

# Lecture 3: galaxy clusters as cosmological tools

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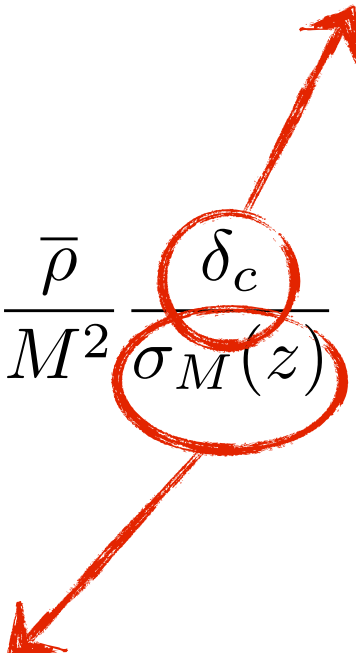
Dipartimento di Astronomia - Università di Bologna

$$\frac{dn(M, z)}{dM} = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M^2} \frac{\delta_c}{\sigma_M(z)} \left| \frac{d \log \sigma_M(z)}{d \log M} \right| \exp \left( -\frac{\delta_c^2}{2\sigma_M(z)^2} \right)$$

Critical density contrast: the overdensity that a perturbation in the initial density field must have for it to end up in a virialized structure

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Mass variance at the scale  $M$   
 linearly extrapolated at redshift  $z$

$$\sigma(M, z) = \sigma_M \delta_+(z)$$

$$\sigma_M^2 = \sigma_R^2 = \frac{1}{2\pi^2} \int dk k^2 P(k) \hat{W}_R^2(k) \quad R \propto \left( \frac{M}{\bar{\rho}} \right)^{1/3}$$

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 linearly extrapolated at redshift  $z$

Lin. Growth factor

$$\delta_+(z) = \frac{5}{2} \Omega_m(z) E(z) \int_z^\infty \frac{1+z'}{E(z')^3} dz'$$

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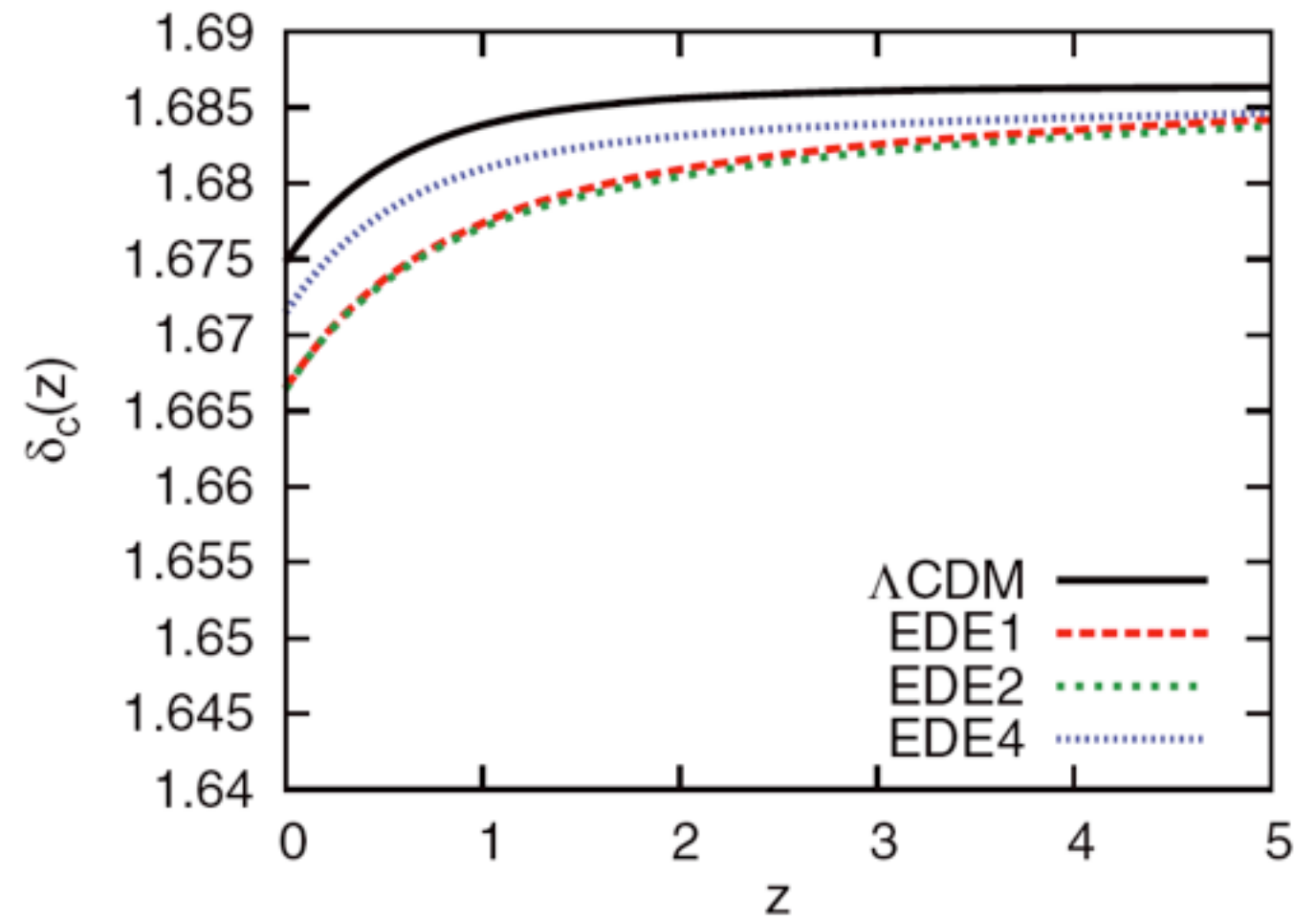
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Power spectrum

# The mass function dependence on cosmology

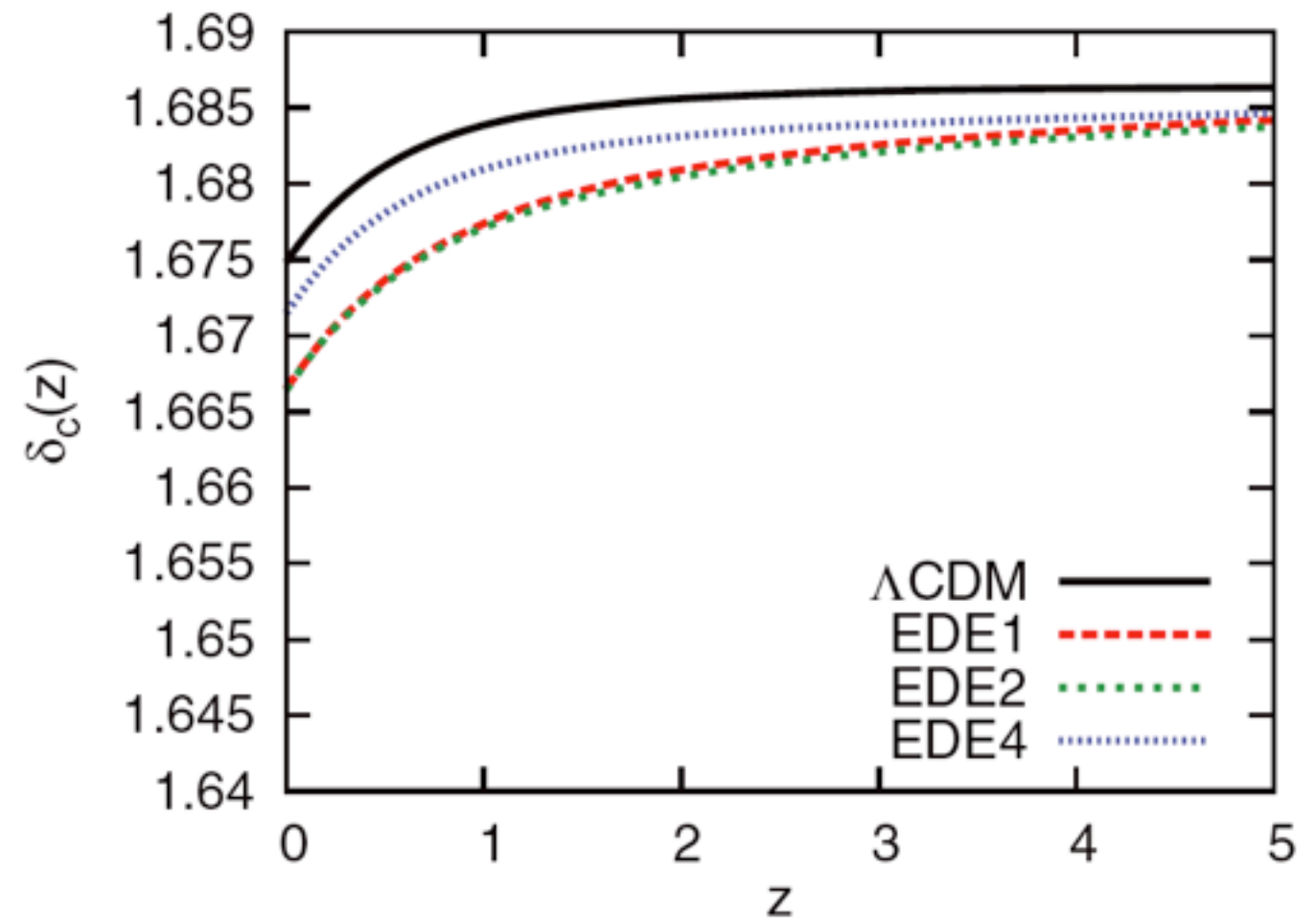
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Pace, Waizmann & Bartelmann (2010)

