July 28^{th} 2011

Resummed Propagators

...with the eikonal approximation

Nicolas Van de Rijt

CEA/Saclay (France)

in collaboration with
F. Bernardeau and F. Vernizzi

Curvature perturbations

Introduction

Curvature perturbations

↓

Density/velocity perturbations

Introduction

Curvature perturbations

↓

Density/velocity perturbations

Propagator: evolution of perturbations

Introduction

Curvature perturbations

Density/velocity perturbations

Propagator: evolution of perturbations

Fluid description $\rightsquigarrow$ Vlasov equation

Fluid equations

continuity equation

$$\left\{ \quad \frac{1}{H}\dot{\delta} + \theta = 0 \right.$$

Fluid equations

continuity equation

$$\begin{cases} \dfrac{1}{H}\dot{\delta} + \theta = 0 \\ \dfrac{1}{H}\dot{\theta} + \left(2 + \dfrac{\dot{H}}{H^2}\right)\theta + \dfrac{3}{2}\Omega_m \delta = 0 \end{cases}$$

Euler equation

Fluid equations

continuity equation

$$\begin{cases} \dfrac{1}{H}\dot{\delta} + \theta = 0 \\ \dfrac{1}{H}\dot{\theta} + \left(2 + \dfrac{\dot{H}}{H^2}\right)\theta + \dfrac{3}{2}\Omega_m \delta = 0 \end{cases}$$

Euler equation

growing mode: $\delta(\mathbf{x}, t) = D_+(t)\delta_+(\mathbf{x})$ $\quad f_+ = \mathrm{d}\ln D_+/\mathrm{d}\ln a$

Fluid equations

$$\Psi_a \equiv \left(\begin{array}{c} \delta \\ \Theta \end{array} \right)$$

$\longleftarrow \quad -\frac{\theta}{f_+}$

$$\frac{\partial}{\partial \eta} \Psi_a(\mathbf{k}) + \Omega_{ab} \Psi_b(\mathbf{k}) = \gamma_{abc}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \Psi_b(\mathbf{k}_1) \Psi_c(\mathbf{k}_2)$$

Fluid equations

$$\Psi_a \equiv \begin{pmatrix} \delta \\ \Theta \end{pmatrix} \longleftarrow -\frac{\theta}{f_+}$$

$$\frac{\partial}{\partial \eta}\Psi_a(\mathbf{k}) + \Omega_{ab}\Psi_b(\mathbf{k}) = \gamma_{abc}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k}_1)\Psi_c(\mathbf{k}_2)$$

$$\Omega_{ab} \rightarrow \begin{pmatrix} 0 & -1 \\ -\frac{3}{2}\frac{\Omega_m}{f_+^2} & \frac{3}{2}\frac{\Omega_m}{f_+^2} - 1 \end{pmatrix}$$

$$\gamma_{abc} \rightarrow \frac{(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{k}_1}{k_1^2}$$

$$\frac{(\mathbf{k}_1+\mathbf{k}_2)^2\mathbf{k}_1\cdot\mathbf{k}_2}{2k_1^2k_2^2}$$

Fluid equations

$$\Psi_a \equiv \left(\begin{array}{c} \delta \\ \Theta \end{array} \right) \longleftarrow -\frac{\theta}{f_+}$$

$$\frac{\partial}{\partial \eta} \Psi_a(\mathbf{k}) + \Omega_{ab} \Psi_b(\mathbf{k}) = \gamma_{abc}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \Psi_b(\mathbf{k}_1) \Psi_c(\mathbf{k}_2)$$

Linear propagator $g_{ab}(\eta, \eta_0)$

$$\frac{\partial}{\partial \eta} g_{ab}(\eta, \eta') + \Omega_{ac}(\eta) g_{cb}(\eta, \eta') = 0$$

Fluid equations

$$\Psi_a \equiv \begin{pmatrix} \delta \\ \Theta \end{pmatrix} \longleftarrow -\frac{\theta}{f_+}$$

$$\frac{\partial}{\partial \eta}\Psi_a(\mathbf{k}) + \Omega_{ab}\Psi_b(\mathbf{k}) = \gamma_{abc}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k}_1)\Psi_c(\mathbf{k}_2)$$

