Light Propagation through exact non-linear inhomogeneities in ΛCDM cosmology

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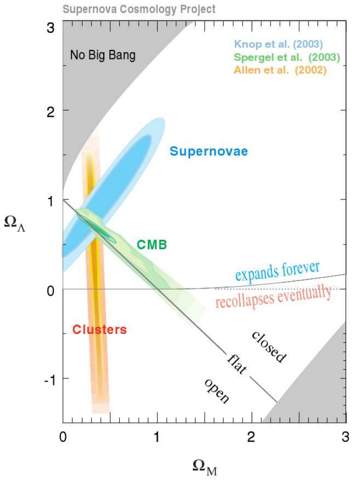
Work with Marco Bruni: arXiv:1103.0501 and arXiv:1107.4433

Outline

1. Motivation
2. Our model
   - The metric and density deviations, arXiv:1103.0501
   - Light tracing equations, arXiv:1107.4433
3. Results
   - Compensated δ
   - Non-compensated δ
4. Summary

Current status of cosmology



WMAP7+BAO+H_0

$\Omega_\Lambda = 0.725 \pm 0.016$

$\Omega_m h^2 = 0.1126 \pm 0.0036$

$w = -1.10 \pm 0.14$

Komatsu et al. (2011)

The distance measure

IN FLRW:

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In general:

$$\begin{aligned} \frac{d^2(d_A)}{d\lambda^2} &= -\left[\frac{1}{2}E^2\bar{\rho}(1+\delta)+|\sigma|^2\right]d_A, \\ \frac{d\sigma}{d\lambda} &+ 2\frac{\frac{d(d_A)}{d\lambda}}{d_A}\sigma = \Psi_0, \end{aligned}$$

where d_A is the angular diameter distance
E is photon energy
σ is shear of light bundle
Ψ_0 is Weyl scalar

Outline

Our metric

Consider here the flat ΛCDM sub-case

$$ds^2 = -dt^2 + S(t)^2 \left[dx^2 + dy^2 + Z(\mathbf{x}, t)^2 dr^2\right]$$

where

$$Z(\mathbf{x}, t) = A(\mathbf{x}) + F(r, t)$$

and **x** stands for all three Cartesian spatial coordinates

The metric deviation Z splits

We find

$$A = 1 + B\beta_+(r)\left\{[x+\gamma(r)]^2 + [y+\omega(r)]^2\right\}$$

and

$$F(r,t) = \beta_+(r)f_+(t) + \beta_-(r)f_-(t)$$

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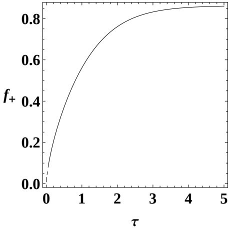
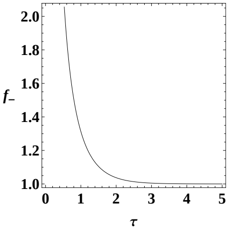
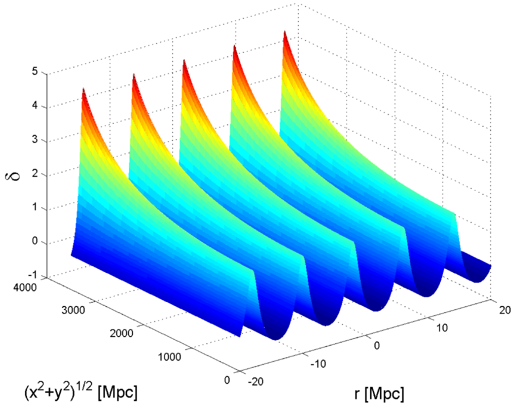
f_+ - the growing mode



f_- - the decaying mode



Density deviation example



Here $\gamma(r) = \omega(r) = \beta_-(r) = 0$ and $\beta_+ \propto sin(kr)$ for $k = 2\pi/8\text{Mpc}^{-1}$.

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The null geodesic equations

The very simplified case for light rays along r-axis

$$-\frac{E'}{E} = \frac{S'}{S} + \frac{F'}{1+F},$$

and

$$r' = \frac{2}{3}\frac{1}{H_0\sqrt{\Omega_\Lambda}SZ},$$

The Sachs optical equations

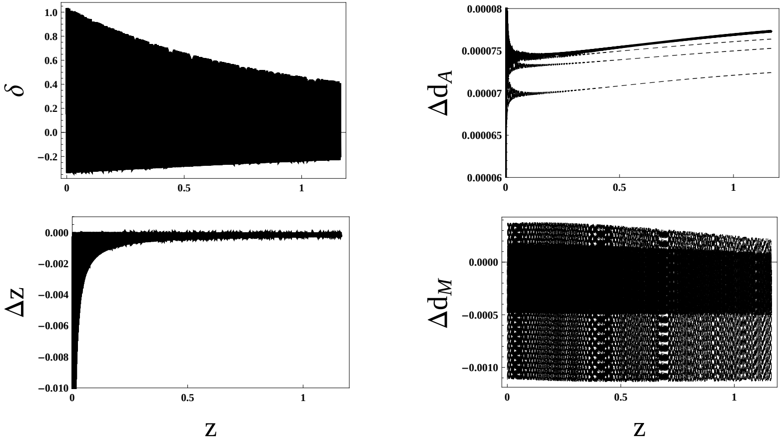
$$d_A'' + d_A' \frac{E'}{E} = \left(-\frac{2}{9} \frac{\rho}{H_0^2 \Omega_\Lambda} - \frac{4}{3} \frac{|\tilde{\sigma}|^2}{E^2} \right) d_A,$$

