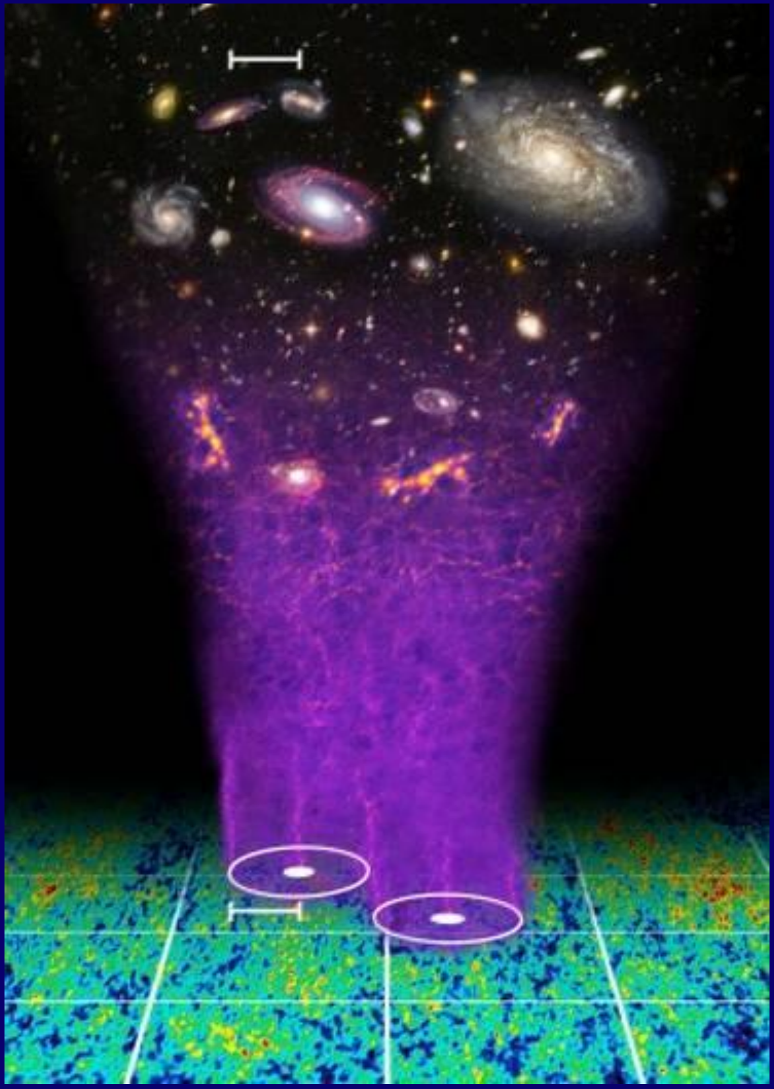
Models for the accelerating Universe



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Summer School

July 2011

Lecture 1

Overview of the accelerating Universe.
Dark Energy models in GR.
Observations of background and structure growth.

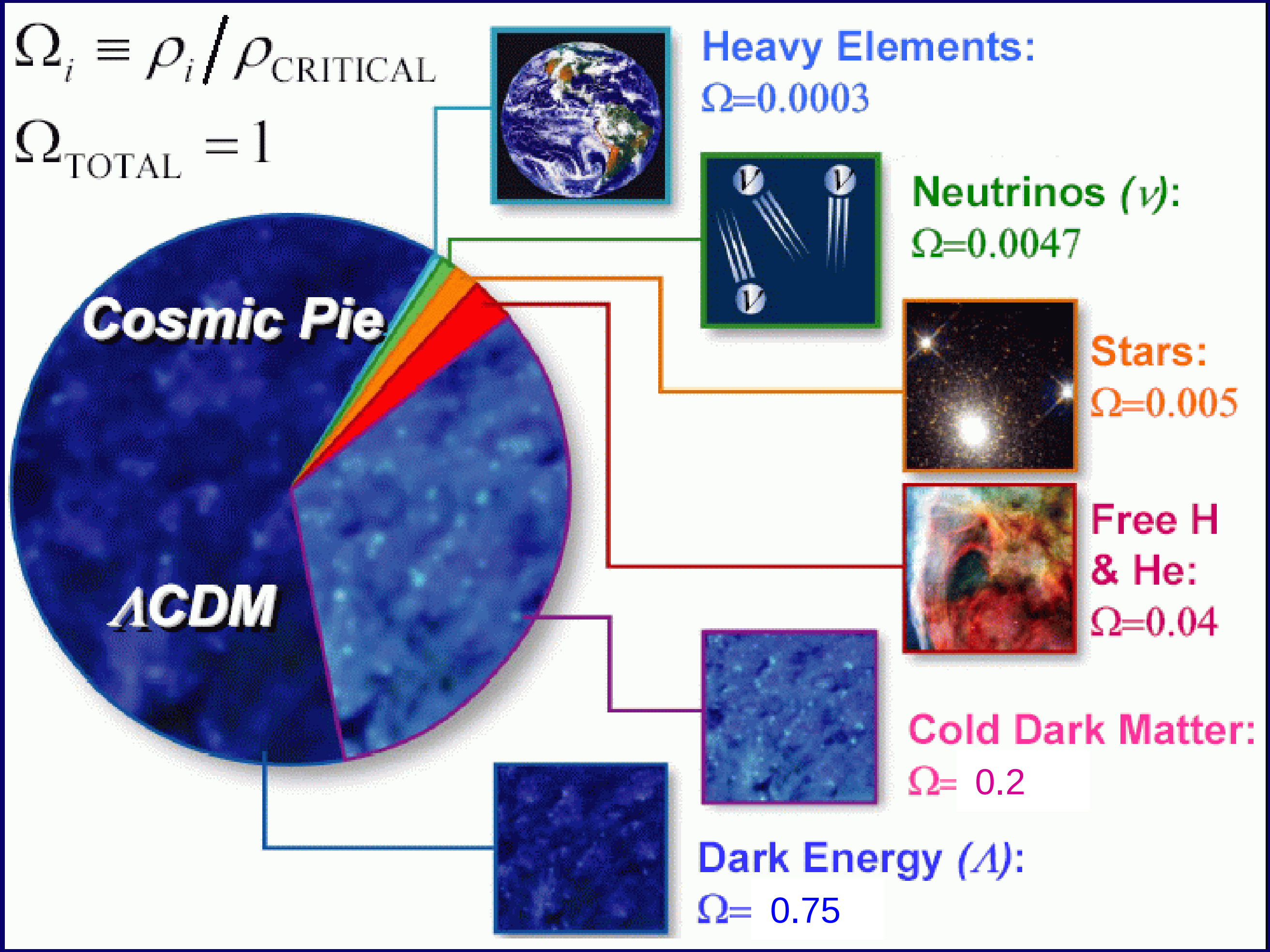
Lecture 2

Modified gravity as an alternative to DE.
f(R) and DGP – simplest models.
Testing GR with cosmology.

Lecture 3

Inhomogeneous models of the accelerating Universe.
Testing the Copernican Principle and homogeneity.

Lecture 1:
Dark energy models in GR

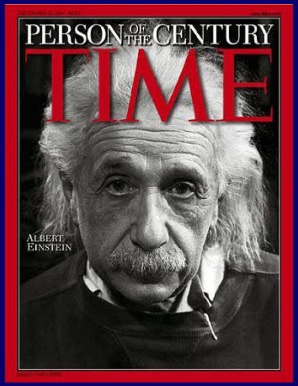


The puzzle of acceleration

- Galaxies are moving apart from each other – the Universe is expanding.

  Historical note:

  Discovered by Lemaitre, then Hubble.
  Einstein in 1915 believed the Universe must be *static*.
  His static Universe model wanted to collapse –
  so he introduced an 'anti-gravity' constant Λ to stop this.

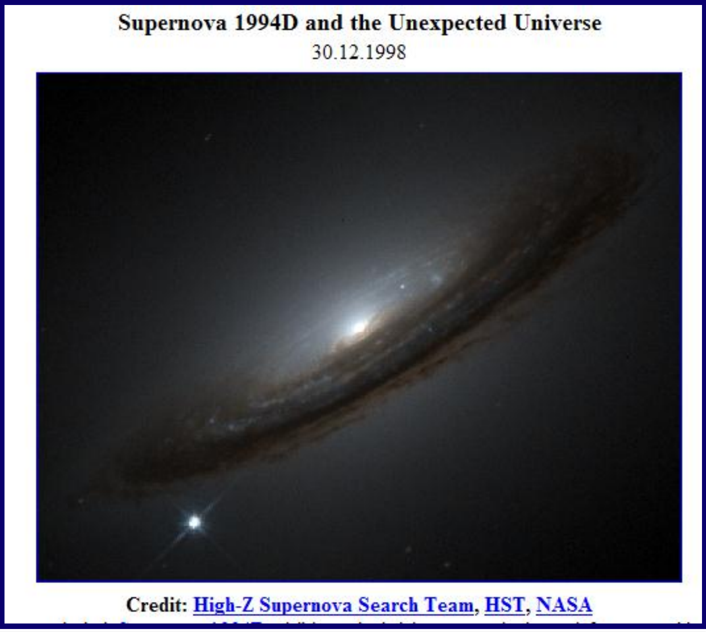


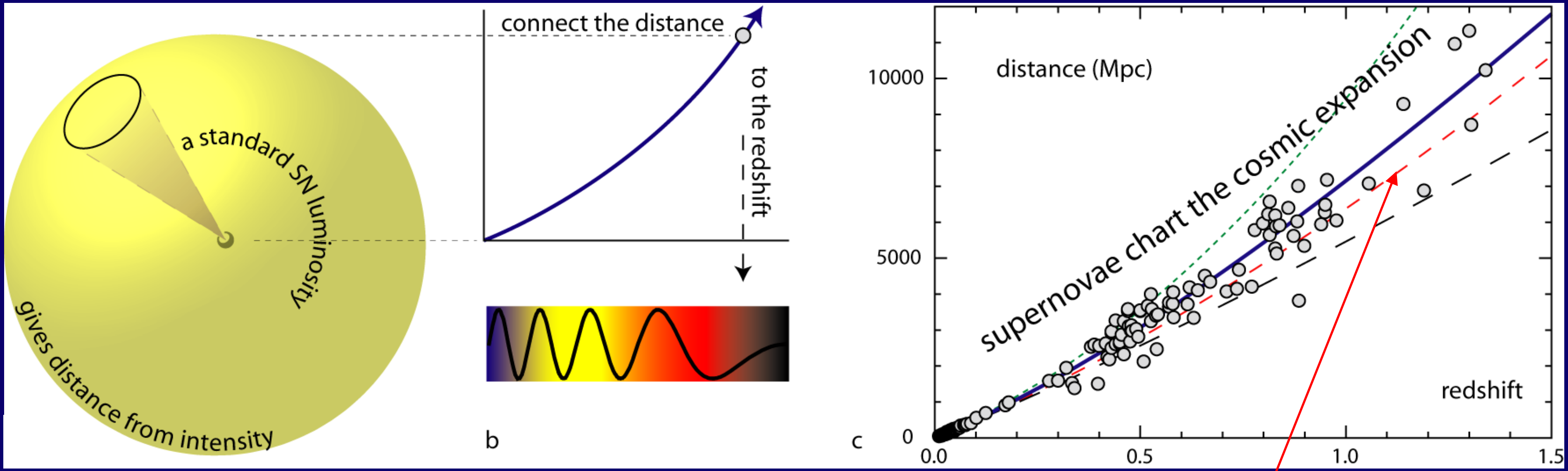
- Galaxies attract each other gravitationally – so we expect the expansion to *slow down.*
  But observations in the 1980s-1990s showed – the expansion is *accelerating.*
- The 'anti-gravity' effect of Λ can cause this: so Λ returned, but for a new reason.
  Λ is the simplest model of DARK ENERGY.

The key evidence for acceleration:

supernovae are more dim than they should be

(this is backed up by other data).

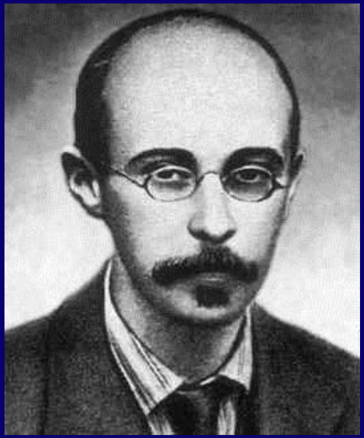


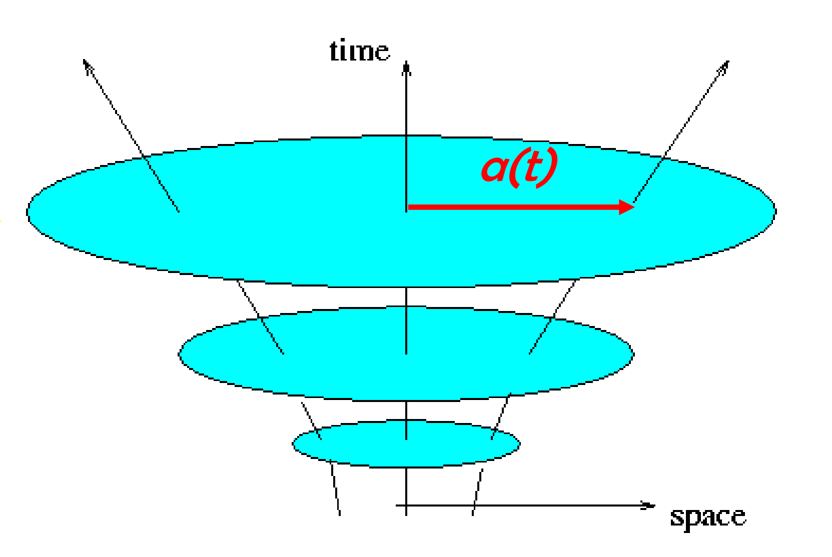


zero-acceleration curve

Friedmann's expanding universe

Suppose that the spatial Universe is as simple as possible, i.e. all points and directions are equivalent:





Each time instant = a 3-space of constant curvature.

a(t) measures the expansion; it obeys the Friedmann equation:

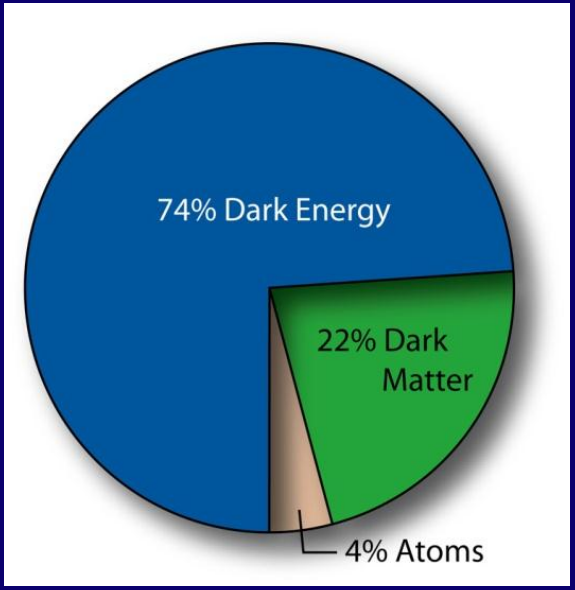
$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}(\rho_m + \rho_r) + \frac{\Lambda}{3} - \frac{K}{a^2}$$

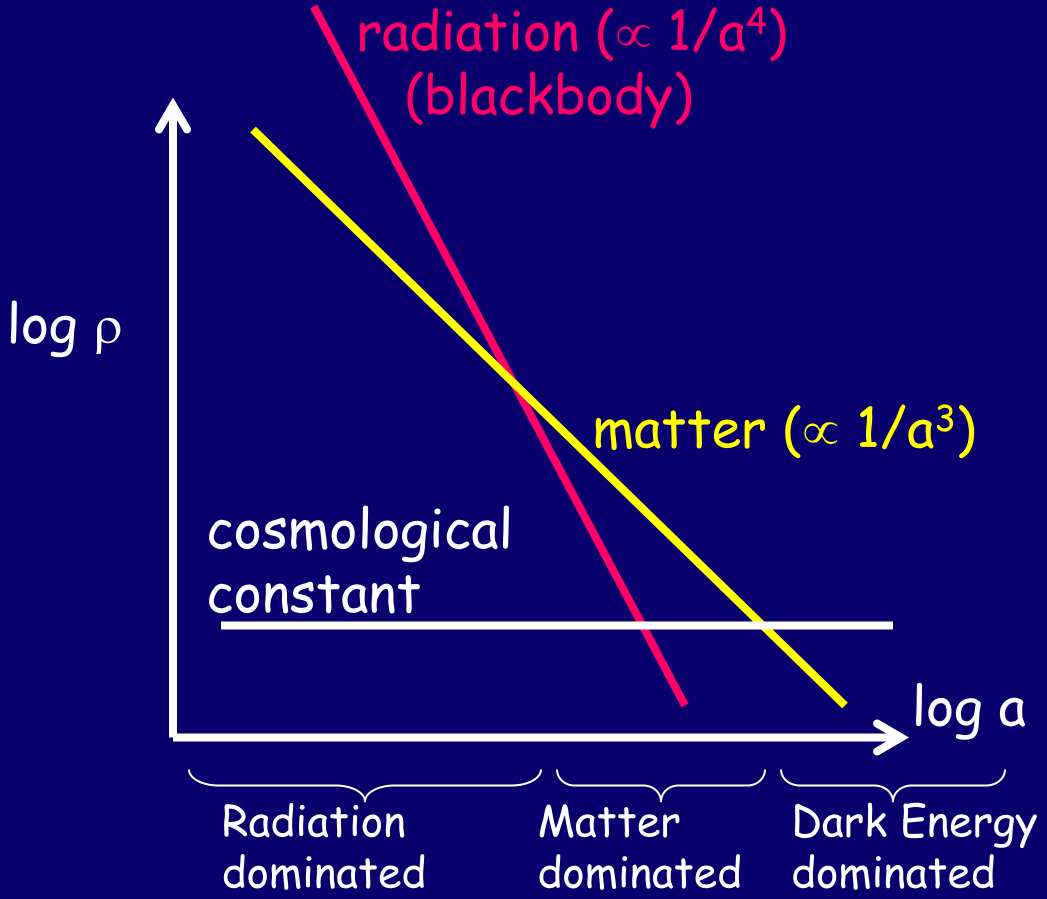
expansion rate = matter/radiation + dark energy + curvature

Evolution of matter, radiation, dark energy

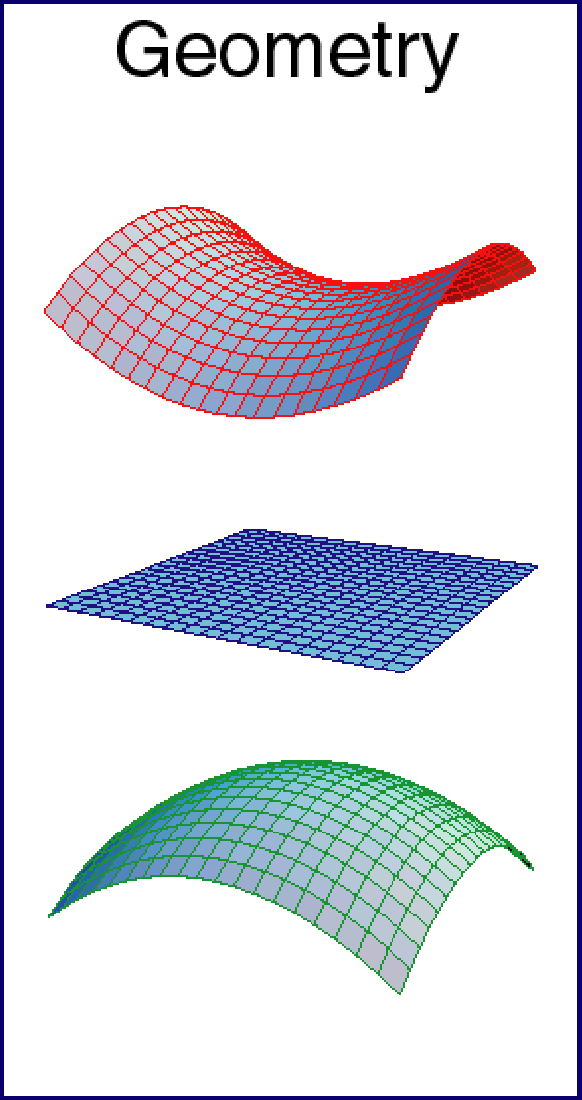
$$\text{energy conservation} \quad \dot{\rho} + 3H(\rho + p) = 0 \;\Rightarrow\; \rho_m \propto a^{-3}, \quad \rho_r \propto a^{-4}$$

ρ fractions today





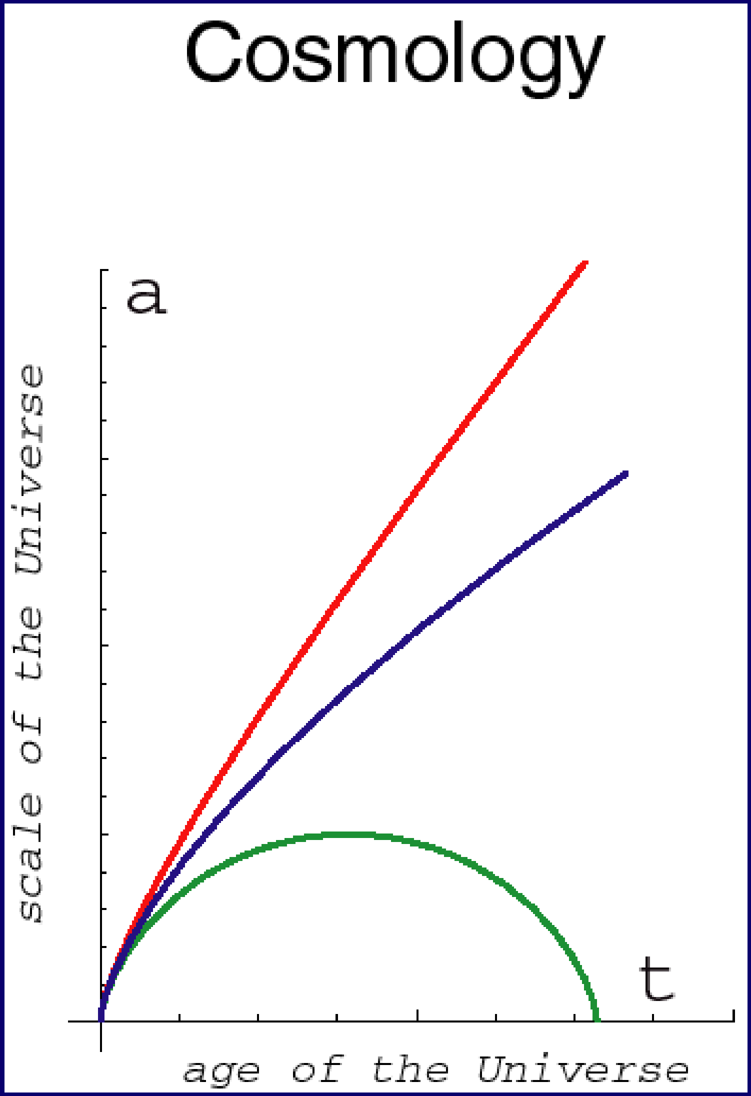
Solutions of Friedmann equation: Λ=0



K=-1

K=0

K=1



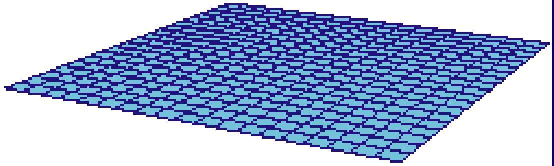
Solutions of Friedmann equation: Λ>0

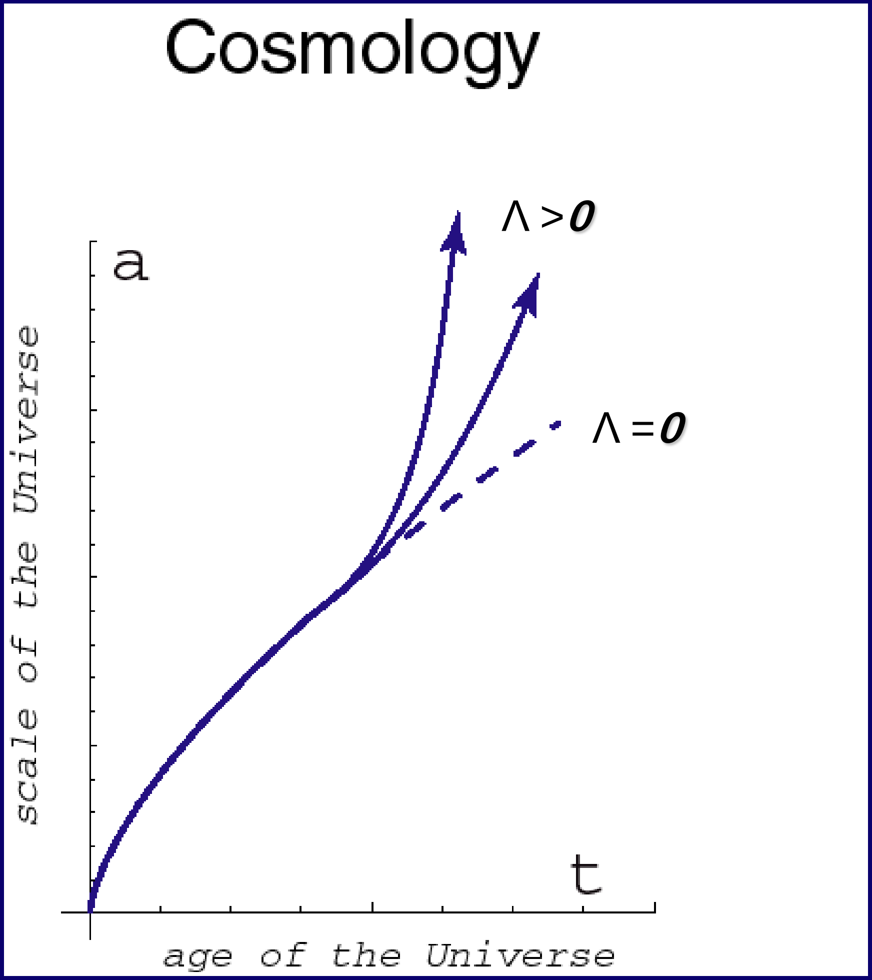
$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left(\rho_m + \rho_r\right) + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_m + 2\rho_r\right) + \frac{\Lambda}{3}$$

Geometry

observations suggest
K=0





Acceleration implies Dark Energy?

If:

- The Universe is well described by a perturbed Friedmann model – with a fixed background (no backreaction).
- GR holds.
- The expansion is accelerating.

