

# Accurate Modeling of the Redshift-Space Distortions of Biased Tracers

based on arXiv:1106.4562

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with

**Atsushi Taruya (RESCEU)**

# Dark Energy? Modified Gravity?



observations → acceleration of cosmic expansion

✓ type-Ia supernova

✓ BAOs

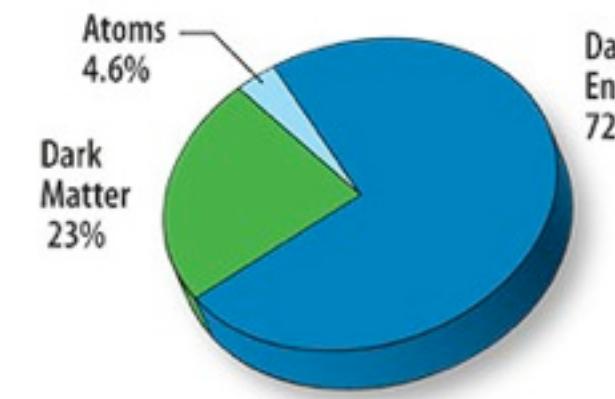
✓ CMB

✓ ...

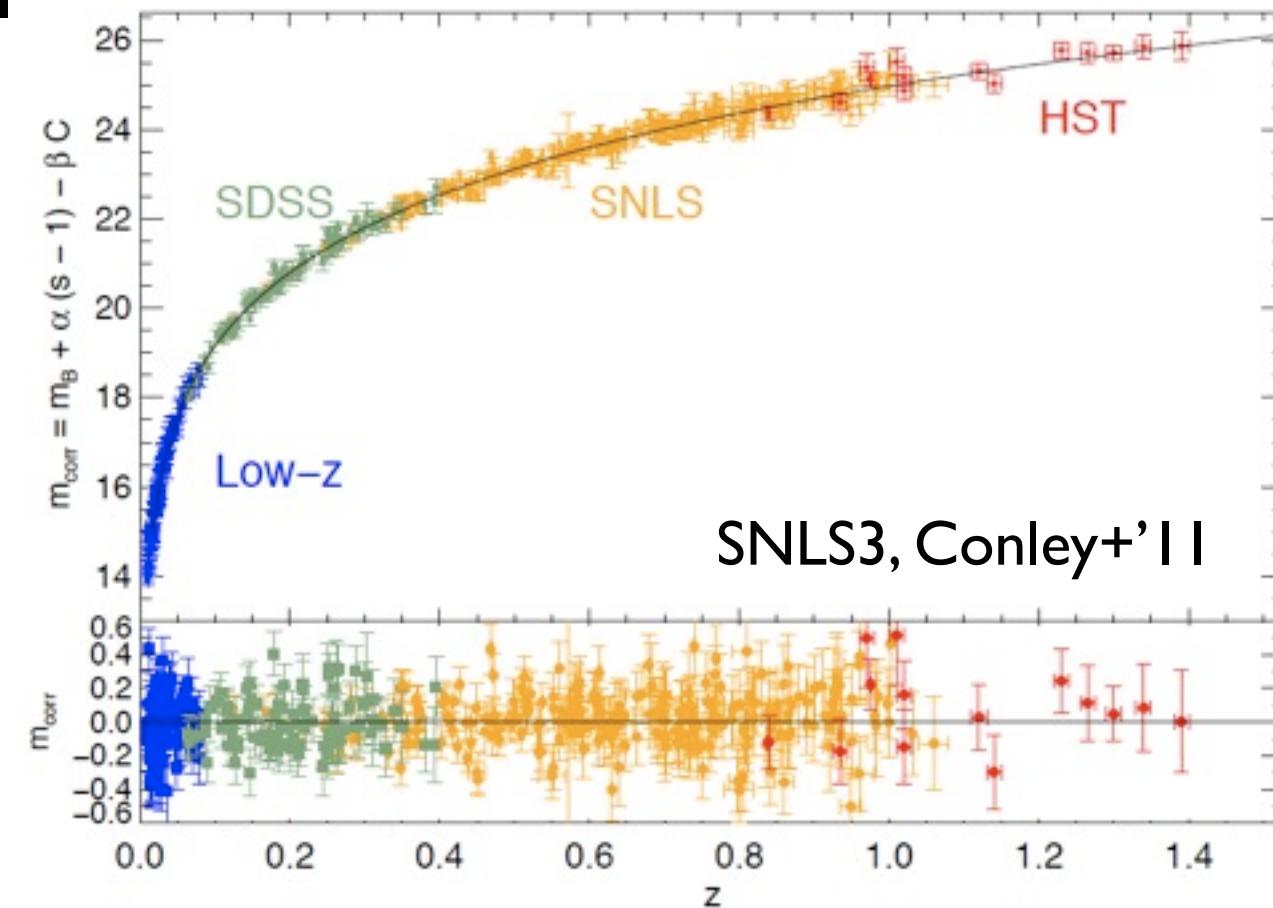
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

f(R), DGP, ...

modified gravity?



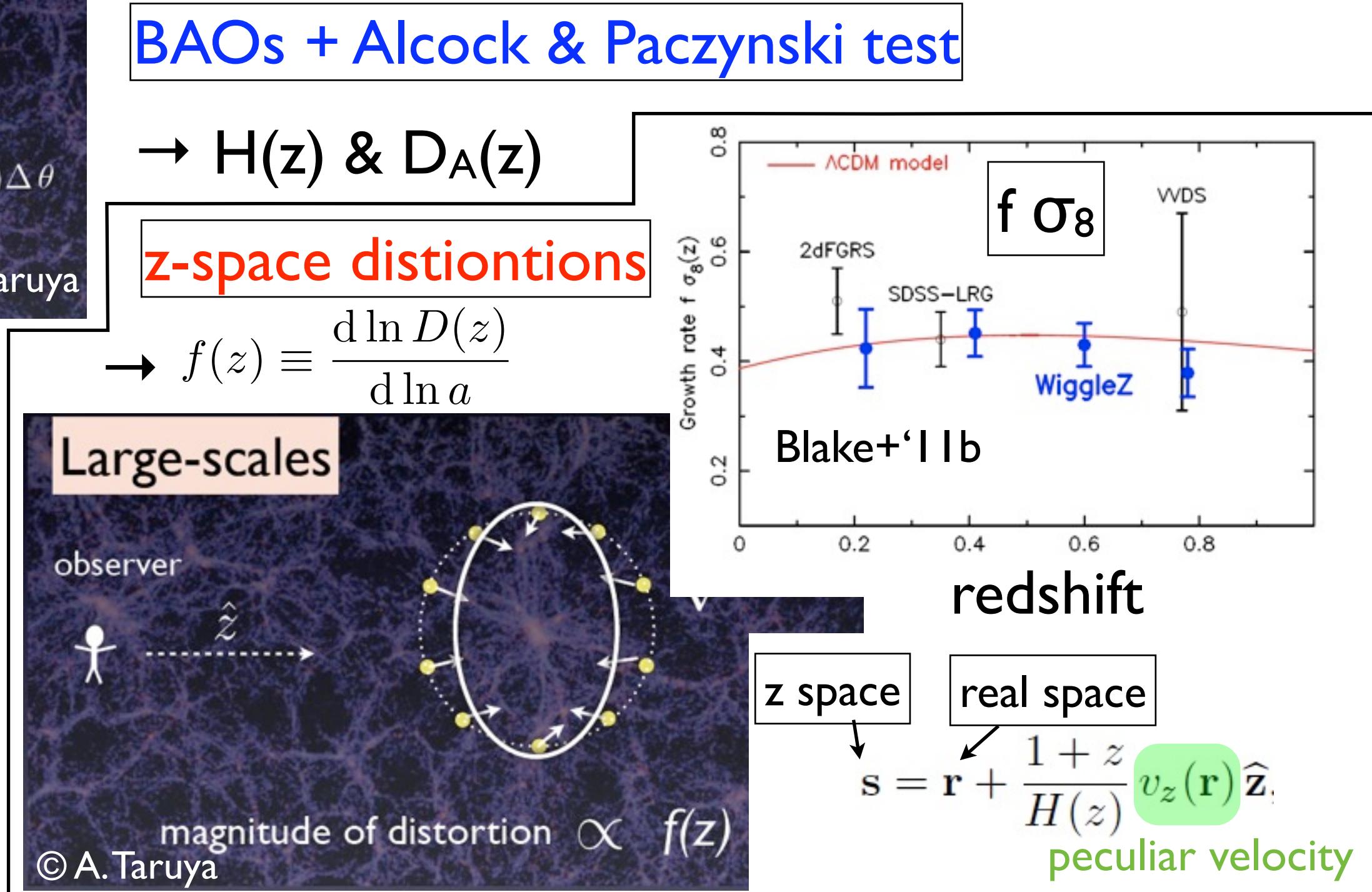
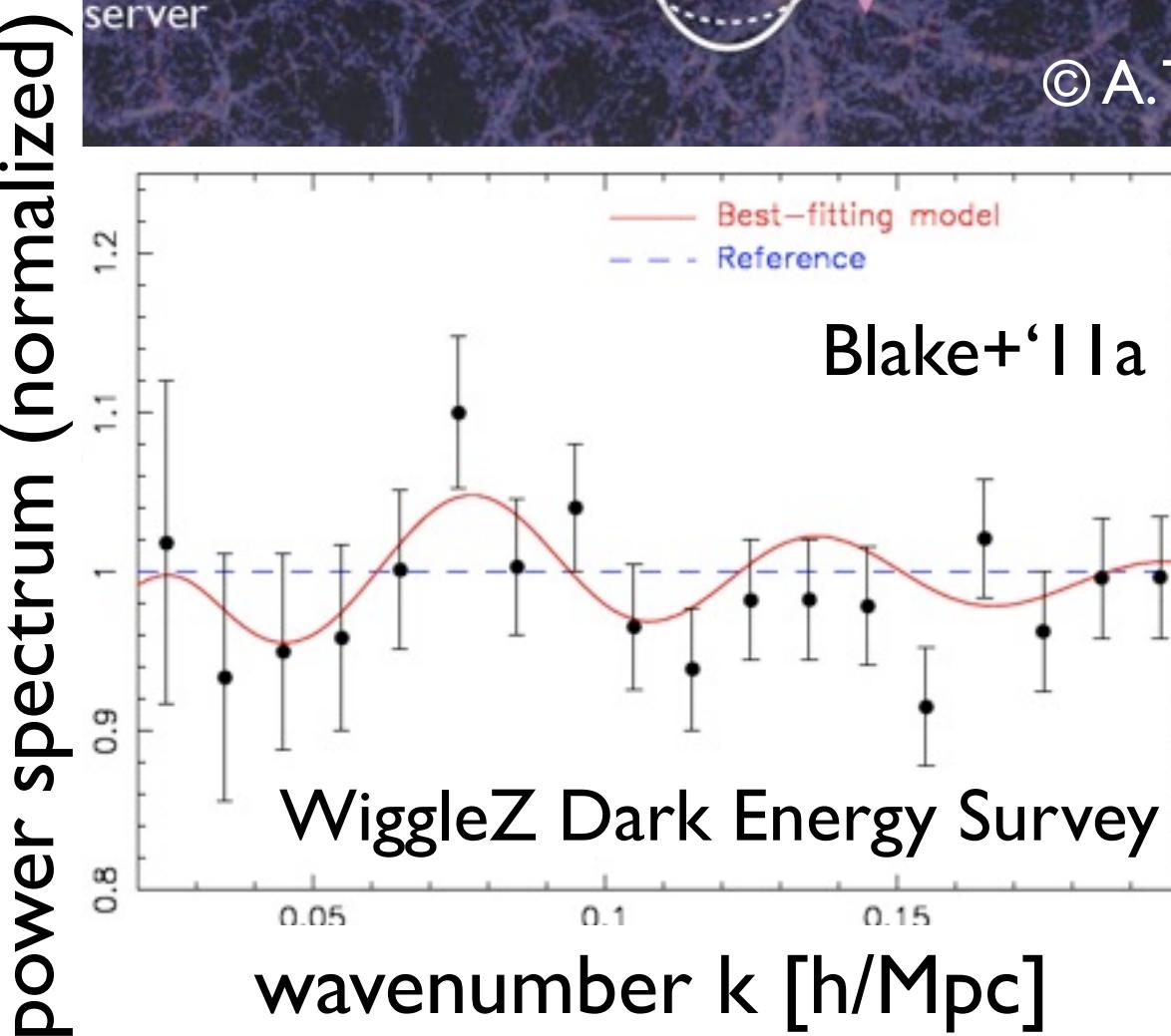
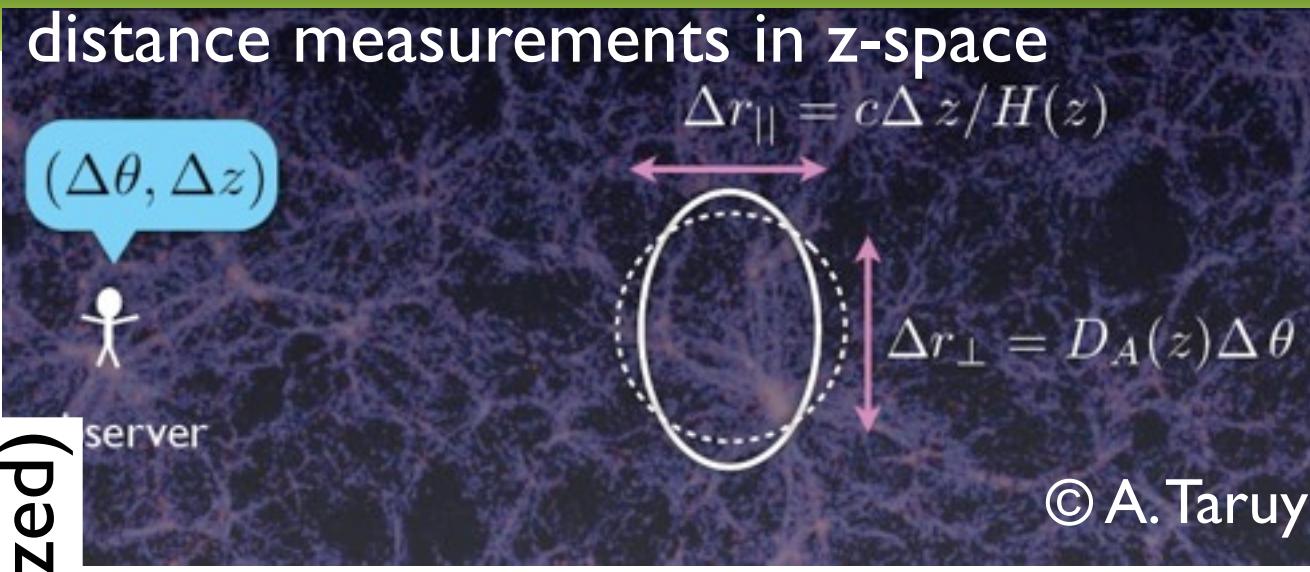
dark energy?



expansion history in a DE model may be mimicked by a MG model

geometrical + growth tests are essential!

