RESCEU/DENET Summer School @ Kumamoto
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Non-Gaussianity from Curvatons Revisited

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based on: arXiv:1107.6011

with Masahiro Kawasaki, Fuminobu Takahashi

The Curvaton Mechanism

Linde, Mukhanov ’97 Enqvist, Sloth ’01 Lyth, Wands ’01 Moroi, Takahashi ’01

- seeds cosmological density perturbations from scalar field fluctuations sourced during inflation (or Horava-Lifshitz gravity, Galilean mechanism, etc.)
- has only been studied for rather trivial curvaton potentials, e.g. quadratic
- however, a quadratic curvaton potential (or more generally, a positively curved potential) cannot produce the red-tilted perturbation spectrum
- concrete microscopic curvaton models also realize intricate energy potentials

The Curvaton Mechanism

Linde, Mukhanov ’97 Enqvist, Sloth ’01 Lyth, Wands ’01 Moroi, Takahashi ’01

- seeds cosmological density perturbations from

We need to go beyond simple quadratic curvatons!

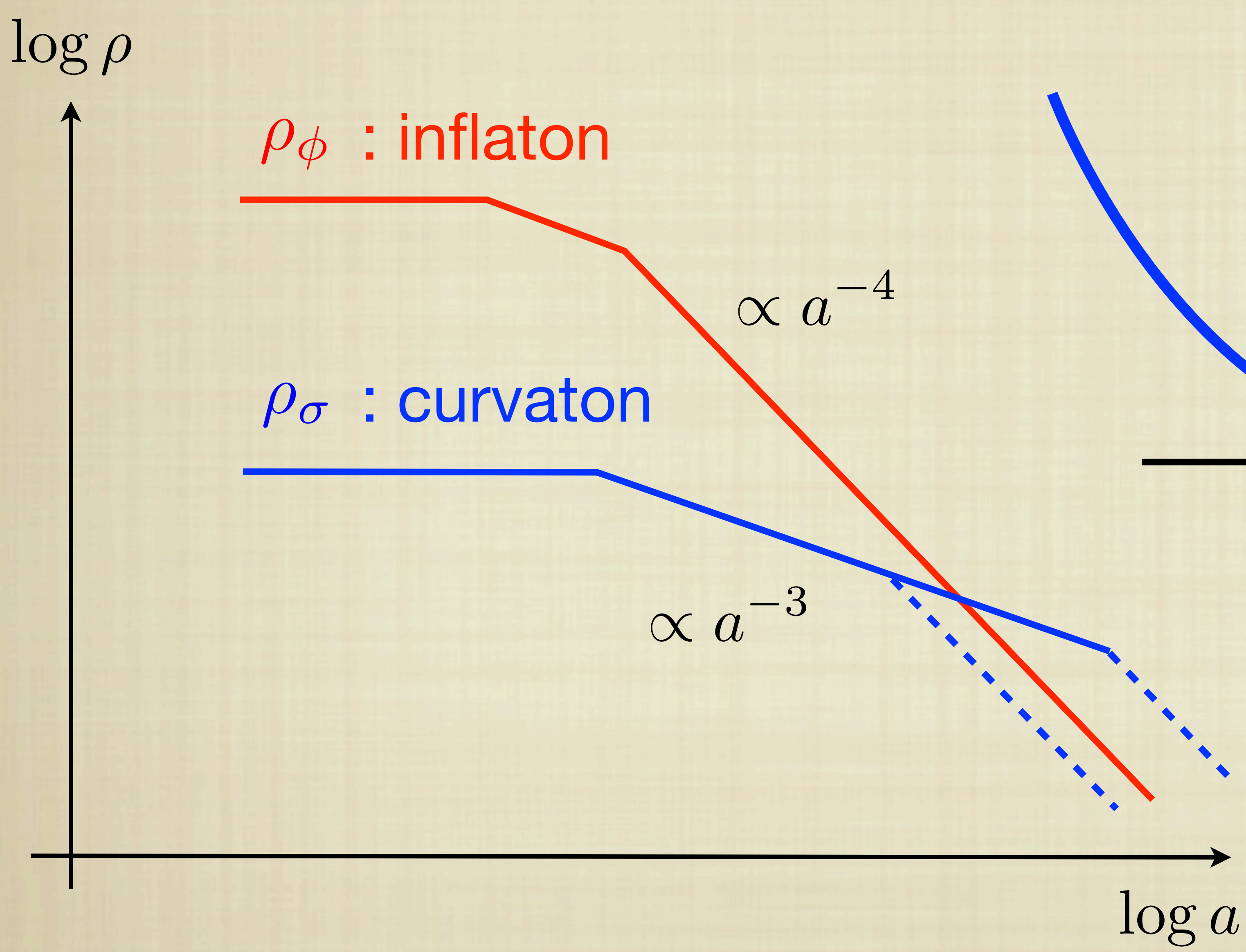
- however, a quadratic curvaton potential (or more generally, a positively curved potential) cannot produce the red-tilted perturbation spectrum

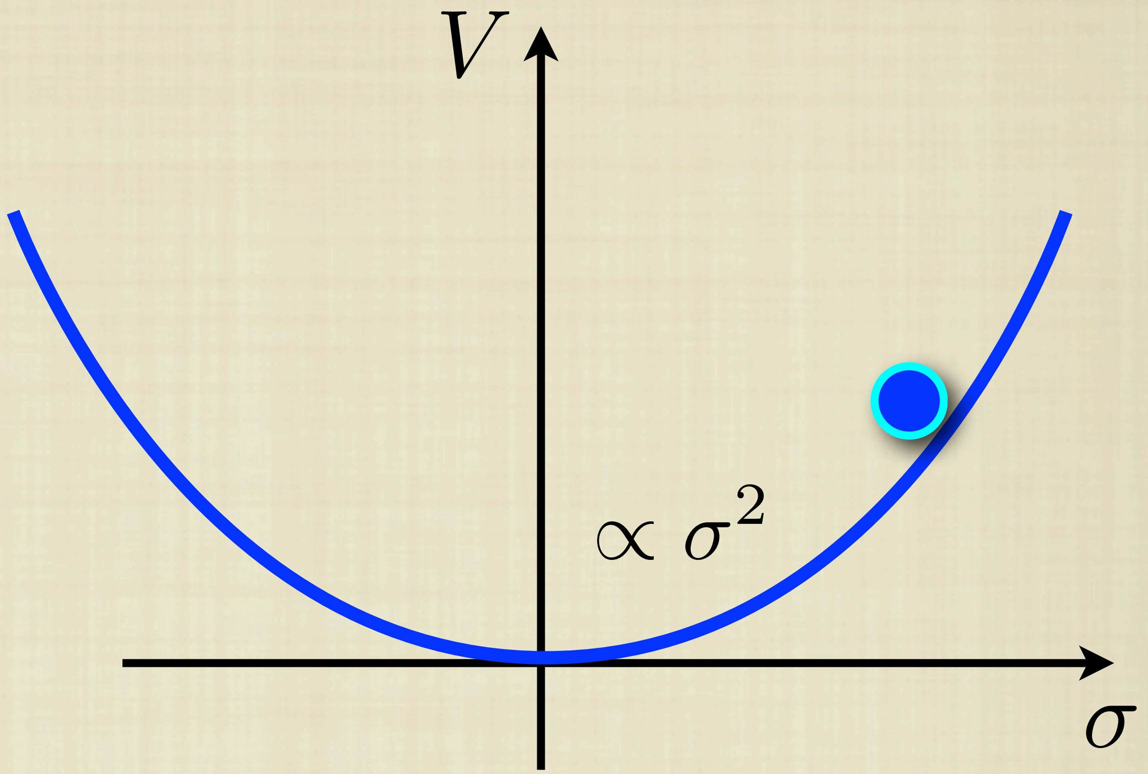
- concrete microscopic curvaton models also realize intricate energy potentials

This Work

- we investigate density perturbations sourced by a curvaton with a generic energy potential
- new features for non-quadratic curvatons
- case study: curvaton = pseudo-NG boson

The Curvaton Scenario





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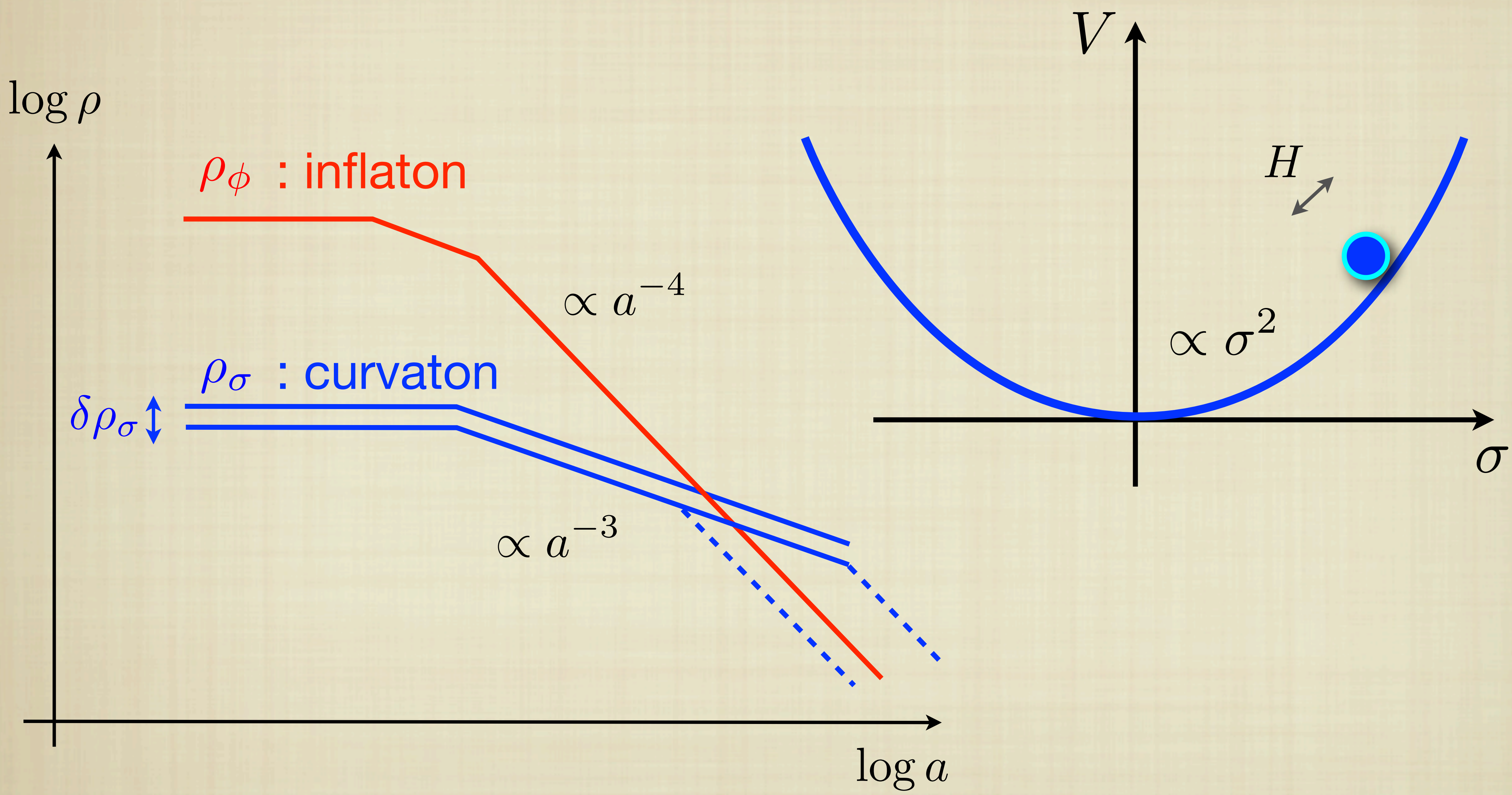

