LECTURE 3: SOME HOT TOPICS

- PBHs and separate universe condition
  (with Harada)
- PBH constraints from Galactic gamma-ray background
  (with Kohri, Sendounda and Yokoyama)
- Higher dimensional black holes
  (with Giddings)
- Black holes and the Generalized Uncertainty Principle
  (with Modesto and Premont-Scwarz)

SEPARATE UNIVERSE PROBLEM

"Black Holes in the Early Universe"
Carr & Hawking, MNRAS,168,39 (1974)
Overdense regions in our universe cannot extend too far

"Separate Universes Do Not Constrain Primordial Black Holes"
Kopp, Hofmann & Weller, PRD 83,124025 (2011)
This condition does not pose any constraint on density fluctuations.

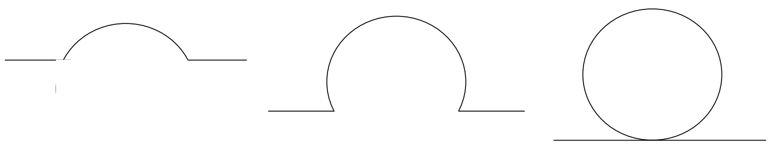
"The separate universe problem: 40 years on"
Carr & Harada (2014)
This is true but argument still gives the maximum scale for PBH

ORIGINAL ARGUMENT (Carr & Hawking 1974)

Overdense region is part of closed FRW model in flat FRW bgd

Curvature of overdense region at max' expansion $\sim G\rho_m / c^2$

=> separate universe scale $R_{SU} \sim c(G\rho_m)^{-1/2} \sim ct \sim R_{PH}$



Collapse to black hole if max radius exceeds Jeans length

$$R_m > R_J \sim \sqrt{k} R_H \qquad \text{(k=w)}$$

where equation of state is $p = k\rho c^2$ (0<k<1)
=> fine-tuning to get primordial black hole

CONSTRAINT ON INITIAL DENSITY PERTURBATION

Consider overdense region with initial size $R_o >> R_{Ho}$

Overdensity $\delta \equiv \frac{\rho - \rho_b}{\rho_b}$ evolves as $\delta = At^{\frac{2(1+3k)}{3(1+k)}}$

Time of max' expansion, region's size and horizon size then are

$$t_m \sim t_o \delta_o^{-\frac{3(1+k)}{2(1+3k)}}, \quad R_m \sim R_o \delta_o^{-\frac{1}{1+3k}}, \quad R_H(t_m) \sim R_{Ho} \delta_o^{-\frac{3(1+k)}{2(1+3k)}}$$

Avoid separate universe for $R_m < R_H(t_m)$

=> $\delta_o < (R/R_{Ho})^{-2} \sim (M/M_{Ho})^{-2/3}$

=> PBH formation for $(M/M_{Ho})^{-2/3} > \delta_0 > k(M/M_{Ho})^{-2/3}$

In terms of overdensity δ_H when region falls inside horizon at t_H

$$t_m \sim t_H \delta_H^{-\frac{3(1+k)}{2(1+3k)}}, \quad R_m \sim R(t_H) \delta_H^{-\frac{1}{1+3k}}, \quad R_H(t_m) \sim R(t_H) \delta_H^{-\frac{3(1+k)}{2(1+3k)}}$$

PBH formation requires $1 > \delta_H > k$ => fine-tuning

PROBLEMS WITH HEURISTIC ANALYSIS

Every quantity in the heuristic analysis is ambiguous!

What is R?

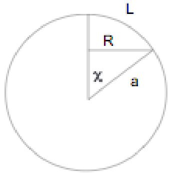
$$ds^2 = -c^2dt^2 + a^2(t)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)]$$

=> proper radius $L = a\chi$

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

=> areal radius $R \equiv ar = (A/4\pi)^{1/2} = a\sin\chi$

$L \to \pi a$ but $R \to 0$ as $\chi \to \pi$



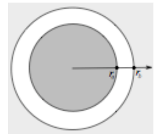
What is M?

Misner-Sharp mass is $M = \frac{4\pi\rho R^3}{3}$

$M \to 0$ and $\frac{2GM}{c^2R} \to 0$ as $\chi \to \pi$ (at any t)

=> separate universe constraint on $\delta_o(M)$ makes no sense!

What is δ?

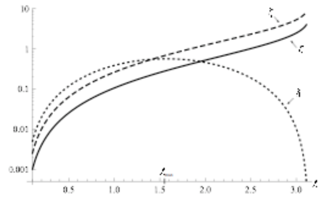


Expect overdense region to be surrounded by compensating void: K=1 FRW model with $0 < \chi < \chi_a$ replaces FRW background with $r < r_b$.

It is more natural to describe region by *curvature* perturbation. Kopp et al. use *central* volume fluctuation ζ, *average* volume fluctuation <ζ>, overdensity when regions enters horizon δ_H

$$\zeta \approx -2\ln\cos\frac{\chi_a}{2} \qquad \bar{\zeta} \approx \frac{1}{3}\ln\frac{2(\chi_a - \sin\chi_a\cos\chi_a)}{2\sin^3\chi_a} \qquad \delta_H = \frac{1}{16}\sin^2\chi_a(8+\sin^2\chi_a)$$

$\bar{\zeta}, \zeta \to \infty$ and $\delta_H \to 0$ as $\chi_a \to \pi$



What is R_H?

Background particle horizon

$$R_{PH}(t) = a(t)\int_0^t \frac{cdt}{a(t)} = \frac{3(1+k)}{1+3k}ct \quad (k > -1/3)$$

Background apparent (Hubble) horizon

$$R_H(t) = \frac{c}{H} = \frac{3(1+k)}{2}ct \quad (k > -1)$$

Local or background horizon scales?