# Anisotropies in galaxy clustering



# Redshift-space distortions



$$\mathbf{s} = \mathbf{r} + \frac{1+z}{H(z)} v_z(\mathbf{r}) \hat{\mathbf{z}}$$

**Kaiser Effect**

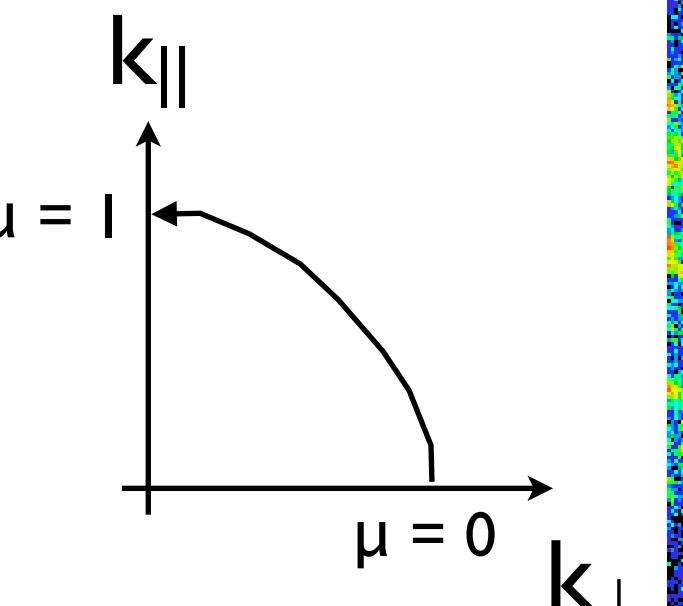
*large-scale coherent motion*

→ enhancement of clustering

**Finger-of-God Effect**

*small-scale random motion*

→ suppression of clustering

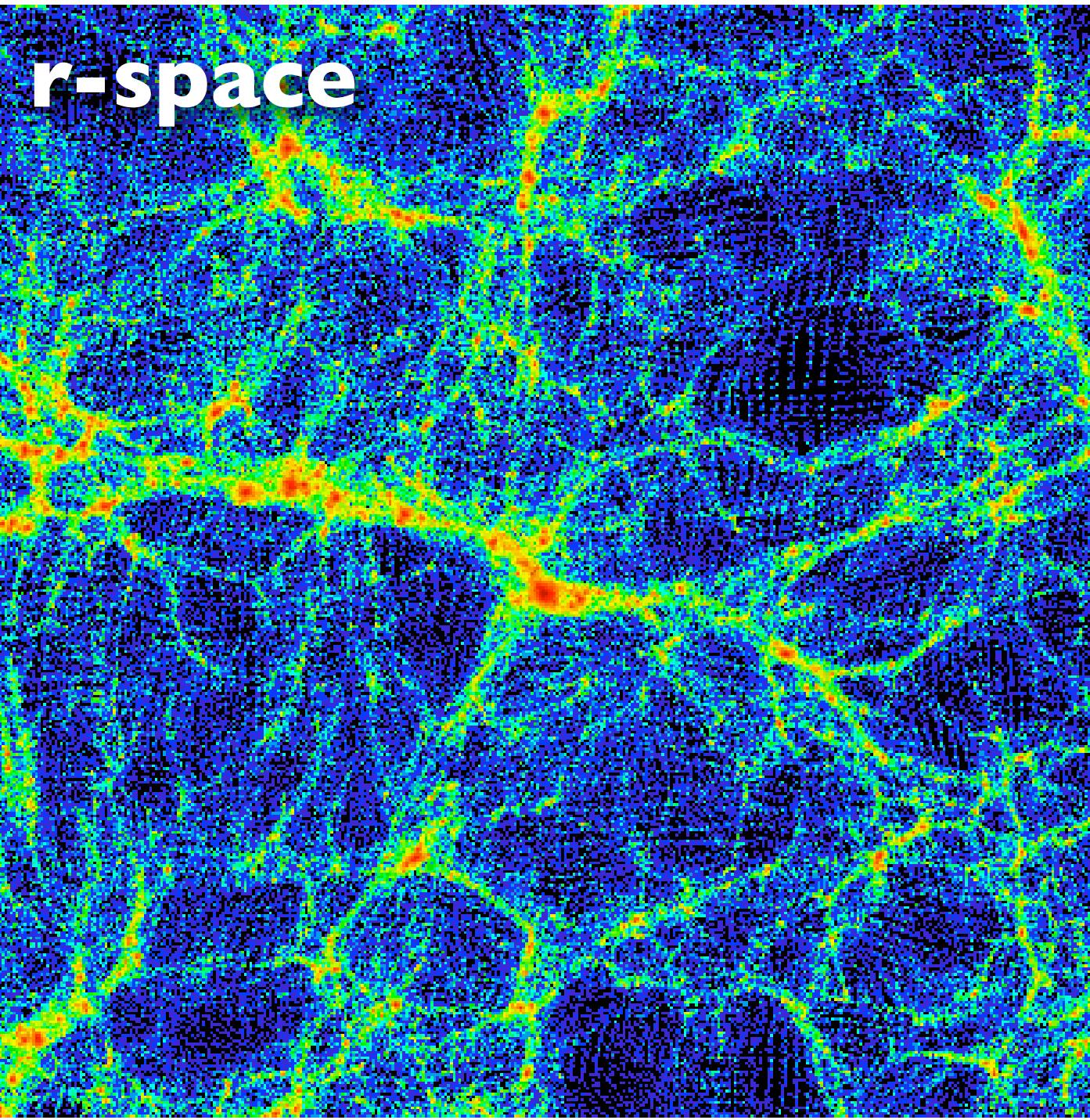


e.g., Scoccimarro'04

$$P(k, \mu) = D_f(k\mu f \sigma_v) \quad \text{streaming model}$$

$$\times [P_{\delta\delta}(k) + 2f\mu^2 P_{\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k)]$$

vel. divergence:  $\theta \equiv -(1+z)/(Hf)\nabla \cdot \mathbf{v}$       vel. dispersion:  $\sigma_v$



