Generalized Galileon and Inflation

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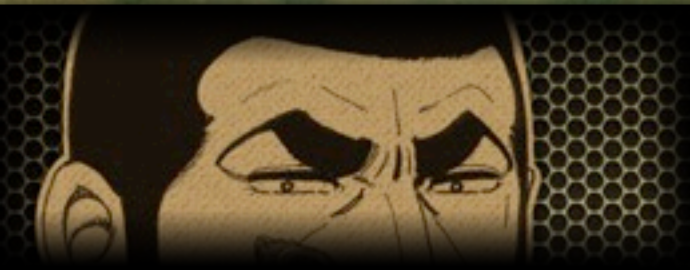
DENET Summer School @ Kochi, 8.31 2010



Last year, I talked about *G-inflation*...

G-inflation

小林 努

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RESCEU, Univ. of Tokyo

Based on work with:

Masahide Yamaguchi (Tokyo Inst. Tech.)

Jun'ichi Yokoyama (RESCEU & IPMU)

arXiv:1008.0603

This year, I will talk about

Generalized G-inflation

↑

G = Galileon

Based on work with **Masahide Yamaguchi** & **Jun'ichi Yokoyama**

arXiv:1105.5723, PTP accepted

My message is:

G^2 is **the most general** single-field inflation model

- ✓ contains *all* the (single-field) inflation models proposed so far as special cases
- ✓ no further generalization is possible

Motivation

So many inflation models...

$$G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$$

$$-\frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\lambda\phi^4$$

$$R + \frac{R^2}{6M^2}$$

$$f(\phi)(\partial\phi)^2$$

$$f(\phi)R$$

$$\xi(\phi)\left(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2\right)$$

$$\sqrt{1+(\partial\phi)^2}$$

The most general thing is definitely valuable!

Consider a gravity + scalar system

Q. What is the most general Lagrangian of the form

$$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \partial g_{\mu\nu}, \partial^2 g_{\mu\nu}, \partial^3 g_{\mu\nu}, \cdots ; \phi, \partial\phi, \partial^2\phi, \partial^3\phi, \cdots)$$

having second-order field equations?

A. It is given by the generalized Galileon

Galileon (in flat space)

Nicolis, Rattazzi, Trincherini (2009)

The **Galileon** is a scalar field with the following properties:

(1) Galilean shift-symmetry

$$\partial_\mu \phi \to \partial_\mu \phi + b_\mu$$

(2) Second order EOM

Unique

$$\mathcal{L}_1 = \phi$$

$$\mathcal{L}_2 = (\partial \phi)^2$$

$$\mathcal{L}_3 = (\partial \phi)^2 \Box \phi$$

$$\mathcal{L}_4 = (\partial \phi)^2 \left[(\Box \phi)^2 - (\partial_\mu \partial_\nu \phi)^2 \right]$$

$$\mathcal{L}_5 = (\partial \phi)^2 \left[(\Box \phi)^3 - 3 \Box \phi (\partial_\mu \partial_\nu \phi)^2 + 2 (\partial_\mu \partial_\nu \phi)^3 \right]$$

Generalized Galileon

The Galileon in flat space can be generalized to give the **most general** theory describing **scalar + gravity** system with **second-order** field equations

Deffayet, Esposito-Farese, Vikman (2009);
Deffayet, Pujolas, Sawicki, Vikman (2010);
Deffayet, Gao, Steer, Zahariade (2011)

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\Big[(\;\ldots$$

$$-3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\Big]$$

4 arbitrary functions of ϕ and $X := -(\partial\phi)^2/2$

The most general single-field inflation model is obtained from the generalized Galileon = **G**eneralized **G**-inflation

Special cases

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$G_4 \supset \frac{M_{\rm Pl}^2}{2}$ → $\mathcal{L}_4 \supset \frac{M_{\rm Pl}^2}{2}R$ Einstein-Hilbert

