The impact of massive neutrino on spherical collapse model

(This work is still in progress. Any comments are welcome!)

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Neutrinos mass!

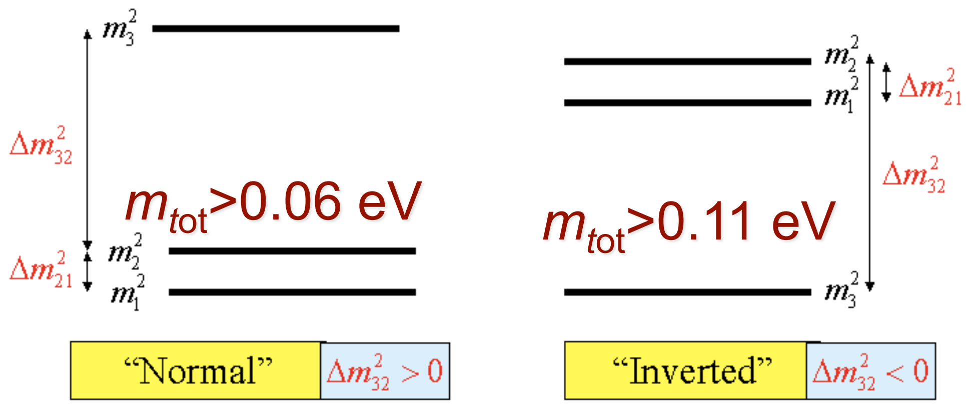
- The experiments (Kamiokande, SK, SNO, KamLAND) imply the total mass, m_tot>0.06 eV; but the mass scale yet unknown
- Neutrinos became non-relativistic at redshift when $T_{\nu,\mathrm{dec}} \sim m_\nu$

$$1 + z_{\mathrm{nr}} \approx 189\left(m_\nu / 0.1\mathrm{eV}\right)$$

  - If *m_nu>0.6eV*, the neutrino became non-relativistic before recombination, therefore larger effect on CMB, vice versa
- The cosmological probes measure the total matter density: CDM + baryon + massive neutrinos

$$\Omega_{\mathrm{m0}} = \Omega_{\mathrm{cdm0}} + \Omega_{\mathrm{baryon0}} + \Omega_{\nu 0}$$

$$f_\nu \equiv \frac{\Omega_{\nu 0}}{\Omega_{\mathrm{m0}}} = \frac{m_{\nu,\mathrm{tot}}}{94.1\mathrm{eV}\Omega_m h^2} > 0.005$$



Suppression in growth of LSS

- A mixed DM model: Structure formation is induced by the density fluctuations of total matter

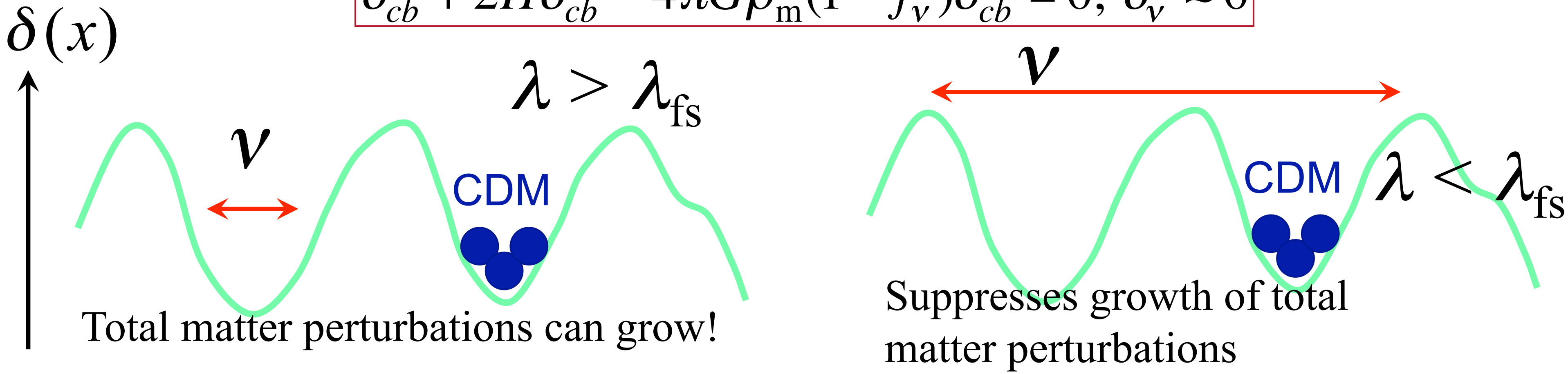
$$\delta_m = \frac{\bar{\rho}_c\delta_c + \bar{\rho}_b\delta_b + \bar{\rho}_\nu\delta_\nu}{\bar{\rho}_c + \bar{\rho}_b + \bar{\rho}_\nu} \equiv f_c\delta_c + f_b\delta_b + f_\nu\delta_\nu$$

- The neutrinos slow down LSS on small scales
  - On large scales $\lambda > \lambda_{fs}$, the neutrinos can grow together with CDM

