Beyond δN-formalism

Resceu, University of Tokyo

Yuichi Takamizu

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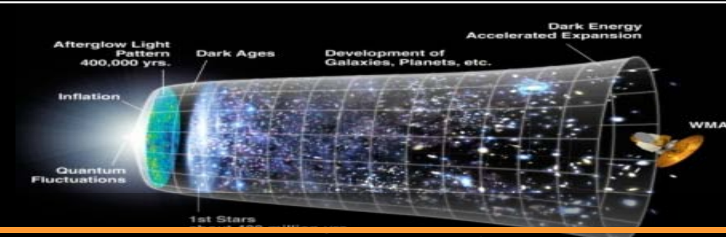
Collaborator: Shinji Mukohyama (IPMU,U of Tokyo),

Misao Sasaki & Yoshiharu Tanaka (YITP,Kyoto U)

Ref: *JCAP06 019 (2010) & JCAP01 013 (2009)*

◆ Introduction



● **Non-Gaussianity from *inflation***

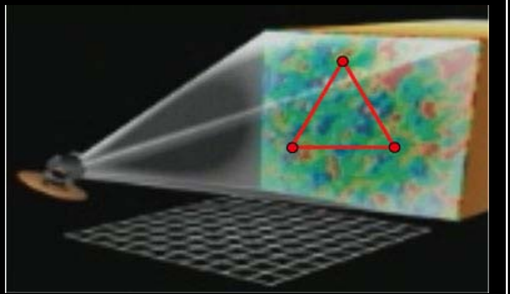
WMAP 7-year		$-10 < f_{\rm NL}^{\rm local} < 74$
PLANK	2009-	$\lvert f_{\rm NL}\rvert \gtrsim 5$

Slow-roll ? Single field ? Canonical kinetic ?

- Standard single slow-roll scalar $f_{\rm NL} = O(10^{-2})$
- Many models predicting Large Non-Gaussianity

  （Multi-fields, DBI inflation & Curvaton） $f_{\rm NL} \gg O(1)$

□ *Non-Gaussianity will be one of powerful tool to discriminate many possible inflationary models with the future precision observations*



● *Nonlinear perturbations on superhorizon scales*

- ➢ *Spatial gradient approach* : $\epsilon = 1/(HL)$ Salopek & Bond (90)
  - ➢ Spatial derivatives are small compared to time derivative
  - ➢ Expand Einstein eqs in terms of small parameter •, and can solve them for nonlinear perturbations iteratively

◆ ***δN formalism** (Separated universe)*

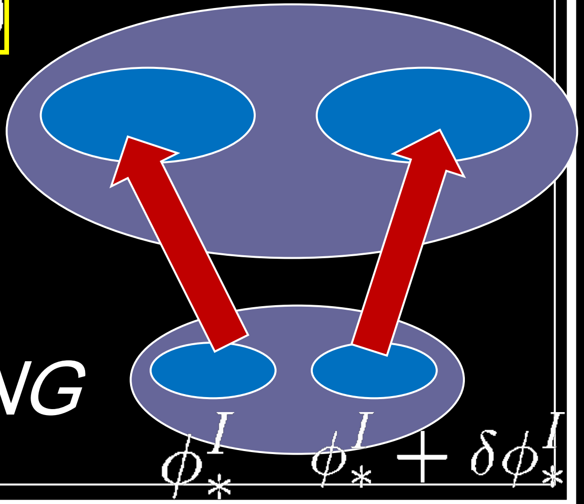
(Starobinsky 85, Sasaki & Stewart 96 Sasaki & Tanaka 98)

$$\zeta(t,\mathrm{x}) = \delta N \equiv N(t,\mathrm{x}) - N_0(t)$$

$$O(\epsilon^0)$$

$$\delta N = N^*_I \delta\phi^I_* + \frac{1}{2} N^*_{IJ} \delta\phi^I_* \delta^J_* + \cdots$$

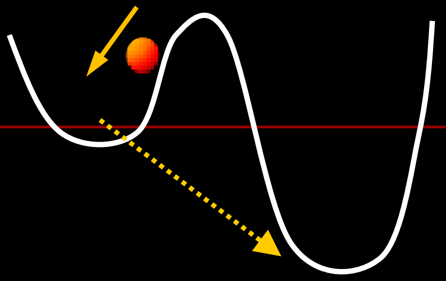
Curvature perturbation = Fluctuations of the local e-folding number