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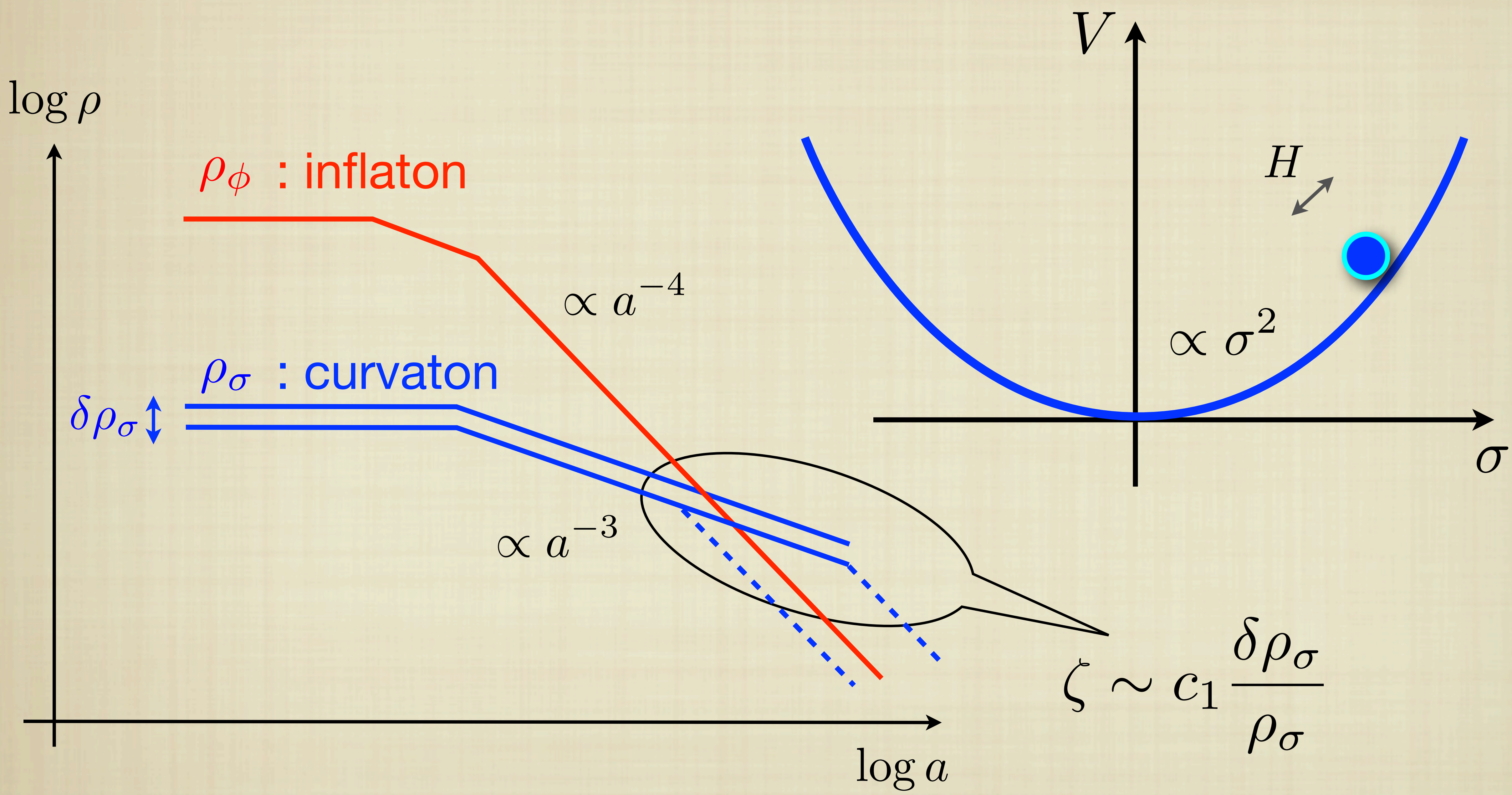
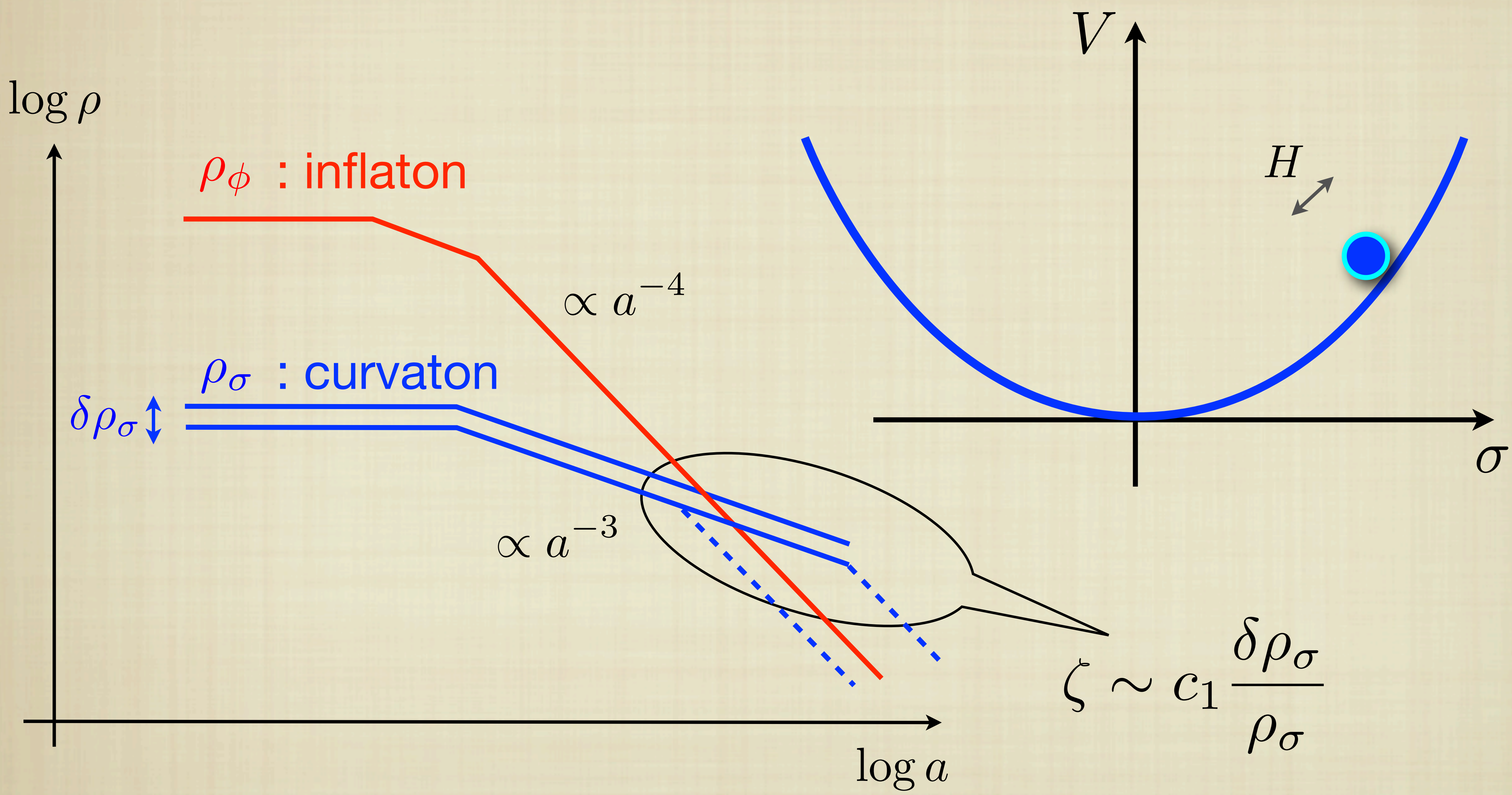


Curvatons with Arbitrary Potentials



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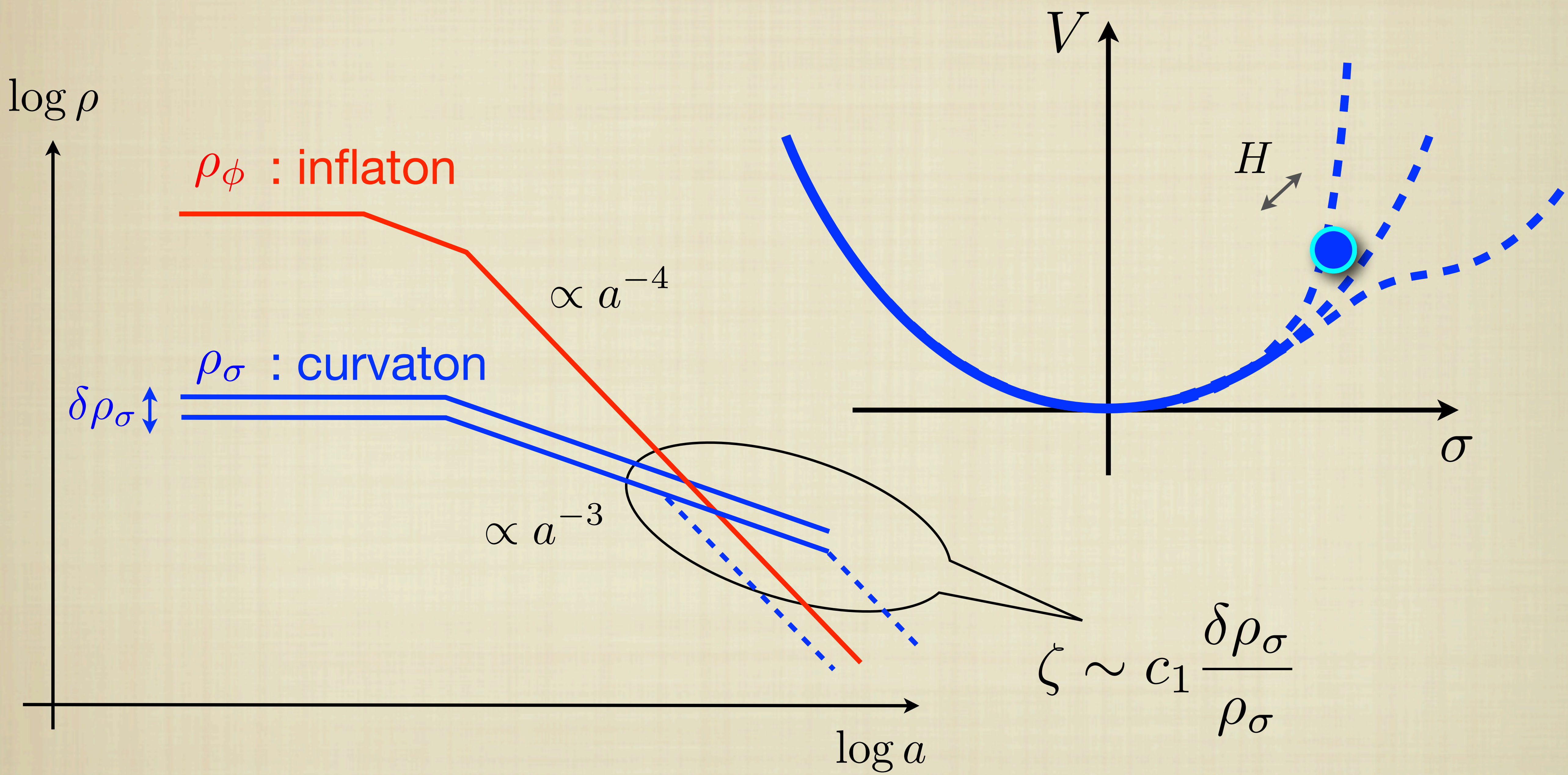
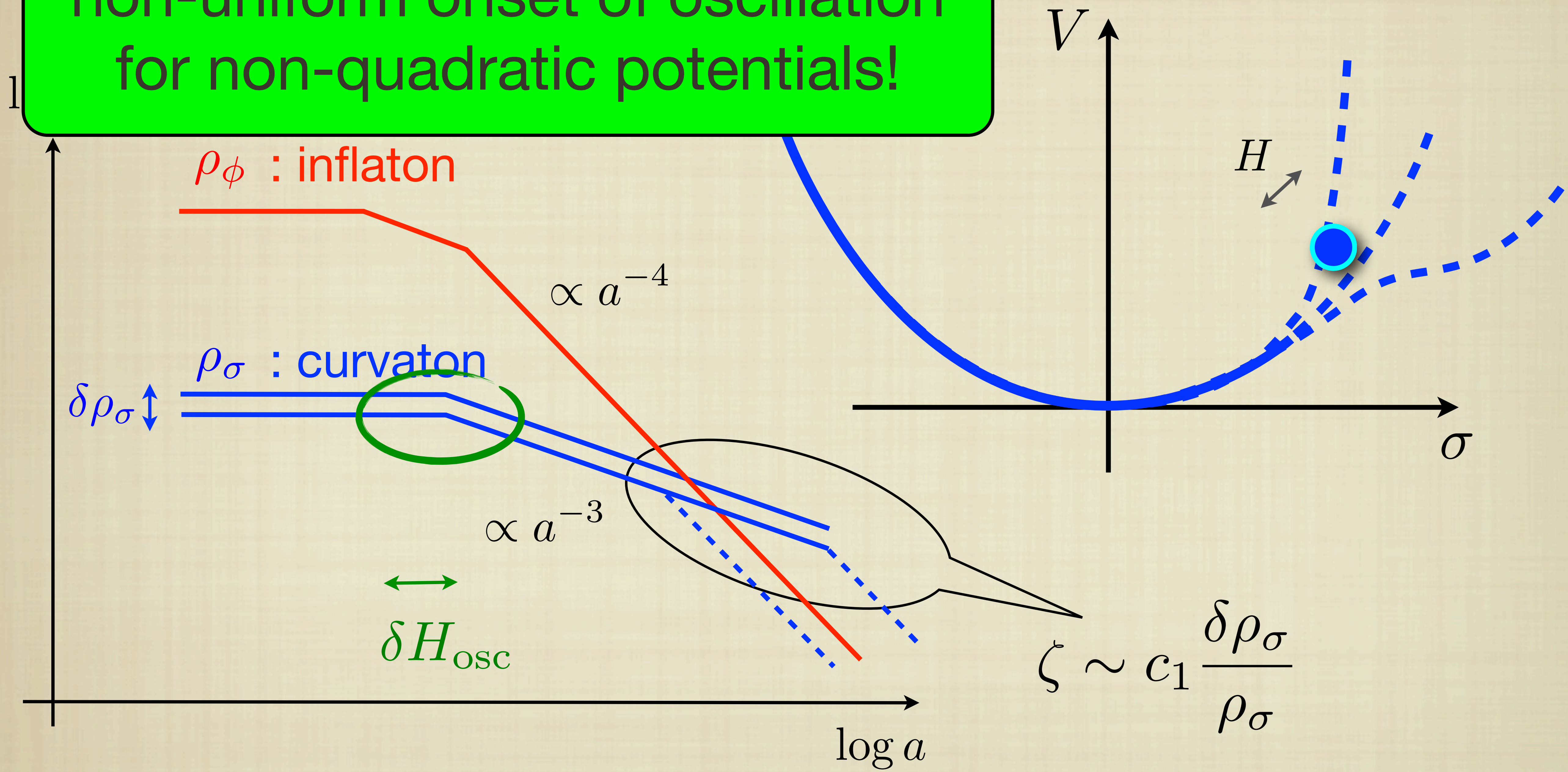


Curvatons with Arbitrary Potentials

non-uniform onset of oscillation
for non-quadratic potentials!