$$\delta_c = \delta_b = \delta_\nu$$

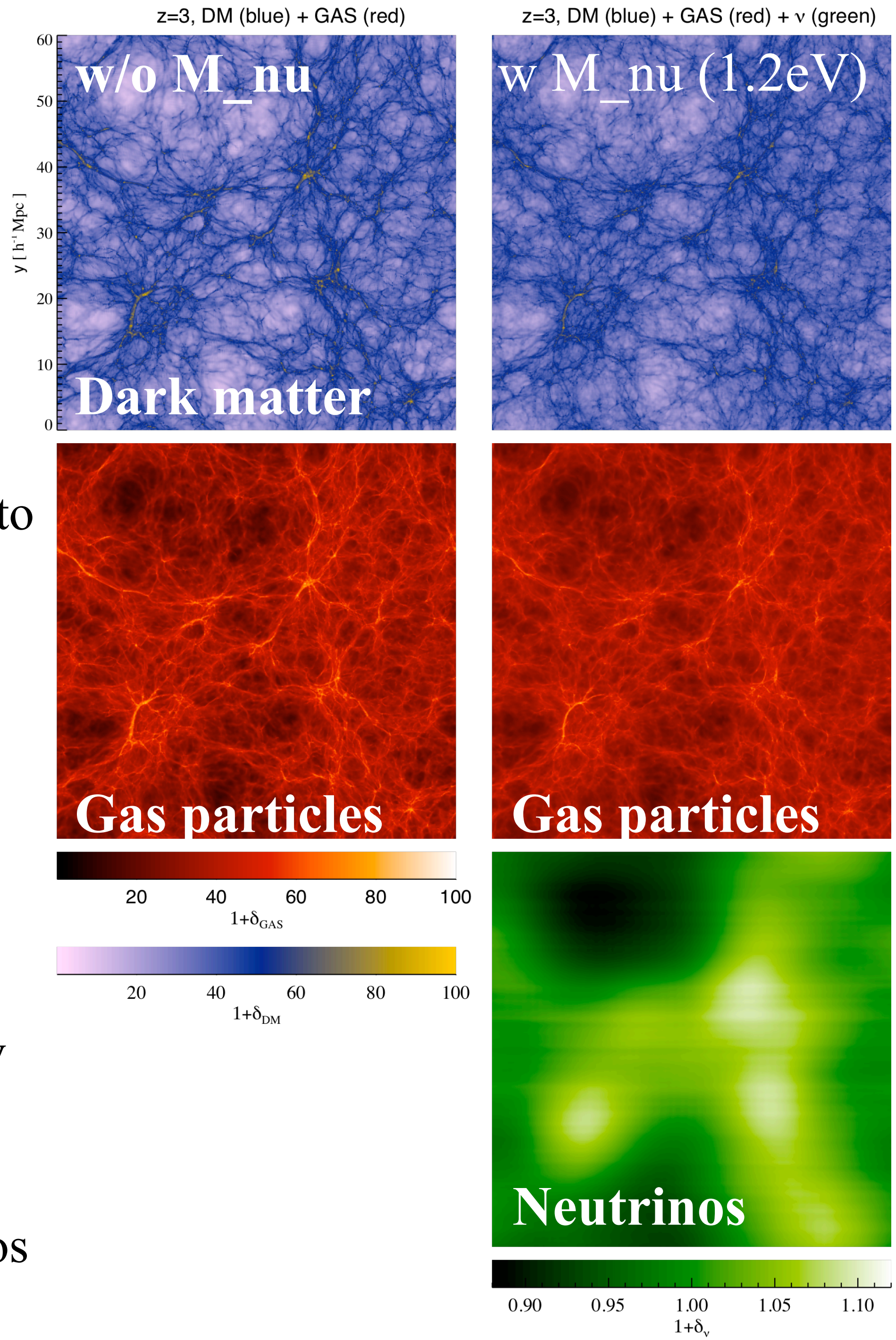
  - On small scales $\lambda < \lambda_{fs}$, the neutrinos are smooth, $\delta_\nu = 0$, therefore weaker gravitational force compared to a pure CDM case

$$\ddot{\delta}_{cb} + 2H\dot{\delta}_{cb} - 4\pi G\bar{\rho}_{\mathrm{m}}(1 - f_\nu)\delta_{cb} = 0,\ \delta_\nu \approx 0$$



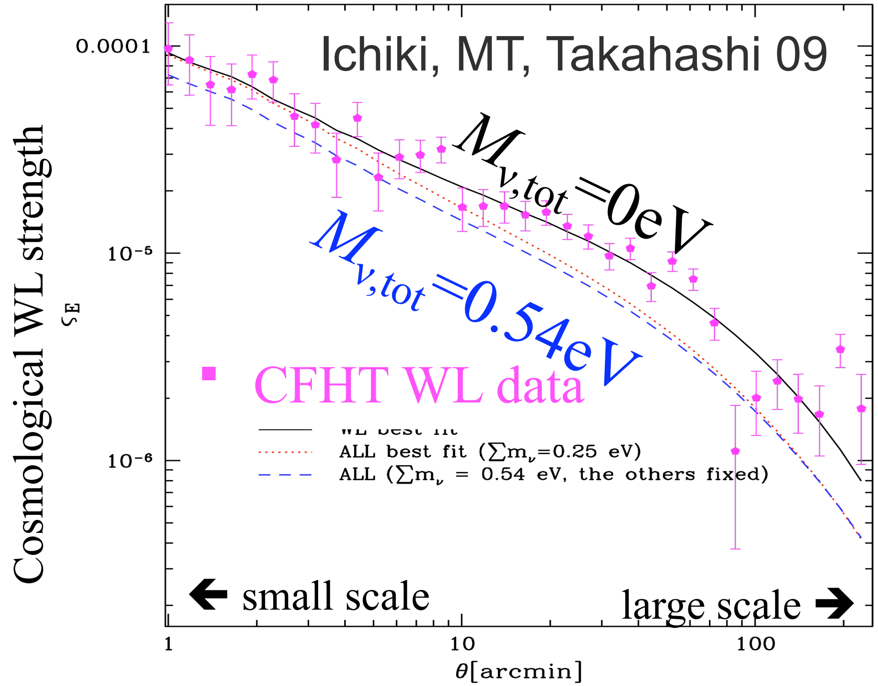
Modeling SF in a mixed DM model

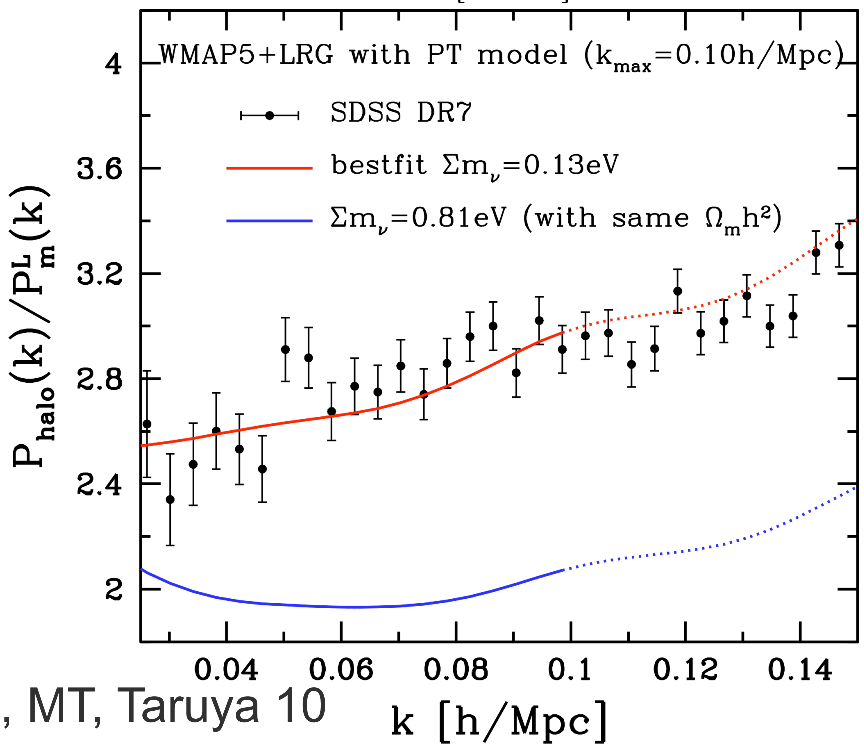
- Need to include the effect of massive neutrinos to interpret the high-precision cosmological data
- Analytical attempts
  - Based on the perturbation theory (Sato et al. 08, 09; Shoji & Komatsu 09; Swanson et a. 10)
  - Only applicable to the weakly NL regime
  - Used to obtain the upper limit: M_nu<0.6 eV (95% C.L.)
- Simulation attempts
  - Several groups have started the study (Brandbyge & Hannestad 08; Viel, Haehnelt, & Springel 10)
  - Still very difficult to include neutrinos with masses <1 eV