# The mass function dependence on cosmology

• critical density contrast



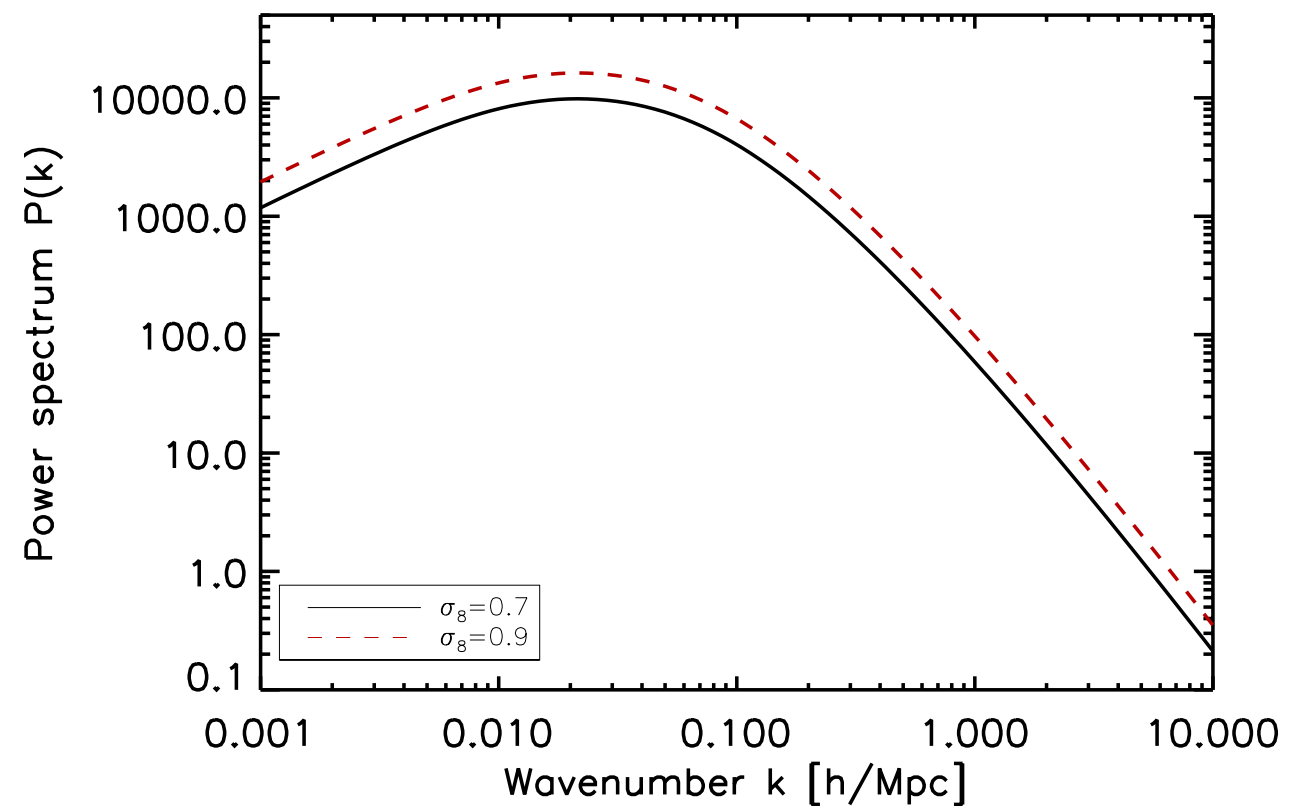
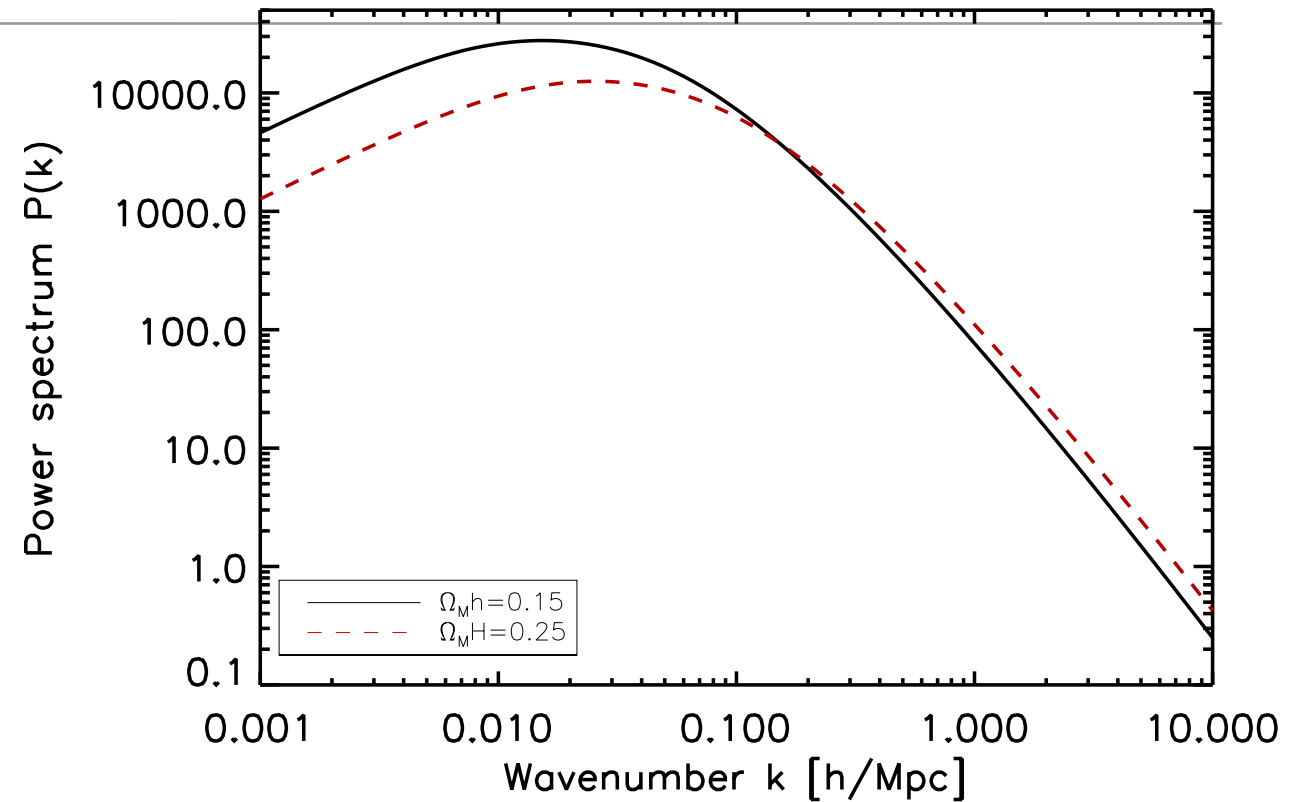
Weak sensitivity to cosmology

Pace, Waizmann & Bartelmann (2010)



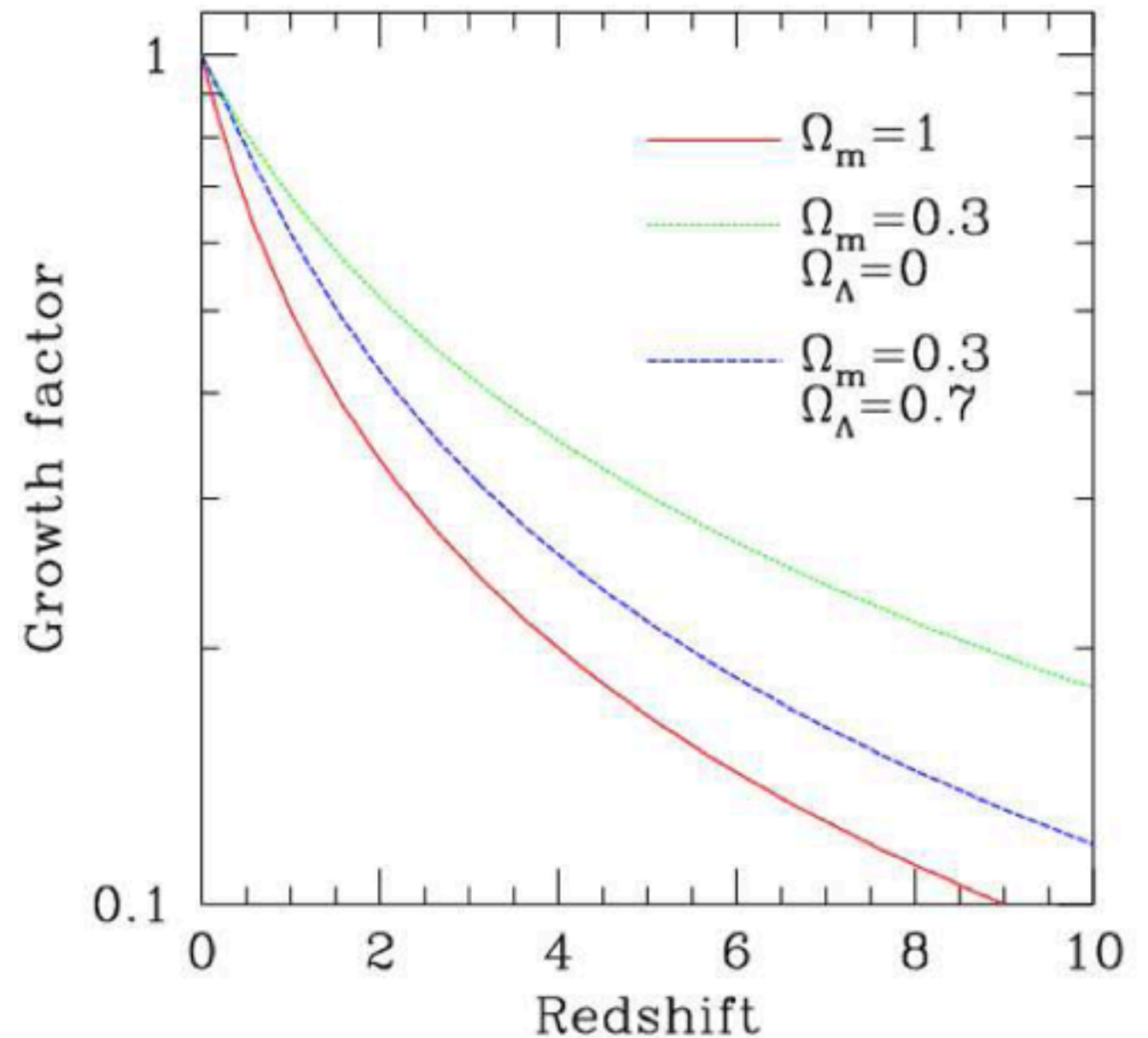
# The mass function dependence on cosmology

- critical density contrast
- Power spectrum (shape and amplitude)



