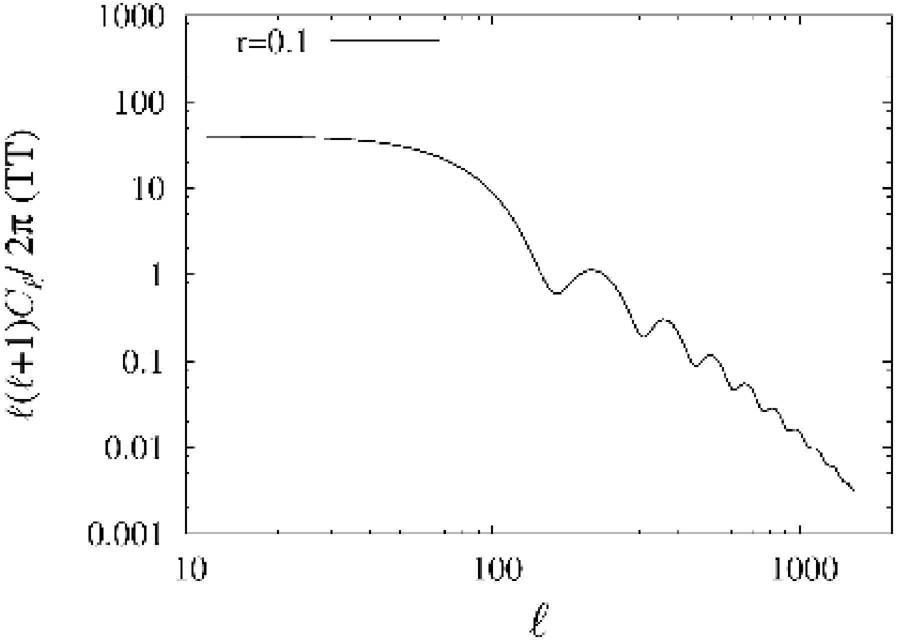
Let's look into the CMB anisotropies

CMB from GWs



CMB from GWs (simulation)

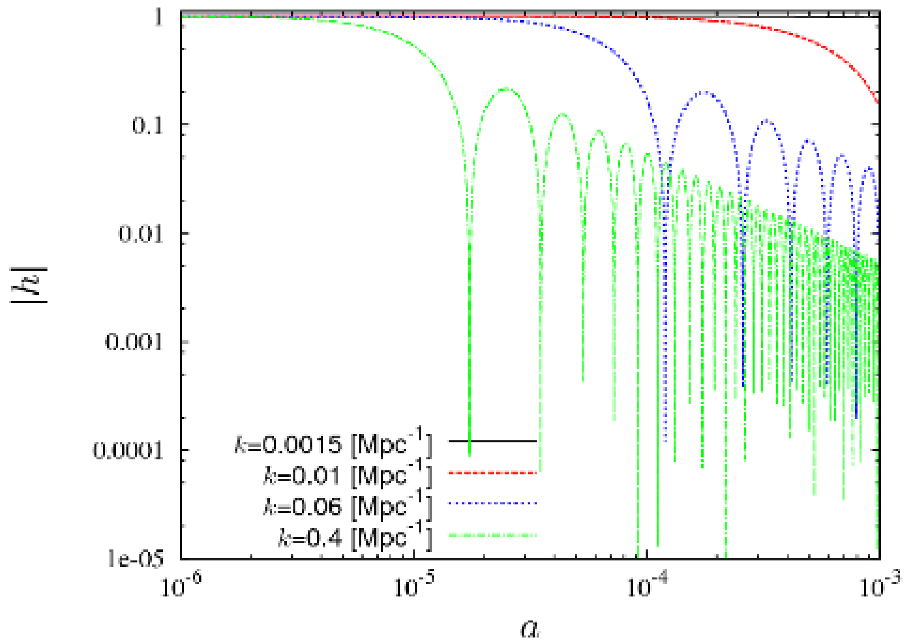




- l<100 flat, l>100: damping with oscillations
- why this shape?

