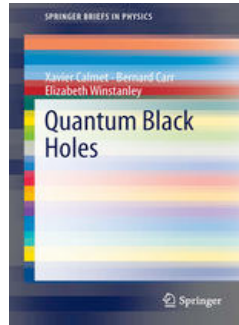


BLACK HOLES IN MACROPHYSICS AND MICROPHYSICS

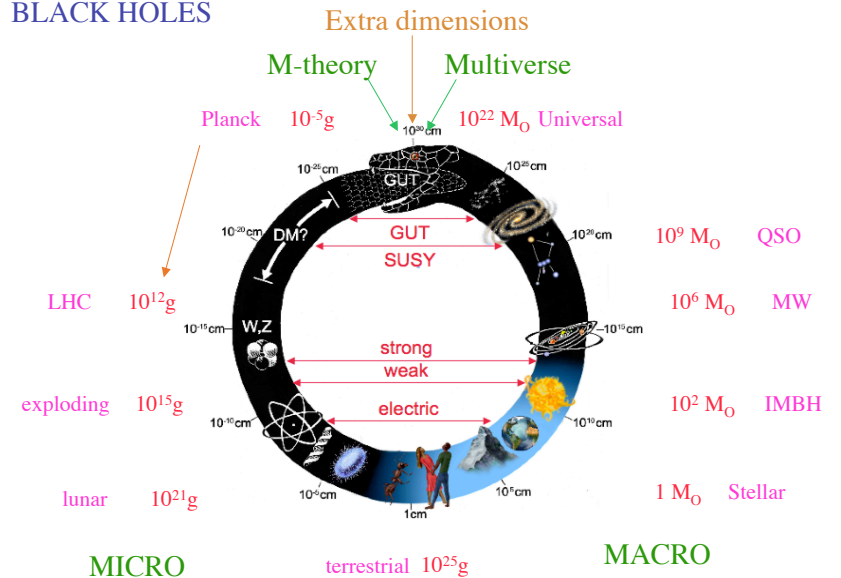
Bernard Carr
Queen Mary, University of London

- Introducing black holes
- Formation of primordial black holes
- Constraints on primordial black holes
- Some hot topics



Lectures at RESCEU summer school 2014

BLACK HOLES



BLACK HOLE FORMATION

$$R_S = 2GM/c^2 = 3(M/M_\odot) \text{ km} \Rightarrow \rho_S = 10^{18}(M/M_\odot)^{-2} \text{ g/cm}^3$$

Stellar BH ($M \sim 10^{1-2} M_\odot$), IMBH ($M \sim 10^{3-5} M_\odot$), SMBH ($M \sim 10^{6-9} M_\odot$) may form at present or recent epochs.

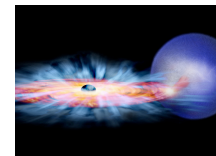
Small “primordial” BHs can only form in early Universe

cf. cosmological density $\rho \sim 1/(Gt^2) \sim 10^6(t/s)^{-2} \text{ g/cm}^3$

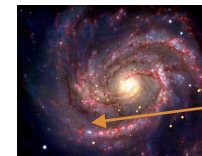
⇒ PBHs have horizon mass at formation

$$M_{\text{PBH}} \sim c^3 t / G = \begin{matrix} 10^{-5} \text{g} & \text{at } 10^{-43} \text{s} & \text{(minimum)} \\ 10^{15} \text{g} & \text{at } 10^{-23} \text{s} & \text{(evaporating now)} \\ 1 M_\odot & \text{at } 10^{-5} \text{s} & \text{(QCD transition)} \\ 10^5 M_\odot & \text{at } 1 \text{s} & \text{(maximum)} \end{matrix}$$

OVERWHELMING EVIDENCE FOR STELLAR BH ($M \sim 10^{1-2} M_\odot$)

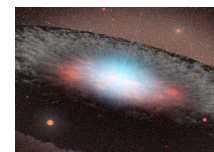


Cygnus X1

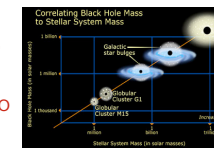


First observ' of BH birth (2010)

OVERWHELMING EVIDENCE FOR SMBH IN AGN ($M \sim 10^{6-9} M_\odot$)



MW $4 \times 10^6 M_\odot$
QSO $10^8 M_\odot$
Max $2 \times 10^9 M_\odot$



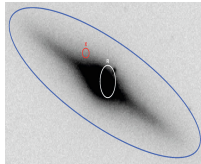
BH mass proport' to stellar mass?

