Tensoron:

spectator scalar field inducing gravitational waves during inflation

Still in progress

Tomohiro Fujita from Kavli IPMU/Tokyo Univ.
with J. Yokoyama(RESCEU) & S. Yokoyama(ICRR)

Introduction & Motivation

INTRODUCTION

BICEP2 has reported

Tensor to scalar ratio: $r \equiv \mathcal{P}_h / \mathcal{P}_\zeta \approx 0.16$

If the observed B-mode polarizations are from inflation, it can be translated into

Inflation Hubble prameter: $H_{\inf} \approx 10^{14}\ \mathrm{GeV} \sqrt{r/0.1}$

But, How robust is this relation？

ANOTHER POSSIBILITY

The production of GWs with $\mathcal{P}_h = \frac{8}{M_{Pl}^2}\left(\frac{H}{2\pi}\right)^2$

is the essential prediction of inflation.

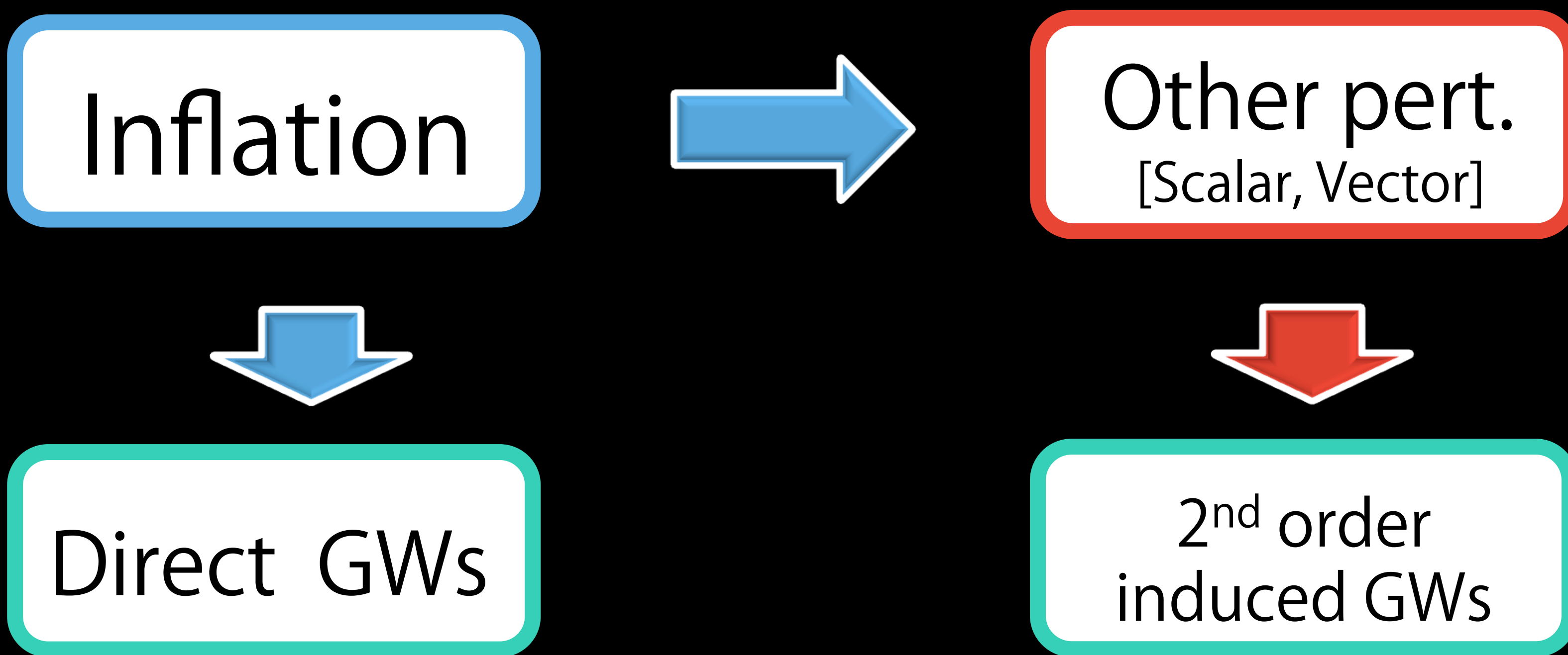
Nevertheless, in principle, it is a lower bound.

Are there any other GW productions？

⇒ If GWs produced by another mechanism are dominant, the estimate of $H_{\inf}$ must be revised.

INDUCED GW

GWs can be induced by 2nd order pert.