MORE PRECISE ANALYSIS (-1/3 < k <1)

$$t_m \sim t_H\delta_H^{-\frac{3(1+k)}{2(1+3k)}}, \quad R_m \sim R(t_H)\delta_H^{-\frac{1}{1-3k}}, \quad R_H(t_m) \sim R(t_H)\delta_H^{-\frac{3(1+k)}{2(1+3k)}}$$

become

overdensity at t_m

$$t_m = t_H\Delta_m^{1/2}\left(\frac{\delta_H}{1+\delta_H}\right)^{-\frac{3(1+k)}{2(1-3k)}}, \quad L_m = R_H\left(\frac{\delta_H}{1+\delta_H}\right)^{-\frac{1}{1+3k}}, \quad R_H(t_m) = R_H\Delta_m^{1/2}\left(\frac{\delta_H}{1+\delta_H}\right)^{-\frac{3(1+k)}{2(1+3k)}}$$

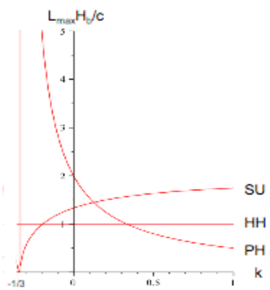
Still have separate universe *length* (but not *mass*) scale

$$c^{-1}H_{bm}L_{max} = \pi\Delta_m^{-1/2} = 2\sqrt{\frac{\pi}{\alpha}}\frac{(1+3k)}{3(1+k)}\frac{\Gamma\left(\frac{2+3k}{1+3k}\right)}{\Gamma\left(\frac{3(1+k)}{2(1+3k)}\right)}$$

$L_{max} > R_{PH}$ for k>0.12 $\quad$ $L_{max} > R_{AH}$ for k>-0.2

$L_{max}H_b = 2$ for infinite k $\quad$ $L_{max}H_b = 0$ for k=-1/3

Maximum scale of PBH is half L_{max}

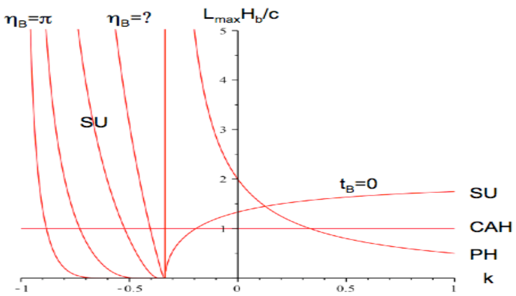


ANALYSIS FOR -1 < k < -1/3 CASE

$$c^{-1} H_b L_{\max} = 2\sqrt{\pi}\,\frac{(1+3k)}{3(1+k)}\,\frac{\Gamma\left(\frac{2+3k}{1+3k}\right)}{\Gamma\left(\frac{3(1+k)}{2(1+3k)}\right)}\left[\frac{A_k(\eta_B)}{A_k(\pi)} - 1\right]^{-1}$$

$$A_k(\eta) \equiv (1-\cos\eta)^{3(1+k)/2(1+3k)}\,F\left(\frac{1}{2}, \frac{3(1+k)}{2(1+3k)}; \frac{5+9k}{2(1+3k)}; \frac{1-\cos\eta}{2}\right)$$

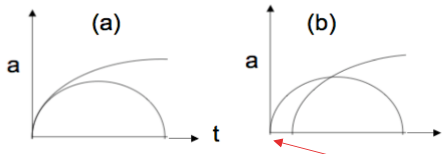
F is hypergeom' fn
η_B is big bang time



But physical interpretation is different in this case

USUAL -1/3 < k < 1

Positive-curvature region has max' expansion and recollapses

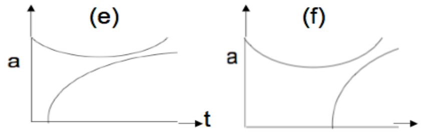


=> primordial black hole

perturbed big bang time

EXOTIC CASE -1 < k < -1/3

Positive-curvature region collapses, bounces and expands



=> baby universe

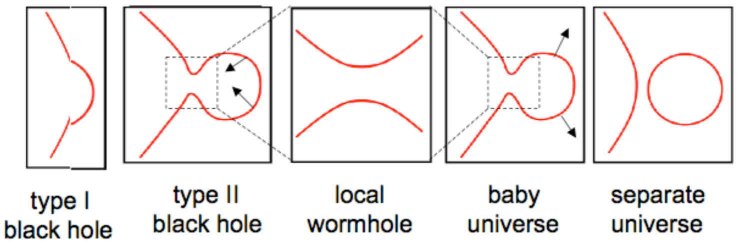
Both cases involve separate universe condition

CONCLUSIONS

* Separate universe condition can be applied but must be interpreted carefully (length not mass scale).

* Can extend analysis to -1 < k < 1 but interpretation for -1 < k < -1/3 is different: baby universe instead of black hole.

* Links concepts of black hole, baby universe, wormhole

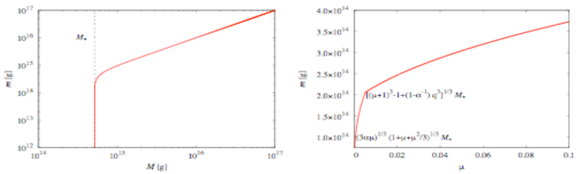


CONSTRAINTS ON PBHS FROM GALACTIC GAMMA-RAY BACKGROUND

Carr, Kohri, Sendouda & Yokoyama (2014)

- Must distinguish between initial mass M and current mass m

$$M = M_*(1+\mu) \qquad m = \begin{cases} \left[(\mu+1)^3 - 1 + (1-\alpha^{-1})\,q^3\right]^{1/3} M_* & (\mu \geq \mu_C) \\ (3\,\alpha\,\mu)^{1/3}\,(1+\mu+\mu^2/3)^{1/3}\,M_* & (0 \leq \mu \leq \mu_C) \end{cases} \qquad \mu_C \approx q^3/(3\alpha) = 0.005$$

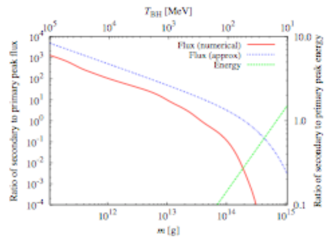


- Must distinguish between primary and secondary emission

$$E^{\rm peak}_{\rm sec}/E^{\rm peak}_{\rm prim} = (100\ {\rm MeV})/(600\ M_{14}^{-1}{\rm MeV}) \approx 0.85(M/M_*)$$

$$\frac{{\rm d}\dot{N}^{\rm pri}_\gamma}{{\rm d}E_\gamma}(E_\gamma = \bar{E}^{\rm pri}_\gamma) \approx 1.6 \times 10^{-3}$$

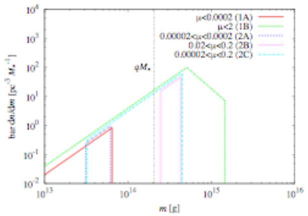
$$\left(\frac{{\rm d}\dot{N}^{\rm sec}_\gamma}{{\rm d}E_\gamma}\right)\Bigg/\left(\frac{{\rm d}\dot{N}^{\rm pri}_\gamma}{{\rm d}E_\gamma}\right) \sim \left(\frac{m}{M_*}\right)^{-1} \quad (m \lesssim M_q)$$



