Parametric Amplification of primordial fluctuations during inflation

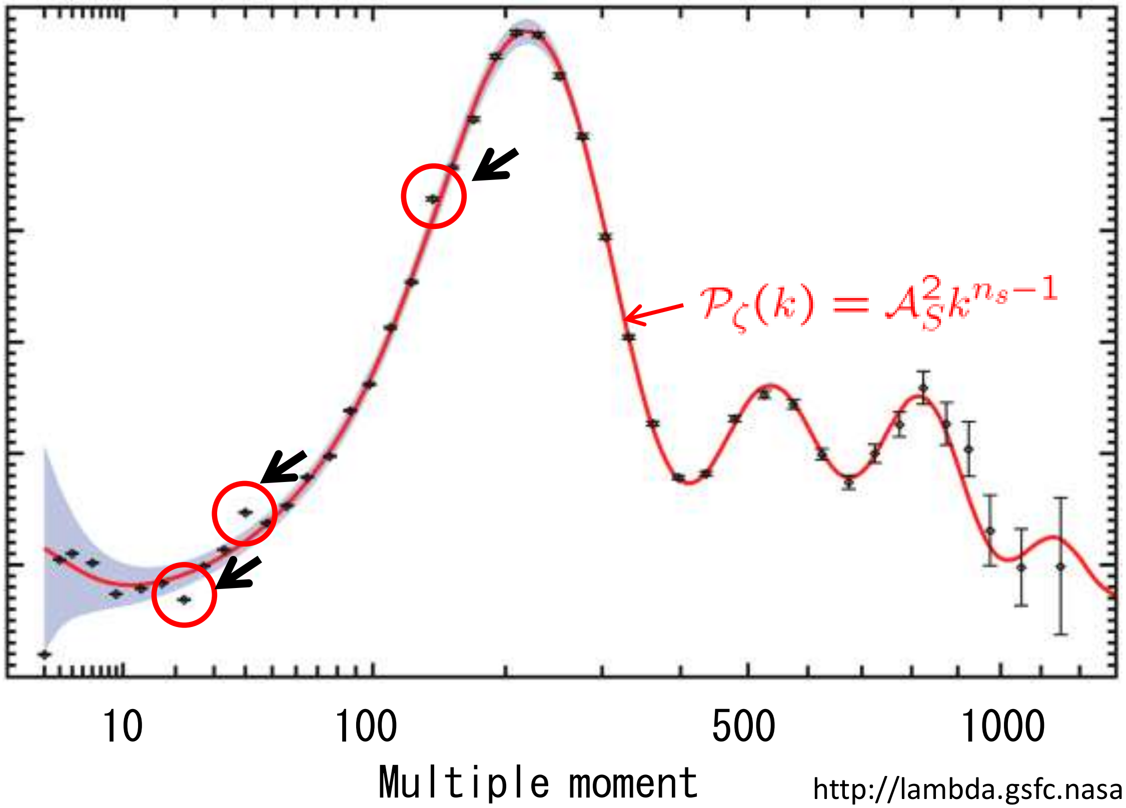
Univ. of Tokyo, RESCEU
Ryo Saito
with Masahiro Nakashima, Yu-ichi Takamizu, and Jun'ichi Yokoyama

Introduction

High precision data on CMB anisotropies

➡ Nearly scale-invariant primordial power spectrum (PPS) + small deviations

$$\mathcal{P}_\zeta(k) = \mathcal{A}_S^2 k^{n_s-1} + \underline{\delta}$$



http://lambda.gsfc.nasa.gov/

Observed *anomalies* in the spectrum of the CMB anisotropy

Systematics ,
or simply Cosmic Variance?

Fine structures in the PPS?

