Numerical approach to the inflationary Langevin equations

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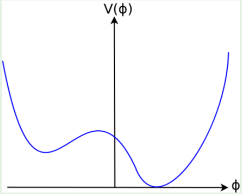
A single scalar field model

$$S = -\int \mathrm{d}^4 x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right]$$

$$\Rightarrow \begin{cases} \rho = \frac{1}{2} \left(\dot{\phi}\right)^2 + V(\phi) \\ p = \frac{1}{2} \left(\dot{\phi}\right)^2 - V(\phi) \end{cases}$$



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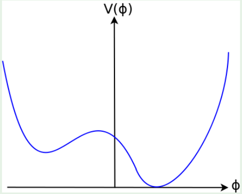

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Slow-Roll Regime

Klein-Gordon equation in this regime: $\dot{\phi} = -\frac{V'(\phi)}{3H}$, with $H^2 = (\dot{a}/a)^2 = V(\phi)/\left(3M_{\mathrm{Pl}}^2\right)$

So $V(\phi) \gg \left(\dot{\phi}\right)^2 \implies M_{\mathrm{Pl}}V'(\phi)/V(\phi) \ll 1$ (**flat potential condition**)

Inflation

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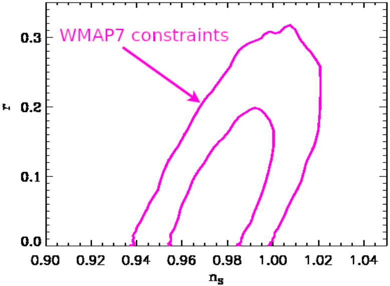
- Is a high energy acceleration of cosmic expansion ($\ddot{a} > 0$)
- Can be implemented with a single scalar field
- Is compatible with observations



Predicts an almost scale invariant power spectrum

Stochastic Inflation

Sub/Super-Horizon Modes

$$\hat{\phi}(\mathbf{x}, t) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} \left[e^{-i\mathbf{k}.\mathbf{x}}\phi_{\mathbf{k}}(t)\hat{a}_{\mathbf{k}} + e^{i\mathbf{k}.\mathbf{x}}\phi^*_{\mathbf{k}}(t)\hat{a}^\dagger_{\mathbf{k}}\right]$$

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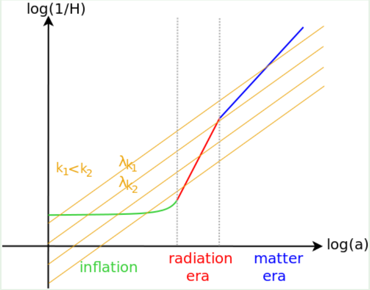

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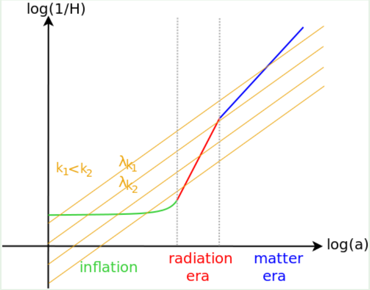
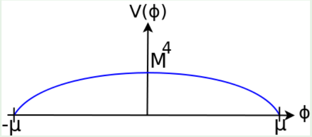


Langevin Equation for the classical field

$\dot{\phi} = -\frac{V'(\phi)}{3H} + \frac{H^{3/2}}{2\pi}\xi$, where ξ is a $\mathcal{N}(0,1)$ white gaussian noise.

- Can be solved perturbatively

Example: Small Field Inflation



$$V\left(\phi\right) = M^4 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right]$$

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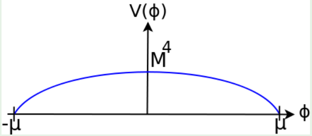

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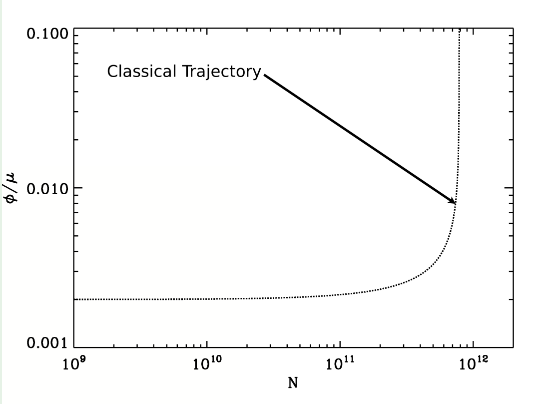

