Phantom Crossing in Modified Gravity Theory

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K. Bamba, CQ Geng and CC Lee JCAP 1008,021
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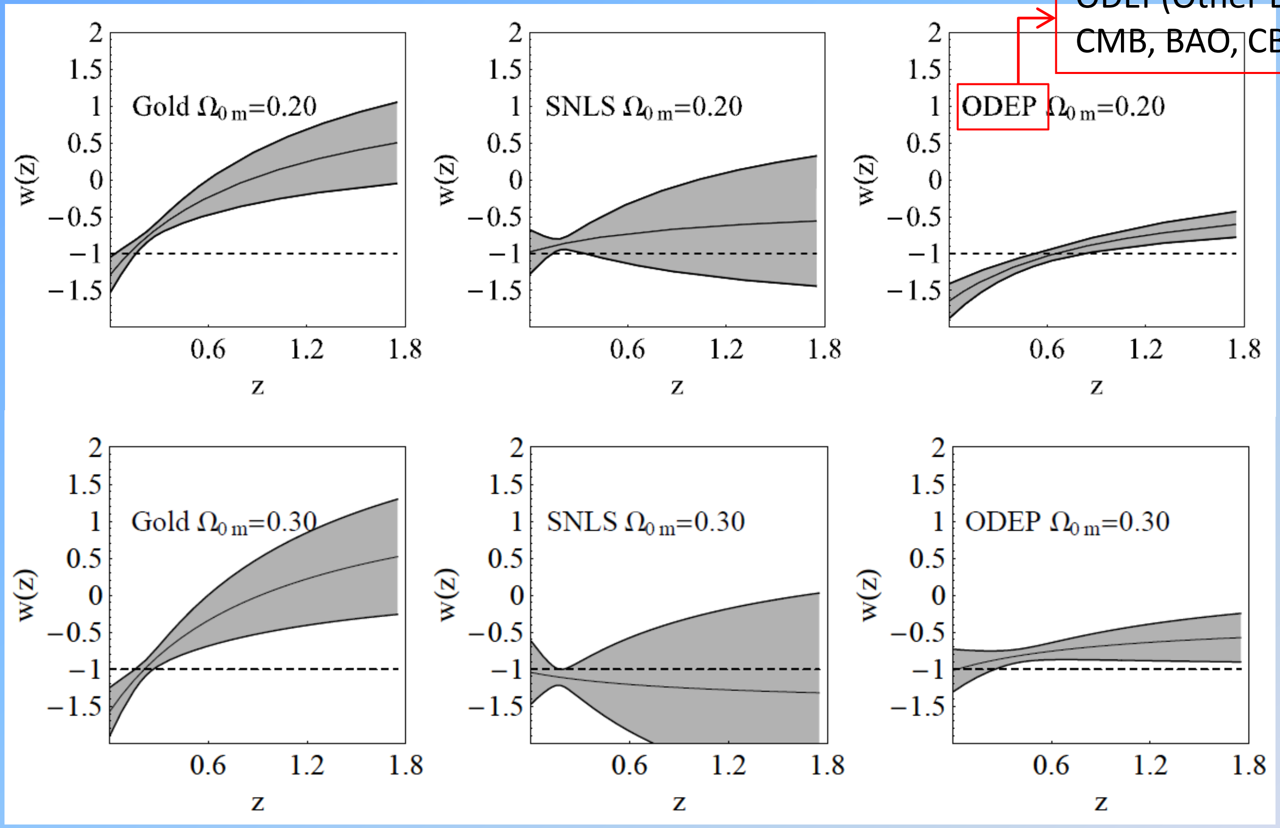
Motivation

- The equation of state($w \equiv \frac{P}{\rho}$) observation analysis

ODEP(Other Dark Energy Probes) contains CMB, BAO, CBF & PGR



S. Nesseris and
L. Perivolaropoulos
JCAP 0701,018(2007)

- The equation of state may not be a constant(In ΛCDM: w=-1)

$f(R)$ gravity

- The Lagrangian of general relativity is given by the Ricci scalar:

$$S = \int \frac{\sqrt{-g}}{2\varkappa^2} \mathcal{L}_g + \mathcal{L}_M d^4x \quad \text{where } \mathcal{L}_g = R \qquad \varkappa^2 = 8\pi G$$