Linear propagator $g_{ab}(\eta,\eta_0)$

$$\Psi_a(\mathbf{k},\eta) = g_{ab}(\eta,\eta_0)\Psi_b(\mathbf{k},\eta_0) + \int_{\eta_0}^{\eta} \mathrm{d}\tilde{\eta}\; g_{ab}(\eta,\tilde{\eta})\gamma_{bde}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\Psi_d(\mathbf{k}_1,\tilde{\eta})\Psi_e(\mathbf{k}_2,\tilde{\eta})$$

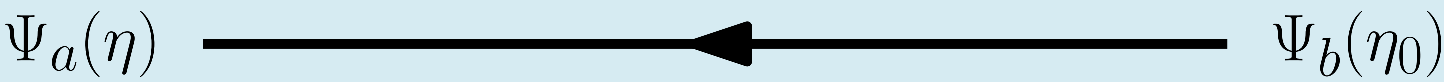
Scoccimarro (2001)

Nonlinear propagator

$$G_{ab}(\eta, \eta_0) \equiv \left\langle \frac{\partial \Psi_a(\eta)}{\partial \Psi_b(\eta_0)} \right\rangle$$

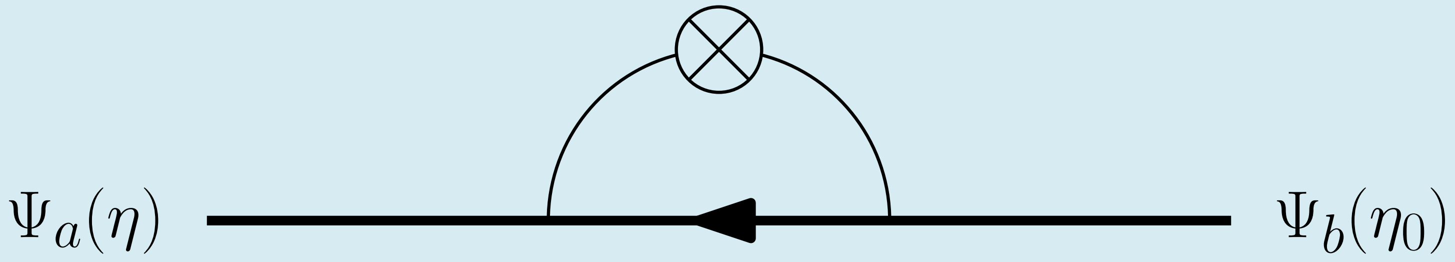
Nonlinear propagator

$$G_{ab}(\eta, \eta_0) \equiv \left\langle \frac{\partial \Psi_a(\eta)}{\partial \Psi_b(\eta_0)} \right\rangle$$