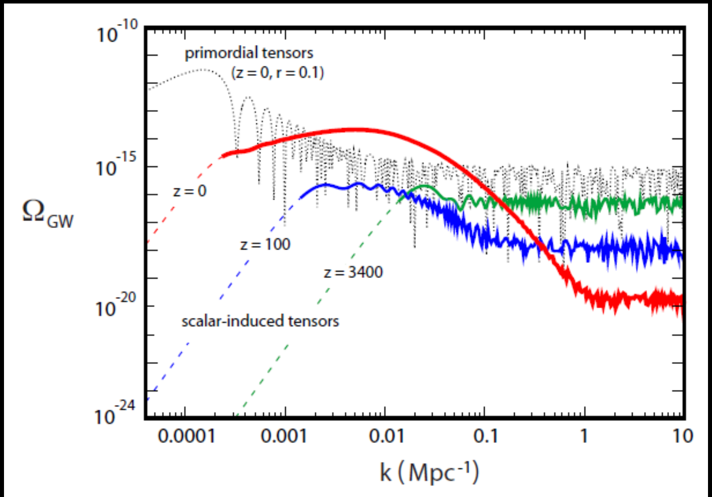
Previous Works

PREVIOUS WORKS

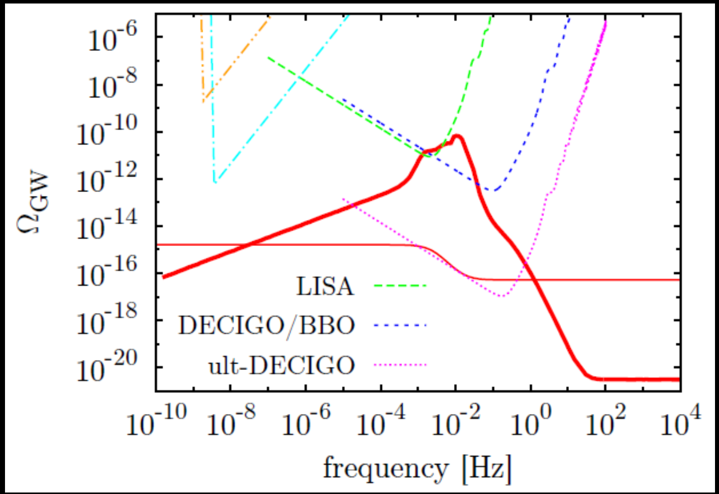
Scalar pert. can induce GWs when it re-enters the horizon.

[Ananda+(2007), Baumann+(2007), Bartolo+(2007), Saito+(2010), Kawasaki+(2013)]

→ Dominant on large scale only for $r<10^{-6}$



[Baumann+(2007)]



[Kawasaki+(2013)]

DURING INFLATION

Scalar pert. also induce GW when it exits the horizon.

→ Large scale GWs (B-mode observable)

Mechanism:

Scalar pert. $\sigma_{\boldsymbol{p}}$ provides 2nd order source for GWs;

EoM: $$h_{ij}'' + 2\mathcal{H} h_{ij}' - \nabla^2 h_{ij} = -4 M_{Pl}^{-2} T_{ij}^{lm} S_{lm}$$

Source: $$S_{lm} = \int \frac{d^3 p\, d^3 q}{(2\pi)^3} \delta(\boldsymbol{p}+\boldsymbol{q}-\boldsymbol{k})\, c_s^2\, p_i q_j\, \sigma_{\boldsymbol{p}} \sigma_{\boldsymbol{q}}$$

where T_{ij}^{lm} is the T.T. projection tensor.