◇ *Powerful tool for the estimation of NG*

◆ *Temporary violating of slow-roll condition*

- e.g. Double inflation, False vacuum inflation
  (**Multi-field** inflation always shows)



For Single inflaton-field (this talk)

◆ *δN formalism*

$O(\epsilon^0)$ $\zeta(t,\mathbf{x}) = \text{const}$

- Ignore the **decaying mode** of curvature perturbation

◆ *Beyond δN formalism*

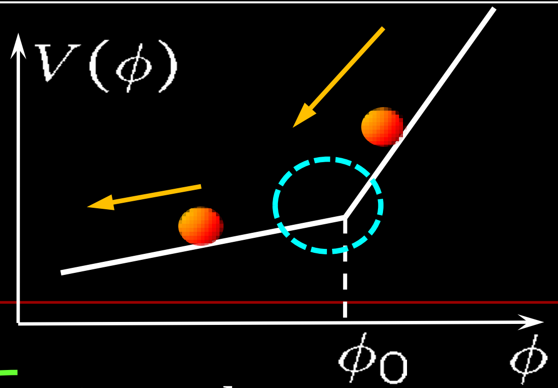
$O(\epsilon^2)$ $\zeta(t,\mathbf{x}) \neq \text{const}$

- **Decaying modes** cannot be neglected in this case
- **Enhancement** of curvature perturbation in the **linear theory**

[Seto et al (01), Leach et al (01)]

Example

- Starobinsky's model (92):
  - There is a stage at which slow-roll conditions are violated



- Leach, Sasaki, Wands & Liddle (01)
  - Linear theory
  - The $O(\epsilon^2)$ in the expansion

Initial: $u - v \ll u$ ⇒ Final: u

Growing mode decaying mode

⇒ $\mathcal{R}(0) = \alpha^{\rm Lin} \mathcal{R}(\eta_*)$

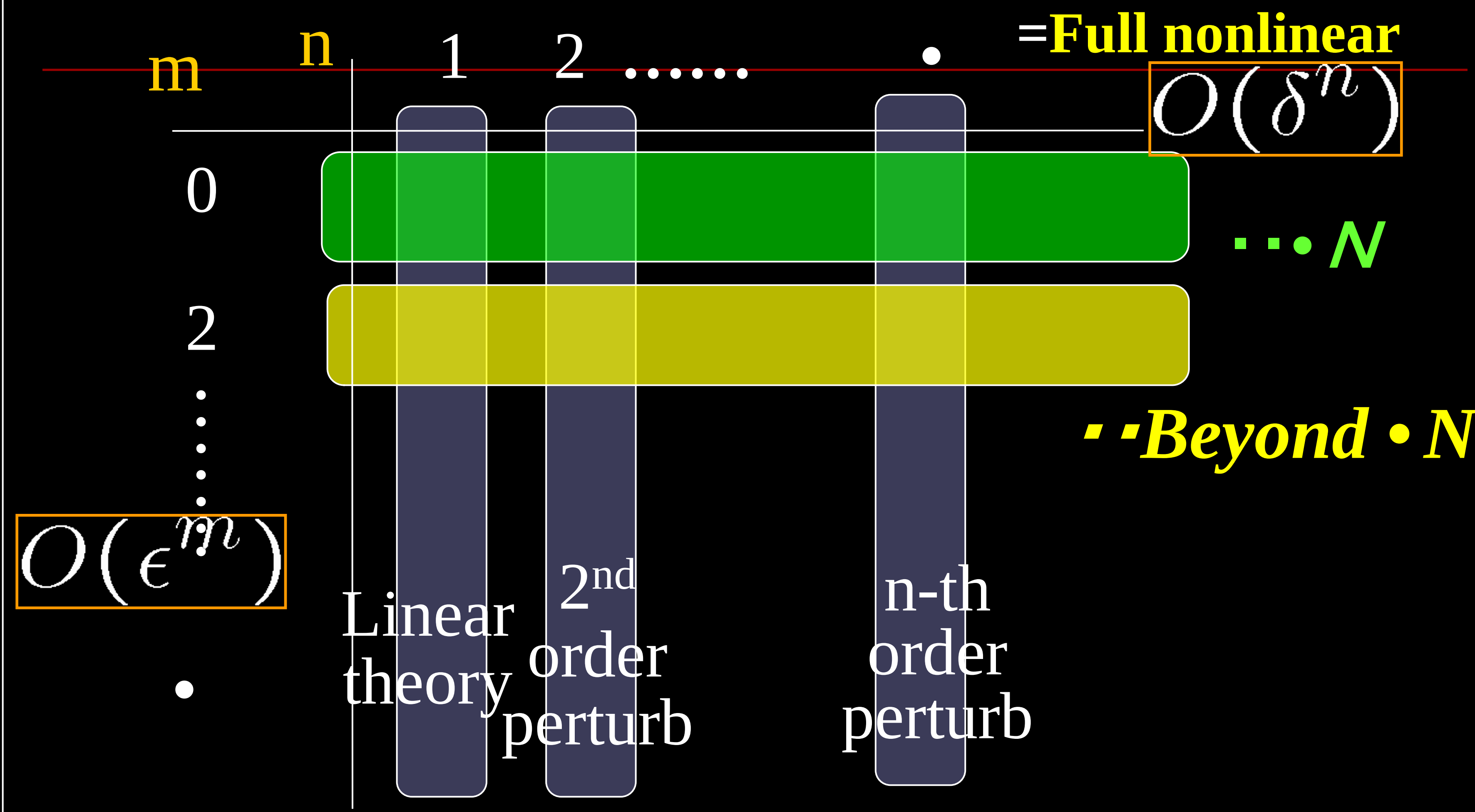
at $\eta = 0$ (Final) at η_* (initial)