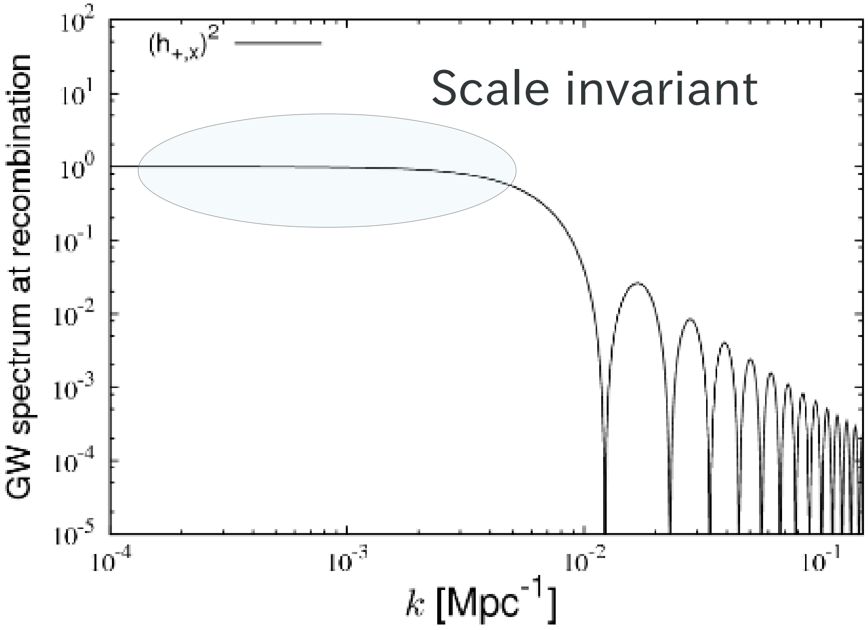
Evolution of GWs

Time evolution of each Fourier mode



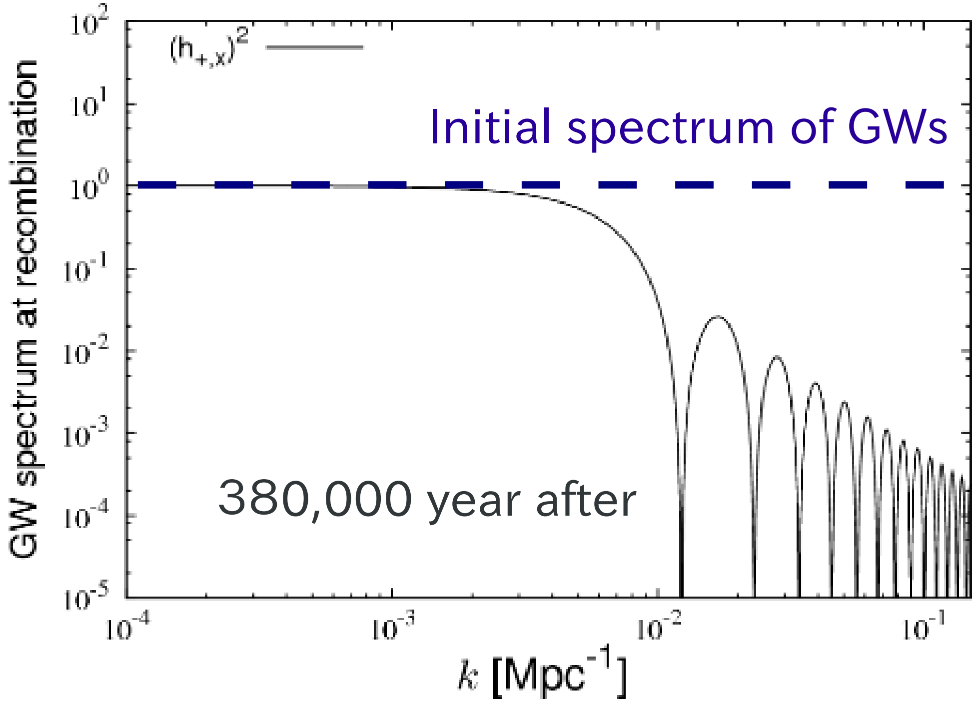
time

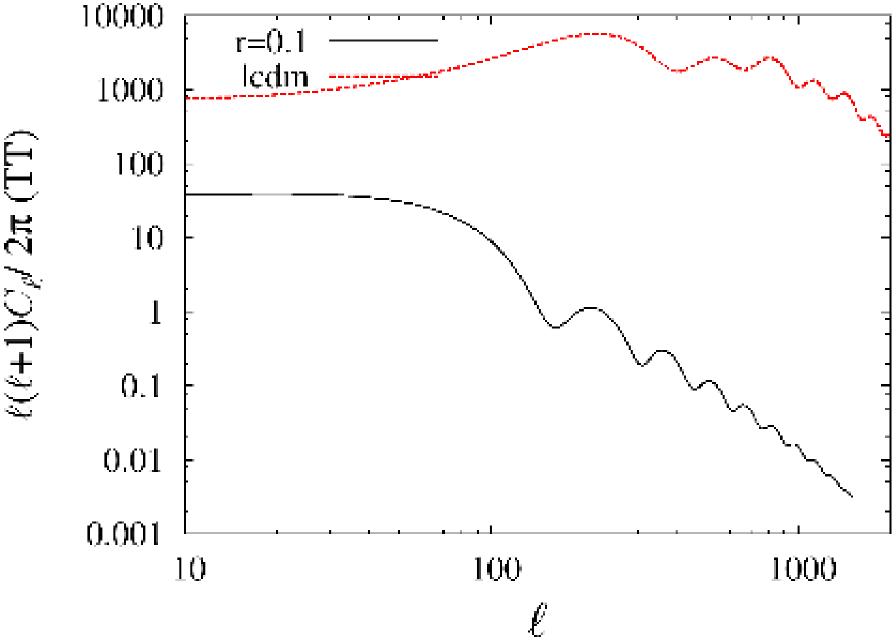
Snapshot of the spectrum at recombination

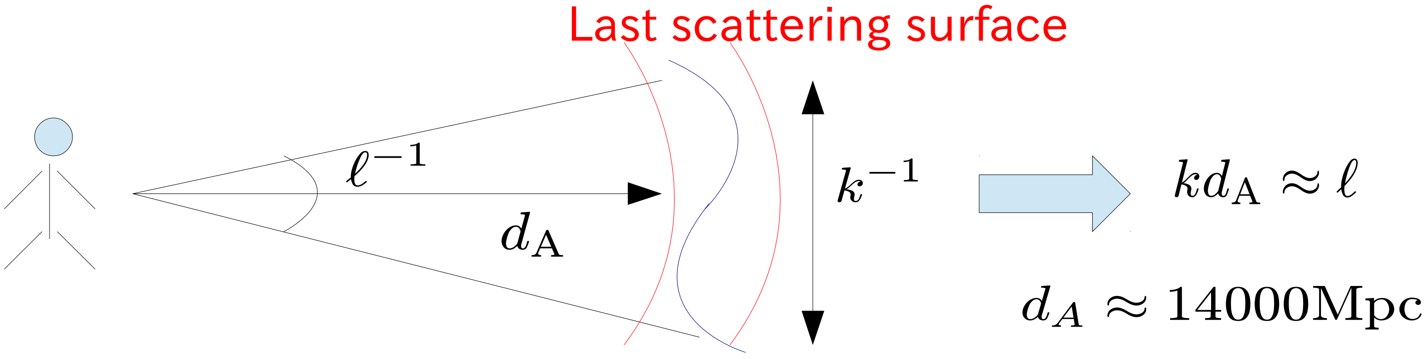


wavenumber

$C_\ell^{\rm TT}$ from GWs



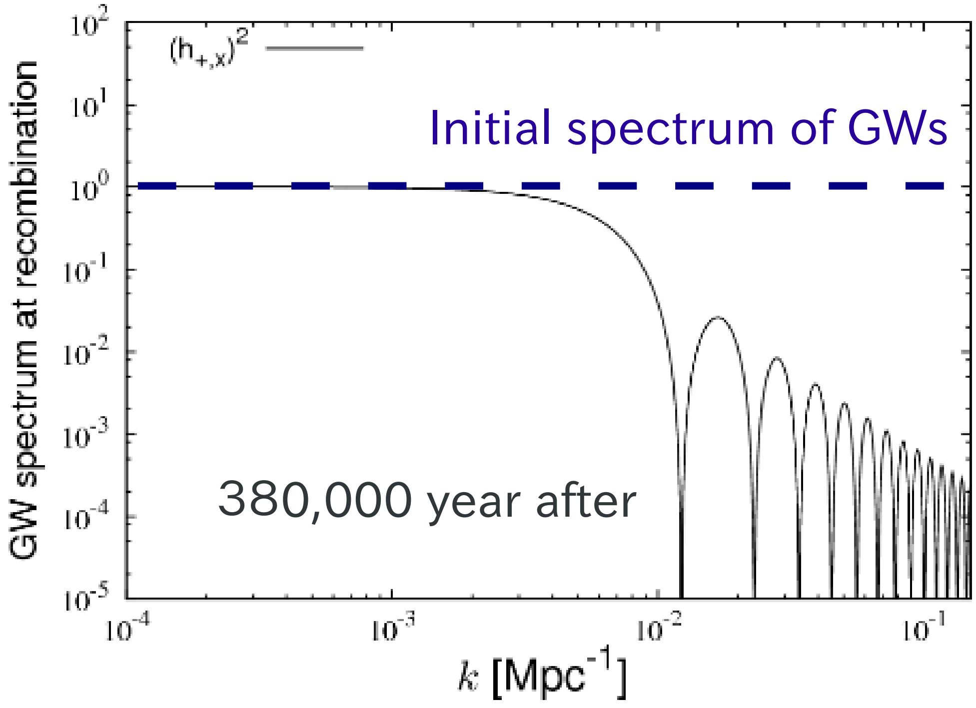




$kd_{\rm A} \approx \ell$

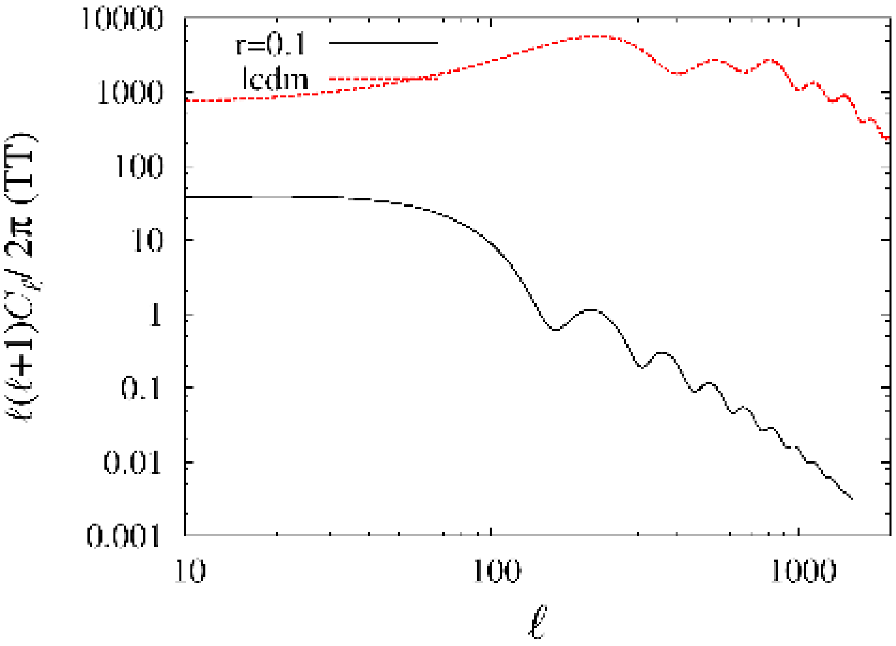
$d_A \approx 14000{\rm Mpc}$

$C_\ell^{\rm TT}$ from GWs



Initial spectrum of GWs

380,000 year after

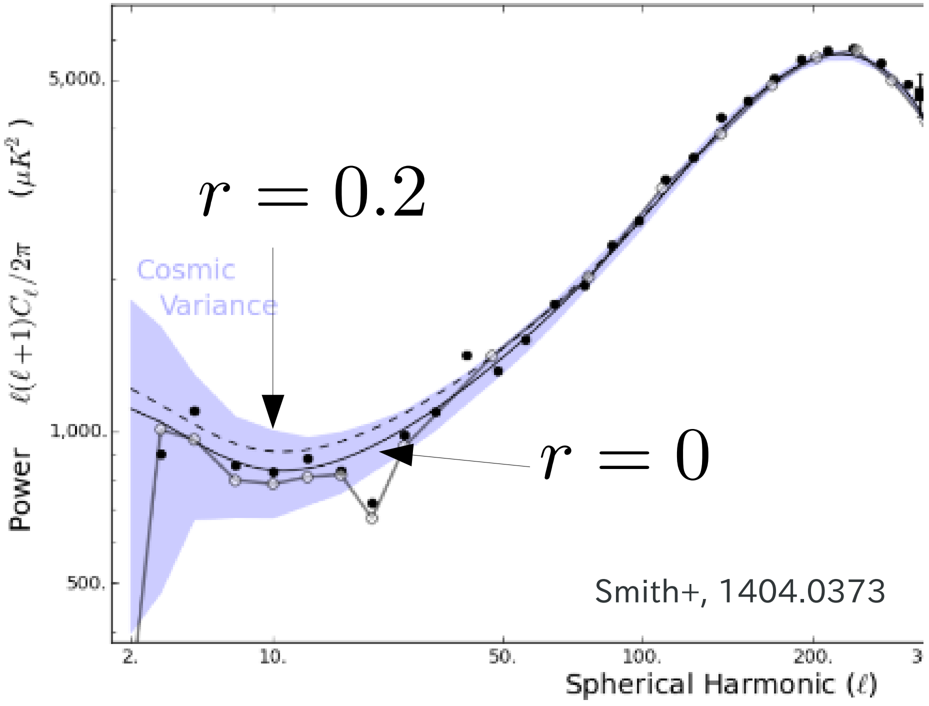


temperature fluctuations from tensors could be important only on large angular scales.