POSSIBLE EVIDENCE FOR IMBH ($M \sim 10^3 - 5 M_{\odot}$)

ULX source NGC1313
may have $500 M_{\odot}$ BH



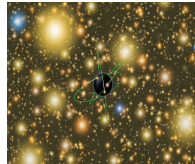
Spiral ESO 243-49
may have $M > 500 M_{\odot}$



QPO in NGC5408 X1
implies $2000 M_{\odot}$ BH

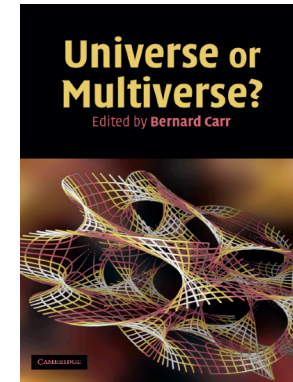


GC Omega Cen
may have $4 \times 10^4 M_{\odot}$ BH



Gamma-ray bursts?

NO EVIDENCE FOR PRIMORDIAL BLACK HOLES!



NO EVIDENCE FOR ACCELERATOR BLACK HOLES!

BLACK HOLE EVAPORATION

Black holes radiate thermally with temperature

$$T = \frac{hc^3}{8\pi GkM} \sim 10^{-7} \left[\frac{M}{M_{\odot}} \right]^{-1} \text{ K} \quad (\text{Hawking 1974})$$

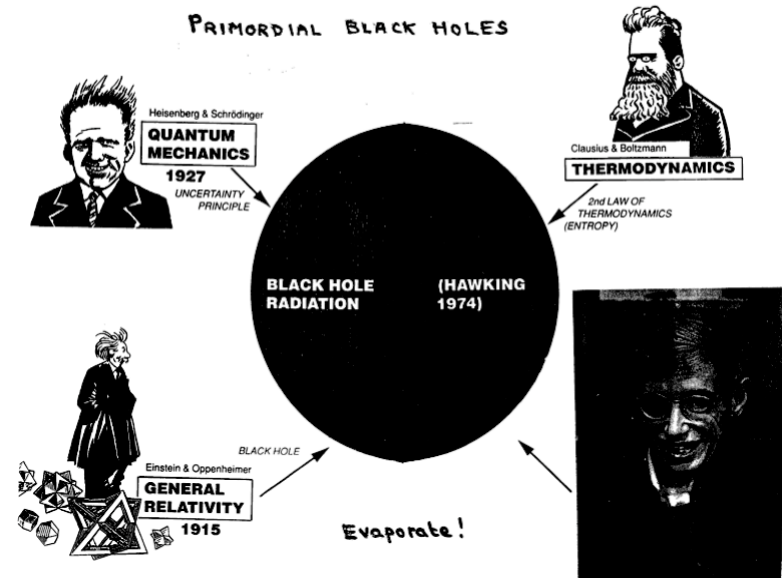
\Rightarrow evaporate completely in time $t_{\text{evap}} \sim 10^{64} \left[\frac{M}{M_{\odot}} \right]^3 \text{ y}$

$M \sim 10^{15} \text{ g} \Rightarrow$ final explosion phase today (10^{30} ergs)

This can only be important for PBHs

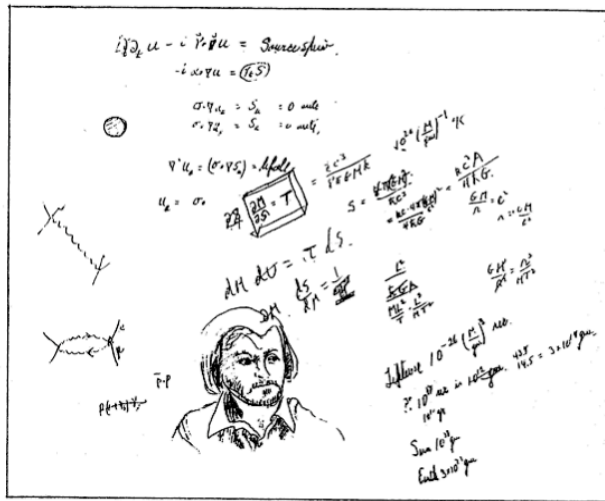
γ -ray bgd at 100 MeV $\Rightarrow \Omega_{\text{PBH}}(10^{15} \text{ g}) < 10^{-8}$
(Page & Hawking 1976)

\Rightarrow explosions undetectable in standard particle physics model



PBHs important even if never formed!