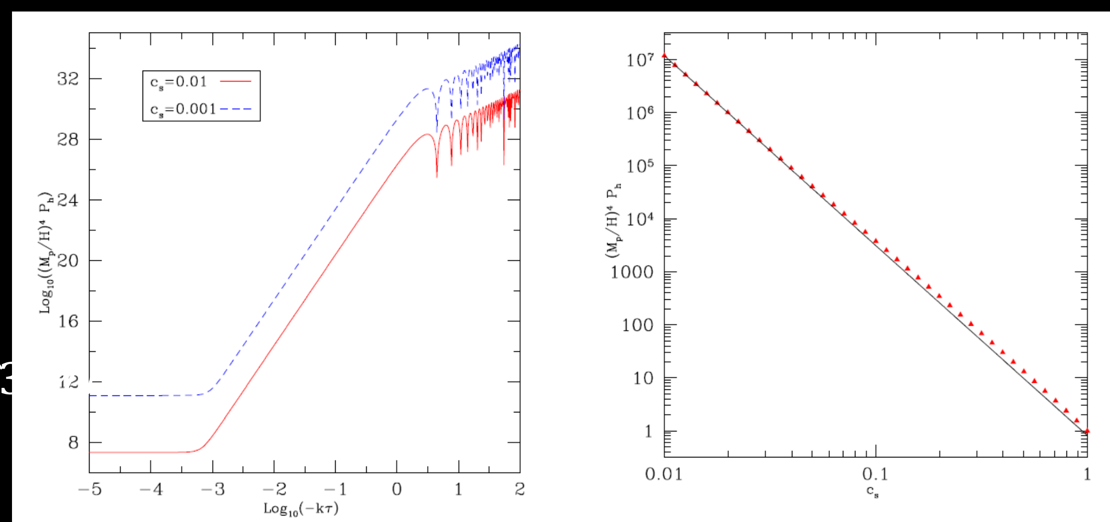
BIAGETTI'S PAPER

"**Enhancing Inflationary Tensor Modes through Spectator Fields**"
Biagetti, Fasiello & Riotto. PRD**88**, 103518 (2013)

- Numerically calculate induced $\mathcal{P}_h$.
- Obtain analytic eq. by fitting the numerical result.
- 4p letter. No UV cutoff or constraints on c_s are mentioned.

$$\mathcal{P}_h = c\frac{H^4}{c_s^{18/5} M_{\rm p}^4},$$

$\mathcal{P}_h^{\rm Our} \simeq 32/15\pi\, H^4 / c_s^3$



SMALL SOUND SPEED

To enhance scalar pert., small c_s is introduced. [Biagetti+(2013)]

$$S_\sigma = \int \mathrm{d}^3x d\eta \sqrt{-g}\left[\frac{1}{2a^2}(\sigma'^2 - c_s^2\partial_i\sigma\partial_i\sigma) - V(\sigma)\right]$$

small c_s has 2 effects. ➡ Power Spec. $\mathcal{P}_\sigma = (\mathrm{H}/2\pi)^2 \times c_s^{-3}$

$$\sigma_{\boldsymbol{p}} = \frac{1}{\sqrt{2c_s p}}\left(1 - \frac{iaH}{c_s p}\right)e^{-ic_s p\eta}$$

Increase BD amplitude

Hasten the horizon crossing

To eliminate vacuum contribution, we approximate

$$\sigma_{\boldsymbol{p}} = \frac{H_{\inf}}{\sqrt{2}\,(c_s p)^{3/2}}\,\theta(1 + c_s p\eta)\,[a_{\boldsymbol{p}} + a_{-\boldsymbol{p}}^\dagger]$$

[Perhaps better treatment exists. Comments are welcome.]

Calculation

EOM

$\sigma \times \sigma$ induces h

$$h_{ij}'' + 2\mathcal{H} h_{ij}' - \nabla^2 h_{ij}$$

$$= -4 M_{Pl}^{-2} T_{ij}{}^{lm} \int \frac{d^3 p \, d^3 q}{(2\pi)^3} \delta(\boldsymbol{p}+\boldsymbol{q}-\boldsymbol{k}) c_s^2 p_i q_j \sigma_{\boldsymbol{p}} \sigma_{\boldsymbol{q}}$$

Solve it and substitute it into power spec. eq.

$$\langle h_{\boldsymbol{k}}^{\pm}(\eta) h_{\boldsymbol{k}'}^{\pm}(\eta) \rangle = \frac{2\pi^2}{k^3} \delta(\boldsymbol{k}+\boldsymbol{k}') \mathcal{P}_h^{\pm}(k,\eta)$$

4-P FUNCTION

Solving the EoM with Green's function, we obtain

$$\frac{2\pi^2}{k^3}\delta(\boldsymbol{k}+\boldsymbol{k'})\mathcal{P}_h^{\pm}(k,\eta) =$$
$$16\frac{c_s^4}{M_{\rm Pl}^4}\int\frac{{\rm d}^3p{\rm d}^3q{\rm d}^3p'{\rm d}^3q'}{(2\pi)^6}\delta(\boldsymbol{p}+\boldsymbol{q}-\boldsymbol{k})\delta(\boldsymbol{p'}+\boldsymbol{q'}-\boldsymbol{k'})e_{ij}^{\pm}(\boldsymbol{k})e_{ml}^{\pm}(\boldsymbol{k'})p_iq_jp'_mq'_l$$
$$\times\int_{-\infty}^{\infty}{\rm d}\tau{\rm d}\tau' g_k(\eta,\tau)g_{k'}(\eta,\tau')\left\langle\sigma_{\boldsymbol{p}}(\tau)\sigma_{\boldsymbol{q}}(\tau)\sigma_{\boldsymbol{p'}}(\tau')\sigma_{\boldsymbol{q'}}(\tau')\right\rangle.$$

