11th RESCEU/DENET Summer School: Dark Energy in the Universe

Observational Constraints on Exponential Gravity

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Reference: Louis Yang, Chung-Chi Lee, Ling-Wei Luo, Chao-Qiang Geng, Phys. Rev. D 82, 103515 (2010)

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Outline

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- Cosmic observations from type Ia supernovae (SNe Ia), large scale structure (LSS), baryon acoustic oscillations (BAO) and cosmic microwave background (CMB) indicate that our universe is undergoing an accelerating expansion

- The reason for this acceleration, the so-called dark energy problem, remains a fascinating question today

- We use the experimental constraints to determine the free parameters in viable $f(R)$ gravity models, in our case, we focus on a viable model called "exponential gravity" with the form of $R + f(R) = R - \beta R_E(1 - e^{-R/R_E})$

- Alternative dark energy models
  - Modified matter
    - Quintessence
    - K-essence
    - Perfect fluid models
  - Modified gravity
    - $f(R)$ gravity (non-linear Lagrangian density in terms of R)
    - Scalar-tensor theories (R couples to ϕ with the form: $F(\phi)R$)
    - Braneworld models
  - Others

- Viable $f(R)$ models:

$$S_{\text{E-H}} = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}R \rightarrow S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left[R + f(R)\right].$$

  - Hu and Sawicki

$$f(R) = -\frac{c1 R_{HS}(R/R_{HS})^p}{c2(R/R_{HS})^p + 1} \text{ with } c1 > 0, c2 > 0, p > 0, R_{HS} > 0,$$

  - Starobinsky

$$f(R) = \lambda R_S\left[(1 + R^2/R_S^2)^{-n} - 1\right] \text{ with } \lambda > 0, n > 2, R_S > 0,$$

  - Tsujikawa

$$f(R) = -\mu R_T \tanh(R/R_T) \text{ with } \mu > 0, R_T > 0,$$

  - Exponential gravity

$$f(R) = -\beta R_E(1 - e^{-R/R_E}) \text{ with } \beta > 1, R_E > 0.$$

Goal: we will test these models with the the observational data (SNe Ia, BAO and CMB).

- Exponential gravity
  - The action is

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}\, [R + f(R)] + S_M ,$$

where $\kappa^2 \equiv 8\pi G$ and

$$f(R) = -\beta R_E (1 - e^{-R/R_E}).$$

  - The modified Friedmann equation is

$$H^2 = \frac{\kappa^2 \rho_M}{3} + \frac{1}{6}(f_R R - f) - H^2 \left(f_R + f_{RR} R'\right),$$

where a subscript R denotes the derivative with respect to R, a prime represents $d/d\ln a$, and $\rho_M = \rho_m + \rho_r$.

  - In flat spacetime, the Ricci scalar is

$$R = 12H^2 + 6HH'.$$

- Exponential gravity
  - Following Hu and Sawicki's parameterization

$$y_H \equiv \frac{\rho_{DE}}{\rho_m^0} = \frac{H^2}{m^2} - a^{-3} \text{ and } y_R \equiv \frac{R}{m^2} - 3a^{-3},$$

with $m^2 \equiv \kappa^2\rho_m^0/3$, we rewrite the modified Friedmann equation and Ricci scalar into two coupled differential equations

$$y_H' = \frac{y_R}{3} - 4y_H$$

and

$$y_R' = 9a^{-3} - \frac{1}{H^2 f_{RR}}\left[y_H + f_R\left(\frac{H^2}{m^2} - \frac{R}{6m^2}\right) + \frac{f}{6m^2}\right].$$