Feynman's envelope!

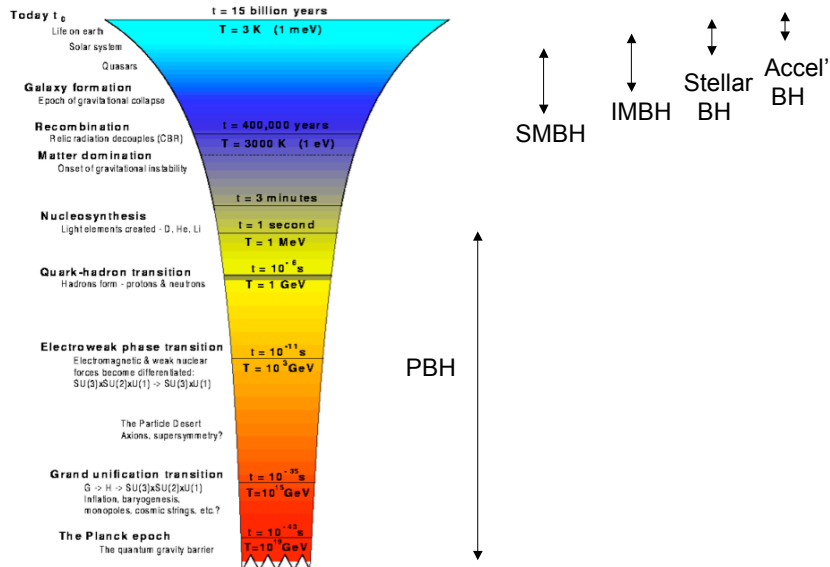


1975

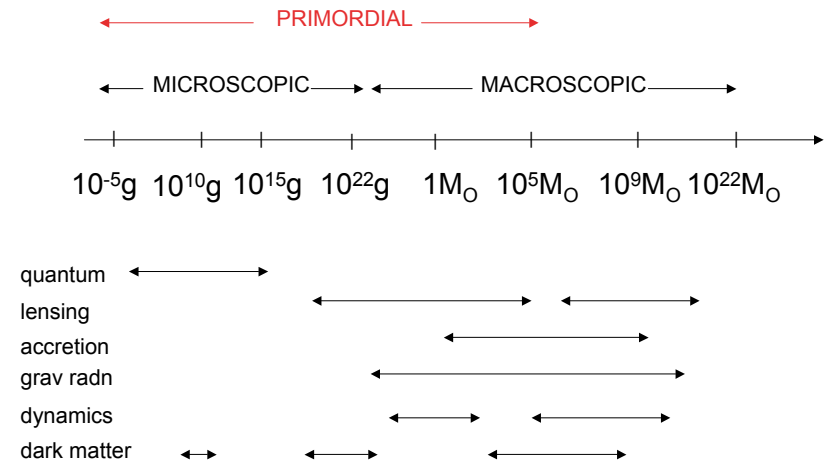
MODES OF BLACK HOLE FORMATION

Name	Mass/ M_\odot	BH formation mechanism/stellar end-state	Location
SMBH	10^5	Collapse to SMBH before H-burning from GR instability. Accretion onto/merger of IMBHs	Quasars/galactic nuclei Intergalactic/halo DM?
IMBH	200	Collapse to IMBH due to electron-positron instability during O-burning	GCs/ULXs/GRBs? Intergalactic/halo DM?
Stellar BH	100	Explode due to electron-positron instability during O-burning (no remnant)	
MO	25	Prompt core collapse after nuclear burning or delayed collapse after failed supernova	Galactic discs
	1	White dwarf or neutron star remnant or total disruption at carbon ignition (no remnant)	
PBH	10^{15} g	Collapse from primordial fluctuations or at cosmological phase transition or in dust era	Intergalactic/halo DM?
	10^{-5} g	Same formation mechanism but evaporation completed and M_\bullet holes exploding today	Local GRBs?
		Planck mass relics of larger evaporated holes or remnants of Planck epoch	Intergalactic/halo DM?