- It is reasonable to modify the gravity in order to solve the Dark Energy problem, one of the simplest ways is $f(R)$ gravity

$$S = \int \frac{\sqrt{-g}}{2\varkappa^2} \mathcal{L}_g + \mathcal{L}_M d^4x \quad \text{where } \mathcal{L}_g = f(R)$$

field equation: $f_R(R)R_{\mu\nu} - f(R)g_{\mu\nu} + (g_{\mu\nu}\nabla^2 - \nabla_\mu \nabla_\nu) f_R(R) = \varkappa^2 T_{\mu\nu}$

- Equation of motion(Friedmann eqn.):

FRW metric $g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & a^2 & & \\ & & a^2 & \\ & & & a^2 \end{pmatrix}$

Friedmann eqn. $\begin{cases} f_R H + \dfrac{\ddot{f}_R - H\dot{f}_R}{2} = -\dfrac{\varkappa^2}{2}(\rho_m + p_m + \rho_r + p_r) \\ f_R H^2 + H f_{RR}\dot{R} + \dfrac{f - Rf_R}{6} = \dfrac{\varkappa^2}{3}(\rho_m + \rho_r) \end{cases}$ where $f_R = \dfrac{\partial f}{\partial R}$

$f(R)$ gravity

- Parameterization (PRD 76, 064004 (2007))

$$y_H = \frac{H^2}{m^2} - a^{-3} \quad \text{and} \quad y_R = \frac{R}{m^2} - 3a^{-3}$$

then Friedmann equation lead to two first order differential equations

$$\begin{cases} y'_H \equiv \dfrac{dy_H}{d\ln a} = \dfrac{y_R}{3} - 4y_H \\ y'_R \equiv \dfrac{dy_R}{d\ln a} = 9a^{-3} - \dfrac{1}{y_H + a^{-3}} \dfrac{1}{m^2 f_{RR}} \left[y_H - (f_R - 1)\left(\dfrac{y_R}{6} - y_H - \dfrac{a^{-3}}{2} \right) + \dfrac{f - R}{6m^2} \right] \end{cases}$$

or a second ODE express as

$$y''_H + J_1 y'_H + J_2 y_H + J_3 = 0$$

where J_1, J_2 and J_3 stand for

$$\begin{cases} J_1 = 4 + \dfrac{1}{y_H + a^{-3}} \dfrac{1 - f_R}{6m^2 f_{RR}} \\ J_2 = \dfrac{1}{y_H + a^{-3}} \dfrac{2 - f_R}{3m^2 f_{RR}} \\ J_3 = -\dfrac{1}{y_H + a^{-3}} \dfrac{(1 - f_R)a^{-3} + (R - f)/3m^2}{6m^2 f_{RR}} - 3a^{-3} \end{cases}$$

Viable $f(R)$ gravity

- The viable conditions in $f(R)$ gravity
  1. $f_R>0$ for $R>R_0$
  2. $f_{RR}>0$ for $R>R_0$
  3. $f(R)\rightarrow R-2\Lambda$ for $R>>R_0$
  4. Exist a stable late time de-Sitter point
  5. Other conditions, such as solar system constraint etc.

- Some viable models

$$1.\ \text{Hu}-\text{Sawicki}: f(R)=R-\mu R_{HS}\frac{c_1\left(R^2/R_{HS}^2\right)^n}{1+c_2\left(R^2/R_{HS}^2\right)^n}$$

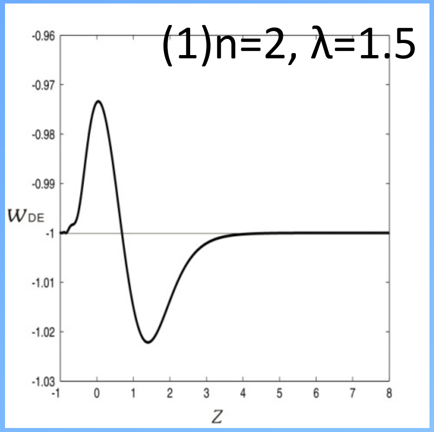
$$2.\ \text{Starobinsky}: f(R)=R-\lambda R_S\left(1-\left(1+R^2/R_s^2\right)^{-n}\right)$$