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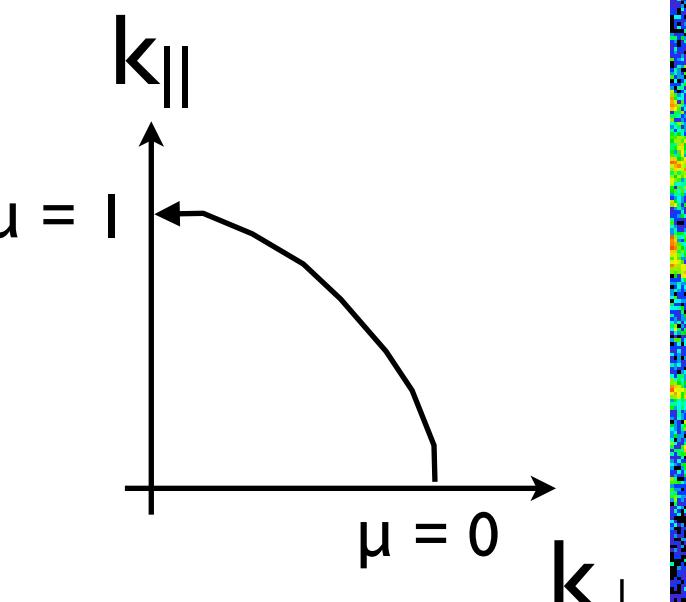
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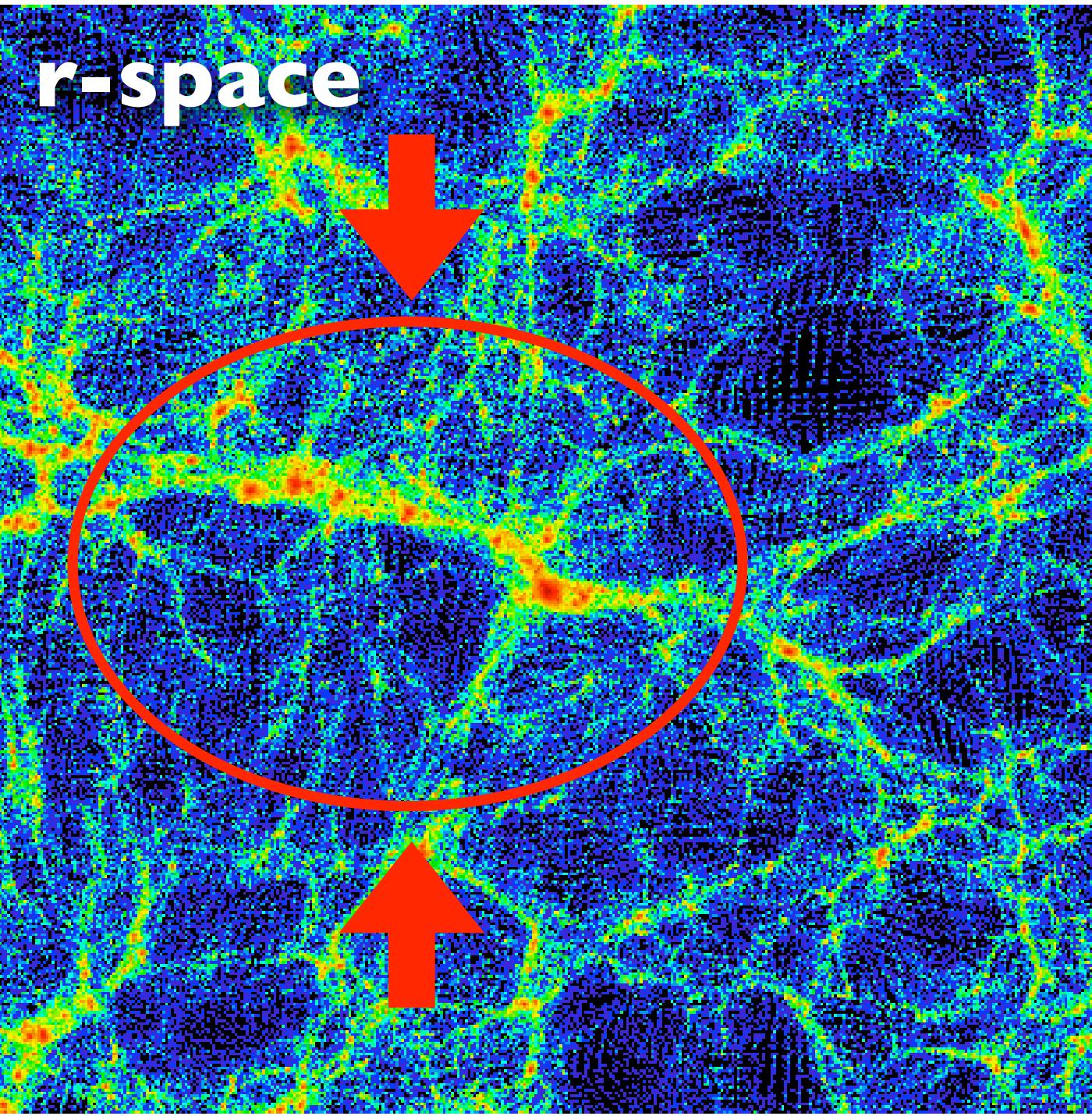
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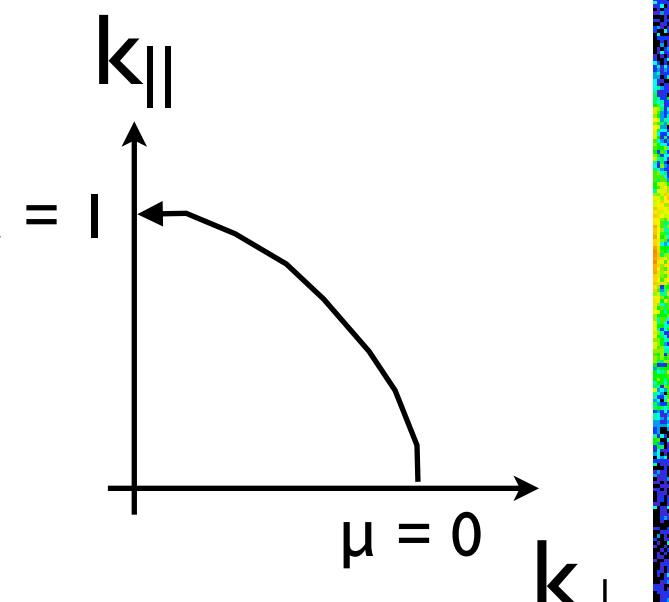
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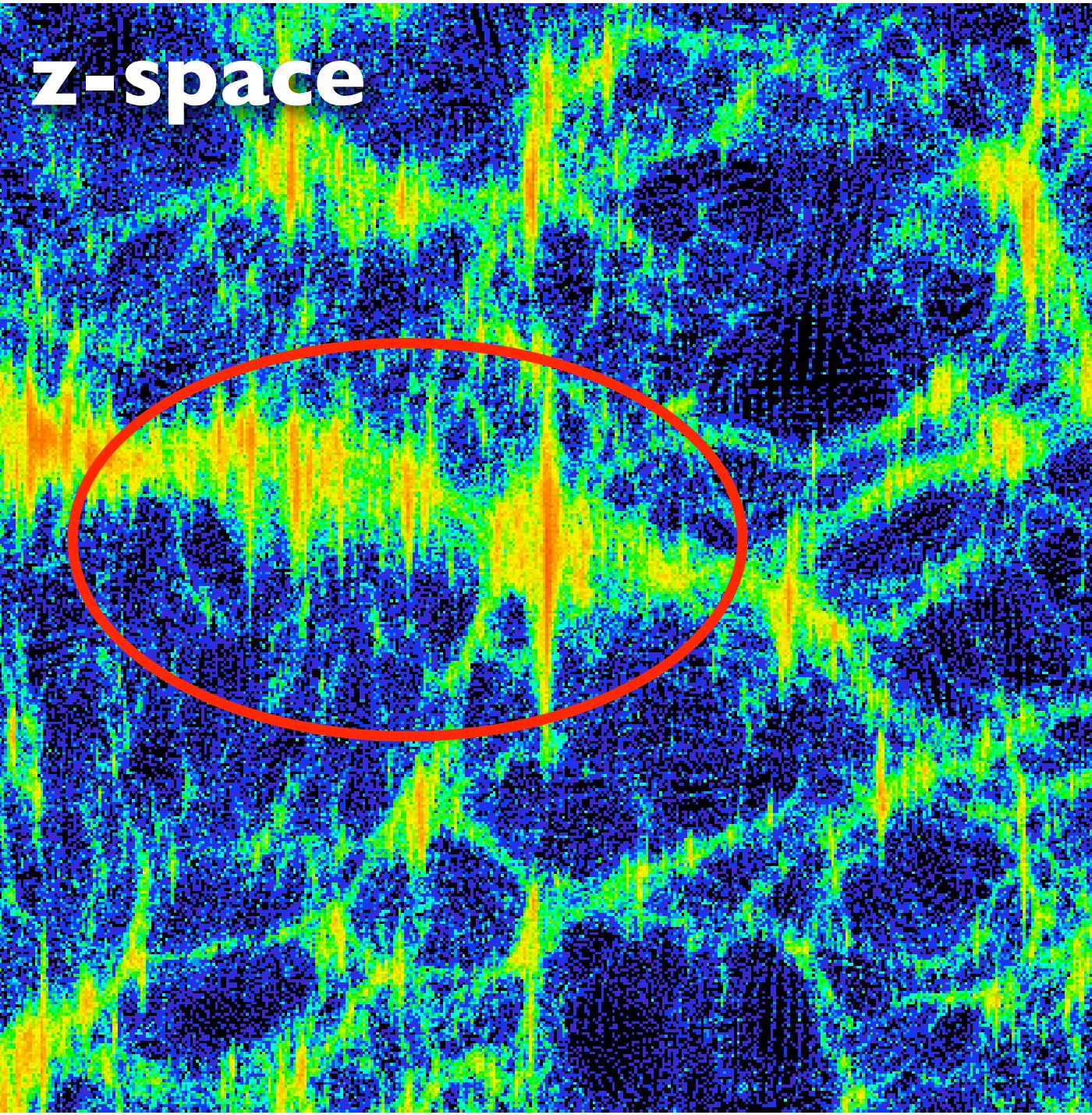
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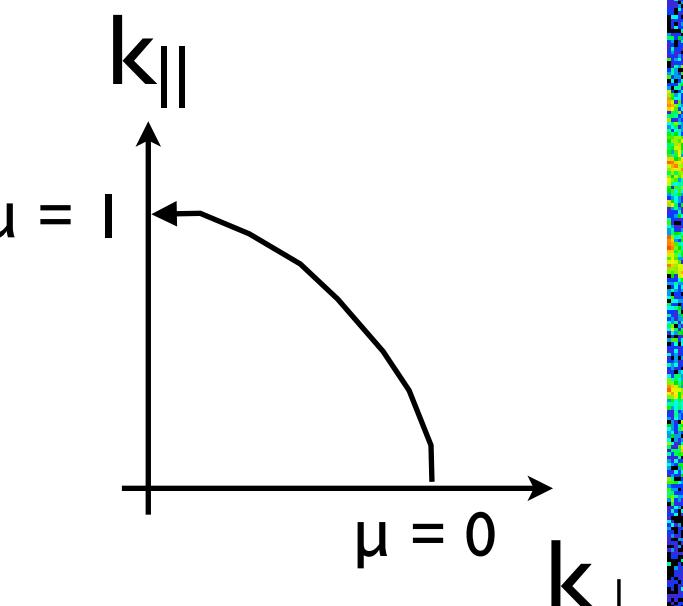
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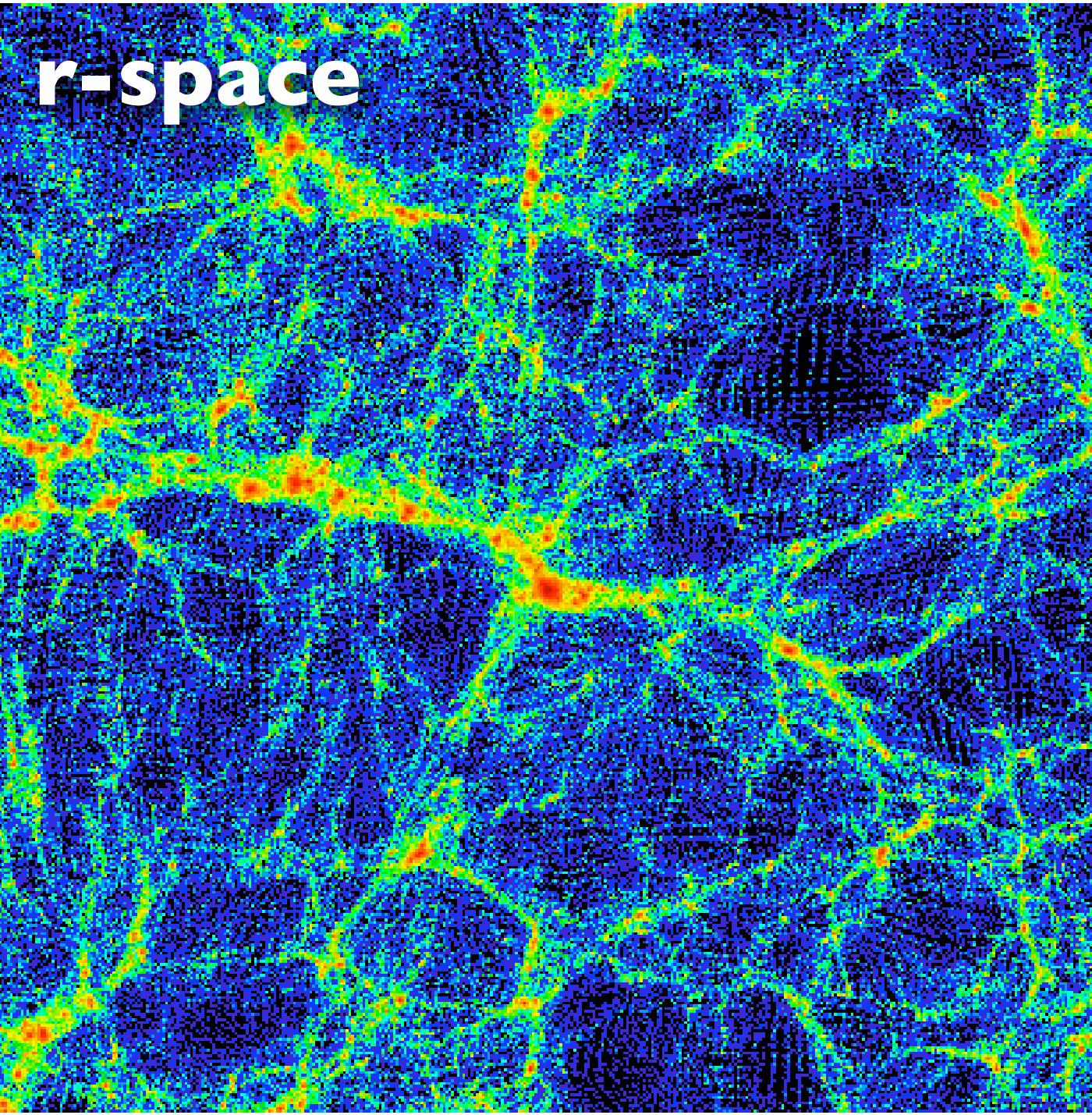
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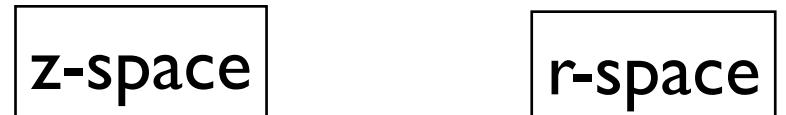
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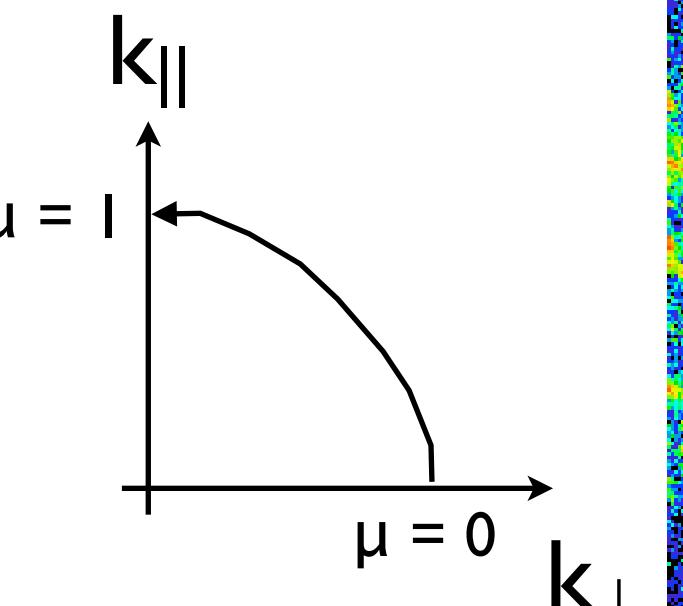
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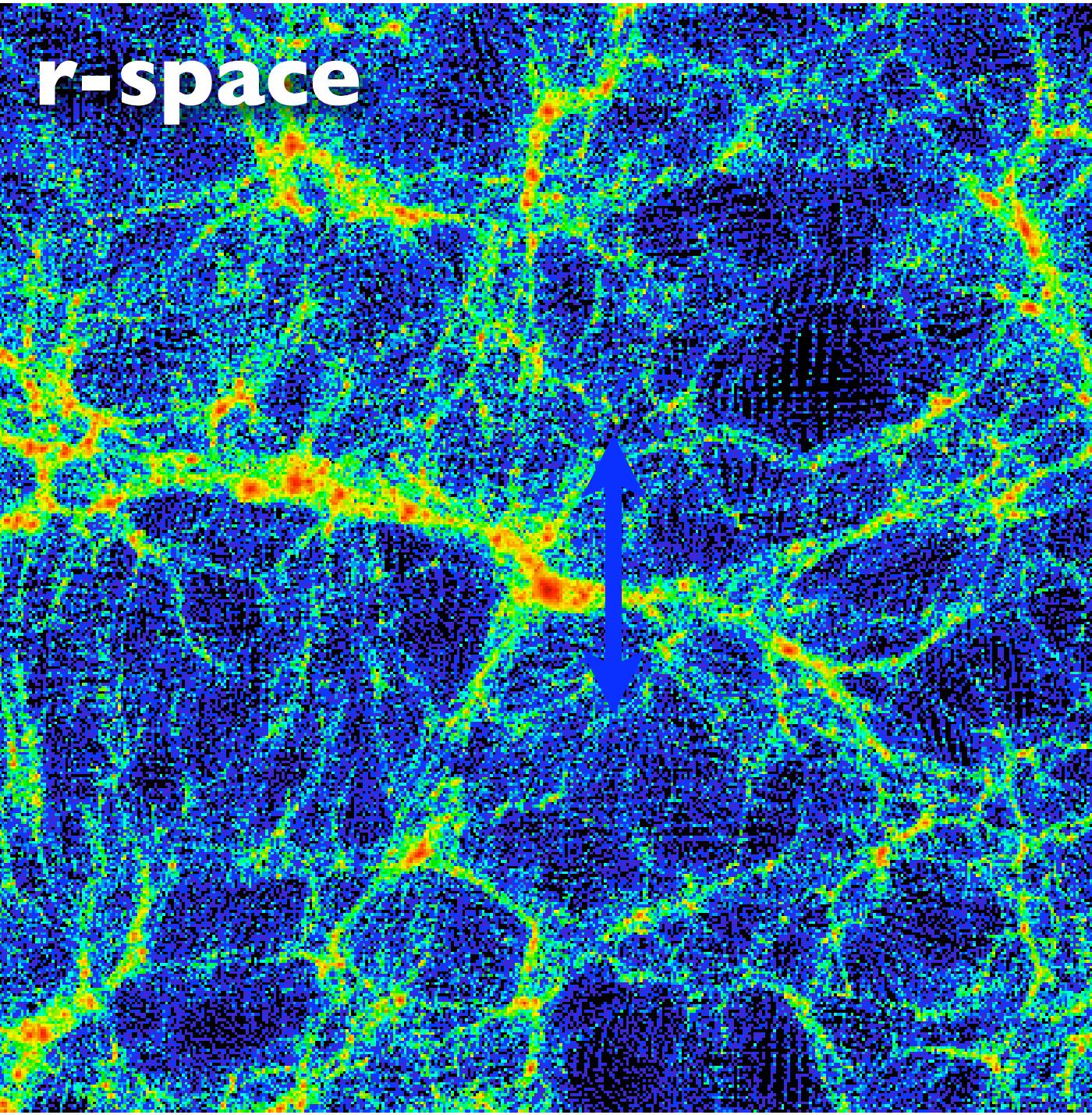