• Must distinguish between initial and current mass function

$$\frac{d\tilde{n}}{dM}(M) = \frac{d\tilde{n}}{dM}\bigg|_{M_*}\left(\frac{M}{M_*}\right)^{-\nu} \qquad (1+\mu_{\min})M_* < M < (1+\mu_{\max})M$$

$$\frac{dn}{dm} = \left(\frac{m}{M_*}\right)^2\left[\frac{1}{1+\mu(m)}\right]^2\left(\frac{dN}{dM}\right) \approx \left(\frac{m}{M_*}\right)^2\left(\frac{dN}{dM}\right)_* \qquad (M_q \le m \ll M_*)$$

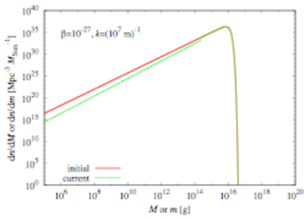


Main contribution from m^2 low mass tail

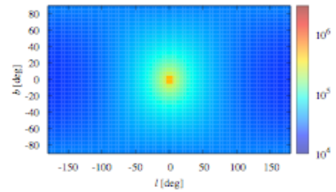
• Most natural choice of mass function associated with critical collapse

$$\frac{dn}{dm} = \beta(M_f)Am^2M(m)^{1/c-3}\exp[-BM(m)^{1/c}]$$

$$\frac{dn}{dM} = \beta(M_f)\frac{a(t_f)^3}{V_f}\frac{1-c}{c}\frac{1}{\gamma M_f}\exp\left[-(1-c)\left(\frac{M}{\gamma M_f}\right)^{1/c}\right]\left(\frac{M}{\gamma M_f}\right)^{1/c-1}$$

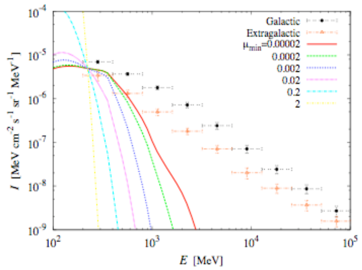


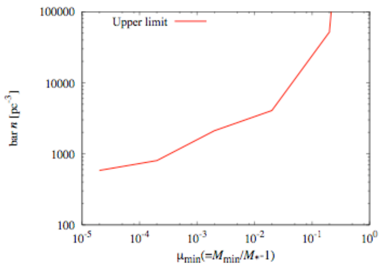
• Must specific density profile of halo and direction of observation



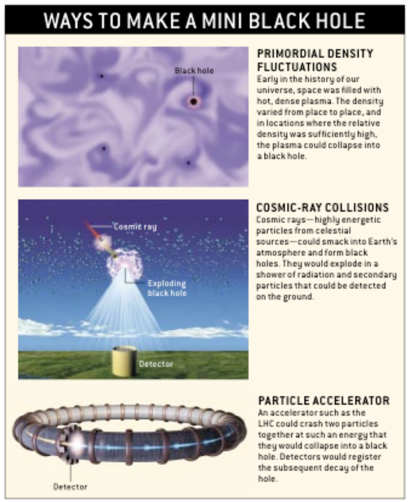
$$\rho_{PBH}(R) = \frac{f\rho_s}{(R/R_s)^\gamma\,[1+(R/R_s)^\alpha]^{(\beta-\alpha)/\alpha}}$$

• Then obtain constraints from FermiLAT observations





BLACK HOLES AS A PROBE OF HIGHER DIMENSIONS



Carr & Giddings (2009)


ブラックホールを製造する

Translated into Japanese by Tetsuya Shiromizu!

BLACK HOLES AND EXTRA DIMENSION

Higher dimensions => $M_D^{n+2} V_n \sim M_p^2$

V_n is volume of compactified or warped space

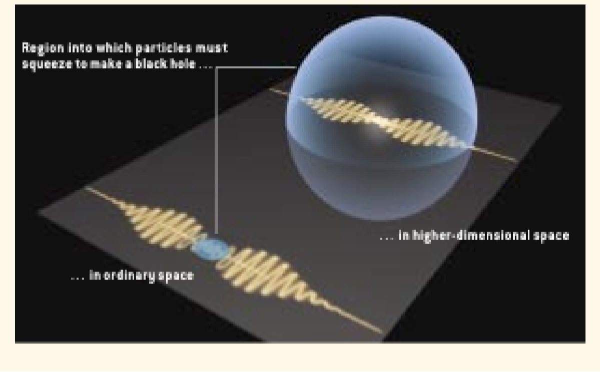
Standard model => $V_n \sim M_P^{-n}$, $M_D \sim M_p$,
Large extra dimensions => $V_n >> M_P^{-n}$, $M_D << M_p$

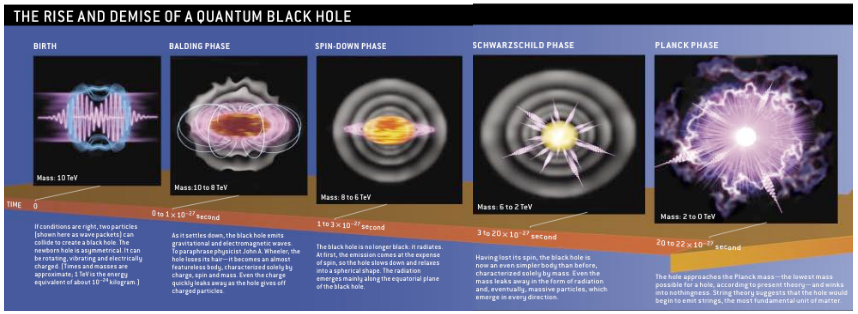
TeV quantum gravity?