$$\alpha^{\rm Lin} \simeq 1 + D(\eta_*) + F(\eta_*)$$

$O(\epsilon^2)$ Decaying mode; $O(\epsilon^2)$ Growing mode

Enhancement of curvature perturbation near $\phi = \phi_0$

● *Nonlinear perturbations on superhorizon scales up to* ***Next-leading order*** *in the expansion*



◆ Beyond • N-formalism for single scalar-field

YT, S.Mukohyama, M.Sasaki & Y.Tanaka JCAP(2010)

Simple result !

- Nonlinear theory in $O(\epsilon^2)$

$$\mathcal{R}_c^{\mathrm{NL}\,\prime\prime} + 2\frac{z'}{z}\mathcal{R}_c^{\mathrm{NL}\,\prime} + \frac{c_s^2}{4}K^{(2)}[\,\mathcal{R}_c^{\mathrm{NL}}\,] = O(\epsilon^4)$$

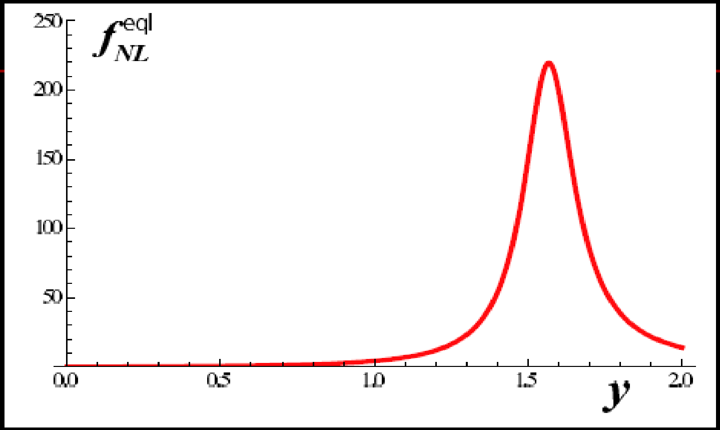
Ricci scalar of spatial metric

- Linear theory

$$\mathcal{R}_c^{\mathrm{Lin}\,\prime\prime} + 2\frac{z'}{z}\mathcal{R}_c^{\mathrm{Lin}\,\prime} + k^2 c_s^2 \mathcal{R}_c^{\mathrm{Lin}} = 0$$

◆ Application of *Beyond δN-formalism*

● To **Bispectrum** for **Starobinsky model**



$$f_{NL}^{eql} \simeq 2T$$: **Ratio of the slope of the potential**

$$y = \sqrt{T}k/k_0 \simeq 1.5$$

Even for $T = 10$

$$f_{NL} \sim 20 \text{ at } k \simeq 0.5k_0$$

● *Temporary violation of slow-rolling* leads to particular behavior (*e.g. sharp spikes*) of the power & bispectrum