WHEN BLACK HOLES FORM



QUANTUM & ASTROPHYSICAL EFFECTS OF BLACK HOLES



HOW PBHS FORM

Inhomogeneities (discussed in more detail later)

Pressure reduction

Khlopov & Polnarev 1980, Widerin & Schmid 1998,
Jedamzik & Niemeyer 1999, Fuller et al 2000

Cosmic strings

Polnarev & Zemboricz 1988, Hawking 1989, Garriga & Sakellariadou
1993, Caldwell & Casper 1996, MacGibbon et al 1998

Bubble collisions

Crawford & Schramm 1982, Hawking et al 1982, La & Steinhardt 1985
Konoplich et al 1998, Khlopov et al 1999

Domain walls

Rubin et al 2000, Khlopov et al 2004

String necklaces

Matsuda 2006, Lake et al 2009

PART II: PRIMORDIAL BLACK HOLES

PBHs as probe of early Universe

inhomogeneities, phase transitions, inflation

PBHs as probe of high energy physics

PBH explosions, cosmic rays, extra dimensions

PBHs as probe of quantum gravity

Planck mass relics, Generalized Uncertainty Principle

WHAT PRIMORDIAL BLACK HOLES DO

Probe fundamental physics ($M \sim 10^{-5}g$)

Planck-mass relics

Extra dimensions and higher dimensional BHs

TeV quantum gravity

Probe early universe ($M < 10^{15}g$)

Baryosynthesis/nucleosynthesis

Gravitino/neutrino/entropy production

Removing monopoles/domain walls

Probe high energy physics ($M \sim 10^{15}g$)

Cosmological and Galactic γ -rays

Cosmic ray antiprotons and positrons

PBH explosions and gamma-ray bursts

Probe gravity ($M > 10^{15}g$)

Non-baryonic cold dark matter candidate

Dynamical/lensing/gravitational-wave effects

Seed large-scale structure and SMBHs in galactic nuclei

Limit on fraction of Universe collapsing

$\beta(M)$ fraction of density in PBHs of mass M at formation

General limit

$$\frac{\rho_{PBH}}{\rho_{CBR}} \approx \frac{\Omega_{PBH}}{10^{-4}} \left[\frac{R}{R_0} \right] \Rightarrow \beta < 10^{-6} \Omega_{PBH} \left[\frac{t}{\text{sec}} \right]^{1/2} < 10^{-18} \Omega_{PBH} \left[\frac{M}{10^{15}g} \right]^{1/2}$$

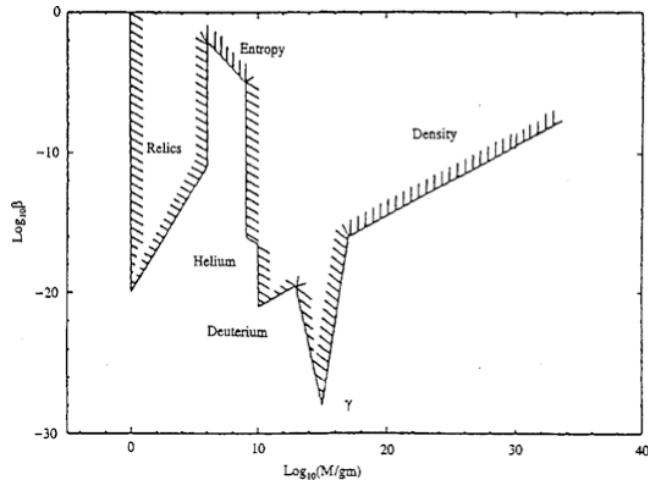
Unevaporated $M > 10^{15}g \Rightarrow \Omega_{PBH} < 0.25$ (CDM)

Evaporating now $M \sim 10^{15}g \Rightarrow \Omega_{PBH} < 10^{-8}$ (GRB)

Evaporated in past $M < 10^{15}g$

\Rightarrow constraints from entropy, γ -background, BBNS

Carr, Gilbert & Lidsey (1994)



PBH FORMATION => LARGE INHOMOGENEITIES

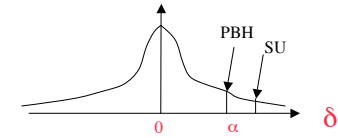
To collapse against pressure, need (Carr 1975)

$$R > \sqrt{\alpha} ct \text{ when } \delta \sim 1 \Rightarrow \delta_H > \alpha \quad (p = \alpha \rho c^2)$$