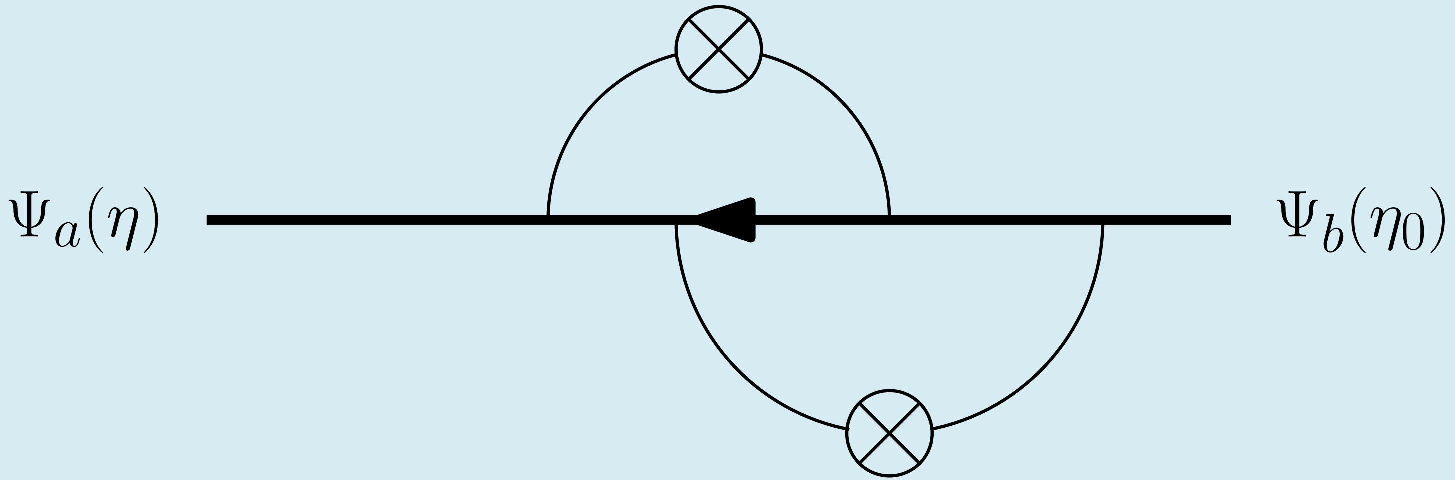
Nonlinear propagator

$$G_{ab}(\eta, \eta_0) \equiv \left\langle \frac{\partial \Psi_a(\eta)}{\partial \Psi_b(\eta_0)} \right\rangle$$



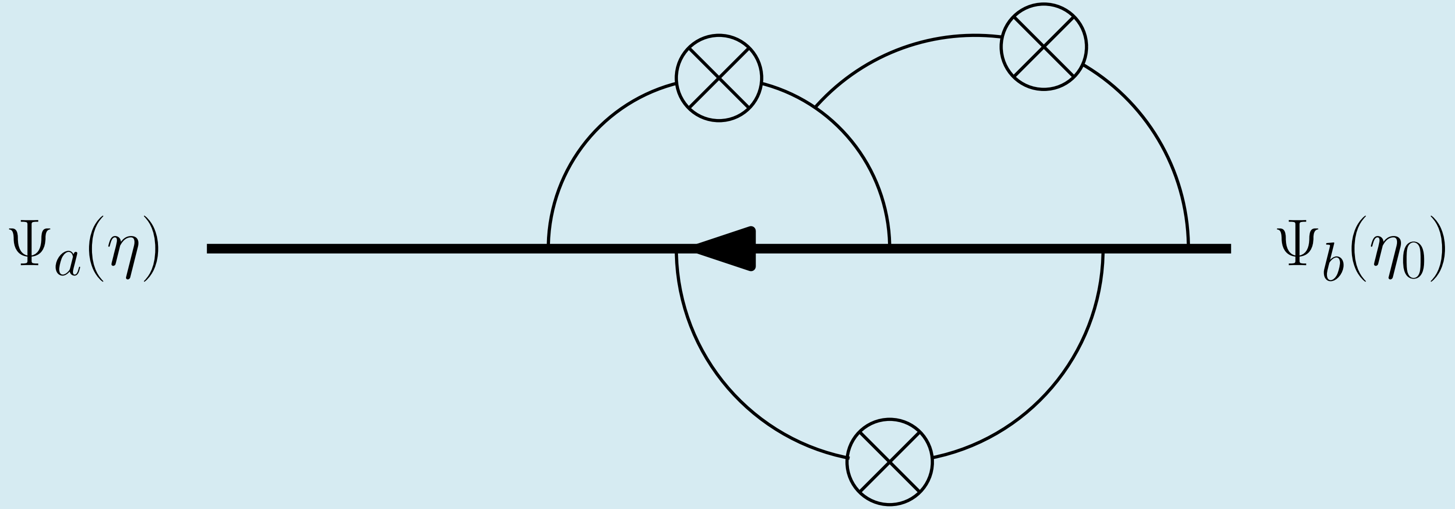
Nonlinear propagator

$$G_{ab}(\eta, \eta_0) \equiv \left\langle \frac{\partial \Psi_a(\eta)}{\partial \Psi_b(\eta_0)} \right\rangle$$