Then: the acceleration is due to DE

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_{\rm tot}(1+3w_{\rm tot})$$

$$\ddot{a} > 0 \Rightarrow w_{\rm tot} < -\tfrac{1}{3} \Rightarrow w_{\rm de} < -\tfrac{1}{3}$$

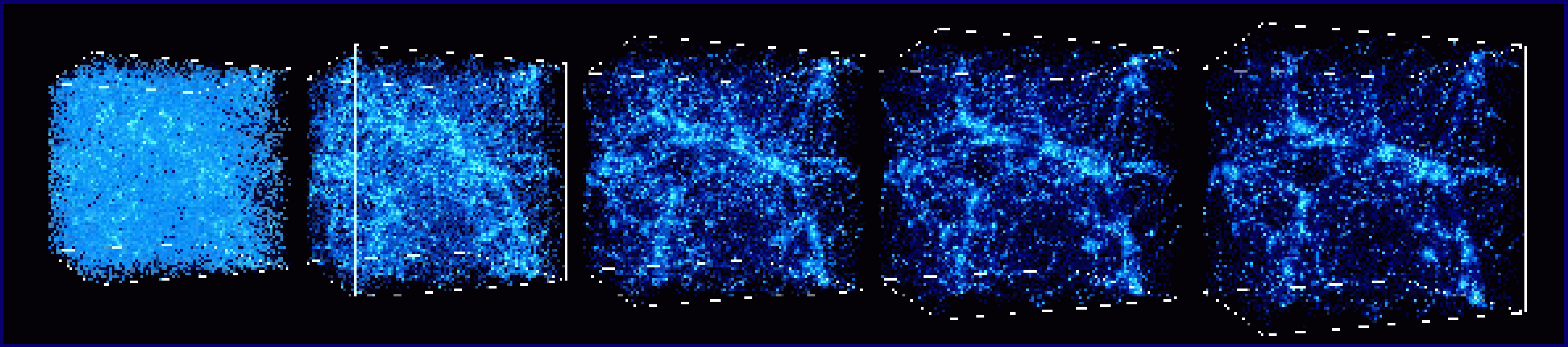
DE is a medium with w<-1/3.
Simplest DE is Λ, with *w=-1* (and *K=0).*

Friedmann perturbed universe

Inflation creates tiny fluctuations in density.

Gravity is *attractive* –

- Small over-densities grow *more* over-dense.
- Small under-densities become *more* under-dense.



- Cold Dark Matter (CDM) forms structure first – baryonic matter is delayed by radiation pressure.
- After photon decoupling, baryonic matter falls into the CDM ‘halos’ (potential wells) – the Cosmic Web begins to take shape.

The growth of structure

Galaxy clusters, voids, filaments, walls
(the Cosmic Web)

time

Galaxies form

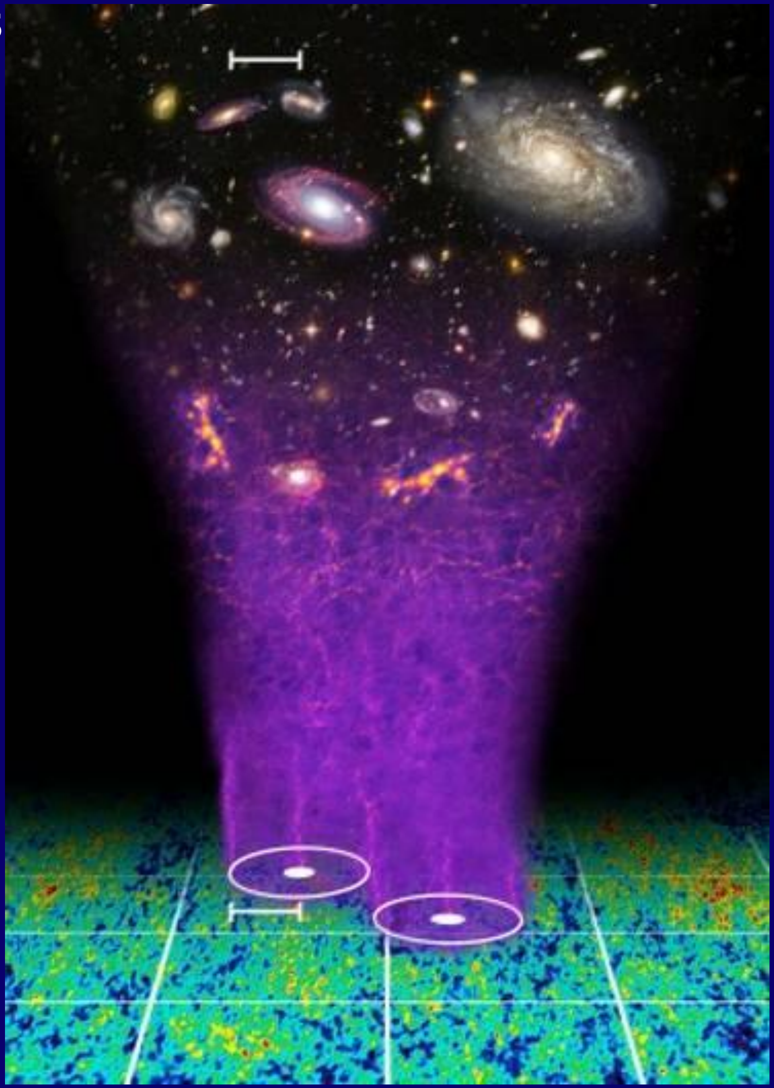
Stars form

Hydrogen condenses in CDM framework

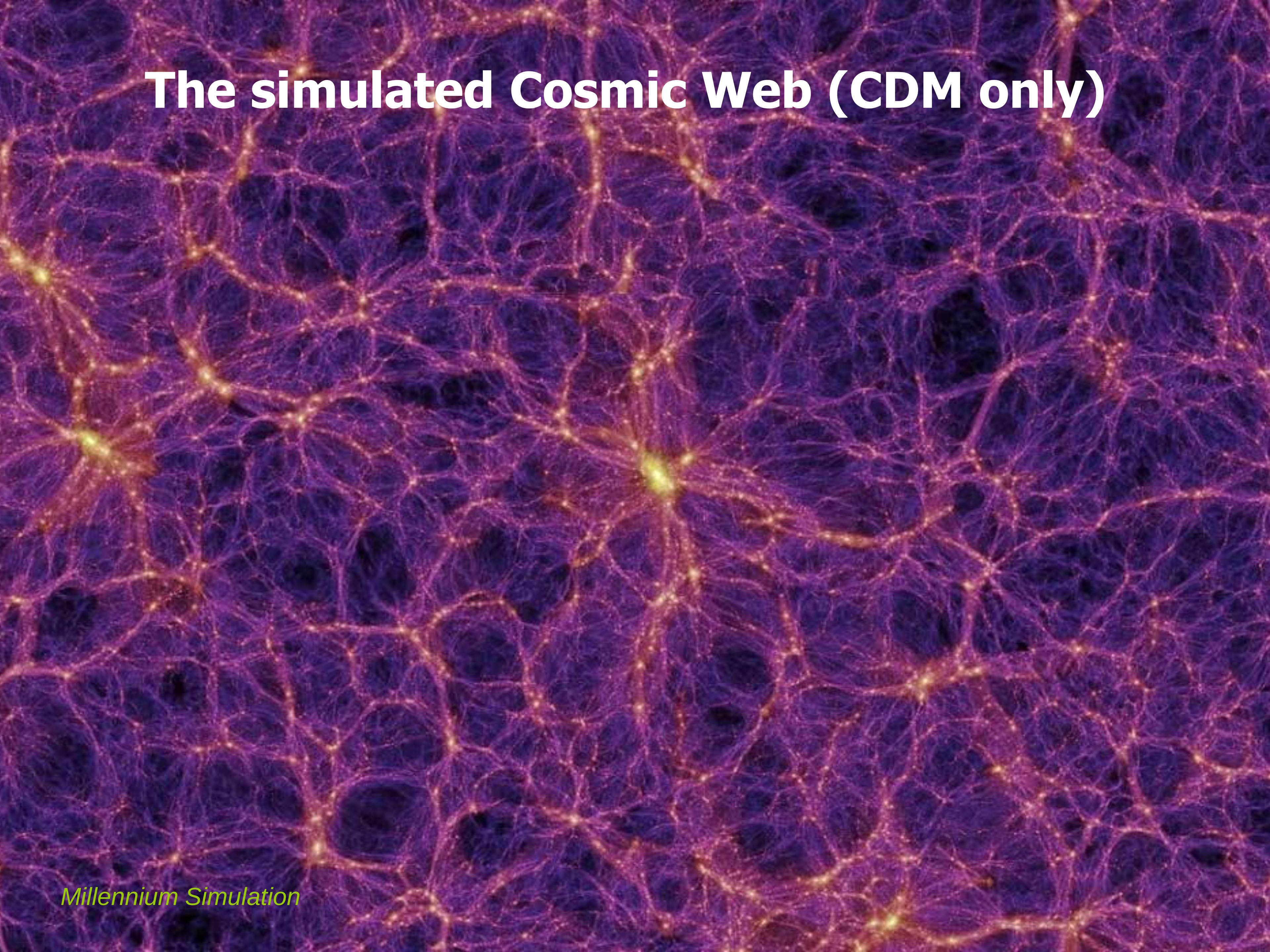
Photon temperature fluctuations
(over- and under-densities in matter)

CDM condenses into structure

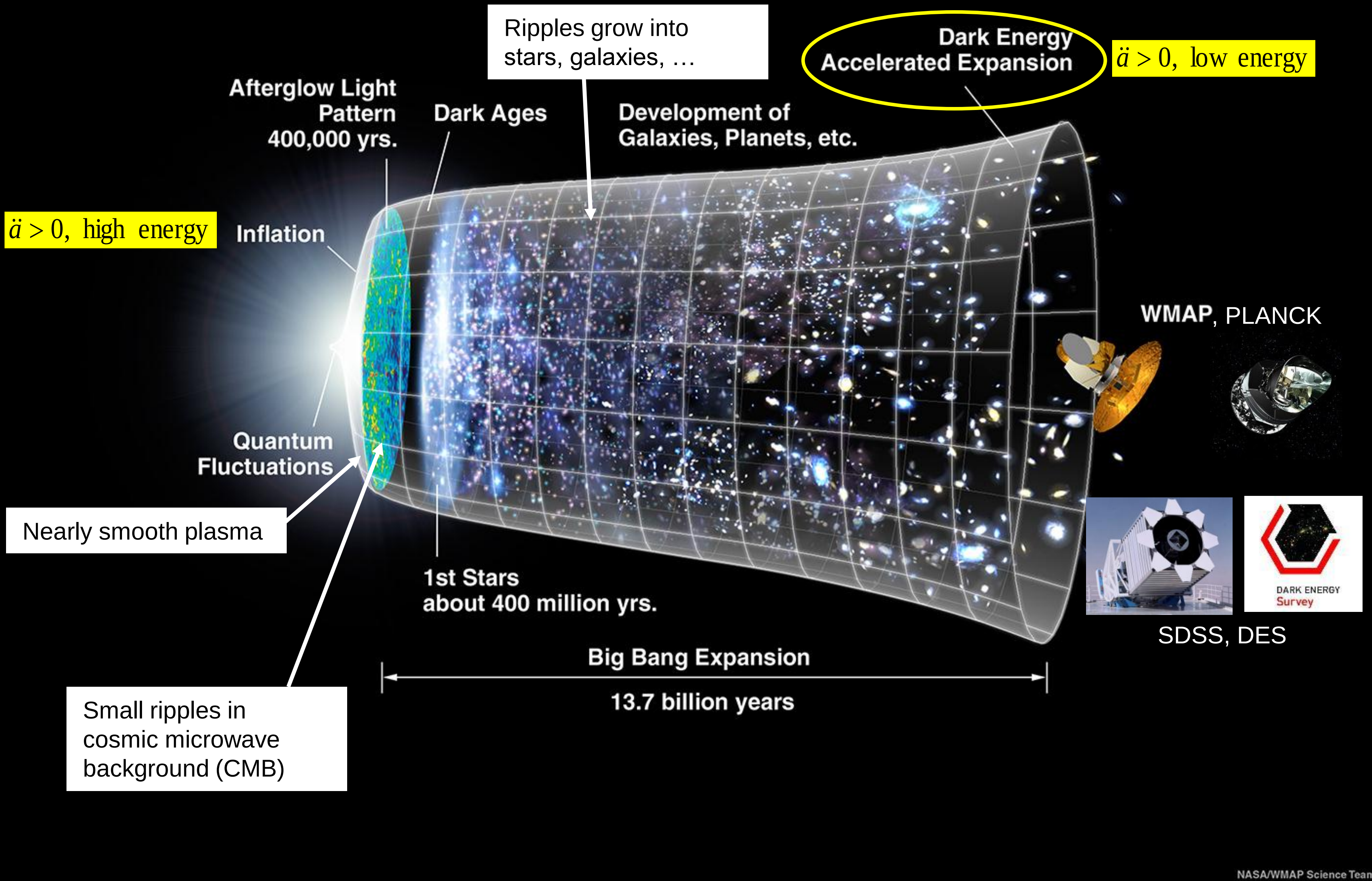
Photons + baryonic matter = plasma



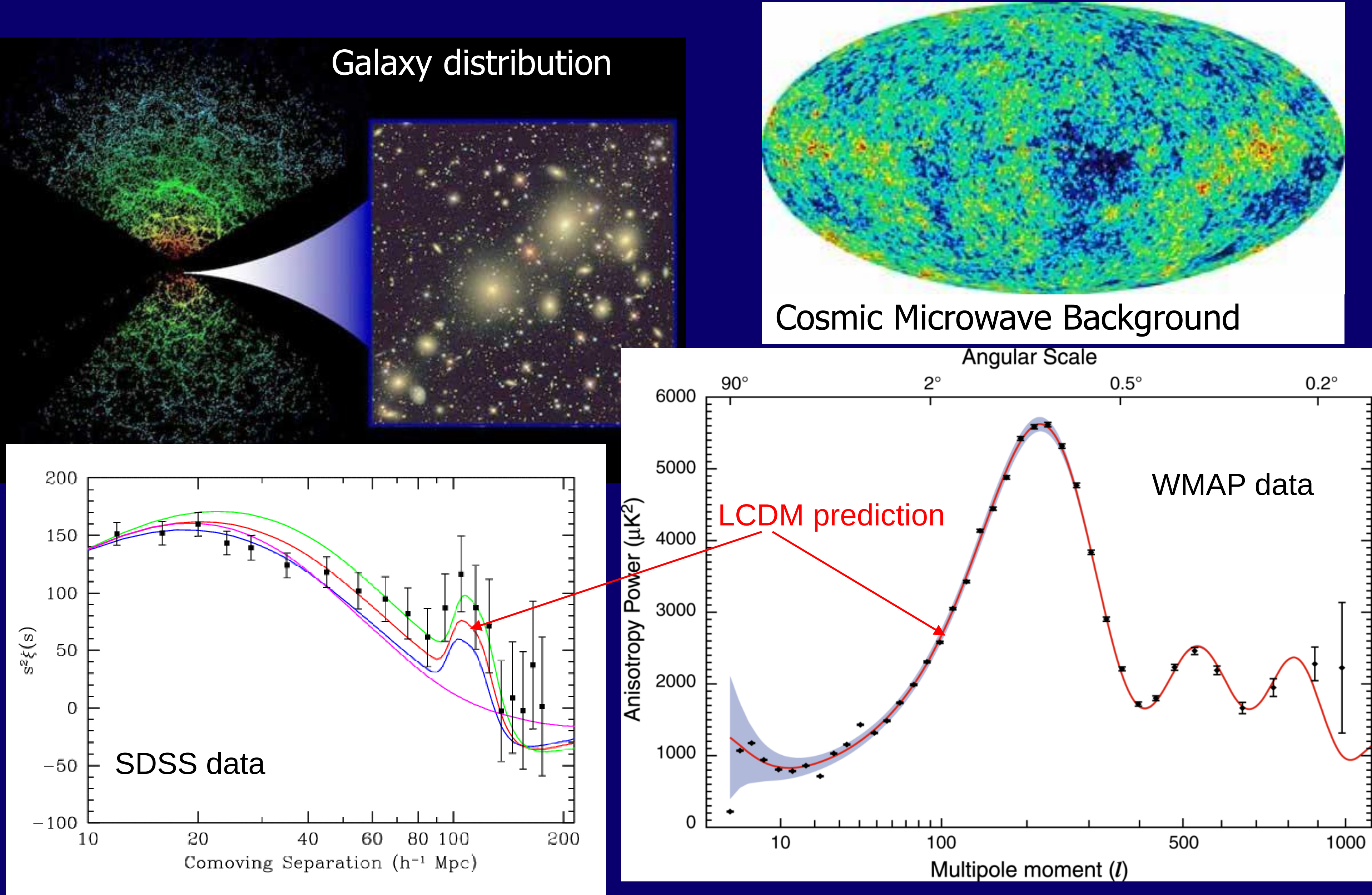
The simulated Cosmic Web (CDM only)

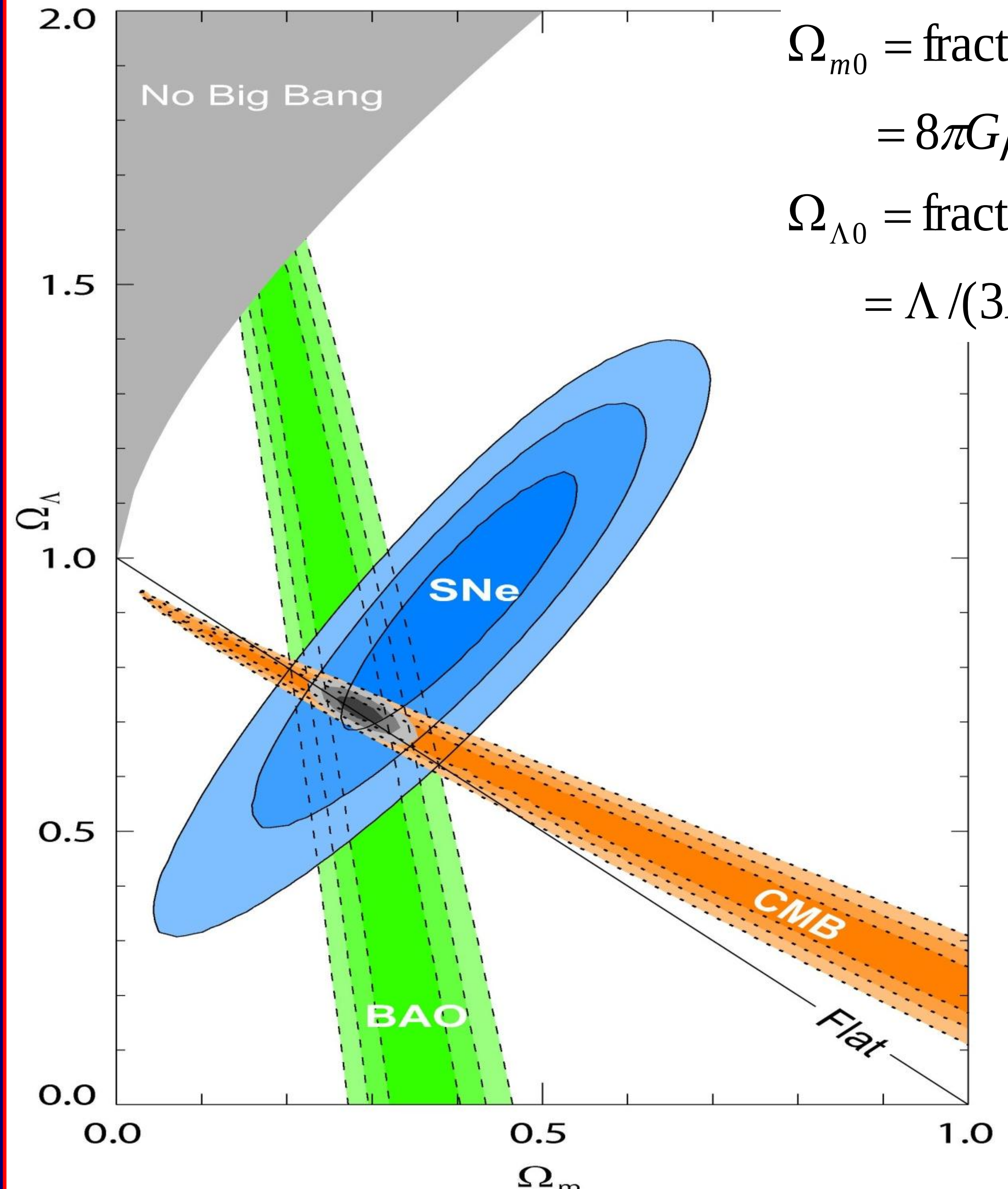


The standard cosmological model (ΛCDM) = smooth background + perturbations



ΛCDM fits the high-precision data





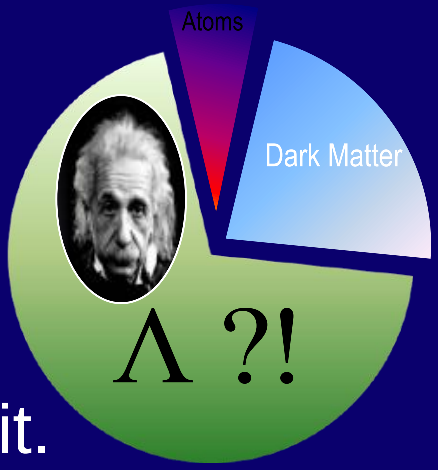
$$\Omega_{m0} = \text{fractional matter density (today)}$$
$$= 8\pi G\rho_{m0} /(3H_0^2)$$
$$\Omega_{\Lambda 0} = \text{fractional DE density (today)}$$
$$= \Lambda /(3H_0^2)$$

$$\Omega_{K0} \equiv -\frac{K}{a_0^2 H_0^2} \approx 0$$

inflation : $\Omega_K \to 0$

(not $K = 0$)

The puzzle of Λ as DE



- It is the simplest model we have.
- It is compatible with all data up to now.
- No other model gives a better statistical fit.
- But …. particle physics theory cannot explain it.

It is incredibly small – much smaller than all known energy scales:

$$\rho_{\Lambda 0}\Big|_{\rm obs} = \frac{\Lambda}{8\pi G} \sim \frac{3H_0^2}{8\pi G} \sim \frac{M_P^2}{H_0^{-2}} \sim (10^{-3}\ \text{eV})^4$$

And very fine-tuned ('coincidence problem'):

$$\rho_{\Lambda 0} \sim \rho_{m0} : \text{crucial for structure formation}$$

$$\text{but } \rho_\Lambda \propto a^0 \text{ while } \rho_m \propto a^{-3}$$

Λ as quantum vacuum energy

Energy-momentum tensor is Lorentz-invariant – and thus indistinguishable from vacuum energy:

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} + T^{\text{vac}}_{\mu\nu}), \quad T^{\text{vac}}_{\mu\nu} = -\rho_{\text{vac}} g_{\mu\nu}$$

Huge clash between 'prediction' (based on QFT) and observation:

$$\rho_\Lambda\big|_{\text{obs}} = \frac{\Lambda}{8\pi G} \sim \frac{3H_0^2}{8\pi G} \sim \frac{M_P^2}{H_0^{-2}} \sim (10^{-3}\ \text{eV})^4$$

$$\rho_\Lambda\big|_{\text{theory}} \sim M^4_{\text{fundamental}} \geq M^4_{\text{susy}} \sim (1\,\text{TeV})^4 >> \rho_\Lambda\big|_{\text{obs}}$$

Some attempts to resolve this:

- Unimodular gravity

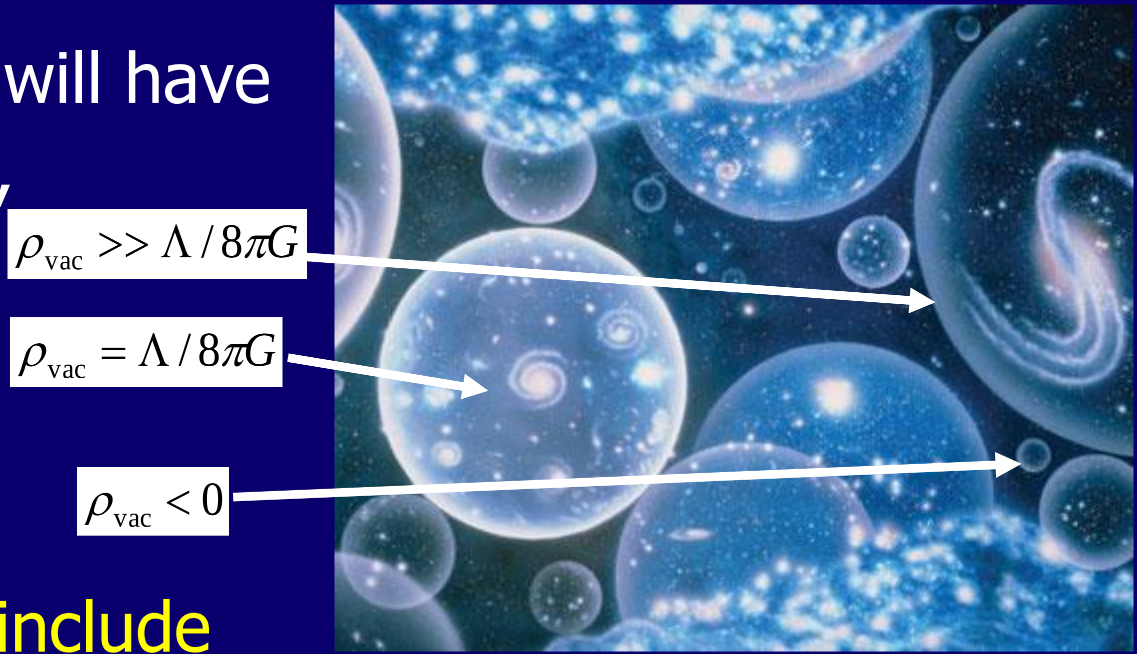
  The vacuum does *not* gravitate – Λ is a new gravitational constant.

- String theory ‘Multiverse’ and ‘Landscape’

  There are a huge number of fields in string theory, and a huge number of possible vacua (‘landscape’).

  Perhaps each vacuum corresponds to a separate ‘universe’ – each with its own value of vacuum energy.

  Some universes will have galaxies and life, others will not.



- Other attempts include ‘Degravitation’

What are the options for cosmology?

- GR + Perturbed Friedmann + Λ
  We let particle physics deal with the Λ problem and we test ΛCDM against the data.
- GR + Perturbed Friedmann + dynamical DE
  We try variable DE (solve coincidence problem?):

$$\Lambda \rightarrow V(\varphi): \ \rho = \tfrac{1}{2}\dot{\varphi}^2 + V(\varphi), \quad p = \tfrac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

- Modified Gravity + Perturbed Friedmann – *no* DE
  We try to do away with DE (must assume that vacuum does not gravitate), *and* we replace GR:

$$G_{\mu\nu} + G_{\mu\nu}^{\text{dark}} = 8\pi G T_{\mu\nu}$$

- GR + Nonlinear effects – *no* DE + *no* acceleration
  We try to do away with DE (must assume vacuum does not gravitate), but we keep GR.

Testing ΛCDM

GR + Perturbed Friedmann + Λ (flat, *K=0)*

– so far it passes all observational tests
– we must continue to test against new observations
– we can devise tests for deviations from *w=-1.*

We can use parametrizations of *w(z),* e.g.