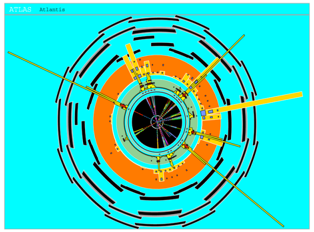
Forming black holes by collisions

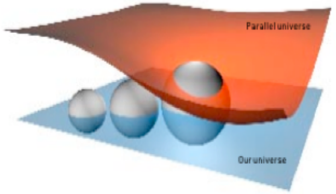
centre of mass energy

Cross-section $\sigma(ij \rightarrow BH) = \pi r_S^2 \Theta(E - M_{BH}^{min})$
Schwarzschild radius $r_S = M_P^{-1}(M_{BH}/M_P)^{1/(1+n)}$
Temperature $T_{BH} = (n+1)/r_S$ < 4D case
Lifetime $\tau_{BH} = M_P^{-1}(M_{BH}/M_P)^{(n+3)/(1+n)}$ > 4D case





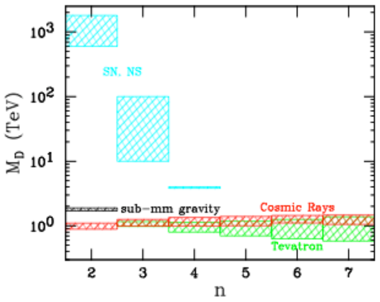




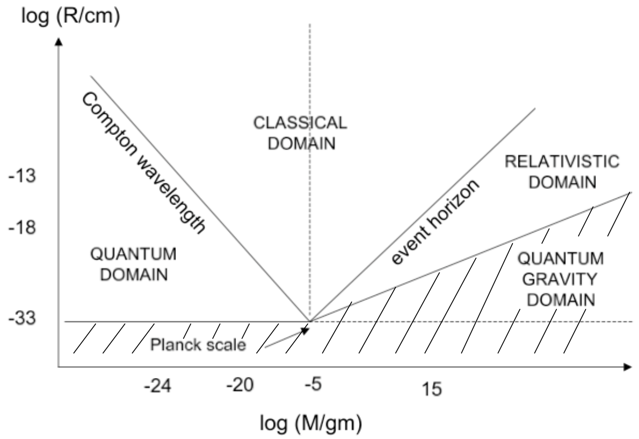
DETECTABLE AT LHC?

$$M_D \sim TeV \Rightarrow R_C \sim 10^{(32/n)-17} cm \sim$$

10^{16} cm	(n=1)	excluded
10^{-1} cm	(n=2)	
10^{-6} cm	(n=3)	
10^{-13} cm	(n=7)	



No evidence from LHC so far



BLACK HOLES AND HIGHER DIMENSIONS

Assume D=3+n dimensions for $R<R_C$

Gauss law

$$F_{grav} = \frac{G_D m_1 m_2}{R^{2+n}} \quad (R<R_C) \qquad G = \frac{G_D}{R_C{}^2}$$

$$F_{grav} = \frac{G\, m_1 m_2}{R^2} \quad (R>R_C)$$

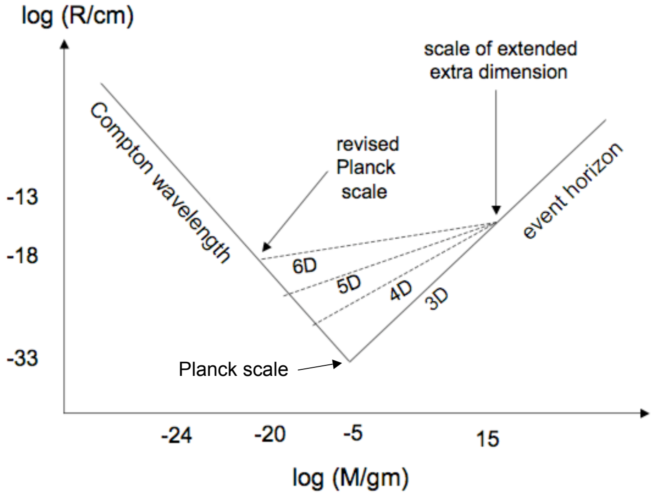
3D black hole smaller than R_C for $M < M_C = \dfrac{c^2 R_C}{2G}$

Black hole condition

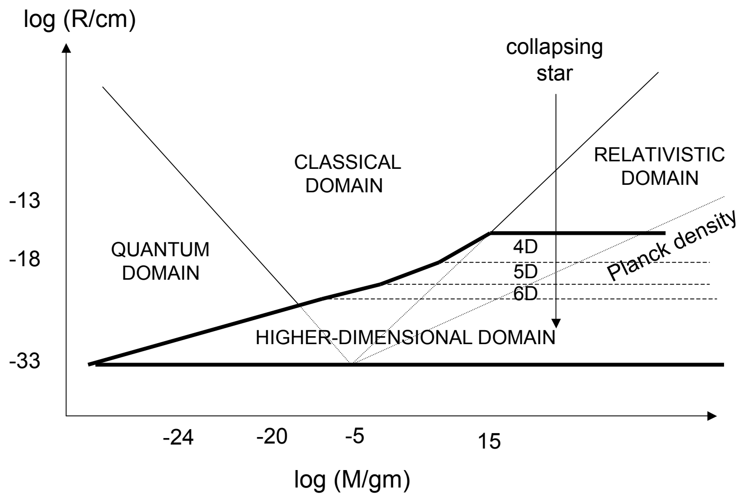
$$R < \begin{cases} R_C (M/M_C)^{1/(n+1)} & (M < M_C) \\ R_C (M/M_C) & (M > M_C) \end{cases}$$

Intersects Compton boundary at higher dimensional Planck scale

$$M_D = M_P \left(\frac{R_P}{R_C}\right)^{n/(n+2)}, \qquad R_D = R_P \left(\frac{R_C}{R_P}\right)^{n/(n+2)}$$



Evaporating black hole is higher-dimensonal if $R_C>10^{-13}$cm!



BLACK HOLES & GENERALIZED UNCERTAINTY PRINCIPLE

(with Modesto and Premont-Scwarz)

L. Modesto & I. Premont-Schwarz, Self-dual Black holes in LQG: Theory and Phenomenology, Phys. Rev. D. 80, 064041 (2009).

B. Carr, L. Modesto & I. Premont-Schwarz, Generalized Uncertainty Principle and Self-Dual Black Holes, arXiv: 1107.0708 [gr-qc] (2011).

B. Carr, Black Holes, Generalized Uncertainty Principle and Higher Dimensions, Phys. Lett. A 28, 134001 (2013).

B. Carr, L. Modesto & I. Premont-Schwarz, Loop Black Holes and the Black Hole Uncertainty Principle Correspondence (2013).