Numerical Solution
(in terms of the number of e-folds $N \simeq \ln\left(a/a_{\mathrm{in}}\right)$)

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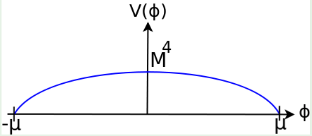

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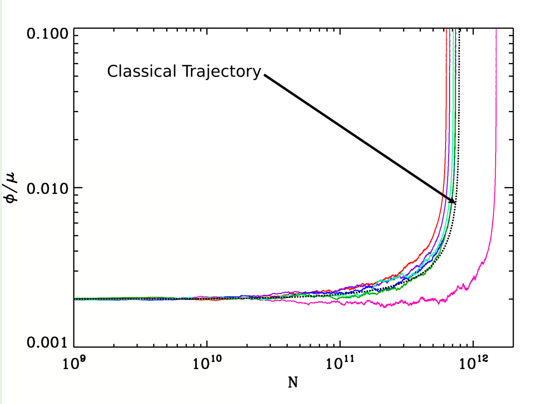

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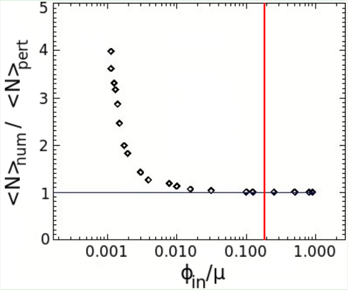

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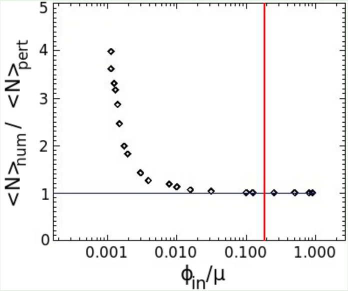
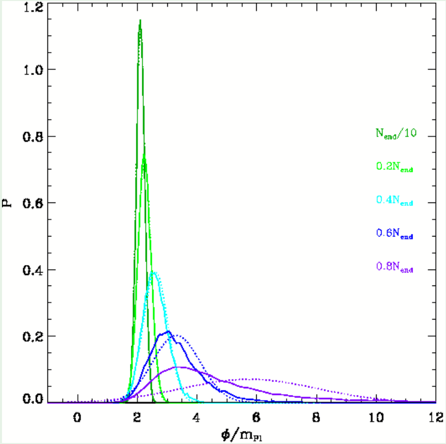
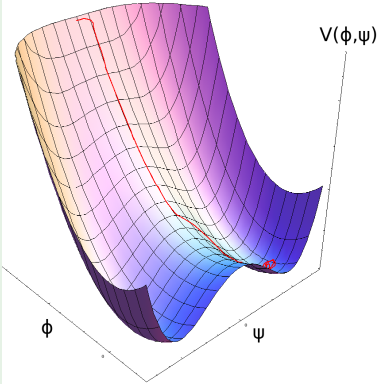




A two-fields model: Hybrid Inflation

$$V\left(\phi,\chi\right)=\Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)^2+\frac{\phi^2}{\mu^2}+2\frac{\phi^2\psi^2}{\phi_c^2 M^2}\right]$$



Non-Perturbative Effects

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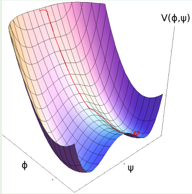
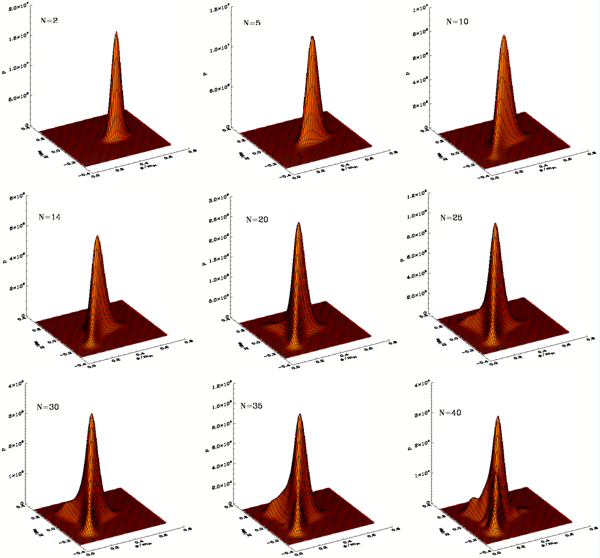