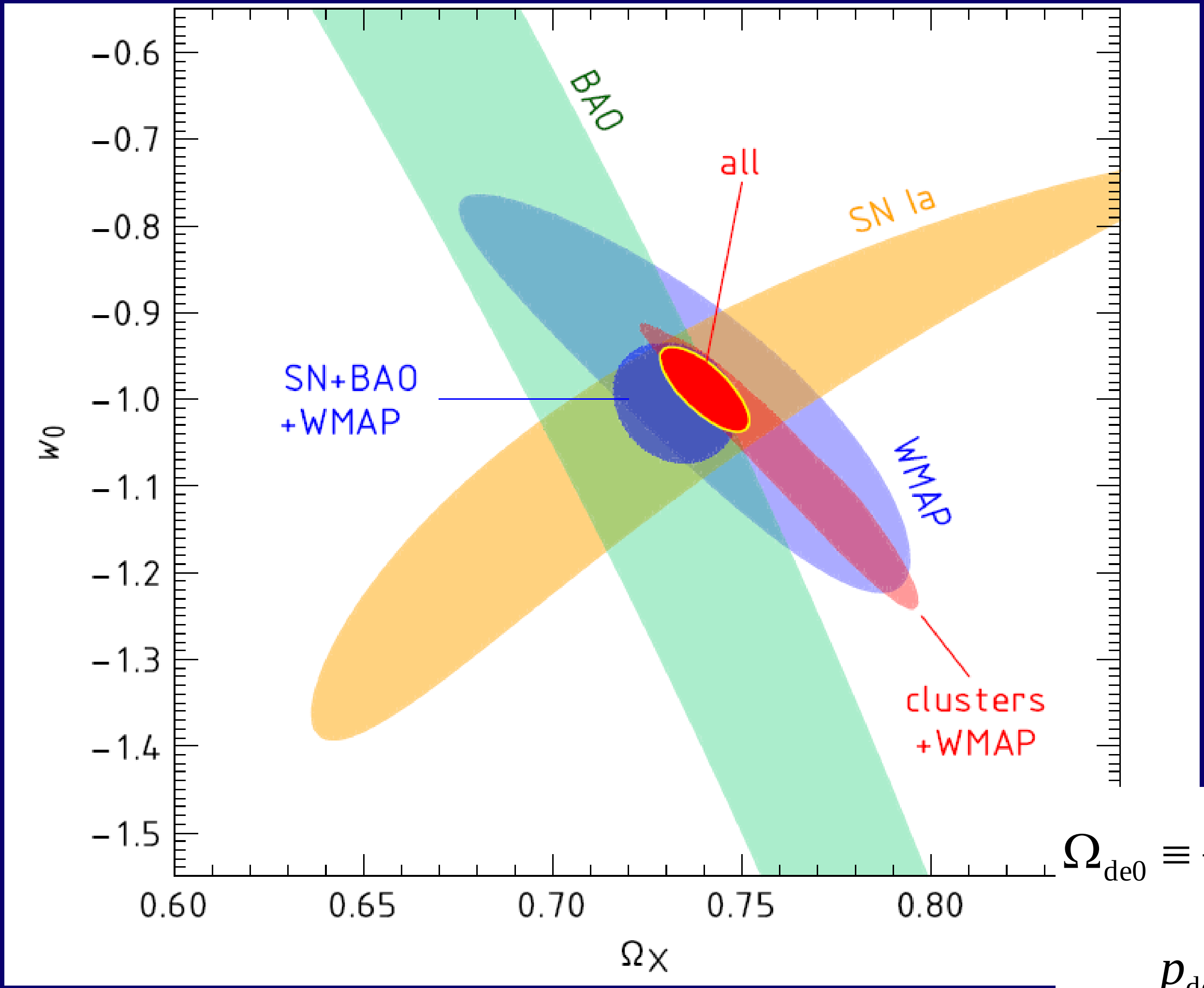
$$w(z) = w_0$$

or

$$w(z) = w_0 + w_a(1-a) = w_0 + w_a z(1+z)^{-1}$$
$$\rho_{de}(z) = \rho_{de0}(1+z)^{-3(1+w_0+w_a)}\exp[-w_a z(1+z)^{-1}]$$

These can be useful – for example, using $w=w_0$ gives an indication whether *w=-1* is violated. But:

* *w(z)* does not describe a *physical* model.
* Observational constraints depend on parametrization.
* We cannot compute DE perturbations.



$$\Omega_{\mathrm{de0}} \equiv \frac{8\pi G \rho_{\mathrm{de0}}}{3H_0^2}$$

$$w \equiv \frac{p_{\mathrm{de}}}{\rho_{\mathrm{de}}} < -\frac{1}{3}$$

$$\frac{H^2}{H_0^2} = \Omega_{\mathrm{m0}}(1+z)^3 + \Omega_{\mathrm{de0}}(1+z)^{3(1+w)}$$

$$+(1-\Omega_{\mathrm{m0}} - \Omega_{\mathrm{de0}})(1+z)^2$$

Null tests of ΛCDM

We should test physically motivated models –
observations cannot determine a model.
Try to devise null tests for the concordance model.

For example, define the observable:

$$\mathbb{O}_m(z) \equiv \frac{H^2(z)/H_0^2 - 1}{(1+z)^3 - 1}$$

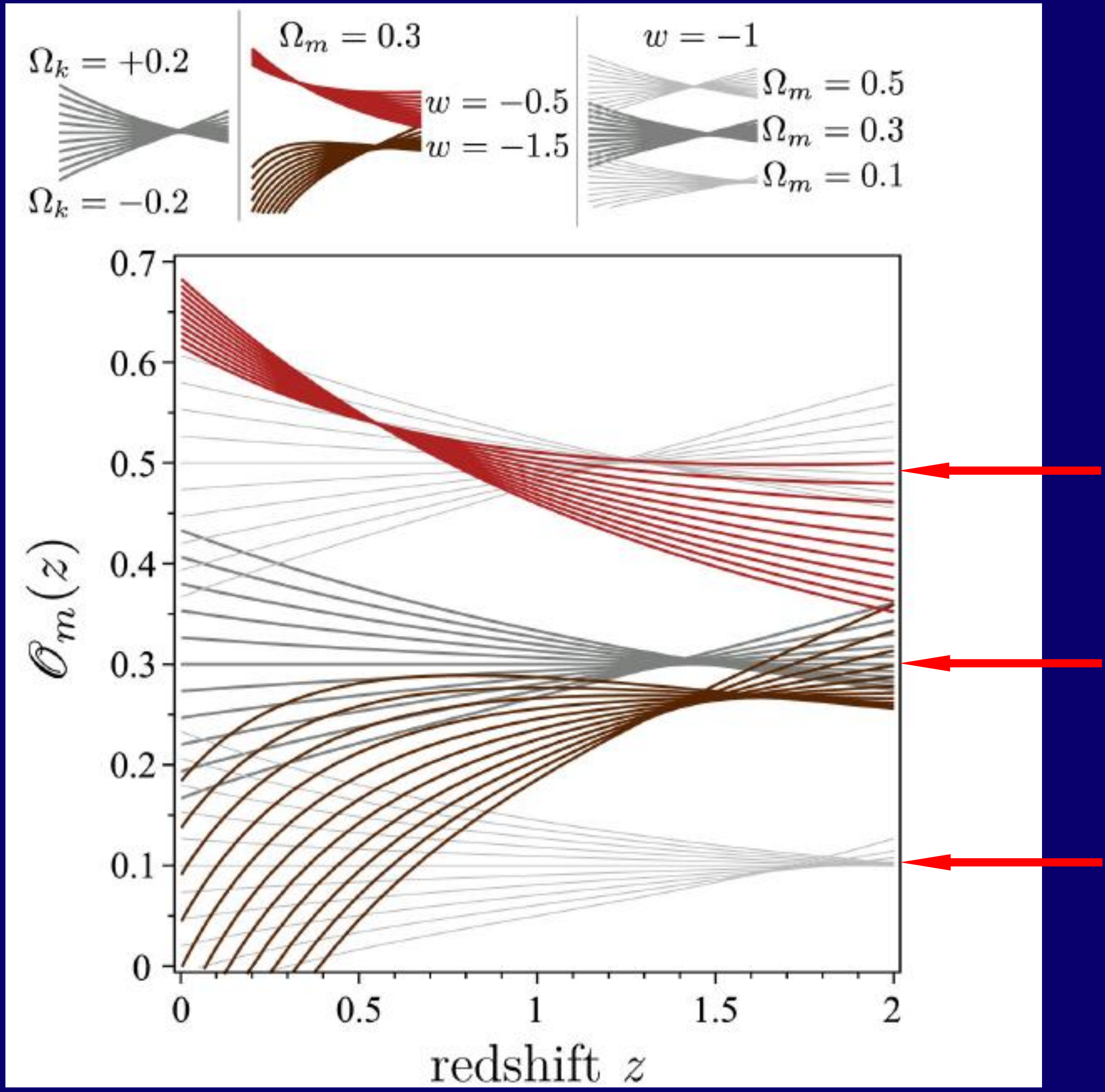
(Zunckel, Clarkson 2008;
Sahni, Shafieloo, Starobinbsky 2008)

It satisfies:

$$\Lambda\text{CDM} \Rightarrow \mathbb{O}_m(z) = \Omega_{m0} = \text{const}$$

$$\frac{d}{dz}\mathbb{O}_m(z) \neq 0 \Rightarrow \text{not } \Lambda\text{CDM}$$

Any model that is not flat ΛCDM has nonzero $\mathbb{O}_m'(z)$
eg curved ΛCDM, *w≠-1* models



(Shafieloo, Clarkson 2010)

In principle this null test can rule out the concordance model (flat ΛCDM).

In practice, the data for *H(z)* is still poor – from cluster counts and galaxy ages.

We can relate *H(z)* to luminosity distance via

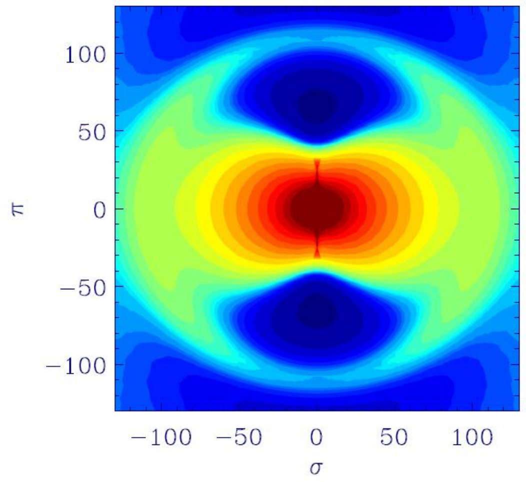
$$D_L(z) = \frac{1+z}{H_0\sqrt{-\Omega_{K0}}} \sin\left(\sqrt{-\Omega_{K0}} \int_0^z \frac{d\tilde{z}}{H(\tilde{z})/H_0}\right)$$

But then we need the derivative of $D_L(z)$ – and data is not good enough yet.

Future BAO surveys should determine *H(z)* – from the radial BAO feature:

$$r_s = \frac{\Delta z(z)}{H(z)}$$

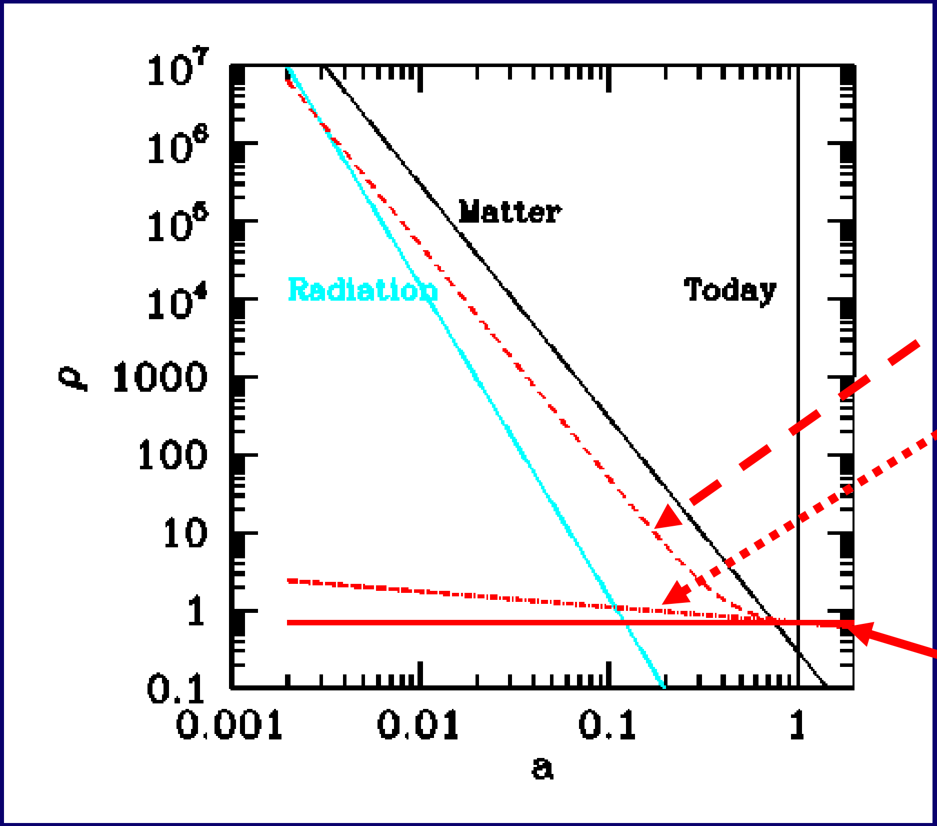
simulation



(Gaztanaga, Cabre, Hui 2009)

Dynamical DE: $\Lambda \rightarrow V(\varphi)$

Motivation: try to solve the coincidence problem (DE should evolve from high value to low).



quintessence (tracker)

quintessence (slow-roll)

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$$

vacuum energy $\dot{\varphi} = 0$

- But this requires highly fine-tuned parameters in *V*.
- Perhaps string theory will explain this?

Observations that probe acceleration

There are 2 kinds of observations.

1. Observations that probe the *background evolution,* i.e. the expansion history *H(z):*
   - SNIa luminosity distance
   - CMB ‘shift’ parameter
   - Baryon Acoustic Oscillation (BAO) scale

   and (less reliable): cluster counts, galaxy ages, GRBs

2. Observations that probe the *growth of structure:*
   - Growth rate of density perturbations
   - CMB: Integrated Sachs-Wolfe effect
   - Gravitational lensing

Observations that probe the background

Evolution is given by *H(z)* – constrained by:

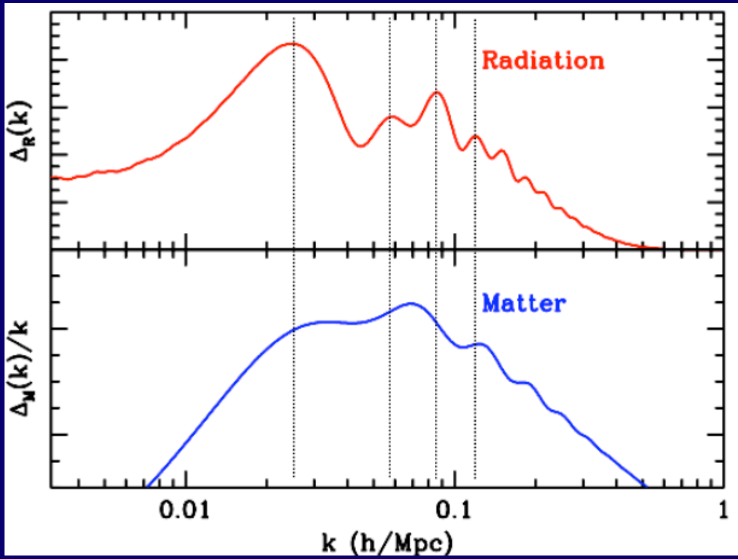
- Standard <u>candles</u> – SNIa luminosity distance

$$D_L(z) = \frac{1+z}{H_0\sqrt{-\Omega_{K0}}}\sin\left(\sqrt{-\Omega_{K0}}\int_0^z \frac{d\tilde{z}}{H(\tilde{z})/H_0}\right)$$

- Standard <u>rulers</u> – acoustic scale at decoupling

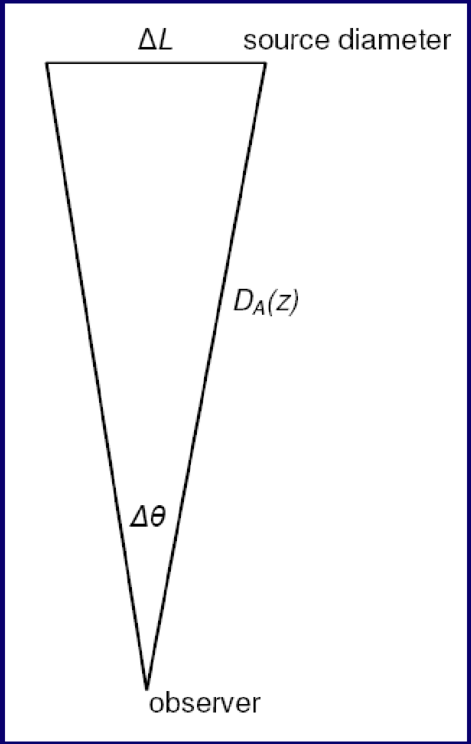
$$r_s = \int_{z_{\rm dec}}^{\infty} \frac{c_s}{H}\,dz, \qquad c_s = \frac{1}{\sqrt{3(1+R)}}, \qquad R = \frac{3\rho_B}{4\rho_\gamma}$$

Probed by CMB 'shift parameter' and by galaxies (Baryon Acoustic Oscillations)



Cosmological distances

Angular diameter distance: $D_A = \frac{\Delta L}{\Delta\theta}$



Luminosity distance: $F_{\text{obs}} = \frac{I_{\text{source}}}{4\pi D_L^2}$

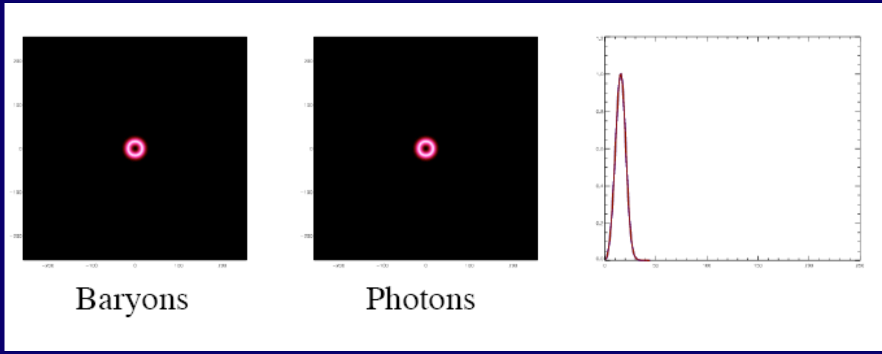
Distance duality:

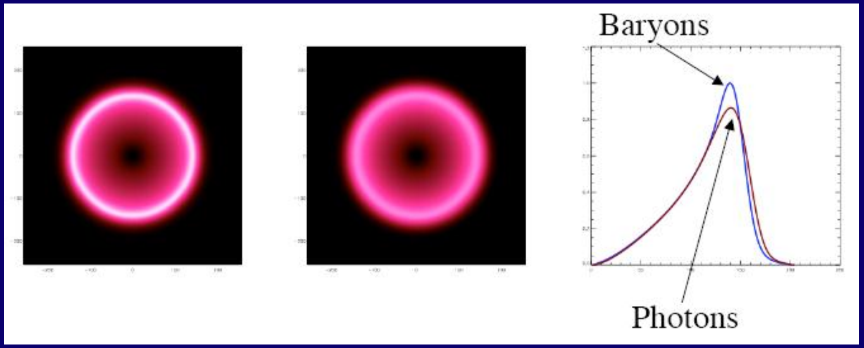
$$D_A(z) = (1+z)^{-2} D_L(z)$$ Etherington

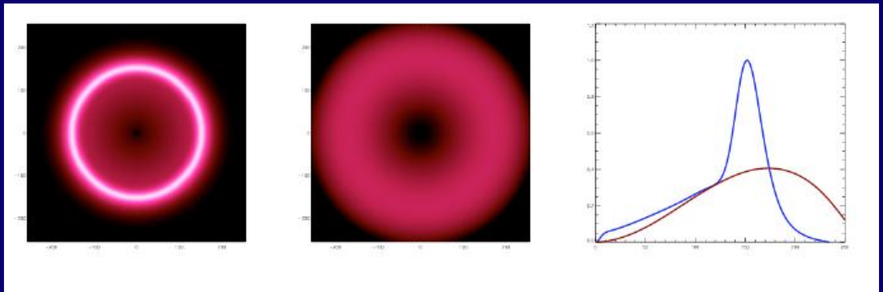
Friedmann:

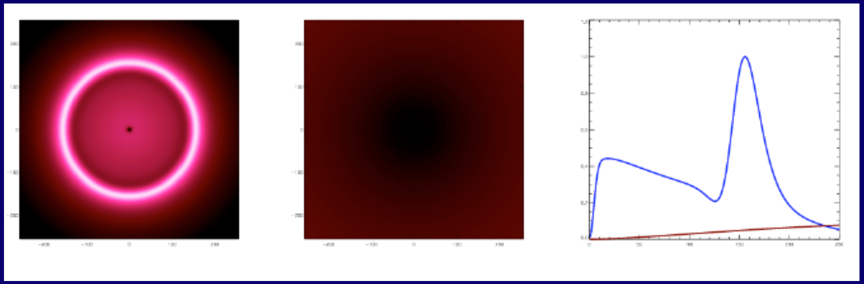
$$D_A(z) = \frac{1}{H_0(1+z)\sqrt{-\Omega_{K0}}} \sin\left(\sqrt{-\Omega_{K0}} \int_0^z \frac{d\tilde{z}}{H(\tilde{z})/H_0}\right)$$

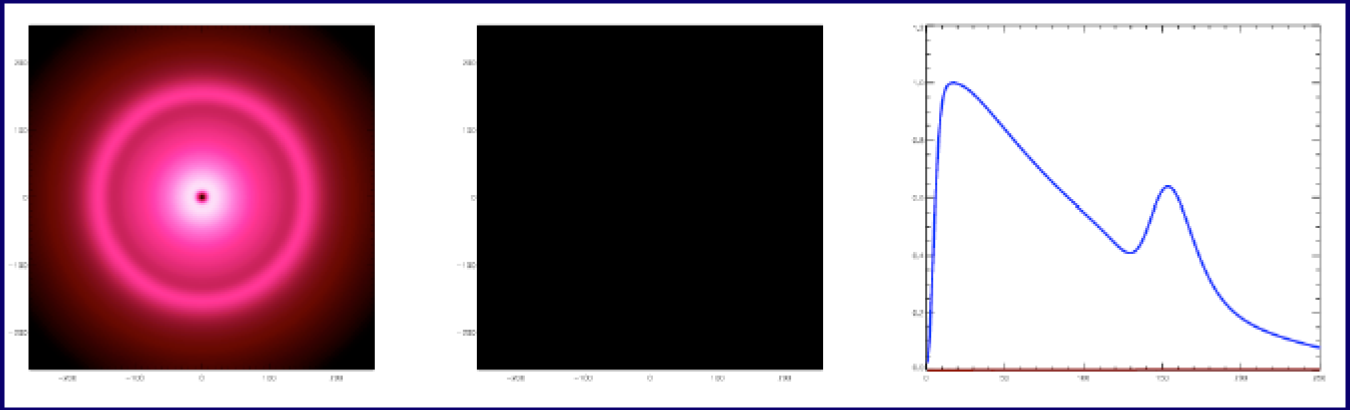
BAO – a fossil record in the galaxy distribution





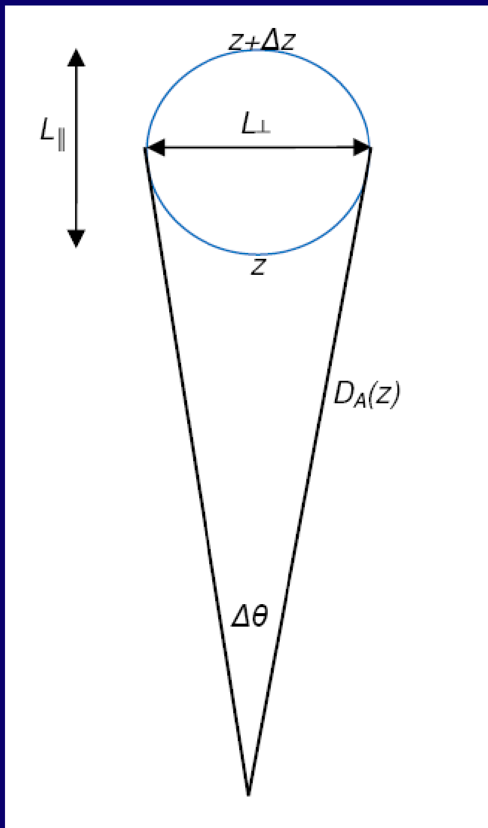






(White 2007)