$$\tilde{\sigma}' + 2 \frac{d_A'}{d_A} \tilde{\sigma} = 0$$

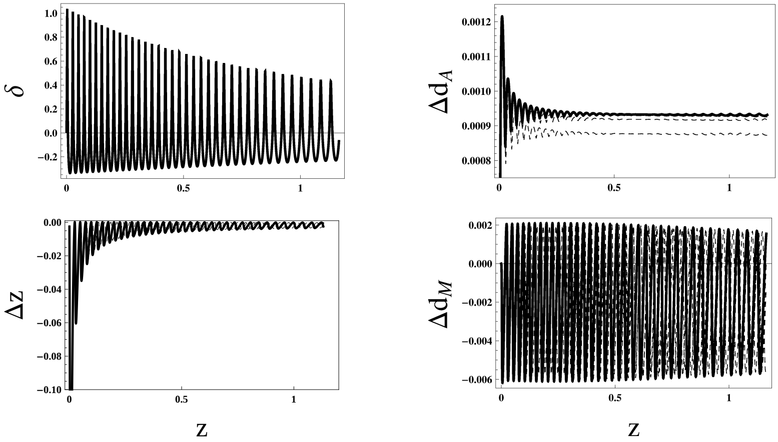
Here, the shear can be set to zero.

Outline

Inhomogeneities periodic on 8 Mpc scale



Inhomogeneities periodic on 100 Mpc scale



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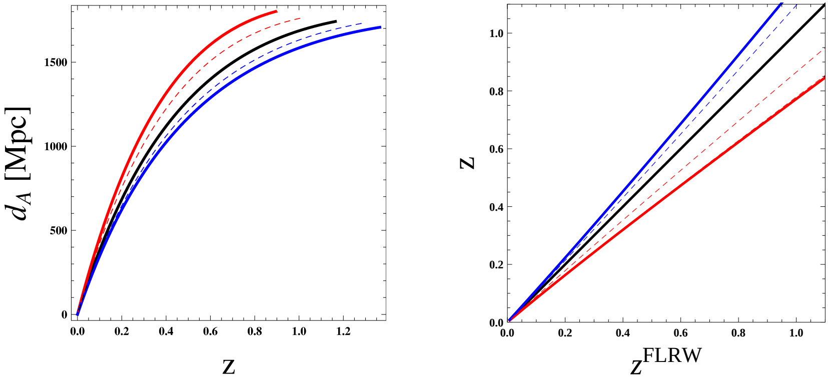


red: over-densities, blue: under-densities, black: FLRW background

Summary

- we have developed an exact inhomogeneous ΛCDM model
- can model non-linear structure growth and mode coupling
- compensated density has negligible effect on d_A
- non-compensated profiles have large effect
- identifying correct background is crucial

Thank you

In our model
The shocker slide

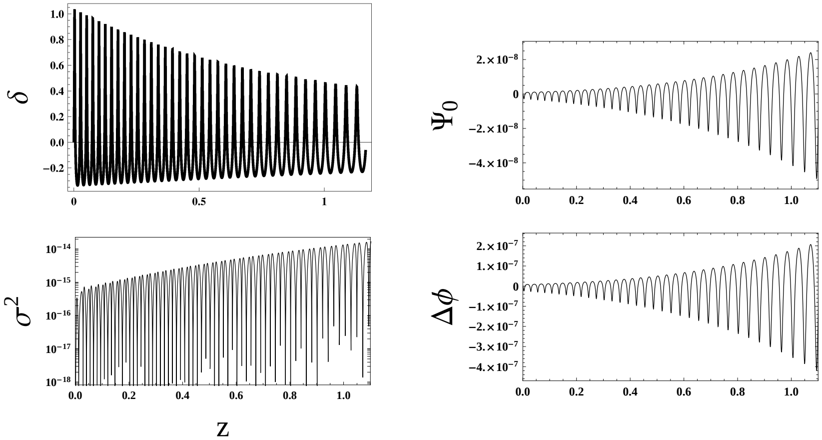
$$-\frac{E_\tau}{E} = S^2\left(\frac{3\Lambda}{4}\right)\left\{r_\tau^2(F+A)\left[(F+A)\frac{S_\tau}{S}+(f_+)_\tau\beta_+\right]+\frac{S_\tau}{S}(x_\tau^2+y_\tau^2)\right\}, \quad (1)$$

$$x_{\tau\tau}+\left(2\frac{S_\tau}{S}+\frac{E_\tau}{E}\right)x_\tau-(F+A)\beta_+(x+\gamma)r_\tau^2=0, \quad (2)$$

$$y_{\tau\tau}+\left(2\frac{S_\tau}{S}+\frac{E_\tau}{E}\right)y_\tau-(F+A)\beta_+(y+\omega)r_\tau^2=0, \quad (3)$$

$$\begin{aligned} r_{\tau\tau} &+ r_\tau^2\frac{(F+A)_r}{F+A} \\ &+ 2r_\tau\left[\frac{S_\tau}{S}+\frac{1}{2}\frac{E_\tau}{E}+\beta_+\frac{(f_+)_\tau+(x+\gamma)x_\tau+(y+\omega)y_\tau}{F+A}\right]=0. \quad (4)\end{aligned}$$

Non-FLRW contributions in the Sachs equations



angular diameter distance at high z