Cosmological upper limits on M_ν

- When combined with CMB information, large-scale structure probes are very powerful to constrain the neutrino mass
  - Weak lensing (Ichiki, MT, Takahashi 09): M_nu<0.54 eV (95% C.L) for WMAP+WL+SN +BAO
  - Galaxy clustering (e.g. Saito et al. 10): M_nu<0.81 eV for WMAP + SDSS LRG (including the DE equation of state w0)





Saito, MT, Taruya 10

Vikhlinin et al. 2009: Chandra

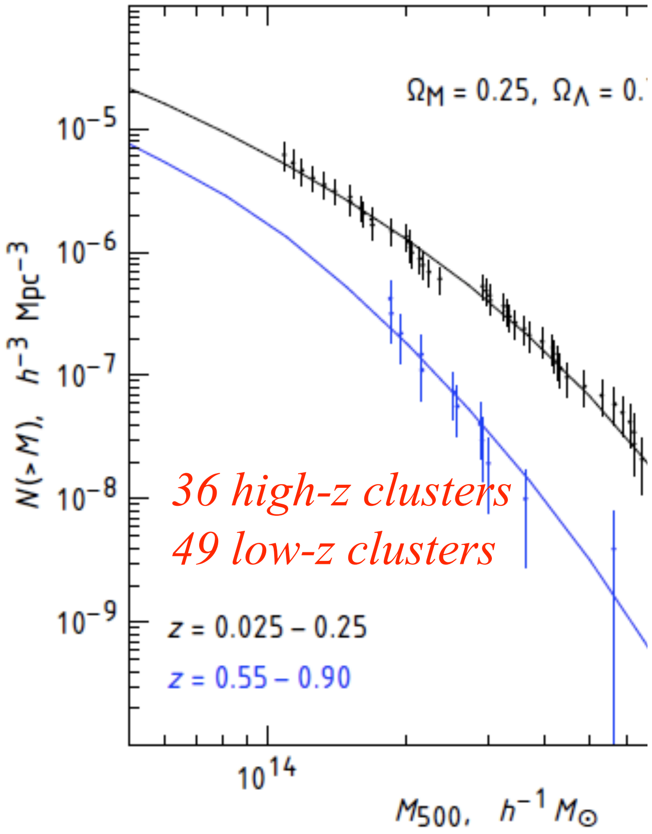
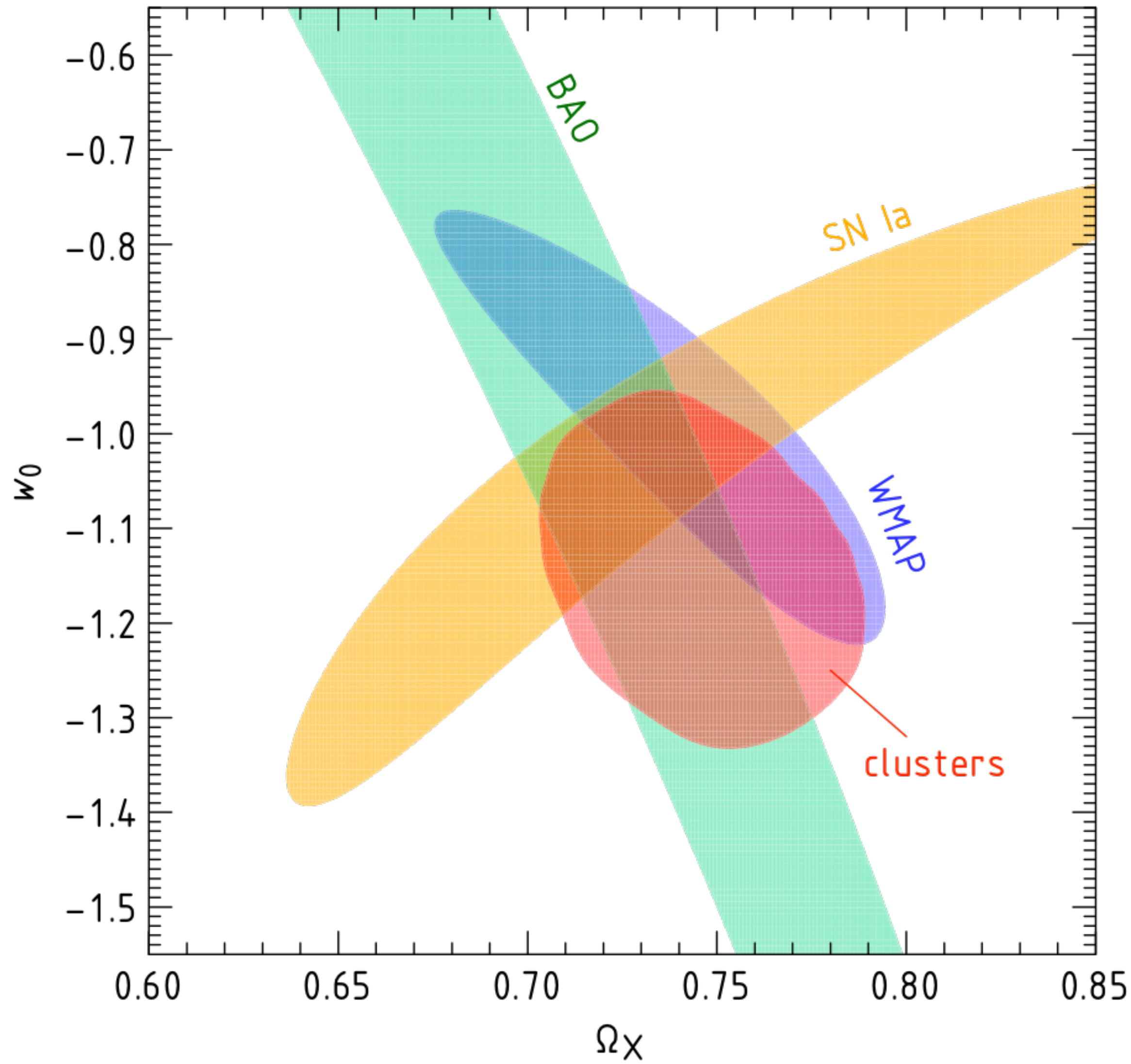
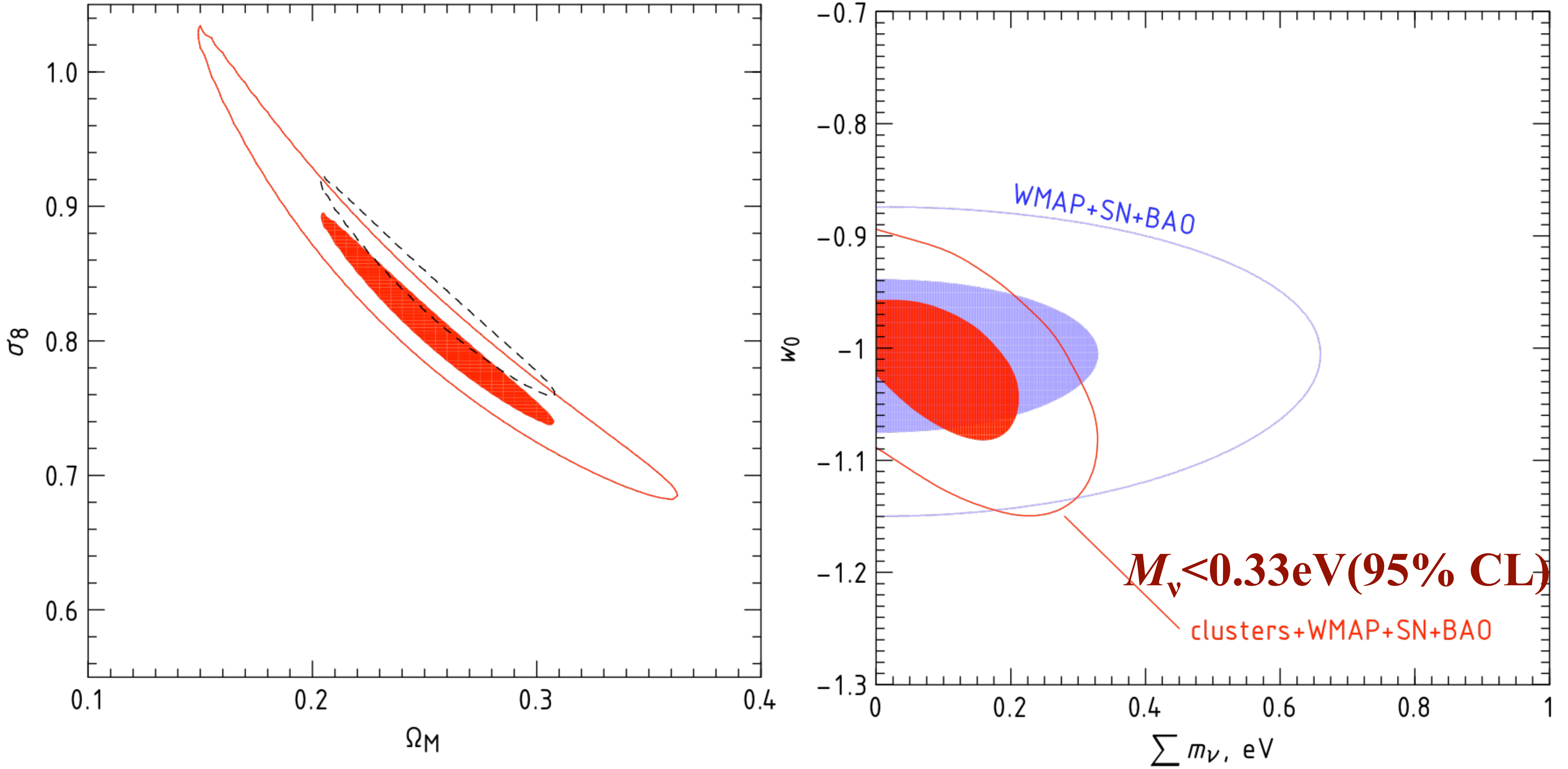