$G_4 \supset f(\phi)$ → $\mathcal{L}_4 \supset f(\phi)R$ Non-minimal inflation

$G_4 \supset X$ → (*integration by parts*) $\mathcal{L}_4 \supset G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ New Higgs inflation

Germani, Kehagias (2010)

$$\begin{aligned} K &= 8\xi^{(4)}X^2(3-\ln X), \\ G_3 &= 4\xi^{(3)}X(7-3\ln X), \\ G_4 &= 4\xi^{(2)}X(2-\ln X), \\ G_5 &= -4\xi^{(1)}\ln X \end{aligned}$$

→ (*integration by parts*) $\xi(\phi)(\text{Gauss-Bonnet})$

The generalized Galileon in 4D was already formulated in 1973!

International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363–384

Revisited by Charmousis *et al.* (2011)

Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

GREGORY WALTER HORNDESKI

Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada

Received: 10 *July* 1973

$$\mathscr{L} = \sqrt{(g)}\,\mathscr{K}_1 \delta^{cde}_{hjk} \phi_{|c}{}^{|h} R_{de}{}^{jk} - \tfrac{4}{3}\sqrt{(g)}\,\dot{\mathscr{K}}_1 \delta^{cde}_{hjk} \phi_{|c}{}^{|h} \phi_{|d}{}^{|j} \phi_{|e}{}^{|k}$$
$$+ \sqrt{(g)}\,\mathscr{K}_3 \delta^{cde}_{hik} \phi_{|c} \phi^{|h} R_{de}{}^{jk} - 4\sqrt{(g)}\,\dot{\mathscr{K}}_3 \delta^{cde}_{hik} \phi_{|c} \phi^{|h} \phi_{|d}{}^{|j} \phi_{|e}{}^{|k}$$

This is equivalent to the generalized Galileon

$$-3\sqrt{(g)}(2\mathscr{F}' + 4\mathscr{W}' + \rho \mathscr{K}_8)\phi_{|c}{}^{|c} + 2\sqrt{(g)}\,\mathscr{K}_8 \delta^{ca}_{fh} \phi_{|c} \phi^{|f} \phi_{|d}{}^{|h}$$
$$+ \sqrt{(g)}\{4\mathscr{K}_9 - \rho(2\mathscr{F}'' + 4\mathscr{W}'' + \rho\mathscr{K}'_8 + 2\dot{\mathscr{K}}_9)\} \qquad (4.21)$$

Horndeski & the Galileon

$$
\begin{aligned}
\mathcal{L}_H \;=\;& \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\Big[\kappa_1 \nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} + \frac{2}{3}\kappa_{1X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi \\
&+\kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma} + 2\kappa_{3X}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi\Big] \\
&+\delta^{\alpha\beta}_{\mu\nu}\Big[(F+2W)R_{\alpha\beta}{}^{\mu\nu} + 2F_X\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\Big] \\
&-6\left(F_\phi + 2W_\phi - X\kappa_8\right)\Box\phi + \kappa_9
\end{aligned}
$$

Horndeski (1974)

$$
\begin{aligned}
K &= \kappa_9 + 4X\int^X \mathrm{d}X'\left(\kappa_{8\phi} - 2\kappa_{3\phi\phi}\right), \\
G_3 &= 6F_\phi - 2X\kappa_8 - 8X\kappa_{3\phi} + 2\int^X \mathrm{d}X'(\kappa_8 - 2\kappa_{3\phi}), \\
G_4 &= 2F - 4X\kappa_3, \\
G_5 &= -4\kappa_1,
\end{aligned}
$$

→ Generalized Galileon

Cosmological Background

"Friedmann equation" (00 equation)

$$(\cdots) + (\cdots)H + (\cdots)H^2 + (\cdots)H^3 = 0$$

$\sim \mathcal{L}_2$ $\sim \mathcal{L}_3$ $\sim \mathcal{L}_4$ $\sim \mathcal{L}_5$