  - Leading to a second order differential equation of y_H in the form

$$y_H'' + J_1 y_H' + J_2 y_H + J_3 = 0.$$

  - And the effective dark energy equation of state w_{DE} is given by

$$w_{DE} = -1 - \frac{y_H'}{3y_H}.$$

Outline

- Type Ia Supernovae (SNe Ia)
  - The distance modulus

$$\mu_{th}(z_i) \equiv 5\log_{10} D_L(z_i) + \mu_0,$$

where $\mu_0 \equiv 42.38 - 5\log_{10} h$ with $H_0 = h \cdot 100km/s/Mpc$.
  - The Hubble-free luminosity distance for the flat universe

$$D_L(z) = (1+z)\int_0^z \frac{dz'}{E(z')},$$

where $E(z) = H(z)/H_0$.
  - The χ^2 for the SNe Ia data is

$$\boxed{\chi^2_{SN} = \sum_i \frac{\left[\mu_{obs}(z_i) - \mu_{th}(z_i)\right]^2}{\sigma_i^2}}.$$

- Type Ia Supernovae (SNe Ia)
  - Since the absolute magnitude of SNe Ia is unknown, we should minimize χ^2_{SN} with respect to μ_0, which relates to the absolute magnitude, and expand it to be

$$\chi^2_{SN} = A - 2\mu_0 B + \mu_0^2 C,$$

where

$$A = \sum_i \frac{\left[\mu_{obs}(z_i) - \mu_{th}(z_i;\mu_0 = 0)\right]^2}{\sigma_i^2},$$

$$B = \sum_i \frac{\mu_{obs}(z_i) - \mu_{th}(z_i;\mu_0 = 0)}{\sigma_i^2}, \quad C = \sum_i \frac{1}{\sigma_i^2}.$$

The minimum of χ^2_{SN} with respect to μ_0 is

$$\boxed{\tilde{\chi}^2_{SN} = A - \frac{B^2}{C}}.$$

- Baryon Acoustic Oscillations (BAO)
  - The distance ratio

$$d_z \equiv r_s(z_d)/D_V(z) \text{ (z_d is redshift at the drag epoach.)}$$

  - The volume-averaged distance

$$D_V(z) \equiv \left[(1+z)^2 D_A^2(z) \frac{z}{H(z)}\right]^{1/3}.$$

  - $D_A(z)$ is the proper angular diameter distance

$$D_A(z) = \frac{1}{1+z} \int_0^z \frac{dz'}{H(z')} \text{ (for flat universe).}$$

  - The comoving sound horizon

$$r_s(z) = \frac{1}{\sqrt{3}} \int_0^{1/(1+z)} \frac{da}{a^2 H(z'=\frac{1}{a}-1)\sqrt{1+(3\Omega_b^0/4\Omega_\gamma^0)a}},$$

here $\Omega_b^0 = 0.022765h^{-2}$ and $\Omega_\gamma^0 = 2.469 \times 10^{-5} h^{-2}$.

- Baryon Acoustic Oscillations (BAO)
  - z_d is the redshift at the drag epoch

$$z_d = \frac{1291(\Omega_m^0 h^2)^{0.251}}{1+0.659(\Omega_m^0 h^2)^{0.828}} \left[1 + b_1 (\Omega_b^0 h^2)^{b2}\right],$$

    with

$$\begin{aligned} b_1 &= 0.313(\Omega_m^0 h^2)^{-0.419}\left[1+0.607(\Omega_m^0 h^2)^{0.674}\right], \\ b_2 &= 0.238(\Omega_m^0 h^2)^{0.223}. \end{aligned}$$

  - The measured distance ratios $d^{obs}_{z=0.2} = 0.1905 \pm 0.0061$ and $d^{obs}_{z=0.35} = 0.1097 \pm 0.0036$ with the inverse covariance matrix:

$$C^{-1}_{BAO} = \begin{pmatrix} 30124 & -17227 \\ -17227 & 86977 \end{pmatrix}.$$

  - The χ^2 for the BAO data is

$$\boxed{\chi^2_{BAO} = (x^{th}_{i,BAO} - x^{obs}_{i,BAO})(C^{-1}_{BAO})_{ij}(x^{th}_{j,BAO} - x^{obs}_{j,BAO})},$$

    where $x_{i,BAO} \equiv (d_{0.2}, d_{0.35})$.