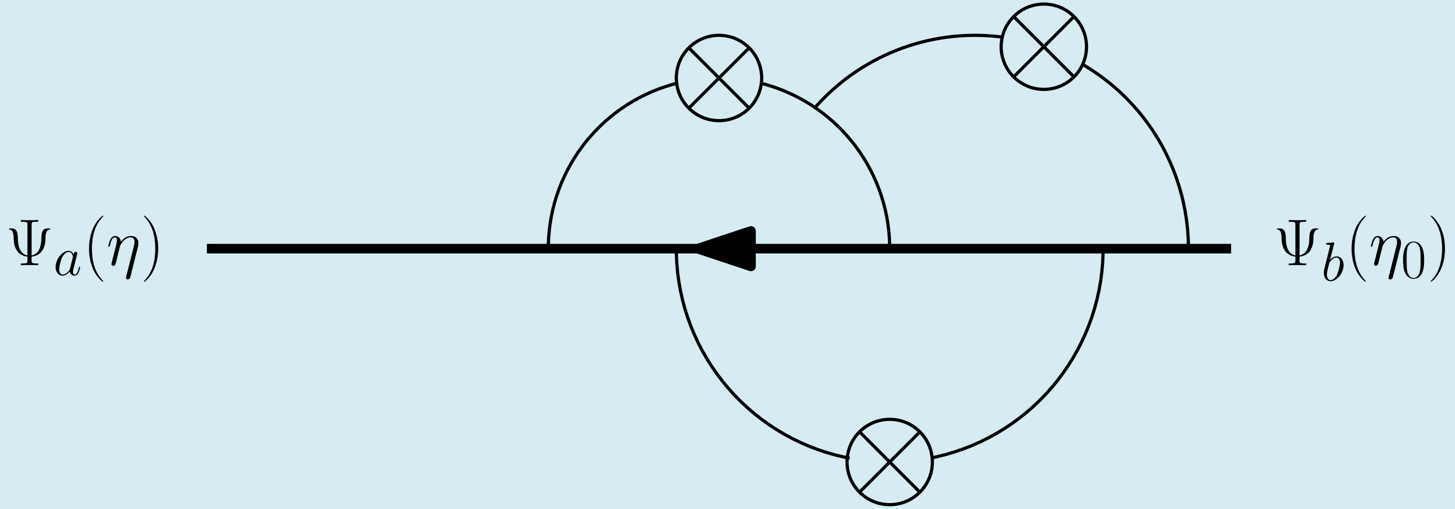
Nonlinear propagator

$$G_{ab}(\eta, \eta_0) \equiv \left\langle \frac{\partial \Psi_a(\eta)}{\partial \Psi_b(\eta_0)} \right\rangle$$



Nonlinear propagator

$$G_{ab}(\eta, \eta_0) \equiv \left\langle \frac{\partial \Psi_a(\eta)}{\partial \Psi_b(\eta_0)} \right\rangle$$