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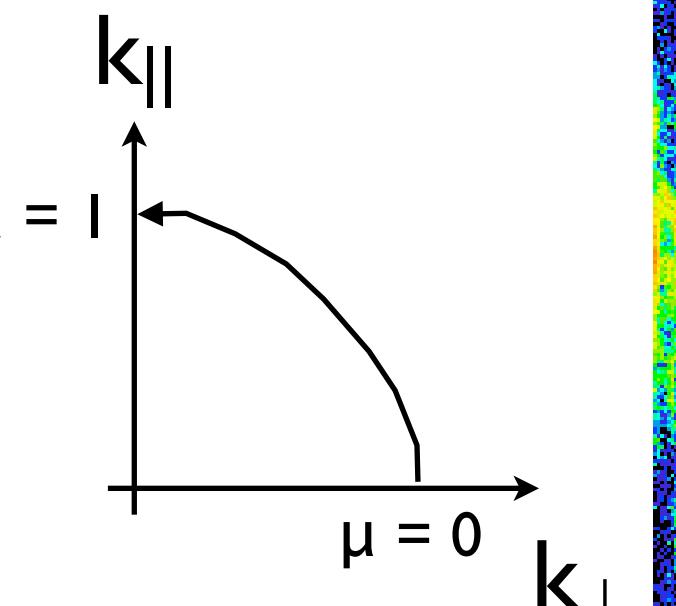
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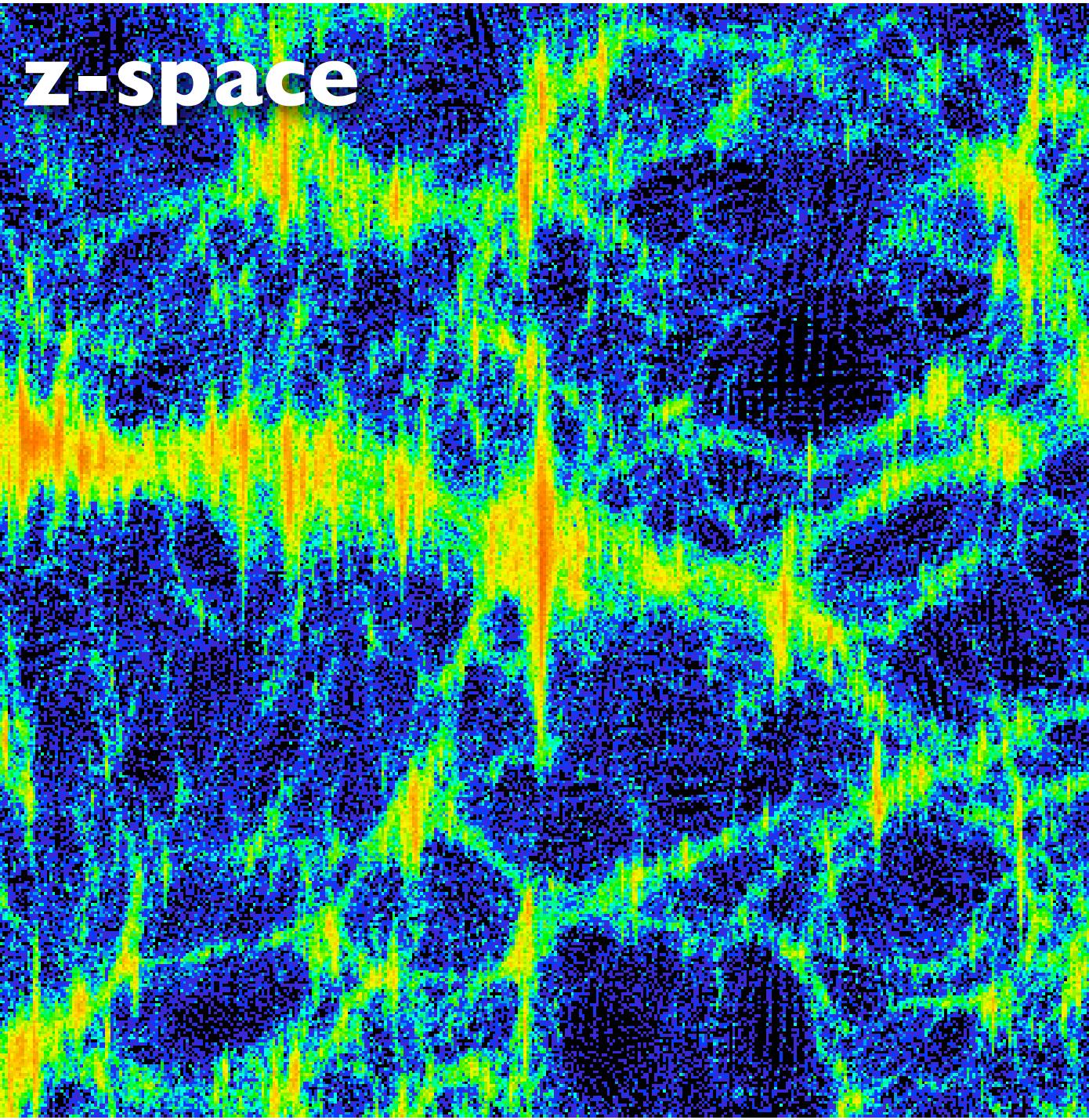
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# Redshift-space distortions (contd.): TNS model



Exact formula for the z-space  $P(k)$

$$P^{(S)}(k) = \int d^3x e^{ik \cdot x} \langle e^{-ik\mu f \Delta u_z} \times \{\delta(r) + f \nabla_z u_z(r)\} \{\delta(r') + f \nabla_z u_z(r')\} \rangle$$

notice  $\langle e^A BC \rangle \neq \langle e^A \rangle \langle BC \rangle$

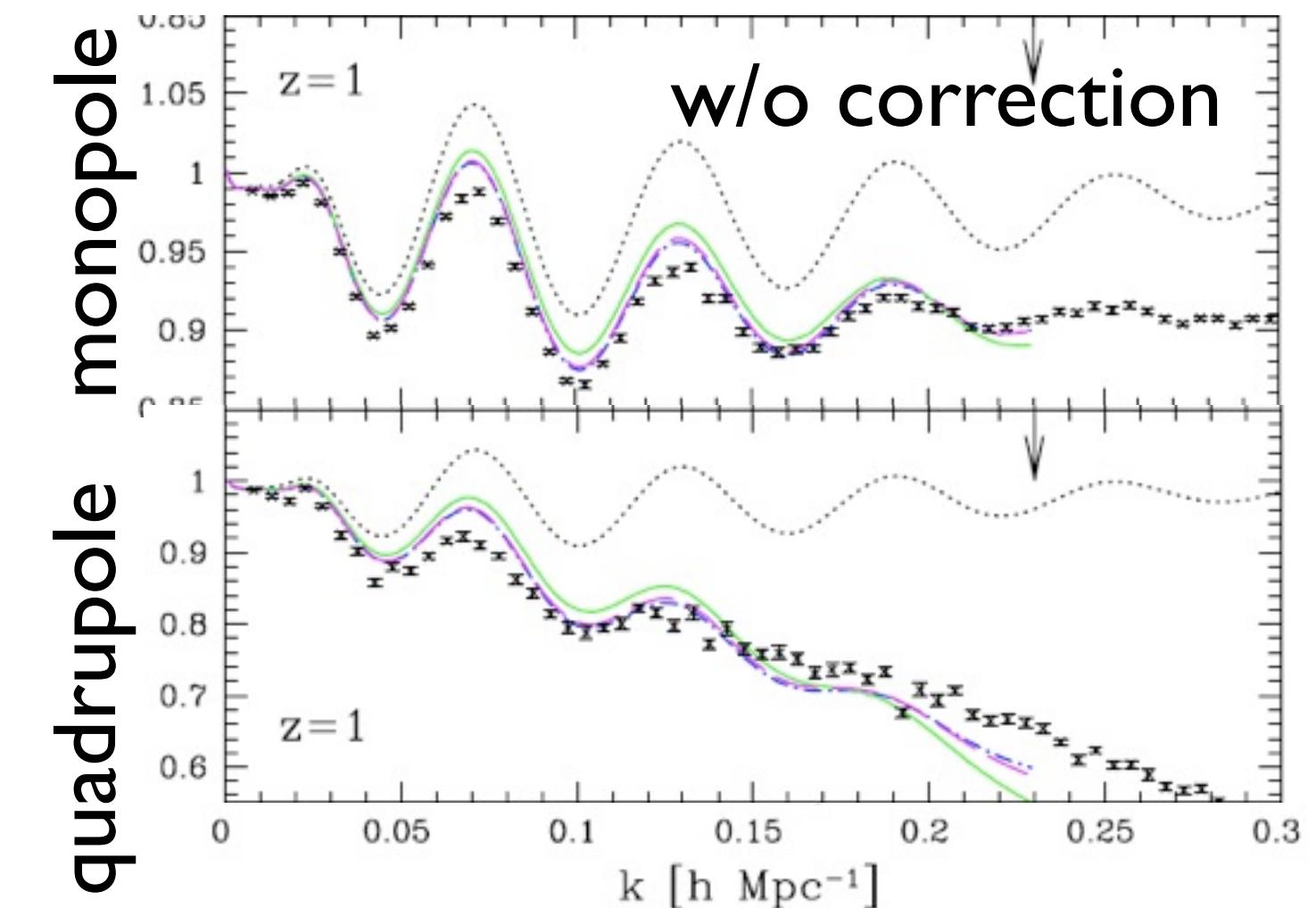
with a help of cumulant expansion theorem

$$P(k, \mu) = D_f(k \mu f \sigma_v) \times [P_{\delta\delta}(k) + 2 f \mu^2 P_{\delta\theta}(k) + f^2 \mu^4 P_{\theta\theta}(k) + \underline{A(k, \mu; f) + B(k, \mu; f)}]$$

A term  $\propto$  cross-bispectrum of  $\delta$  &  $\theta$  **new terms!**

B term  $\propto$  sum of convolutions of  $P_{\delta\theta}$  &  $P_{\theta\theta}$

Taruya, Nishimichi, Saito ('10)



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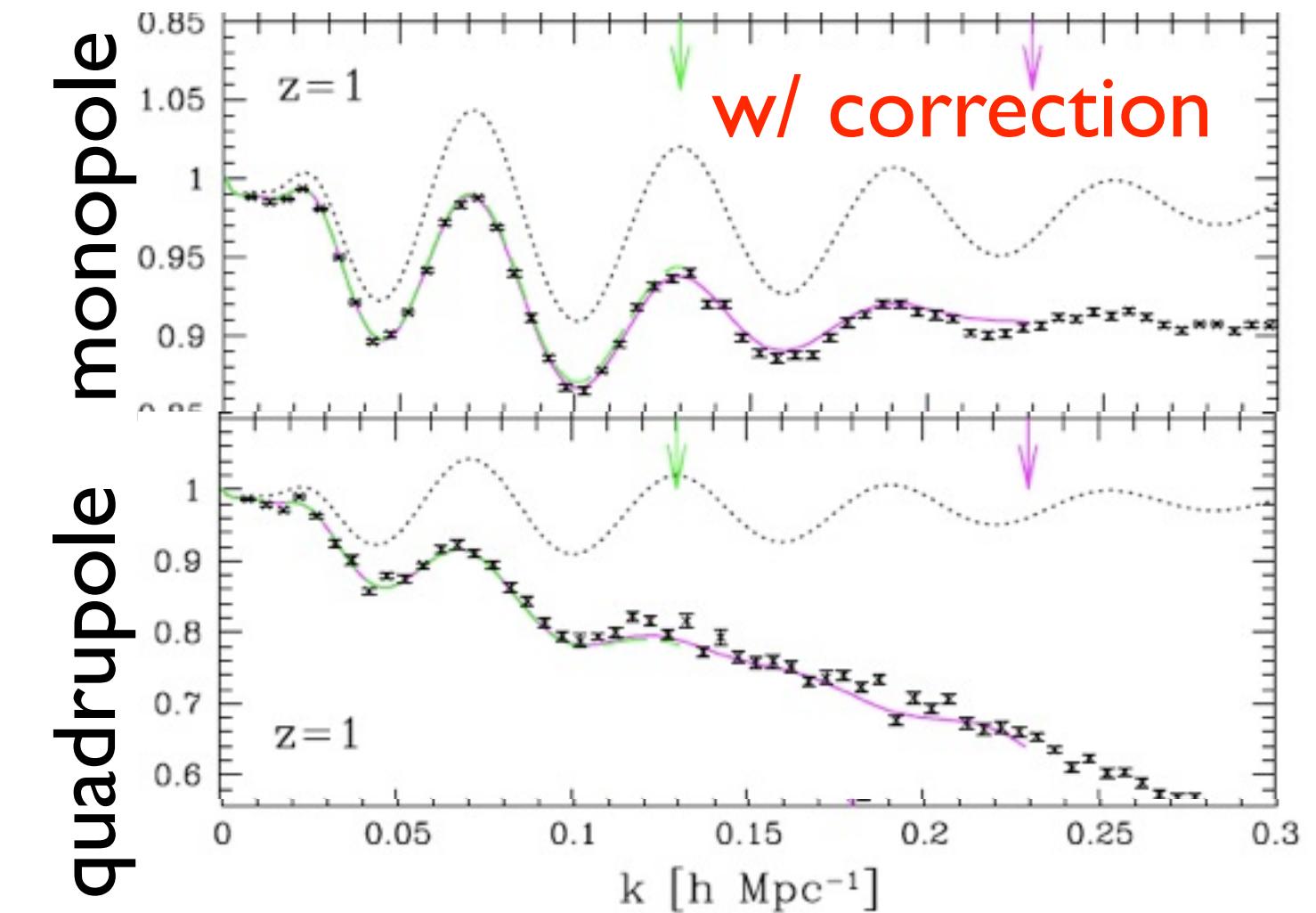
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# RSDs for biased tracers?