"Kinetic gravity braiding"
Deffayet *et al.* (2010)

ij and scalar-field equations

$$\dot{H} = (\cdots)\ddot{\phi} + \cdots$$
$$\ddot{\phi} = (\cdots)\dot{H} + \cdots$$

Not diagonal in second derivatives

In general, this mixing cannot be undone through conformal transformation

Cf. Usual (k-)inflation

T_{ij} does not contain second derivatives of ϕ

Scalar-field EOM does not contain second derivatives of $g_{\mu\nu}$

Background example 1

$$K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \cdots ,$$
$$G_i(\phi, X) = g_i(\phi) + h_i(\phi)X + \cdots .$$

Slowly-rolling ϕ

Potential-dominated inflation $H^2 \simeq \frac{1}{6}\frac{V(\phi)}{g_4(\phi)}$

$$\mathcal{L}_4 = g_4(\phi)R + \cdots$$

Modified friction term in scalar-field EOM

$$3H\left[\mathcal{K}\dot{\phi} + 6\left(Hh_3X + H^2h_4\dot{\phi} + H^3h_5X\right)\right] \simeq -V_\phi + 12H^2 g_{4\phi}$$

can enhance friction

Germani, Kehagias (2010); Kamada *et al.* (2011)

Background example 2

Shift symmetry: $\phi \to \phi + c$, i.e., $K = K(X)$, $G_i = G_i(X)$

Inflation can be driven by constant kinetic energy

Scalar-field EOM

$$\dot{J} + 3HJ = 0 \longrightarrow J \propto a^{-3} \to 0$$

where

$$J := \dot{\phi} K_X + 6HXG_{3X} + 6H^2\dot{\phi}\left(G_{4X} + 2XG_{4XX}\right) + 2H^3X\left(3G_{5X} + 2XG_{5XX}\right)$$

de Sitter attractor

$$H = \text{const.}$$
$$X = \text{const.}$$

Use for dark energy, see Deffayet, Pujolas, Sawicki, Vikman (2010)

h_{ij}

ζ

Perturbations

Tensor perturbations

$$g_{ij} = a^2(t)(\delta_{ij} + h_{ij})$$

Quadratic action for tensor perturbations

$$S_T^{(2)} = \frac{1}{8}\int dt d^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla} h_{ij})^2\right]$$

$$\mathcal{F}_T > 0 \qquad + G_{5\phi}\Big)\Big],$$

Stability conditions

$$\mathcal{G}_T > 0 \qquad X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\Big]$$

Scalar perturbations

$$g_{ij} = a^2(t)e^{2\zeta}\delta_{ij}$$

Quadratic action for scalar perturbations

$$S_S^{(2)} = \int dt d^3x\, a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\vec{\nabla}\zeta)^2\right]$$

$$\mathcal{F}_S > 0$$

$$\mathcal{G}_S > 0$$

Stability conditions

$$\begin{aligned}
\Sigma \;:=\; & XK_X + 2X^2K_{XX} + 12H\dot{\phi}XG_{3X} \\
& +6H\dot{\phi}X^2G_{3XX} - 2XG_{3\phi} - 2X^2G_{3\phi X} - 6H^2G_4 \\
& +6\Big[H^2\left(7XG_{4X} + 16X^2G_{4XX} + 4X^3G_{4XXX}\right) \\
& -H\dot{\phi}\left(G_{4\phi} + 5XG_{4\phi X} + 2X^2G_{4\phi XX}\right)\Big] \\
& +30H^3\dot{\phi}XG_{5X} + 26H^3\dot{\phi}X^2G_{5XX} \\
& +4H^3\dot{\phi}X^3G_{5XXX} - 6H^2X\big(6G_{5\phi} \\
& +9XG_{5\phi X} + 2X^2G_{5\phi XX}\big), \\
\Theta \;:=\; & -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} \\
& -8HX^2G_{4XX} + \dot{\phi}G_{4\phi} + 2X\dot{\phi}G_{4\phi X} \\
& -H^2\dot{\phi}\left(5XG_{5X} + 2X^2G_{5XX}\right) \\
& +2HX\left(3G_{5\phi} + 2XG_{5\phi X}\right)
\end{aligned}$$