and it reads

$$\mathcal{P}_h^{\pm}(\eta,k) = \pm\frac{4}{\pi^2}c_s^{-2}\frac{H_{\rm inf}^4}{M_{\rm Pl}^4}k^3\int{\rm d}^3p{\rm d}^3p'\delta(\boldsymbol{p}-\boldsymbol{p'}-\boldsymbol{k})e_{ij}^{\pm}(\boldsymbol{k})e_{ml}^{\pm}(\boldsymbol{k})\frac{p_ip'_jp'_mp_l}{(pp')^3}$$
$$\times\left[\int_{-\infty}^{\infty}{\rm d}\tau g_k(\eta,\tau)\theta(\gamma+c_sp\tau)\theta(\gamma+c_sp'\tau)\right]^2$$

with

$$g_k(\eta,\tau) = \frac{\theta(\eta-\tau)}{k^3\tau^2}\Re e\left[e^{ik(\eta-\tau)}(1-ik\eta)(-i+k\tau)\right]$$

RESULT

The main contribution comes from $c_s p \sim k$, namely same wave-number as the GW's.

The induced GW power spectrum is

M_{Pl}^4

If it is dominant, the tensor-to-scalar ratio is

$3\ (H_{\inf} /$

Thus for $c_s \lesssim 10^{-3}$, $H_{\inf}$ can be lower than 10^{14} GeV.

Constraints

CONSTRAINT-1

c_s cannot be arbitrarily small:

1. Curvature Pert. (curvaton)

$$\mathcal{P}_\zeta^{(\sigma)} \simeq \frac{c_s^{-3}\sigma_*^2 H_{\rm inf}^2}{576\pi^2 M_{\rm Pl}^4}\frac{\Gamma_\phi}{\Gamma_\sigma} \ll \mathcal{P}_\zeta^{obs} \approx 2.2\times 10^{-9}$$

$$\Longrightarrow c_s > 0.01\left(\frac{\sigma_*}{0.1M_{\rm Pl}}\right)^{\frac{2}{3}}\left(\frac{H_{\rm inf}}{10^{14}{\rm GeV}}\right)^{\frac{2}{3}}\left(\frac{\Gamma_\phi}{\Gamma_\sigma}\right)^{\frac{1}{3}}.$$

2. Non-gaussianity

$$f_{\rm NL} \simeq \frac{5}{3\hat{r}}\left(\frac{\mathcal{P}_\zeta^{(\sigma)}}{\mathcal{P}_\zeta^{obs}}\right)^2 \simeq \frac{5c_s^{-6}(\mathcal{P}_\zeta^{obs})^{-2}}{165888\pi^4}\frac{\sigma_*^2 H_{\rm inf}^4}{M_{\rm Pl}^6}\left(\frac{\Gamma_\phi}{\Gamma_\sigma}\right)^{\frac{3}{2}} < 14.3,$$

$$\Longrightarrow c_s = 0.024\left(\frac{f_{\rm NL}}{10}\right)^{-\frac{1}{6}}\left(\frac{\sigma_*}{0.1M_{\rm Pl}}\right)^{\frac{1}{3}}\left(\frac{H_{\rm inf}}{10^{14}{\rm GeV}}\right)^{\frac{2}{3}}\left(\frac{\Gamma_\phi}{\Gamma_\sigma}\right)^{\frac{1}{4}}.$$

➡ They require small σ_*

CONSTRAINT-2

3. Validity of Perturbation

$\delta\sigma \ll \sigma_*$ requires large σ_* ➡ $\langle \delta\sigma \delta\sigma \rangle \ll \sigma_*^2$

$$N_{\rm obs}\mathcal{P}_{\delta\sigma}(\eta,\cancel{k}) = N_{\rm obs}\frac{H_{\rm inf}^2}{(2\pi)^2 c_s^3} < \sigma_*^2\,,$$

$$\Longrightarrow c_s > \left(\frac{N_{\rm obs}H_{\rm inf}^2}{4\pi^2\sigma_*^2}\right)^{\frac{1}{3}} \approx \left(\frac{N_{\rm obs}}{50}\right)^{\frac{1}{3}}\left(\frac{H_{\rm inf}}{\sigma_*}\right)^{\frac{2}{3}}$$