Many people are working hard on this!

e.g., Okumura & Jing'II  
Tang, Kayo & Takada'II  
Reid & White'II  
Sato & Matsubara 'II

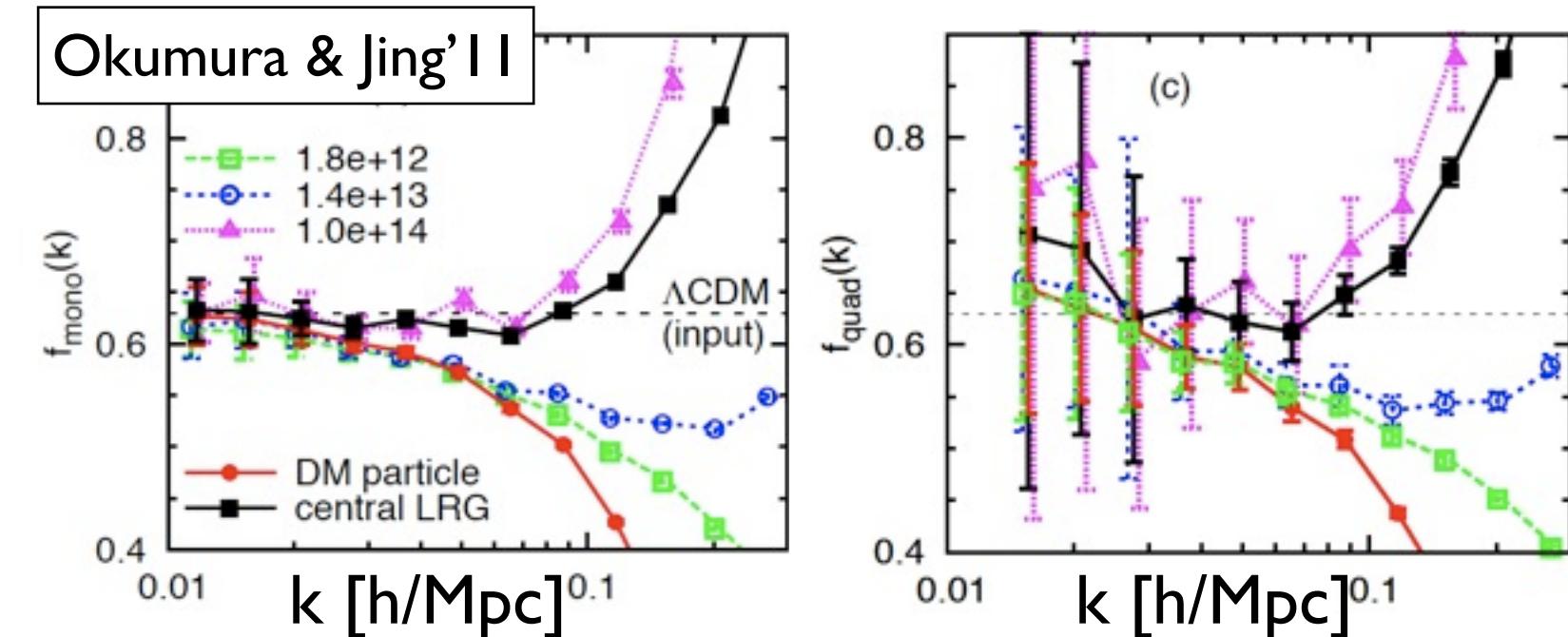
biased tracer?

assume  $\delta_g = b\delta$

$$P(k, \mu) = D_f(k \mu f \sigma_v) \\ \times [P_{\delta\delta}(k) + 2 f \mu^2 P_{\delta\theta}(k) + f^2 \mu^4 P_{\theta\theta}(k) \\ + A(k, \mu; f) + B(k, \mu; f)]$$

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B term  $\propto$  sum of convolutions of  $P_{\delta\theta}$  &  $P_{\theta\theta}$



$$P_h(k, \mu) = D_f(k \mu f \sigma_v) \\ \times b^2 [P_{\delta\delta}(k) + 2 \beta \mu^2 P_{\delta\theta}(k) + \beta^2 \mu^4 P_{\theta\theta}(k) \\ + b A(k, \mu; \beta) + b^2 B(k, \mu; \beta)]$$

$$\beta = f / b$$

Are correction terms *enhanced* by bias?

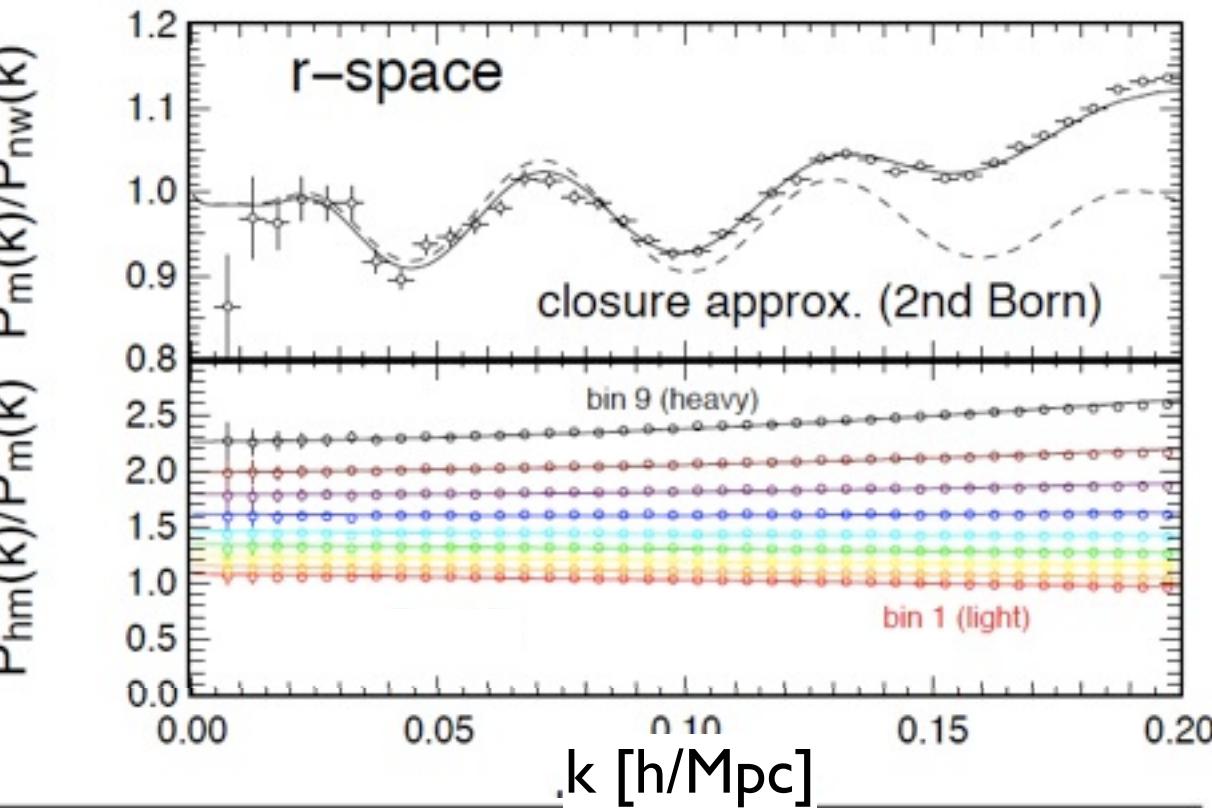
# Analysis



Large N-body simulations ( $L=1.14\text{Gpc}/h$ ,  $N=1,280^3$ )  
starting with 2LPT initial conditions  $\times$  15 realizations

- ✓ 9 halo catalogs over a wide mass range @  $z=0.35$
- ✓ volume & number density  $\doteq$  SDSS DR7 LRGs
- ✓  $b(k)$  is directly measured from r-space clustering
- ✓  $\sigma_v$  is treated as a free fitting parameter

mass:  $h^{-1}\text{M}_{\text{sun}}$ , density:  $h^3\text{Mpc}^{-3}$



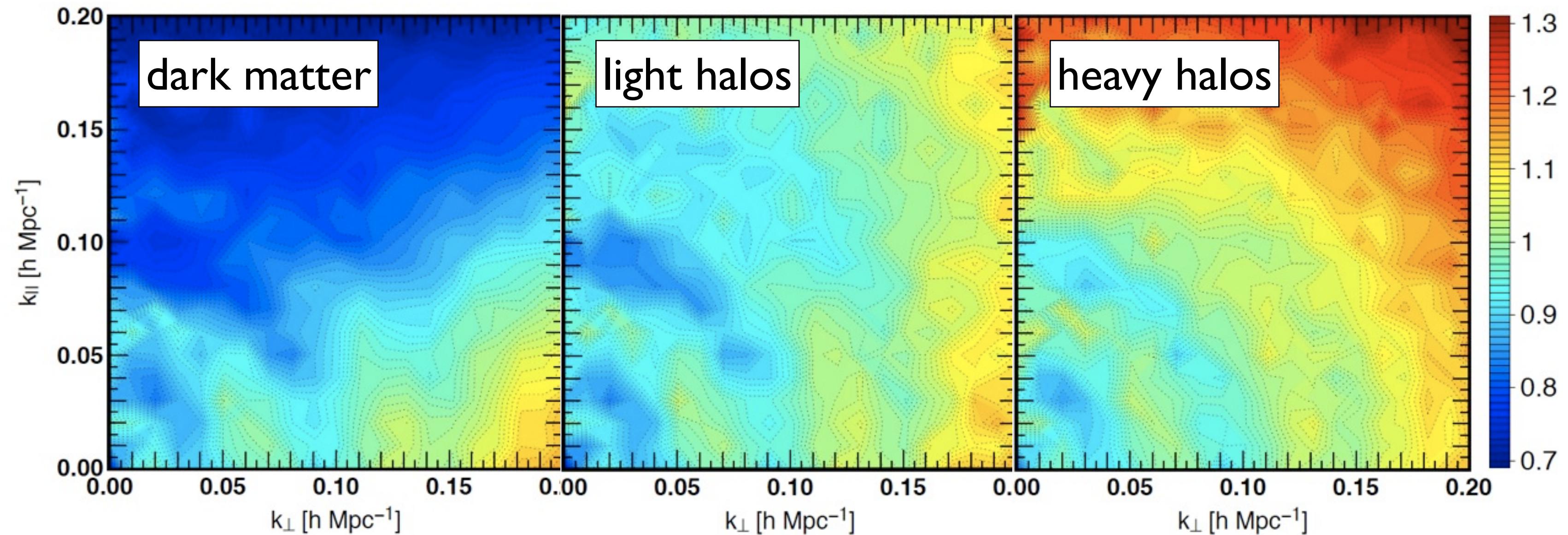
| Sample      | bin 1 (light)         | bin 2                 | bin 3                 | bin 4                 | bin 5                 | bin 6                 | bin 7                 | bin 8                 | bin 9 (heavy)         |
|-------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $M_{\min}$  | $1.77 \times 10^{12}$ | $2.49 \times 10^{12}$ | $3.54 \times 10^{12}$ | $4.98 \times 10^{12}$ | $7.09 \times 10^{12}$ | $1.00 \times 10^{13}$ | $1.42 \times 10^{13}$ | $2.01 \times 10^{13}$ | $2.84 \times 10^{13}$ |
| $M_{\max}$  | $5.54 \times 10^{12}$ | $1.02 \times 10^{13}$ | $1.74 \times 10^{13}$ | $2.66 \times 10^{13}$ | $4.04 \times 10^{13}$ | $6.76 \times 10^{13}$ | $1.19 \times 10^{14}$ | $2.08 \times 10^{14}$ | -                     |
| $\bar{M}_h$ | $2.96 \times 10^{12}$ | $4.65 \times 10^{12}$ | $7.08 \times 10^{12}$ | $9.37 \times 10^{12}$ | $1.47 \times 10^{13}$ | $2.18 \times 10^{13}$ | $3.21 \times 10^{13}$ | $4.63 \times 10^{13}$ | $7.03 \times 10^{13}$ |
| $n_h$       | $1.57 \times 10^{-3}$ | $1.26 \times 10^{-3}$ | $9.46 \times 10^{-4}$ | $6.87 \times 10^{-4}$ | $4.87 \times 10^{-4}$ | $3.47 \times 10^{-4}$ | $2.43 \times 10^{-4}$ | $1.64 \times 10^{-4}$ | $1.09 \times 10^{-4}$ |
| $b_0$       | 1.08                  | 1.16                  | 1.25                  | 1.35                  | 1.47                  | 1.62                  | 1.80                  | 1.99                  | 2.26                  |