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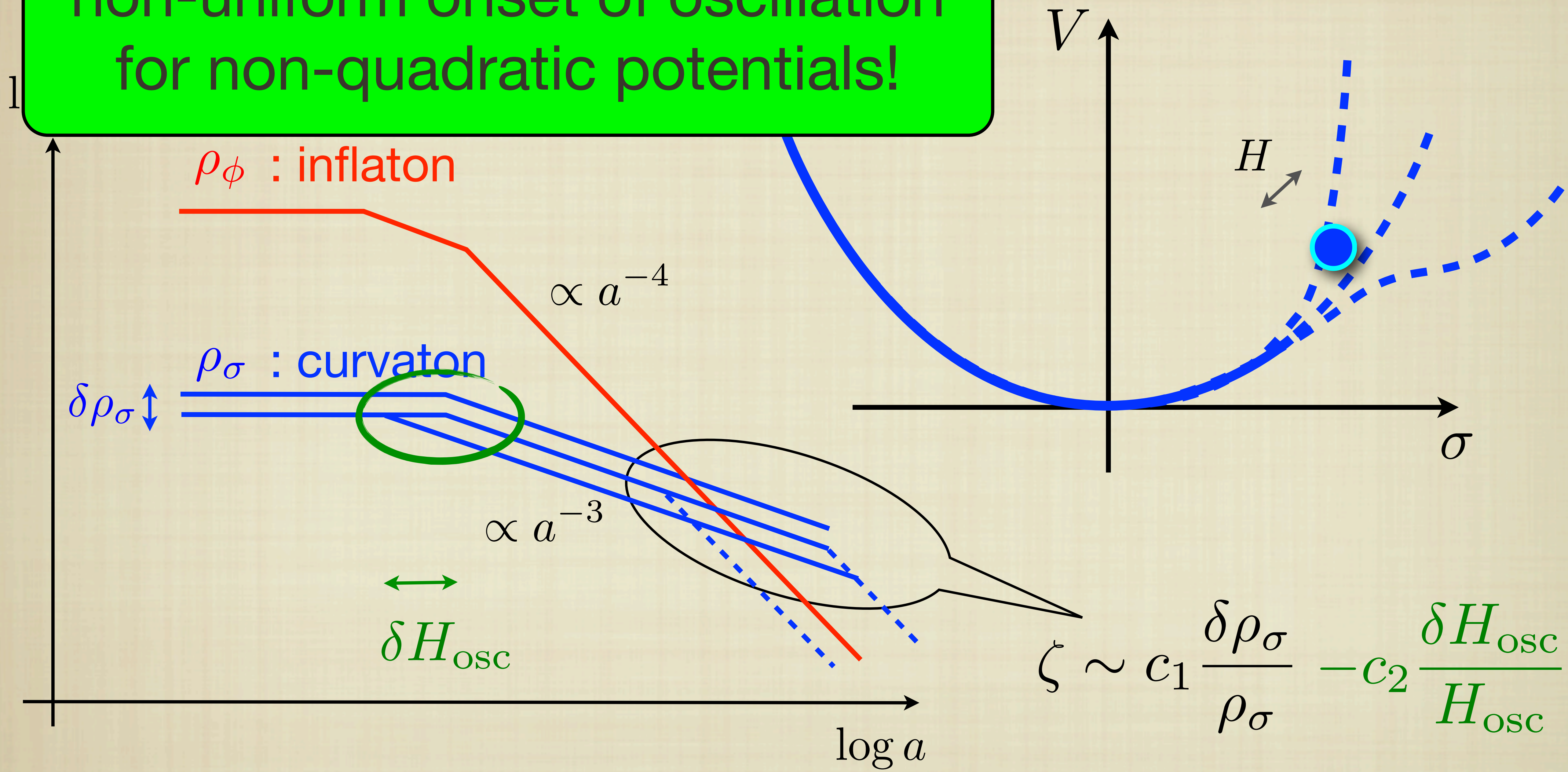

Additional contributions to the density perturbations!

Density Perturbations

$$\mathcal{P}_\zeta = \left(\frac{\partial \mathcal{N}}{\partial \sigma_*} \frac{H_*}{2\pi} \right)^2$$

$$\frac{\partial \mathcal{N}}{\partial \sigma_*} = \frac{r}{4+3r} \left(1 - X(\sigma_{\rm osc})\right)^{-1} \left\{ \frac{V'(\sigma_{\rm osc})}{V(\sigma_{\rm osc})} - \frac{3X(\sigma_{\rm osc})}{\sigma_{\rm osc}} \right\} \frac{V'(\sigma_{\rm osc})}{V'(\sigma_*)}$$

$r \equiv \dfrac{\rho_\sigma}{\rho_r}$ @ curvaton decay

$*$: @ horizon exit

$\rm osc$: @ onset of curvaton oscillation

$$X(\sigma_{\rm osc}) \equiv \frac{1}{2(c-3)} \left(\frac{\sigma_{\rm osc} V''(\sigma_{\rm osc})}{V'(\sigma_{\rm osc})} - 1 \right)$$

: effects due to non-uniform onset of oscillation

Density Perturbations

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spectral index $\quad n_s - 1 \equiv \frac{d \ln \mathcal{P}_\zeta}{d \ln k} = \frac{2}{3} \frac{V''(\sigma_*)}{H_*^2} + 2 \frac{\dot{H}_*}{H_*^2}$

observational data $\quad n_s = 0.963 \pm 0.012 \quad$ (WMAP7, 68%CL)

requires the curvaton to be tachyonic during inflation

(or $\dot{H}/H^2 \sim -0.01$, implying large-field inflation)

Non-Gaussianity

$$f_{\rm NL} = \frac{40(1+r)}{3r(4+3r)} + \frac{5(4+3r)}{6r}\left\{\frac{V'(\sigma_{\rm osc})}{V(\sigma_{\rm osc})} - \frac{3X(\sigma_{\rm osc})}{\sigma_{\rm osc}}\right\}^{-1}\Bigg[(1-X(\sigma_{\rm osc}))^{-1}X'(\sigma_{\rm osc})$$
$$+\left\{\frac{V'(\sigma_{\rm osc})}{V(\sigma_{\rm osc})} - \frac{3X(\sigma_{\rm osc})}{\sigma_{\rm osc}}\right\}^{-1}\left\{\frac{V''(\sigma_{\rm osc})}{V(\sigma_{\rm osc})} - \frac{V'(\sigma_{\rm osc})^2}{V(\sigma_{\rm osc})^2} - \frac{3X'(\sigma_{\rm osc})}{\sigma_{\rm osc}} + \frac{3X(\sigma_{\rm osc})}{\sigma_{\rm osc}^2}\right\}$$
$$+\frac{V''(\sigma_{\rm osc})}{V'(\sigma_{\rm osc})} - (1-X(\sigma_{\rm osc}))\frac{V''(\sigma_*)}{V'(\sigma_{\rm osc})}\Bigg]$$

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Non-Gaussianity

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$$+\left\{\frac{V'(\sigma_{\mathrm{osc}})}{V(\sigma_{\mathrm{osc}})} - \frac{3X(\sigma_{\mathrm{osc}})}{\sigma_{\mathrm{osc}}}\right\}^{-1}\left\{\frac{V''(\sigma_{\mathrm{osc}})}{V(\sigma_{\mathrm{osc}})} - \frac{V'(\sigma_{\mathrm{osc}})^2}{V(\sigma_{\mathrm{osc}})^2} - \frac{3X'(\sigma_{\mathrm{osc}})}{\sigma_{\mathrm{osc}}} + \frac{3X(\sigma_{\mathrm{osc}})}{\sigma_{\mathrm{osc}}^2}\right\}$$

$$\left. + \frac{V''(\sigma_{\mathrm{osc}})}{V'(\sigma_{\mathrm{osc}})} - (1-X(\sigma_{\mathrm{osc}}))\frac{V''(\sigma_*)}{V'(\sigma_{\mathrm{osc}})}\right]$$

$r \equiv \frac{\rho_\sigma}{\rho_r}$ @ curvaton decay

$*$: @ horizon exit

osc : @ onset of curvaton oscillation

(

cf. quadratic curvatons

$$f_{\mathrm{NL}} = \frac{5}{12}\left(-3 + \frac{4}{r} + \frac{8}{4+3r}\right)$$

$f_{\mathrm{NL}} \gg 1$ only for curvatons decaying when subdominant ($r \ll 1$)

)

Non-Gaussianity

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Large f_{NL} (with either sign) possible for both dominant/subdominant curvatons!

$r \equiv \dfrac{\rho_\sigma}{\rho_r}$ @ curvaton decay

$*$: @ horizon exit

$\rm osc$: @ onset of curvaton oscillation

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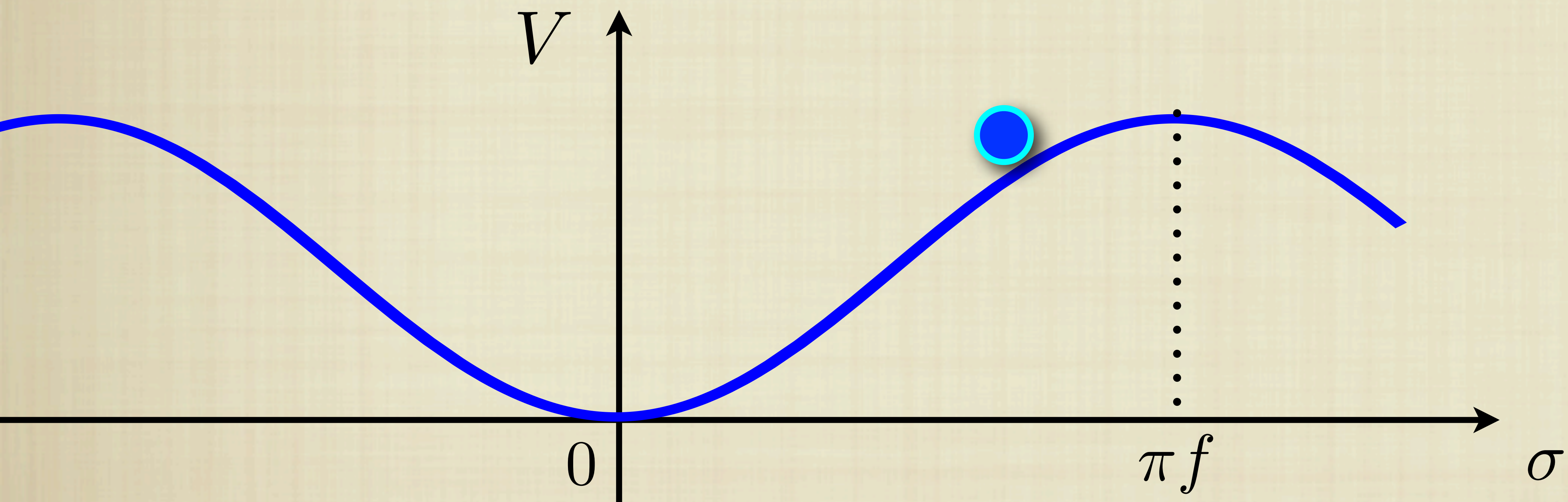
$f_{\rm NL} \gg 1$ only for curvatons decaying when subdominant ($r \ll 1$)

case study : Curvaton = pseudo-NG boson of a broken U(1) symmetry

$$V(\sigma) = \Lambda^4 \left[1 - \cos\left(\frac{\sigma}{f}\right)\right]$$



curvaton decay rate : $\Gamma_\sigma \sim \frac{1}{16\pi}\frac{m^3}{f^2} = \frac{1}{16\pi}\frac{\Lambda^6}{f^5}$

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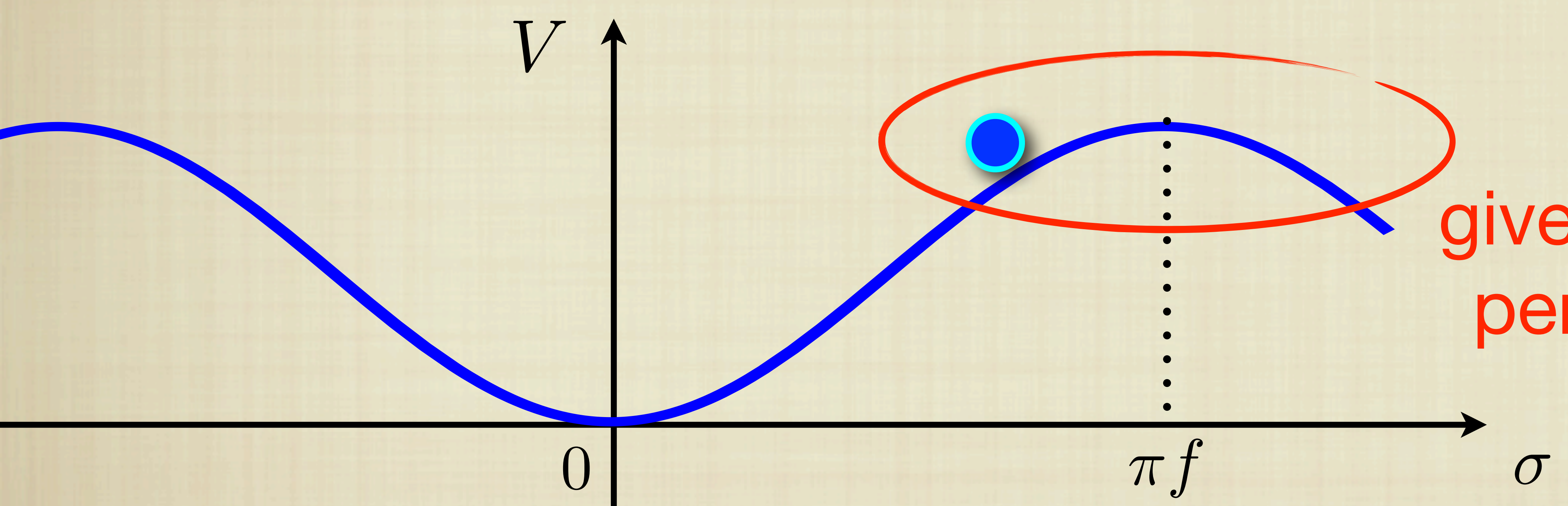