constraints on r from C_ℓ^{TT}

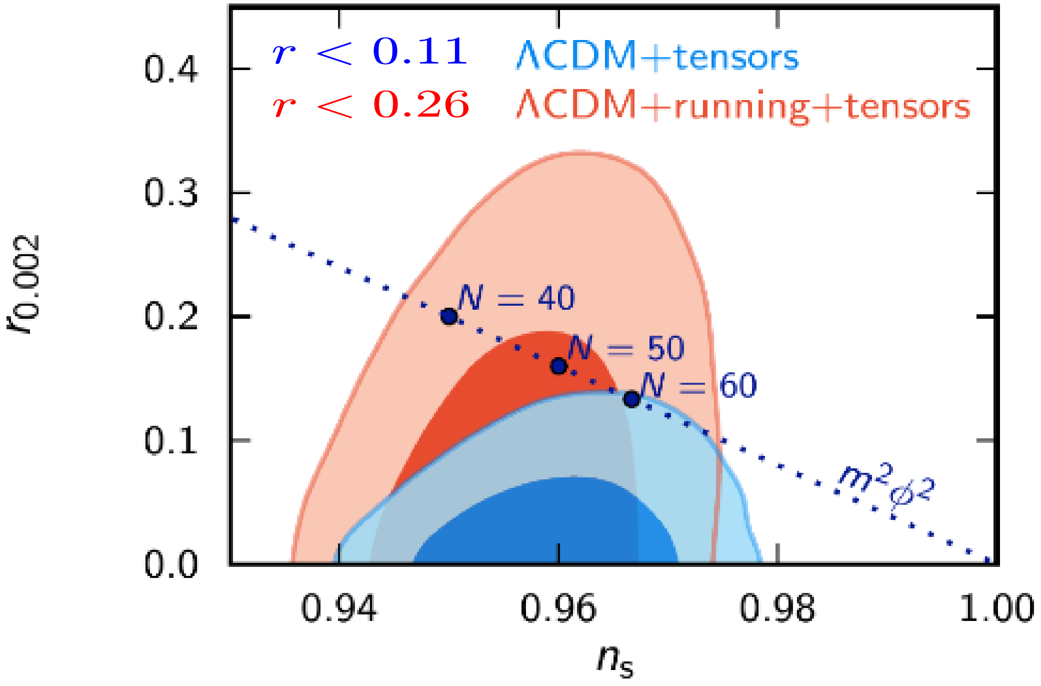
- Tensor-to-Scalar ratio
  - “r”=(tensor mode)/ (scalar mode)
  - being related to the energy scale of inflation

$$V_{\mathrm{inf}} \approx \left(2 \times 10^{16}\ \mathrm{GeV}\right)^4 \left(\frac{r}{0.12}\right)$$



Smith+, 1404.0373

(Arxiv 1303.5076, PLANCK)



outline

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- The sources of B-mode polarization
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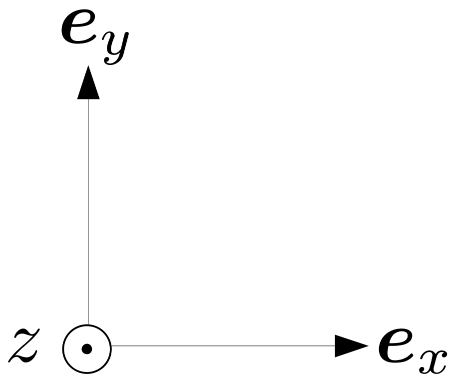
- summary

Stokes Parameters Q&U

- Observable: Electric Fields
  - *Define* a tensor (matrix)

$$T_{ab} \equiv \langle E_a^* E_b \rangle$$



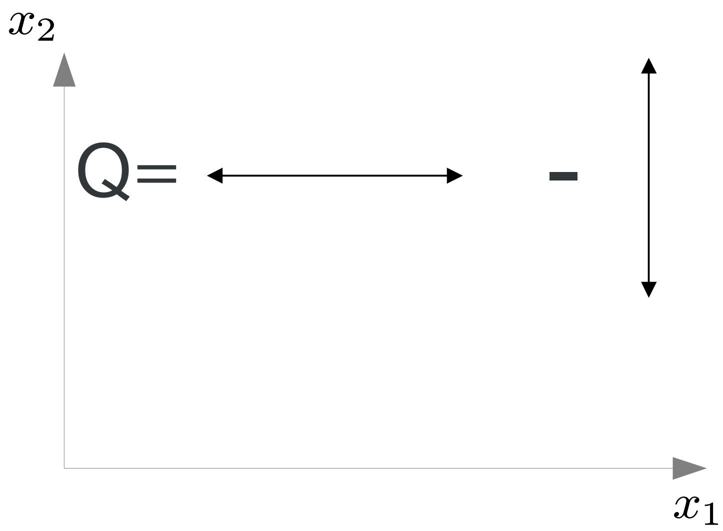
- in the local (x,y,z) coordinate, this is

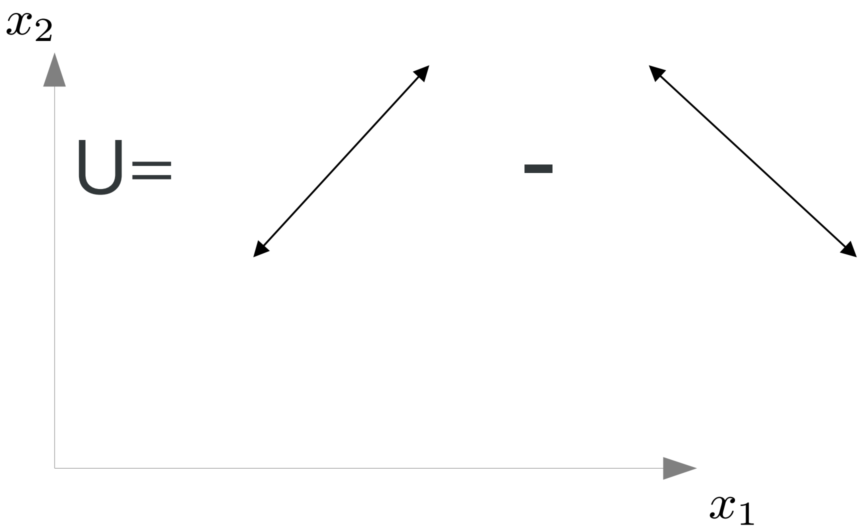
$$T_{ab} = I\delta_{ab} + \underbrace{\frac{1}{2}\begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}}_{\equiv P_{ab}^{(x,y)}}$$

(polarization tensor
in the (x,y) coordinate)

(see, e.g., Lewis&Challinor, PhysRep 2006)

The meaning of (Q,U)

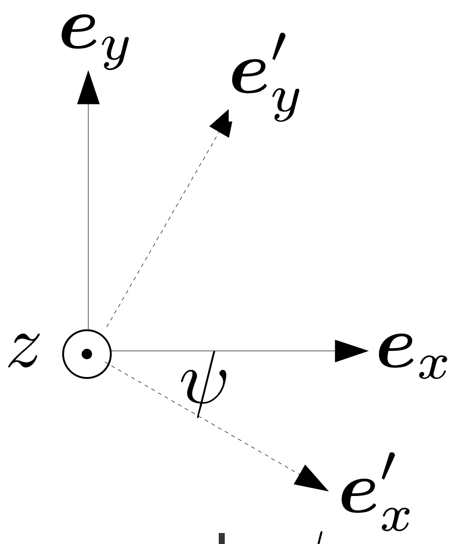




- Q,U depend on the coordinate system you choose
  - Q&U interchange by 45 deg rotation along x^3 axis
  - Q&U changes sign by 90 deg rotation