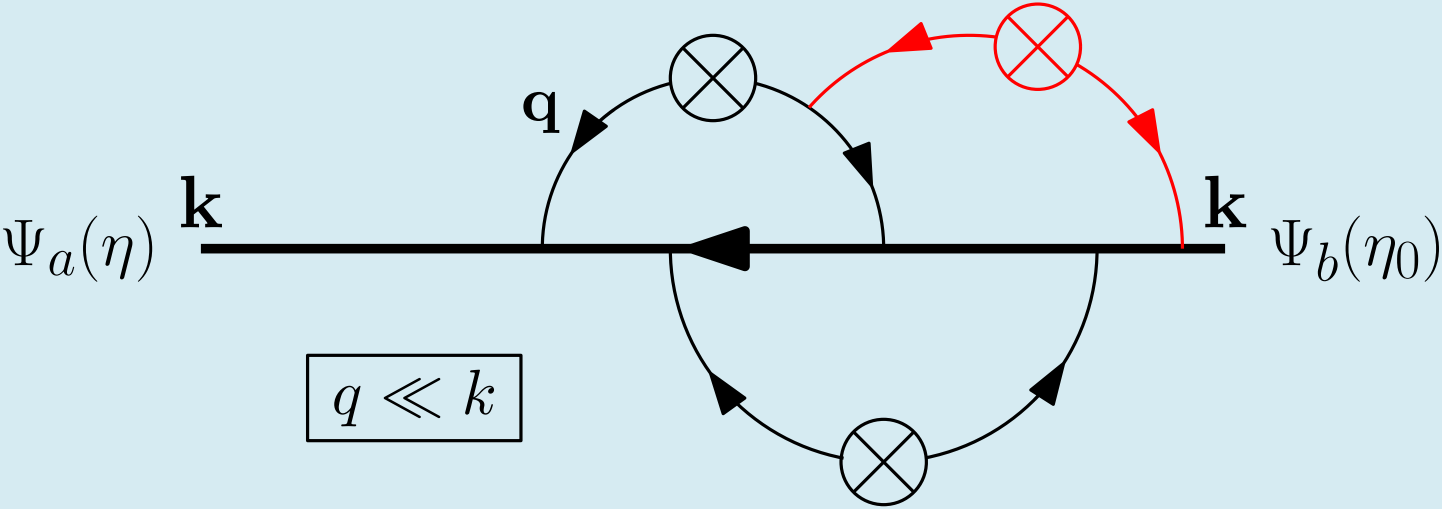


$$G_{ab}(\eta, \eta_0) = g_{ab}(\eta, \eta_0) \exp\left[-k^2 \sigma_d^2 (e^{\eta} - e^{\eta_0})^2/2\right]$$

Crocce, Scoccimarro (2006)

Nonlinear propagator

$$G_{ab}(\eta,\eta_0) \equiv \left\langle \frac{\partial \Psi_a(\eta)}{\partial \Psi_b(\eta_0)} \right\rangle$$



$$G_{ab}(\eta,\eta_0) = g_{ab}(\eta,\eta_0)\exp\left[-k^2\sigma_d^2(e^{\eta}-e^{\eta_0})^2/2\right]$$

Crocce, Scoccimarro (2006)

Eikonal approximation

$$\frac{\partial}{\partial\eta}\Psi_a(\mathbf{k}) + \Omega_{ab}\Psi_b(\mathbf{k}) = \gamma_{abc}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2)\Psi_b(\mathbf{k}_1)\Psi_c(\mathbf{k}_2)$$

Eikonal approximation

$$\frac{\partial}{\partial\eta}\Psi_a(\mathbf{k}) + \Omega_{ab}\Psi_b(\mathbf{k}) = \gamma_{abc}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k}_1)\Psi_c(\mathbf{k}_2)$$

$$[\gamma_{abc}\Psi_b\Psi_c]_S + [\gamma_{abc}\Psi_b\Psi_c]_H$$

Eikonal approximation

$$\frac{\partial}{\partial \eta}\Psi_a(\mathbf{k}) + \Omega_{ab}\Psi_b(\mathbf{k}) = \gamma_{abc}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k}_1)\Psi_c(\mathbf{k}_2)$$

$$[\gamma_{abc}\Psi_b\Psi_c]_S + [\gamma_{abc}\Psi_b\Psi_c]_H$$

$$\Xi_{ab}(\mathbf{k},\cancel{\mathbf{k}_1},\cancel{\mathbf{k}_2})\Psi_b(\mathbf{k})$$