BAO – a powerful future probe of *H(z)* and *D_A(z)*



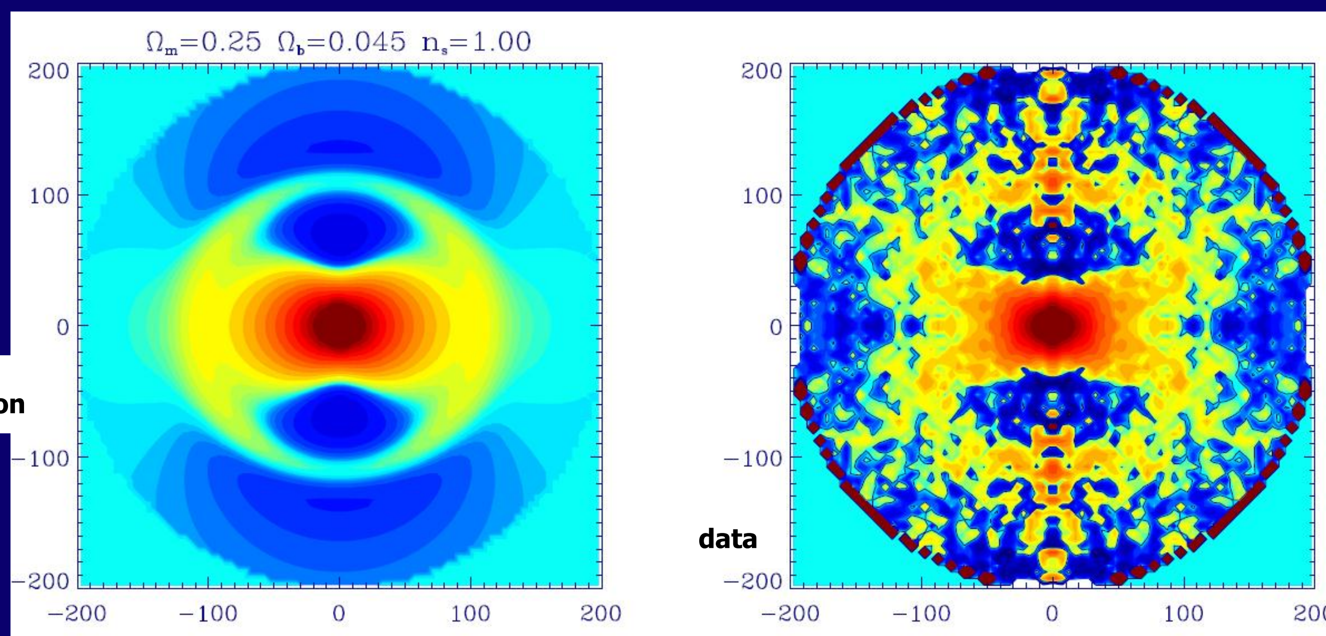
$$L_\perp = D_A \Delta\theta = \frac{r_s}{1+z}$$

$$L_\parallel = \frac{\Delta z}{(1+z)H} = \frac{r_s}{1+z}$$

Currently only a volume average is found:

$$D_V \propto r_s \left[\Delta z (\Delta\theta)^2\right]^{-1/3} \propto (D_A^2 H^{-1})^{1/3}$$

simulation

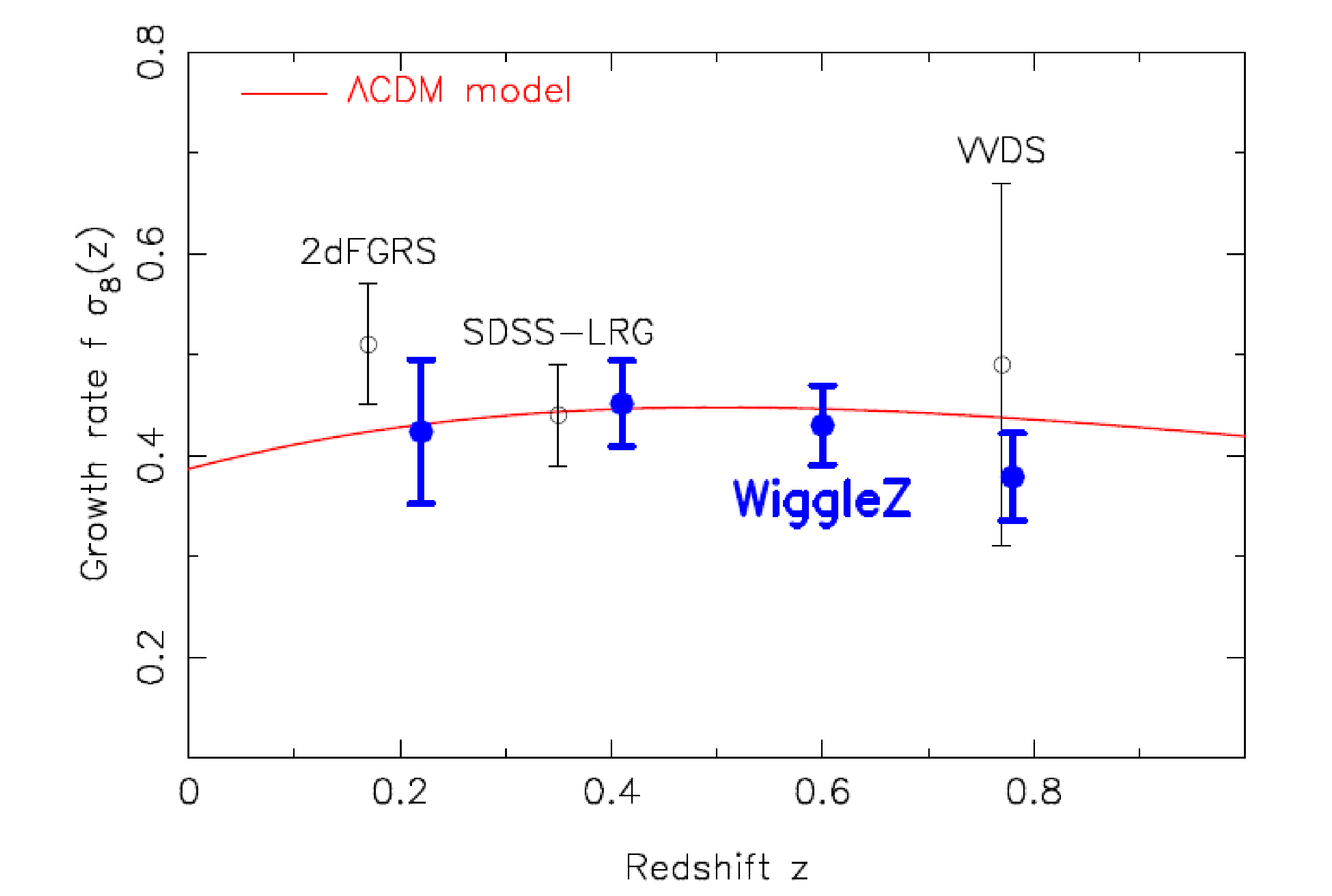


(Gaztanaga, Cabre, Hui 2009)

Observations that probe the growth of structure. 1

Rate of growth of structure

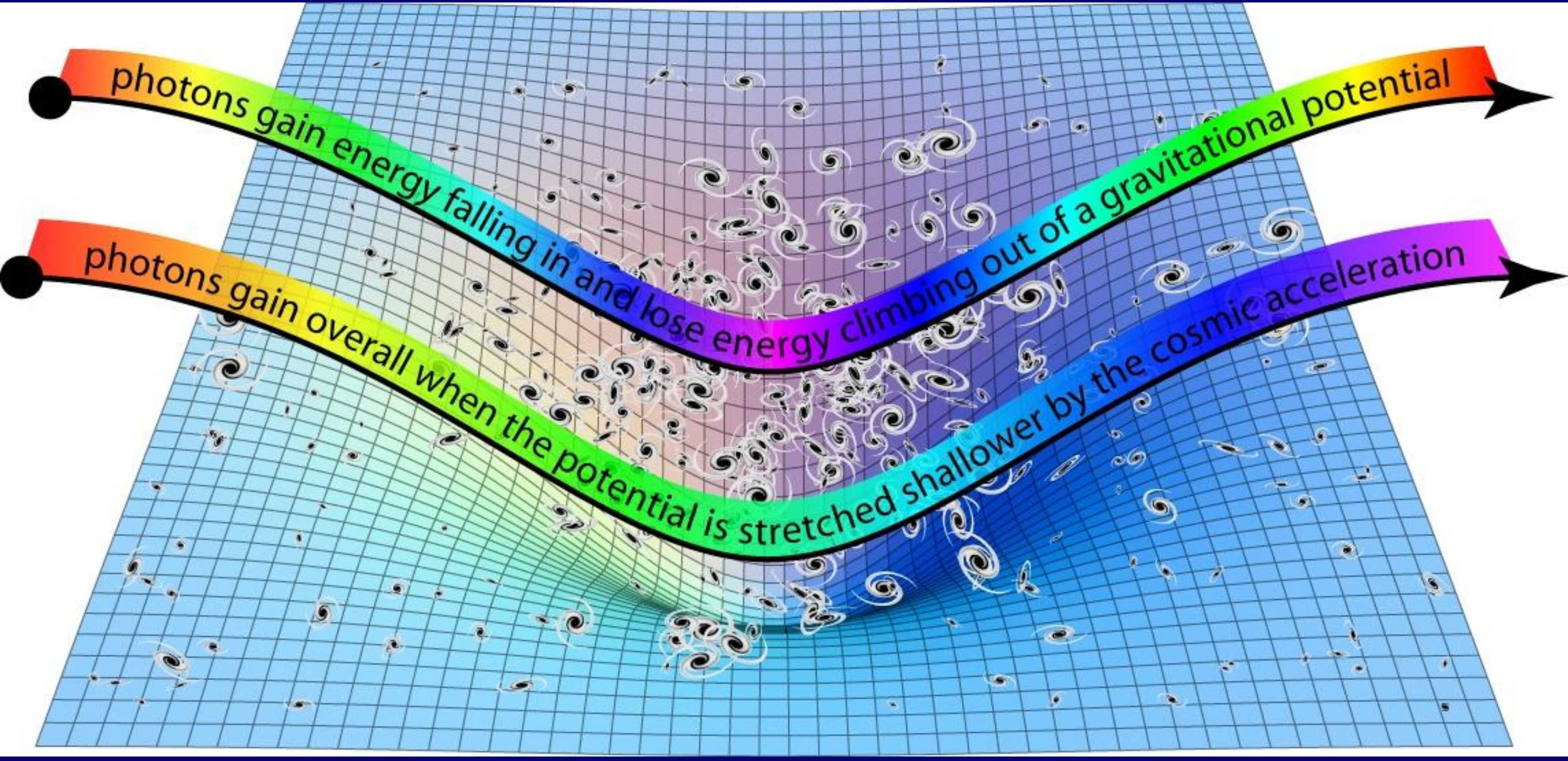
$$\delta = \frac{\delta\rho}{\rho}, \quad f = \frac{d\ln\delta}{d\ln a} = \Omega_m^{\gamma}, \quad \Lambda\mathrm{CDM}: \gamma \approx 0.55$$

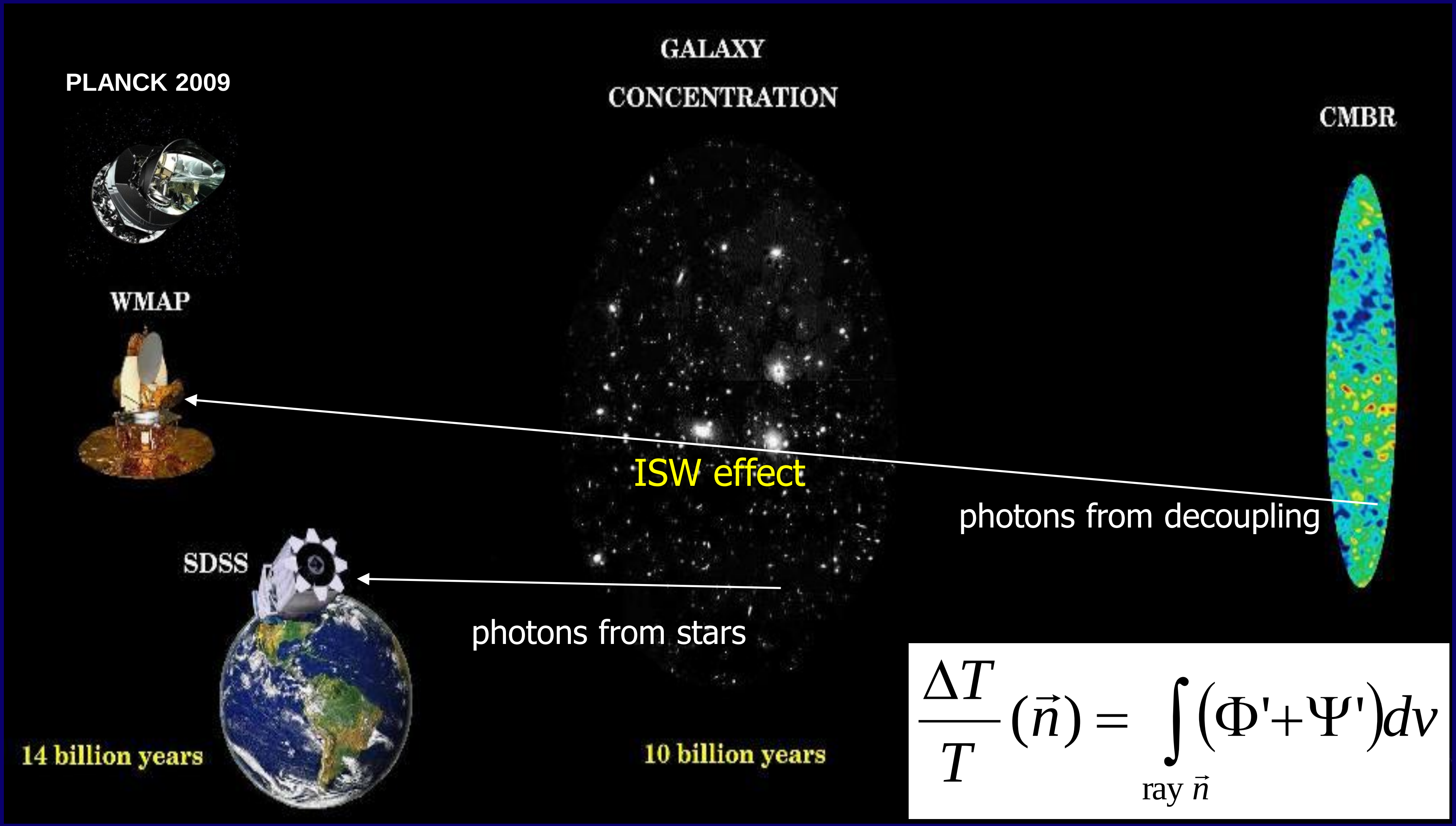


(Blake et al 2011)

Observations that probe the growth of structure. 2

The ISW effect:
CMB photons carry the signature of acceleration

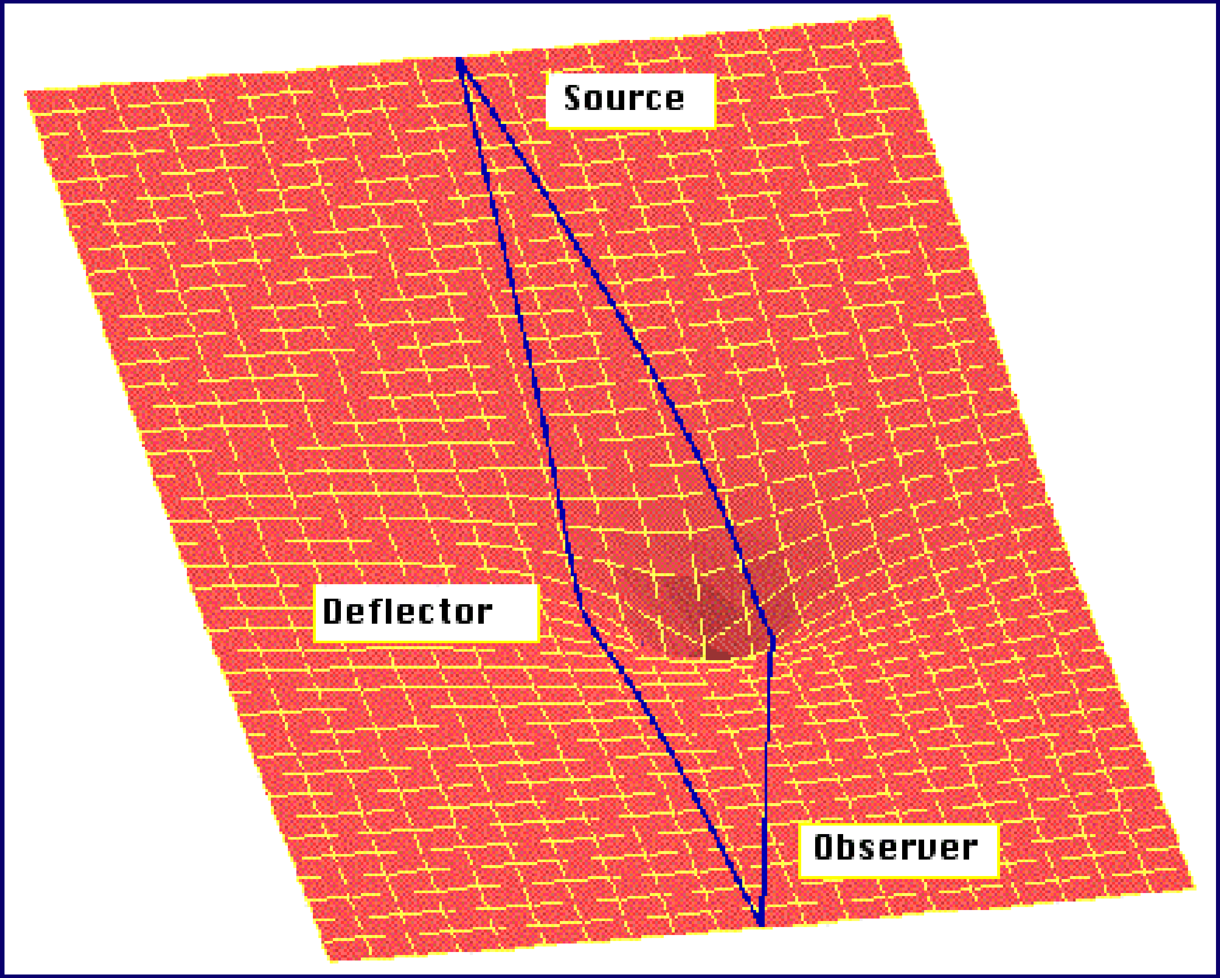




$$\frac{\Delta T}{T}(\vec{n}) = \int_{\text{ray } \vec{n}} \left(\Phi' + \Psi'\right) dv$$

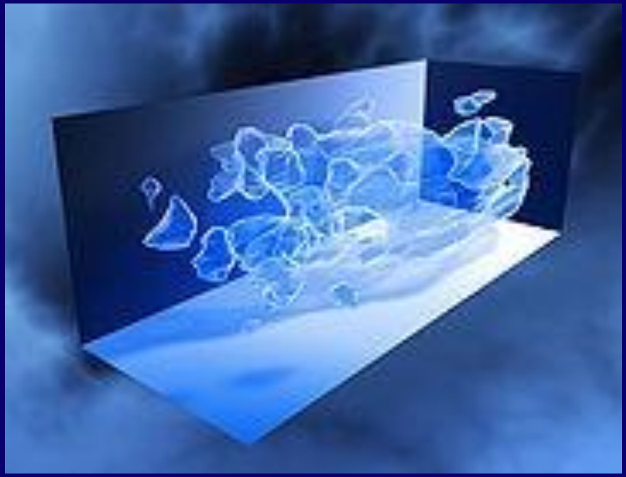
Observations that probe the growth of structure. 3

Einstein: bending of light by matter



Light rays from a distant galaxy are bent by the matter between the galaxy and us.

‘Gravitational lensing’ gives us a measure of the total matter – which is sensitive to DE.



Deflection angle:

$$\vec{\alpha} = \int_{\text{ray } \vec{n}} \vec{\nabla}_{\perp} \left(\Phi + \Psi \right) dv$$

Friedmann background

For *K=0*

$$ds^2 = -a^2[d\tau^2 + d\vec{x}^2]$$

$$h \equiv \frac{a'}{a} = aH$$

$$T^{\mu}_{\nu} = \begin{pmatrix} -\rho & 0 \\ 0 & p\,\delta^i_j \end{pmatrix}$$

$$\rho = \rho_{\mathrm{b}} + \rho_{\mathrm{c}} + \rho_{\mathrm{de}} = \rho_{\mathrm{m}} + \rho_{\mathrm{de}} \qquad (\rho_{\mathrm{r}} \approx 0)$$

$$p = p_{\mathrm{m}} + p_{\mathrm{de}} = p_{\mathrm{de}} = w\rho_{\mathrm{de}}$$

Metric and matter perturbations

Newtonian gauge (any metric gravity theory):

$$ds^2 = -a^2[(1+2\Phi)d\tau^2 + (1-2\Psi)d\vec{x}^2]$$

$$\delta T^{\mu}_{\nu} = \begin{pmatrix} -\delta\rho & (\rho+p)\partial_i V \\ -(\rho+p)\partial_j V & \delta p\, \delta^i_j \end{pmatrix}$$

Φ = Newtonian potential

Ψ = curvature perturbation

V = velocity potential

Φ, Ψ, V are gauge-invariant. δρ, δp are not.

Gauge-invariant denisty perturbation: $\Delta \equiv \delta + \frac{\rho'}{\rho} V$

We neglect anisotropic stress since we neglect radiation (late universe).

Perturbed conservation equations

$$\delta\rho' + 3h(\delta\rho + \delta p) = (\rho + p)[3\Psi' - \nabla^2 V]$$
$$[(\rho + p)V]' + \delta p = -(\rho + p)[\Phi + 4hV]$$

Pressure perturbations

$$\delta p = c_s^2 \delta\rho + \delta p_{\mathrm{nad}}$$

$$c_s^2 = \frac{p'}{\rho'}$$ adiabatic sound speed

$$\delta p_{\mathrm{nad}} = (c_{eff}^2 - c_s^2)\rho\Delta$$ non-adiabatic pressure

$$c_{\mathrm{eff}}^2 = \left.\frac{\delta p}{\delta\rho}\right|_{\mathrm{rest\ frame}}$$ effective, physical sound speed

Adiabatic medium: $c_{\mathrm{eff}}^2 = c_s^2$

Examples:

Matter and radiation (after decoupling) – adiabatic:

$$c_{\mathrm{eff}}^2 = c_s^2 \quad (= w \ \text{since} \ \ w' = 0)$$

Quintessence – non-adiabatic and self-consistent:

$$c_s^2 = \frac{p'_\varphi}{\rho'_\varphi} = 1 + \frac{2a^2 V_\varphi(\varphi)}{3h\varphi'}$$

$$c_s^2 \neq w = \frac{p_\varphi}{\rho_\varphi} = \frac{\frac{1}{2}a^{-2}\varphi'^2 - V(\varphi)}{\frac{1}{2}a^{-2}\varphi'^2 + V(\varphi)}$$

$$c_{\mathrm{eff}}^2 = \left.\frac{\delta p_\varphi}{\delta \rho_\varphi}\right|_{\delta\varphi=0} = \frac{a^{-2}\varphi'\delta\varphi'}{a^{-2}\varphi'\delta\varphi'} = 1 \neq c_s^2$$

'Fluid' DE, w =const – non-adiabatic, not self-consistent:

$$c_s^2 = w \ \ (\text{since} \ \ w' = 0) \ \Rightarrow \ c_s^2 < 0\,!$$

$$c_{\mathrm{eff}}^2 = ? \quad \text{No consistent model: put } c_{\mathrm{eff}}^2 = 1 \text{ by hand (or any } c_{\mathrm{eff}}^2 > 0)$$

Perturbed Einstein equations

$$k^2\Psi + 3h(\Psi' + h\Phi) = -4\pi G a^2 \delta\rho$$

$$\Psi' + h\Phi = -4\pi G a^2 \rho(1+w)V$$

$$\Psi'' + 2h\Psi' + h\Phi' + (2h' + h^2)\Phi = 4\pi G a^2 \delta p$$

$$\Psi - \Phi = 0$$

GR Poisson

$\Pi_{ij} = 0$

GR Poisson equation in Newtonian form:

$$k^2\Psi = -4\pi G a^2 \rho\Delta$$

Since $\Psi = \Phi$, we can derive:

$$\Phi'' + 3(1+c_s^2)h\Phi' + \left[2h' + (1+3c_s^2)h^2 + c_{\mathrm{eff}}^2 k^2\right]\Phi = 0$$

(This verifies that c_{eff} = the physical sound speed.)

Perturbed Einstein equations – implications

Growth rate: on sub-Hubble scales *(k>>h)*

$$k^2\Phi \approx -4\pi G a^2 \rho\delta$$
$$= -4\pi G a^2\left[\rho_{\rm m}\delta_{\rm m} + \rho_{\rm de}\delta_{\rm de}\right] \approx -4\pi G a^2 \rho_{\rm m}\delta_{\rm m}$$

since DE does not cluster (exactly true for Λ).

Also $$\delta_{\rm m}' \approx k^2 V_{\rm m}, \quad V_{\rm m}' + hV_{\rm m} \approx -\Phi$$

Thus $$\delta_{\rm m}'' + h\delta_{\rm m}' - 4\pi G a^2 \rho_{\rm m}\delta_{\rm m} = 0$$

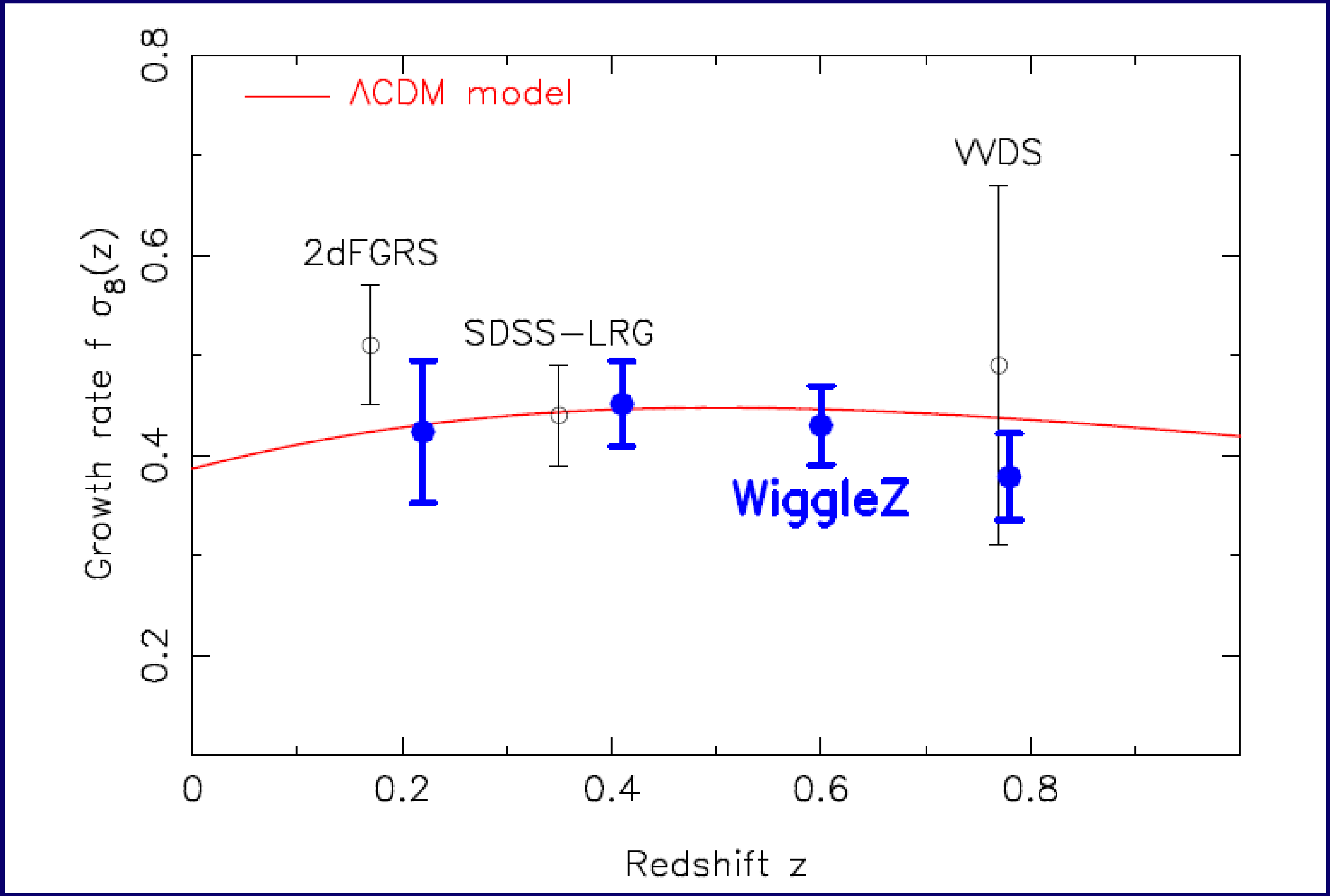
DE only affects growth via the background.