# The mass function dependence on cosmology

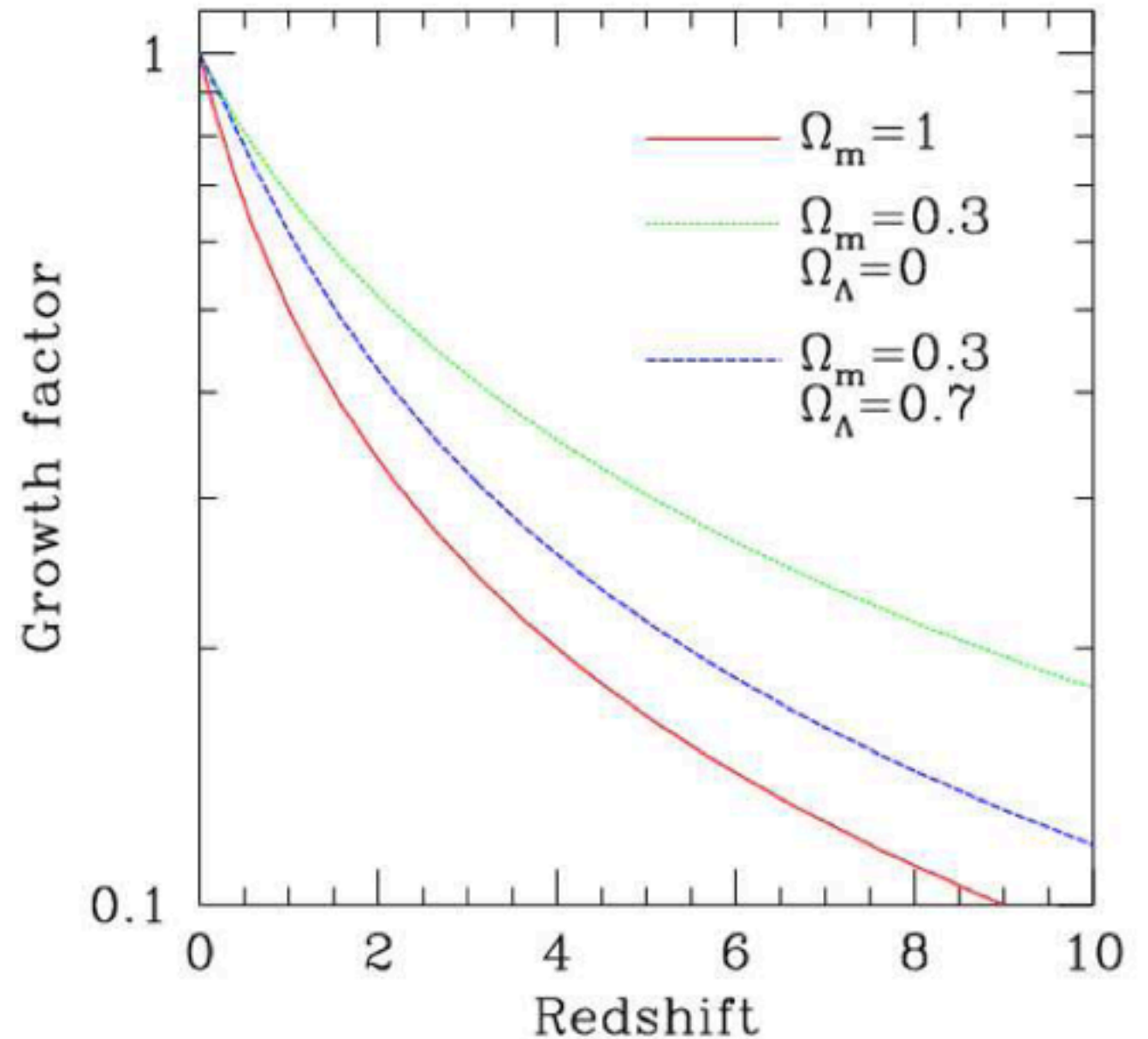
- critical density contrast
- Power spectrum (shape and amplitude)
- Growth factor



# The mass function dependence on cosmology

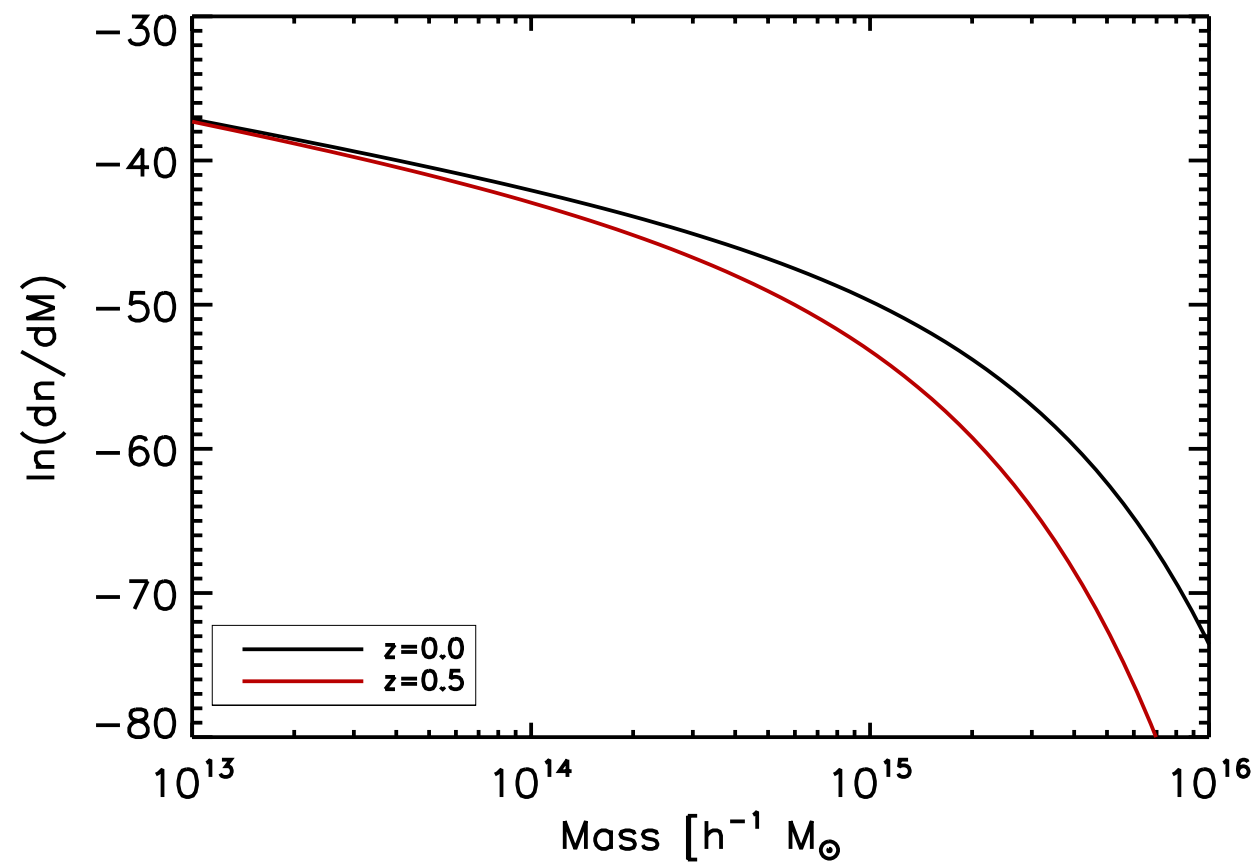
- critical density contrast
- Power spectrum (shape and amplitude)
- Growth factor

The evolution of the mass function reflects the growth of the cosmic structures:  
additional sensitivity to  $\Omega_{DE}$



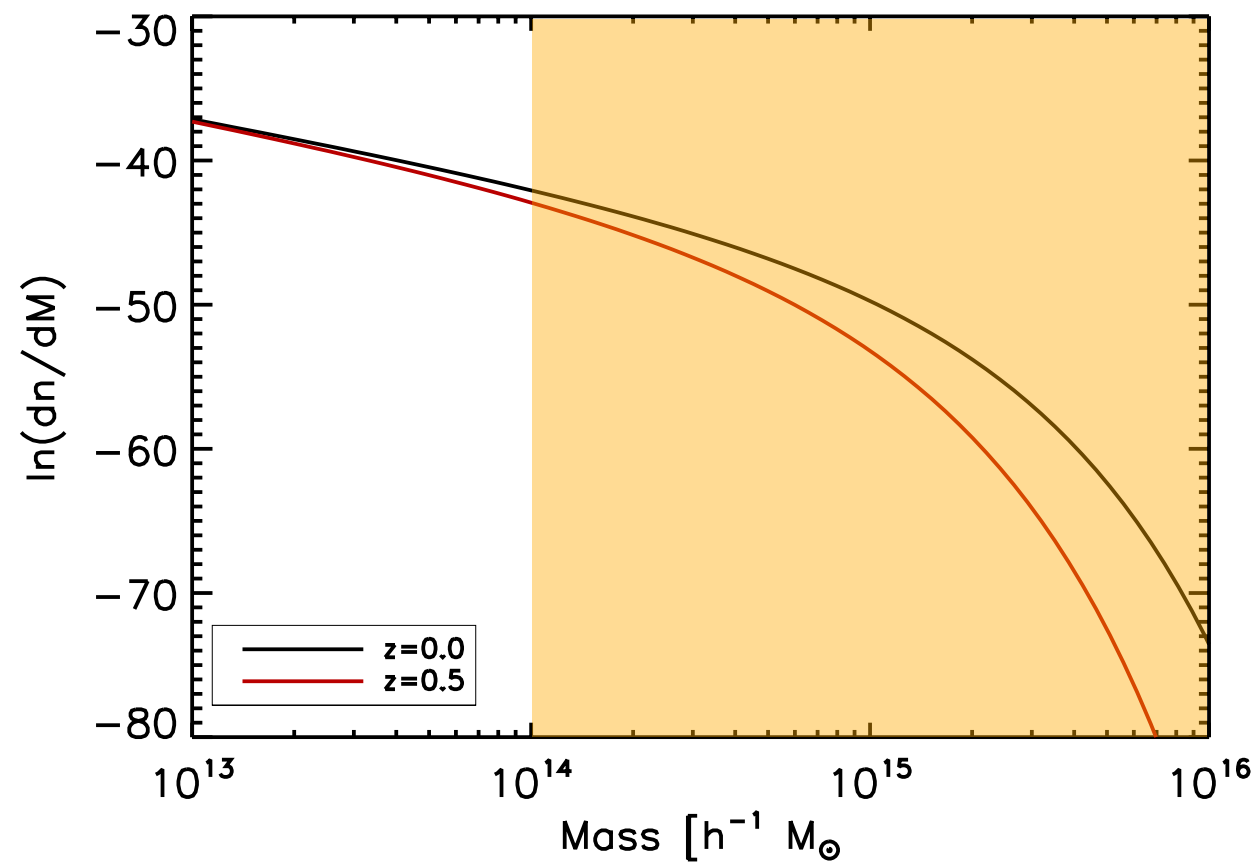
# Sensitivity of the cluster mass function to cosmological models