→ Localized feature models

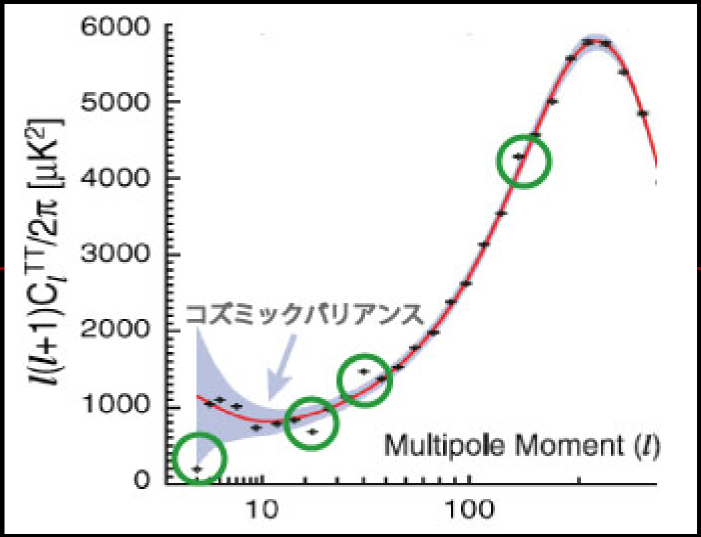
□ *These features may be one of powerful tool to discriminate many inflationary models with the future precision observations*

◆ Localized feature models

(breaking scale-invariant)

Observation (CMB spectrum)

- Ichiki et.al.(09,10) $l \sim 120$
- Cline et.al.(03) $l \sim 20 - 40$



Model building

- Inflaton rescattered by particle production (Barnaby, 09,10)
- Brane-inflation: Collision of branes (e.g. Monodoromy)
- Varying sound velocity (with Nakashima 10, with Saito 10, in progress)

Observation (CMB bispectrum)

NG shapes: Local, equilateral, ･･･

Should not assume the form of shapes

- Estimator for any shapes (Shellard et.al. 09,10)

→ Apply to NG of feature models?

(With Yokoyama-san, in progress)

◆ Beyond δN-formalism

System : $I = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G_N} + P(X,\phi)\right]$, $X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

➢ *Single scalar field with general potential & kinetic term*

including K- inflation & DBI etc

● **ADM decomposition & Gradient expansion**

Small parameter: $\epsilon = 1/(HL)$ $\partial_i\psi = \psi \times O(\epsilon)$

◆ **Background is the flat FLRW universe**

Basic assumption:

$\beta^i = O(\epsilon),\quad v^i = O(\epsilon),\quad \partial_t\tilde{\gamma}_{ij} = O(\epsilon)$ ⇒ $\partial_t\tilde{\gamma}_{ij} = O(\epsilon^2)$

➢ Absence of any decaying at leading order

➢ Can be justified in most inflationary model with $N \gg 60$

Solve the Einstein equation after ADM decomposition

In the gauge as

Uniform Hubble slicing

$$K=-3H(t),\quad H(t)\equiv\frac{\partial_t a}{a}.$$

Trace of Extrinsic curvature

+ Time-orthogonal $\beta^i=O(\epsilon^3)$ Shift

● General solution

valid up to $O(\epsilon^2)$

YT& Mukohyama, JCAP 01(2009)

● *Curvature perturbation*

Spatial metric

$$\gamma_{ij}=a^2\psi^4\tilde{\gamma}_{ij};\quad \det(\tilde{\gamma}_{ij})=1$$

$$\psi\simeq(\mathrm{const})+\ (\mathrm{time-dep})+O(\epsilon^4),$$

Constant (δ N)

$O(\epsilon^2)$

$$\delta P=c_s^2\delta\rho+\rho\Gamma\delta\phi,$$

- Variation of Pressure (speed of sound etc)
- Scalar decaying mode

◆ Nonlinear curvature perturbation

YT, Mukohyama, Sasaki & Tanaka , JCAP06 (2010)

Complete gauge fixing $\tilde{\gamma}_{ij} \to \delta_{ij} \, (t \to \infty)$

Focus on only **Scalar-type** mode

Define nonlinear curvature perturbation $\psi^4 = e^{2\zeta}$

Perturbation of the e-folding (• N)