FIG. 2.— Illustration of sensitivity of the cluster mass fun
models (with only the overall normalization at $z = 0$ adjuste
from Fig. 1, which for the high-z cluster we show only the r
models are computed for a cosmology with $\Omega_\Lambda = 0$. Both t
function is changed because it is derived for a different dist
thresholds corresponding to $\Delta_{crit} = 500$ are different. Whe
of $z > 0.55$ clusters is in strong disagreement with the data



M_500 estimated from Ch

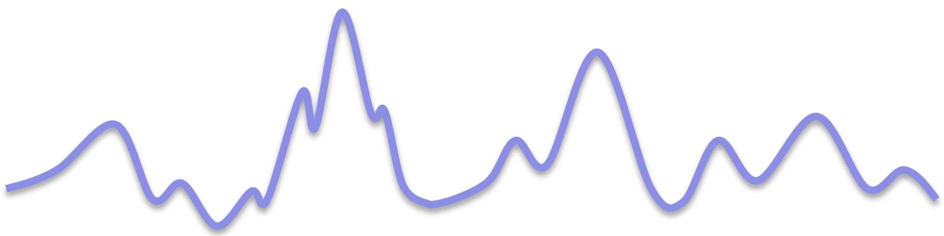
Vikhlinin et al. 09



- Note: the neutrino mass constraints are translated from the constraint on σ_8 (the cluster counts $\rightarrow \sigma_8 \rightarrow$ M_nu)
- The CDM-based prediction of mass function, i.e. w/o neutrinos, was used to obtain the constraint on σ_8

Cosmological Use of Clusters: Halo Mass Function

Tiny density fluctuations at z~1000: δ_m~10^-3

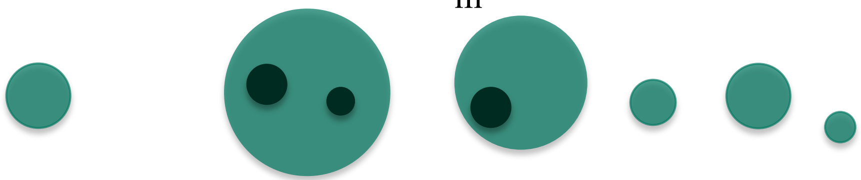


Gravitational instability

(gravity ⇔ cosmic expansion)