B. Carr, Black Hole Uncertainty Principle Correspondence (2014).

B. Carr, J. Mureika, P. Nicolini, Sub-Planckian black holes and Generalized Uncertainty Principle (2014)

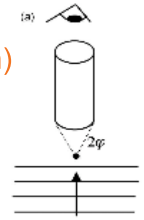
UNCERTAINTY PRINCIPLE

$h \to \hbar$

Photon of momentum p determines position to precision

$\Delta x > \lambda = h/p$ but imparts momentum $\Delta p \sim p$

$\Rightarrow \Delta x > \dfrac{h}{(2)\Delta p} \Rightarrow R_C = \dfrac{h}{Mc}$ (Compton wavelength)



Particle production for $R < R_C \Rightarrow$ QFT

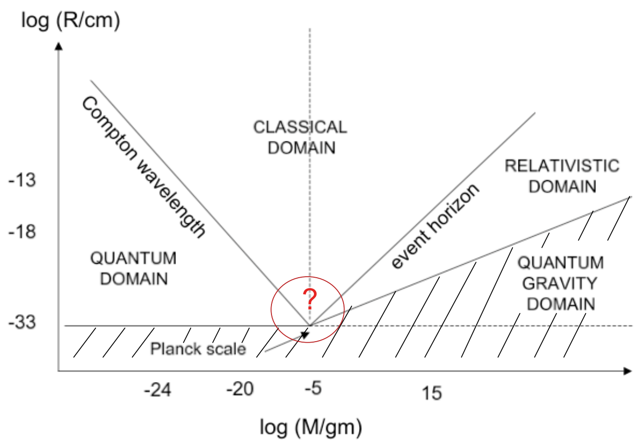
BLACK HOLE EVENT HORIZON

$R < R_S = 2GM/c^2$ (Schwarzschild radius)

Intersect at Planck scales

$$R_P = \sqrt{Gh/c^3} \sim 10^{-33} cm, \quad M_P = \sqrt{hc/G} \sim 10^{-5} g, \quad \rho_P = \frac{c^5}{h^2 G} \sim 10^{94} g cm^{-3}$$

What happens to Compton and Schwarzschild lines near M_P…



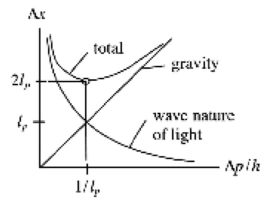
…is important feature of theory of quantum gravity.

GENERALIZED UNCERTAINTY PRINCIPLE

Newtonian heuristic argument Adler, Am. J. Phys. 78, 925 (2010)

Photon of frequency ω approaching to distance R induces
=> acceleration $a \sim G h\omega/(cR)^2$ over time $t \sim R/c$
=> uncertainty in momentum $\Delta p \sim p \sim h\omega/c$ and in position

$$\Delta x_g > at^2 \sim Gh\omega/c^4 \sim G\Delta p/c^3 \sim R_P{}^2 \Delta p/h$$



Einstein heuristic argument

Metric fluctuation $\delta g_{\mu\nu}$ on scale R is

$$\frac{\delta g_{\mu\nu}}{R^2} = \left(\frac{8\pi G}{c^4}\right)\frac{pc}{R^3} \Rightarrow \Delta x_g \sim R\delta g_{\mu\nu} \sim G\Delta p/c^3 \sim R_P{}^2 \Delta p/h$$

Both suggest $\Delta x > \dfrac{h}{\Delta p} + R_P{}^2 \dfrac{\Delta p}{h} > 2R_P$ (GUP)

GUP in string theory (Veneziano 1986, Witten 1996)

$$\Delta x > \frac{h}{\Delta p} + \alpha \frac{\Delta p}{h}$$ string tension $\alpha \sim (10R_P)^2$

Minimal length considerations (Maggiore 1993)

Link with back holes (Scardigli 1999, Calmet et al. 2004)

Modifications of commutator [x,p] (Magueio & Smolin 2002)

Principle of relative locality (Doplicher 2010)

Polymer corrections in loop quantum gravity

$$\Delta x > \frac{h}{2\Delta p}\left[1 - \frac{\lambda^2}{2}(\Delta p)^2 + O(\lambda^4)\right]$$ (Hossain et al. 2010)

Experimental probes of GUP (Pikowski et al 2012))

Pikowski et al. Nature Phys. 8 39 (2012)
Probing Planck-scale physics with quantum optics

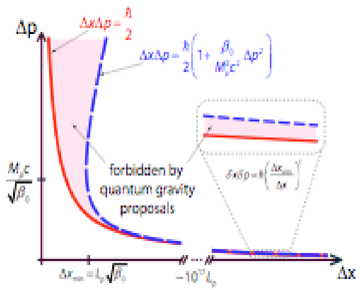


Figure 1: The minimum Heisenberg uncertainty (red curve) is plotted together with a modified uncertainty relation (dashed blue curve) with modification strength β_0. M_P and L_P are the Planck mass and Planck length, respectively. The shaded region represents states that are allowed in regular quantum mechanics but are forbidden in theories of quantum gravity that modify the uncertainty relation. The inset shows the two curves far from the Planck scale at typical experimental position uncertainties $\Delta x \gg \Delta x_{min}$. An experimental precision of $\delta x\, \delta p$ is required to distinguish the two curves, which is beyond current experimental possibilities. However, this can be overcome by our scheme that allows to probe the underlying commutation relation in massive mechanical oscillators and its quantum gravitational modifications.

GENERALIZED COMPTON WAVELENGTH

These arguments suggest $\Delta x > \frac{h}{\Delta p} + \alpha R_P{}^2 \frac{\Delta p}{h} = \frac{h}{\Delta p}\left[1 + \alpha\left(\frac{\Delta p}{cM_P}\right)^2\right]$

Putting $\Delta x \to R$ and $\Delta p \to cM$ gives

$$R > R_C{}' = \frac{h}{Mc} + \frac{\alpha GM}{c^2} \approx \begin{cases} \frac{h}{Mc}\left[1 + \alpha\left(\frac{M}{M_P}\right)^2\right] & (M << M_P) \\ \frac{\alpha GM}{c^2}\left[1 + \frac{1}{\alpha}\left(\frac{M_P}{M}\right)^2\right] & (M >> M_P) \end{cases}$$ α=2?

Compton scale becomes Schwarzschild scale for $M>>M_P$?

Compton irrelevant for $M>>M_P$ since $R_C<<R_P$?

Cannot localize on scale below R_S? Firewalls?

BH radiation => quantum boundary becomes BH boundary?