For no DE, growing mode is

$$\delta_{\rm m} \propto D(a), \quad D(a) = a$$

DE *suppresses* growth: $$D_{\rm de}(a_0) \sim 0.75 D_{\rm node}(a_0)$$

$$\delta_{\mathrm{m}} = \frac{\delta\rho_{\mathrm{m}}}{\rho_{\mathrm{m}}}, \quad f = \frac{d\ln\delta_{\mathrm{m}}}{d\ln a} = \Omega_m^{\gamma}, \quad \Lambda\mathrm{CDM}: \gamma \approx 0.55$$



Perturbed Einstein equations – implications. 2

ISW:

$$\left.\frac{\Delta T}{T}\right|_{\mathrm{ISW}} = \int \left(\Phi' + \Psi'\right) dv = 2\int \Phi' dv$$

where

$$\Phi'' + 3(1 + c_s^2) h\Phi' + \left[2h' + (1 + 3c_s^2) h^2 + c_{\mathrm{eff}}^2 k^2\right]\Phi = 0$$

With no DE:

$$\Phi' = 0 \Rightarrow \left.\frac{\Delta T}{T}\right|_{\mathrm{ISW}} = 0$$

DE generates a nonzero ISW –

because the matter potentials decay.

Perturbed Einstein equations – implications. 3

Weak lensing:

$$\vec{\alpha} = \int \vec{\nabla}_{\perp}\left(\Phi + \Psi\right)dv = 2\int \vec{\nabla}_{\perp}\Phi dv$$

where

$$\Phi'' + 3(1 + c_s^2)h\Phi' + \left[2h' + (1 + 3c_s^2)h^2 + c_{\text{eff}}^2 k^2\right]\Phi = 0$$

DE generates a different WL signal to no-DE.

Note:

- Ψ is determined by growth rate *(k>>h)*
- Ψ + Φ is determined by ISW *(k<<h)* and WL *(k>>h)*
- In GR, the two potentials are the same, Ψ = Φ, and all 3 observables are closely tied together.

Summary: GR DE models

- Simplest model = ΛCDM (with $K=0$)
- It is predictive and passes all observational tests
- But there is no satisfactory explanation from fundamental physics
- Dynamical DE does not solve the problems of Λ – and observations do not require it (up to now)
- No dynamical DE model is better motivated than Λ
- DE affects perturbations mainly via the background

We can use new observations to:

- Test for deviations from ΛCDM (good physics)
- Test physically-motivated dynamical DE (good physics)
- Test parametrizations of dynamical DE (can be useful)

Combining background and structure growth observations gives the best contraints.

Lecture 1

Overview of the accelerating Universe.
Dark Energy models in GR.
Observations of background and structure growth.

Lecture 2

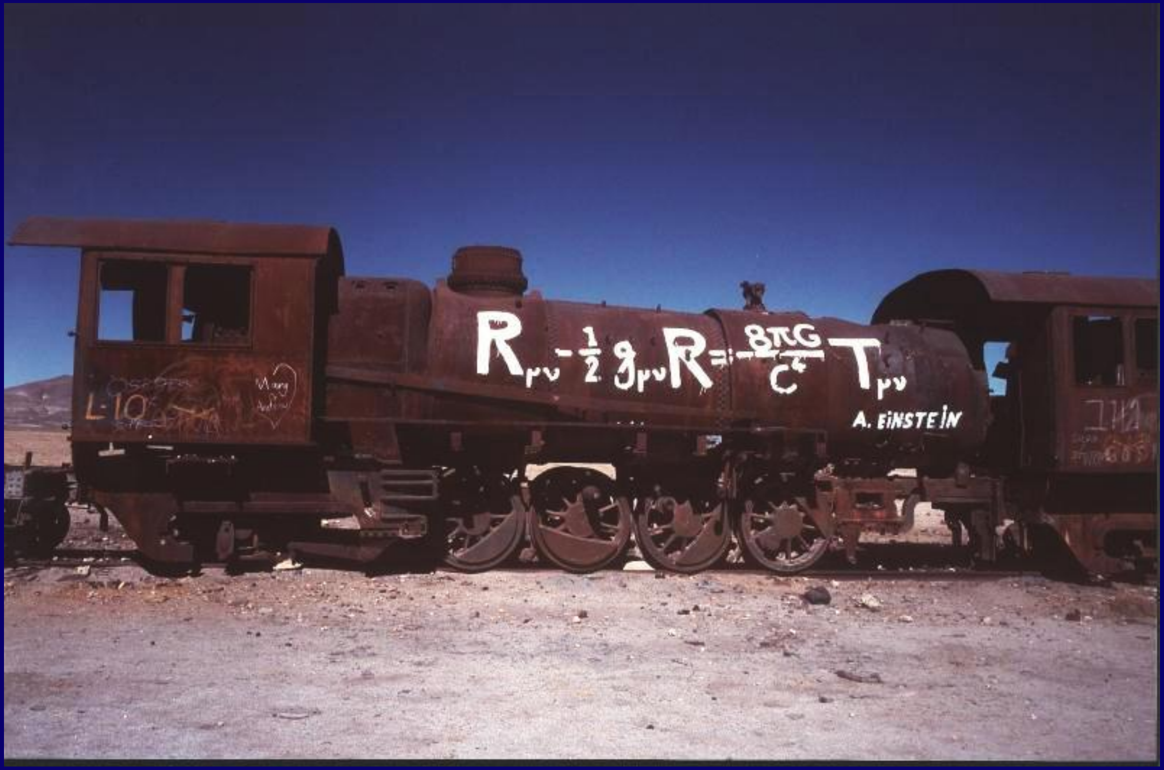
Modified gravity as an alternative to DE.
f(R) and DGP – simplest models.
Testing GR with cosmology.

Lecture 3

Inhomogeneous models of the accelerating Universe.
Testing the Copernican Principle and homogeneity.

Models for the accelerating Universe

Lecture 2:
Modified gravity and testing GR



Roy Maartens
Western Cape
& Portsmouth

Is DE actually a gravitational effect in disguise?

1. If we *keep GR* as our gravity theory, then:

   we must abandon the perturbed Friedmann model

- We can abandon the Copernican Principle and consider inhomogeneous models like LTB
- We can try to use backreaction from structure formation to explain acceleration as an apparent, not real, effect

(see lecture 3)

2. If we *keep the perturbed Friedmann* model, then:

   we must abandon GR on large scales/ low energies

Modified gravity

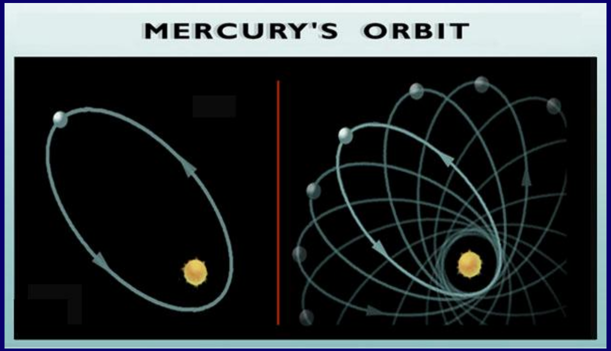
In Einstein's General Relativity, in a Friedmann Universe, acceleration is caused by the anti-gravity of Dark Energy.

Suppose there is NO Dark Energy – but instead, the Universe accelerates because gravity weakens on large scales. This would mean a modification of GR.

Example from history:

Mercury's perihelion

– Newtonian gravity + 'dark' planet?
No – modified gravity, ie Einstein's GR



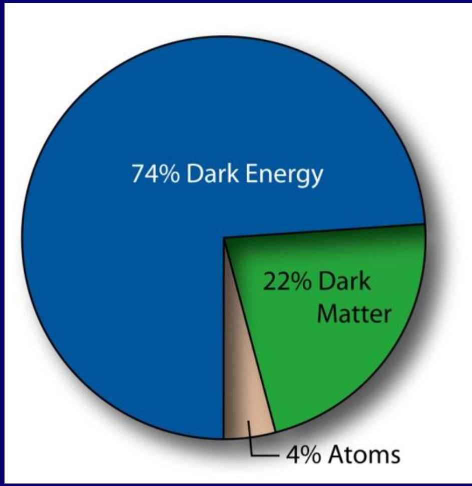
Maybe 'Dark Energy' is just a signal of the breakdown of GR?
No good modified gravity has been found yet –
but in any case, we need to test GR.

NB – The MG alternative must still explain why the vacuum energy does not gravitate: $\rho_{\text{vac}} \equiv 0$

Dark Energy dynamics

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + 8\pi G T_{\mu\nu}^{\text{dark}}$$

$$T_{\mu\nu}^{\text{dark}} = \text{DE field - 'anti - gravity'}$$



Modified Gravity dynamics

$$G_{\mu\nu} + G_{\mu\nu}^{\text{dark}} = 8\pi G T_{\mu\nu}$$

$$G_{\mu\nu}^{\text{dark}} = \text{new gravity degree of freedom to induce } \ddot{a} > 0$$

$$\rightarrow 0 \text{ on small scales (UV)}$$

We assume $\nabla_\nu T^{\mu\nu} = 0$

In the Friedmann background –

same energy conservation: $\dot{\rho} + 3H(\rho + p) = 0$

modified Friedmann equation: $H^2 + H^2_{\text{mod}} = \frac{8\pi G}{3}\rho$

Examples:

f(R) modified gravity (*R* = Ricci scalar)

$$L_{\text{grav}} = f(R)$$

$$H^2_{\text{mod}} = \frac{f - R}{6} + (1 - f_R)(H^2 + \dot{H}) + f_{RR} H \dot{R}$$

DGP modified gravity (braneworld model)

$$H^2_{\text{mod}} = \frac{-H}{r_c}, \quad r_c \sim H_0^{-1}$$

Note: MG theory must satisfy solar system and binary pulsar constraints – very close to GR.

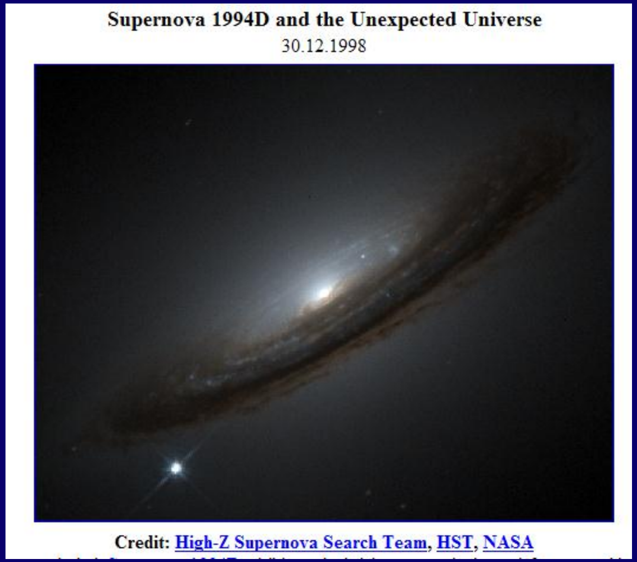
We can find a GR model of DE to mimic the *H(z)* of a modified gravity theory:

GR + DE $\quad H^2 = \frac{8\pi G}{3}(\rho + \rho_{\mathrm{DE}})$

Mod. gravity $\quad H^2 + H^2_{\mathrm{mod}} = \frac{8\pi G}{3}\rho$

choose $\quad \rho_{\mathrm{DE}}(z) = -\frac{3H^2_{\mathrm{mod}}(z)}{8\pi G}$

then $\quad H_{GR}(z) \equiv H_{MG}(z)$



This can match SNIa data (but it is not a physical DE model).

How to distinguish MG and DE models that both fit the observed *H(z)* ?

They predict different growth of structure.

Structure formation is suppressed by acceleration in different ways in GR and modified gravity:

* in GR – because DE dominates over matter

* in MG – because gravity weakens

$$\delta = \frac{\rho - \rho_{\rm background}}{\rho_{\rm background}}$$

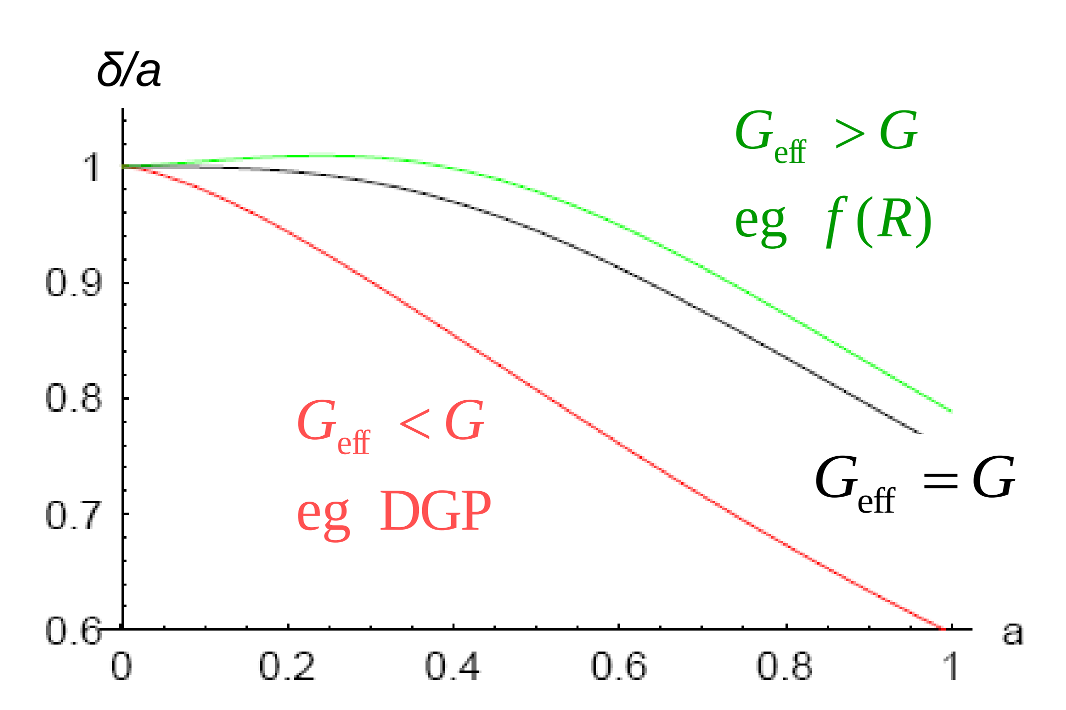
$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_{\rm eff}\,\rho\delta$$

$$\text{DE}:\ G_{\rm eff} = G$$

$$\text{MG}:\ G_{\rm eff} > G \rightarrow \delta \text{ increases}$$

$$G_{\rm eff} < G \rightarrow \delta \text{ decreases}$$

$$G_{\rm eff} - G \text{ could change sign}$$



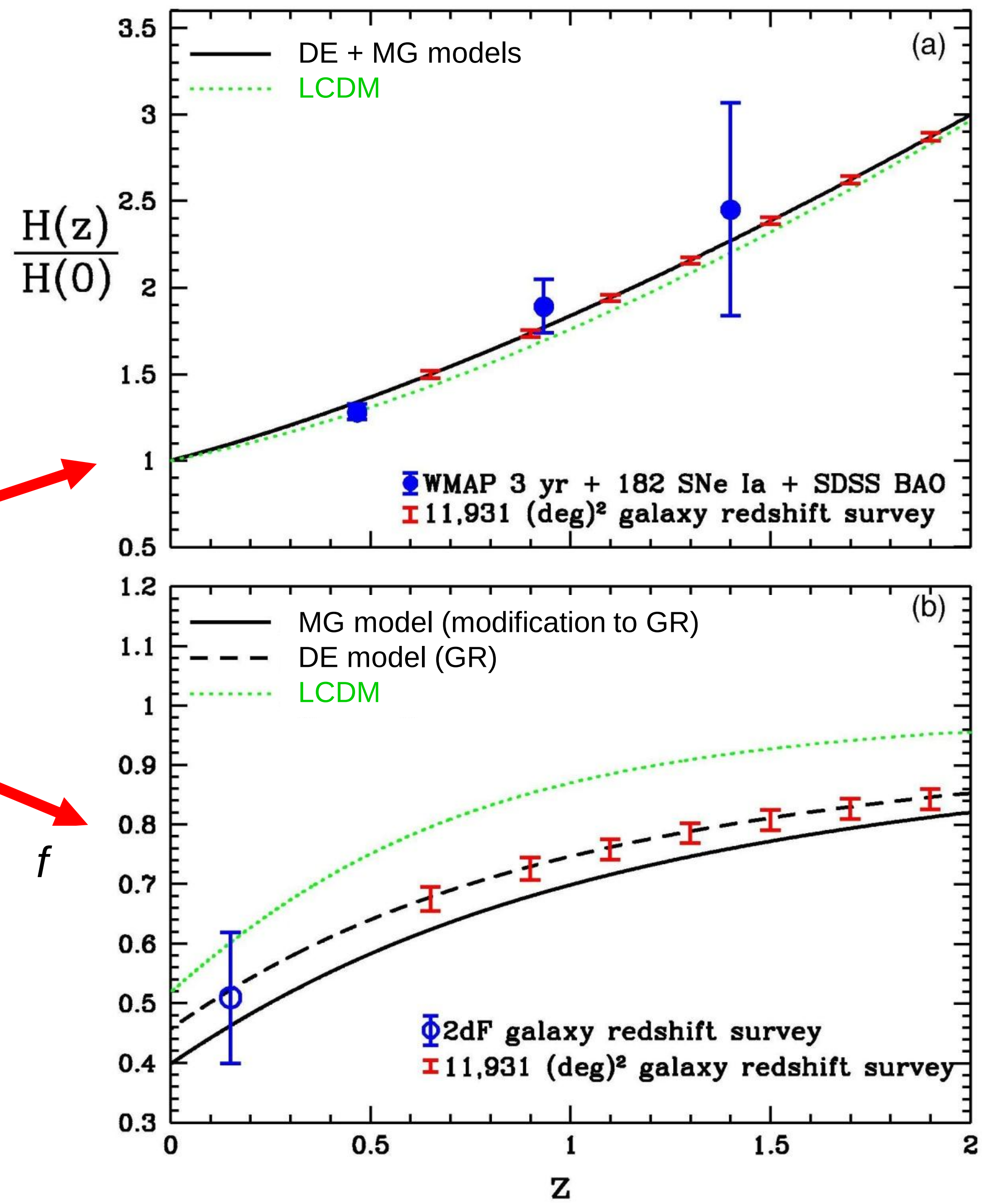
Distinguish DE from MG via growth of structure

DE and MG with the same *H(z)*

rates of growth of structure *f(z)* differ

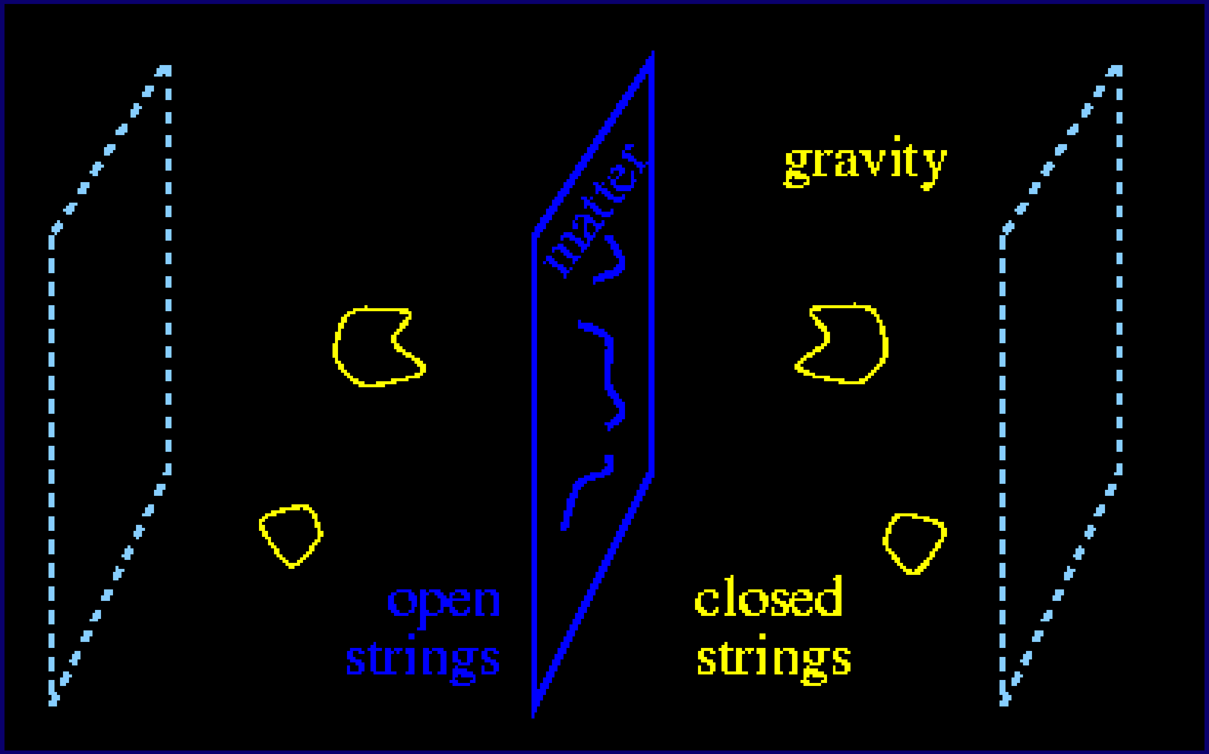
$$f = \frac{d \ln \delta}{d \ln a}$$

Also use ISW + WL



Braneworld universe

- Our 4D universe may be moving in 10D spacetime
- Motivated by string theory



unifies the 4 interactions

Modified gravity from braneworlds

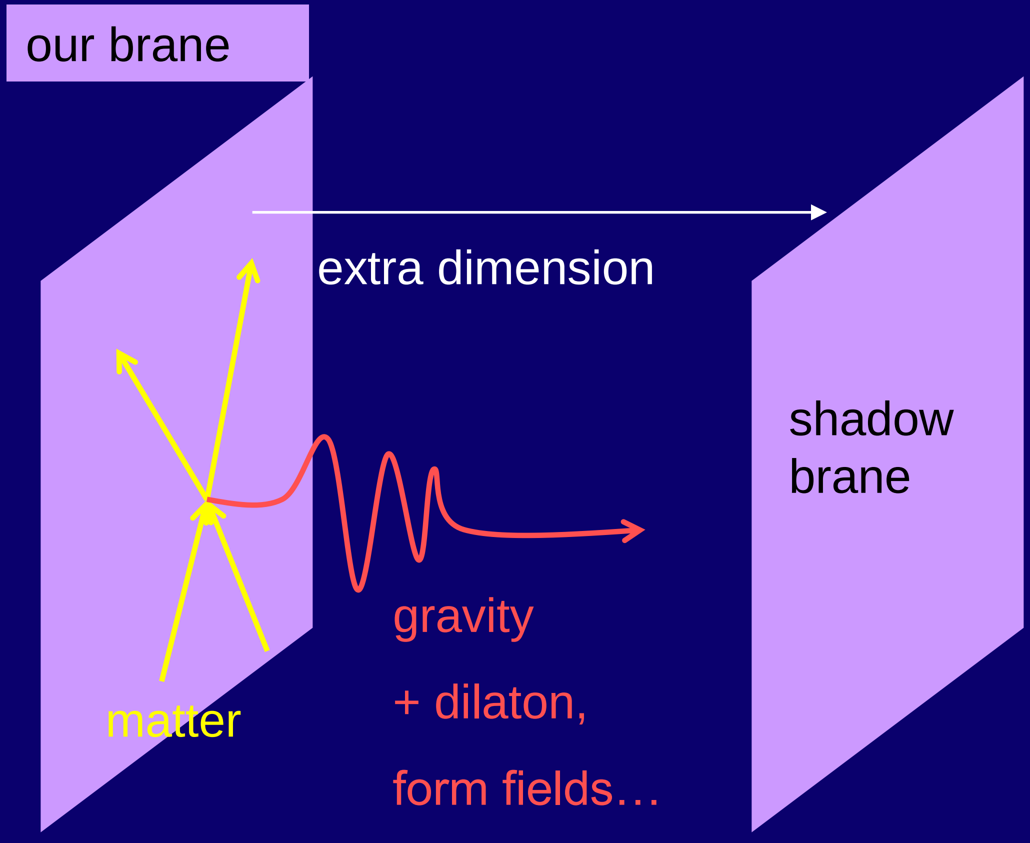
- New massive graviton modes
- New effects from higher-D fields and other branes

Could these dominate at low energies?