$\dot{H}$ and stability

k-inflation: $\mathcal{F}_S = M_{\rm Pl}^2\epsilon$ Garriga, Mukhanov (1999)

$$\epsilon = -\frac{\dot{H}}{H^2} > 0 \quad \Longleftrightarrow \quad \text{Stable}$$

In more general cases, the sign of $\dot{H}$
and the stability criteria are not correlated

→ Stable cosmology with $\dot{H} > 0$ is possible

Interesting scenarios with null energy condition violation:

Creminelli *et al.* (2006); Creminelli, Nicolis, Trincherini (2010)

Scalar power spectrum

Normalized mode: $z\zeta = \frac{\sqrt{\pi}}{2}\sqrt{-y}H_\nu^{(1)}(-ky)$

$$z := \sqrt{2}a(\mathcal{F}_S\mathcal{G}_S)^{1/4}$$

Useful time coordinate: $\mathrm{d}y := \frac{c_s}{a}\mathrm{d}t$

$$\nu := \frac{3-\epsilon+g_S}{2-2\epsilon-f_S+g_S}$$

Sound speed: $c_s^2 = \mathcal{F}_S/\mathcal{G}_S$

$$f_S := \frac{\dot{\mathcal{F}}_S}{H\mathcal{F}_S}, \quad g_S := \frac{\dot{\mathcal{G}}_S}{H\mathcal{G}_S}$$

Power spectrum:

$$\mathcal{P}_\zeta \simeq \frac{1}{2}\frac{\mathcal{G}_S^{1/2}}{\mathcal{F}_S^{3/2}}\frac{H^2}{4\pi^2}\Bigg|_{-ky_S=1}$$

Spectral index:

$$n_s - 1 = 3 - 2\nu$$

Tensor power spectrum

Power spectrum:

$$\mathcal{P}_T \simeq 8 \, \frac{\mathcal{G}_T^{1/2}}{\mathcal{F}_T^{3/2}} \frac{H^2}{4\pi^2}\bigg|_{-ky_T=1}$$

Spectral index:

$$n_T = 3 - 2\nu_T$$

$$\nu_T := \frac{3 - \epsilon + g_T}{2 - 2\epsilon - f_T + g_T}$$

can be blue in general

Tensor-to-scalar ratio:

$$r = 16 \left(\frac{\mathcal{F}_S}{\mathcal{F}_T}\right)^{3/2} \left(\frac{\mathcal{G}_S}{\mathcal{G}_T}\right)^{-1/2} = 16 \frac{\mathcal{F}_s}{\mathcal{F}_T} \frac{c_S}{c_T}.$$

Consistency relation

$\mathcal{L}_2,\ \mathcal{L}_4$

$\mathcal{L}_3,\ \mathcal{L}_5$

Potential dominated inflation

$$\mathcal{F}_S \simeq \frac{X}{H^2}(\mathcal{K}+6H^2h_4)+\frac{4\dot{\phi}X}{H}(h_3+H^2h_5)$$

$$\mathcal{G}_S \simeq \frac{X}{H^2}(\mathcal{K}+6H^2h_4)+\frac{6\dot{\phi}X}{H}(h_3+H^2h_5)$$

$$\mathcal{F}_T \simeq \mathcal{G}_T \simeq 2g_4$$

Taylor coefficients in K and G_i

New consistency relation

$$c_s^2 \simeq \frac{2}{3}$$

$$r \simeq -\frac{32\sqrt{6}}{9}n_T$$

Usual consistency relation

$$c_s^2 \simeq 1$$

$$r \simeq -8n_T$$

Conclusion

G^2 is **the most general** single-field inflation model

✓ contains *all* the (single-field) inflation models proposed so far as special cases

✓ no further generalization is possible

Thank you!