GENERALIZED EVENT HORIZON

Rewrite GUP in the form

$$R > R_C{}' = \frac{\beta h}{Mc} + \frac{2GM}{c^2}$$

For $M>>M_P$ we obtain generalized event horizon

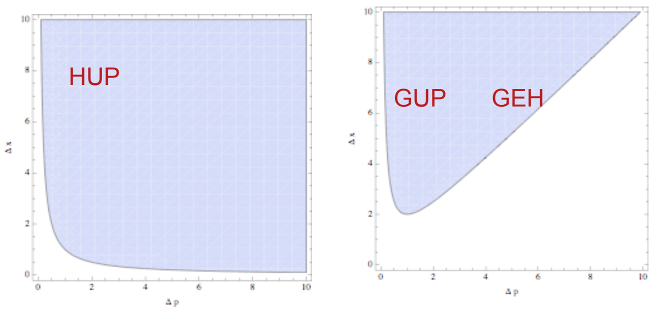
$$R > R_S{}' = \frac{2GM}{c^2}\left[1 + \beta\left(\frac{M_P}{M}\right)^2\right]$$ (small correction to Schwarzschild)

This becomes Compton wavelength for $M<<M_P$, suggesting

"Black Hole Uncertainty Principle Correspondence"

Generalize/unify Compton/Schwarzschild expressions such that

$$R_C{}' \equiv R_S{}' \approx \begin{cases} h/(Mc) & (M << M_P) \\ 2GM/c^2 & (M >> M_P) \end{cases}$$



DO GUP UNCERTAINTIES ADD LINEARLY?

Root-mean-square error would give

$$\Delta x > \sqrt{\left(\frac{h}{\Delta p}\right)^2 + \left(\alpha R_P{}^2 \frac{\Delta p}{h}\right)^2} \Rightarrow R_C{}' = \sqrt{\left(\frac{h}{Mc}\right)^2 + \left(\frac{\alpha GM}{c^2}\right)^2} \approx \frac{h}{Mc}\left[1 + \frac{\alpha^2}{2}\left(\frac{M}{M_P}\right)^4\right]$$

$$\Rightarrow R_S{}' = \sqrt{\left(\frac{2GM}{c^2}\right)^2 + \left(\frac{\beta h}{Mc}\right)^2} \approx \frac{2GM}{c^2}\left[1 + \frac{\beta^2}{8}\left(\frac{M_P}{M}\right)^4\right]$$

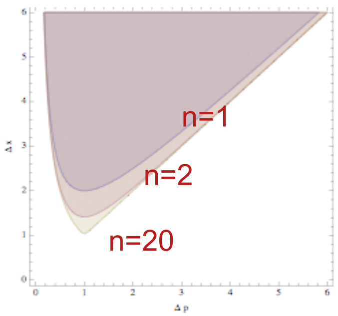
More generally

$$\Delta x > \left[\left(\frac{h}{\Delta p}\right)^n + \left(\alpha R_P{}^2 \frac{\Delta p}{h}\right)^n\right]^{1/n} \Rightarrow R_C{}' = \left[\left(\frac{h}{Mc}\right)^n + \left(\frac{\alpha GM}{c^2}\right)^n\right]^{1/n} \approx \frac{h}{Mc}\left[1 + \frac{\alpha^n}{n}\left(\frac{M}{M_P}\right)^{2n}\right]$$

$$\Rightarrow R_S{}' = \left[\left(\frac{2GM}{c^2}\right)^n + \left(\frac{\beta h}{Mc}\right)^n\right]^{1/n} \approx \frac{2GM}{c^2}\left[1 + \frac{\beta^n}{n}\left(\frac{M_P}{M}\right)^{2n}\right]$$

Minimum at $R_{\min} = 2^{1/n}\sqrt{2\beta}R_P, \quad M_{\min} = \sqrt{\beta/2}\,M_P$

POSSIBLE FORMS OF BHUP CORRESPONDENCE

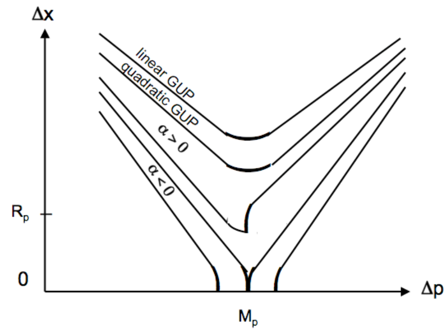


Could allow <u>any</u> form for $R_C{}'(M) \equiv R_S{}'(M)$ with usual asym' limits

- Does form of Δx(Δp) determines $R_S(M)$?
- Do we need $M \leftrightarrow 1/M$ symmetry (t-duality)?
- Can there be black holes for $M<M_P$ since $R_S<R_C$?

INTERESTING ALTERNATIVE

$\alpha < 0$ => cusp rather than smooth minimum

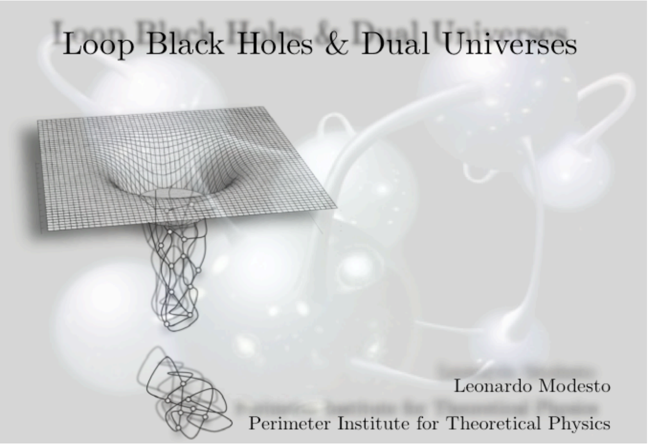


$G \to 0 \Rightarrow$ asymptotic safety (Bonanno & Reuter 2006)

$\hbar \to 0 \Rightarrow$ classical theory (Scardigli et al. 2009)

"Generalized Uncertainty and Self-dual Black Holes"

Carr, Modesto & Premont-Schwarz, arXiv: 1107.0708



LOOP BLACK HOLES

Metric $ds^2 = -G(r)dt^2 + \frac{dr^2}{F(r)} + H(r)d\Omega^{(2)}$,

$$G(r) = \frac{(r-r_+)(r-r_-)(r+r_*)^2}{r^4 + a_o^2},$$
$$F(r) = \frac{(r-r_+)(r-r_-)r^4}{(r+r_*)^2(r^4 + a_o^2)},$$
$$H(r) = r^2 + \frac{a_o^2}{r^2}.$$

where $r_+ = 2Gm/c^2$, $r_- = 2GmP^2/c^2$, $r_* \equiv \sqrt{r_+ r_-}$ and $a_o = A_{\min}/8\pi = \sqrt{3}\,\gamma\zeta R_P^2/2$