Since we derive the lower bound on c_s,
we can find the maximum $\mathcal{P}_h$, or

$$R \equiv \mathcal{P}_h^{\sigma} / \mathcal{P}_h^{\rm inf}$$

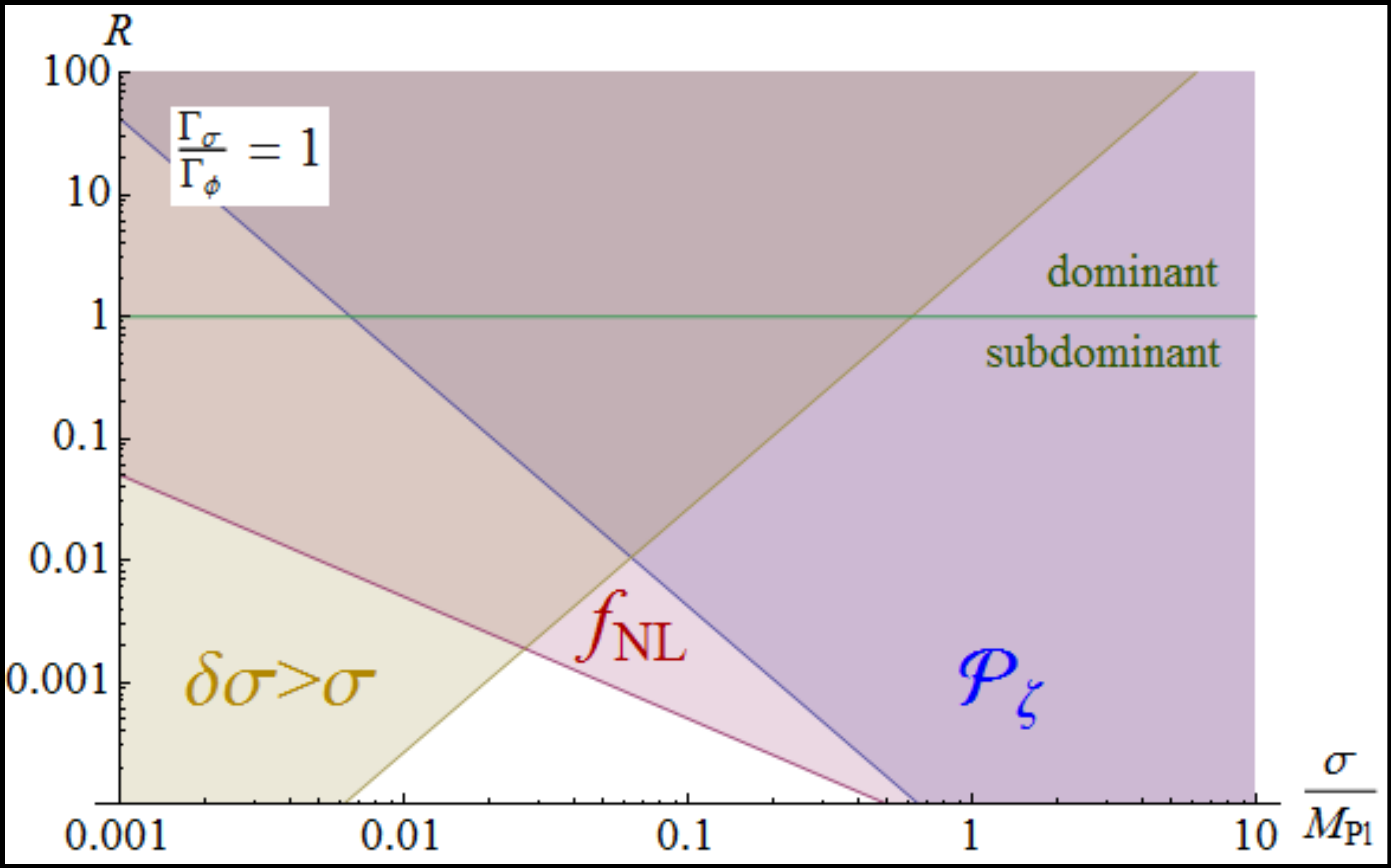
CONSTRAINT-3

Constraint on R

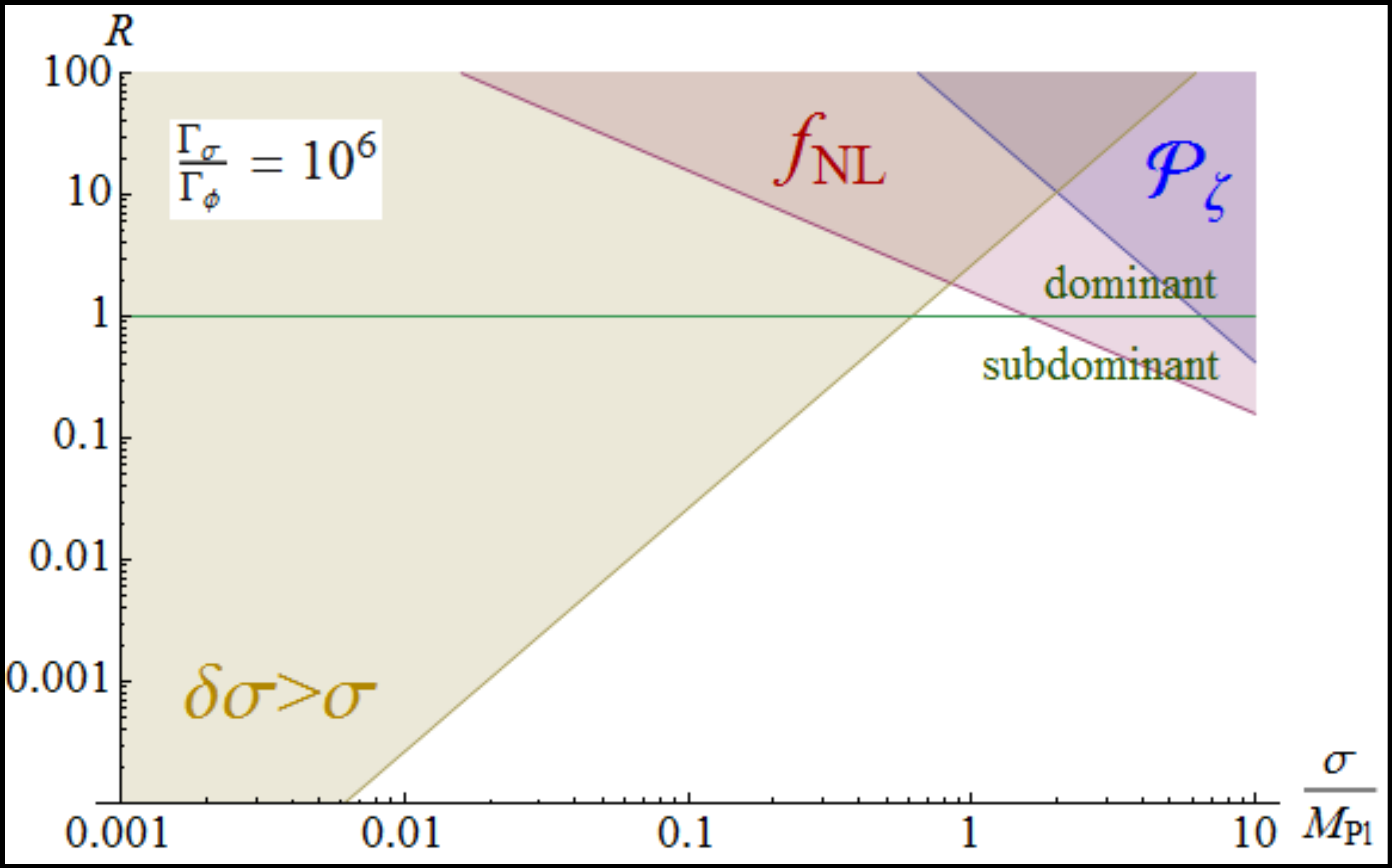
1. $\mathcal{P}_{\zeta}(\sigma)$: $$R \lesssim 4\times 10^{-5} \frac{M_{Pl}^2}{\sigma_*^2} \frac{\Gamma_\sigma}{\Gamma_\phi}$$
2. $f_{\rm NL}$: $$R \simeq 1\times 10^{-5} f_{NL}^{-1} \frac{M_{Pl}}{\sigma_*} \left(\frac{\Gamma_\sigma}{\Gamma_\phi}\right)^{3/4}$$
3. $\delta\sigma/\sigma_*$: $$R \lesssim 2.6 \frac{\sigma_*^2}{M_{Pl}^2}$$