We have to know how the rotation affects the definition of polarizations

Stokes Parameters Q&U



- Consider the helicity basis:

$$\boldsymbol{e}_\pm \equiv \boldsymbol{e}_x \pm i\boldsymbol{e}_y$$

- Why helicity basis? Under a rotation by an angle ψ clockwise, the helicity basis vectors transform as

$$\begin{aligned}\boldsymbol{e}_\pm &\to \boldsymbol{e}'_x \pm i\boldsymbol{e}'_y \\ &= (\cos\psi\,\boldsymbol{e}_e - \sin\psi\,\boldsymbol{e}_y) \pm i(\sin\psi\,\boldsymbol{e}_x + \cos\psi\,\boldsymbol{e}_y) \\ &= e^{\pm i\psi}\boldsymbol{e}_\pm\end{aligned}$$

Stokes Parameters Q&U

- Let us consider a projection of $P_{ab}^{(x,y)}$ onto the helicity basis:

$$P_{++} \equiv (\boldsymbol{e}_+)^a (\boldsymbol{e}_+)^b P_{ab}^{(x,y)} = Q + iU$$

$$P_{--} \equiv (\boldsymbol{e}_-)^a (\boldsymbol{e}_-)^b P_{ab}^{(x,y)} = Q - iU$$

- In the rotated basis

$$\begin{aligned} P_{++} \to P'_{++} &= (\boldsymbol{e}'_+)^a (\boldsymbol{e}'_+)^b P_{ab}^{(x,y)} \\ &= e^{2i\psi} (\boldsymbol{e}_+)^a (\boldsymbol{e}_+)^b P_{ab}^{(x,y)} = e^{2i\psi} \left(Q + iU\right) \end{aligned}$$

$Q \pm iU$ has spin ± 2 and thus basis dependent

Scalar Functions E&B

- Instead, let us construct P_{ab} from basis-independent scalar functions $E(\hat{n})$ and $B(\hat{n})$:

$$P_{ab} = \underbrace{\nabla_{\langle a}\nabla_{b\rangle}}_{\text{symmetric trace-free}} E(\hat{n}) + \underbrace{\epsilon^{c}{}_{(a}\nabla_{b)}}_{\text{symmetric}} \nabla_c B(\hat{n})$$

2D vector analogue: $V_a = \nabla_a G(x,y) + \epsilon_a^{\ b} \nabla_b C(x,y)$

- Because $E(\hat{n})$ and $B(\hat{n})$ are scalar functions on the sphere, they can be expanded by the scalar harmonics

$$E(\hat{n}) = \sum_{(\ell,m)} E_{\ell m} Y_{\ell m}(\hat{n}), \quad B(\hat{n}) = \sum_{(\ell,m)} B_{\ell m} Y_{\ell m}(\hat{n})$$

Scalar Functions E&B

$$\boxed{\begin{aligned} &P_{ab} = \nabla_{\langle a}\nabla_{b\rangle} E(\hat{n}) + \epsilon^{c}_{\ (a}\nabla_{b)}\nabla_{c} B(\hat{n}) \\ &E(\hat{n}) = \sum_{(\ell,m)} E_{\ell m} Y_{\ell m}(\hat{n}), \ B(\hat{n}) = \sum_{(\ell,m)} B_{\ell m} Y_{\ell m}(\hat{n}) \end{aligned}}$$

- Using these definitions, we get

$$P_{++} \equiv Q + iU = \boldsymbol{e}^{a}_{+}\boldsymbol{e}^{b}_{+} P_{ab}$$

$$= \ldots\ldots\ldots$$

some normalization constant

$$= \sum_{\ell m} N \left(E_{\ell m} + i B_{\ell m} \right) \boldsymbol{e}^{a}_{+}\boldsymbol{e}^{b}_{+} \nabla_{a}\nabla_{b} Y_{\ell m}(\hat{n})$$

Scalar Functions E&B

- The final formula

$$(Q+iU)(\hat{n}) = \sum_{\ell m} (E_{\ell m} + iB_{\ell m})\, \underbrace{N \boldsymbol{e}_+^a \boldsymbol{e}_+^b \nabla_a \nabla_b Y_{\ell m}(\hat{n})}_{\equiv\, {}_2Y_{\ell m}(\hat{n})}$$

spin-weighted spherical harmonics

- In a similar way,

$$(Q-iU)(\hat{n}) = \sum_{\ell m} (E_{\ell m} - iB_{\ell m})\, {}_{-2}Y_{\ell m}(\hat{n})$$

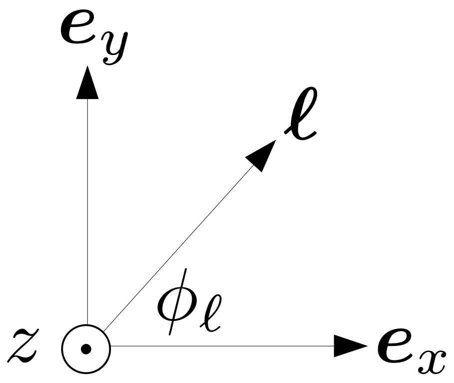
Q+iU field is expanded by the spin-weighted spherical harmonics, which are just the doubly covariant derivatives of the usual spherical harmonics along the helicity basis

(Q,U) & (E,B) on the flat sky

- On the flat sky (x,y), the harmonics is $e^{\mathrm{i}\boldsymbol{\ell}\cdot\mathrm{x}}$ and the covariant derivatives become just partial derivatives

$$
{}_2Y_{\ell m} = N\boldsymbol{e}_+^a\boldsymbol{e}_+^b\nabla_a\nabla_b Y_{\ell m}(\hat{n})
$$

$$
\rightarrow N(\partial_x + i\partial_y)^2 e^{\mathrm{i}\boldsymbol{\ell}\cdot\mathrm{x}} \propto \underline{-e^{2\mathrm{i}\phi_\ell}e^{\mathrm{i}\boldsymbol{\ell}\cdot\mathrm{x}}}
$$



(Q,U) & (E,B) on the flat sky

- Therefore, we expand the (Q+iU) field as

$$(Q \pm iU)(\boldsymbol{x}) = -\int \frac{\mathrm{d}^2\ell}{2\pi} \left(E(\boldsymbol{\ell}) \pm iB(\boldsymbol{\ell})\right) \underline{e^{\pm 2\mathrm{i}\phi_\ell} e^{\mathrm{i}\boldsymbol{\ell}\cdot\mathrm{x}}}$$

$$\Longleftrightarrow \quad Q(\boldsymbol{\ell}) \pm iU(\boldsymbol{\ell}) = -\left(E(\boldsymbol{\ell}) \pm iB(\boldsymbol{\ell})\right) e^{\pm 2i\phi_\ell}$$

- The inverse is, for example,

$$(E + iB)(\boldsymbol{\ell}) = -\int \frac{\mathrm{d}^2\mathrm{x}}{2\pi} (Q + iU)(\boldsymbol{x}) e^{-2\mathrm{i}\phi_\ell} e^{-\mathrm{i}\boldsymbol{\ell}\cdot\mathrm{x}}$$

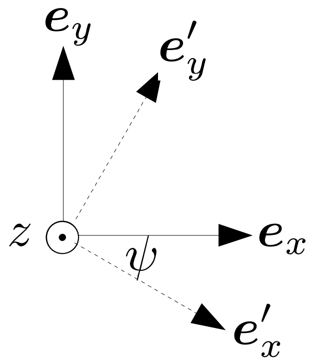
Physical Meaning of E&B

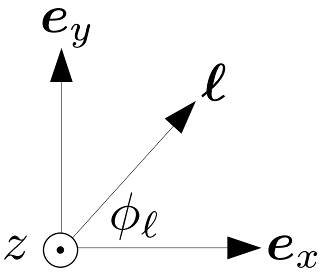
- Now you have two formulae

$$\begin{cases} Q\,(\boldsymbol{\ell}) \pm iU\,(\boldsymbol{\ell}) = -\left(E(\boldsymbol{\ell}) \pm iB(\boldsymbol{\ell})\right) e^{\pm 2i\phi_\ell} \\ Q'(\boldsymbol{\ell}) \pm iU'(\boldsymbol{\ell}) = \quad \left(Q(\boldsymbol{\ell}) \pm iU(\boldsymbol{\ell})\right) e^{\pm 2i\psi} \end{cases}$$

$$\Downarrow$$

$$\left(Q'(\boldsymbol{\ell}) \pm iU'(\boldsymbol{\ell})\right) e^{\mp 2i\psi} = -\left(E(\boldsymbol{\ell}) \pm iB(\boldsymbol{\ell})\right) e^{\pm 2i\phi_\ell}$$