Possibilities

* 'bulk' fields as effective DE on the brane (eg ekpyrotic/ cyclic)

* no bulk fields - effective 4D gravity on the brane modified on large scales (eg DGP)



DGP – the simplest example

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^{(5)}}\; R^{(5)} + \frac{r_c}{8\pi G_5}\int_{\text{brane}} d^4x\sqrt{-g}\; R$$

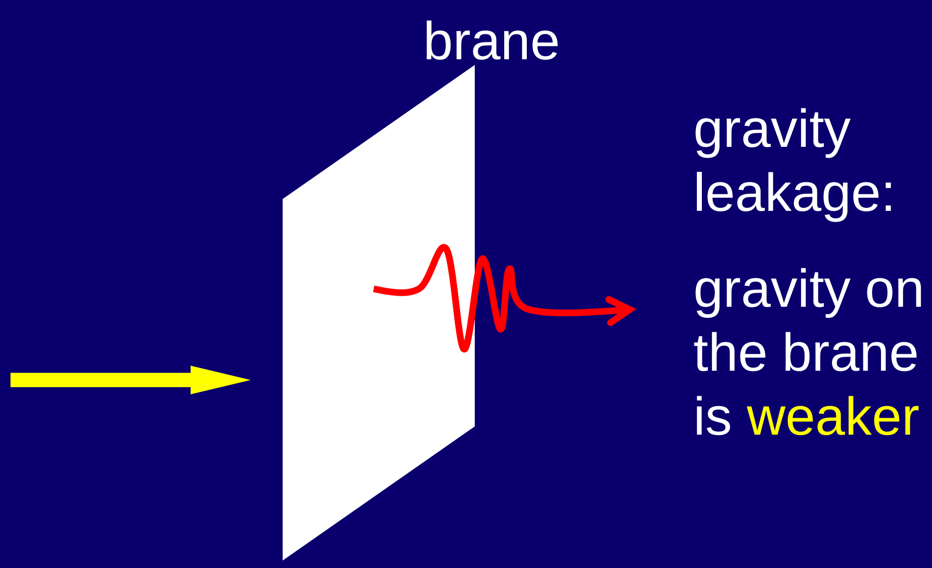
- * DGP was NOT constructed to solve the DE problem
- * NO free functions, 1 parameter – same as LCDM
- * no massless graviton – ultralight graviton + scalar

$$r_c = \frac{G_5}{2G}$$ crossover scale

Weak field static regime

$$r << r_c \Rightarrow \Phi \propto \frac{1}{r} \quad \rightarrow \quad 4D$$

$$r >> r_c \Rightarrow \Phi \propto \frac{1}{r^2} \quad \rightarrow \quad 5D$$



DGP self-acceleration

$$H^2 + \frac{K}{a^2} - \frac{1}{r_c}\sqrt{H^2 + \frac{K}{a^2}} = \frac{8\pi G}{3}\rho_m$$

$$\text{late time}: \quad \rho_m \to 0 \Rightarrow H \to \frac{1}{r_c}$$ de Sitter

$$\text{early time}: \quad H^2 + \frac{K}{a^2} >> \frac{1}{r_c^2} \Rightarrow H^2 + \frac{K}{a^2} \approx \frac{8\pi G}{3}\rho_m$$ like GR

early universe – recover the GR *H(z)*: 4D gravity dominates
late universe – acceleration without DE: 5D gravity dominates
* gravity "leaks" off the brane
* therefore gravity on the brane weakens

- Passes the solar system/ binary pulsar tests:
  since DGP ⟶ GR on small scales.
- The background is very simple – like LCDM

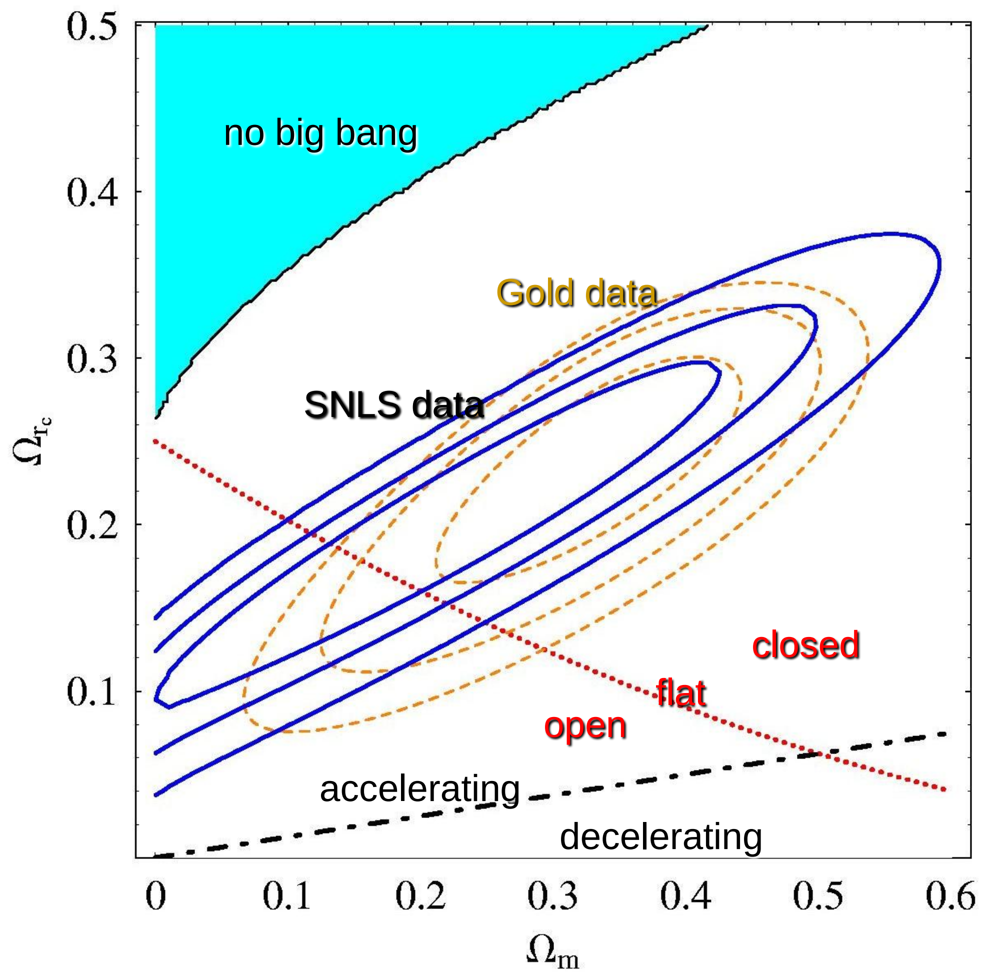
$$H^2 + \frac{K}{a^2} - \frac{1}{r_c}\sqrt{H^2 + \frac{K}{a^2}} = \frac{8\pi G}{3}\rho_m$$

Modified Friedmann

Passes the SNIa test:

$$1 = \left(\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_m}\right)^2 + \Omega_K$$

$$\Omega_{r_c} = \frac{H_0^{-2}}{4 r_c^{\,2}}$$

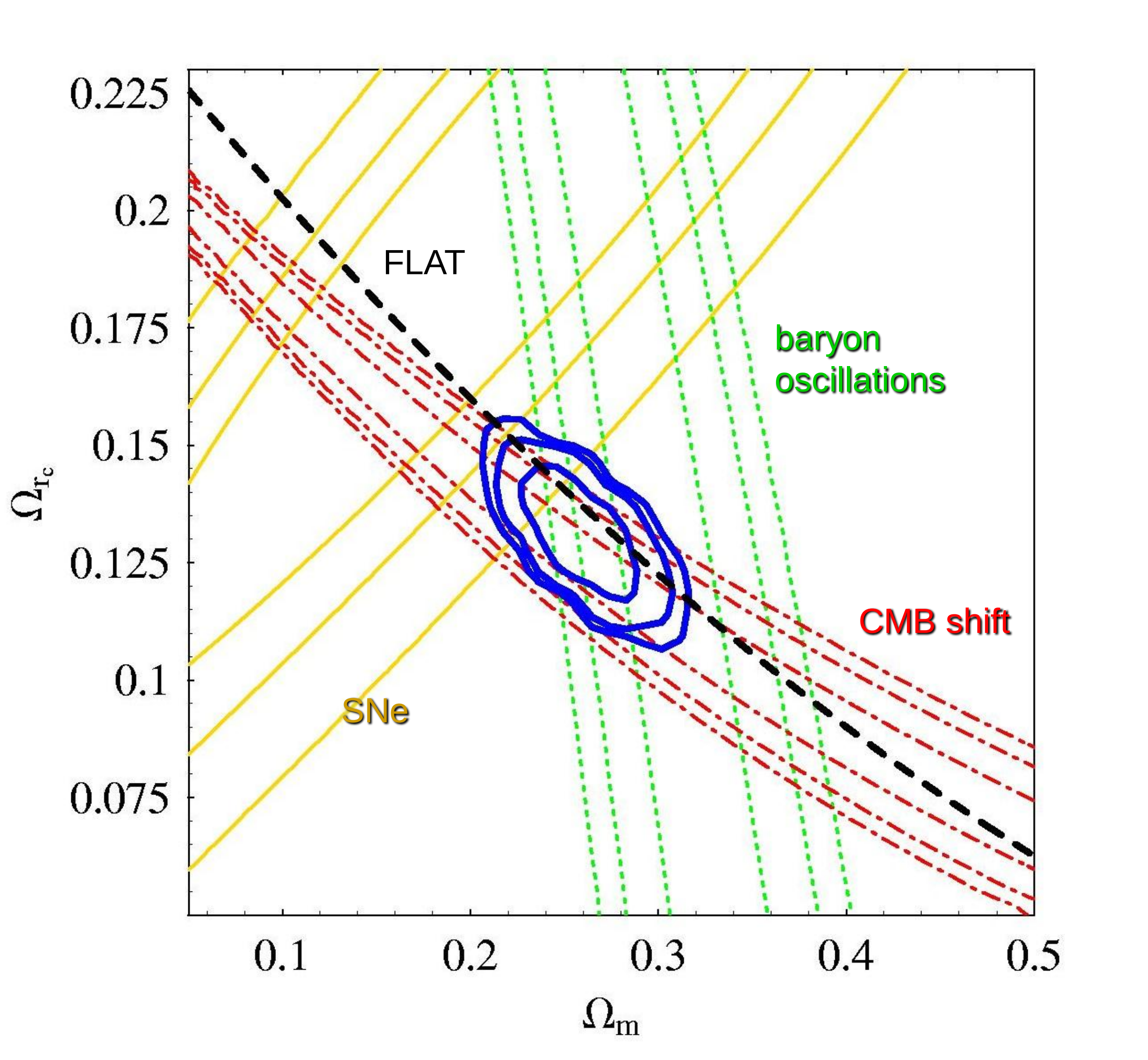


(RM, Majerotto 2006)

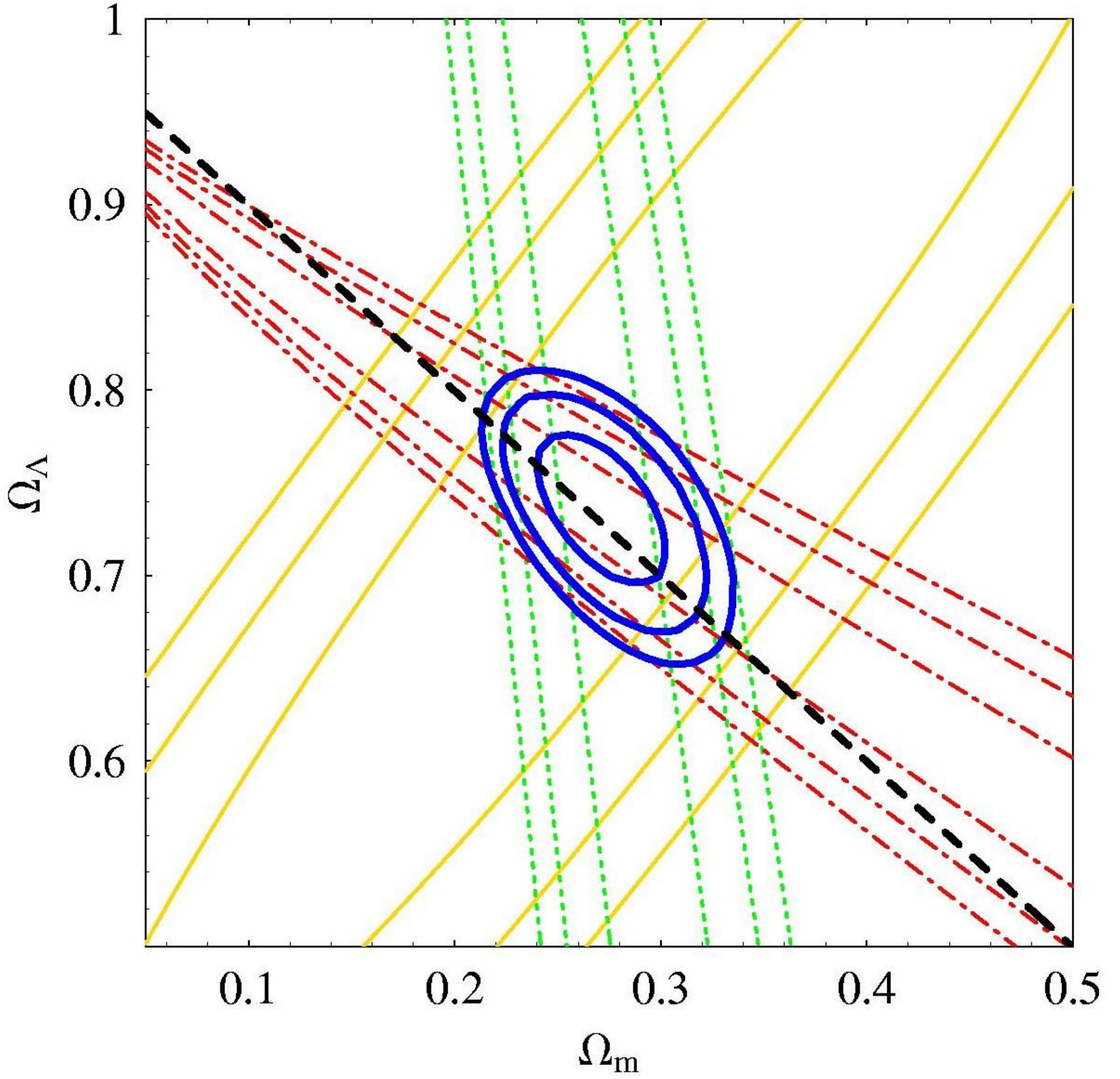
Further tests of the background expansion history:

Tension between SNe, BAO and CMB shift.
DGP struggles to fit the data.

Unlike LCDM…



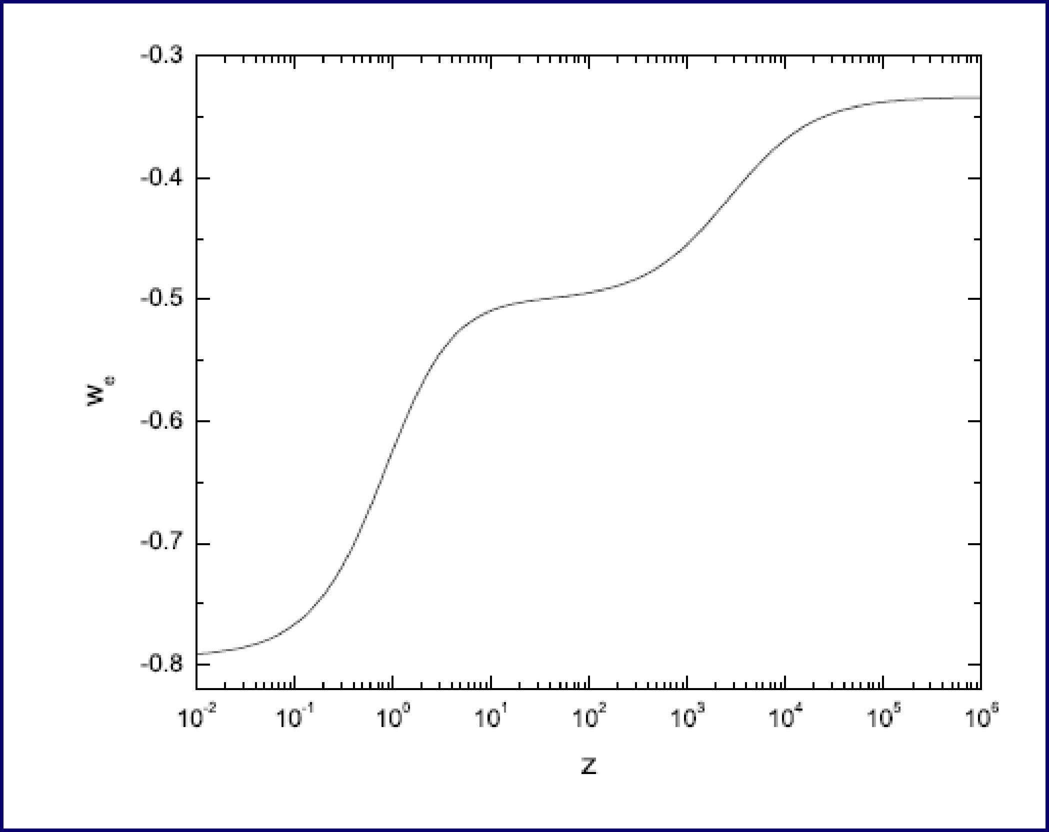
(RM, Majerotto 2006)


1
0.9
0.8
0.7
0.6
Ω_Λ
0.1
0.2
0.3
0.4
0.5
Ω_m

A key problem is the effective equation of state

$$w_{\mathrm{eff}} = \frac{-1}{1+\Omega_m(z)}$$

too large



Structure formation in DGP

Metric perturbations on the brane:

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)d\vec{x}^2$$

$$\text{GR}: \quad \Psi = \Phi$$

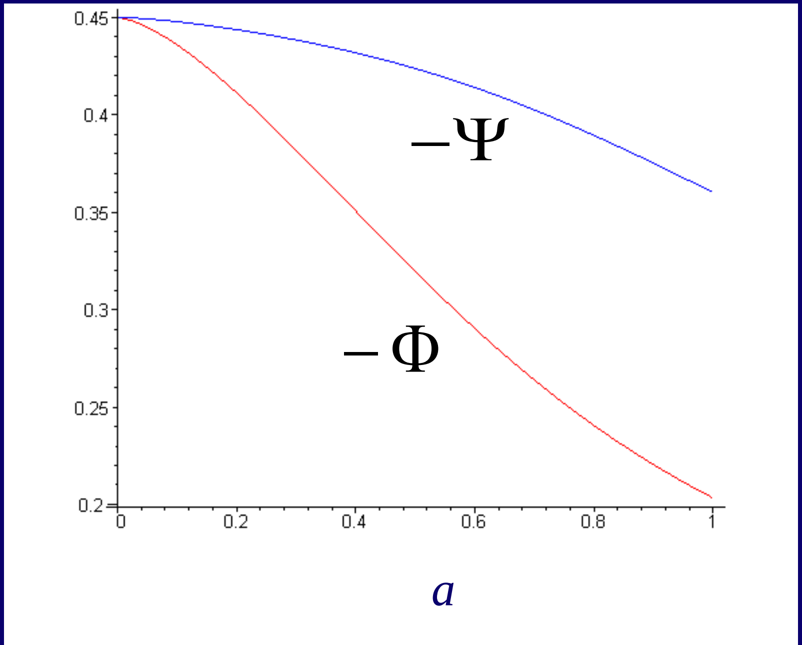
DGP:

$$\frac{k^2}{a^2}\Phi = -4\pi G\left(1+\frac{1}{3\beta}\right)\rho\delta$$

$$\frac{k^2}{a^2}\Psi = -4\pi G\left(1-\frac{1}{3\beta}\right)\rho\delta$$

where

$$\beta = 1 - 2Hr_c\left(1+\frac{\dot{H}}{3H^2}\right)$$



Effective anisotropic stress:

$$\Pi_{\text{eff}} \propto \Phi - \Psi \propto \beta^{-1}\delta$$

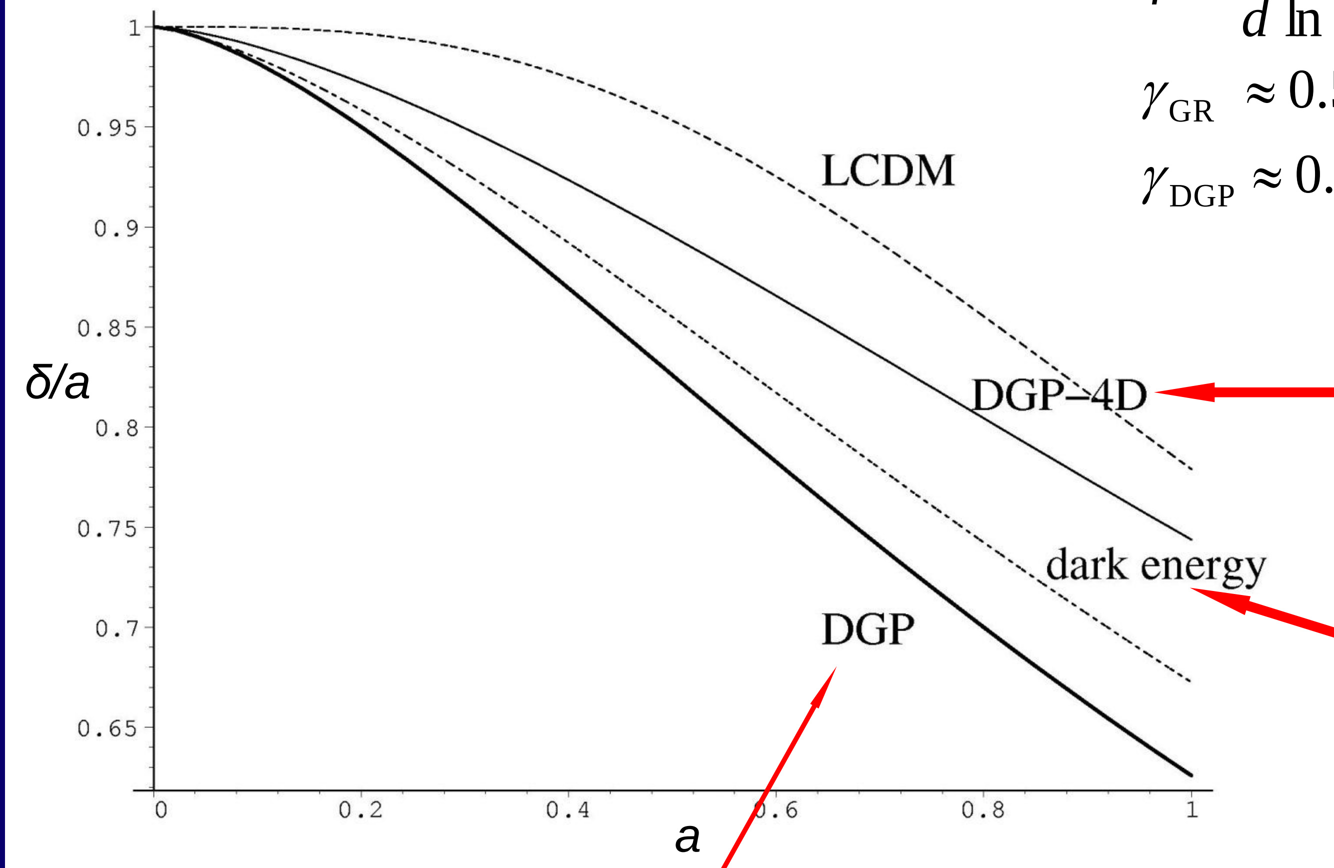
Like Brans-Dicke with

$$\omega_{BD} = \frac{3}{2}(\beta - 1) < 0$$

$$f = \frac{d \ln \delta}{d \ln a} = \Omega_m^{\gamma}$$

$$\gamma_{\rm GR} \approx 0.55 + 0.05\left[1 + w(z=1)\right]$$

$$\gamma_{\rm DGP} \approx 0.68$$



ad hoc 4D treatment of perturbations gives incorrect results – violates 4D Bianchi identity

DE model with same expansion history

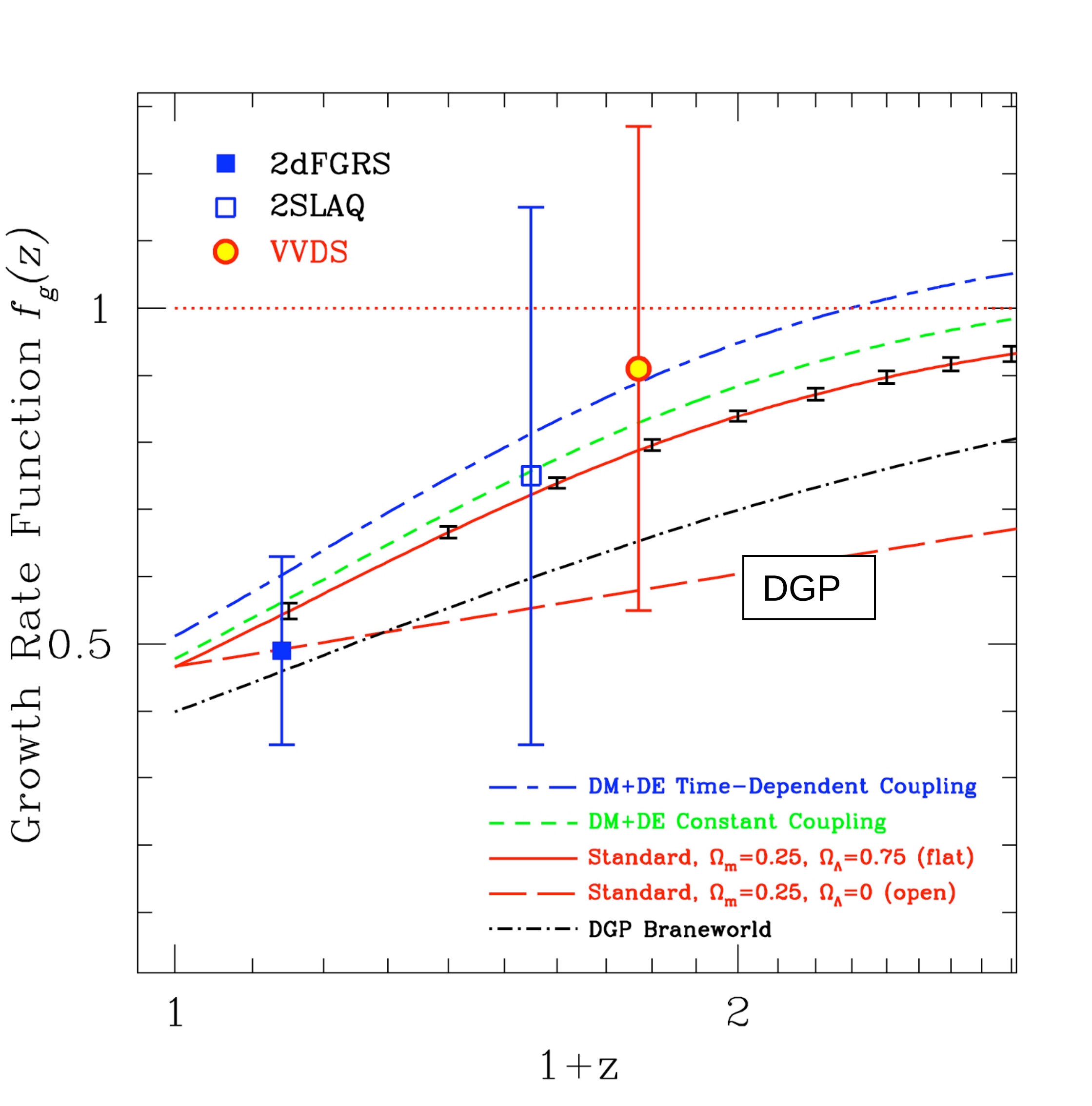
(Koyama, RM 2006)

Very strong suppression of growth from 5D effects – from $w_{\rm eff}$ and Φ

This could violate observational constraints ...

Growth factor data

Need to look at the CMB …..