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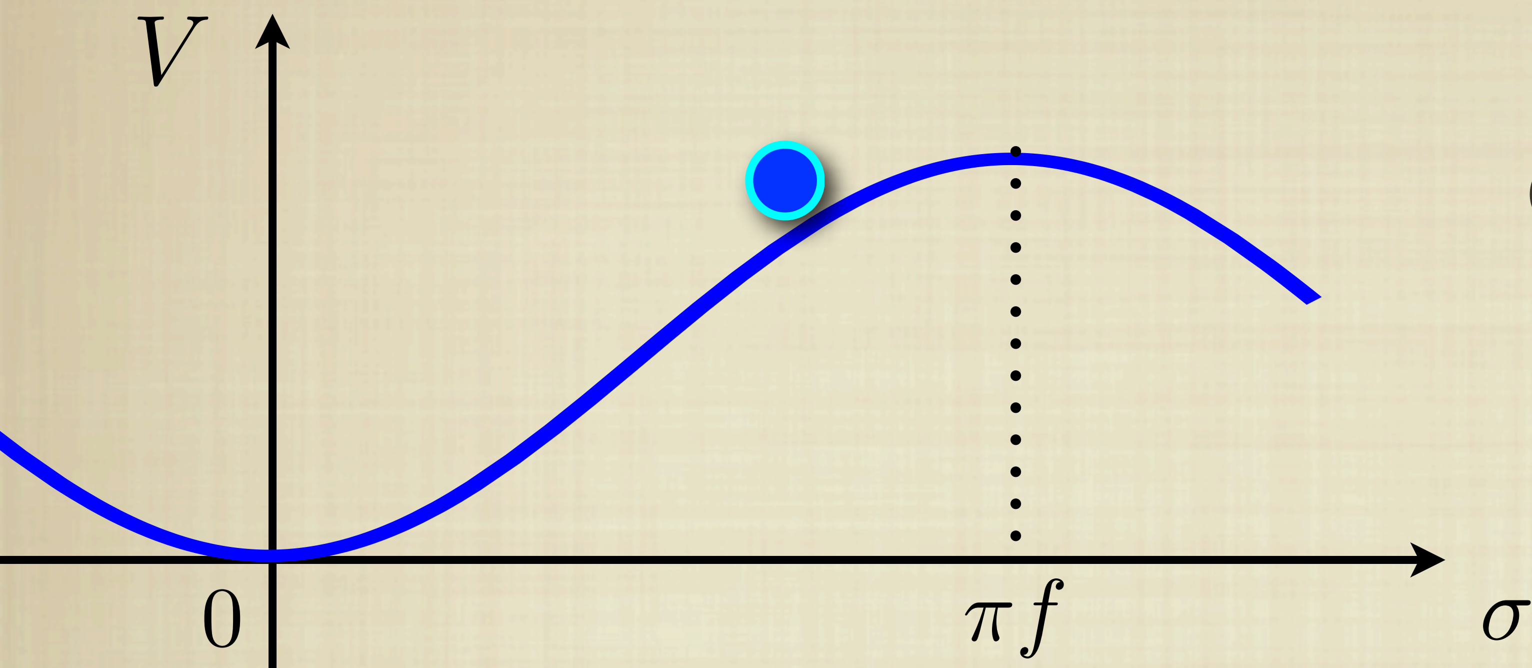
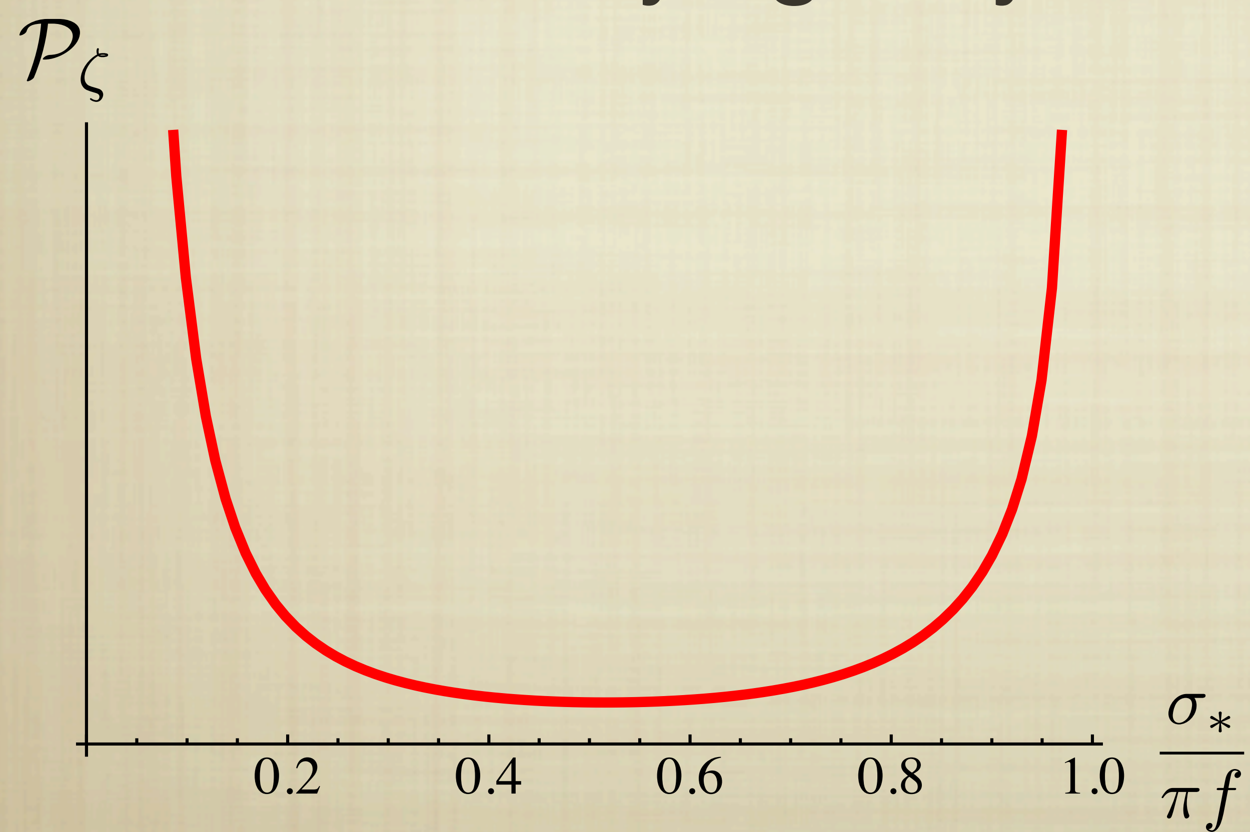
Density Pert. from a NG-Curvaton

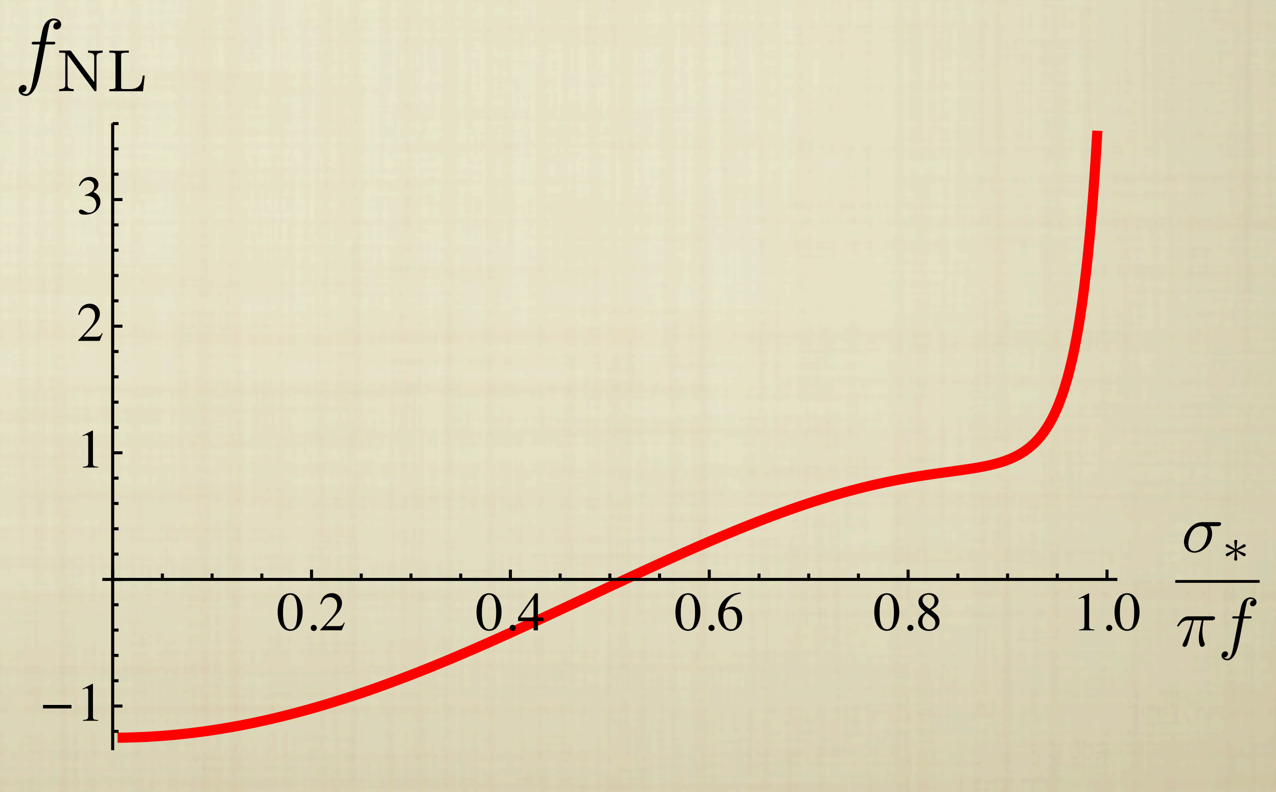


curvaton dominant case,

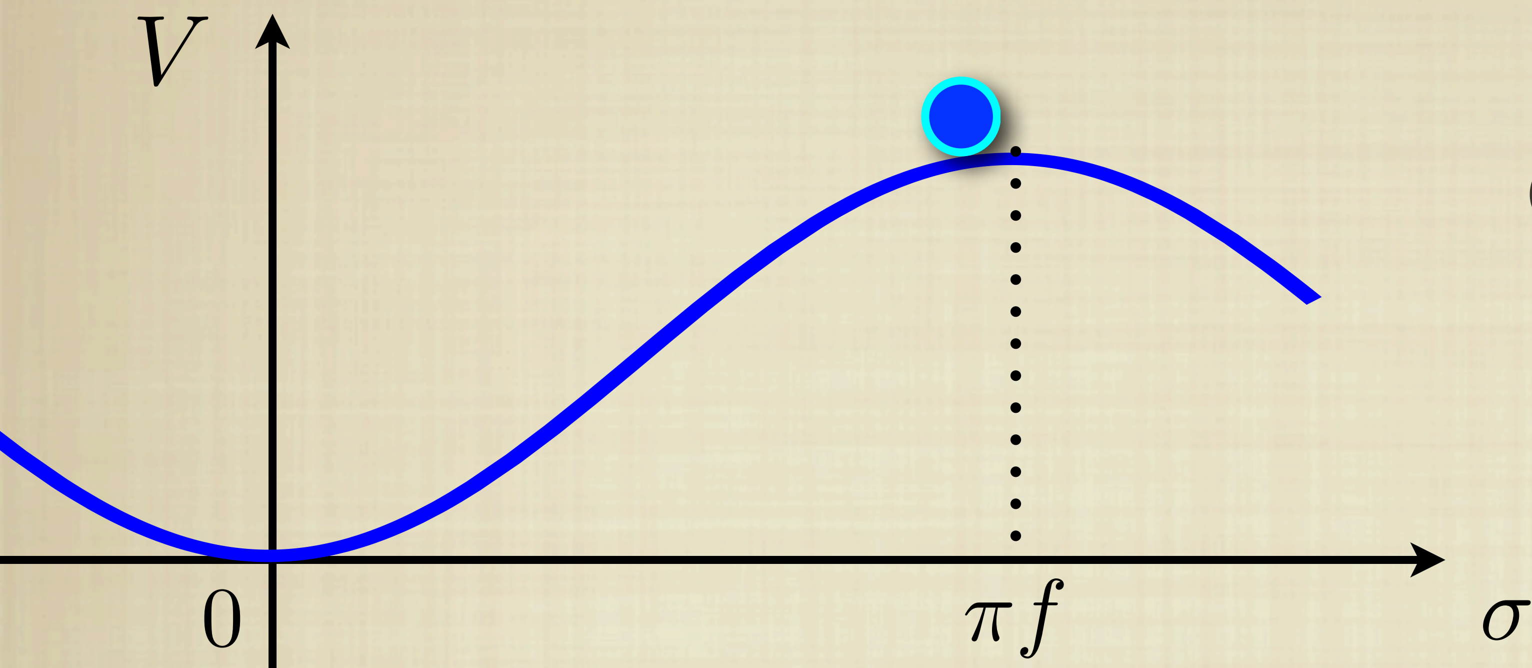
i.e. $r \equiv \left.\frac{\rho_\sigma}{\rho_r}\right|_{\rm dec} \gg 1$

When varying only σ_* :