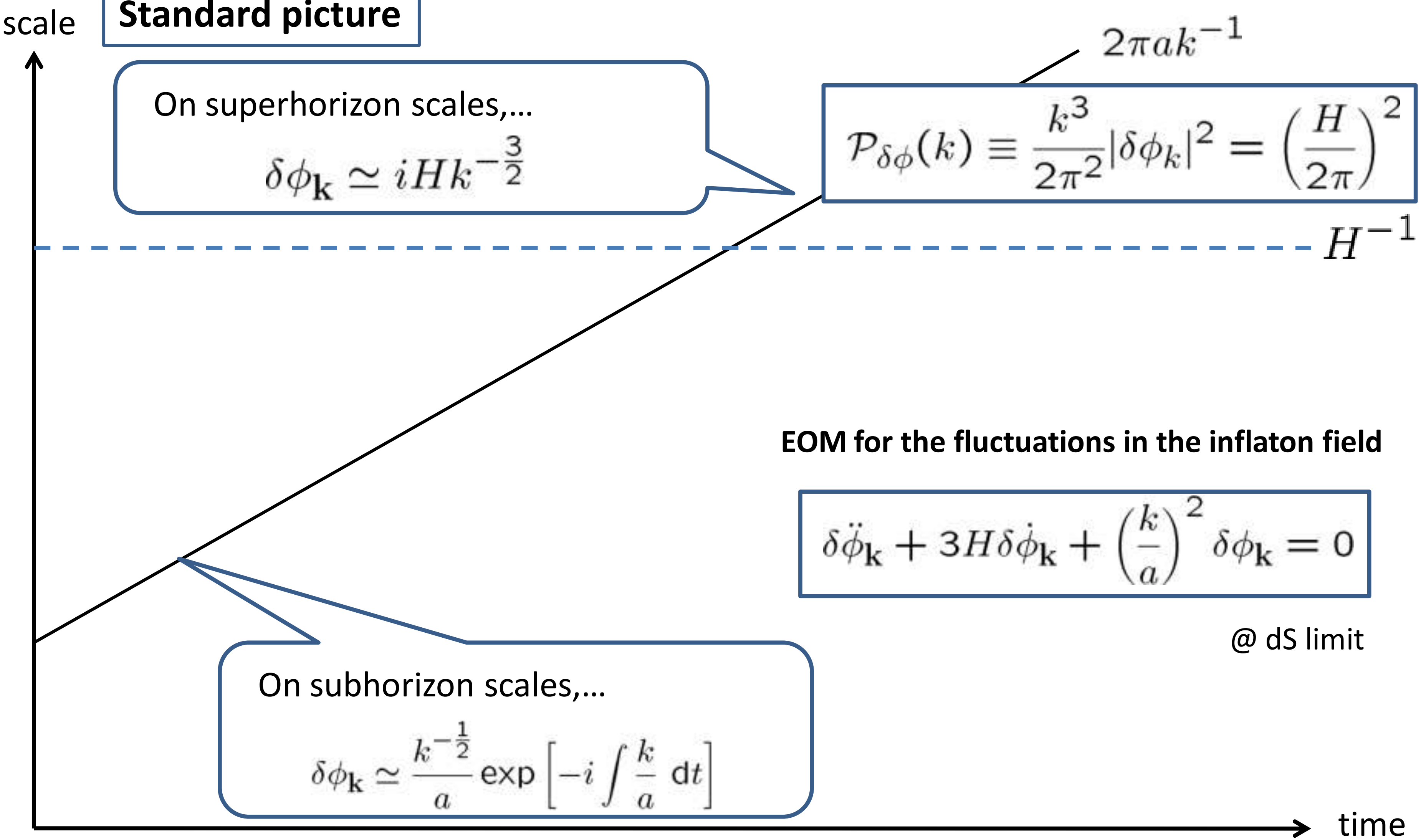
$$\mathcal{P}_\zeta(k) = \mathcal{A}_S^2 k^{n_s-1} + \boxed{\delta}$$

Imprint of high energy physics?

Oscillations of a heavy scalar field

Basic idea of our work

Standard picture



On superhorizon scales,...

$$\delta\phi_{\mathbf{k}} \simeq iHk^{-\frac{3}{2}}$$

$$\mathcal{P}_{\delta\phi}(k) \equiv \frac{k^3}{2\pi^2}|\delta\phi_k|^2 = \left(\frac{H}{2\pi}\right)^2$$