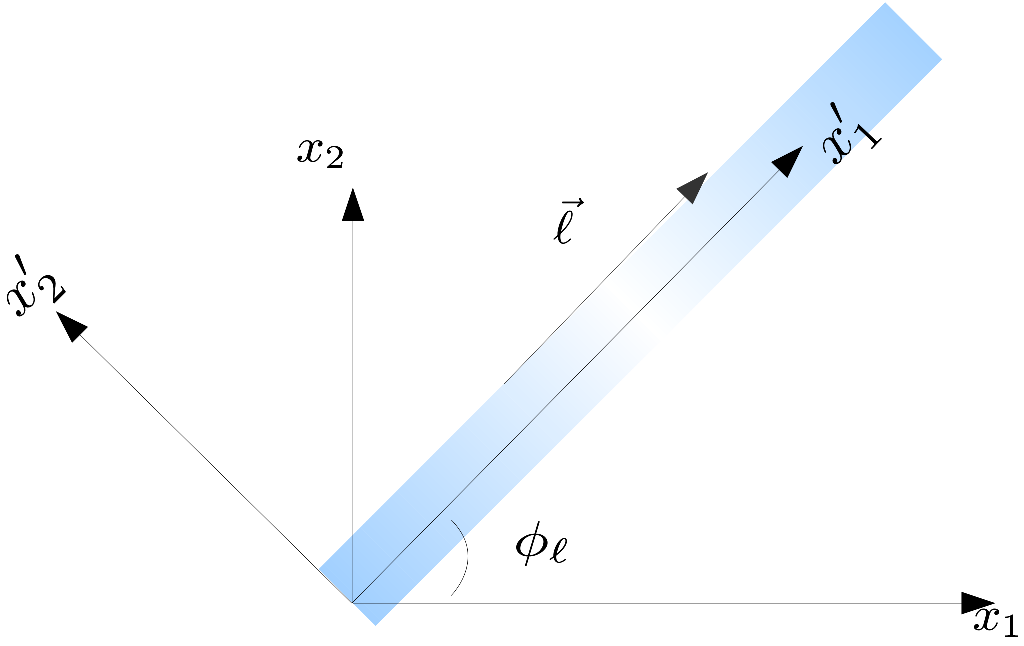
Physical Meaning of E&B

$$\left(Q'(\boldsymbol{\ell}) \pm iU'(\boldsymbol{\ell})\right) e^{\mp 2i\psi} = -\left(E(\boldsymbol{\ell}) \pm iB(\boldsymbol{\ell})\right) e^{\pm 2i\phi_\ell}$$

Therefore if you rotate coordinate system by an angle $\psi = -\phi_\ell$ depending on the direction of the Fourier mode, you get the correspondence

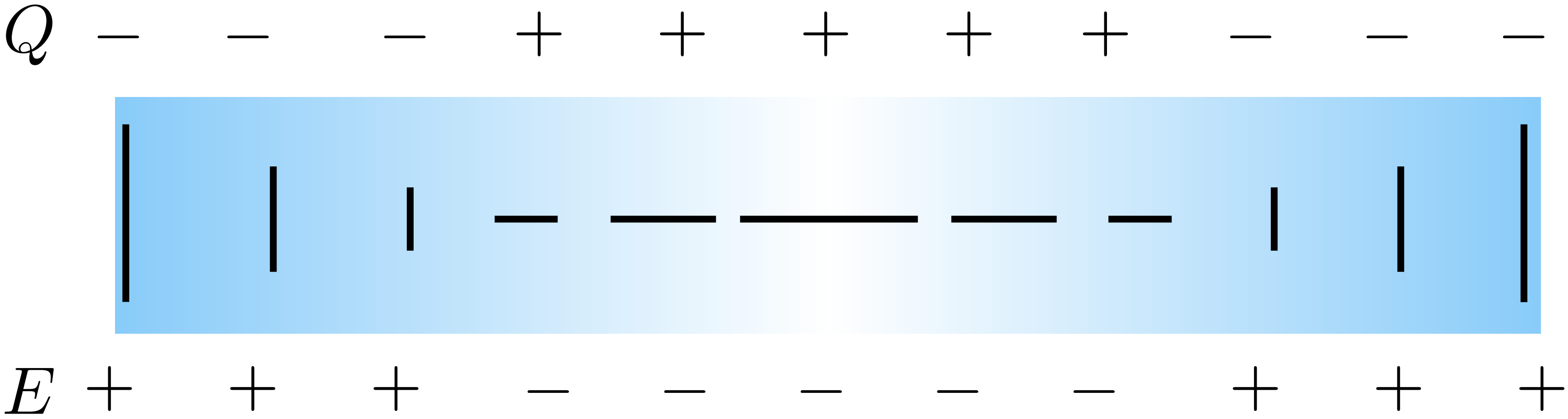
$Q \leftrightarrow -E$ and $U \leftrightarrow -B$

which determines a natural basis for polarization



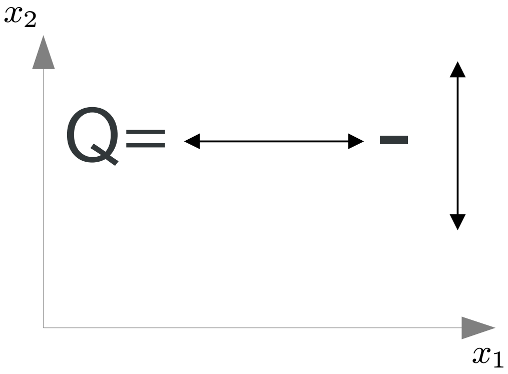
If you have only the E-mode...

- E is $-Q$ in the coordinate system such that $\boldsymbol{\ell} \parallel \boldsymbol{e}_1$



$Q \quad - \quad - \quad - \quad + \quad + \quad + \quad + \quad + \quad - \quad - \quad -$

$E \quad + \quad + \quad + \quad - \quad - \quad - \quad - \quad - \quad + \quad + \quad +$



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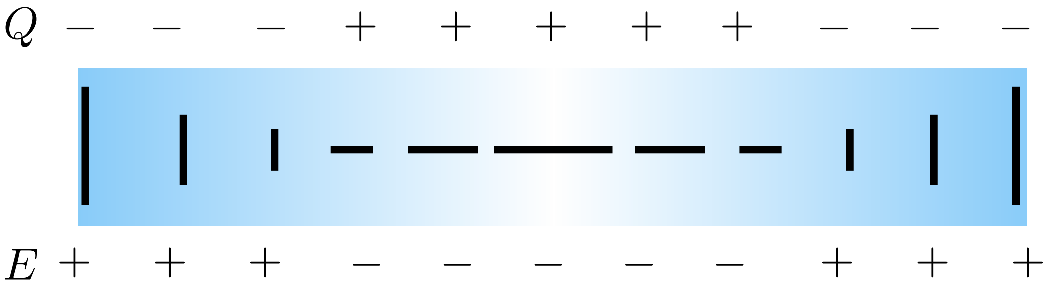

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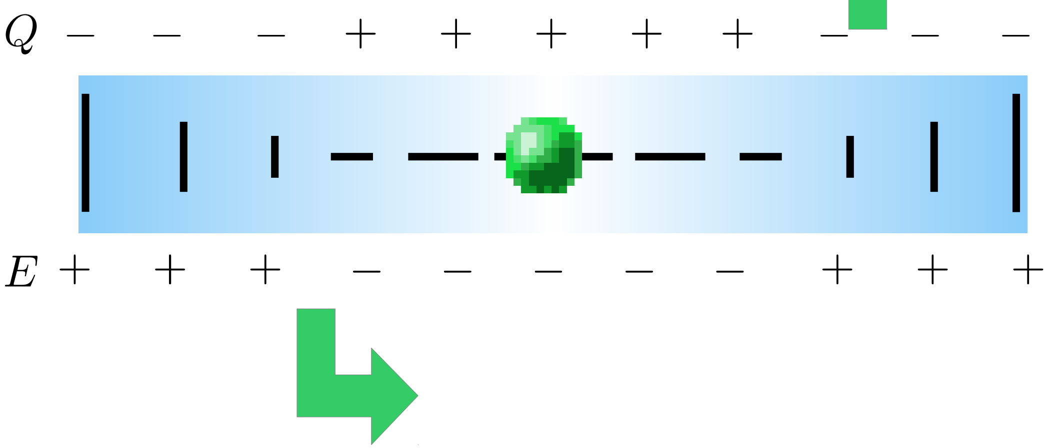