Immirzi parameter (γ); $1<\zeta<4$

Polymeric function $P \equiv \frac{\sqrt{1+\epsilon^2}-1}{\sqrt{1+\epsilon^2}+1} \sim \epsilon^2/4 \ll 1$

At large r
$$G(r) \to 1 - \frac{2M}{r}(1-\epsilon^2),$$
$$F(r) \to 1 - \frac{2M}{r},$$
$$H(r) \to r^2.$$
implies $M = m(1+P)^2$ (ADM mass)

Physical radial coordinate $R = \sqrt{H(r)} = \sqrt{r^2 + \frac{a_o^2}{r^2}}$

$$\Rightarrow R_S = \sqrt{\left(\frac{2Gm}{c^2}\right)^2 + \left(\frac{c^2 a_o}{2Gm}\right)^2} \approx \begin{cases} \frac{2Gm}{c^2} & (m > M_P) \\ \frac{\sqrt{3}\gamma\beta}{4}\frac{h}{mc} & (m < M_P) \end{cases}$$

This removes the singularity, permits existence of black holes with m << M_P, and corresponds to the quadratic GEH.

Introduce dual coordinates

$$\bar{r} = a_o/r \Rightarrow R = \sqrt{r^2 + \bar{r}^2} \qquad \bar{t} = t r_*^{\,2}/a_o$$

Metric has self-duality with dual radius $\bar{r} = r = \sqrt{a_o}$

=> another asymptotic infinity (r=0) with BH mass M_P^2/m

However, sub-Planckian black hole hidden within wormhole.

GUP AND BLACK HOLE THERMODYNAMICS

Heuristic argument

$$kT_{BH} = \eta c\Delta p = \frac{\eta hc}{\Delta x} = \frac{\eta hc^3}{2GM} \sim \frac{M_P^{\,2}}{M} \quad (M >> M_P) \qquad \eta=1/(4\pi)$$

Putting $\Delta p \sim T$ and $\Delta x \sim GM/c^2$ in <u>linear</u> GUP (Adler & Chen)

$$\Rightarrow \frac{2GM}{c^2} = \frac{\eta hc}{kT} + \frac{\alpha R_P^{\,2} kT}{h\eta c}$$

$$\Rightarrow T_{BH} = \frac{\eta Mc^2}{\alpha k}\left(1 - \sqrt{1 - \frac{\alpha M_P^{\,2}}{M^2}}\right) \approx \frac{hc^3}{8\pi GkM}\left[1 + \frac{\alpha M_P^{\,2}}{4M^2}\right] \quad (M >> M_P)$$

Complex for $M < \sqrt{\alpha}M_P$ => Planck mass relics.

<u>Quadratic</u> GUP

$$\Rightarrow \frac{2GM}{c^2} = \left[\left(\frac{\eta hc}{kT}\right)^2 \overset{-}{+} \left(\frac{\alpha R_P{}^2 kT}{h\eta c}\right)^2\right]^{1/2}$$

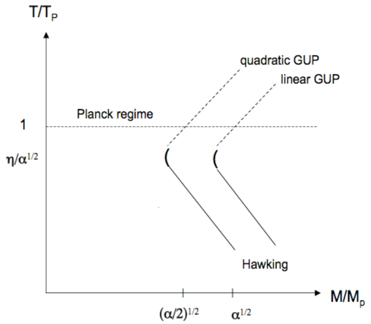
$$\Rightarrow T_{BH} = \frac{\sqrt{2}\eta Mc^2}{\alpha k}\left(1-\sqrt{1-\frac{\alpha^2 M_P{}^4}{4M^4}}\right)^{1/2} \approx \frac{hc^3}{8\pi GkM}\left[1+\frac{\alpha^2 M_P{}^4}{32M^4}\right](M >> M_P)$$

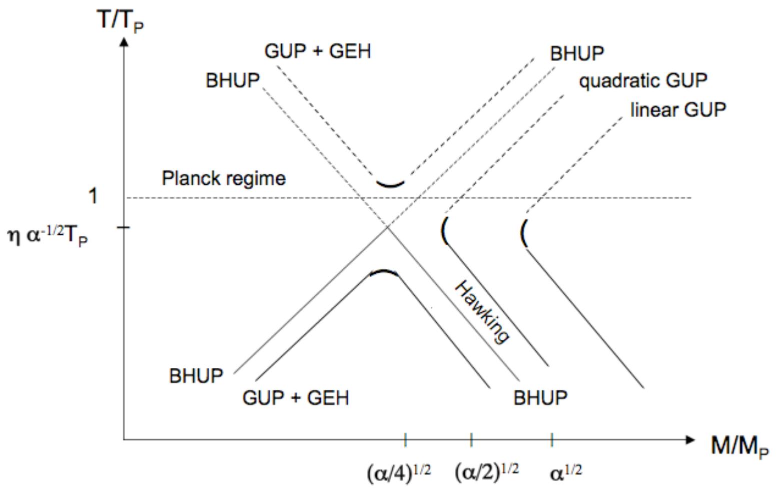
Complex for $M < \sqrt{\alpha/2}M_P$

=> smaller relics

$$T_{\max} = \eta M_P c^2/\sqrt{\alpha}$$

Minus sign gives $T>T_P$
which may be unphysical





Quadratic GUP + GEH   Regard α are β as independent

$$\Rightarrow \left[\left(\frac{2GM}{c^2}\right)^2 + \left(\frac{h\beta}{Mc}\right)^2\right]^{1/2} = \left[\left(\frac{\eta hc}{kT}\right)^2 + \left(\frac{\alpha R_P{}^2 kT}{h\eta c}\right)^2\right]^{1/2}$$

$$\Rightarrow T_{BH} = \frac{\sqrt{2}\eta Mc^2}{\alpha k}\left(1+\frac{\beta^2 M_P{}^4}{4M_P{}^4} - \sqrt{1+\frac{(2\beta^2-\alpha^2)M_P{}^4}{4M^4}+\frac{\beta^4 M_P{}^8}{16M_P{}^8}}\right)^{1/2}$$

Real for all M if $\alpha < 2\beta$

$$\Rightarrow T_{BH} \approx \begin{cases} \dfrac{\eta hc^3}{2GkM}\left[1+\left(\dfrac{\alpha^2-4\beta^2}{32}\right)\dfrac{M_P{}^4}{M^4}\right] & (M >> M_P) \\ \dfrac{\eta Mc^2}{\beta k}\left[1+\left(\dfrac{\alpha^2-4\beta^2}{2\beta^4}\right)\dfrac{M^4}{M_P{}^4}\right] & (M << M_P) \end{cases}$$

BHUP => $\alpha = 2\beta \Rightarrow T_{BH} = \dfrac{h\eta c^3}{2kGM}$ or $T_{BH} = \dfrac{\eta}{\beta}Mc^2$ Exact!