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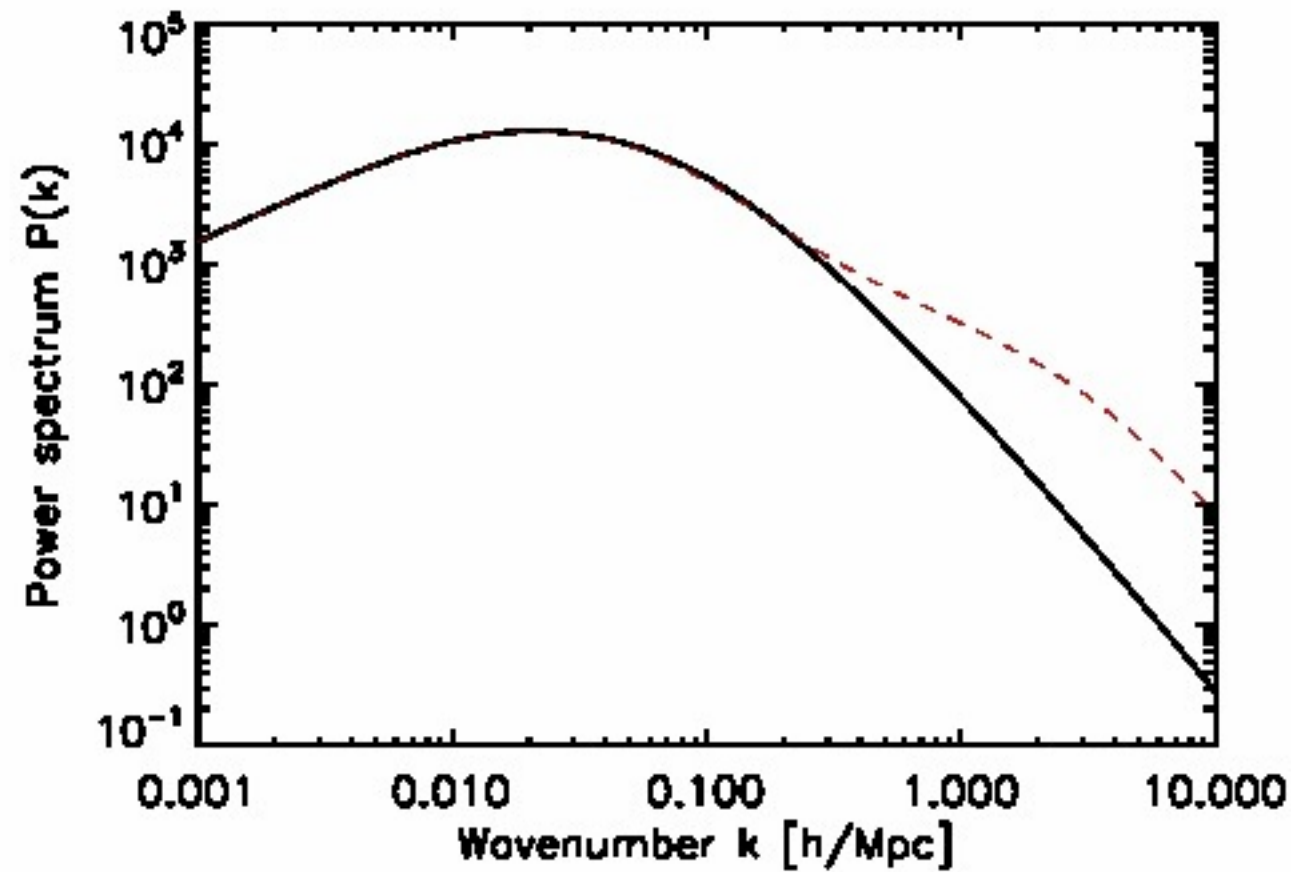
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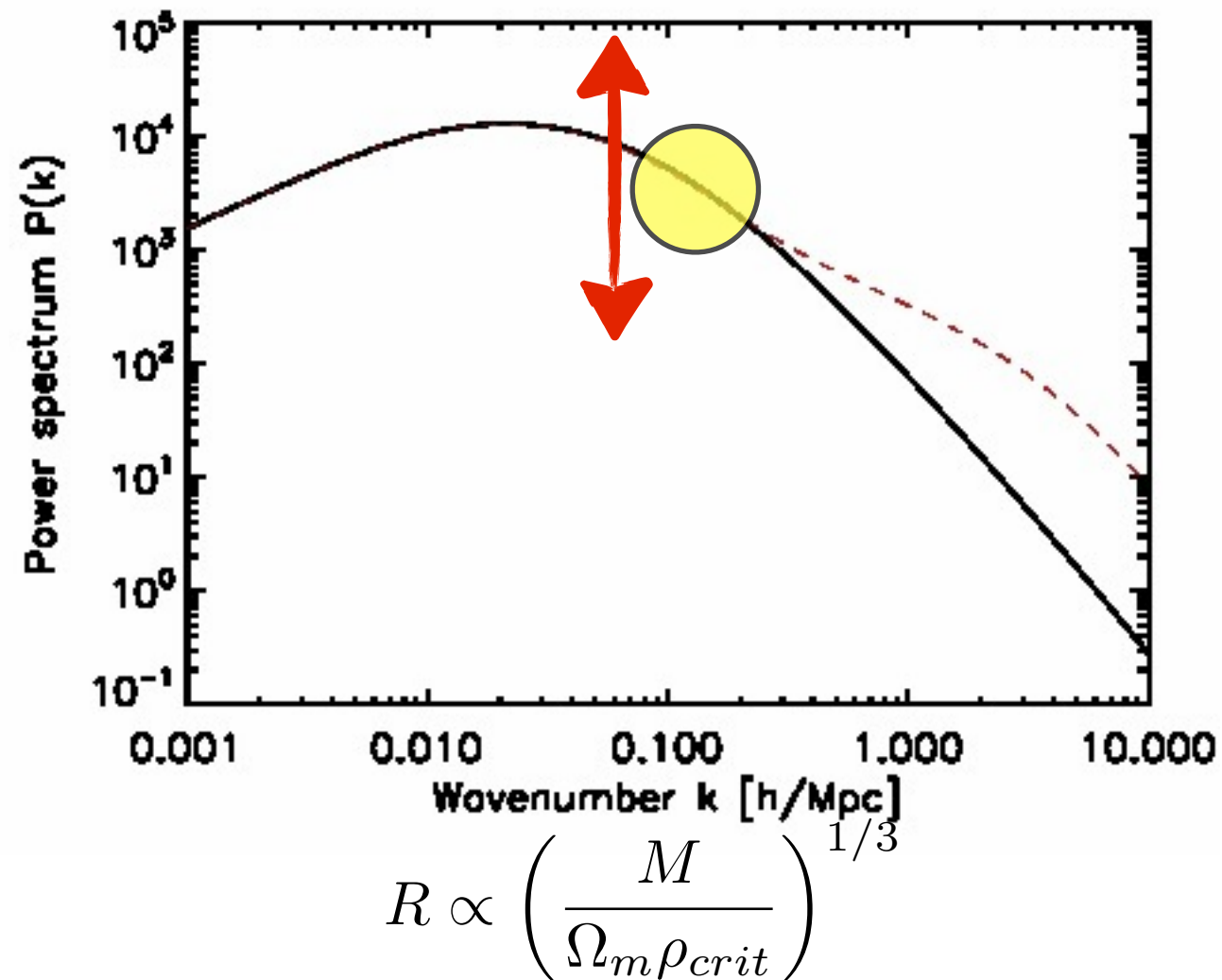
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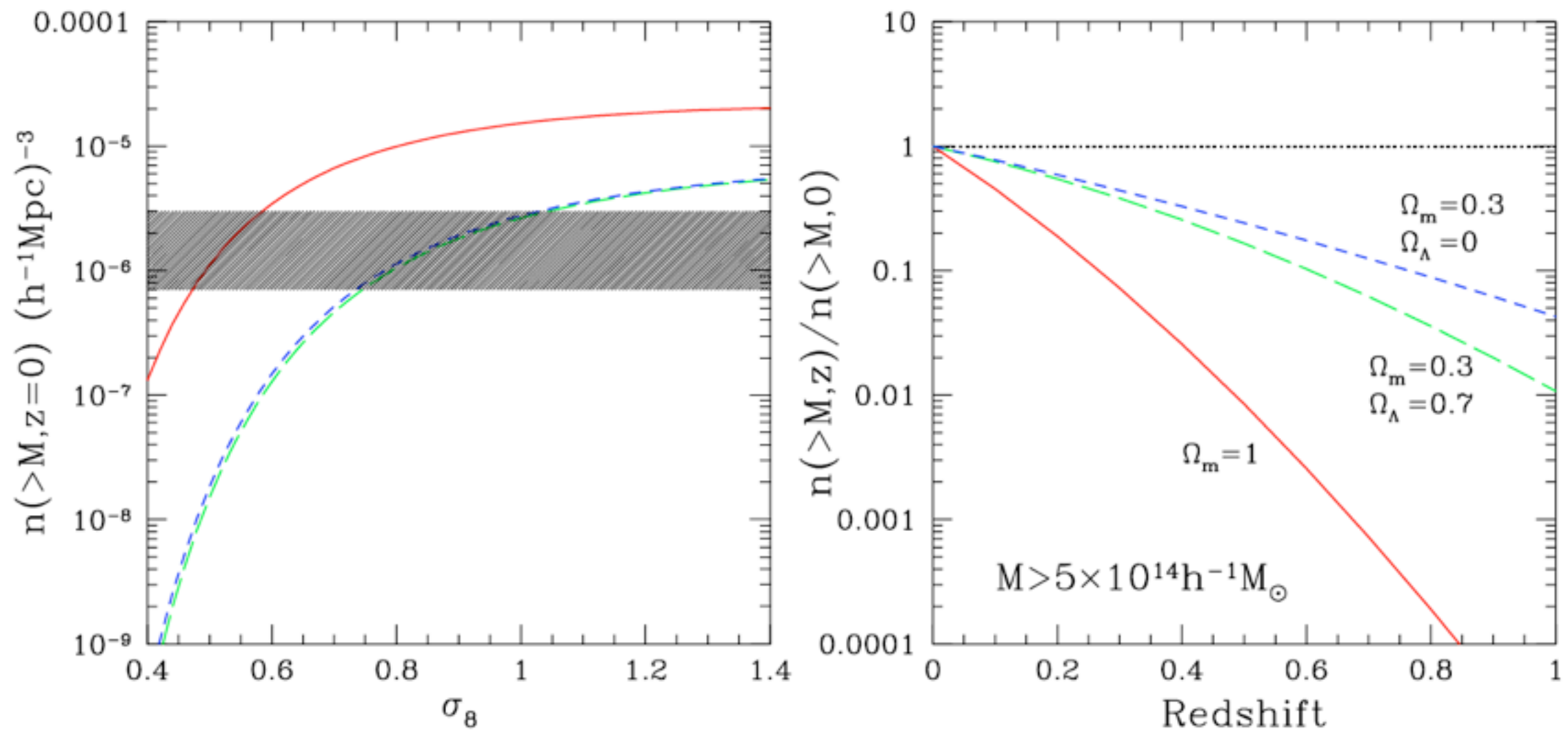
# Sensitivity of the cluster mass function to cosmological models

Clusters probe a narrow range of scales:



The scale  $R$  depends on both  $M$  and  $\Omega_m$ , thus the mass function of nearby clusters is only able to constrain a relation of  $\sigma_8$  and  $\Omega_m$ .