$R^{\rm max}$ is independent of $H_{\rm inf}$

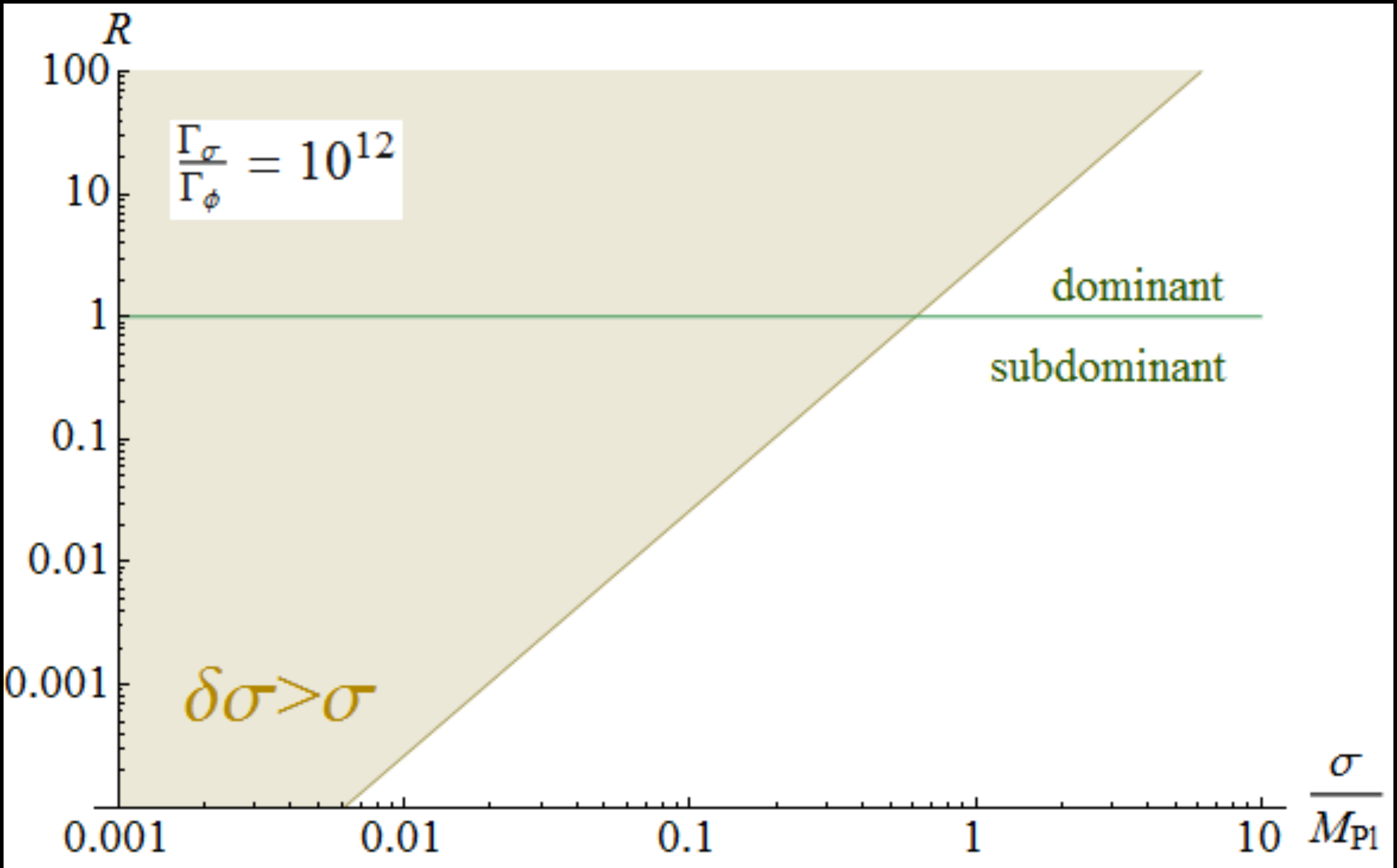
CONSTRAINT-3



CONSTRAINT-3



CONSTRAINT-3



RESULT-2

We can maximize Tensoron's GWs by

$$\sigma_* \sim M_{Pl}, \qquad \Gamma_\sigma > 10^6\,\Gamma_\phi$$

However, it cannot be dominant

$$R \lesssim 3$$

Thus $\rho_{\inf}$ doesn't significantly change.