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\bar{\rho}_m\delta_m = 0$$

Halo formation at z~0: δ_m>>1



Gaussian seed density fluctuations

+

Spherical collapse model (or N-body simulation)

Mass function:

$$\frac{dn}{dM} \propto \exp\left(-\frac{\delta_c^2}{2\sigma^2(M)}\right)$$

@cluster mass scales

The mass function can be a powerful probe of cosmology (e.g. DE)

This work

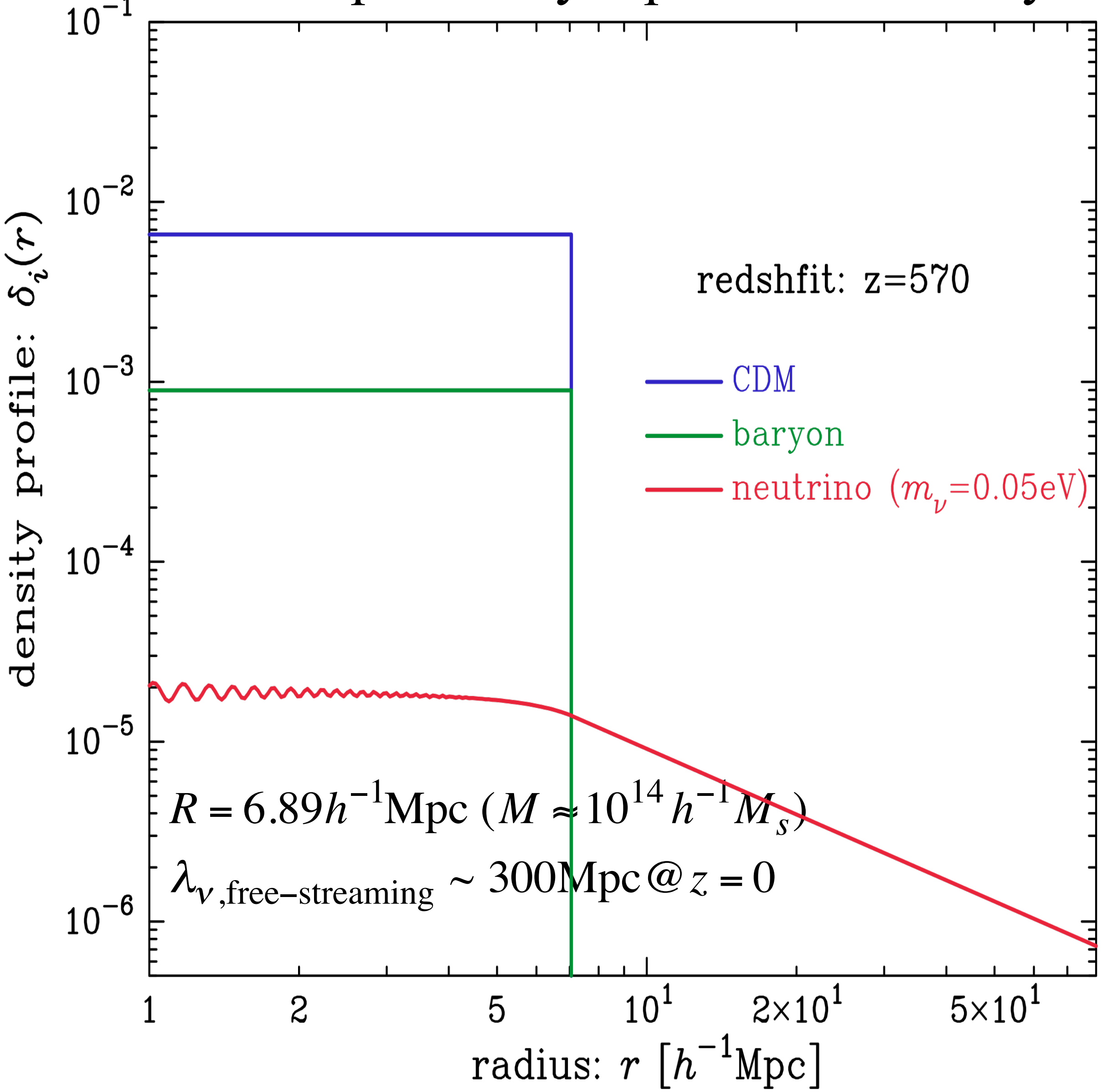
- Study the impact of massive neutrinos on nonlinear structure formation
- As the first step, study a spherical, top-hat collapse model in a mixed dark matter model
  - Enable to solve the nonlinear evolution analytically
  - Include all the components (photons, baryons, neutrinos, CDM)
- By plugging the spherical collapse model in the model mass function, we can estimate the impact of massive neutrinos on the abundance of massive clusters

Towards the spherical collapse model - the initial conditions -

- The initial conditions of structure formation are now well constrained by WMAP (z~1100) (in combination with linear perturbation theory)
- Need to know the different initial conditions on the density fields for different components (photon, CDM, baryon, neutrinos)
- These physics also depend on the scale of neutrino mass and the length scale of density fluctuations

The initial condition (contd.)

Consider a spherically top-hat overdensity region of CDM perturbation



- The different amplitudes of different components
- CDM, baryon: cold components
  - Note: for baryon, we just plot the region for the top-hat region
- Neutrino (0.05 eV, the lower limit of the NMH) has smaller amplitude, and is extended beyond the top-hat region

Equations

- CDM and baryons: the time-differential equation for the radius of the top-hat region

$$\frac{\ddot{R}_i(t)}{R_i(t)} = -\frac{4\pi G}{3}\left[\bar{\rho}_{\rm tot}(t) + \bar{P}_{\rm tot}(t)\right] - \frac{G\delta M_{\rm tot}(< R_i)}{R_i} \quad ; i = \text{CDM or baryon}$$