Probability Density Function



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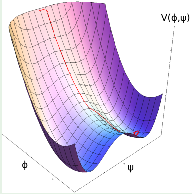

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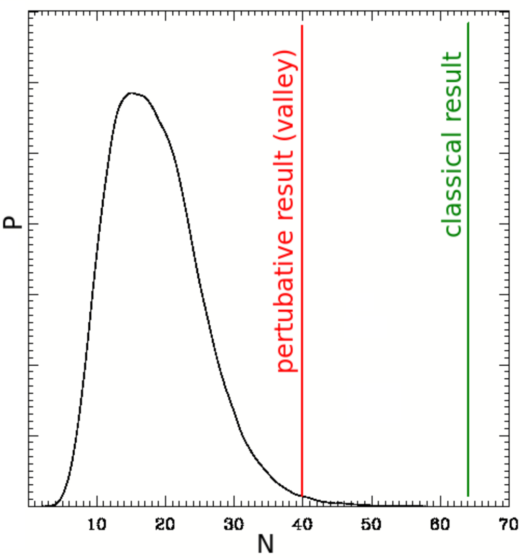
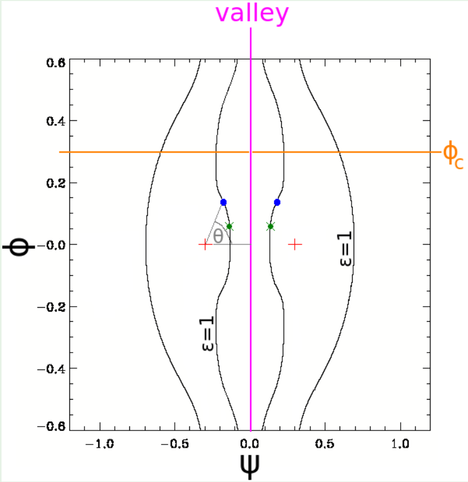


Non-Perturbative Effects

$\epsilon = 1$ Map



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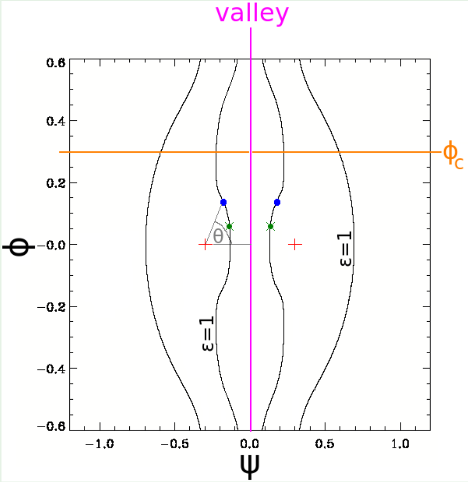
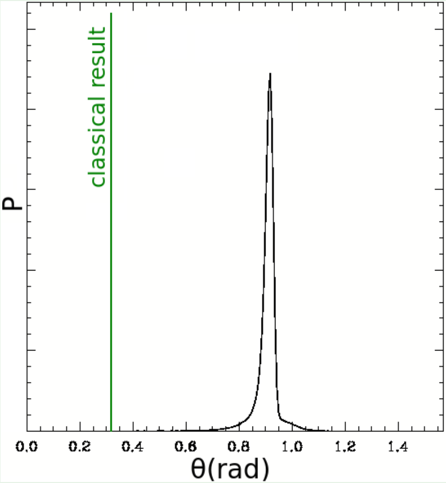


Exit Angle



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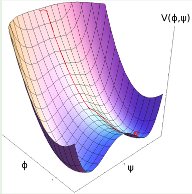

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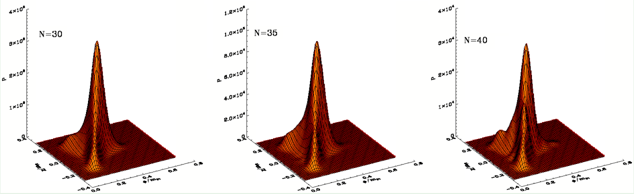

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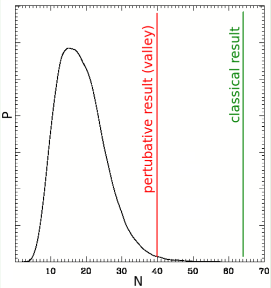