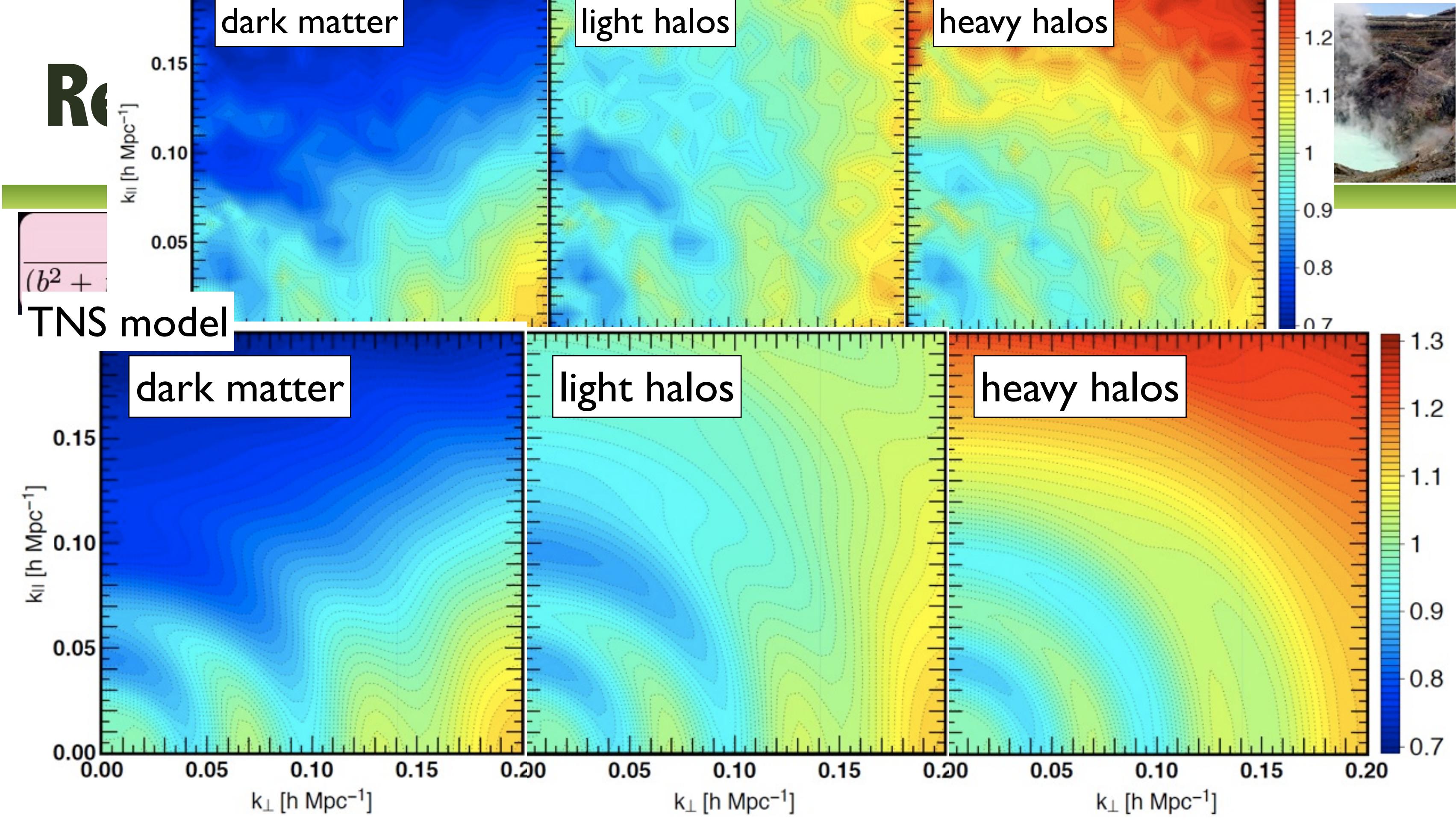
# Result

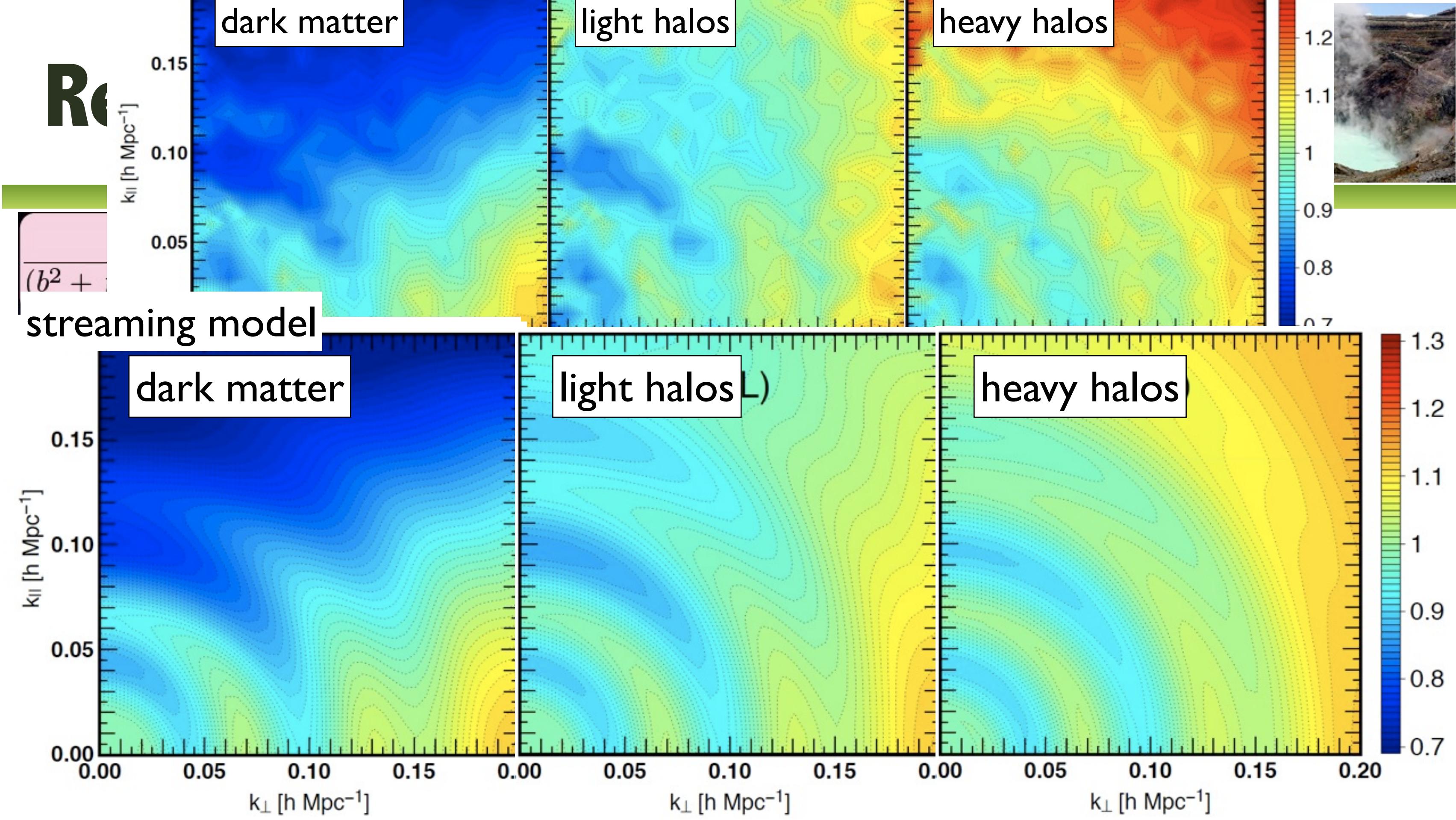


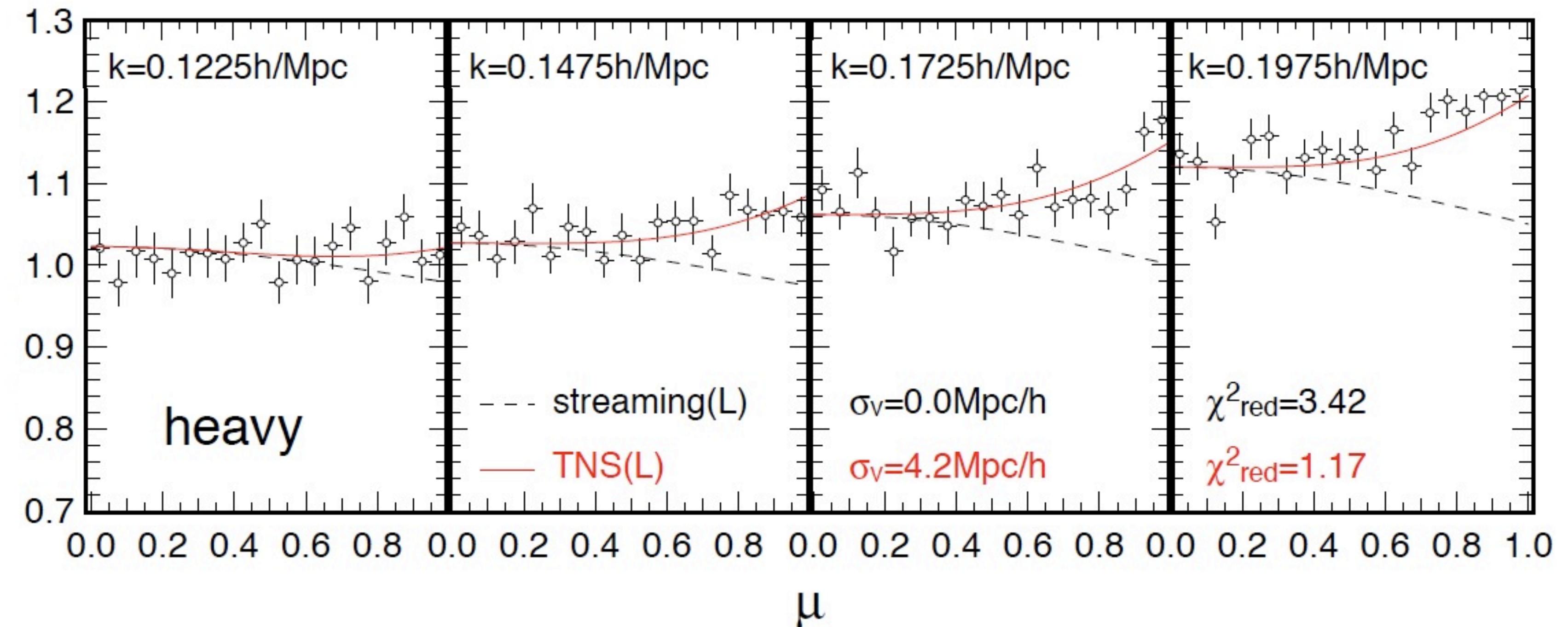
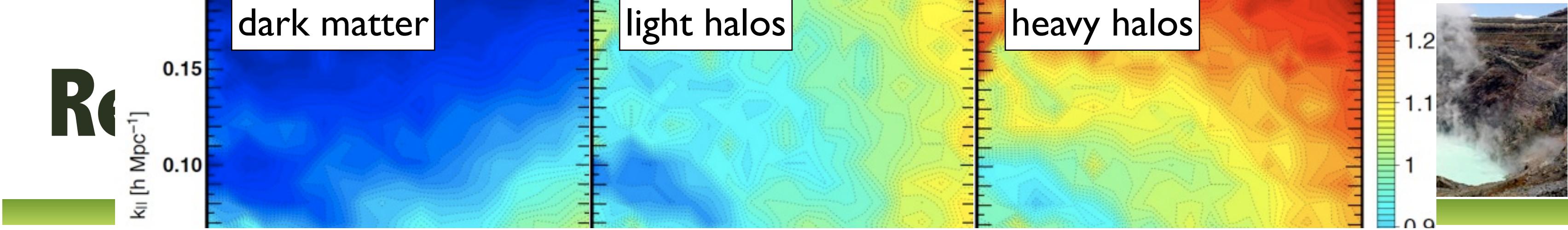
$$\frac{P_{\text{halo}}(k_{||}, k_{\perp})}{(b^2 + f \mu^2)^2 P_{\text{lin, no-wiggle}}(k)}$$

N-body simulations









# Goodness of fits



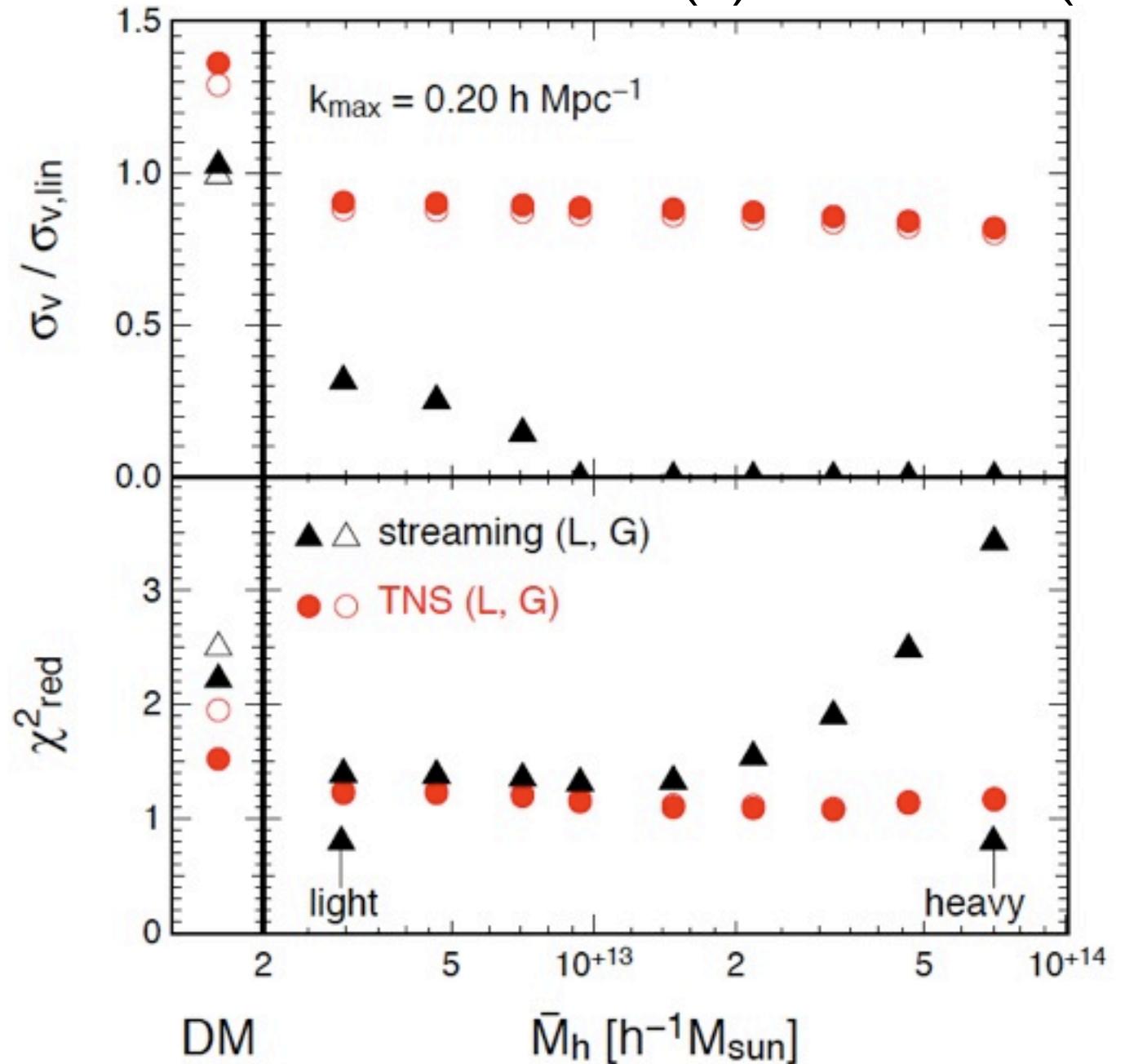
best-fit values of  $\sigma_v$ :

- smaller for streaming model
- consistent with 0 for massive halos
- **consistent with the linear theory for TNS**
- **does not depend the halo mass**

goodness of fit:

- worse for streaming model
- especially for massive halos
- **reduced  $\chi^2$  are close to 1 for TNS**
- **independent of the halo mass**

FoG function: Lorentzian (L), Gaussian (G)