Uniform Hubble & time-orthogonal gauge → **Gauge transformation** → Comoving & time-orthogonal gauge

● ***What is a suitable definition of NL curvature perturbation ?***

$$\mathcal{R}^{\rm NL} \equiv \zeta + \frac{E}{3}$$

$$\gamma_{ij} = a^2 \psi^4 \tilde{\gamma}_{ij};$$

?

$$E \equiv \frac{3}{4} \Delta^{-1} \left[\partial^i \psi^{-6} \partial^j \psi^6 (\ln \tilde{\gamma})_{ij} \right]$$

Need this term in the formula in $O(\epsilon^2)$

Cf) δN-formalism $\mathcal{R}^{\rm Lin} = \left(H_L^{\rm Lin} + \frac{H_T^{\rm Lin}}{3} \right) = \zeta_s \, \delta N$

◆ Second-order differential equation

YT, Mukohyama, Sasaki & Tanaka (10)

$$\mathcal{R}_c^{\rm NL}(\eta) = \ell^{(0)} + \ell^{(2)} + \frac{1}{4}\left[F(\eta) - F_*\right]K^{(2)} + \left[D(\eta) - D_*\right]C^{(2)}$$

Constant (δ N) η_* (initial) Some integrals Scalar decaying mode

Decaying mode $D'' + 2\frac{z'}{z}D' = 0$ $O(\epsilon^2)$

Growing mode $F'' + 2\frac{z'}{z}F' + c_s^2 = 0$ $O(\epsilon^2)$

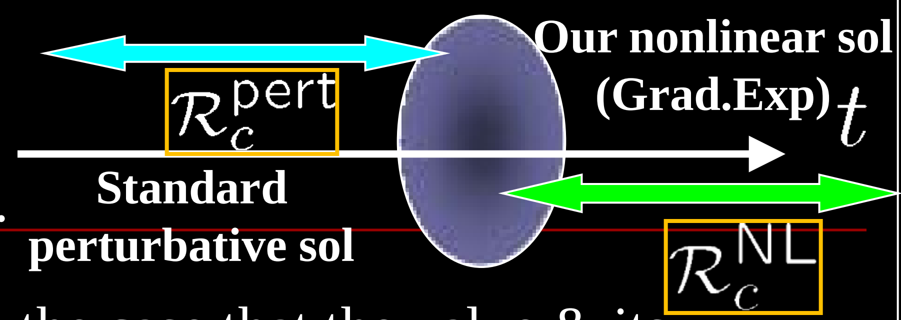
satisfies
$$\mathcal{R}_c^{\rm NL}{}'' + 2\frac{z'}{z}\mathcal{R}_c^{\rm NL}{}' + \frac{c_s^2}{4}K^{(2)}[\mathcal{R}_c^{\rm NL}] = O(\epsilon^4)$$

Ricci scalar of 0th spatial metric

- *Natural extension of well-known linear version*

$$\mathcal{R}_c^{\rm Lin}{}'' + 2\frac{z'}{z}\mathcal{R}_c^{\rm Lin}{}' - c_s^2\,\Delta[\mathcal{R}_c^{\rm Lin}] = 0$$

◆ Matching conditions



In order to determine the initial cond.

General formulation: Valid in the case that the value & its deri. of the *N-th order perturb. Sol are given*

◆ Matched Nonlinear sol to linear sol

Approximate Linear sol around horizon crossing @ $\eta = \eta_*$

● Final result

$$\mathcal{R}_c^{\mathrm{NL}}(0) = u^{(0)} - (1 - \alpha^{\mathrm{Lin}})u^{(0)} - F_k \left[\Delta u^{(0)} + \frac{K^{(2)}[u^{(0)}]}{4} \right] + O(\epsilon^4) .$$

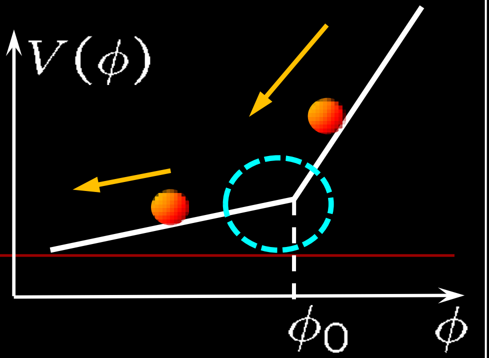
• N

Enhancement in Linear theory

Nonlinear effect

Application to Starobinsky model

$$V(\phi) = \begin{cases} V_0 + A_+(\phi - \phi_0) & \text{for } \phi > \phi_0 \\ V_0 + A_-(\phi - \phi_0) & \text{for } \phi < \phi_0 \end{cases}$$