Density Pert. from the Hilltop



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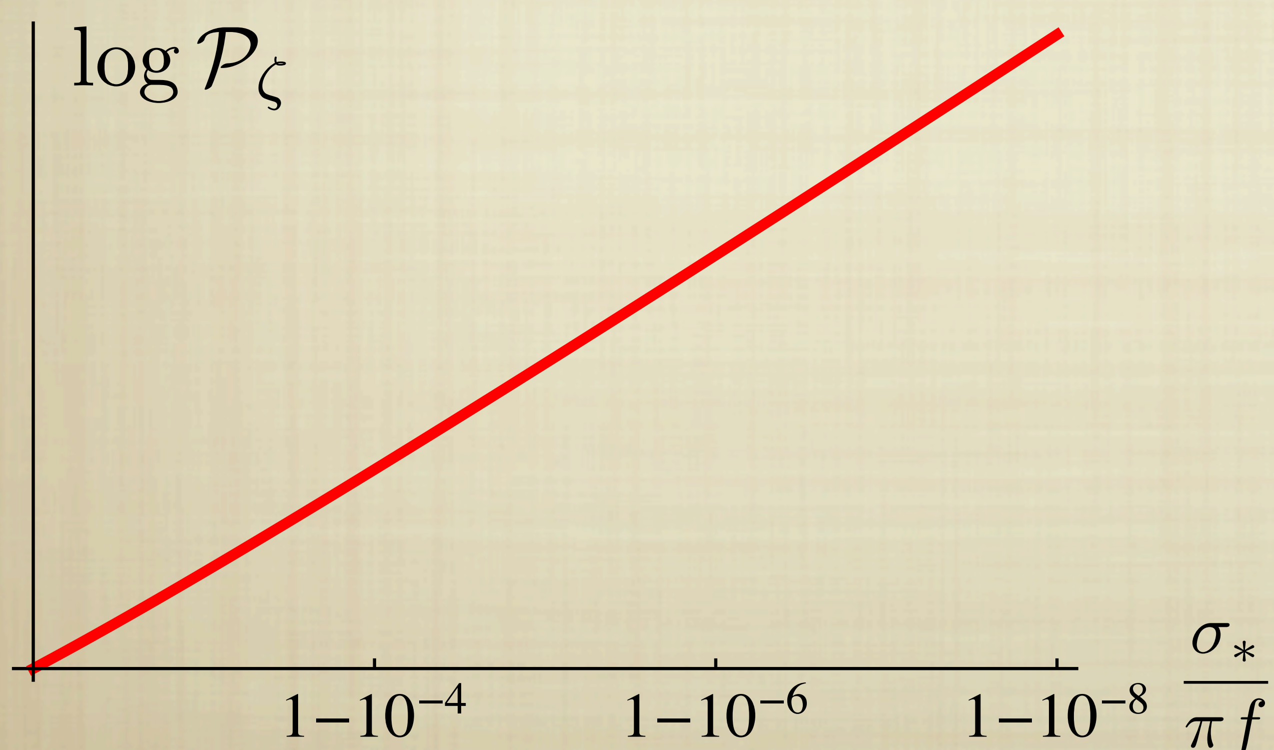

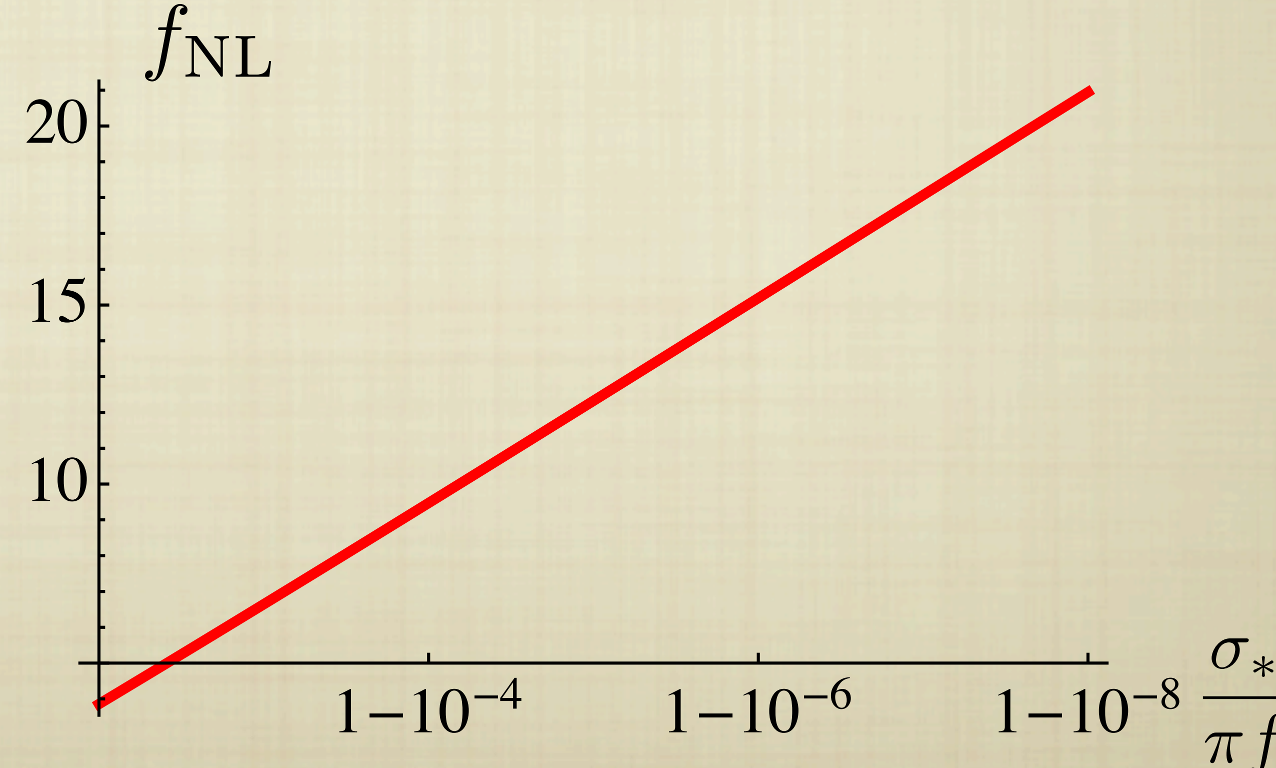

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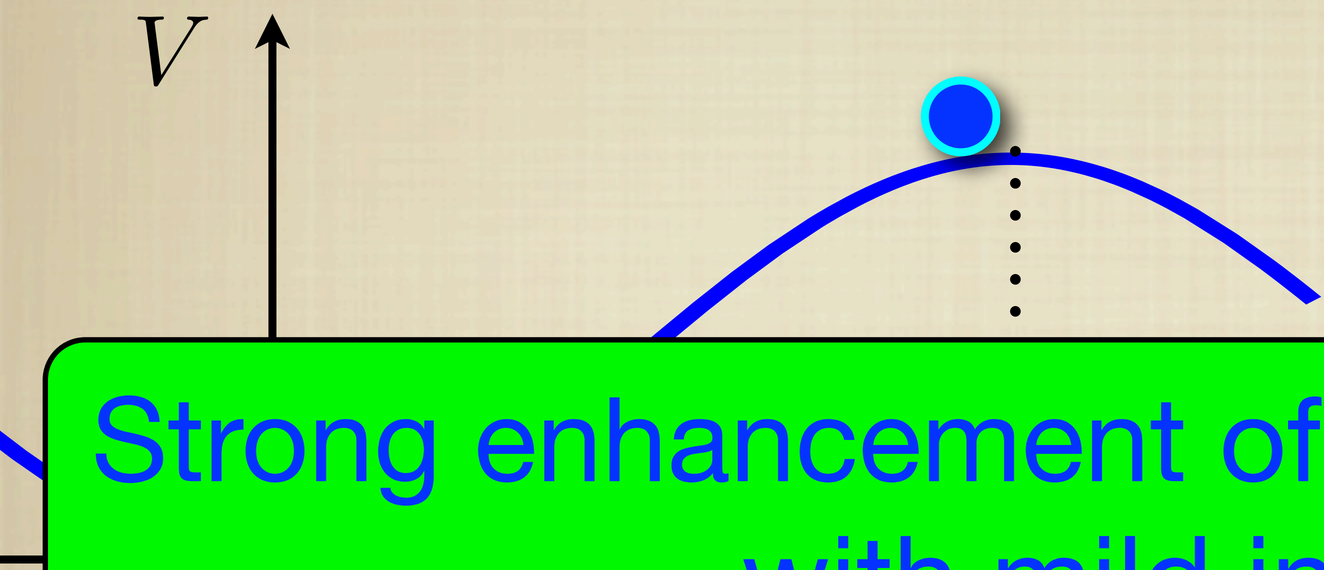
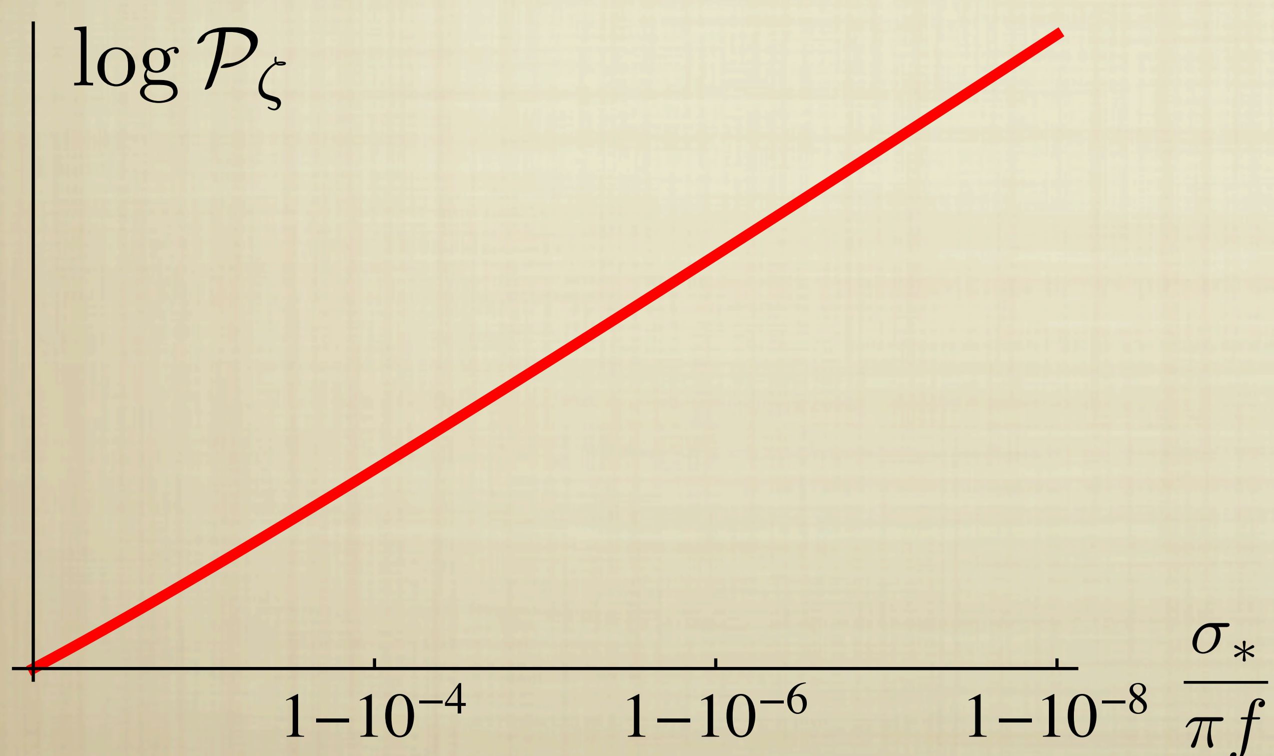
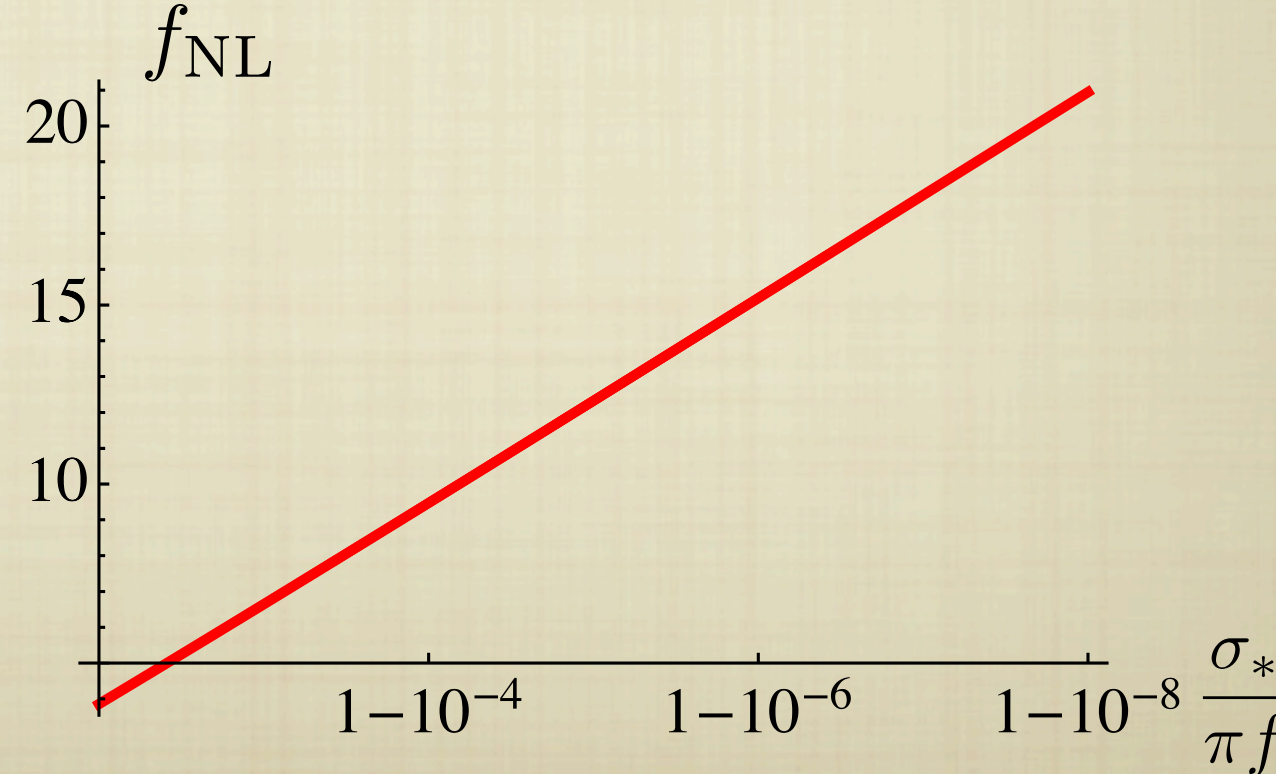