Discussions

K-ESSENCE

[Armendariz-Picon+(1999),Appignani+(2012)]

K-essence model:

$$\mathcal{L} = P(X,\phi)\ , \qquad 2\,X = \nabla^{\alpha}\phi\,\nabla_{\alpha}\phi$$

$$T_{\mu\nu} = P_{X}\nabla_{\mu}\phi\,\nabla_{\nu}\phi - g_{\mu\nu}\,P$$

$$c^2_{\delta\phi} = \frac{P^{(0)}_{X}}{P^{(0)}_{X} + P^{(0)}_{XX}\,\dot{\phi}_0^2}$$

Modified EoM of $\delta\phi$:

$$\left(P^{(0)}_{X} + P^{(0)}_{XX}\,\dot{\phi}_0^2\right)\delta\ddot{\phi} - \frac{P^{(0)}_{X}}{a^2}\,\delta\phi_{,ii} + C\,\delta\dot{\phi} + D\,\delta\phi = 0$$

Perturbed EMT:

$$\delta^{(2)}T_{ij} = P^{(0)}_{X}\left(1 - \frac{\delta_{ij}}{2}\right)\delta\phi_{,i}\,\delta\phi_{,j}$$ +(Non-T.T. terms)

Not c_s^2 !!

New parameter naturally appears.
Constraint may be relaxed.

LARGE $\delta\sigma$

Is $\delta\sigma \ll \sigma_*$ mandatory??

➡ No! But $\delta\sigma \gg \sigma_*$ looks a bit unnatural.

$\delta\sigma_p$ produced at $N > 50$ should be absorbed in σ_* .

If $\mathcal{P}_\sigma$ is scale invariant, at least $\delta\sigma_p \sim \sigma_*$.

However, if $c_s \neq 0$

$\mathcal{P}_\sigma$ is not scale invariant. ➡ $\delta\sigma \gg \sigma_*$ realized

The constraint from f_{NL} should be reconsidered.

Summary

- We study Tensoron σ with small c_s and obtain analytic solution.

- For $\sigma_* \sim M_{Pl}$, $\Gamma_\sigma > 10^6 \Gamma_\phi$, induced GWs are comparable to intrinsic GWs but cannot be far bigger within $\delta\sigma < \sigma_*$.

- If a more generalized theory is considered, Induced GWs may be dominant.

Backup slides

INTERACTION TERM

Original Action:

$$\frac{1}{2}\sqrt{-g}\; g^{\mu\nu}\,\partial_\mu \phi\,\partial_\nu \phi$$

Perturb it w.r.t. graviton $h_{\mu\nu}$

$$g_{\mu\nu}=a^2\,(\eta_{\mu\nu}+h_{\mu\nu}), \qquad g^{\mu\nu}=a^{-2}\,(\eta^{\mu\nu}-h^{\mu\nu}+O(h^2))$$

Substituting them, we rewrite the action as

$$\frac{1}{2}\,a^2\,[\eta^{\mu\nu}+h^{\mu\nu}+O(h^2)]\partial_\mu \phi\,\partial_\nu \phi$$

QUICK REVIEW

Let us briefly review the derivation of inflationary GWs.

- Introduce the metric pert. $ds^2 = a^2(\eta)\left[d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j\right]$
- Adopt the T.T. gauge $h_{ii} = 0$ (Traceless), $\partial_i h_{ij} = 0$ (Transverse)

At 2nd order pert., Einstein-Hilbert action is reduced to

$$S_h = \frac{M_{Pl}^2}{8}\int d\eta\, d^3x\; a^2(\eta)\left[h_{ij}'^2 - (\partial_l h_{ij})^2\right].$$

h_{ij} is decomposed into mode func. and C/A operator in Fourier space,

$$\frac{M_{Pl}}{2} \times a h_{ij}(\mathbf{k},\eta) = \sum_{\lambda=+,\times}\left[a_{\mathbf{k}}^{\lambda} v_k^*(\eta)\, e_{ij}^{\lambda}(\mathbf{k}) + a_{-\mathbf{k}}^{\lambda\dagger} v\right.$$

where $e_{ij}^{\lambda}(\mathbf{k})$ is the polarization basis tensor. Thus the power spectrum of each GW polarization is $\times \frac{M_{Pl}}{2}$ of the scalar PS, $\mathcal{P}_\phi = (H/2\pi)^2$:

$\mathcal{P}_h = \frac{8}{M_{Pl}^2}\left(\frac{H}{2\pi}\right)^2$ & $\mathcal{P}_\zeta^{obs} \approx 2.2 \times 10^{-9}$ → $H_{inf} \approx 10^{14}\,\text{GeV}\sqrt{r/0.1}$