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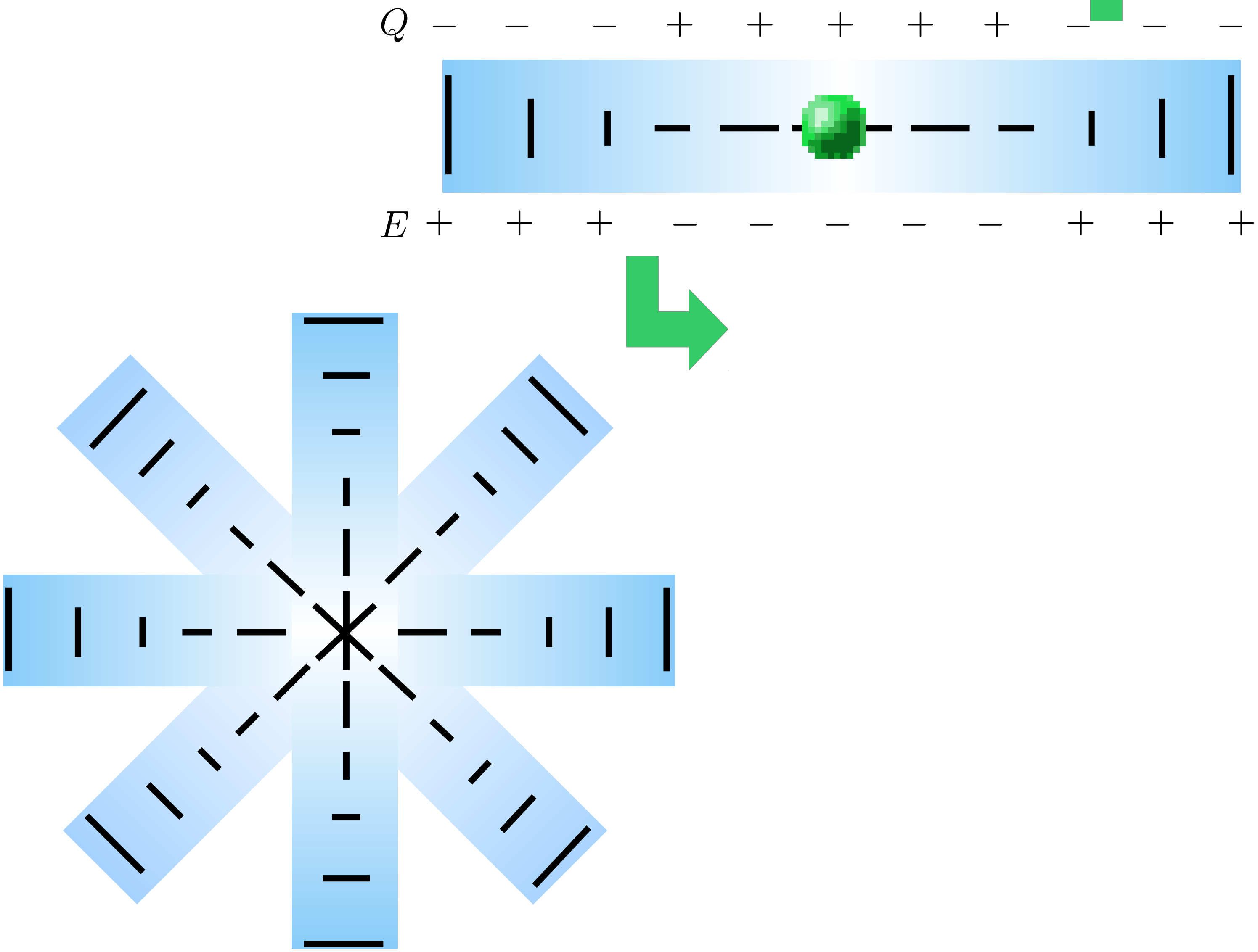