curvaton dominant case,

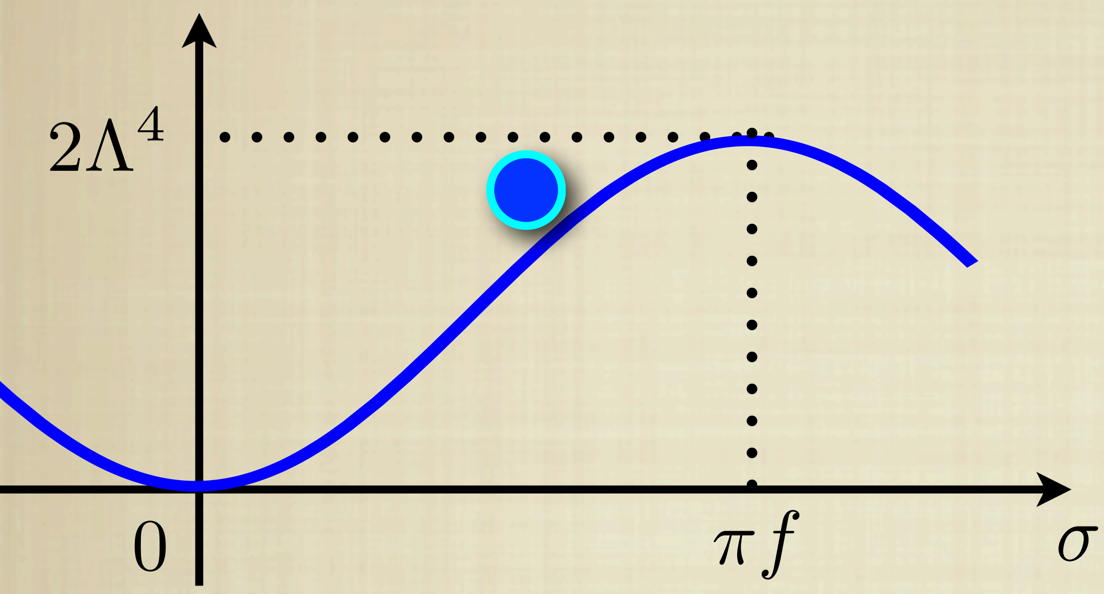
Strong enhancement of linear-order density pert. with mild increase of f_{NL} towards the hilltop.

When varying only σ_* :



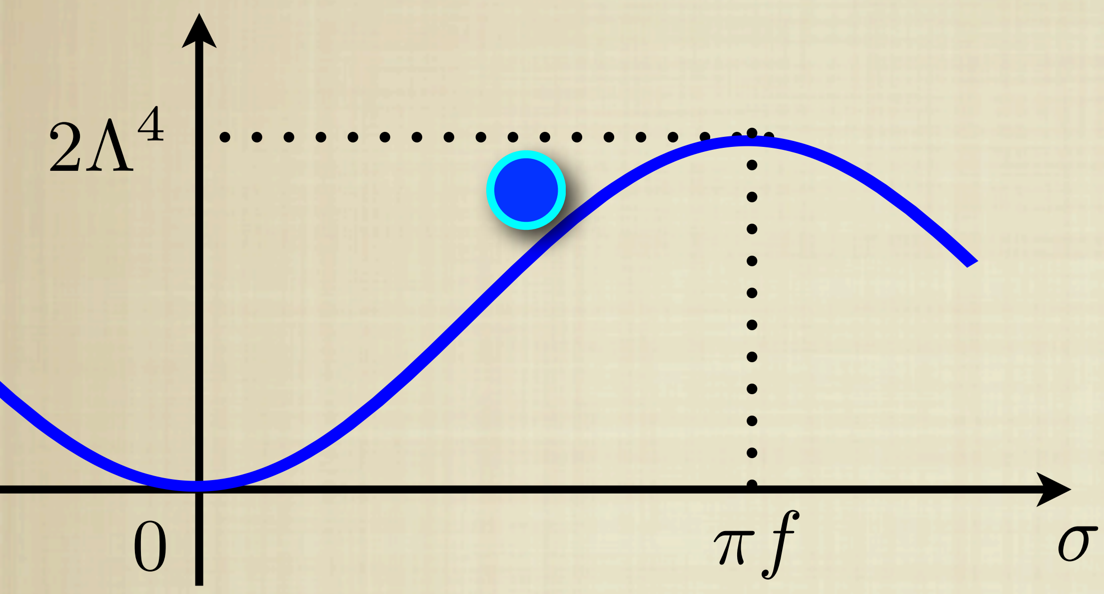


Windows for Inflation/Reheating Scales



$$(f,\ \Lambda,\ H_{\mathrm{inf}},\ T_{\mathrm{reh}},\ \sigma_*)$$

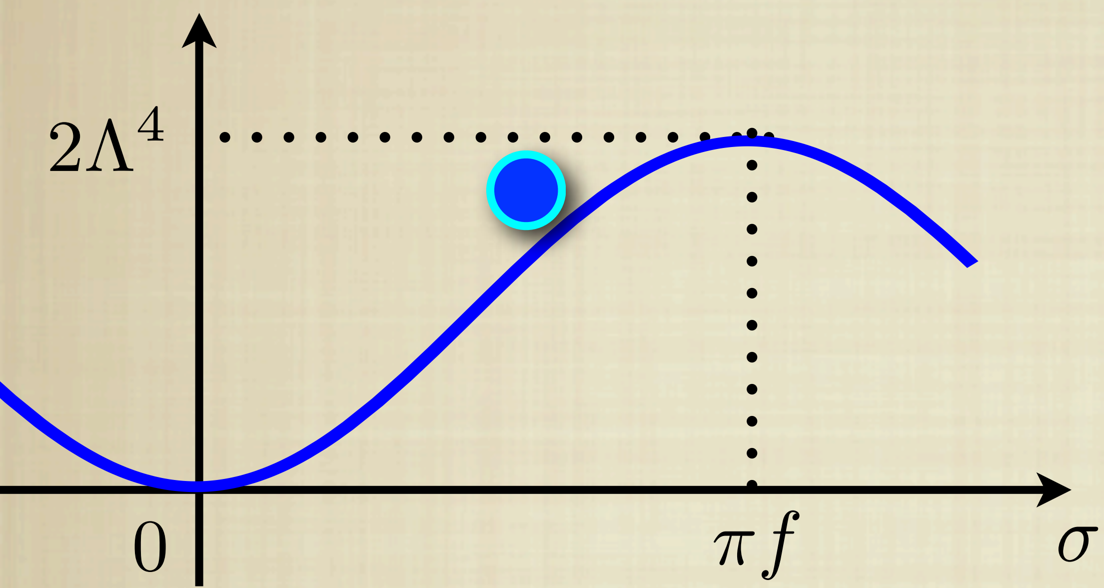
Windows for Inflation/Reheating Scales



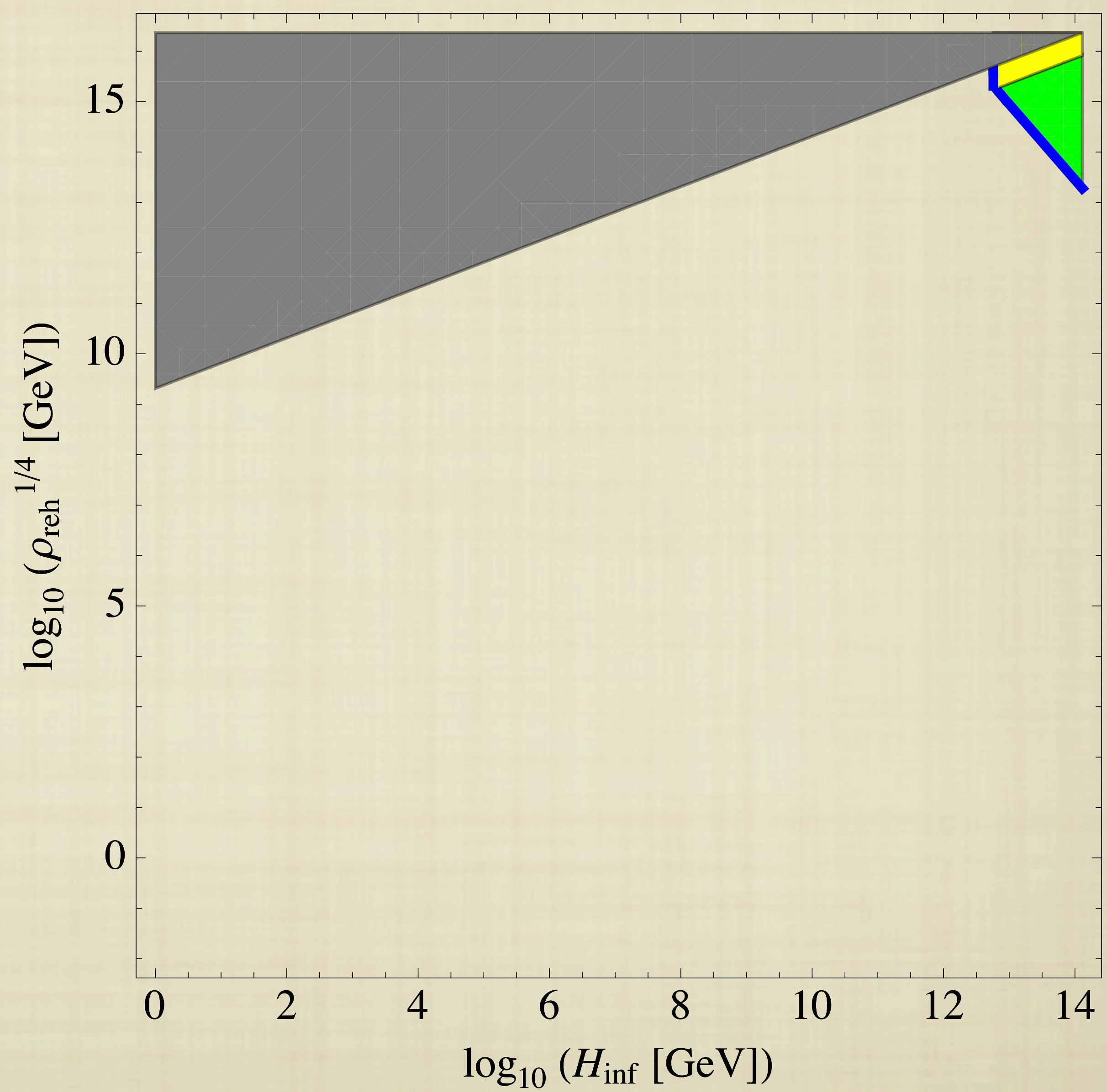
$$(\cancel{f,}\ \cancel{\Lambda,}\ H_{\rm inf}\,,\ T_{\rm reh}\,,\ \sigma_*)$$