EOM for the fluctuations in the inflaton field

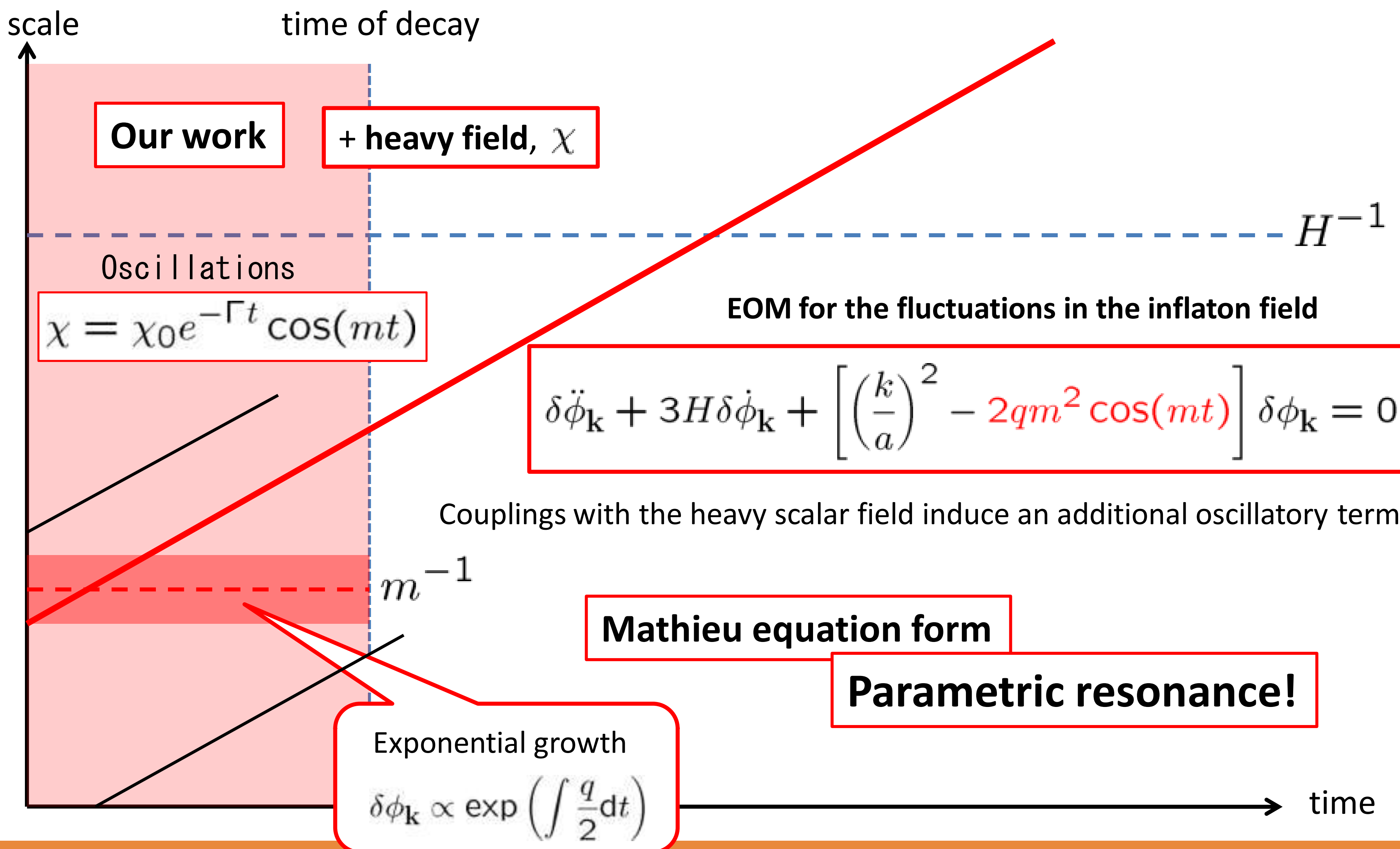
$$\delta\ddot{\phi}_{\mathbf{k}} + 3H\delta\dot{\phi}_{\mathbf{k}} + \left(\frac{k}{a}\right)^2\delta\phi_{\mathbf{k}} = 0$$

@ dS limit

On subhorizon scales,...

$$\delta\phi_{\mathbf{k}} \simeq \frac{k^{-\frac{1}{2}}}{a}\exp\left[-i\int\frac{k}{a}\,\mathrm{d}t\right]$$

Basic idea of our work



Our model

Couplings with a heavy field ⇒ **Feature in the primordial power spectrum**

But,…

Does the oscillations affect the background evolution? ~~**Inflation**~~

- Couplings would violate slow-roll conditions.

$$\lambda\chi^2\phi^2 \to \delta\eta_V = \frac{2\lambda\bar{\chi}^2}{3H^2} \ll \mathcal{O}(10^{-2})$$

parametric resonance is inefficient.