T peaks at $M = \sqrt{\beta/2}M_P$ with $T_{\max} = \eta T_P/\sqrt{2\beta}$

SURFACE GRAVITY ARGUMENT

$$T \propto \frac{GM}{R_S{}^2} \propto \left[\frac{M^{3n/2}}{M^{2n}+(\beta/2)^n M_P{}^{2n}}\right]^{2/n} \propto \begin{matrix} M^{-1}(M >> M_P) \\ M^3(M << M_P) \end{matrix}$$

=> different prediction in sub-Planckian range!

Both arguments predict deviations from Hawking formula
and imply that T never exceeds T_P but which one is correct?

RESOLUTION: THERE ARE TWO ASYMPTOTIC SPACES

Emission looks different in two spaces!

So different asymptotic spaces in $M>M_P$ and $M<M_P$ cases.

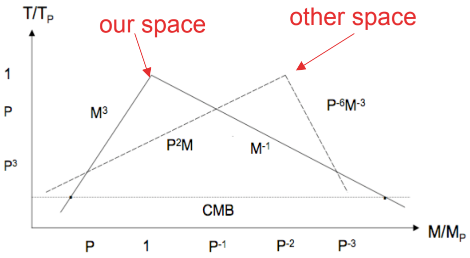
Need asymptotic space on same side of throat as horizon
=> our space for $M>M_P$, other space for $M<M_P$.

Three mass regimes

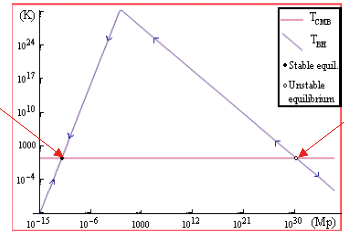
$$M > P^{-2}M_P \Rightarrow r_P < r_- < r_+ \Rightarrow T_\infty \propto M^{-1}, T_0 \propto P^{-6}M^{-3}$$

$$M < M_P \Rightarrow r_- < r_+ < r_P \Rightarrow T_\infty \propto M^{3}, T_0 \propto P^{2}M$$

$$M_P < M < P^{-2}M_P \Rightarrow r_- < r_P < r_+ \Rightarrow T_\infty \propto M^{-1}, T_0 \propto P^{2}M$$



CAN SUB-PLANCKIAN RELICS PROVIDE DARK MATTER?



$T \propto M^3 \Rightarrow$ cooler than CMB for $M < \left(\frac{T_{CMB}}{T_P}\right)^{1/3} M_P \sim 10^{-16} g$

$\frac{dM}{dt} \propto R^{-2}T^4 \propto M^{10} \Rightarrow M(t) \propto t^{-1/9}$

=> never evaporate but current mass $M \sim \left(\frac{t_P}{t_0}\right)^{1/9} M_P \sim 10^{-12} g$

=> current temperature $T \sim \left(\frac{t_P}{t_o}\right)^{1/3} T_P \sim 10^{12} K$

=> same observational effects as PBHs with M~10^{15}g!

A RAINBOW-INSPIRED BLACK HOLE SOLUTION

Carr, Mureika, Nicolini (2014)

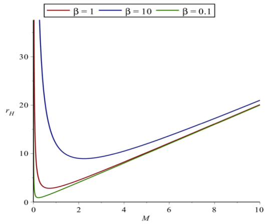
$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega^2 \qquad f(r) = 1 - \frac{2}{M_{Pl}^2}\frac{M}{r}\left(1 + \frac{\beta}{2}\frac{M_{Pl}^2}{M^2}\right) = 1 - \frac{R_S'}{r}$$

$$r_H = \frac{2}{M_{Pl}^2}\frac{M^2 + \frac{\beta}{2}M_{Pl}^2}{M}$$

$$M \gg M_{Pl} \implies r_H \sim \frac{2M}{M_{Pl}^2}$$

$$M \sim M_{Pl} \implies r_H \sim \frac{3\sqrt{\beta}}{M_{Pl}}$$

$$M \ll M_{Pl} \implies r_H \sim \frac{\beta}{M}$$

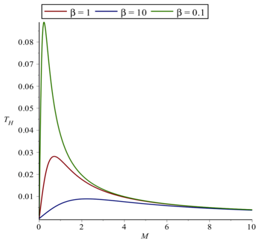


$$T = \frac{\sqrt{2}\,\eta M c^2}{\alpha k}\left(1 - \sqrt{1 - \frac{\alpha^2}{4}\left(\frac{M_P}{M}\right)^4}\right)^{1/2} \approx \frac{\eta \hbar c^3}{2GkM}\left[1 + \frac{\alpha^2}{32}\left(\frac{M_P}{M}\right)^4\right]$$

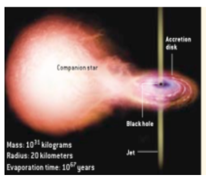
$$M \gg M_{Pl} \implies T \sim \frac{M_{Pl}^2}{8\pi M} \qquad \tau \sim L/\tau \sim M^3(1 + M_{Pl}^2/2M^2)^2.$$

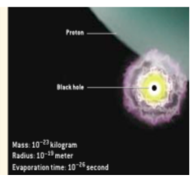
$$M \sim M_{Pl} \implies T \sim \frac{M_{Pl}}{12\pi} \qquad M_* \sim (t_0/t_{Pl})^{1/3} M_{Pl} \sim 10^{20} M_{Pl} \sim 10^{15}\ \mathrm{g}$$

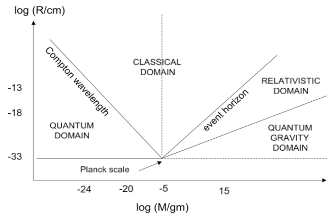
$$M \ll M_{Pl} \implies T \sim \frac{M}{4\pi\beta} \qquad M_{**} \sim (t_0/t_{Pl})^{-1} M_{Pl} \sim 10^{-60} M_{Pl} \sim 10^{-65}\ \mathrm{g}.$$



CONCLUSIONS







- Both black holes and Generalized Uncertainty Principle provide important link between micro and macro physics.
- They are themselves linked and black holes with sub-Planckian mass may play a role in quantum gravity.