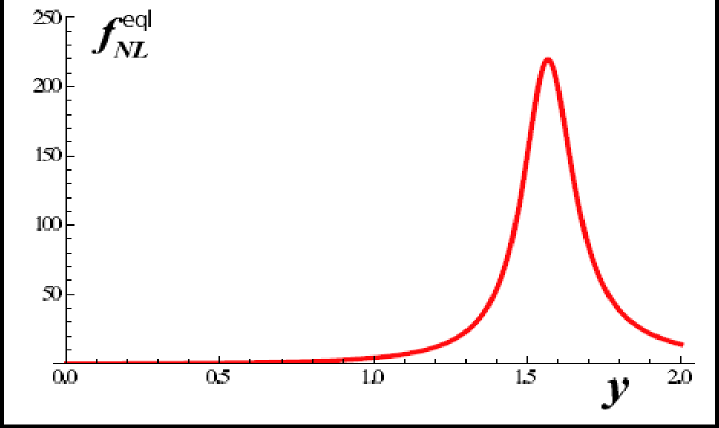


- There is a stage at which slow-roll conditions are violated

- In Fourier space, calculate Bispectrum

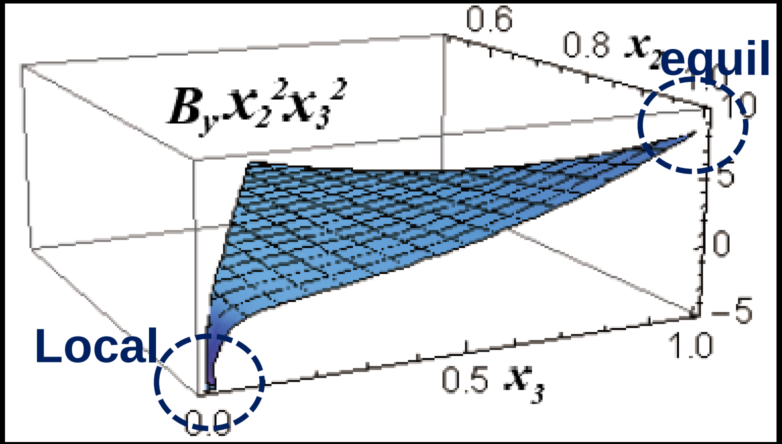
$$T = \left(\frac{A_+}{A_-} - 1\right) = 10^2$$

- Equilateral $k = k_1 = k_2 = k_3$



$$f_{NL}^{eql} \simeq 2T \quad y = \sqrt{T}k/k_0 \simeq 1.5$$

- Shape of bispectrum



$$y = 10$$

◆ Summary

- We develop a theory of non-linear cosmological perturbations on superhorizon scales for a scalar field with a general potential & kinetic terms

- We employ the **ADM formalism** and the spatial **gradient expansion** approach to obtain general solutions valid up through second-order $O(\epsilon^2)$

- This Formulation can be applied to k-inflation and DBI inflation to investigate superhorizon evolution of non-Gaussianity **beyond • N-formalism**

$$\mathcal{R}^{\mathrm{NL}} \equiv \zeta + \frac{E}{3}$$

- Show the **simple 2nd order diff eq** for **nonlinear variable:**

- We formulate a general method to match the n-th order perturbative sol

- Can applied to Non-Gaussianity *in temporary violating of slow-rolling*

- **Calculate the bispectrum** for the Starobinsky model

◆ *Future work*

- Can applied to Non-Gaussianity on superhorizon scales matched to Non-Guussianity generated by DBI inflation on subhorizon scales
- Extension to the models of multi-scalar field (with Naruko,in progress)

(naturally gives temporary violating of slow-roll cond)

- Extension to nonlinear Gravitational waves
- To Trispectrum of the feature models
- Nonlinear 2nd order differential eq can be applied to the case of reentering the horizon ?