- Cosmic Microwave Background (CMB)
  - The acoustic scale

$$l_A(z_*) \equiv (1+z_*)\frac{\pi D_A(z_*)}{r_S(z_*)}.$$

  - The shift parameter

$$R(z_*) \equiv \sqrt{\Omega_m^0} H_0 (1+z_*) D_A(z_*).$$

  - The decoupling epoch

$$z_* = 1048\left[1 + 0.00124(\Omega_b^0 h^2)^{-0.738}\right]\left[1 + g_1(\Omega_m^0 h^2)^{g2}\right],$$

where

$$g_1 = \frac{0.0783(\Omega_b^0 h^2)^{-0.238}}{1 + 39.5(\Omega_b^0 h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1(\Omega_b^0 h^2)^{1.81}}.$$

- Cosmic Microwave Background (CMB)
  - $l_A(z_*) = 302.09$, $R(z_*) = 1.725$ and $z_* = 1091.3$ with the inverse covariance matrix:

$$C_{CMB}^{-1} = \begin{pmatrix} 2.305 & 29.698 & -1.333 \\ 29.698 & 6825.27 & -113.180 \\ -1.333 & -113.180 & 3.414 \end{pmatrix}.$$

  - The χ^2 for the CMB data is

$$\boxed{\chi^2_{CMB} = (x^{th}_{i,CMB} - x^{obs}_{i,CMB})(C^{-1}_{CMB})_{ij}(x^{th}_{j,CMB} - x^{obs}_{j,CMB})},$$

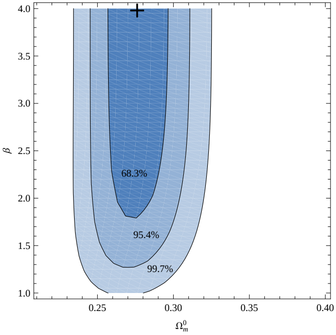
where $x_{i,CMB} \equiv (l_A(z_*), R(z_*), z_*)$.

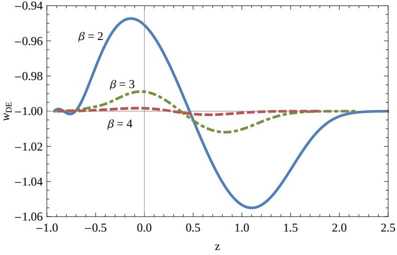
Outline

- The $\boxed{\chi^2 = \tilde{\chi}^2_{SN} + \chi^2_{BAO} + \chi^2_{CMB}}$

Exponential Gravity: $R + f(R) = R - \beta R_E (1 - e^{-R/R_E})$

Model		Ω_m^0	χ^2
Exponential Gravity	$\beta = 2$	$0.274^{+0.014}_{-0.013}$	546.7136
	$\beta = 3$	$0.276^{+0.014}_{-0.013}$	545.3836
	$\beta = 4$	$0.276^{+0.014}_{-0.013}$	545.1721
ΛCDM		$0.276^{+0.014}_{-0.013}$	545.1522





- Solving $y''_H + J_1 y'_H + J_2 y_H + J_3 = 0$ numerically in the low redshift regime($z = 0 \sim 4$) and getting the evolution of the Hubble parameter
- In high red shift regime ($z \gtrsim 4$), the factor $e^{-R/R_S} (< 10^{-5})$ become negligible, and the exponential gravity model behaves like a ΛCDM model with $\Omega_\Lambda = \beta R_s / 6H_0^2$ and the Hubble parameter is $H(z) = H_0 \sqrt{\Omega_m^0 (1+z)^3 + \Omega_r^0 (1+z)^4 + \frac{\beta R_S}{6H_0^2}}$
- In high curvature regime, $f(R) \to -\beta R_E$ is regarded as the cosmological constant
- There is no upper limit for parameter β
- Ω_m^0 is constrained to the concordance value
- $\beta \to \infty$ corresponds to the ΛCDM model
- From the plot of effective dark energy equation of state w_{DE}, the deviation from cosmological constant phase ($w_{DE} = -1$) becomes smaller for larger value of β

Outline

- We have studied the modified gravity, especially the exponential gravity
- We have done the data fitting by using the SNe Ia, BAO and CMB data
- In the low redshift regime, we follow Hu and Sawicki's parameterization to form the differential equation for the exponential gravity and solve it numerically
- In the high redshift regime, we take advantage of the asymptotic behavior of the exponential gravity toward an effective cosmological constant
- Current observational data can not distinguish the ΛCDM and exponential gravity model

End

- Thank you!