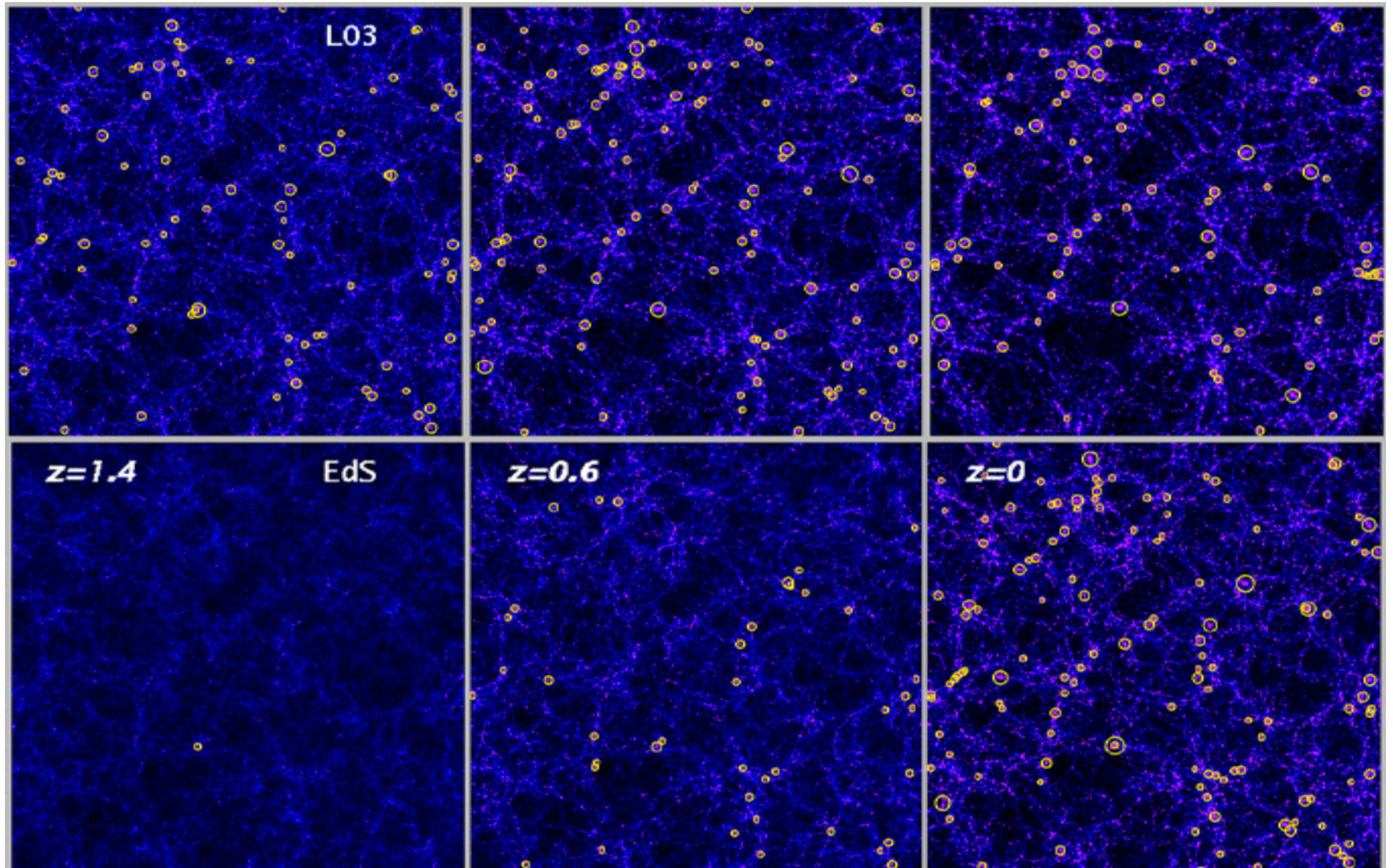
# Sensitivity of the cluster mass function to cosmological models



Borgani (2006)



# Sensitivity of the cluster mass function to cosmological models





# Therefore...

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- find clusters
- measure their masses
- compare to theory

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Cosmology with galaxy clusters

# How to find galaxy clusters?

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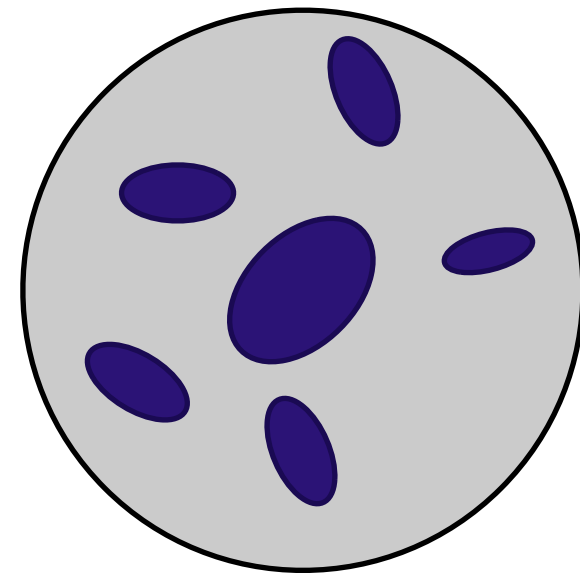
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- optical selection
- X-ray selection
- lensing selection
- SZ selection

# Optical selection

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- the first statistically complete sample of galaxy clusters (Abell 1958, 1989)
- clusters were identified as galaxy overdensities and classified on the basis of their "Richness"
- several algorithms have been developed, which try to enhance the contrast of galaxy overdensity at a given position (e.g. Postman et al. 1996)
- an extension of these techniques is the MaxBCG method (Koester et al. 2007a,b: 13823 clusters in the SLOAN)



Abell radius =  $1.5 \text{ Mpc}/h$   
Count galaxies within  $R_A$  with mag between  $m_3$  and  $m_{3+2}$

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$$n_m(\boldsymbol{\theta}, m) = n_f(m) + \Lambda n_c(\boldsymbol{\theta}, m) = n_f(m) + \Lambda P(\boldsymbol{\theta} - \boldsymbol{\theta}_c) \phi(m)$$

$$\begin{aligned} \mathcal{L} &= - \int \frac{[n_d(\boldsymbol{\theta}, m) - n_m(\boldsymbol{\theta}, m)]^2}{n_m(\boldsymbol{\theta}, m)} d\Omega dm \\ &= - \int \frac{[n_d(\boldsymbol{\theta}, m) - n_f(m) - \Lambda n_c(\boldsymbol{\theta}, m)]^2}{n_f(m)} d\Omega dm \end{aligned}$$

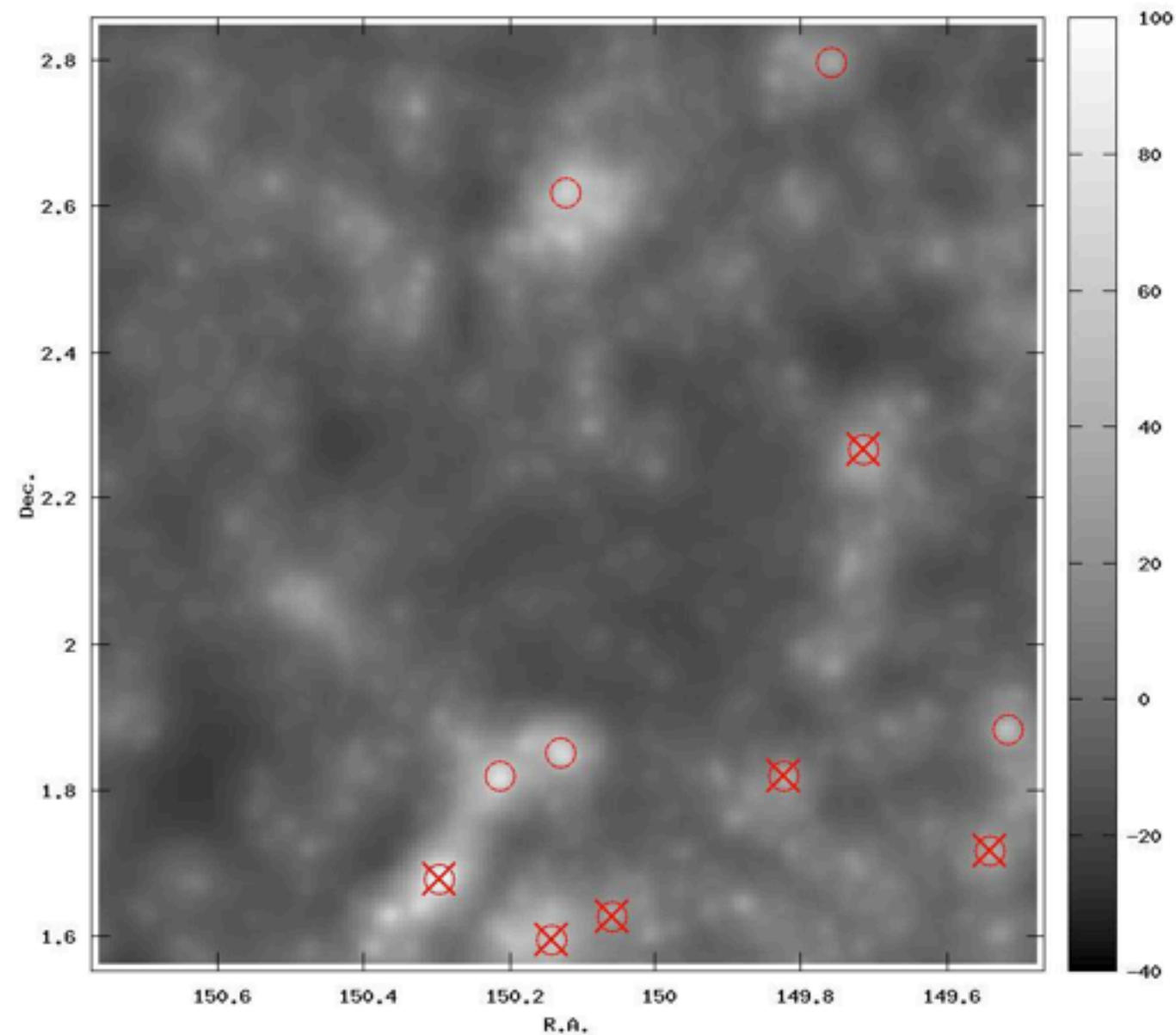
$$\Lambda = \int \Phi(\boldsymbol{\theta} - \boldsymbol{\theta}_c, m) n_d(\boldsymbol{\theta}, m) d\Omega dm - B$$

$$\Phi(\boldsymbol{\theta} - \boldsymbol{\theta}_c, m) = \left( \int \frac{n_c^2}{n_f} d\Omega dm \right)^{-1} \frac{n_c(\boldsymbol{\theta}, m)}{n_f(m)}$$