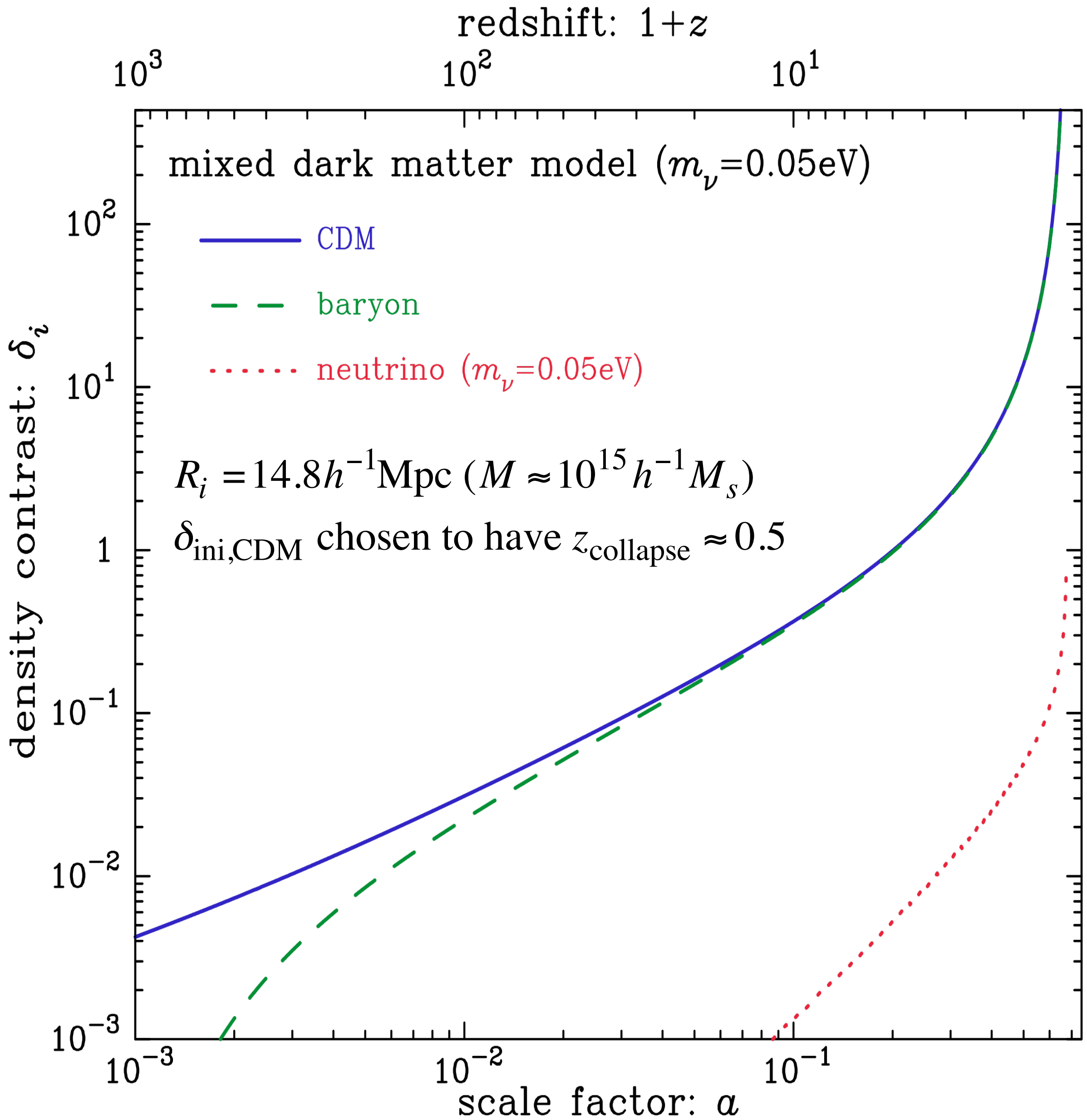
Note: the initial shell velocity is different for CDM and baryon

- The neutrino perturbations: solving the Boltzmann equation hierarchies (used the modified CAMB code)

$$\begin{cases} \dfrac{df_\nu}{dt} = \dfrac{\partial f_\nu}{\partial t} + \dfrac{\hat{p}_i}{a}\dfrac{\partial f_\nu}{\partial x^i} - p\dfrac{\partial f_\nu}{\partial p}\left[H + \dfrac{\partial \Phi}{\partial t} - \dfrac{\hat{p}_i}{a}\dfrac{\partial \Phi}{\partial x^i}\right] = 0 \\ k^2\phi = 4\pi G a^2\left[\underline{\bar{\rho}_{cb}\delta_{cb}^{NL}} + \bar{\rho}_\nu \delta_\nu\right] \end{cases}$$

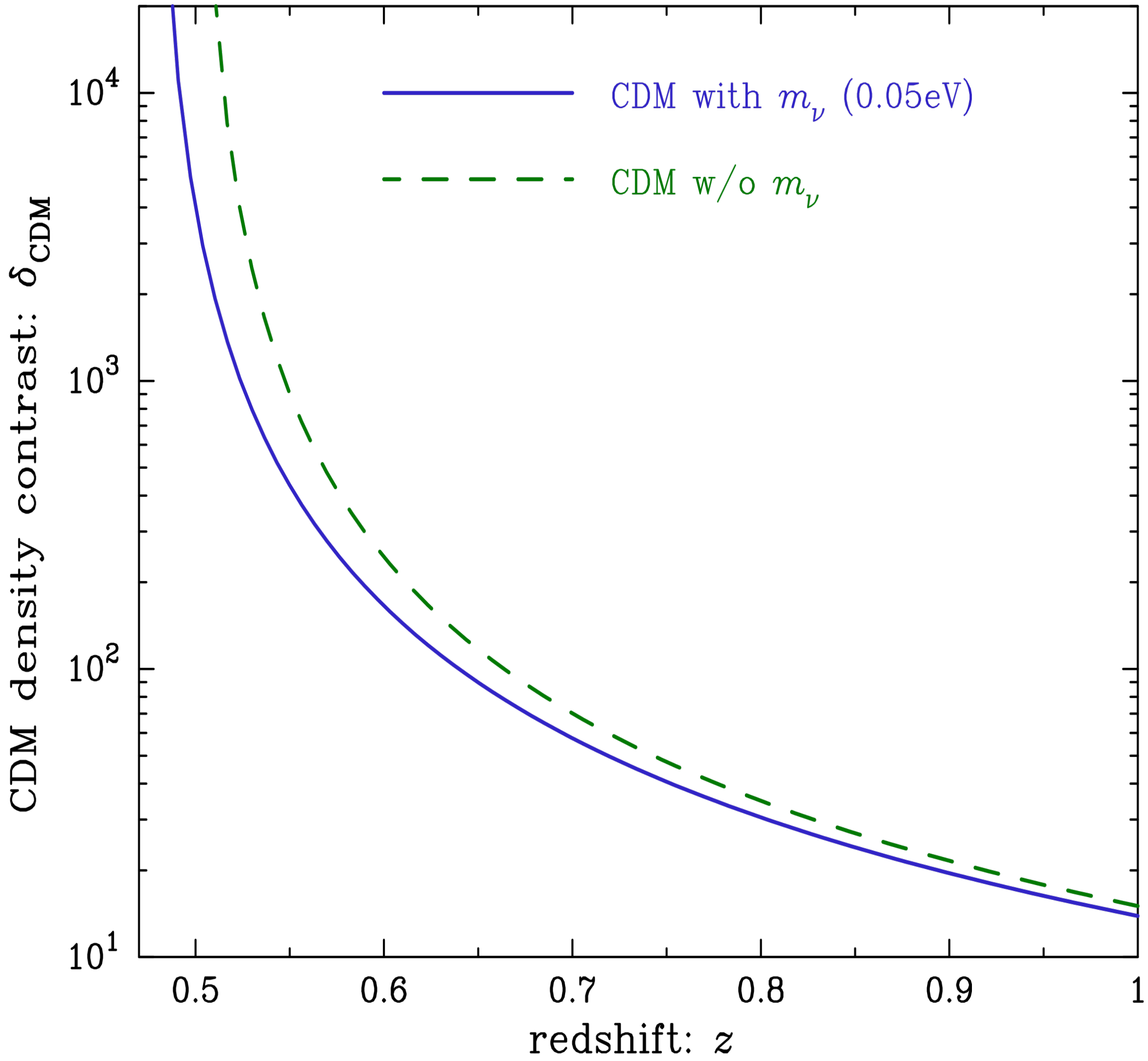
$$\Rightarrow \quad \bar{\rho}_\nu\left[1 + \delta_\nu\right] \sim \int d\boldsymbol{p}^3 \sqrt{p^2 + m_\nu^2}\, f_\nu(p) \rightarrow \delta M_\nu(< R)$$