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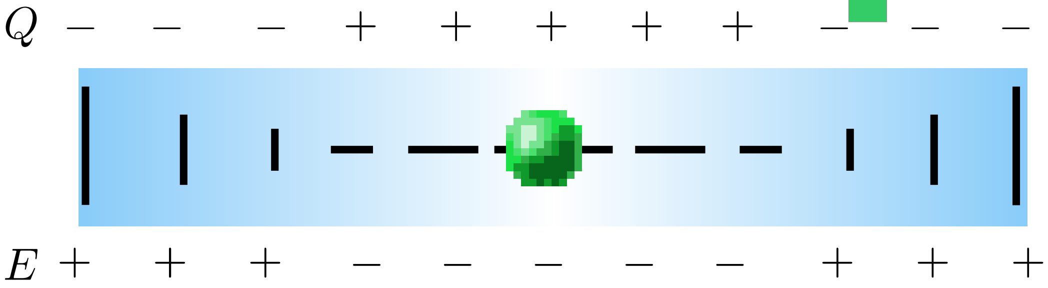
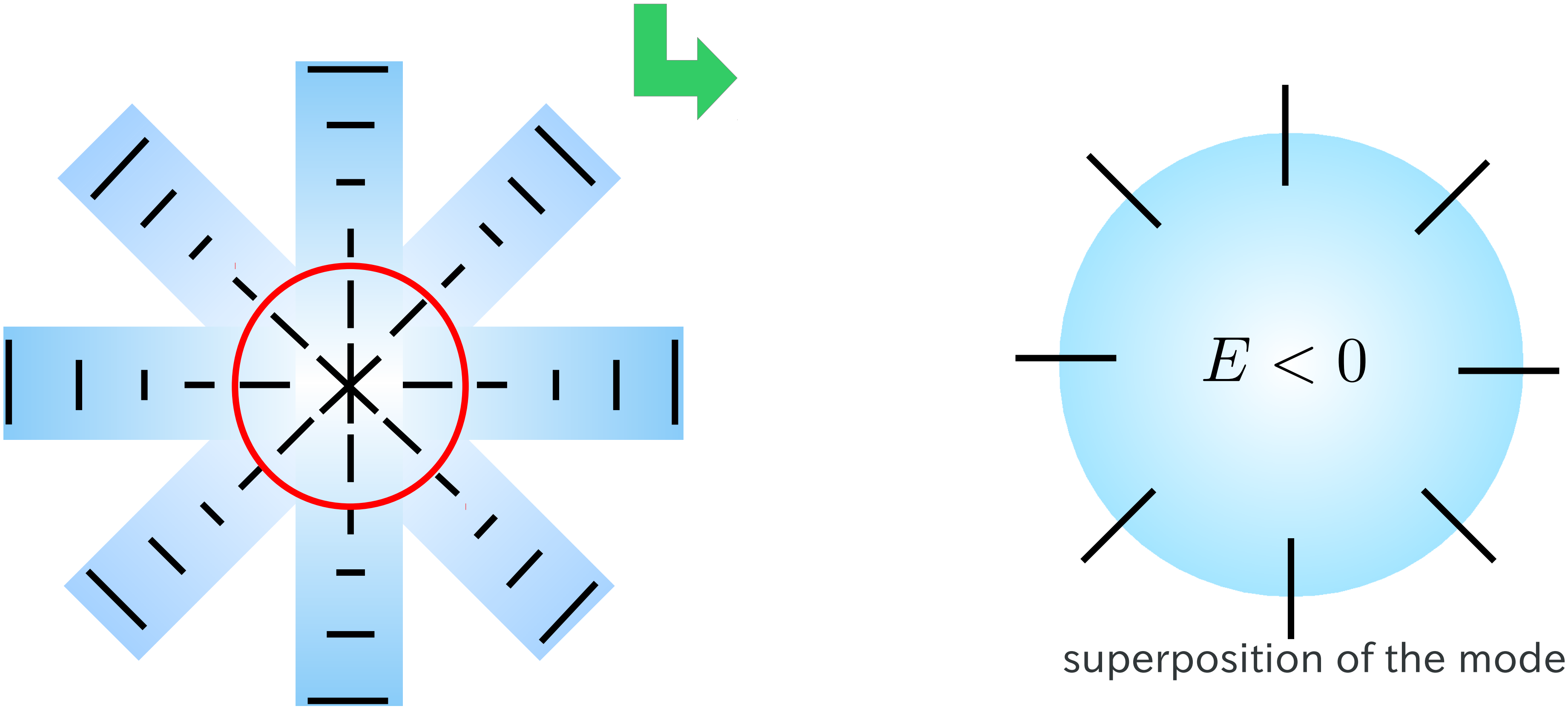
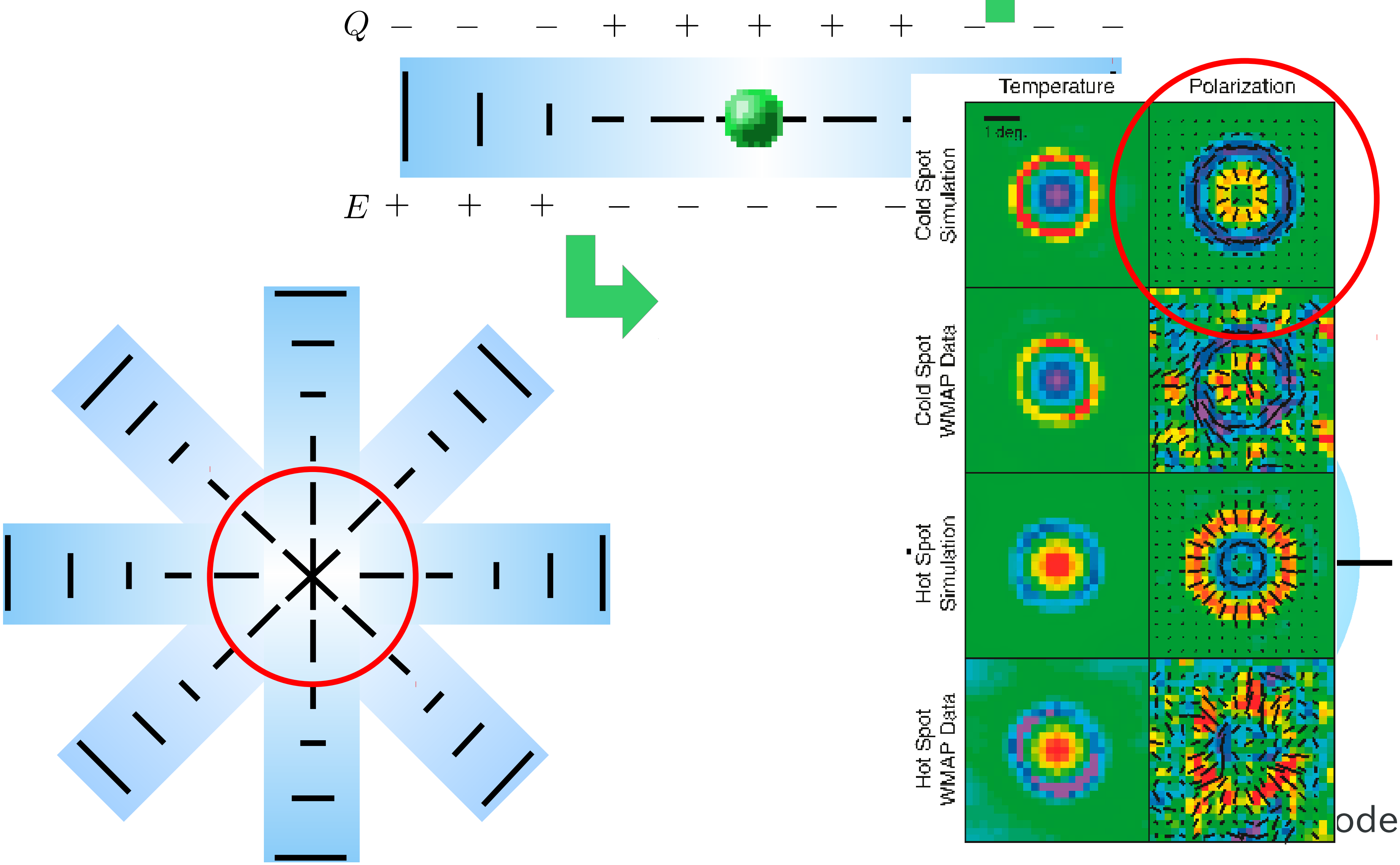




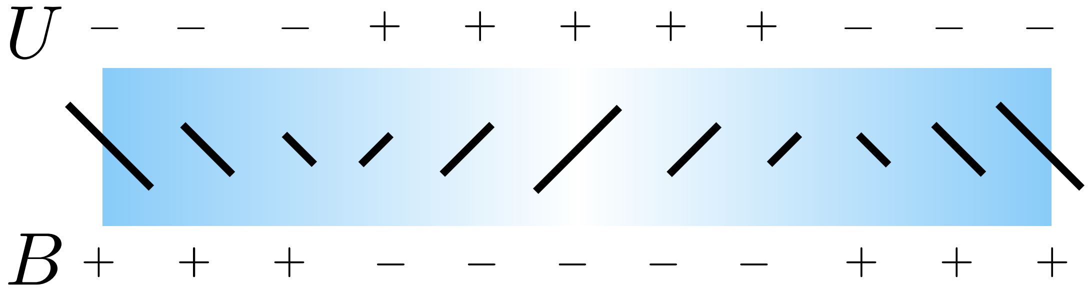
If you have only the E-mode…

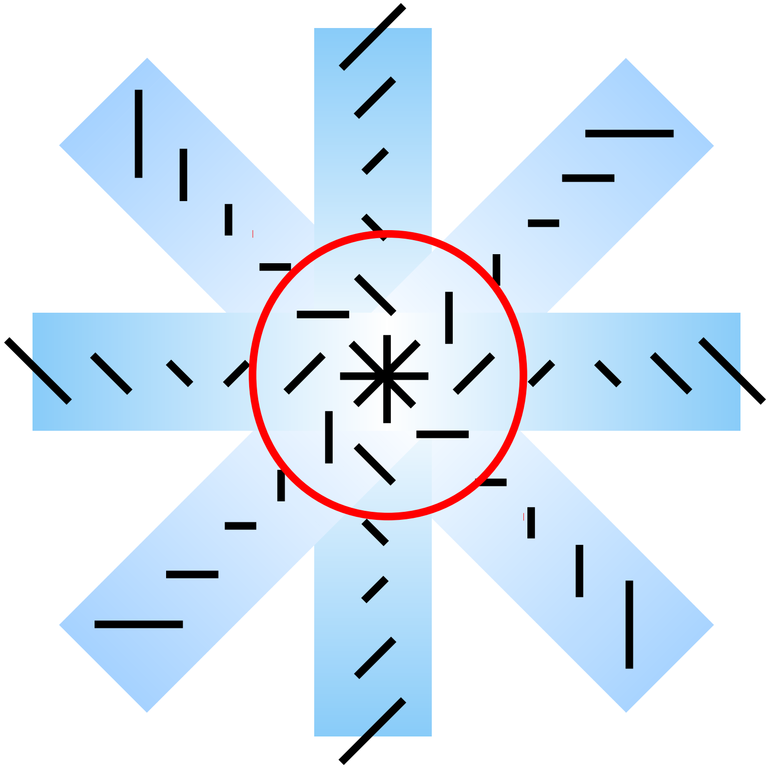
- E is $-Q$ in the coordinate system such that $\boldsymbol{\ell} \parallel \boldsymbol{e}_1$

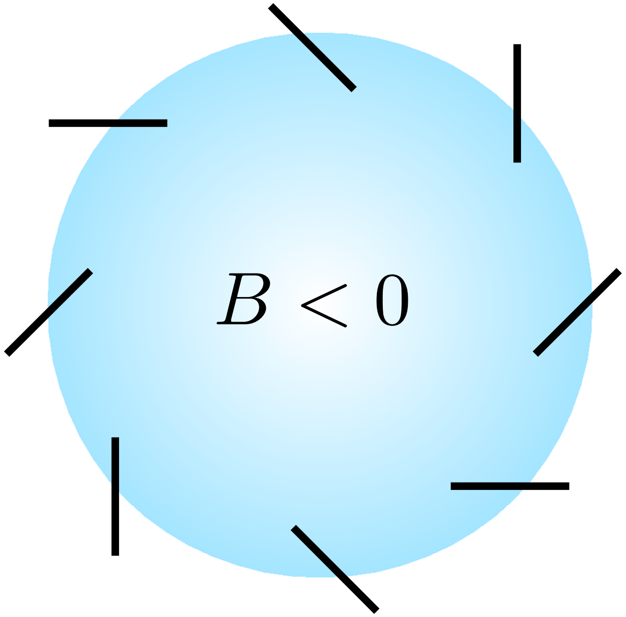


If you have only the B-mode...