# Optical selection

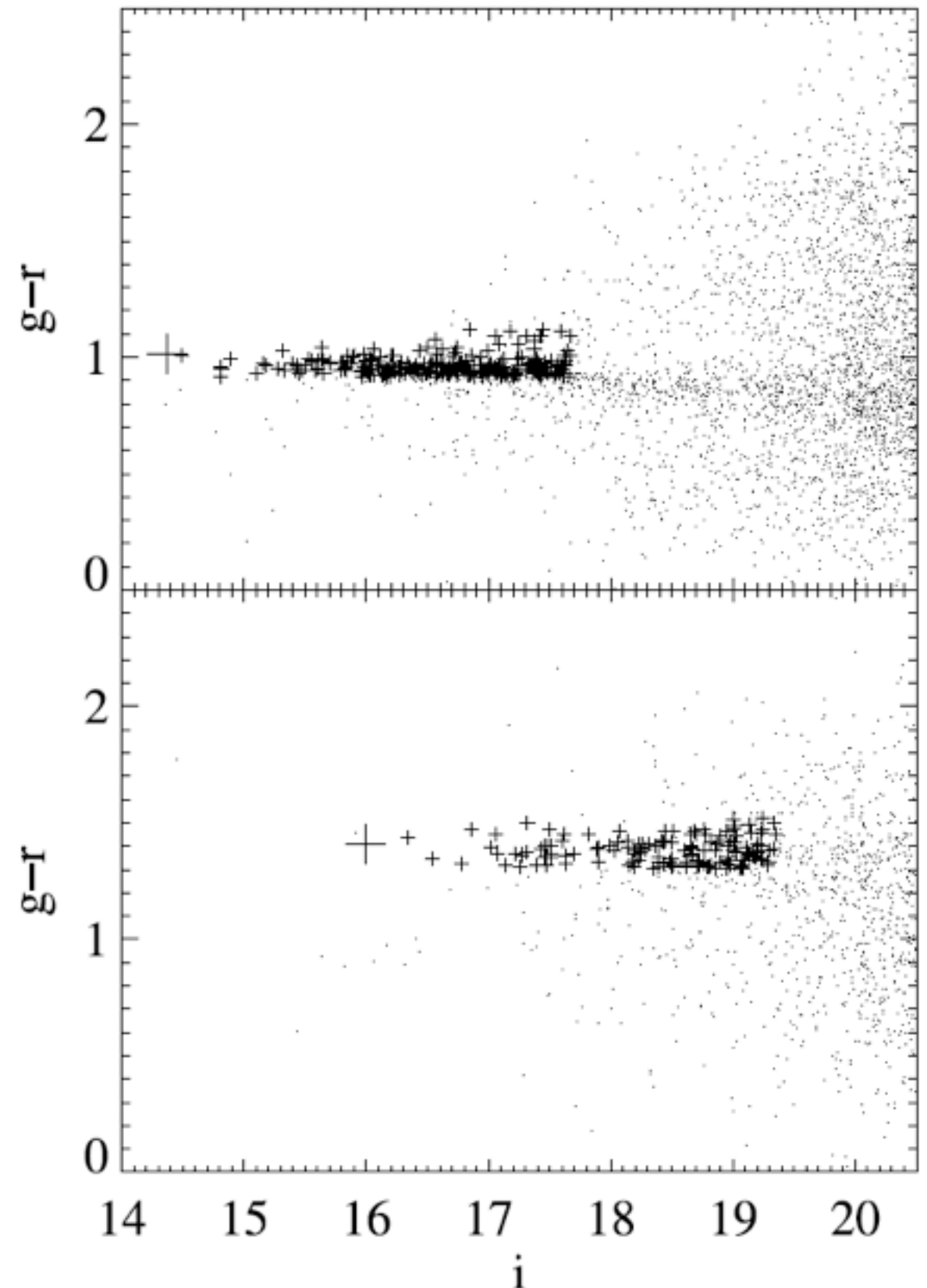
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Bellagamba et al. 2010

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# X-ray selection

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- Clusters are bright X-ray sources: thermal bremsstrahlung from optically thin plasma at the temperature of several keV
- Clusters can then be searched as extended X-ray sources on the sky
- Advantages: 1) X-ray emission comes from physically bound systems 2) the emissivity is proportional to  $\rho^2$  3) easy selection function and 4) X-ray lum. is well correlated with mass

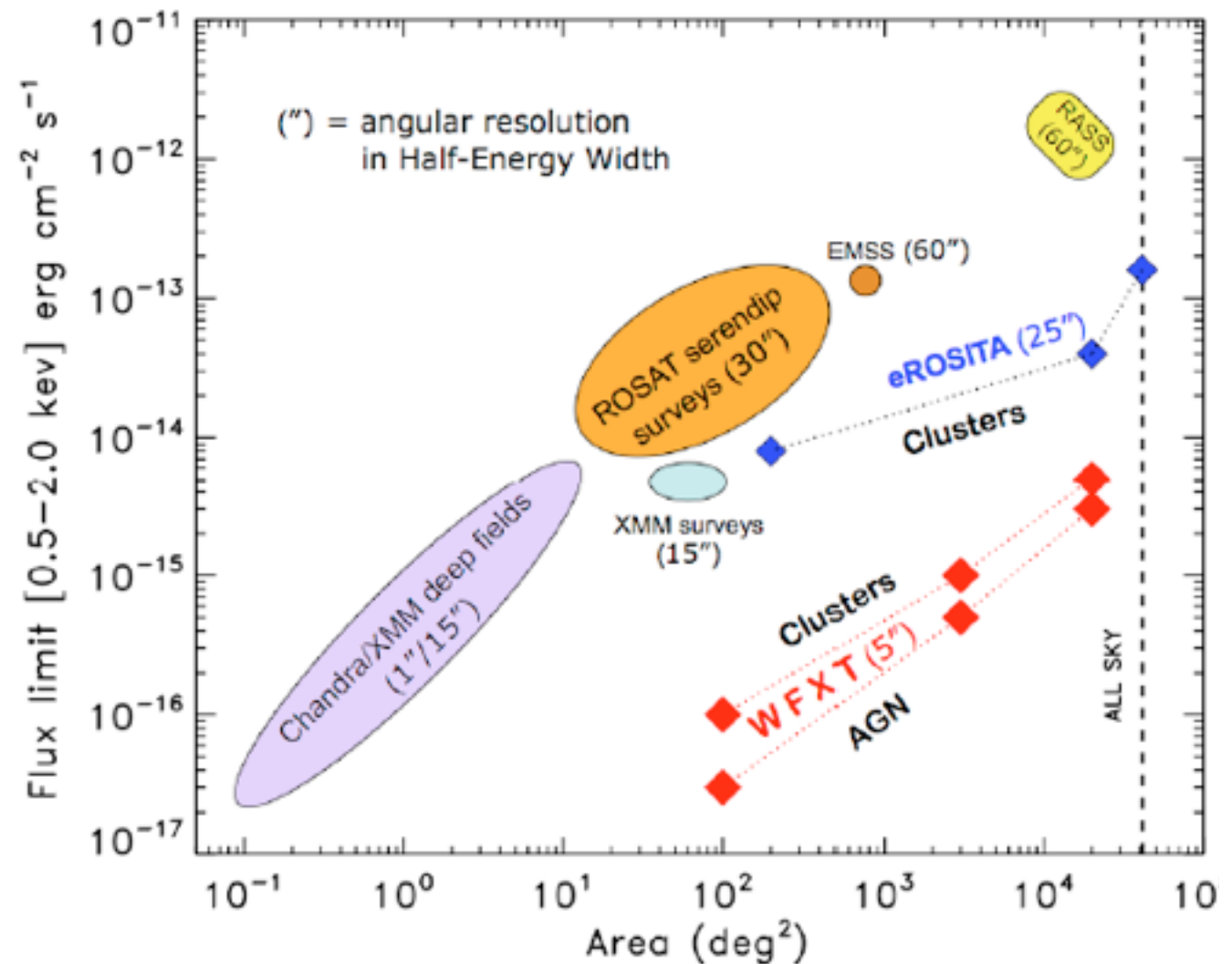


Credit: X-ray: NASA/CXC/MIT/E.-H Peng et al; Optical: NASA/STScI



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# Lensing selection

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- As we have seen, clusters are the most powerful lenses in the universe
- Clusters can then be searched through their lensing signal
- One can quantify the lensing signal by means of the "mass in apertures"
- Big problem: projection effects
- Possible solution: optimal filtering (see e.g. Maturi et al. 2006)

$$M_{\text{ap}}(\theta_0) = \int d^2\varphi \gamma_t(\varphi; \theta_0) Q(\varphi)$$

$$\sigma_{M_{\text{ap}}}^2 = \frac{\pi \sigma_\varepsilon^2}{n} \int_0^\theta d\vartheta \vartheta Q^2(\vartheta)$$

$$S(\theta; \theta_0) = \sqrt{\frac{n}{\pi \sigma_\varepsilon^2}} \frac{\int_0^\theta d^2\vartheta \gamma_t(\vartheta; \theta_0) Q(\vartheta)}{\sqrt{\int_0^\theta d\vartheta \vartheta Q^2(\vartheta)}}$$

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$$D(\theta) = S(\theta) + N(\theta) = A\tau(\theta) + N(\theta)$$

$$A_{\text{est}}(\theta) = \int D(\theta') \Psi(\theta - \theta') d^2\theta'$$

Construct the filter such that it gives unbiased estimates and minimizes the noise

$$b \equiv \langle A_{\text{est}} - A \rangle = A \left[ \int \Psi(\theta) \tau(\theta) d^2\theta - 1 \right]$$

$$\sigma^2 \equiv \langle (A_{\text{est}} - A)^2 \rangle = b^2 + \frac{1}{(2\pi)^2} \int |\hat{\Psi}(k)|^2 P_N(k) d^2k$$



$$\hat{\Psi}(k) = \frac{1}{(2\pi)^2} \left[ \int \frac{|\hat{\tau}(k)|^2}{P_N(k)} d^2k \right]^{-1} \frac{\hat{\tau}(k)}{P_N(k)}$$

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In case of lensing by clusters:

- signal =  $g$
- shape of signal = NFW
- Noise = LSS + intrinsic ellipt. + ...

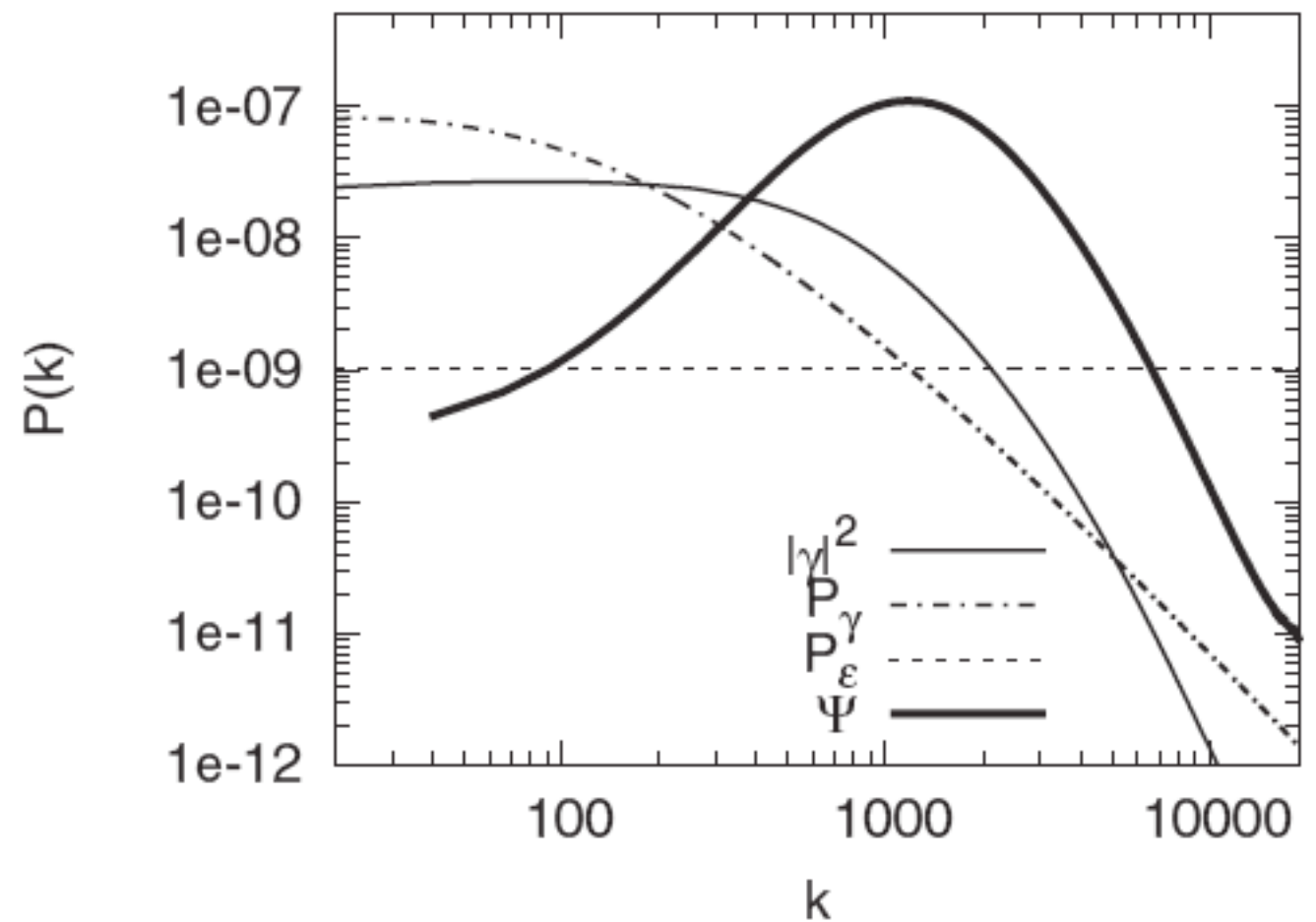
$$\hat{\tau}(k) \equiv \hat{g}(k) = \int d^2x g(x) \exp(ix \cdot k)$$