Results



- Baryon can catch up with the CDM overdensity at low redshifts
  - Note that, for halos at much earlier collapse time (e.g. first stars), baryon can't catch up (Naoz & Barkana 05)
- Hence the CDM and baryon can collapse
- Neutrinos can't catch up
  - The neutrino overdensity is still in the regime, δ_ν<1 even at the collapse redshift
  - This is also true for M_nu~0.1eV, the lower limit of IMH

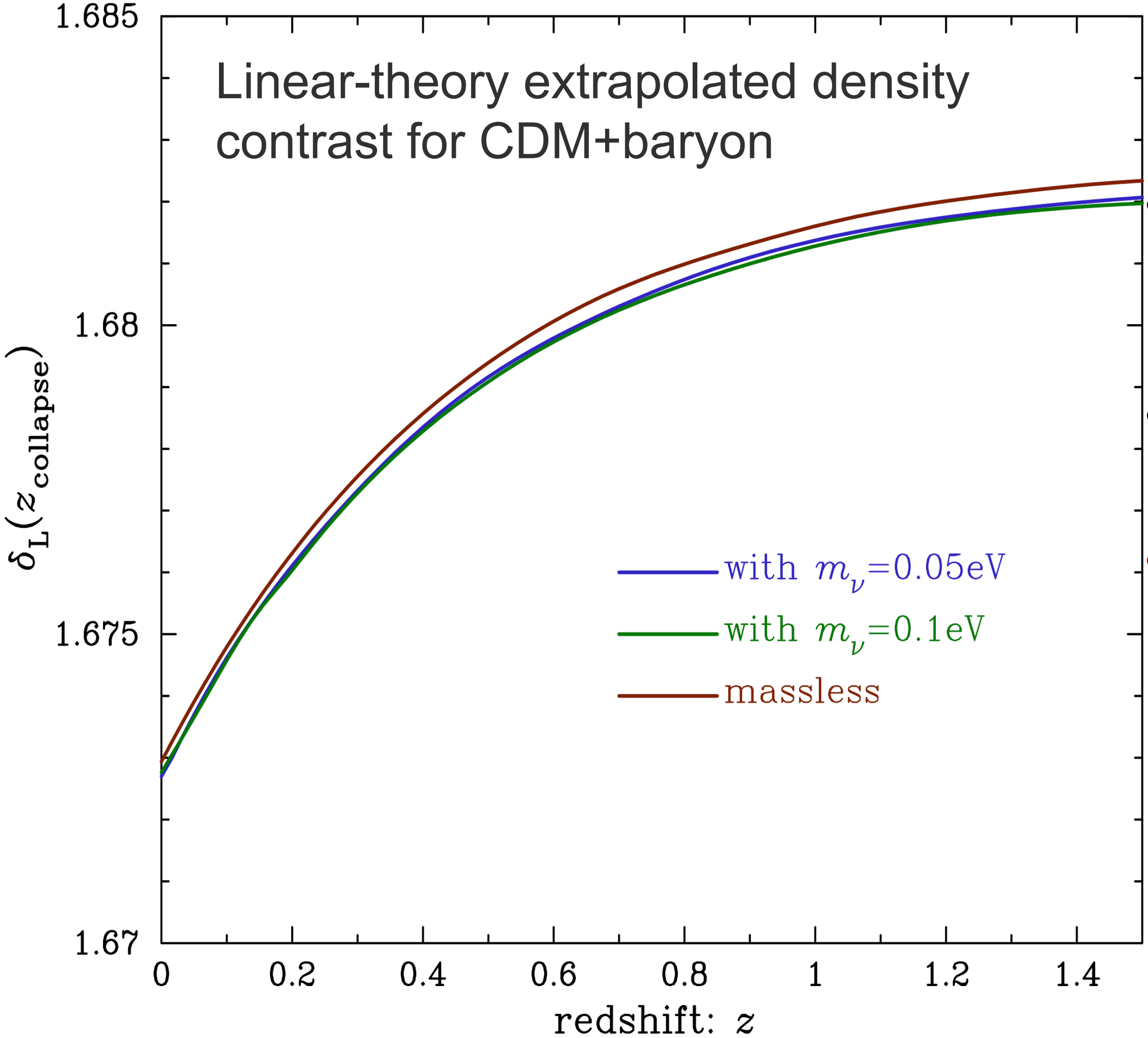
Result: M_ν vs. massless ν



- The same initial overdensity of CDM perturbation for the two cases with and w/o massive neutrinos
- The presence of massive neutrino delays the collapse: z_c≈0.51 ➔ 0.49

The collapsing time

- The linear-theory extrapolated density contrast can be used to know the collapsing redshift : e.g. $\delta_m^L(z_c) \approx 1.68$ for ΛCDM model