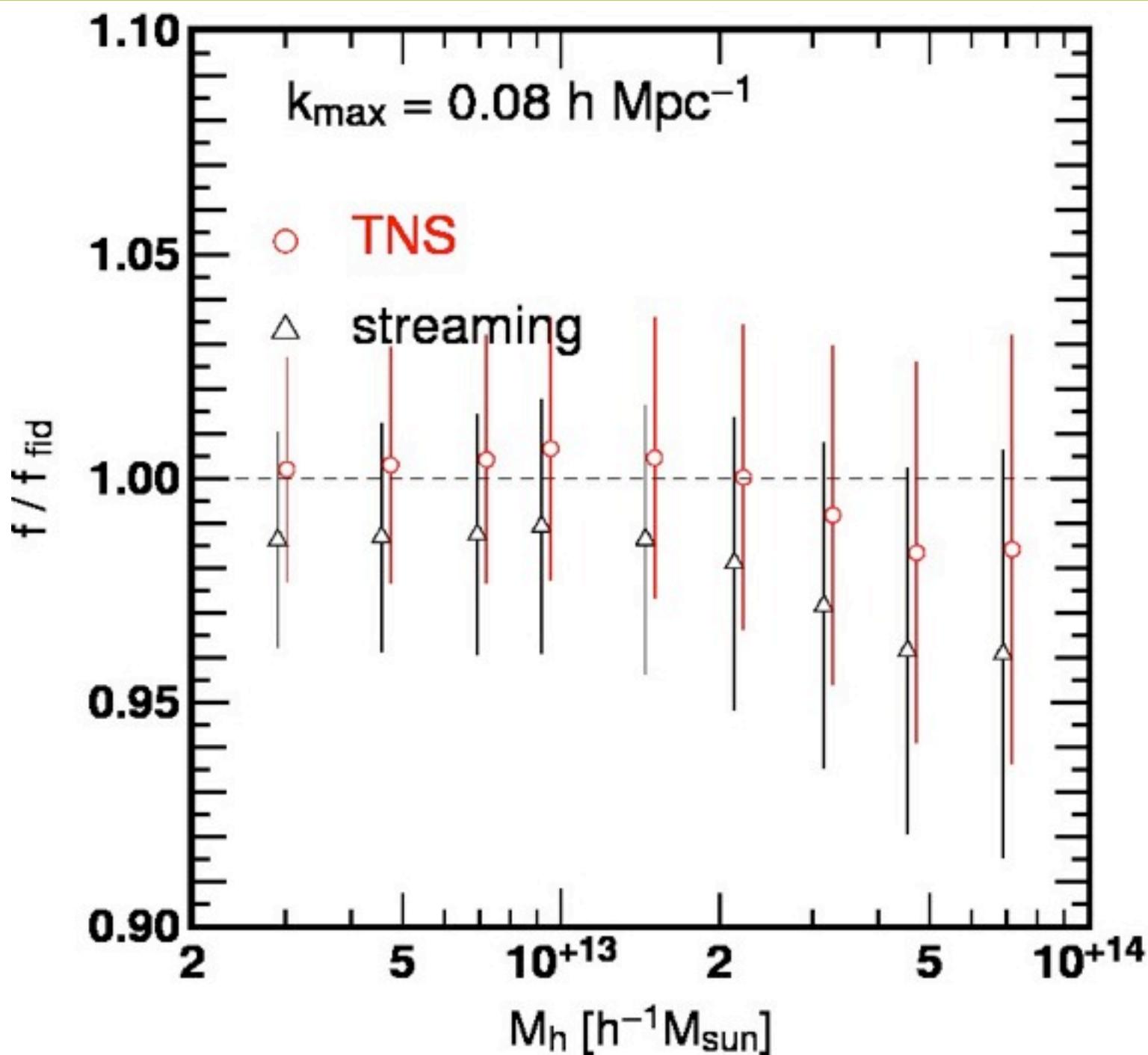
# Recovery of $f(z)$



2 parameter fit to N-body data:  $f$  and  $\sigma_v$

- ✓ streaming model
  - seams OK only at  $k_{\max} < 0.1 \text{ h/Mpc}$
  - typically  $\sim 5\%$  underestimate of  $f$

- ✓ TNS model
  - gives unbiased estimate of  $f$
  - up to  $k_{\max} \sim 0.2 \text{ h/Mpc}$



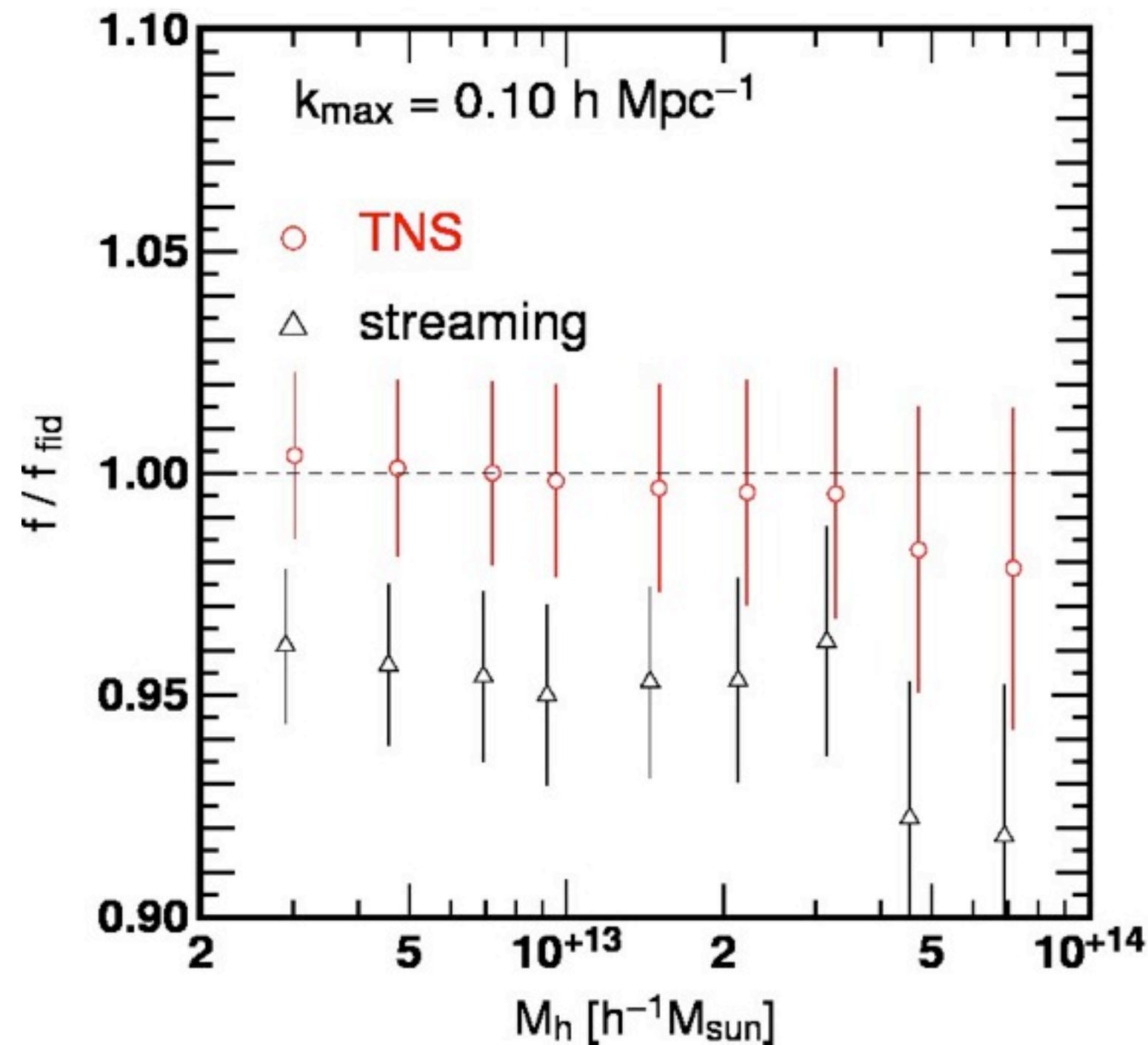
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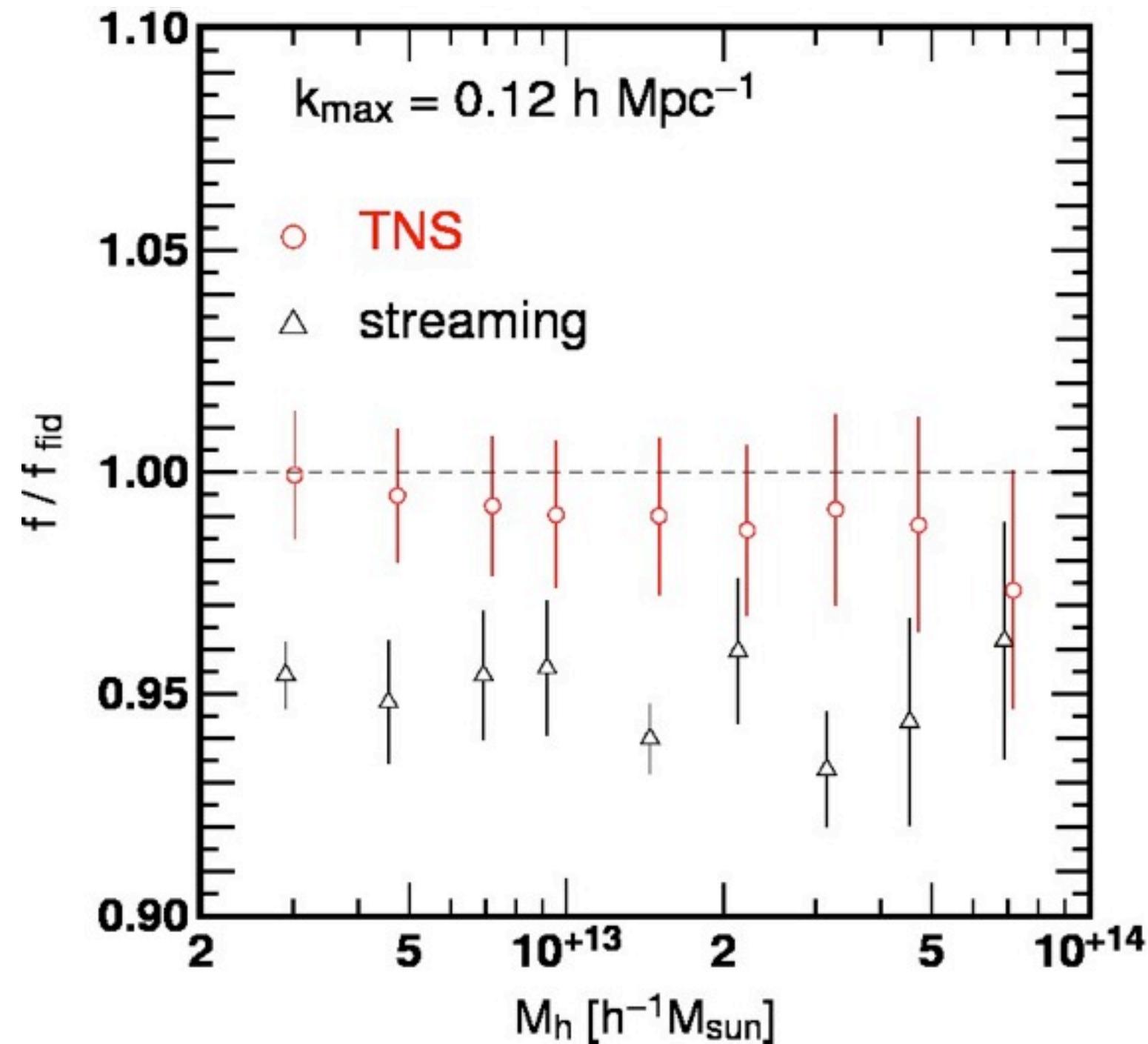
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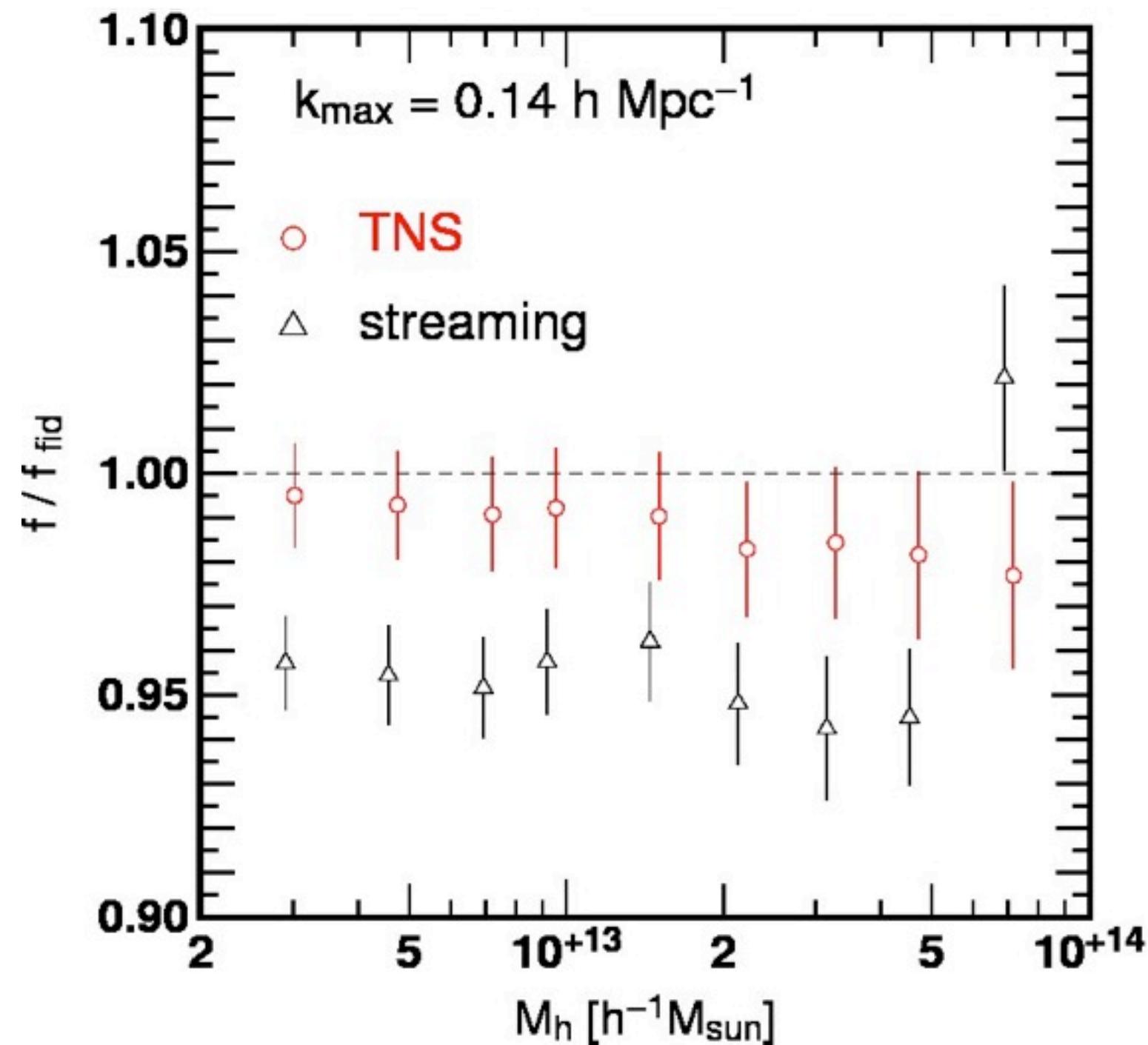
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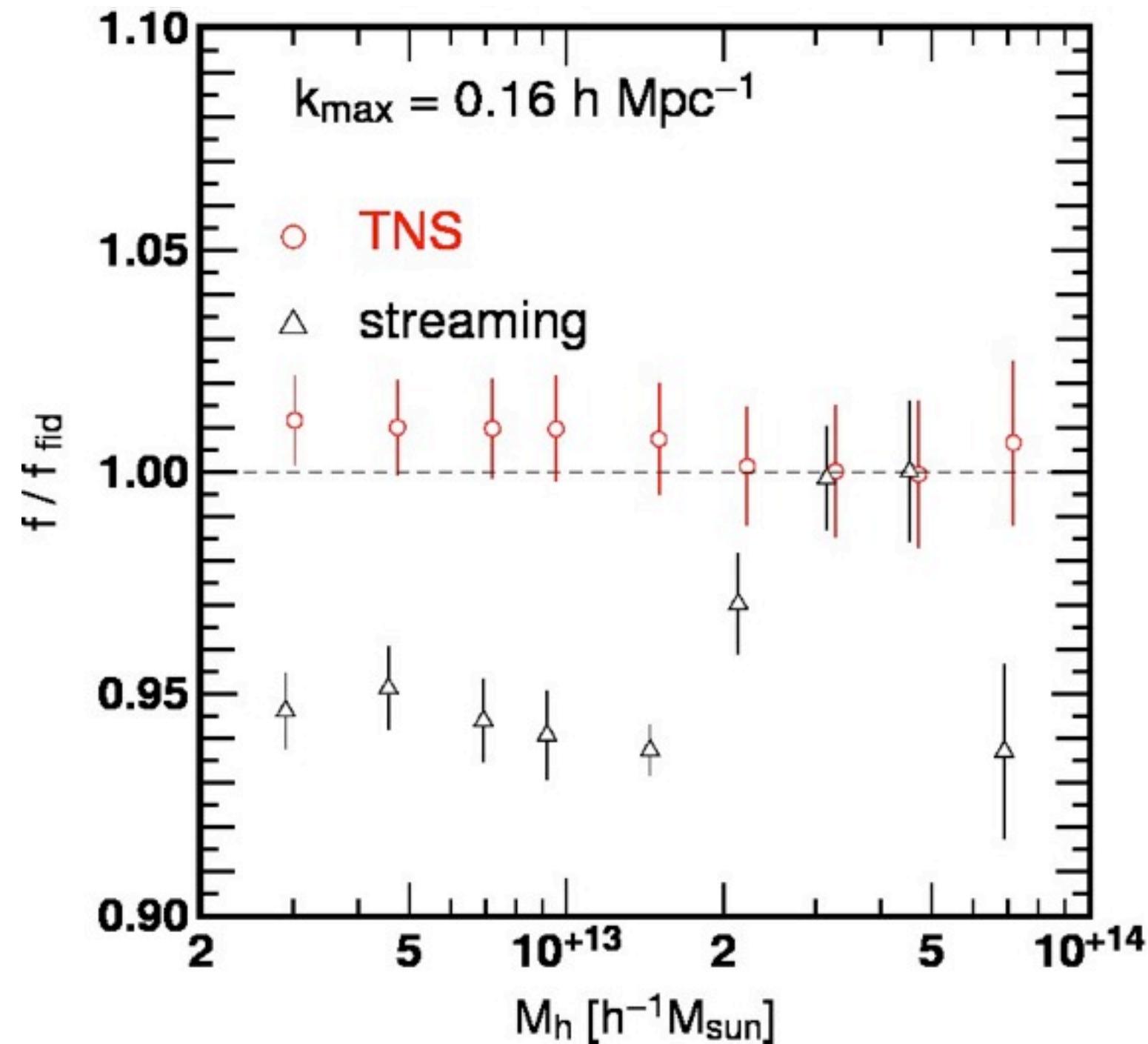
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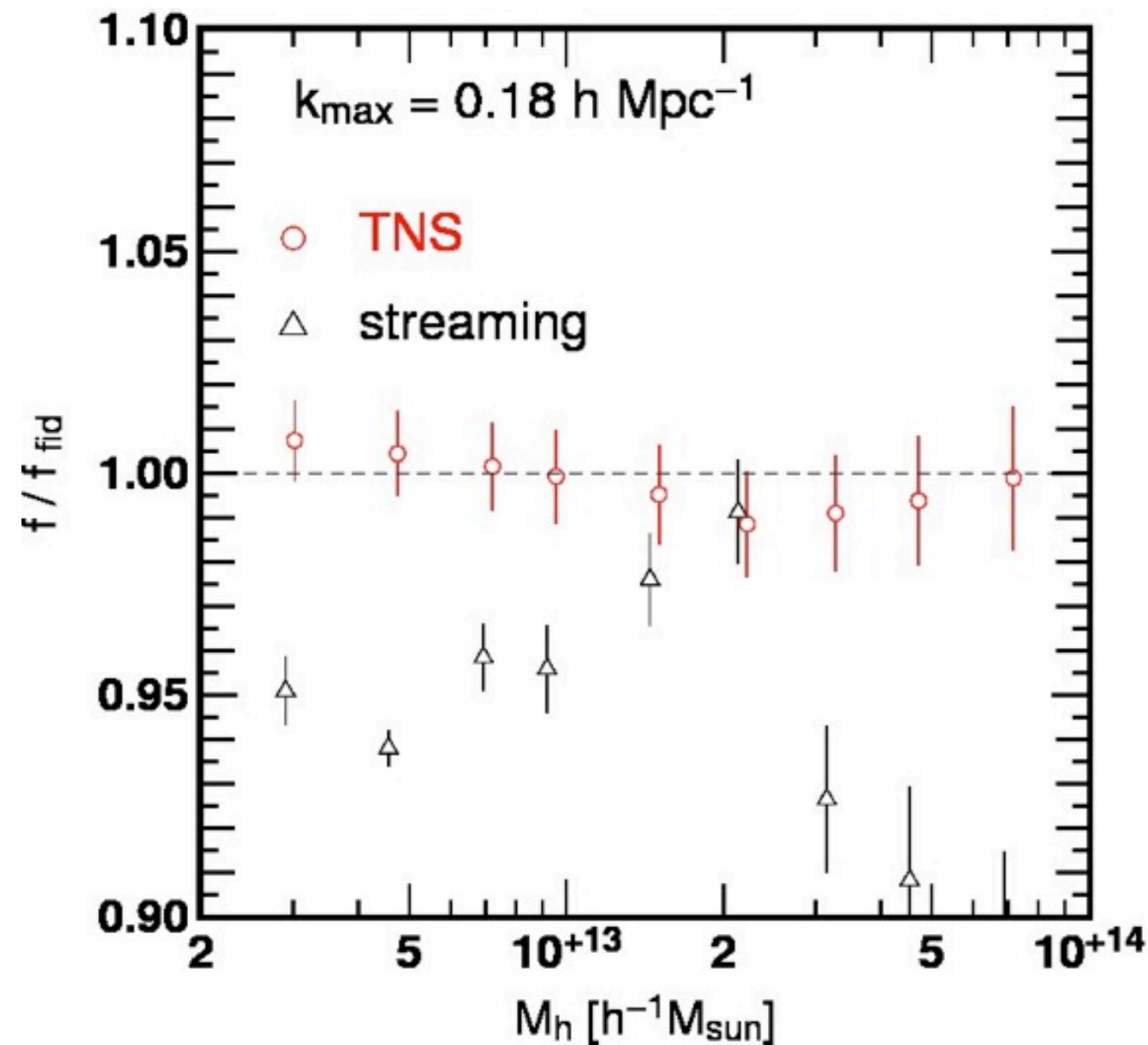
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# Future/on-going surveys