$$P_{\kappa}(k) = \frac{9H_0^2 \Omega_m^2}{4c^2} \int_0^{w_H} dw \frac{\bar{W}^2(w)}{a^2(w)} P_{\delta}\left(\frac{k}{f_K(w)}, w\right)$$

$$P_{\epsilon}(k) = \frac{1}{2} \frac{\sigma_{\epsilon_s}^2}{n_g}$$

# Lensing selection

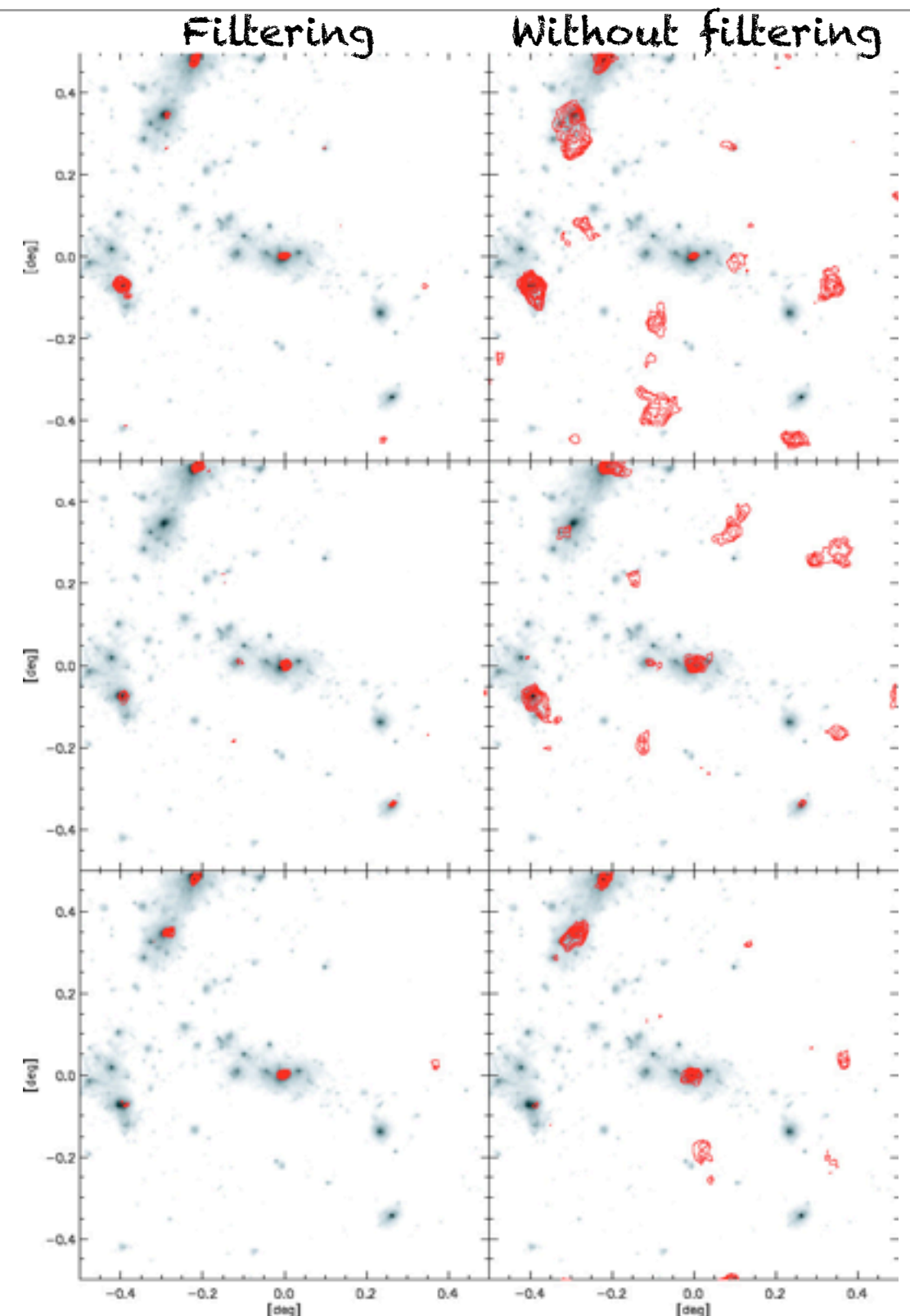
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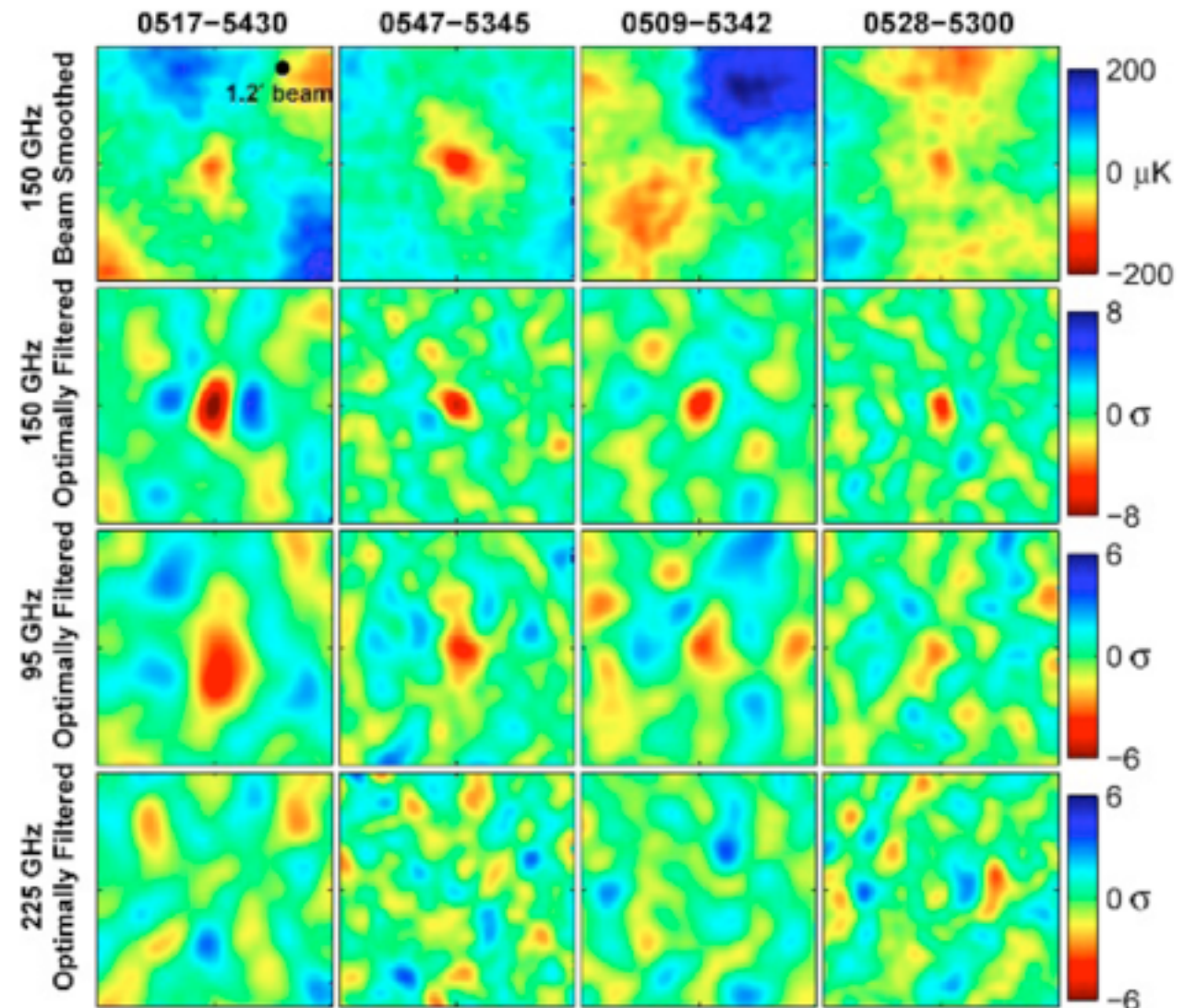