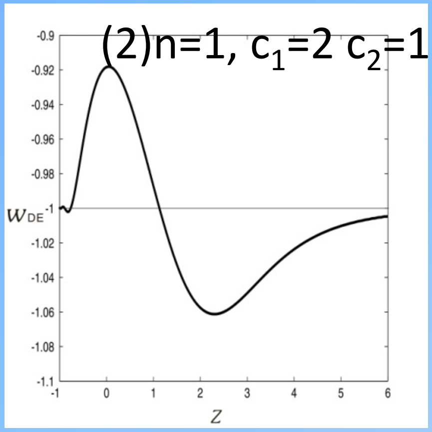
$$3.\ \text{Tsujikawa}: f(R)=R-\mu R_T \tanh\left(R/R_T\right)$$

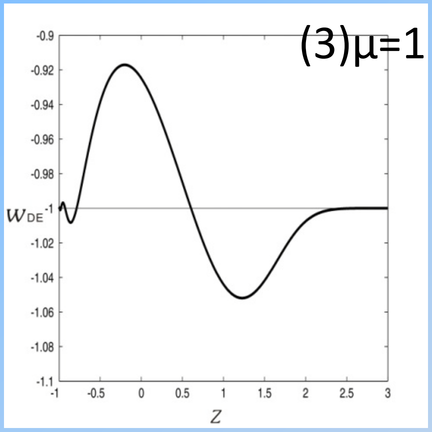
$$4.\ \text{Exponential}: f(R)=R-\beta R_E\left(1-e^{-R/R_E}\right)$$

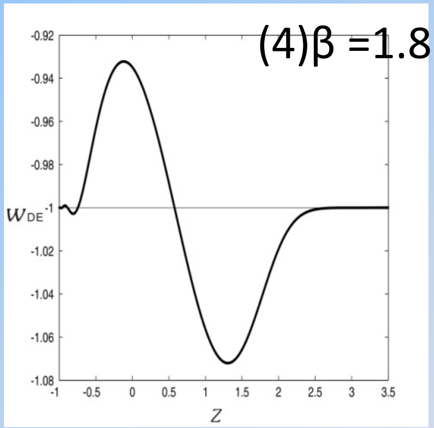
Phantom crossing behavior in $f(R)$ gravity