- ***Derivative*** couplings *do not* violate slow-roll conditions.

$$(\partial\phi)^2(\partial\chi)^2, \quad (\partial\phi\cdot\partial\chi)^2, \ldots$$

Our model

Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\chi)^2 + \frac{\lambda_{d1}}{4\Lambda^4}(\partial\phi)^2(\partial\chi)^2 + \frac{\lambda_{d2}}{4\Lambda^4}(\partial\phi\cdot\partial\chi)^2 + \cdots$$
$$- V(\phi) - \frac{m^2}{2}\chi^2,$$
$$\equiv \Lambda^4 K(X^{IJ}) - V(\phi) - \frac{m^2}{2}\chi^2.$$
$$(X^{IJ} \equiv -g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J/(2\Lambda^4),\ \phi^1\equiv\phi,\ \phi^2\equiv\chi)$$

Standard terms + *derivative* couplings between the inflaton and the heavy scalar field

- ***Derivative*** couplings *do not* violate slow-roll conditions.

$$(\partial\phi)^2(\partial\chi)^2, \quad (\partial\phi\cdot\partial\chi)^2, \ldots$$

We analyze the model under an assumption,

$$\frac{\dot\chi^2}{\Lambda^2} < 1 \longrightarrow \text{~~Higher order terms~~}$$

Background evolution

Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\chi)^2 + \frac{\lambda_{d1}}{4\Lambda^4}(\partial\phi)^2(\partial\chi)^2 + \frac{\lambda_{d2}}{4\Lambda^4}(\partial\phi\cdot\partial\chi)^2 + \cdots$$
$$- V(\phi) - \frac{m^2}{2}\chi^2,$$
$$\equiv \Lambda^4 K(X^{IJ}) - V(\phi) - \frac{m^2}{2}\chi^2.$$

$$(X^{IJ} \equiv -g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J/(2\Lambda^4),\ \phi^1 \equiv \phi,\ \phi^2 \equiv \chi)$$

Background EOM

$$\dot{\pi}_1 + 3H\pi_1 + V' = 0.$$

$\pi_I \equiv K_{IJ}\dot{\phi}^J$: Conjugate momentum of the fields

derivatives w.r.t. X^{IJ}

Slow-roll solution →

$$\pi_1 \simeq -\frac{V'}{3H}.$$

Background evolution

Slow-roll solution

$$\pi_1 \equiv K_{1J}\dot{\phi}^J \simeq -\frac{V'}{3H}, \qquad \chi \simeq \chi_0 e^{-\Gamma t}\cos(mt)$$

- **Inflation?**

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1 \ ?$$

$$\epsilon = \frac{K_{IJ}X^{IJ}}{M_p^2 H^2}, \\ \simeq \epsilon_V + 6 f_\chi \sin^2(mt), \quad (f_\chi : \text{ energy fraction of } \chi)$$

- **Consistency of the slow-roll solution**

$$\frac{\dot{\pi}_1}{3H\pi_1} \ll 1 \ ?$$

$$\frac{\dot{\pi}_\phi}{3H\pi_\phi} \simeq \frac{V''}{V'}\frac{\dot{\phi}}{H} - \frac{\dot{H}}{H^2}, \\ \simeq -\eta_V + \epsilon.$$

Note

$\pi_1 \simeq \dot{\phi}$, but $\dot{\pi}_1 \not\simeq \ddot{\phi}$.

$\dot{K}_{1J} \sim \mathcal{O}(m)$ ⇒ $\frac{\ddot{\phi}}{3H\dot{\phi}} \sim \mathcal{O}\left(\frac{m}{H}\right)$

Background evolution

Inflation is realized, and the inflaton slowly rolls down its potential with a tiny oscillatory correction.

- **Inflation?**

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1 \ ?$$

$$\epsilon = \frac{K_{IJ}X^{IJ}}{M_p^2 H^2}, \simeq \epsilon_V + 6 f_\chi \sin^2(mt), \quad (f_\chi : \text{ energy fraction of } \chi)$$

- **Consistency of the slow-roll solution**

$$\frac{\dot{\pi}_1}{3H\pi_1} \ll 1 \ ?$$

$$\frac{\dot{\pi}_\phi}{3H\pi_\phi} \simeq \frac{V''}{V'}\frac{\dot{\phi}}{H} - \frac{\dot{H}}{H^2}, \simeq -\eta_V + \epsilon.$$

Note

$\pi_1 \simeq \dot{\phi}$, but $\dot{\pi}_1 \not\simeq \ddot{\phi}$.