- δ_c(z) is not largely changed from the model (w/o massive neutrinos)
- The effect of massive neutrino is ~0.1% in δ_c
- The delay in the collapsing time is mostly captured by the linear growth rate

The impact on the abundance of massive clusters

- The abundance of massive clusters is well modeled by the halo mass function at the exponential tail
- The halo mass function is given in terms of the peak height (e.g. Press & Schechter 74)

$$\frac{dn}{dM}(M,z) = \frac{\bar{\rho}_m}{M} f\left(\nu \equiv \frac{\delta_c(z)}{\sigma(M,z)}\right)\frac{d\nu}{dM}$$

$f(x)$: the fitting function calibrated by simulations

$\bar{\rho}_m$: the mean mass density of collapsing matter

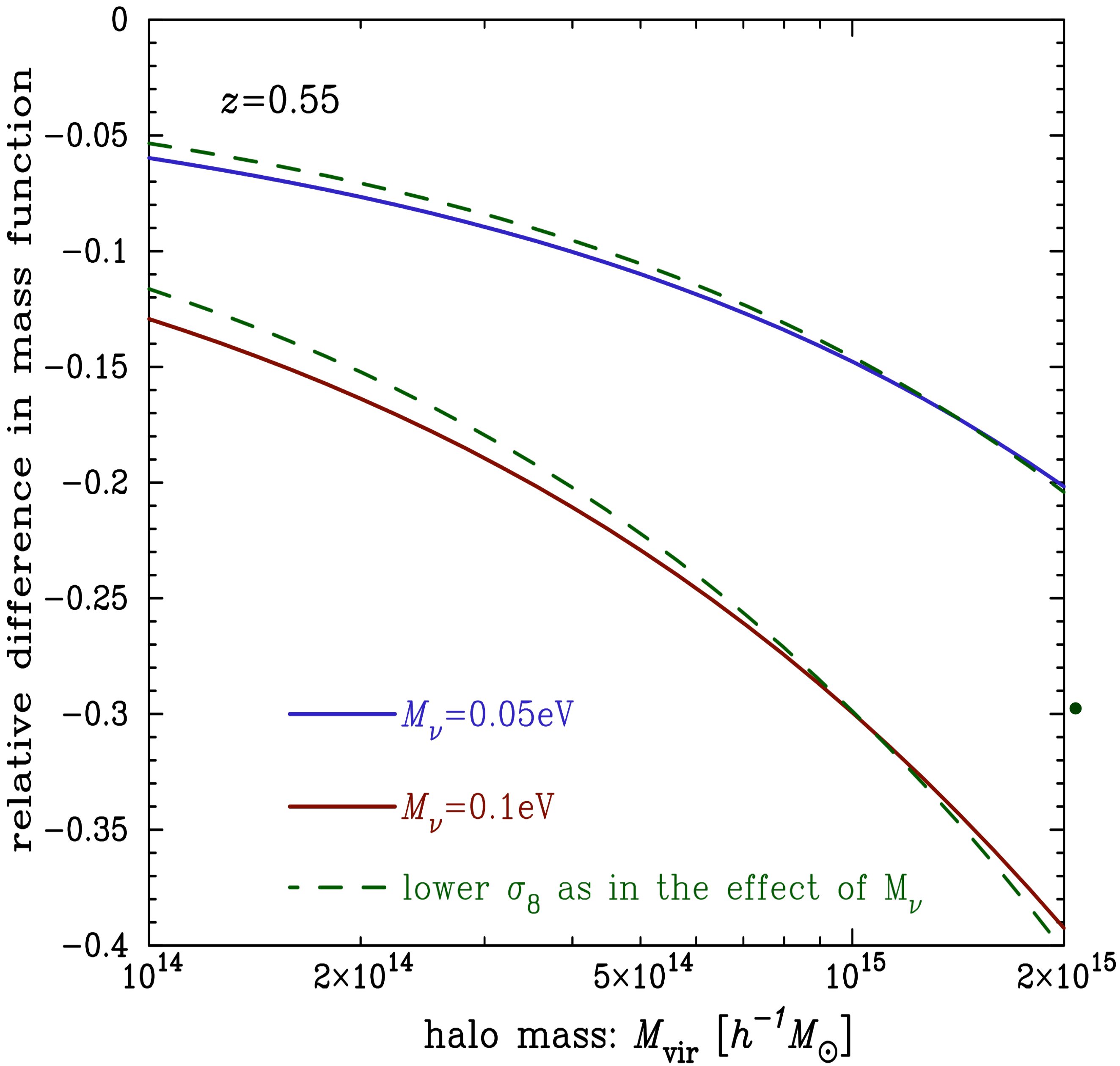
$\sigma(M,z)$: the rms mass fluctuations of the halo mass scale M

- Our results imply that the halo mass function for a mixed dark matter can be estimated as

$$\left.\frac{dn}{dM}\right|_{m_\nu \neq 0} = \frac{\bar{\rho}_{\rm c+b}}{M} f\left(\nu \equiv \frac{\delta_c(z)}{\sigma_{\rm c+b}(M,z)}\right)\frac{d\nu}{dM}$$

$\bar{\rho}_{\rm c+b}, \sigma_{\rm c+b}(M,z)$: the quantities of CDM + baryon (w/o massive neutrinos)

Result



- The presence of massive neutrino decreases the abundance of massive halos
  - Normal Mass Hierarchy (>0.05eV): the decrease is more than 15% for M~10^15Msun
  - Inverted Mass Hierarchy (>0.1eV): >30%
  - Note: the effect on σ_8 is >4% or 8%.
- *The neutrino effect can be mimicked by the model w/o massive neutrino, but with lower σ_8*

Summary

- Developed a spherical-tophat collapse model for a mixed dark matter model (CDM + massive neutrino)
  - Included the proper initial conditions: different amplitudes in the initial density amplitudes for different components (CDM, baryon, neutrino)
  - Solved the nonlinear spherical top-hat model for the cold component (CDM, baryon)
  - Solved the Boltzmann equation hierarchies for massive neutrino
- The presence of massive neutrino delays the collapse of CDM overdensity region
  - CDM + baryon can collapse for cluster-scale halos
  - Neutrinos can't catch up
  - This effect is well captured by the linear growth rate
- The abundance of massive halos is decreased:
  - Normal mass hierarchy (>0.05eV): >15% for M~10^15Msun
  - Inverted mass hierarchy (>0.1eV): >30% for M~10^15Msun
  - The effect can be absorbed by the lowered-σ_8 model w/o M_nu
- The larger effect would be expected for high-z halos (like first stars)