Eikonal approximation

$$\frac{\partial}{\partial \eta}\Psi_a(\mathbf{k}) + \Omega_{ab}\Psi_b(\mathbf{k}) = \gamma_{abc}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2)\Psi_b(\mathbf{k}_1)\Psi_c(\mathbf{k}_2)$$

$$[\gamma_{abc}\Psi_b\Psi_c]_S + [\gamma_{abc}\Psi_b\Psi_c]_H$$

$$\Xi_{ab}(\mathbf{k}, \cancel{\mathbf{k}_1}, \cancel{\mathbf{k}_2})\Psi_b(\mathbf{k})$$

$$\Xi(\mathbf{k})\Psi_a$$

Eikonal approximation

$$\frac{\partial}{\partial \eta}\Psi_a(\mathbf{k}) + \Omega_{ab}\Psi_b(\mathbf{k}) = \gamma_{abc}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k}_1)\Psi_c(\mathbf{k}_2)$$

$$[\gamma_{abc}\Psi_b\Psi_c]_S + [\gamma_{abc}\Psi_b\Psi_c]_H$$

$$\Xi_{ab}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k})$$

$$\Xi(\mathbf{k})\Psi_a$$

$$\left(\frac{\partial}{\partial \eta} - \Xi(\mathbf{k})\right)\Psi_a(\mathbf{k}) + \Omega_{ab}\Psi_b(\mathbf{k}) = [\gamma_{abc}\Psi_b\Psi_c]_H$$

Resummed propagator

$$\left(\frac{\partial}{\partial\eta} - \Xi(\mathbf{k})\right)\xi_{ab}(\mathbf{k}) + \Omega_{ac}\xi_{cb}(\mathbf{k}) = 0$$

Resumed propagator

$$\left(\frac{\partial}{\partial\eta} - \Xi(\mathbf{k})\right)\xi_{ab}(\mathbf{k}) + \Omega_{ac}\xi_{cb}(\mathbf{k}) = 0$$

$$\boxed{\xi_{ab}(\mathbf{k},\eta,\eta_0) = g_{ab}(\mathbf{k},\eta,\eta_0)\exp\left[\int_{\eta_0}^{\eta}\mathrm{d}\tilde{\eta}\ \Xi(\mathbf{k},\tilde{\eta})\right]}$$

Resumed propagator

$$\left(\frac{\partial}{\partial\eta} - \Xi(\mathbf{k})\right)\xi_{ab}(\mathbf{k}) + \Omega_{ac}\xi_{cb}(\mathbf{k}) = 0$$

$$\boxed{\xi_{ab}(\mathbf{k},\eta,\eta_0) = g_{ab}(\mathbf{k},\eta,\eta_0)\exp\left[\int_{\eta_0}^{\eta}\mathrm{d}\tilde{\eta}\;\Xi(\mathbf{k},\tilde{\eta})\right]}$$

$$G_{ab}(\mathbf{k},\eta) = \langle\xi_{ab}(\mathbf{k},\eta)\rangle_{\Xi}$$

Multi-component fluid

$$\delta \longrightarrow \delta_\alpha \qquad \theta \longrightarrow \theta_\alpha \qquad \frac{3}{2}\Omega_m \delta \longrightarrow \frac{3}{2}\Omega_m \delta$$

Multi-component fluid

$$\delta \longrightarrow \delta_\alpha \qquad \theta \longrightarrow \theta_\alpha \qquad \frac{3}{2}\Omega_m\delta \longrightarrow \frac{3}{2}\Omega_m\delta$$

- adiabatic growing / decaying modes
- isodensity modes $\left(\delta = \sum \frac{\Omega_\alpha}{\Omega_m}\delta_\alpha = 0\right)$

Multi-component fluid

$$\delta \longrightarrow \delta_\alpha \qquad \theta \longrightarrow \theta_\alpha \qquad \frac{3}{2}\Omega_m \delta \longrightarrow \frac{3}{2}\Omega_m \delta$$

- adiabatic growing / decaying modes
- isodensity modes $\left(\delta = \sum \frac{\Omega_\alpha}{\Omega_m}\delta_\alpha = 0\right)$

$$\Xi_{ab} = \Xi_{ab}^{\mathrm{ad}} + \Xi_{ab}^{\mathrm{iso}}$$

Conclusion

- We describe a new method to derive the resummed propagators
- Applicable to non-standard cosmologies and initial conditions
- Applicable to multi-component fluids