Gaussian fluctuations with $\langle \delta_H^2 \rangle^{1/2} = \epsilon(M)$

=> fraction of PBHs

$$\beta(M) \sim \epsilon(M) \exp\left[-\frac{\alpha^2}{2\epsilon(M)^2}\right]$$

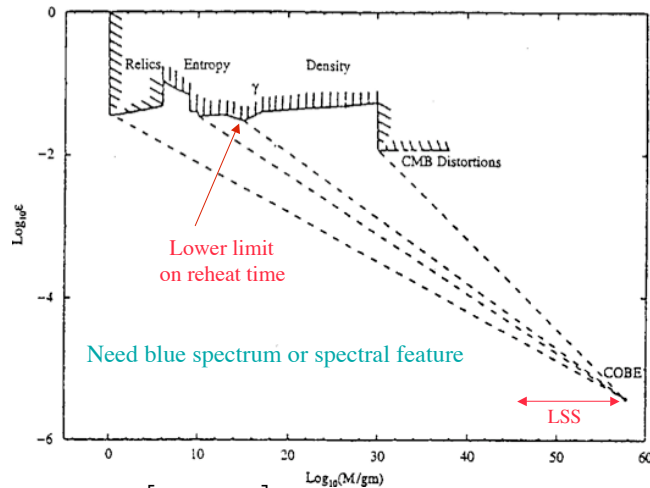


$$\epsilon(M) \text{ constant} \Rightarrow \beta(M) \text{ constant} \Rightarrow dN/dM \propto M^{-\left(\frac{1+3\alpha}{1+\alpha}\right)-1}$$

$\epsilon(M)$ decreases with M => exponential upper cut-off

Separate universe for $\delta_H > 1$ but recently disputed

Constraints on amplitude of density fluctuations at horizon epoch



$$\beta(M) \sim \epsilon(M) \exp\left[-\frac{1}{18\epsilon(M)^2}\right]$$

PBHs are unique probe of ϵ on small scales

Analytic prediction ($\delta > 0.3$ for $\alpha = 1/3$) confirmed by early numerical studies but pressure gradient means PBHs are somewhat smaller than horizon (Nadezhin et al 1978)

CRITICAL PHENOMENA

$$\text{Mass scaling} \Rightarrow M_{\text{PBH}} = k M_H (\delta - \delta_C)^\gamma \quad (\text{Choptuik 1993})$$

In Friedmann background (Niemeyer & Jedamzik 1998)

$$2.4 < k < 11.9, \quad 0.67 < \delta_C < 0.71, \quad 0.34 < \gamma < 0.37$$

=> PBH mass spectrum peaks at horizon mass but has extended low mass tail

$$dN/dM \propto M^{1/\gamma-1} \exp[-(M/M_f)^{1/\gamma}] \quad (\text{Yokoyama 1998, Kribs 1999, Green 2000})$$

Later studies => $0.43 < \delta_C < 0.47$ (Musco et al 2005)

Peak analysis versus Press-Schechter => $\delta > 0.3$ again! (Green et al 2004)

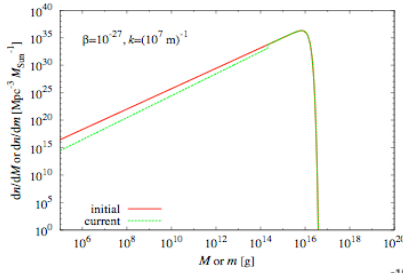
Constraints on PBHs from near-critical collapse

(Yokoyama 1998)

JCAP 01 (2014) 037

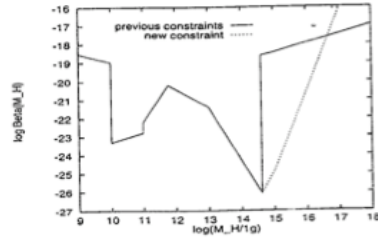
Identifying the most crucial parameters of the initial curvature profile for primordial black hole formation

Tomohiro Nakama,^{a,b} Tomohiro Harada,^c A.G. Polnarev^d and Jun'ichi Yokoyama^{b,c}

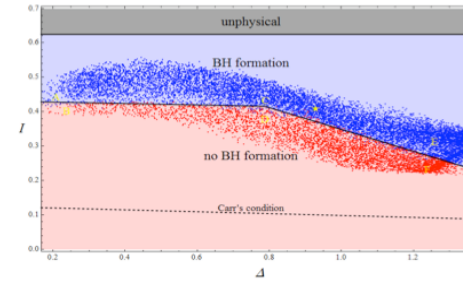


=> broad mass spectrum
=> strong constraints above 10¹⁴g

GRBs and DM from 10¹⁶g PBHs?
(Green 2000)



Numerically solve evolution of spherically symmetric highly perturbed configuration using curvature profiles characterized by 5 parameters. Only two are important: average amplitude of central overdensity (\bar{l}) and width of transition region at boundary (Δ).