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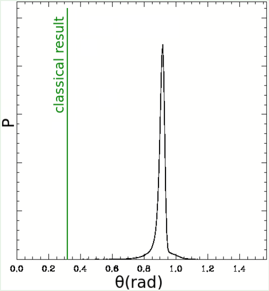

Conclusion

Main results: numerical solutions of inflationary Langevin equations

- Good agreement with the perturbative treatment in the dedicated regimes
- Limits of such a treatment:
  - Regimes where it is not valid
  - Multiple fields models
- Necessity to have a numerical code
- Significant changes in predicted relevant physical quantities

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Prospects

- Impacts on the spectrum
- Non Gaussianities
- Towards a numerical signature of eternal inflation
- Systematical Exploration of the parameter space

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Horizon Problem

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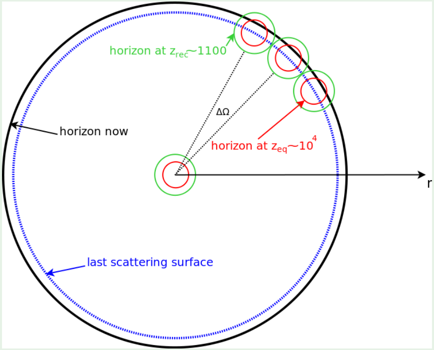

Horizons sketch without inflation

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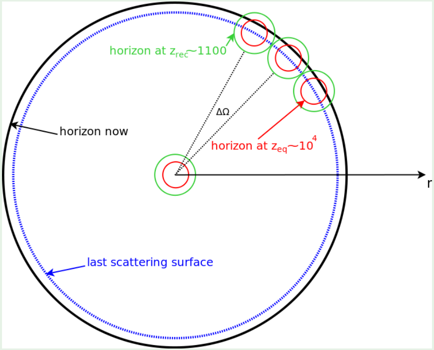

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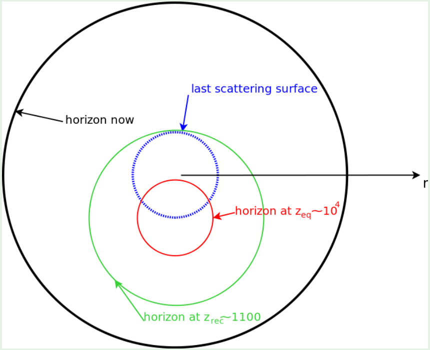

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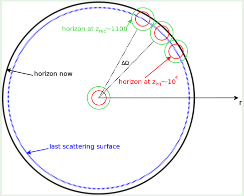

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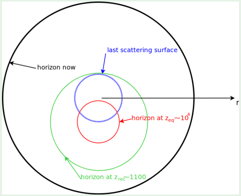

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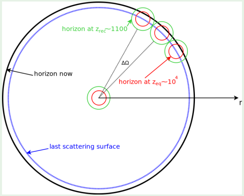

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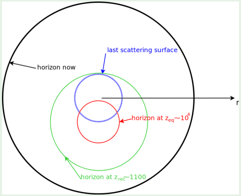

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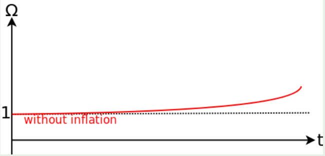

A fine tuning issue

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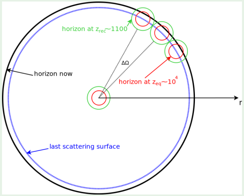

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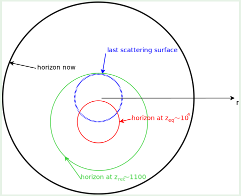

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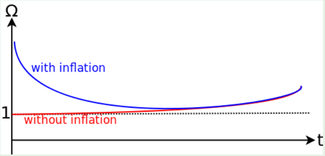

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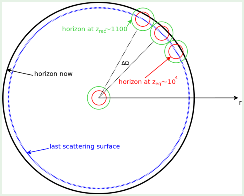

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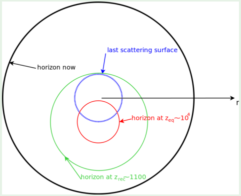

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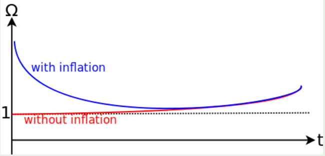

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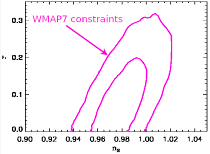
Unexpected Prediction



A scale invariant power spectrum