$\dot{K}_{1J} \sim \mathcal{O}(m)$ ⇒ $\frac{\ddot{\phi}}{3H\dot{\phi}} \sim \mathcal{O}\left(\frac{m}{H}\right)$

Perturbations

Deep inside the Hubble horizon, $k/a \gg H$ ➡ ~~Fluctuations in the metric tensor~~

Small couplings & large mass, ➡ ~~Fluctuations in the heavy scalar field~~

EOM for the fluctuations in the inflaton field (on subhorizon scale)

$$\ddot{v}_{\mathbf{k}} + \left[c_s^2 \left(\frac{k}{a}\right)^2 - \frac{\ddot{z}_\phi}{z_\phi}\right] v_{\mathbf{k}} = 0, \quad \left(v \equiv a^{\frac{3}{2}} \delta\phi\right)$$

Here,

$$z_\phi^2 \equiv \frac{a^3 \bar{K}_{11}}{c_s^2},$$

$$c_s^2 \equiv \frac{\bar{K}_{11}}{2\bar{K}_{1K,L1}\bar{X}^{KL} + \bar{K}_{11}}.$$ **(speed of sound)**

$$K \equiv \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 + \frac{\lambda_{d1}}{4\Lambda^4}(\partial\chi)^2(\partial\phi)^2 + \frac{\lambda_{d2}}{4\Lambda^4}(\partial\chi\cdot\partial\phi)^2$$

Perturbations

EOM for the fluctuations in the inflaton field (on subhorizon scale)

$$\frac{\mathrm{d}^2 v_{\mathbf{k}}}{\mathrm{d}z^2} + [A_k - 2q\cos(2z)]\, v_{\mathbf{k}} = 0, \qquad (z \equiv mt)$$

Here,

$$A_k \equiv \left(\frac{k}{am}\right)^2 \quad \text{for } k \sim am,$$

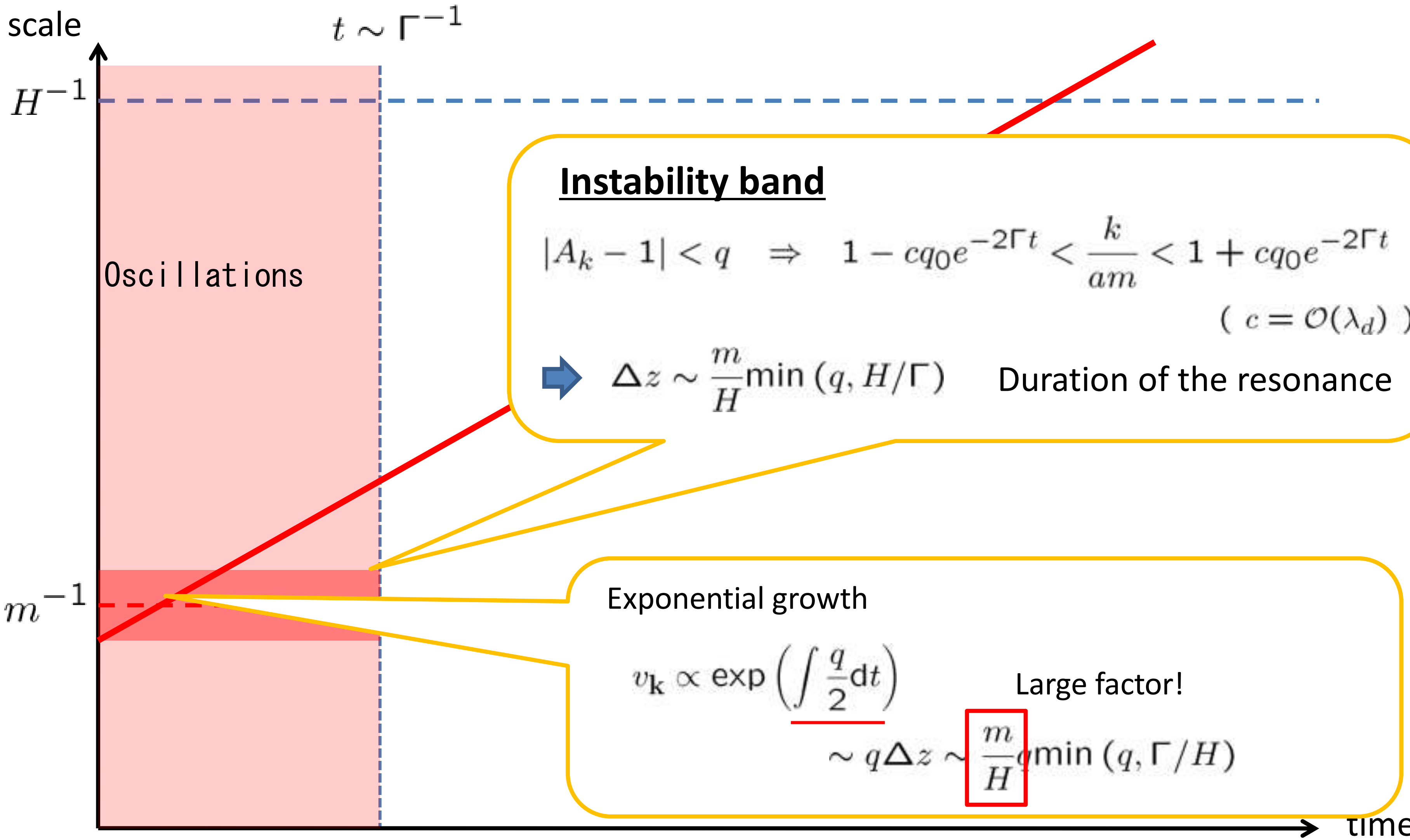
$$q \equiv \frac{m^2\chi_0^2}{8\Lambda_d^4} e^{-2\Gamma t}\left[-(\lambda_{d1} - 2\lambda_{d2})\left(\frac{k}{am}\right)^2 + 2(\lambda_{d1} + 2\lambda_{d2})\right]$$