Outline

- The method of maximum likelihood - likelihood function
  - A set of N measure quantities $\boldsymbol{x} = (x_1, ..., x_N)$ describe by a joint p.d.f. $f(\boldsymbol{x};\boldsymbol{\theta})$, where $\boldsymbol{\theta} = (\theta_1, ..., \theta_n)$ is a set of n parameters whose value are unknown.
  - The *likelihood function* $\mathcal{L}(\boldsymbol{\theta}) \equiv f(\boldsymbol{x};\boldsymbol{\theta})$.
  - If the measurements x_i are statistically independent and each follow the p.d.f. $f(x_i;\boldsymbol{\theta})$, then the joint p.d.f. for x factorizes and the likelihood function is

$$\mathcal{L}(\boldsymbol{\theta}) = \prod_{i=1}^{N} f(x_i;\boldsymbol{\theta}) \ .$$

  - It is usually easier to work with $\ln \mathcal{L}$, and since both are maximized for the same parameter value $\boldsymbol{\theta}$, the maximum likelihood (ML) estimators can be found by solving the *likelihood equations*,

$$\frac{\partial \ln \mathcal{L}}{\partial \theta_i} = 0 \ , \qquad i = 1, ..., n \ .$$

- The method of least squares - χ^2 function
  - The *method of least squares* (LS) coincides with the method of maximum likelihood in the following special case.
  - Consider a set of N independent measurements y_i at known points x_i. The measurement y_i is assumed to be Gaussian distributed with mean $F(x_i;\boldsymbol{\theta})$ and known variance σ_i^2.
  - The goal is to construct estimators for the unknown parameters $\boldsymbol{\theta}$,

$$\chi^2(\boldsymbol{\theta}) = -2\ln\mathcal{L}(\boldsymbol{\theta}) + \mathrm{const} = \sum_{i=1}^{N}\frac{(y_i - F(x_i;\boldsymbol{\theta}))^2}{\sigma_i^2} + \mathrm{const}\,.$$

  - The set of parameters $\boldsymbol{\theta}$ which maximize $\mathcal{L}$ is the same as those which minimize χ^2.

- The method of least squares - χ^2 function
  - In general, the measurements y_i are not Gaussian distributed as long as they are not indepentent, If they are not independent but rather have a covariance matrix $V_{ij} = \text{cov}[y_i, y_j]$, then the LS estimators are determined by the minimum of

$$\chi^2(\boldsymbol{\theta}) = (\boldsymbol{y} - \boldsymbol{F}(\boldsymbol{\theta}))^T V^{-1} (\boldsymbol{y} - \boldsymbol{F}(\boldsymbol{\theta})) \ .$$

    where $\boldsymbol{y} = (y_1, ..., y_N)$ is the vector of measurements, $\boldsymbol{F}(\boldsymbol{\theta})$ is the corresponding vector of predicted values.

- Best-fit
  Small value of χ^2 indicate a good fit. The parameters $\boldsymbol{\theta}^*$ that minimize χ^2 are called the best-fit parameters.

- Confidence Level

Table 32.2: $\Delta\chi^2$ or $2\Delta \ln L$ corresponding to a coverage probability $1-\alpha$ in the large data sample limit, for joint estimation of m parameters.

$(1-\alpha)$ (%)	$m=1$	$m=2$	$m=3$
68.27	1.00	2.30	3.53
90.	2.71	4.61	6.25
95.	3.84	5.99	7.82
95.45	4.00	6.18	8.03
99.	6.63	9.21	11.34
99.73	9.00	11.83	14.16

PDG2008