- COBE norm.
- spectral index

Windows for Inflation/Reheating Scales



$$\frac{\sigma_*}{\pi f} = \frac{3}{4}$$

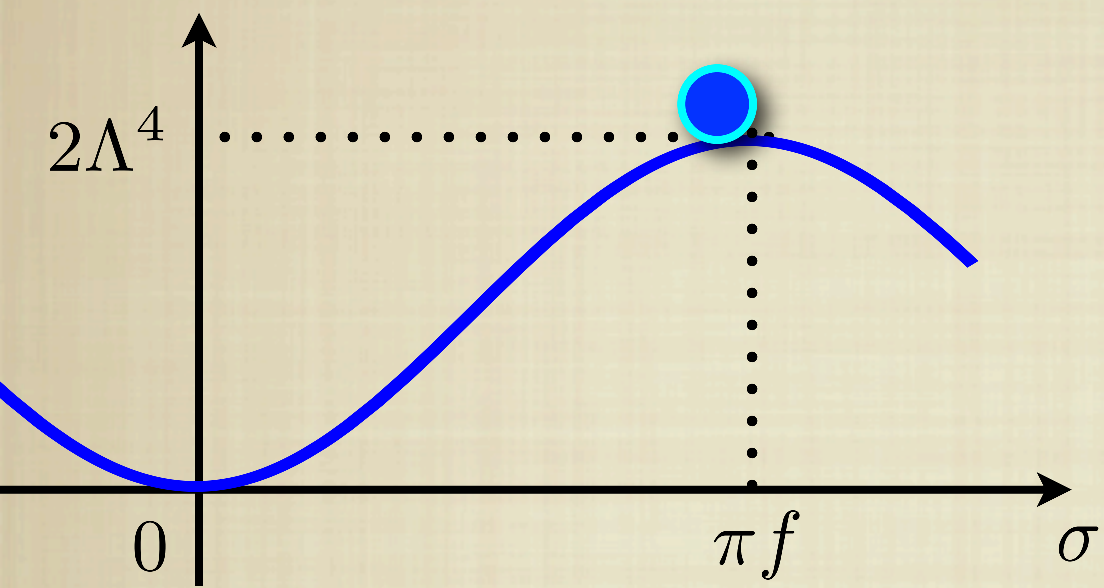


$$(\cancel{f,}\ \cancel{\Lambda,}\ H_{\rm inf},\, T_{\rm reh},\, \sigma_*)$$

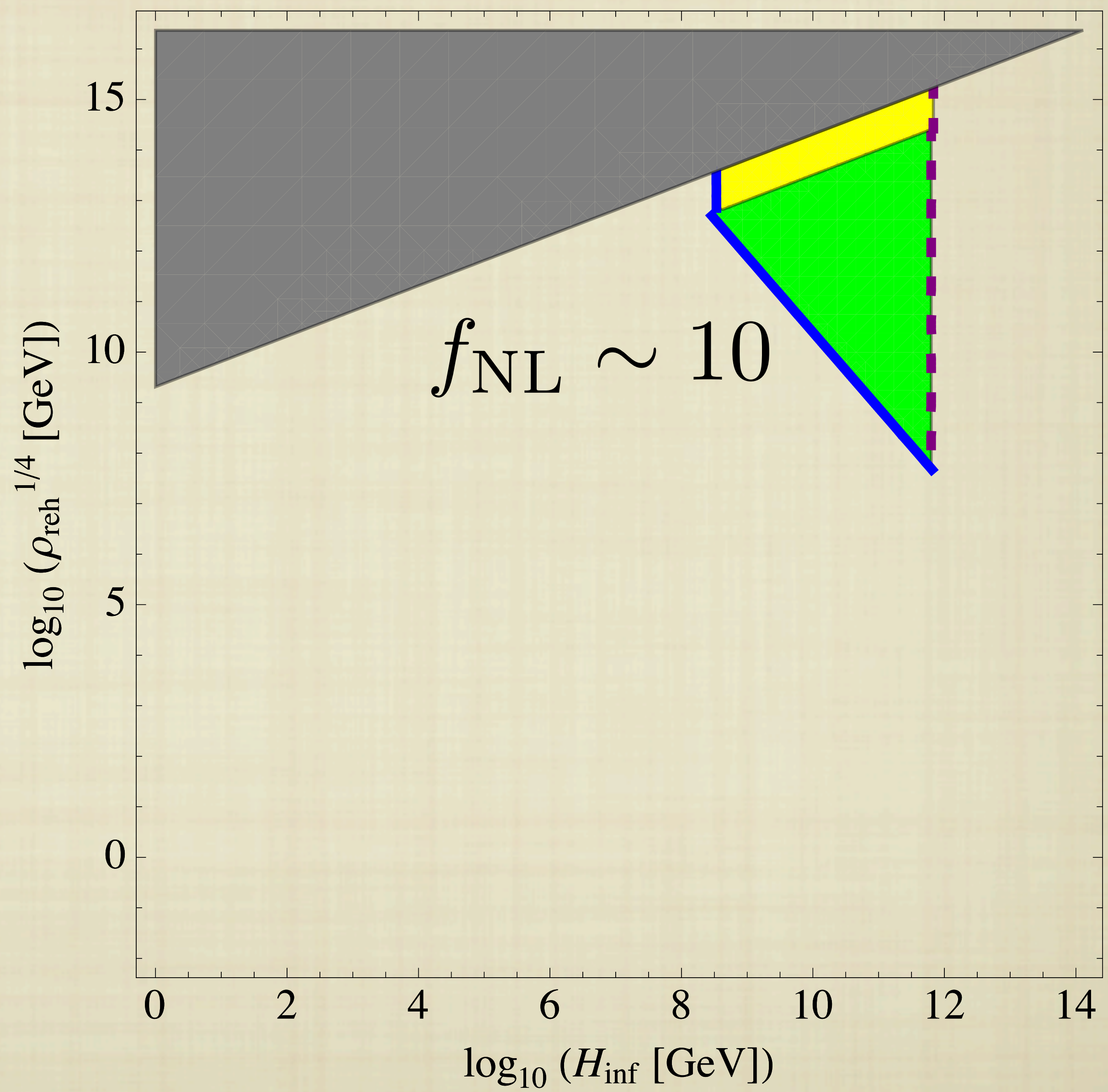
- COBE norm.
- spectral index

curvaton dominant case

Windows for Inflation/Reheating Scales



$$\frac{\sigma_*}{\pi f} = 1 - 10^{-4}$$



$$(\cancel{f,}\ \cancel{\Lambda,}\ H_{\rm inf},\ T_{\rm reh},\ \sigma_*)$$

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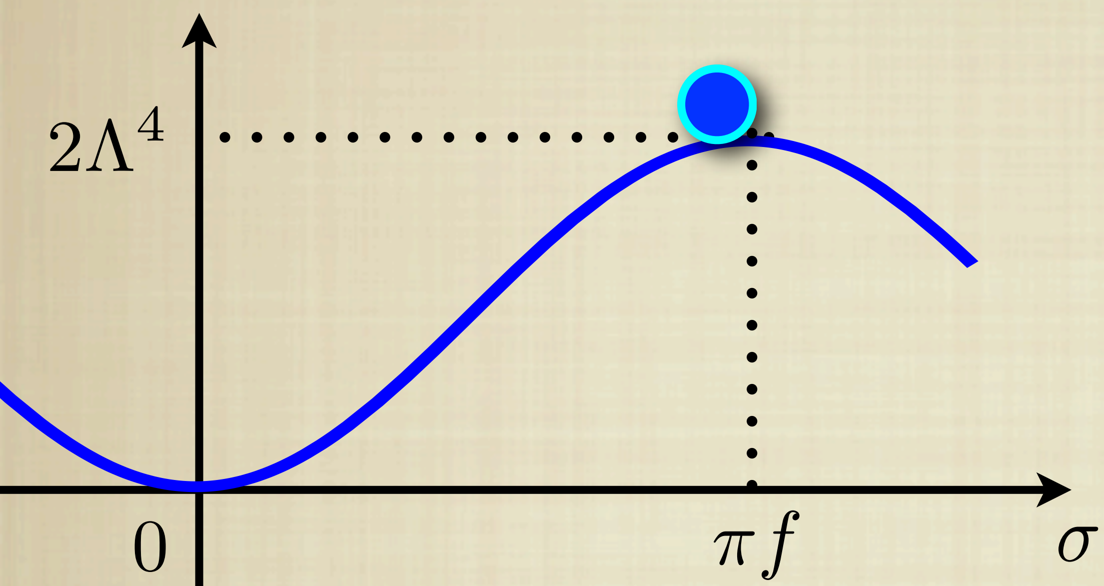
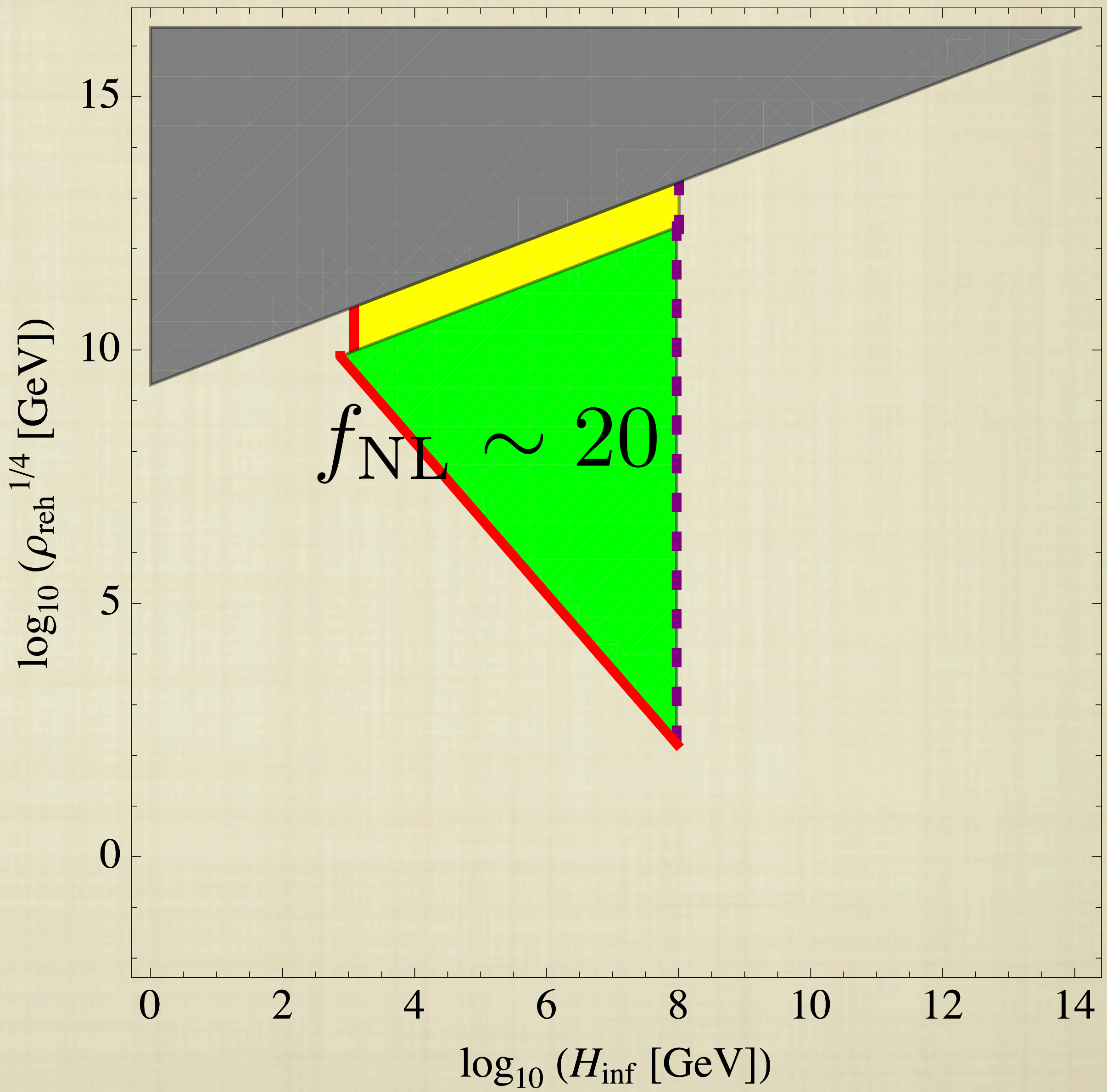
Windows for Inflation/Reheating Scales



$$\frac{\sigma_*}{\pi f} = 1 - 10^{-8}$$

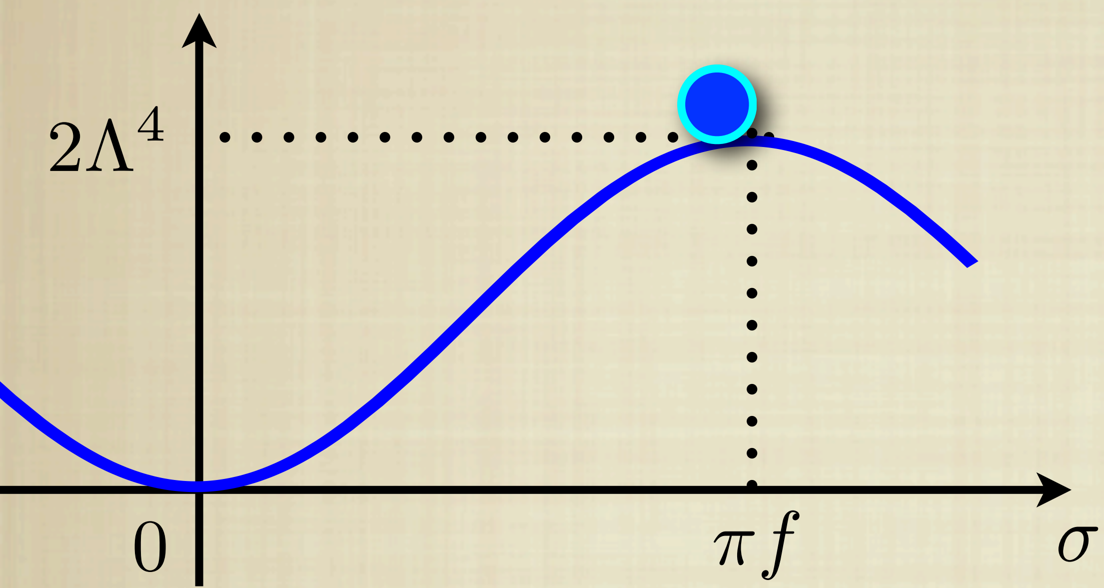
$$(\cancel{f}, \cancel{\Lambda}, H_{\rm inf}, T_{\rm reh}, \sigma_*)$$