NON-GAUSSIAN FLUCTUATIONS

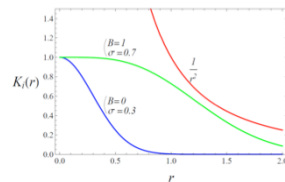
Expected whenever fluctuations are large
(Bullock & Primack 1997, Ivanov 1998, Hidalgo 2007, Young & Byres 2013)

Quantum field theory => n-point correlation function
Slow-roll correction using inflation 3-point correlator

$$P(\delta) = \frac{1}{\sqrt{2\pi}\Sigma} \left[1 - \left(\frac{\delta^3}{\Sigma^6} - \frac{3\delta}{\Sigma^4} \right) \right] \exp \left[-\frac{\delta^2}{2\Sigma^2} \right] \quad (\text{Seery & Hidalgo 2006})$$

2-parameter curvature profile (Polnarev & Musco 2007, Polnarev et al 2012)

$$K_l(r) = \left[1 + \frac{B}{2} \left(\frac{r}{\sigma} \right)^2 \right] \exp \left[-\frac{1}{2} \left(\frac{r}{\sigma} \right)^2 \right]$$



5-parameter curvature profile (Nakama et al. 2014)

$$K_l(r) = A \left[1 + B \left(\frac{r}{\sigma_1} \right)^{2n_1} \right] \exp \left[-\left(\frac{r}{\sigma_1} \right)^{2n_1} \right] + (1-A) \exp \left[-\left(\frac{r}{\sigma_2} \right)^2 \right]$$

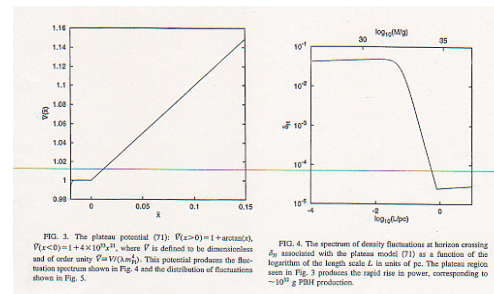
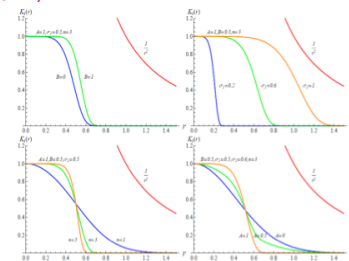


FIG. 3. The plateau potential (71): $V(r>0) = 1 + \arctan(r)$, $V(r<0) = 1 + 4 \times 10^6 (r)^3$, where V is defined to be dimensionless and of order unity $V \sim V(r)/M_{pl}^2$. This potential produces the fluctuation spectrum shown in Fig. 4 and the distribution of fluctuations shown in Fig. 5.

FIG. 4. The spectrum of density fluctuations at horizon crossing δ_{H^*} associated with the plateau model (71) as a function of the logarithm of the length scale L in units of pc . The plateau region seen in Fig. 3 produces the rapid rise in power, corresponding to $\sim 10^{11}$ g PBH production.

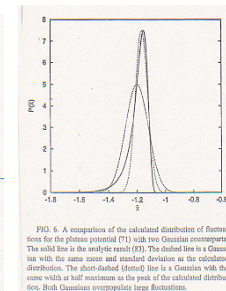
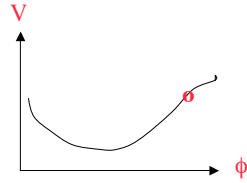


FIG. 6. A comparison of the calculated distribution of fluctuations for the plateau potential (71) with two Gaussian counterparts. The solid line is the analytic result (3). The dotted line is a Gaussian with the same mean and standard deviation as the calculated distribution. The short-dashed (dotted) line is a Gaussian with the same width at half maximum as the peak of the calculated distribution. Both Gaussian counterparts large fluctuations.