(Guzzo et al 2008)

Large-angle CMB (ISW)

$$\left.\frac{\Delta T}{T}\right|_{\rm ISW} = a\int\left(\dot{\Phi}+\dot{\Psi}\right)dv$$

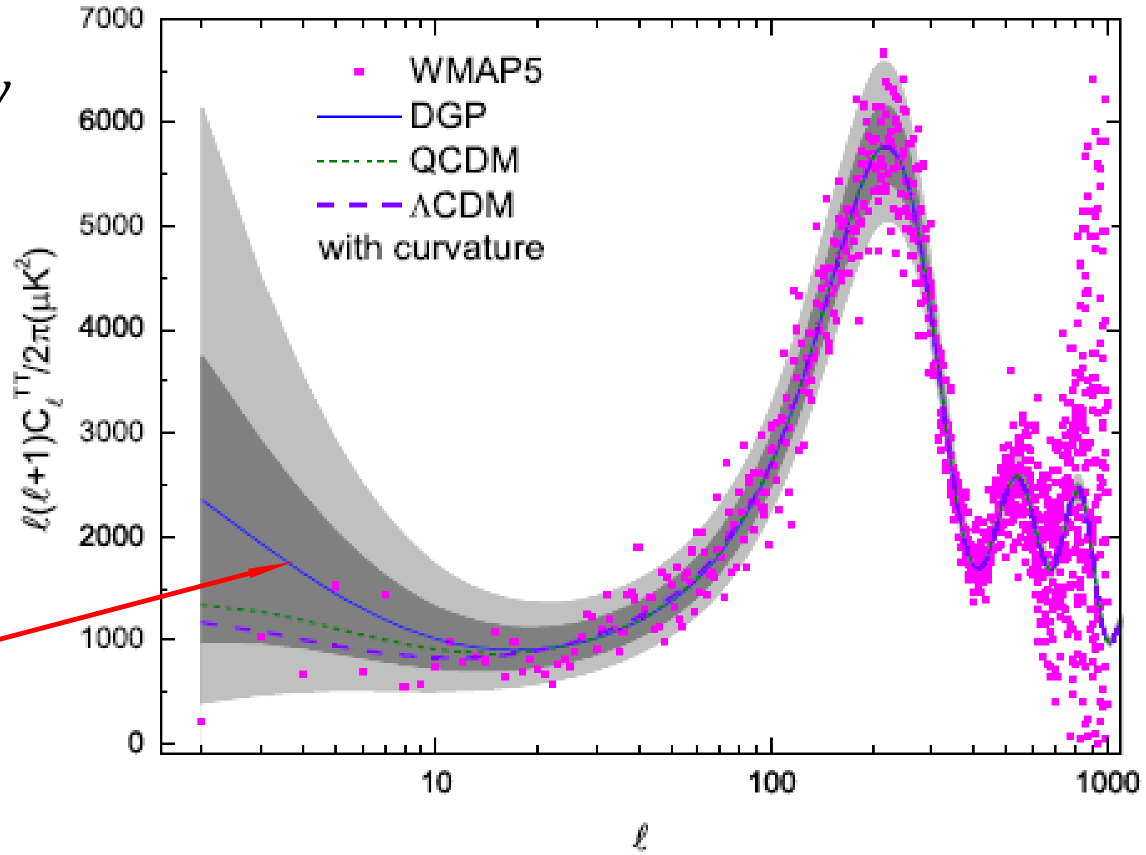
Steeper

$\Phi + \Psi$

implies

stronger ISW

than LCDM.



(Fang et al 2008)

QCDM = DE with the same expansion history as DGP.

Together with geometric data: DGP is a poorer fit than LCDM (~5 sigma).

DGP seriously challenged

- DGP – simplest MG model from braneworlds
  – probably the simplest MG model of all
  – no free functions – same as LCDM
- But it is seriously challenged by data:
  – both background and structure-formation

Key problem = the scalar degree of freedom:
DGP is like Brans-Dicke with $\omega_{BD} < 0$

This leads to drastic suppression of growth
Furthermore: $\omega_{BD} < 0$ indicates a ghost -
confirmed by detailed analysis
(The ghost makes the quantum vacuum unstable)

DGP lessons

- Despite the challenge from data and the ghost –
  DGP is a key example of how to combine geometric and structure data to test GR

- Can we avoid the crisis of data and the ghost?
- Ghost-free self-accelerating models:
  * we must go to higher dimensions
  * up to now, no ghost-free cosmological model (?)

Other modified gravity models?

Modified gravity from a nonlinear Lagrangian?

Simplest extension of the Einstein-Hilbert Lagrangian:

$$L_{\mathrm{grav,GR}} = R \;\rightarrow\; L_{\mathrm{grav}} = f(R)$$

- Avoids the Ostrogradski instability (ghost)
- *f''(R)* nonzero implies a new scalar DOF in gravity
- Starobinsky's inflation model

$$f(R) = R + \alpha R^2$$

  is a high-energy (UV) modification of GR
- Here we want a low-energy (IR) modification, eg

$$f(R) = R + \beta R^{-1}$$

- *f(R)* is equivalent to a scalar-tensor theory with Brans-Dicke parameter $\omega_{BD} = 0$
- This signals a major problem: how to escape solar system/ binary pulsar constraints?

$$\omega_{BD} > 40{,}000$$

eg. $$f(R) = R - \frac{\mu^4}{R}, \quad \mu \sim H_0$$

At low energy, $1/R$ dominates –

and this produces late-time self-acceleration.

But the light scalar strongly violates solar system/ binary pulsar constraints.

The simplest *f(R)* models fail –

because the light scalar means we cannot recover the Newtonian limit on solar system scales.

- Problem – the scalar mass must be ultra-light on cosmological scales to induce acceleration:

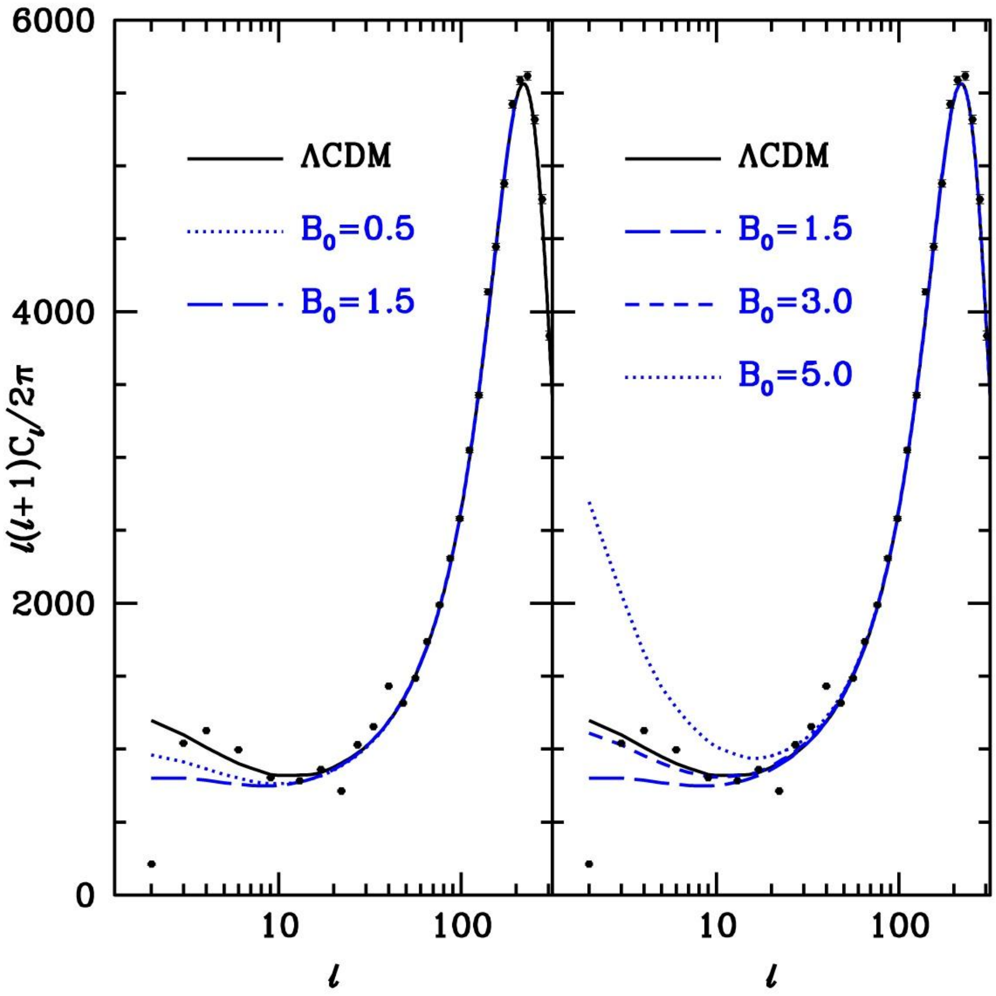
$$m \sim H_0 \approx 10^{-33}\ \mathrm{eV}$$

but it must be heavy near massive objects to overwhelm the Brans-Dicke behaviour.

- Solution: the 'chameleon' mechanism: requires a fine-tuned *f(R)*.
- Then solar system and basic cosmological tests can be passed. But the models are fine-tuned and very close to LCDM in the background.

Large-angle CMB in *f(R)* models

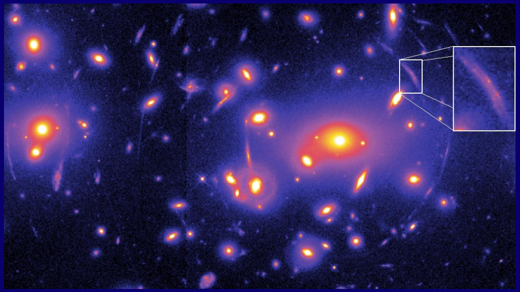
$$B_0 = \left. \frac{d \ln f_R}{d \ln H} \right|_{\text{today}}$$



(Song, Peiris, Hu 2007)

- Effective anisotropic stress

$$\Pi_{\text{eff}} \propto \Phi - \Psi \propto \frac{f''(R)}{f'(R)}$$



- Weak lensing more complicated:
  it involves the transition from
  linear regime (large scales, Brans-Dicke like) to
  nonlinear regime (small scales, Newtonian like).

  Need new N-body simulations as a guide – cannot use the GR formulation based on GR N-body results.

  Up to now, *f(R)* models can be constructed to meet the observations – but they are 'tailor-made' for data, i.e. they are not really predictive.

The simplest models are not successful

- *f(R)* and DGP – simplest in their class
  – simplest modified gravity models
- Both suffer because of their scalar degree of freedom:
  f(R) needs fine-tuning (‘tailor-made’)
  DGP over-suppresses δ and has a ghost

Either GR is the correct theory on large scales
Or Modified gravity is more complicated

But *f(R)* and DGP are valuable toy models.

Now we turn to MG perturbations in general.

Modified gravity: Perturbed field equations

$$k^2\Psi = -4\pi(G + G_{\mathrm{mod}})a^2\rho_{\mathrm{m}}\delta_{\mathrm{m}}$$

modified Poisson

$$\Psi' + h\Phi = -4\pi G a^2\rho_{\mathrm{m}}(V_{\mathrm{m}} + V_{\mathrm{mod}})$$

$$\Phi - \Psi = -8\pi G a^2\Pi_{\mathrm{mod}}$$

and

$$\delta_{\mathrm{m}}'' + h\delta_{\mathrm{m}}' - 4\pi(G + G_{\mathrm{mod}})a^2\rho_{\mathrm{m}}\delta_{\mathrm{m}} = 0$$

Modified gravity in general produces:

* Change in strength of gravity G_{mod}
* Effective gravitational velocity V_{mod}
* Effective gravitational anisotropic stress Π_{mod}

Testing modified gravity

Observations (growth rate, ISW, WL) put constraints on G_{mod} , ν_{mod} , Π_{mod}

Currently – no evidence for modified gravity, but the data is improving.

Which modified gravity theory should we test?

- There is no convincing and natural candidate.
- It is very hard to parametrize modifications to GR – the parametrization implies assumptions about the theory.
- But – we can try to test GR itself, without choosing any alternative.

For example, if we find $\Psi - \Phi$ nonzero then GR is violated.

A consistency probe

Combining growth rate and WL, which are tightly linked in GR, define:

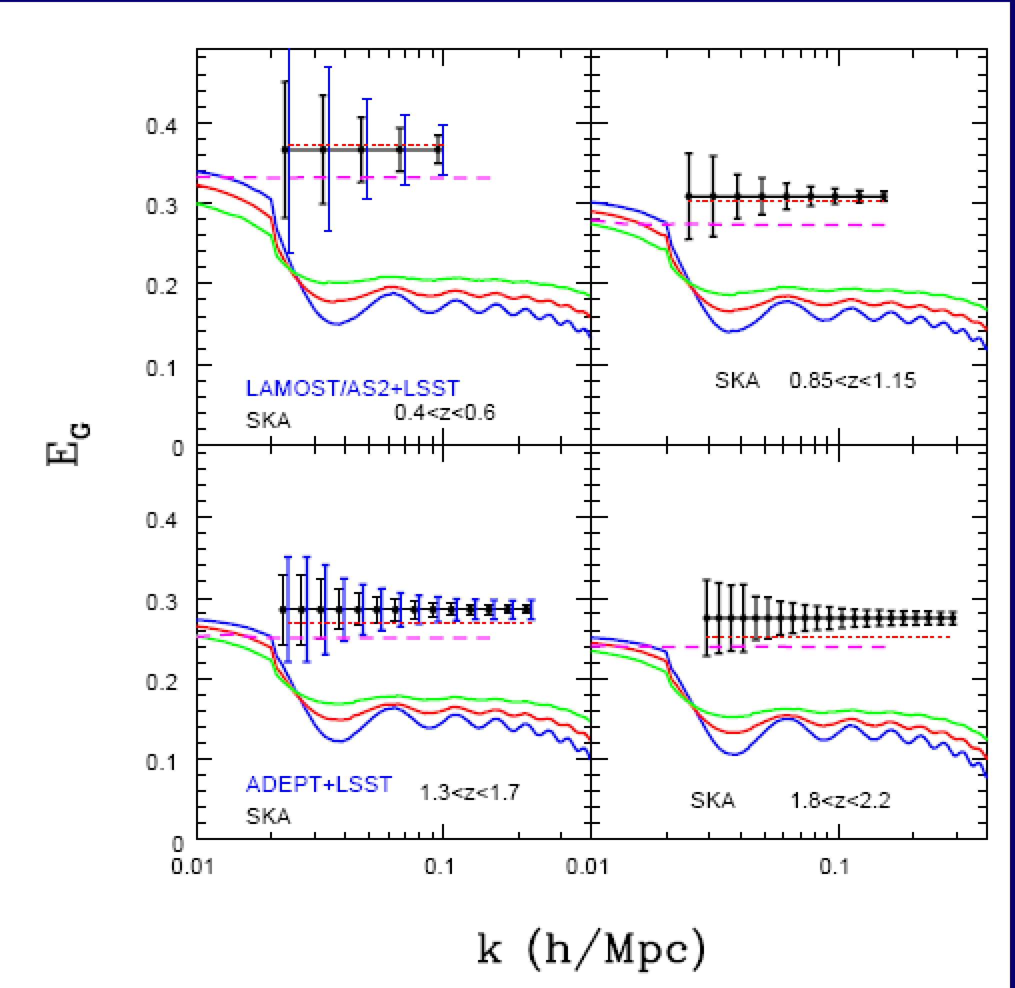
$$E_G \equiv \frac{\nabla^2(\Phi+\Psi)}{3H_0^2(1+z)f\delta}$$

$$\mathrm{GR}: \quad E_G = \frac{\Omega_0}{f_0}$$

If E_G deviates from the GR value, then this signals a breakdown of GR. Illustration →

Current data is not yet good enough, and GR survives the tests (based on SDSS).

(Zhang et al 2008)



Summary

- Lesson: it is incredibly difficult to repeat the successes of GR across the huge range of scales:
  0.1mm to thousands of Gigaparsecs
- Modifying GR in the infrared can lead to problems on smaller scales and problems with scalar gravitons.
- The simplest models *f(R)* and DGP do not succeed as alternatives to DE but are useful toy models.
- Uncovering the problems with MG models gives deep insights into:
  - properties of GR
  - how observations form a "web" of consistency
  - how to test the validity of GR itself on large scales – even if we do not have a viable alternative yet

Lecture 1

Overview of the accelerating Universe.
Dark Energy models in GR.
Observations of background and structure growth.

Lecture 2

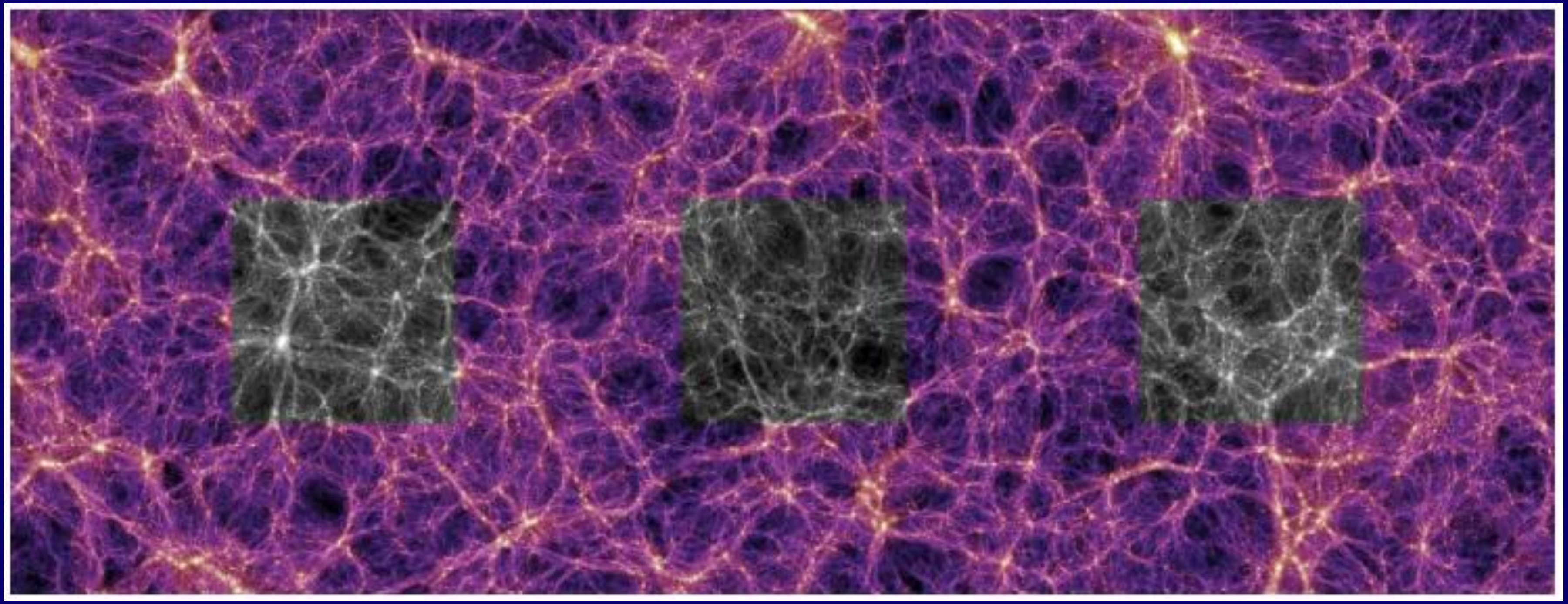
Modified gravity as an alternative to DE.
f(R) and DGP – simplest models.
Testing GR with cosmology.

Lecture 3

Inhomogeneous models of the accelerating Universe.
Testing the Copernican Principle and homogeneity.

Models for the accelerating Universe

Lecture 3:
The problem of inhomogeneity



Roy Maartens
Western Cape
& Portsmouth

Large-scale homogeneity is part of the foundation

- of the standard model of cosmology with GR
- of all Modified Gravity models

Testing GR with cosmology therefore *assumes* homogeneity.

The homogeneous Friedmann model is successful – simple, predictive, compatible with all observations so far.

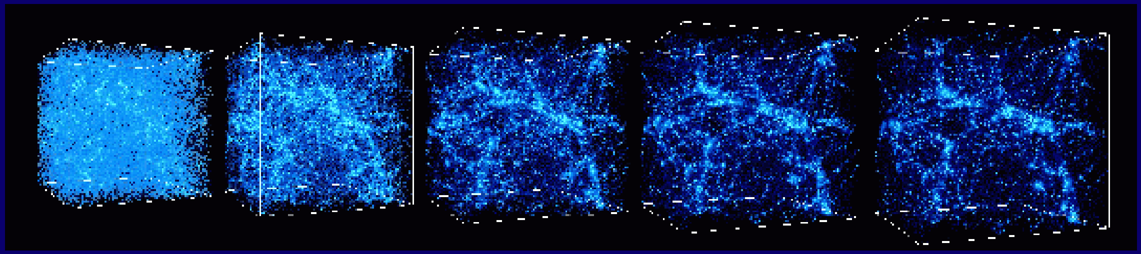
But the dark sector puzzle alerts us to possible weaknesses and inconsistencies.

We need to probe the foundations of the standard model, to test its robustness and advance our understanding of it.

- Probe the theory of gravity
- Probe models of matter – especially the averaging problem
- Probe models of light propagation
- Probe homogeneity

How to model matter and light rays: the averaging/ backreaction problem

As nonlinear structure forms (voids, filaments, walls) – is there any backreaction, i.e. when we average over density at different redshifts, do we always get the same fixed Friedmann background?

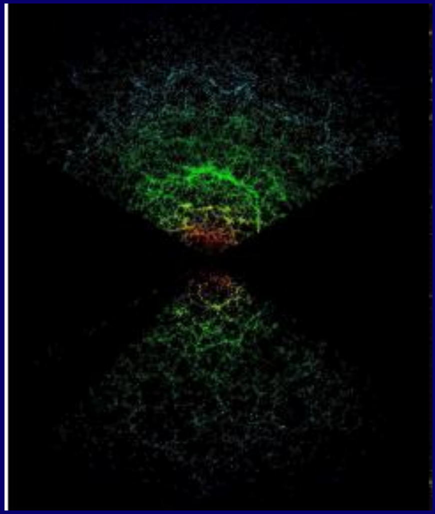


Strong claim in favour:
Backreaction is nonlinear, enough to mimic DE.
Strong claim against:
Backreaction is zero/ negligible/ irrelevant.

The problem:

We don’t know the answer, because:

1. We cannot perform the self-consistent GR calculation of the growth of structure – N-body simulations are not self-consistent.
2. We do not know how to average in GR in a covariant way – and averaging does not commute with the field equations.



Reasonable approach:

Backreaction is not strong enough to replace DE – but it is also not negligible.

Hypothesis based on the reasonable approach:

Backreaction could operate at O(1%) level – and thus would affect ‘precision cosmology’.

Conclusion based on the reasonable approach:
We should take backreaction/ averaging seriously since it could overturn attempts at O(1%) 'precision cosmology'.

This point is reinforced by the 'twin' problem of light propagation in a universe with nonlinear structure:

1. Light rays do not travel through the average density, since a significant part of matter is in bound structures.
2. This affects distances – and we do not understand how to calculate this effect, nor how to average over it.

Opposite strong claims:
We can ignore nonlinearities and treat light as propagating in the background geometry   versus
Nonlinear structure means that we badly misinterpret many observations and arrive at the wrong background model.

A reasonable approach:
We should take light propagation seriously since
it could overturn attempts at O(1%) ‘precision cosmology’.

Tentative overall conclusion:

The backreaction/averaging of nonlinear structure, and the associated effects on light propagation are not strong enough to mimic DE – but they are likely to affect our interpretation of observations (maybe at the percent level), and we need to give them more serious attention.

This is a very difficult problem. Therefore it is also useful to look at ‘toy’ models of inhomogeneous solutions of Einstein’s GR, to understand nonlinearity better.

Testing homogeneity with observations

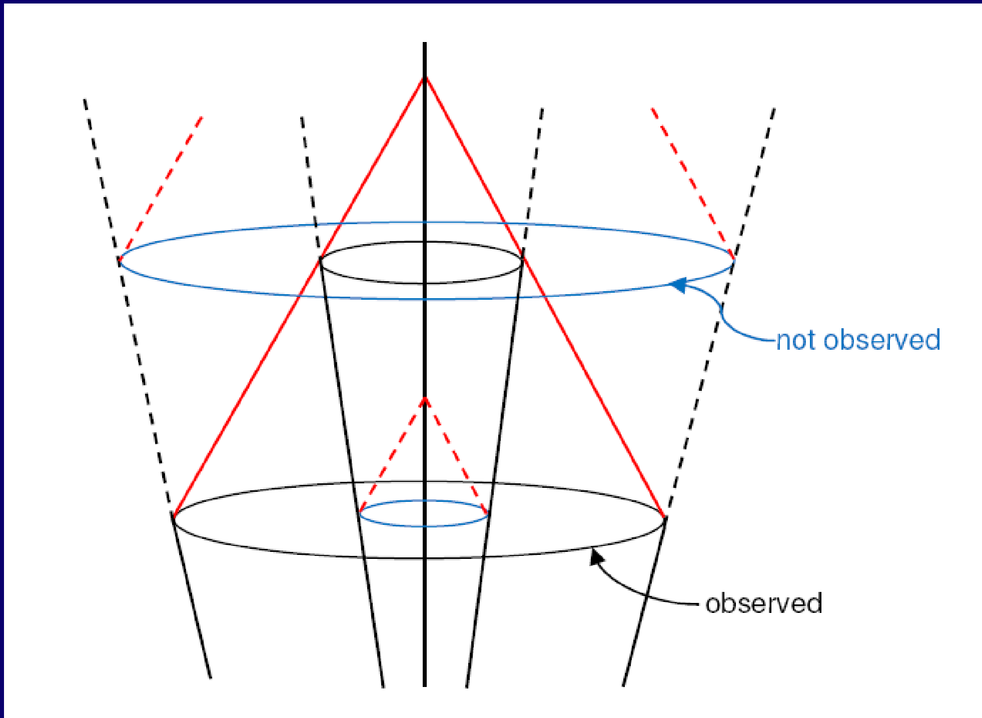
A common misconception: 'homogeneity is obvious from the CMB and galaxy distribution'.

Not true – without extra assumptions that are not observationally based.

What is the *observational* basis for homogeneity?

We cannot *directly observe* homogeneity – only isotropy. We need the Copernican Principle to deduce homogeneity from isotropy.

CP: we are not at a special position in the universe



Without the CP – what can we say from isotropy of observations?

1. Isotropic matter observations

What is the minimal set of observables that we need to produce isotropic geometry? Einstein's equations show:

Matter isotropy on lightcone gives isotropy of geometry

If one observer comoving with matter sees isotropic angular diameter distances, number counts, bulk velocities and lensing, in a dust Universe with Λ, then spacetime is isotropic about the observer, i.e. LTB (Lemaitre-Tolman-Bondi)

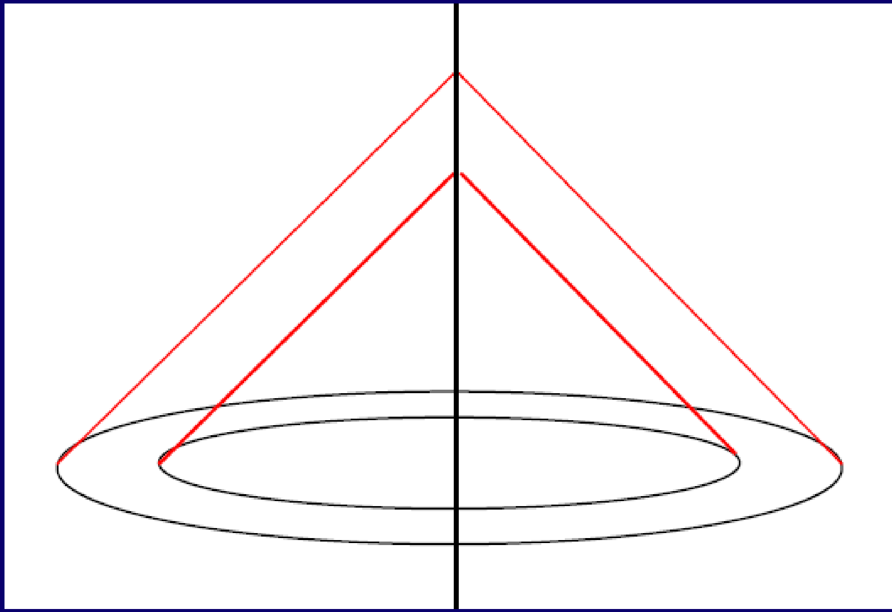
(RM 1980; RM, Matravers 1994)

2. Isotropy of the CMB

- It seems 'obvious' that this enforces isotropy of the spacetime.
- It is plausible: we expect that the decoupling surface is isotropic, and that it evolves isotropically to the future.
- But this has not been shown from the Einstein-Liouville equations up to now.