$$f_\chi \frac{3M_p^2 H^2}{4\Lambda^4}\bigg|_{t=0} \equiv q_0 < 1$$

Mathieu equation form!

$|q| < 1$ ⇒ Narrow resonance, instability band $|A_k - 1| < q$

Parametric resonance



Instability band

$$|A_k - 1| < q \quad \Rightarrow \quad 1 - cq_0 e^{-2\Gamma t} < \frac{k}{am} < 1 + cq_0 e^{-2\Gamma t}$$

$(\ c = \mathcal{O}(\lambda_d) \ )$

$\Rightarrow$ $\Delta z \sim \frac{m}{H} \mathrm{min}\,(q, H/\Gamma)$ Duration of the resonance

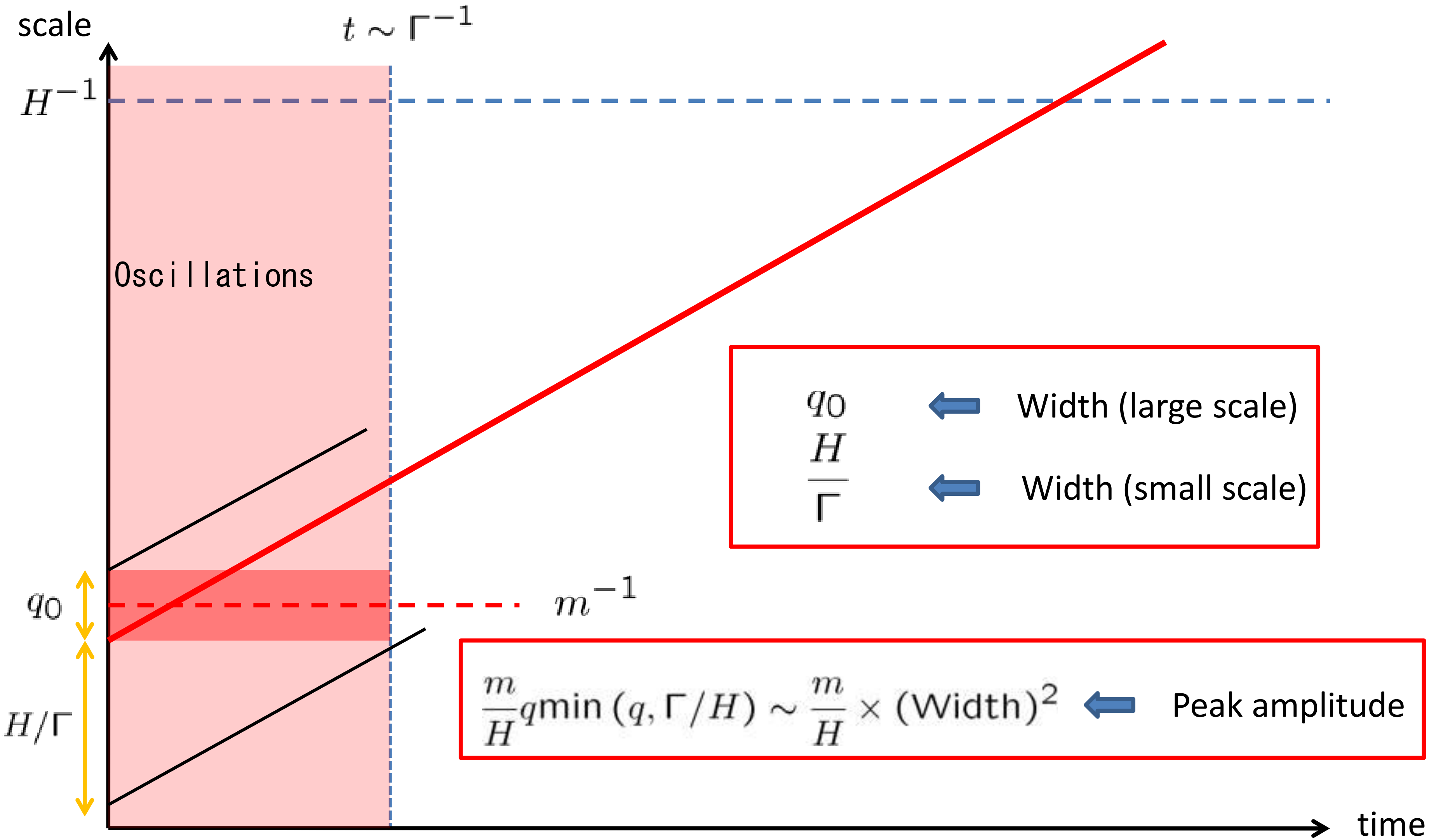
Exponential growth

$$v_{\mathbf{k}} \propto \exp\left(\int \frac{q}{2} \mathrm{d}t\right)$$

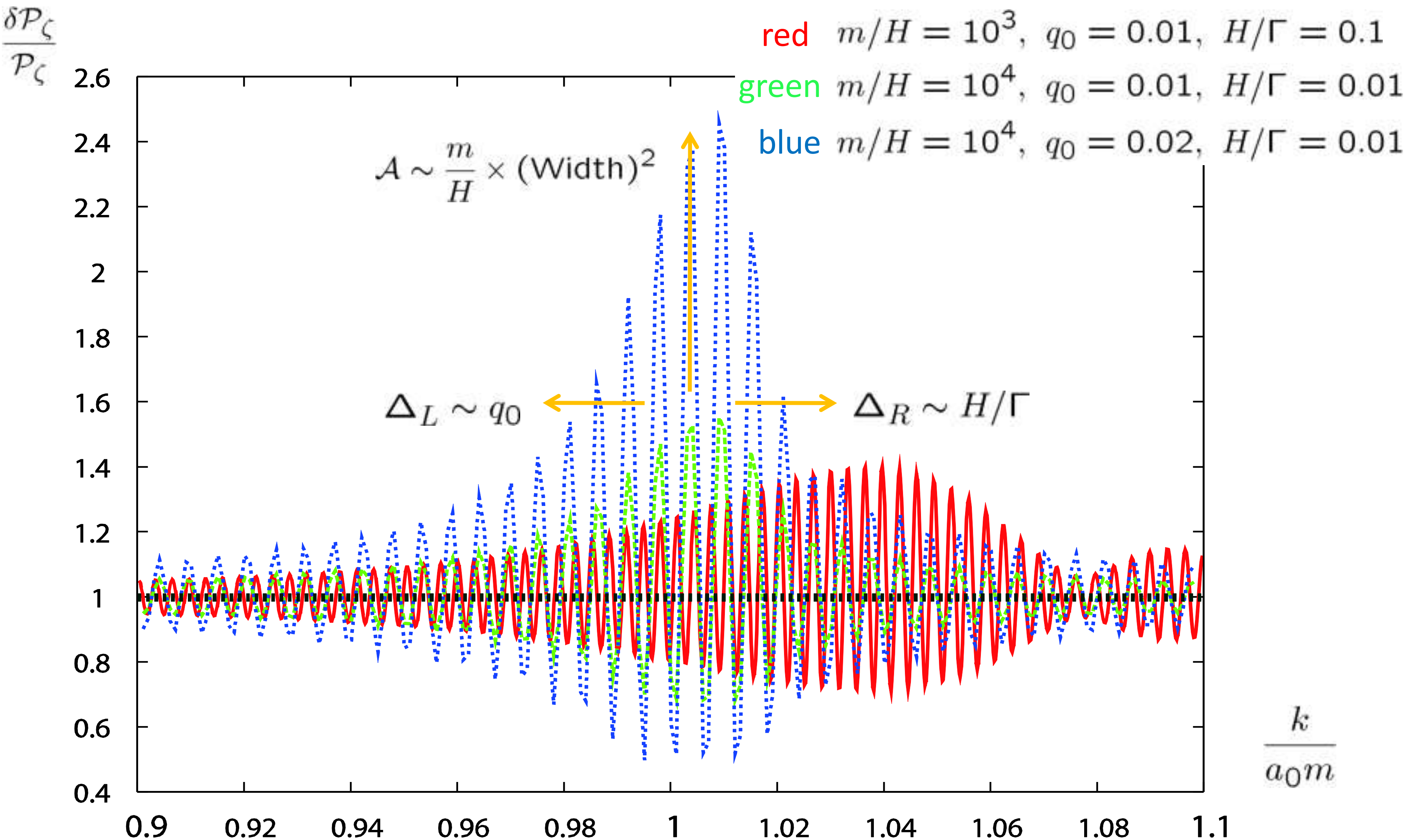
Large factor!

$$\sim q\Delta z \sim \frac{m}{H} q \, \mathrm{min}\,(q, \Gamma/H)$$

Parametric resonance



Modulated primordial power spectrum