PBHS AND INFLATION

PBHs formed before reheat inflated away \Rightarrow

$$M > M_{\min} = M_{\text{Pl}}(T_{\text{reheat}} / T_{\text{Pl}})^{-2} > 1 \text{ gm}$$



since CMB quadrupole $\Rightarrow T_{\text{reheat}} < 10^{16} \text{ GeV}$

But inflation generates fluctuations $\frac{\delta\rho}{\rho} \sim \left[\frac{V^{3/2}}{M_{\text{Pl}}^3 V'} \right]_H$

Can these produce PBHs?

Slow roll plus friction-domination

$$\xi = (M_{\text{Pl}} V' / V)^2 \ll 1, \quad \eta = M_{\text{Pl}} V'' / V \ll 1$$

\Rightarrow nearly scale-invariant fluctuations

$$|\delta_k|^2 \sim k^n, \quad \delta_H \sim M^{(1-n)/4} \text{ with } n = 1 - 3\xi + 2\eta \approx 1$$

Can we believe $\frac{\delta\rho}{\rho} \sim \left[\frac{V^{3/2}}{M_{\text{Pl}}^3 V'} \right]_H$ at end of inflation?

New sources of curvature fluct'ns at end of inflation
(Kolb et al 2005, Lyth 2005, Salem 2005)

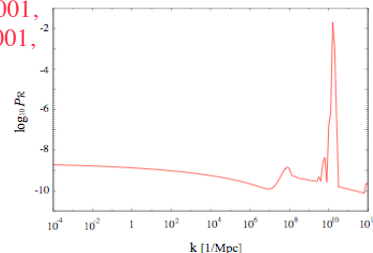
PBH formation on sub-horizon scales (Lyth et al 2005)

Feature in $V(\phi) \Rightarrow$ discrete PBH mass spectrum on any scale
(Hodges & Blumenthal 1990, Ivanov et al 1994)

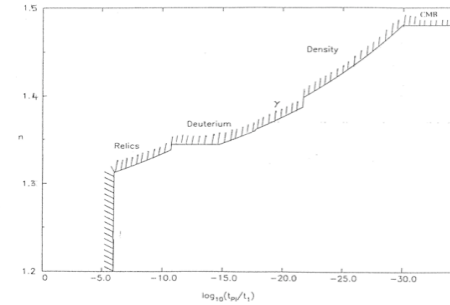
PBHs and preheating

(Easter & Parry 2000, Bassett & Tsujikawa 2001, Green & Malik 2001, Finelli & Khlebnikov 2001, Suyama et al 2005, Torres-Lomas et al. 2013)

Second inflationary phase can produce PBHs in any mass range
(Yokohama, Frampton et al. 2010)



CMB $\Rightarrow \delta_H \sim 10^{-5} \Rightarrow n > 1$ for PBHs $\Rightarrow V''V/V^2 > 3/2$.
PBH limits then constrain n for each reheat time t_R



Observe $n < 1$ on horizon scale \Rightarrow need running index for PBHs.

BICEP2 gives $\frac{d \ln n}{dk} \approx -0.02 \pm 0.01$ (wrong sign!)

Can reasonable inflation model allow $n > 1$ at large k ?

PBH CONSTRAINTS ON INFLATION MODELS

Designer (Hodge et al 1990, Ivanov et al 1994, Yokoyama 1999, Blais et al 2003)

Chaotic (Carr et al 1994, Green & Liddle 1997, Kim et al 2000, Bringmann et al 2001, Lyth et al 2006, Zaballa et al 2007)

Supernatural (Randall, Soljatic & Guth 1996)

Supersymmetry/supergravity (Green et al 1997, Green 1999, Kanazawa et al. 2000)

Hybrid (Garcia-Bellido, Linde & Wands 1996, Kanazawa et al. 2000)

Multiple/oscillating (Yokoyama 1998, Taruya 1999, Saito et al 2008)

Running (Leach, Grivell & Liddle 2000, Kawasaki et al 2007, Kawaguchi et al 2008, Bugaev & Klimai 2009, Kohri et al 2009)

Locked/saddle (Easter, Houry & Schalm 2004)

Hill top (Kohri, Lin & Lyth 2007, Alabidi & Kohri 2009)

Preheating (Easter & Parry 2000, Green & Malik 2001, Bassett & Tsujikawa 2001, Finelli & Khlebnikov 2001, Suyama et al 2005)