We *cannot* deduce isotropy of the geometry, without further assumptions to tie in the matter.

CMB isotropy is not enough.



With the Copernican Principle

Without the CP, we cannot establish homogeneity:

because homogeneity cannot be directly observed in the matter or CMB.

We have to adopt the CP.

1. What do isotropic matter observations tell us?

Matter isotropy on <u>all</u> lightcones gives homogeneity

If all observers comoving with matter see isotropic angular distances, number counts, bulk velocities and lensing, in a dust region with Λ, then that region is Friedmann

---

(This is an observational basis for the Cosmological Principle.)

A more powerful result

We don't need isotropy of all 4 observables!

Isotropy of *distances alone*, and only for *small* z, about all observers - implies homogeneity:

In a dust region of a Universe with Λ, if all fundamental observers measure isotropic distances to $O(z^3)$, then that region is Friedmann.

(Hasse, Perlick 1999; Clarkson, RM 2010)

Note: we do *not* get isotropic spacetime if only *one* observer sees isotropic distances to $O(z^3)$

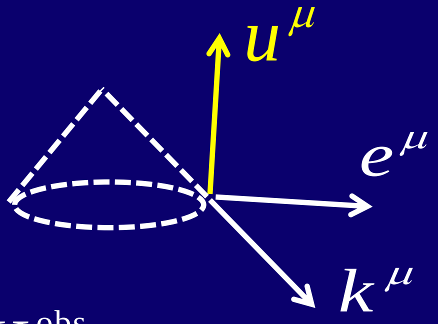
Series expansion (Kristian, Sachs 1966):

$$z = \left[k^\mu k^\nu \nabla_\mu u_\nu\right]_O D_A + \frac{1}{2}\left[k^\mu k^\nu k^\alpha \nabla_\mu \nabla_\nu u_\alpha\right]_O D_A^2$$

$$+\frac{1}{6}\left[k^\mu k^\nu k^\alpha k^\beta \nabla_\mu \nabla_\nu \nabla_\alpha u_\beta + \frac{1}{2} k^\mu k^\nu k^\alpha k^\beta R_{\mu\nu} \nabla_\alpha u_\beta\right]_O D_A^3 + \ldots.$$

where

$$\nabla_\mu u_\nu = \frac{1}{3}\Theta h_{\mu\nu} + \sigma_{\mu\nu} - \omega_{\mu\nu} - u_\mu \dot{u}_\nu$$



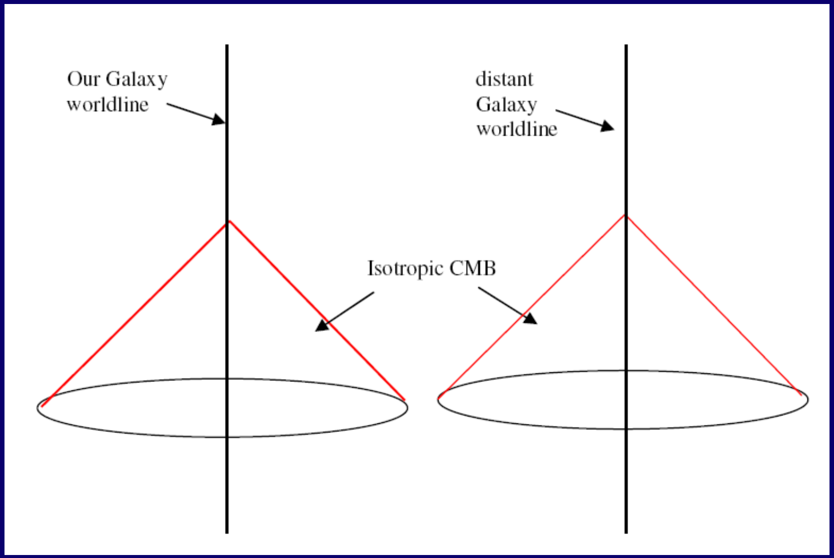
At *O(z)*:

$$\left(k^\mu k^\nu \nabla_\mu u_\nu\right)_0 = \left(\frac{1}{3}\Theta + \dot{u}_\mu e^\mu + \sigma_{\mu\nu} e^\mu e^\nu\right)_0 = H_0^{\text{obs}}$$

Then isotropy at *O(z)* gives $\dot{u}_\mu = 0, \;\; \sigma_{\mu\nu} = 0$

At *O(z²)*, *O(z³)* we get enough further information to deduce Friedmann.

2. What do isotropic CMB observations tell us?



It seems obvious that we should get Friedmann – and is often stated. But these statements *assume* the result!

We have to *show* it using the *general, fully nonlinear* Einstein-Liouville equations.

Nonlinear perturbations are not an option – we cannot assume the Friedmann background that we are trying to prove.

History: 1968 mathematical theorem by Ehlers, Geren, Sachs (EGS)

Update: Generalized to include baryons, CDM and DE

(Stoeger, RM, Ellis 1995; Clarkson, RM 2010)

CMB isotropy + Copernican Principle gives Friedmann

In a region, if

* collisionless radiation is exactly isotropic,
* the radiation 4-velocity is geodesic and expanding,

$$\dot{u}^a = 0, \quad \Theta > 0,$$

* baryons and CDM are pressure-free, and DE is Λ or quintessence or perfect fluid,

then the region is Friedmann, and

$$u^a_{\text{bar}} = u^a_{\text{cdm}} = u^a_{\text{de}} = u^a, \qquad \pi^a_{\text{bar}} = \pi^a_{\text{cdm}} = \pi^a_{\text{de}} = 0$$

Liouville equation in any spacetime

$$\frac{\mathrm{d}f}{\mathrm{d}\tau} = p^a \frac{\partial f}{\partial x^a} + \frac{\mathrm{d}p^a}{\mathrm{d}\tau}\frac{\partial f}{\partial p^a} = 0$$

where $p^a = E(u^a + e^a) \quad (= hk^a)$

(here the indices *a,b,...* refer to a tetrad or coordinates).

Covariant harmonics (tracefree):

$$f(x,p) = \sum_{\ell=0}^{\infty} F_{A_\ell}(x,E)e^{A_\ell} = F(x,E) + F_a(x,E)e^a + F_{ab}(x,E)e^a e^b + \ldots$$

$$(A_\ell \equiv a_1 \ldots a_\ell)$$

Intensity multipoles:

$$I_{A_\ell}(x) \propto \int_0^\infty \mathrm{d}E\, E^3 F_{A_\ell}(x,E)$$

$$I = \rho_\gamma,\ I^a = q_\gamma^a,\ I^{ab} = \pi_\gamma^{ab},\ \ldots$$

- Liouville equation is decomposed into covariant multipoles
- Integrate these over photon energy
- Gives the hierarchy of tracefree evolution equations for the intensity multipoles, *in a general spacetime:*

$$0 = \dot{I}_{\langle A_\ell \rangle} + \frac{4}{3}\Theta I_{A_\ell} + \frac{\ell}{(2\ell+1)}\overline{\nabla}_{\langle a_\ell} I_{A_{\ell-1}\rangle} + \overline{\nabla}^b I_{bA_\ell} + \frac{\ell(\ell+3)}{(2\ell+1)}\dot{u}_{\langle a_\ell} I_{A_{\ell-1}\rangle}$$
$$- (\ell-2)\dot{u}^b I_{bA_\ell} - \ell\,\omega_{c\,(a_\ell} I_{A_{\ell-1})}{}^c - (\ell-1)\sigma^{bc} I_{bcA_\ell}$$
$$+ \frac{5\ell}{(2\ell+3)}\sigma^b{}_{\langle a_\ell} I_{A_{\ell-1}\rangle b} - \frac{(\ell-1)\ell(\ell+2)}{(2\ell-1)(2\ell+1)}\sigma_{\langle a_\ell a_{\ell-1}} I_{A_{\ell-2}\rangle}$$

(Ellis, Treciokas, Matravers 1984; RM, Gebbie, Ellis 1999)

where $\overline{\nabla}_a \equiv (\nabla_a)_\perp$

This is the basis for a covariant proof of EGS
(Stoeger, RM, Ellis 1995; Clarkson, RM 2010)

1. *ℓ=2:* photon quadrupole evolution

$$\dot{\pi}_\gamma^{ab} + \tfrac{4}{3}\Theta\pi_\gamma^{ab} + \tfrac{8}{15}\rho_\gamma\sigma^{ab} + \overline{\nabla}^{<a}q_\gamma^{b>} + (\tfrac{10}{7}\sigma_c^{<a} - \omega_c^{<a})\pi_\gamma^{b>c}$$

$$+\overline{\nabla}_c I^{abc} - \sigma_{cd} I^{abcd} = 0$$

Thus we get zero shear: $\sigma_{ab} = 0$

2. *ℓ=1:* photon momentum conservation

$$\dot{q}_\gamma^a + \tfrac{4}{3}\Theta q_\gamma^a + \tfrac{1}{3}\overline{\nabla}_a\rho_\gamma + \overline{\nabla}_b\pi_\gamma^{ab} = -(\sigma_b^a + \omega_b^a)q_\gamma^b$$

$$\Rightarrow \overline{\nabla}_a\rho_\gamma = 0$$

Take the covariant curl:

$$\overline{\nabla}_{[a}\overline{\nabla}_{b]}\rho_\gamma = \dot{\rho}_\gamma\omega_{ab} \quad \Rightarrow \quad \omega_{ab} = 0$$

3. *ℓ=0:* photon energy conservation

$$\dot{\rho}_\gamma + \tfrac{4}{3}\Theta\rho_\gamma + \overline{\nabla}_a q_\gamma^a + \sigma_{ab}\pi_\gamma^{ab} = 0 \quad \Rightarrow \quad \overline{\nabla}_a\Theta = 0$$

etc.

More powerful result:

CMB partial isotropy + Copernican Principle → FLRW

In a region, if

- collisionless radiation has vanishing dipole, quadrupole and octupole, $F_a = F_{ab} = F_{abc} = 0$,
- the radiation four-velocity is geodesic and expanding,
- there are pressure-free baryons and CDM, and dark energy in the form of Λ, quintessence or a perfect fluid,

then the metric is FLRW in that region.

Ellis, Treciokas, Matravers 1985 (ETM theorem – generalized in Clarkson, RM 2010)

This is the best basis we have for (exact) homogeneity.

Major open question: generalize the EGS-ETM results to derive near-homogeneity from near-isotropy.

(partial result: Stoeger, RM, Ellis 1995)

Testing the Copernican Principle and homogeneity

The CP is the foundation of homogeneity: can we test it?

- If we find no violation – this strengthens the evidence for homogeneity (but cannot prove homogeneity).
- If we find even one violation, this can disprove homogeneity.

1. Geometric consistency test

Luminosity distance in FLRW

$$D_L(z) = \frac{(1+z)}{H_0\sqrt{-\Omega_{K0}}} \sin\left(\sqrt{-\Omega_{K0}} \int_0^z \frac{\mathrm{d}z'}{H(z')/H_0}\right)$$

$$\Longrightarrow \quad \Omega_{K0} = \frac{H^2(z)[(1+z)D_L'(z) - D_L(z)]^2 - H_0^2(1+z)^4}{H_0^2(1+z)^2 D_L^2(z)}$$

Now differentiate with respect to *z:*

$$\begin{aligned}\mathcal{K}(z) &:= H_0^2(1+z)^4 + H^2(z)\left[(1+z)^2\left\{D_L(z)D_L''(z) - D_L'^2(z)\right\} + D_L^2(z)\right] \\ &\quad + (1+z)H(z)H'(z)D_L(z)\left[(1+z)D_L'(z) - D_L(z)\right] \\ &= 0 \text{ for FLRW geometry.}\end{aligned}$$

Thus we have a null test of homogeneity:

$$\mathcal{K}(z) \text{ significantly different from } 0 \quad \Rightarrow \quad \text{non-FLRW universe.}$$

(Clarkson, Bassett, Lu 2009)

If $\mathscr{K}$ is consistent with 0 – then this strengthens support for the CP (but it cannot prove homogeneity).

This test can already be applied – using the *same* data that is needed for determining the DE equation of state *w(z)*.

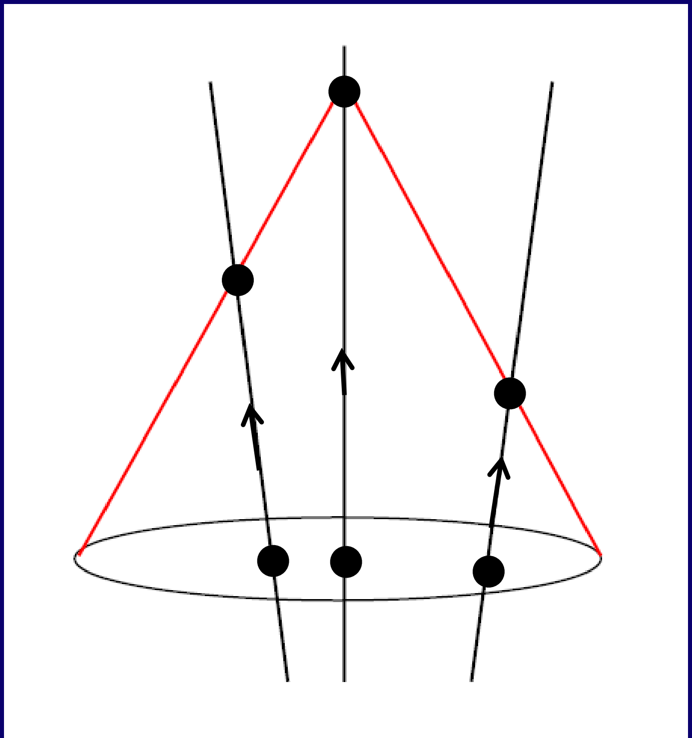
The data is not yet good enough to determine derivatives of *H(z)* and $D_L(z)$ with enough accuracy.

2. Probing inside the past lightcone

2A. Remote sources: information from the causal past

<u>BAO – probe the sound horizon at last scattering</u>

Indirect probe inside our past lightcone:



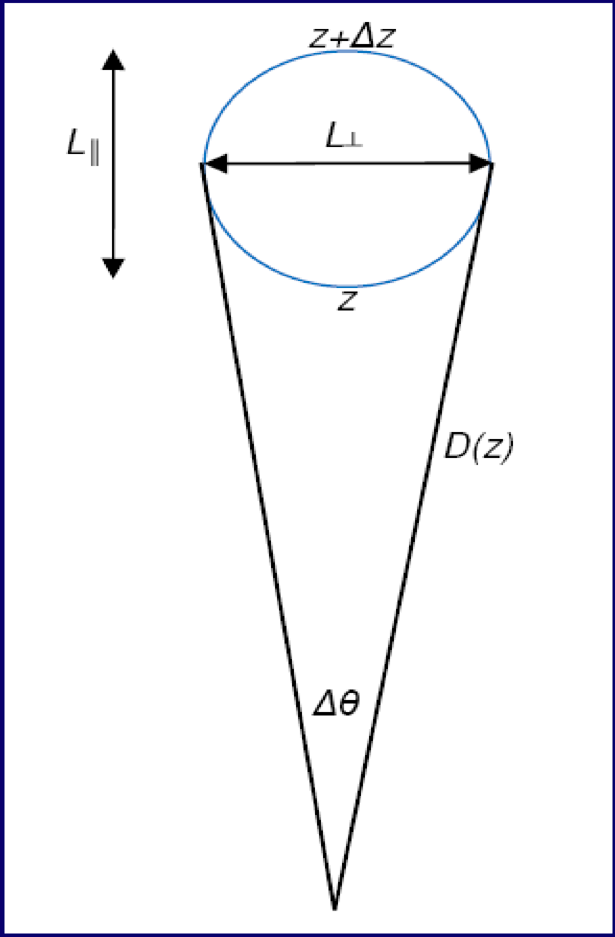
The proper radial and transverse BAO scales, in any spacetime:

$$L_{||} = \frac{\Delta z}{(1+z)H^{\text{obs}}(z)}, \qquad L_{\perp} = D_A(z)\Delta\theta$$

where

$$H^{\text{obs}} = \tfrac{1}{3}\Theta + \dot{u}_{\mu}e^{\mu} + \sigma_{\mu\nu}e^{\mu}e^{\nu}$$

(generalization of Friedmann *H* to any spacetime)



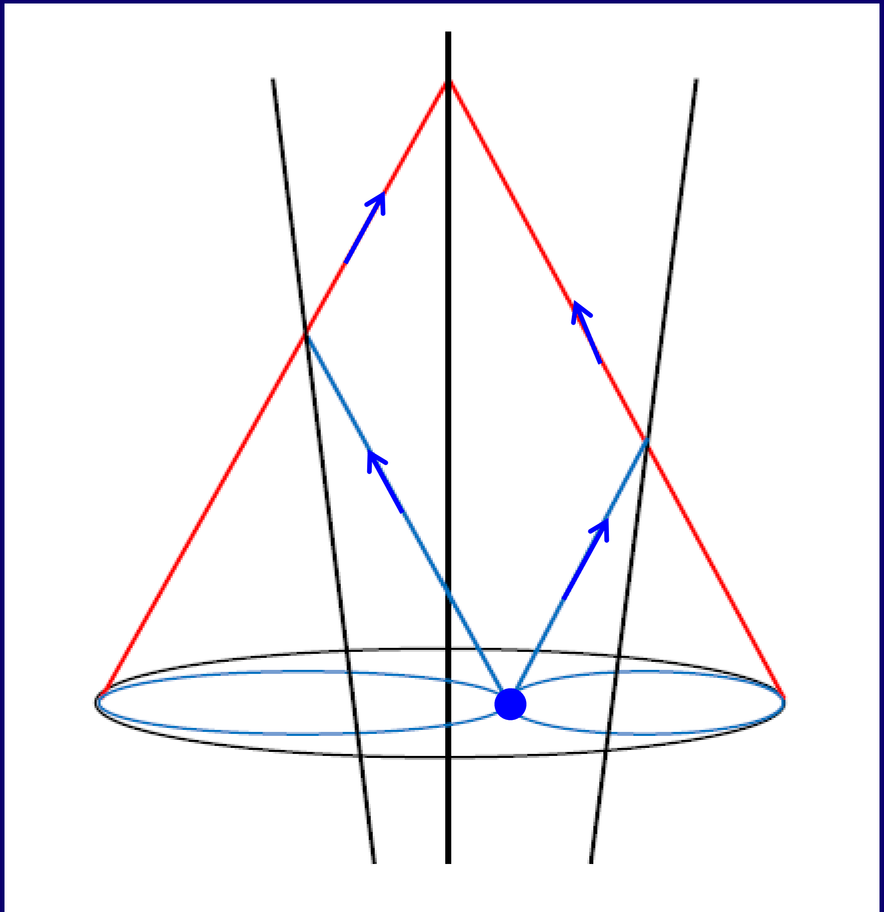
A null test for homogeneity:

$$\frac{L_{\|}}{L_{\perp}} - 1 \ \ \text{significantly different from } 0$$

implies violation of homogeneity

2B. Ionized gas in galaxy clusters scatters CMB

Clusters act like giant mirrors that give us a glimpse of the last scattering surface inside our past lightcone. This probes remote multipoles of the CMB.



Sunyaev-Zeldovich effect on CMB temperature

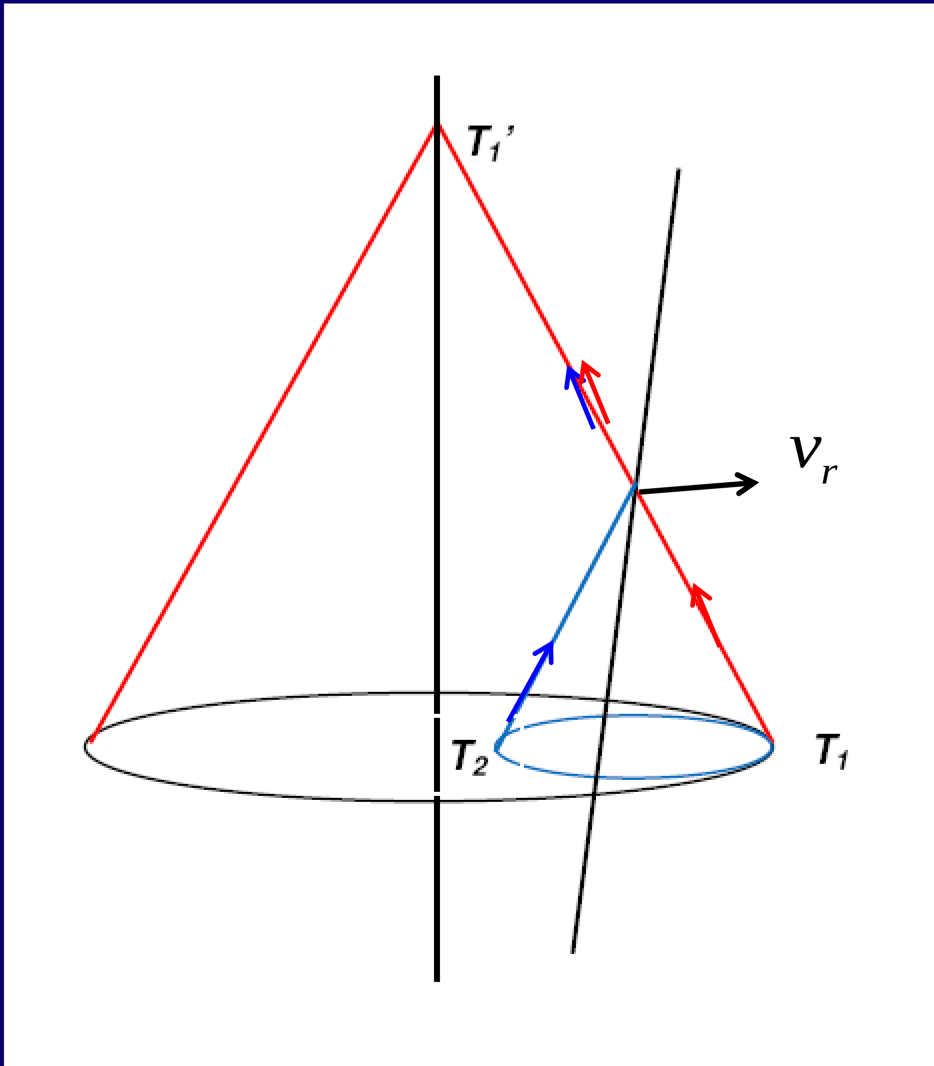
Scattered CMB photons at distant clusters:

* thermal SZ distorts the blackbody spectrum – a large effect if there is large anisotropy at the cluster

(Goodman 1995; Caldwell, Stebbins 2008)

* kinetic SZ probes bulk radial velocity

(Garcia-Bellido, Haugbolle 2008; Zhang, Stebbins 2011)

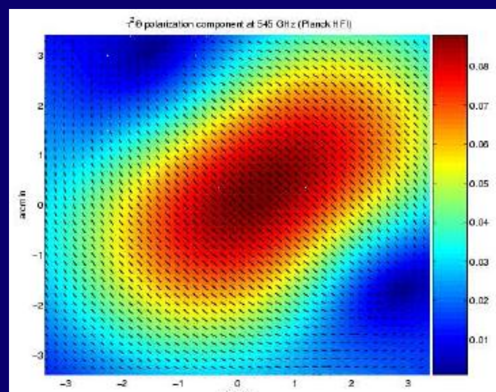


Non-perturbative SZ effect implies violation of CP.

Cluster polarization of CMB photons

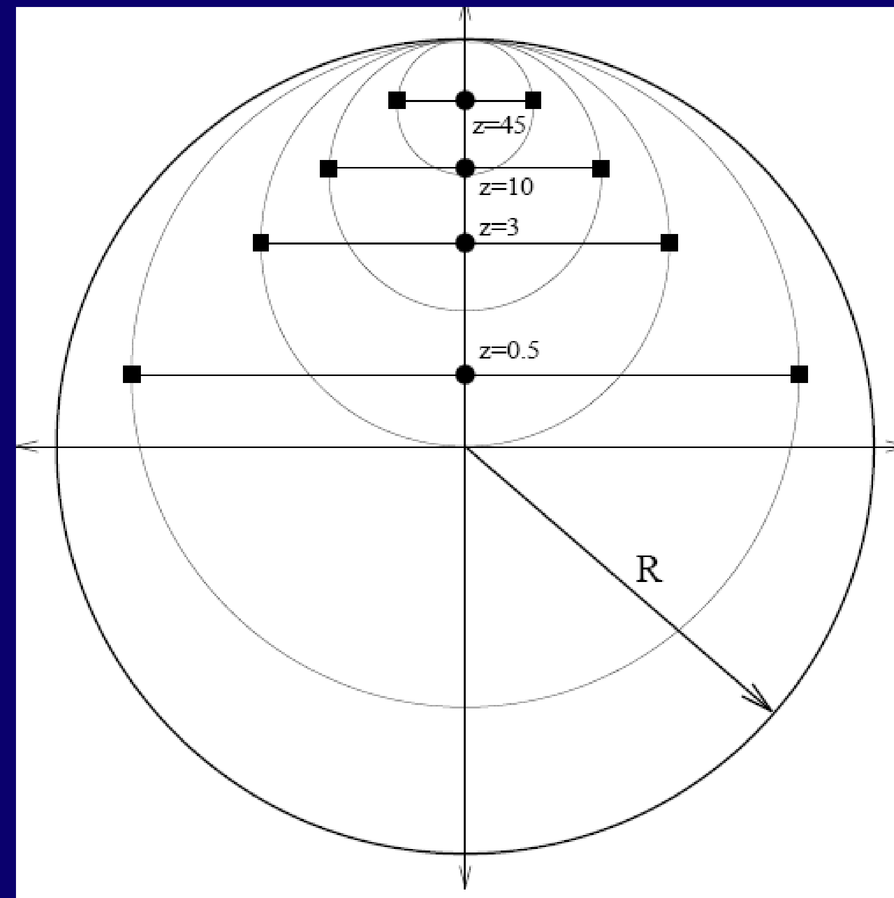
Scattered CMB photons at distant clusters are polarized:

* by the CMB quadrupole – a large effect if there is a large quadrupole at the cluster



* by the transverse velocity of the cluster

(Sunyaev, Zeldovich 1980; Kamionkowski, Loeb 1997)



Non-perturbative polarization implies violation of CP.

Summary

Homogeneity makes a good/ successful model.
But what is the *observational* basis for homogeneity?

(I) Without assuming the Copernican Principle:

* What can we say from isotropy?
  - isotropy of D_A , N, velocities, lensing gives isotropic (LTB) geometry
  - CMB isotropy does *not* enforce isotropic geometry

(II) With the Copernican Principle – strongest results:

* isotropic $D_A(z)$ up to $O(z^3)$ gives Friedmann
* isotropic CMB up to the octupole gives Friedmann

(III) We can test the Copernican Principle :

* consistency tests on distances and BAO
* clusters as remote probes of CMB anisotropy