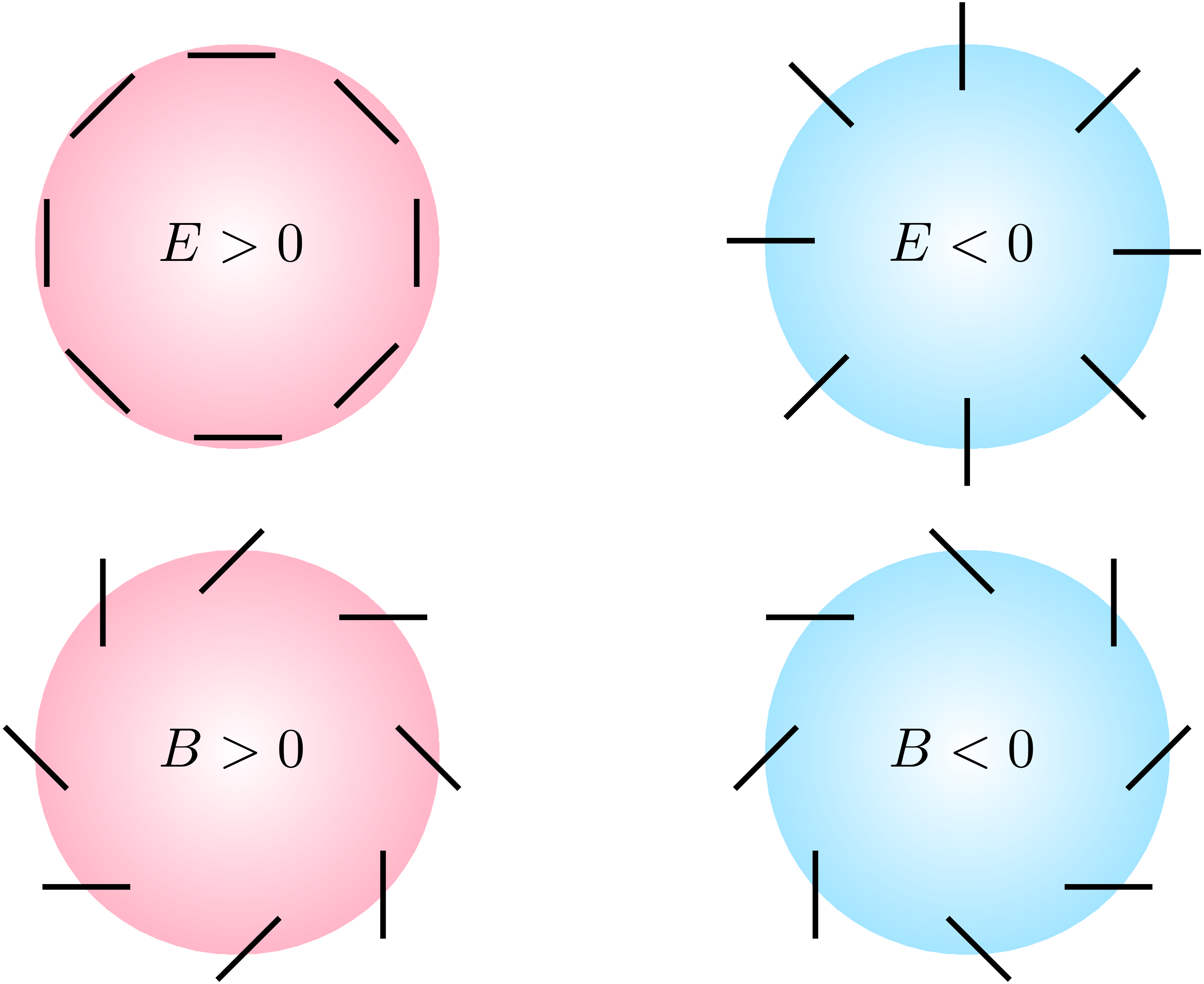
- B is $-U$ in the coordinate system such that $\boldsymbol{\ell} \parallel \boldsymbol{e}_1$





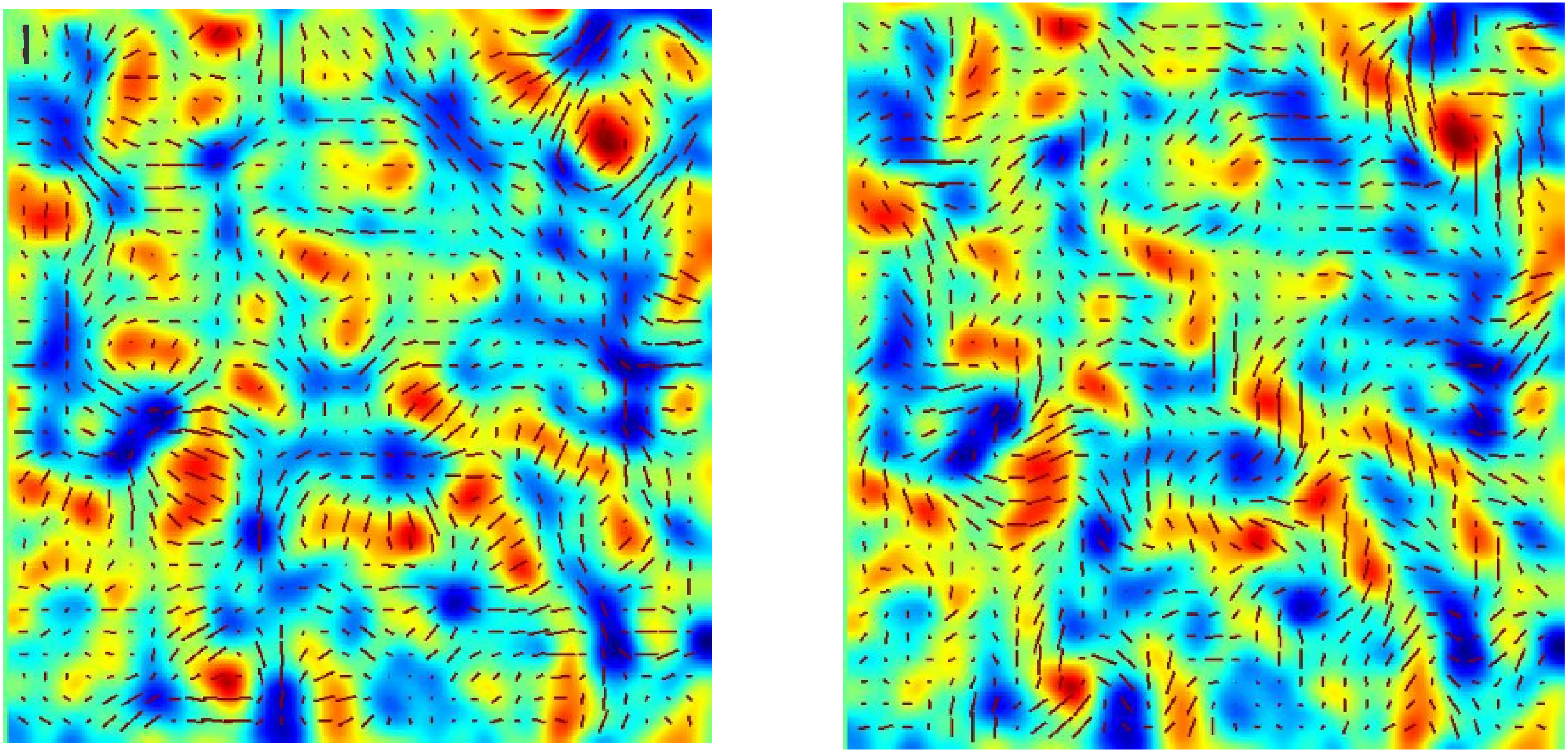


superposition of the modes



Note: sign convention matched to R. Durrer's text book

Get it? (Seljak&Zaldarriaga,'98)



Q：which is the E-mode and which is the B-mode?

If right pattern (B-mode) is observed, it suggests the existence of primordial GWs (Seljak&Zaldarriaga, PRL, '97)