- EOS ($w=P/\rho$) evolution in different viable models

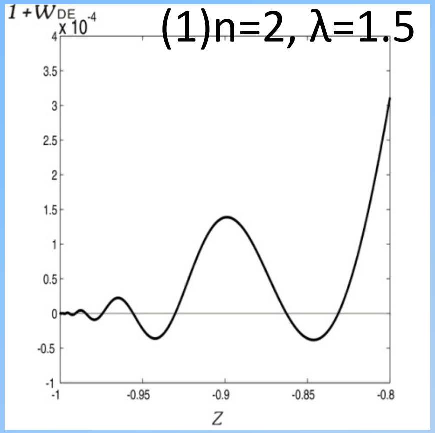


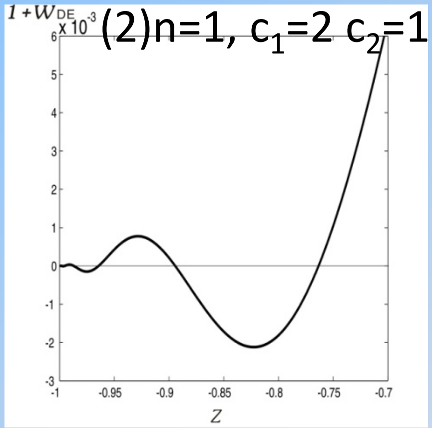


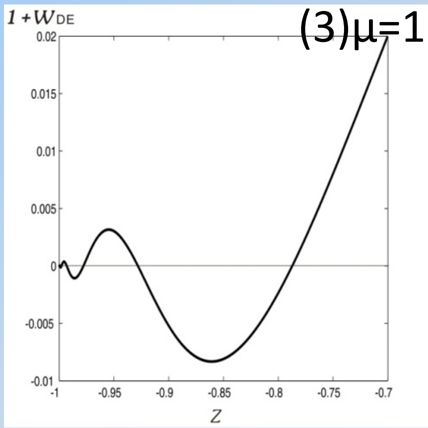


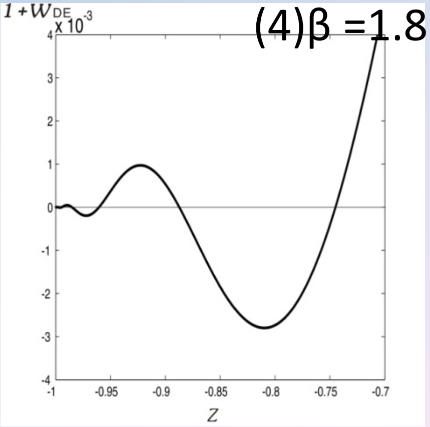


- Generic oscillation feature in EOS









Future oscillation behavior

- The oscillation of scalaron in de-Sitter stage
  (H. Motohashi A. A. Starobinsky and J. Yokoyama JCAP 1106,006)

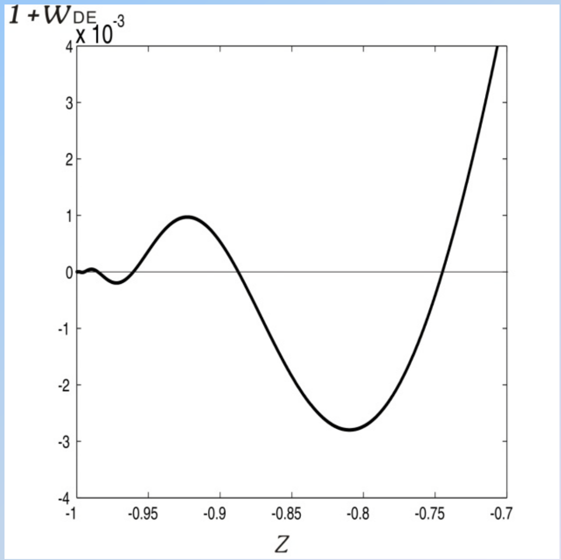
  The de-Sitter universe appears when $2f = Rf_R$ and ρ_m=0 in $f(R)$ gravity

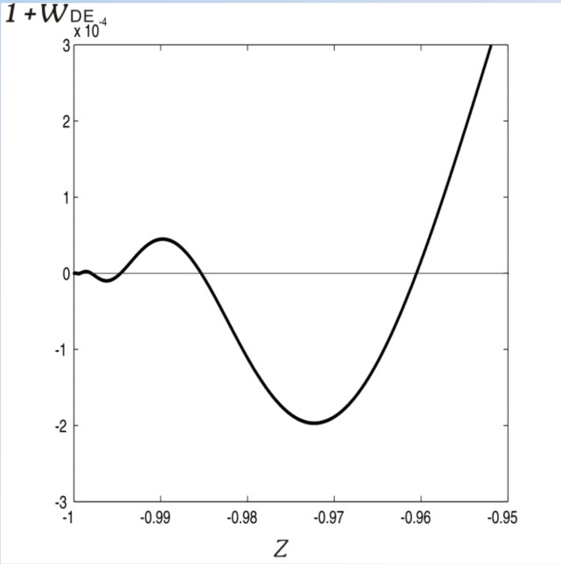
  $$\left(\partial_0^2 + 3H\partial_0\right)\delta R + \frac{1}{3}\left(\frac{f_R}{f_{RR}} - R\right)\delta R = 0$$

  The solution of this ODE with H=const. is $\delta R = Ae^{-3Ht/2}\cos\left(\omega_1 t + \phi\right) = Ae^{-3N/2}\cos\left(\omega_2 N + \phi\right)$

  where
  $$\begin{cases} \omega_1 = \left[\frac{1}{3}\left(\frac{f_R}{f_{RR}} - R\right) - \frac{9}{4}H^2\right]^{1/2} \\ \omega_2 = 2\left(\frac{f_R}{Rf_{RR}} - \frac{25}{16}\right)^{1/2} \\ a = e^{Ht} \Rightarrow N = \ln a = Ht \end{cases}$$

(4)Exponential model with β =1.8





Thank you for your attention