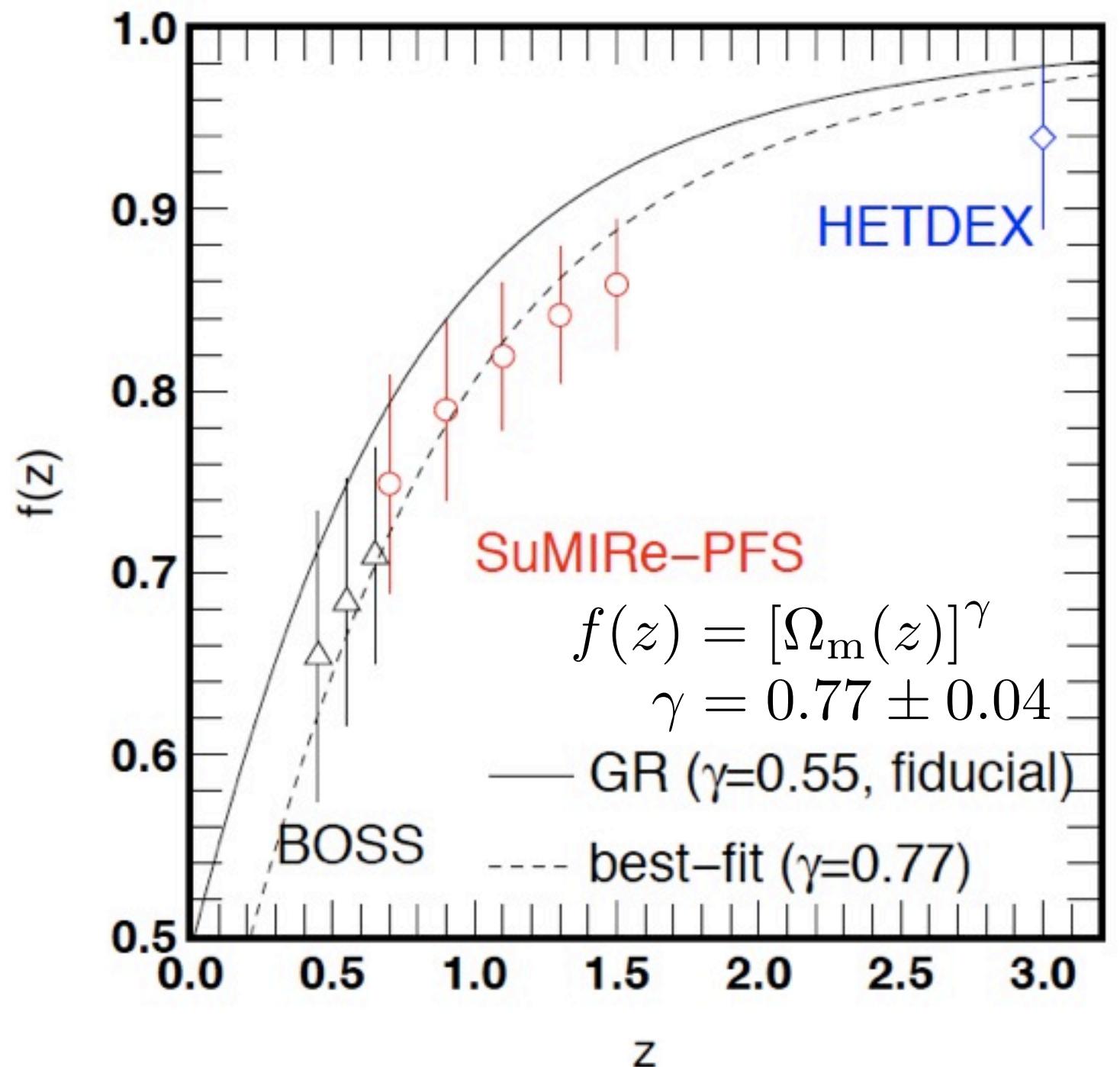
Fisher matrix analysis with 5 parameters

$(b, \sigma_v, H, D_A, f)$

**Assumption**

TNS model is true, but  
adopt streaming model

|            | $z_c$ | $V$ | $n_g$                | $b$ | $k_{\max}$ |
|------------|-------|-----|----------------------|-----|------------|
| BOSS       | 0.45  | 1.1 | $3 \times 10^{-4}$   | 2.2 | 0.15       |
|            | 0.55  | 1.5 | $3 \times 10^{-4}$   | 2.2 | 0.15       |
|            | 0.65  | 1.9 | $3 \times 10^{-4}$   | 2.2 | 0.15       |
| SuMIRe-PFS | 0.7   | 0.8 | $3 \times 10^{-4}$   | 1.5 | 0.2        |
|            | 0.9   | 1.1 | $3 \times 10^{-4}$   | 1.5 | 0.2        |
|            | 1.1   | 1.4 | $4 \times 10^{-4}$   | 1.5 | 0.2        |
|            | 1.3   | 1.6 | $4 \times 10^{-4}$   | 1.5 | 0.2        |
|            | 1.5   | 1.7 | $4 \times 10^{-4}$   | 1.5 | 0.2        |
| HETDEX     | 3.0   | 3.0 | $2.5 \times 10^{-4}$ | 2.5 | 0.4        |



# Summary



- tested the clustering of halos in z-space by N-body simulations ...
  - frequently used phenomenological model is not sufficient  
 $\sim$ 5% systematic bias in  $f(z)$
  - correction terms in TNS model  
more prominent for more massive halos or more biased objects
- codes for our model are **publicly available!!**  
visit CPT library: [http://www-utap.phys.s.u-tokyo.ac.jp/~ataruya/cpt\\_pack.html](http://www-utap.phys.s.u-tokyo.ac.jp/~ataruya/cpt_pack.html)