Energy scales $\longleftarrow$ Observations (rough estimation)

Observations ➡ $\mathcal{A} \sim \frac{m}{H} q_0^2,\ \Delta_L \sim q_0, \Delta_R \sim H/\Gamma$

Anomaly in the CMB power spectrum at $l \simeq 124$ (Ichiki, Nagata, Yokoyama `09)

$\mathcal{A} = 1.0,\ \Delta_L = \Delta_R = 0.04$

$$\Gamma = 10^{13}\text{GeV}\left(\frac{H}{10^{12}\text{GeV}}\right)\left(\frac{\Delta_R}{0.04}\right)^{-1}$$

$$m = 10^{14}\text{GeV}\left(\frac{H}{10^{12}\text{GeV}}\right)\left(\frac{\mathcal{A}}{1.0}\right)\left(\frac{\Delta_L}{0.04}\right)^{-2}$$

$$\Lambda = 10^{15}\text{GeV}\left(\frac{f_\chi}{0.01}\right)^{1/4}\left(\frac{H}{10^{12}\text{GeV}}\right)^{1/2}\left(\frac{\Delta_L}{0.04}\right)^{-1/4}$$

$$q_0 \equiv f_\chi \frac{3M_p^2 H^2}{4\Lambda^4}\bigg|_{t=0}$$

Feature in the primordial power spectrum ➡ **Scalar field with mass** $\sim \mathcal{O}(10^{14})\text{GeV}$ **?**

Summary

• A heavy scalar field give an oscillatory correction to the dispersion relation of the fluctuations in the inflaton field.

• EOM of the fluctuations takes a form of Mathieu equation.
⇨ Parametric resonance between oscillations of the heavy field and the fluctuations in the inflaton field.
⇨ Features in the power spectrum of the primordial curvature perturbations.

• Derivative couplings → Sufficient amplification without violation of the slow-roll conditions.

• High energy physics ↔ Features in the CMB power spectrum ?

• Non-Gaussianity induced by the resonance? (Chen, Easther, & Lim `08, Chen `10)