- COBE norm.
- spectral index



curvaton dominant case

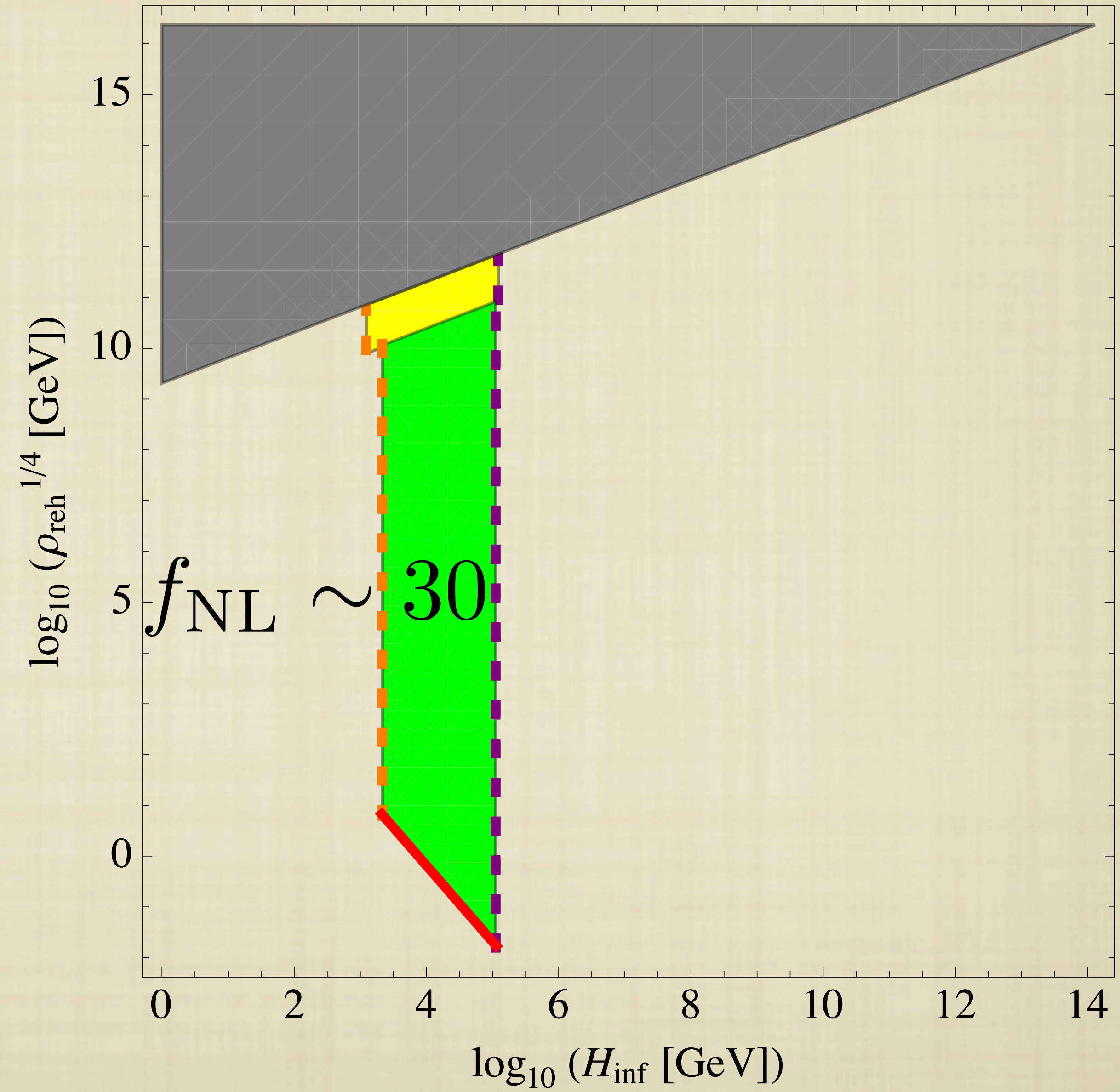
Windows for Inflation/Reheating Scales



$$\frac{\sigma_*}{\pi f} = 1 - 10^{-11}$$

$$(\cancel{f}, \cancel{\Lambda}, H_{\rm inf}, T_{\rm reh}, \sigma_*)$$

- COBE norm.
- spectral index



curvaton dominant case

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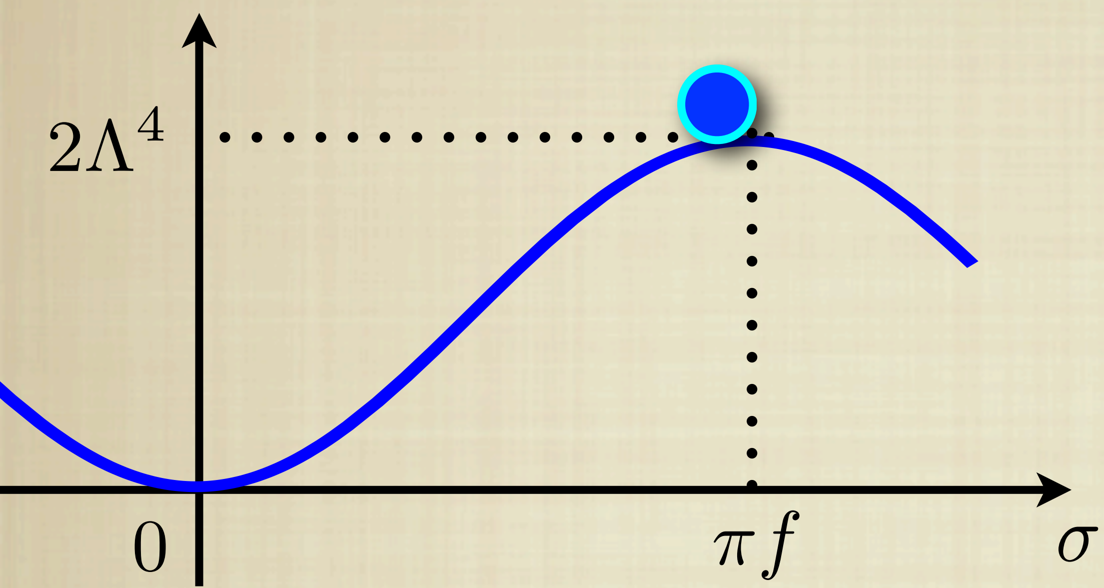
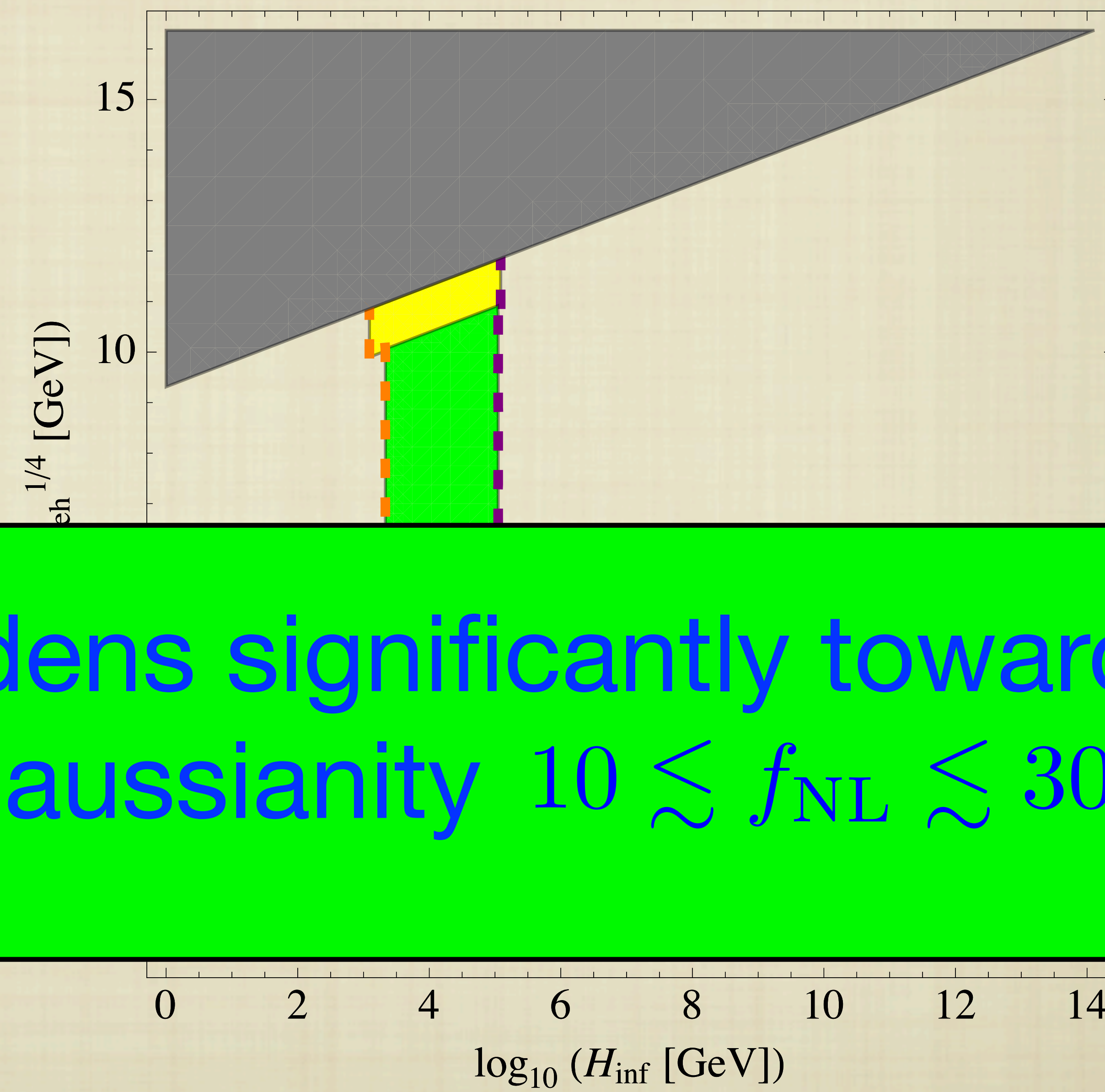


$$\frac{\sigma_*}{\pi f} = 1 - 10^{-11}$$



Allowed window broadens significantly towards the hilltop,with non-Gaussianity $10 \lesssim f_{\rm NL} \lesssim 30$.

- spectral index

curvaton dominant case

Summary

- We investigated density perturbations sourced by a curvaton with a generic potential.
- A non-quadratic curvaton experiences a non-uniform onset of its oscillations, which can strongly enhance/suppress the density perturbations.
- f_{NL} can be large with either sign, no matter the curvaton dominates/subdominates the universe upon decay.
- NG-curvatons at the hilltop work with a wide range of inflation/reheating scales, while predicting $10 \lesssim f_{\rm NL} \lesssim 30.$
- Opens up new possibilities for the curvaton paradigm.

Backup Slides

δN

case $t_{\rm reh} < t_{\rm osc}$:

$$\frac{\partial \mathcal{N}}{\partial \sigma_*} = \frac{r}{4+3r}\frac{\partial}{\partial \sigma_*}\left(\ln \rho_{\sigma \rm osc} - \frac{3}{4}\ln H_{\rm osc}^2\right)$$

$$\frac{\partial^2 \mathcal{N}}{\partial \sigma_*^2} = \frac{16(1+r)}{(4+3r)r}\left(\frac{\partial \mathcal{N}}{\partial \sigma_*}\right)^2 + \frac{r}{4+3r}\frac{\partial^2}{\partial \sigma_*^2}\left(\ln \rho_{\sigma \rm osc} - \frac{3}{4}\ln H_{\rm osc}^2\right)$$

case $t_{\rm reh} > t_{\rm osc}$:

$$\frac{\partial \mathcal{N}}{\partial \sigma_*} = \frac{r}{4+3r}\frac{\partial}{\partial \sigma_*}\left(\ln \rho_{\sigma \rm osc} - \ln H_{\rm osc}^2\right)$$

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Curvaton Dynamics

EOM : $\ddot{\sigma} + 3H(t)\dot{\sigma} + \frac{\partial V(\sigma)}{\partial \sigma} = 0$

$\longrightarrow$

$$cH(t)\dot{\sigma} \simeq -\frac{\partial V(\sigma)}{\partial \sigma}$$

where $c = 3 + \frac{3(w+1)}{2}$

necessary condition : $\left|\frac{V''}{cH^2}\right| \ll 1,$

under which the above approximation is a stable attractor

Onset of Curvaton Oscillation

$$\boxed{\left|\frac{\dot{\sigma}}{H\sigma}\right|_{\mathrm{osc}} = 1} \quad \longrightarrow \quad H_{\mathrm{osc}}^2 = \frac{V'(\sigma_{\mathrm{osc}})}{c\sigma_{\mathrm{osc}}}$$

$$\frac{1}{H_{\mathrm{osc}}}\frac{\partial H_{\mathrm{osc}}}{\partial \sigma_{\mathrm{osc}}} = \frac{1}{2}\left(\frac{V''(\sigma_{\mathrm{osc}})}{V'(\sigma_{\mathrm{osc}})} - \frac{1}{\sigma_{\mathrm{osc}}}\right)$$

$$\begin{aligned}\frac{\partial \sigma_{\mathrm{osc}}}{\partial \sigma_*} &= \left\{1 - \frac{1}{c(c-3)}\frac{V'(\sigma_{\mathrm{osc}})}{H_{\mathrm{osc}}^3}\frac{\partial H_{\mathrm{osc}}}{\partial \sigma_{\mathrm{osc}}}\right\}^{-1}\frac{V'(\sigma_{\mathrm{osc}})}{V'(\sigma_*)} \\ &= \left\{1 - \frac{1}{2(c-3)}\left(\frac{\sigma_{\mathrm{osc}}V''(\sigma_{\mathrm{osc}})}{V'(\sigma_{\mathrm{osc}})} - 1\right)\right\}^{-1}\frac{V'(\sigma_{\mathrm{osc}})}{V'(\sigma_*)}\end{aligned}$$

Hilltop Curvatons

$$V(\sigma) = V_0 - \frac{1}{2}m^2(\sigma - \sigma_0)^2$$

under $$\frac{\sigma_{\rm osc}}{\sigma_0 - \sigma_{\rm osc}} \gg 1, \quad \frac{V_0}{m^2(\sigma_{\rm osc} - \sigma_0)^2} \gg 1,$$

$$\mathcal{P}_\zeta^{1/2} \simeq \frac{3r}{4+3r}\frac{\sigma_0 - \sigma_{\rm osc}}{\sigma_0 - \sigma_*}\frac{H_{\rm inf}}{2\pi\sigma_{\rm osc}}$$

$$f_{\rm NL} \simeq \frac{5(4+3r)}{18r}\frac{\sigma_{\rm osc}}{\sigma_0 - \sigma_{\rm osc}}$$

$$n_s - 1 = -\frac{2}{3}\frac{m^2}{H_{\rm inf}^2}$$