# SZ selection

- The SZ effect allows to observe clusters by measuring the distortion of the CMB spectrum owing to the hot ICM (inverse compton scattering of CMB photons by ICM electrons)
- Below 217GHz, clusters are revealed as intensity/temperature decrements of the CMB radiation
- The decrement is

$$\frac{\Delta T}{T} \propto y = \int n_e(r) \sigma_T \frac{k_B T_e(r)}{m_e c^2} dl$$

$$Y_{SZ} = \frac{\mu_e m_p m_e c^2}{\sigma_T} D_A^2 \int y d\Omega$$



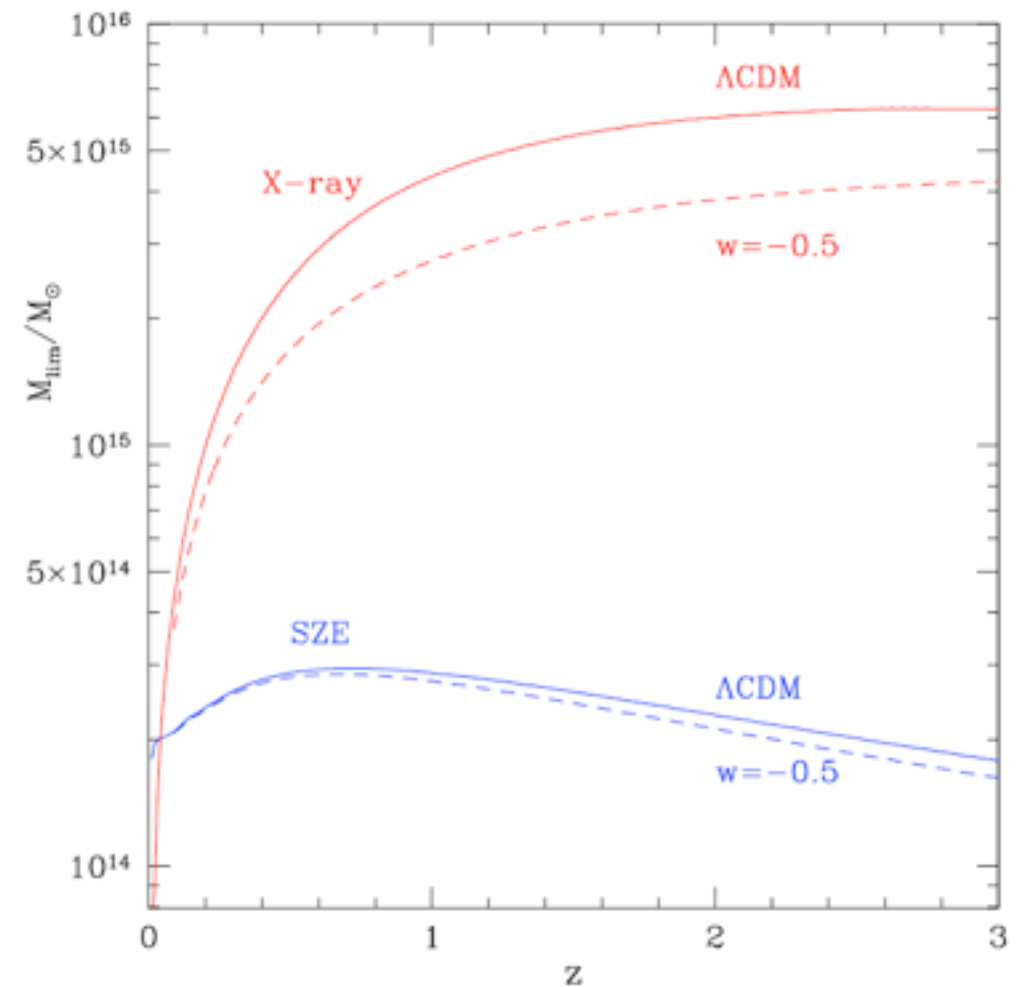
Staniszewski et al. 2009

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Advantages:

1. Independent of redshift! Lower mass limit
2. YSZ has a tight correlation with the mass

Disadvantages:

Similarly to lensing, possible contamination from background/foreground structures and point sources

# Methods to measure the mass of clusters

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- gravitational lensing
- X-ray
- Dynamical mass estimates

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- gravitational lensing
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- Dynamical mass estimates
- mass proxies

# X-ray mass estimates

---

The condition for hydrostatic equilibrium determines the balance between the pressure force and the gravitational force

$$\nabla P_{gas} = -\rho_{gas} \nabla \phi$$

Under the assumption of spherical symmetry this becomes

$$\frac{dP}{dr} = -\rho_{gas} \frac{d\phi}{dr} = -\rho_{gas} \frac{GM(< r)}{r^2}$$

Further using the equation of state of ideal gas to relate pressure to gas density and temperature, we obtain

$$M(< r) = -\frac{r k_B T}{G \mu m_p} \left( \frac{d \ln \rho_{gas}}{d \ln r} + \frac{d \ln T}{d \ln r} \right)$$



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# Dynamical masses

---

Other methods to measure the cluster masses are based on the assumption that the cluster is spherical and in dynamical equilibrium. Galaxies are bound by gravity, i.e. they trace the gravitational potential of the cluster.

Applying the virial equilibrium:  $\frac{GM}{R} = \sigma^2 \Rightarrow M = \frac{\sigma^2 R}{G}$

If a large number of galaxy spectra is available to measure the velocity dispersion profile, we can apply the Jeans equation for steady-state spherical systems.

$$M(< r) = -\frac{\langle v_r^2 \rangle r}{G} \left[ \frac{d \ln \rho_m}{d \ln r} + \frac{d \ln \langle v_r^2 \rangle}{d \ln r} + 2\beta(r) \right]$$

$$\beta(r) = 1 - \frac{\langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle}{2\langle v_r^2 \rangle}$$

Problems: requires the assumption of a relation between galaxy number density profile and mass density profile and we usually don't know  $\beta(r)$ .

# Self-similar model

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The simplest model to explain the physics of the ICM is based on the assumption that gravity only determines the thermodynamical properties of the hot diffuse gas.

Gravity has no preferred scale, thus, under this approximation galaxy clusters should be self-similar (Kaiser 1986), and clusters of different sizes should be scaled versions of each other.

# Self-similar scaling relations

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At redshift  $z$ , we define the mass

$$M_{\Delta_c} \propto \rho_c(z) \Delta_c r_{\Delta_c}^3 \quad \rho_c(z) = \rho_{c,0} E^2(z) \quad E(z) = [(1+z)^3 \Omega_m + \Omega_\Lambda]^{1/2}$$

Thus, the cluster size scales as  $r_{\Delta_c} \propto M_{\Delta_c}^{1/3} E^{-2/3}(z)$

Assuming hydrostatic equilibrium, this implies:

$$M_{\Delta_c} \propto T^{3/2} E^{-1}(z) \quad \text{M-T relation}$$

The X-ray luminosity is

$$L_X = \int_V \left( \frac{\rho_{gas}}{\mu m_p} \right)^2 \Lambda(T) dV$$

$$L_X \propto M_{\Delta_c} \rho_c T^{1/2} \propto T^2 E(z) \quad \text{L-T relation}$$

$$\Lambda(T) \propto T^{1/2}$$

$$L_X \propto M_{\Delta_c}^{4/3} E^{7/3}(z)$$

L-M relation

Assuming  $\rho_{gas}(r) \propto \rho_m(r)$

# Self-similar scaling relations

---

As for the SZ signal:

$$Y_{SZ} \propto D_A^2 \int y d\Omega \propto \int T n_e d^3r \propto M_{gas} T \propto f_{gas} M_{\Delta_c} T$$

And we obtain:

$$Y_{SZ} \propto f_{gas} T^{5/2} E^{-1}(z)$$

$Y$ - $T$  relation

$$Y_{SZ} \propto f_{gas} M_{\Delta_c}^{5/3} E^{2/3}(z)$$

$Y$ - $M$  relation

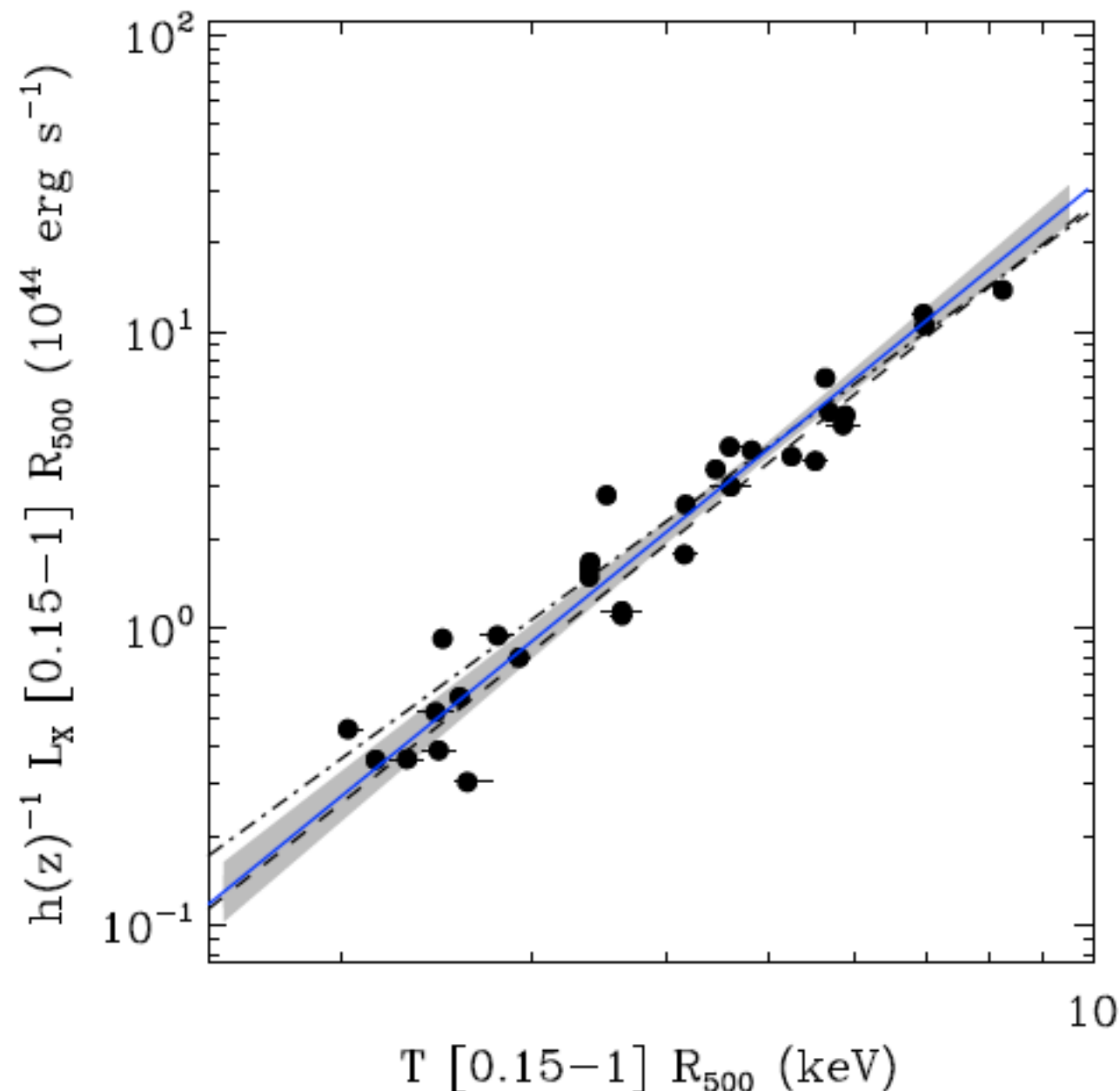
$$Y_{SZ} \propto f_{gas}^{-2/3} M_{gas}^{5/3} E^{2/3}(z)$$

$Y$ - $M_{gas}$  relation

If clusters were self-similar, we might use several observables ( $L_x$ ,  $T_x$ ,  $M_{gas}$ ,  $Y_{SZ}$ ) to infer the mass using these scaling relations, but...

# Phenomenological scaling relations

...several evidences AGAINST self-similarity!



Ex.: the L-T relation is found to be steeper than predicted from the self similar model.

$$E^{-1}(z)L_X \propto T^\alpha$$

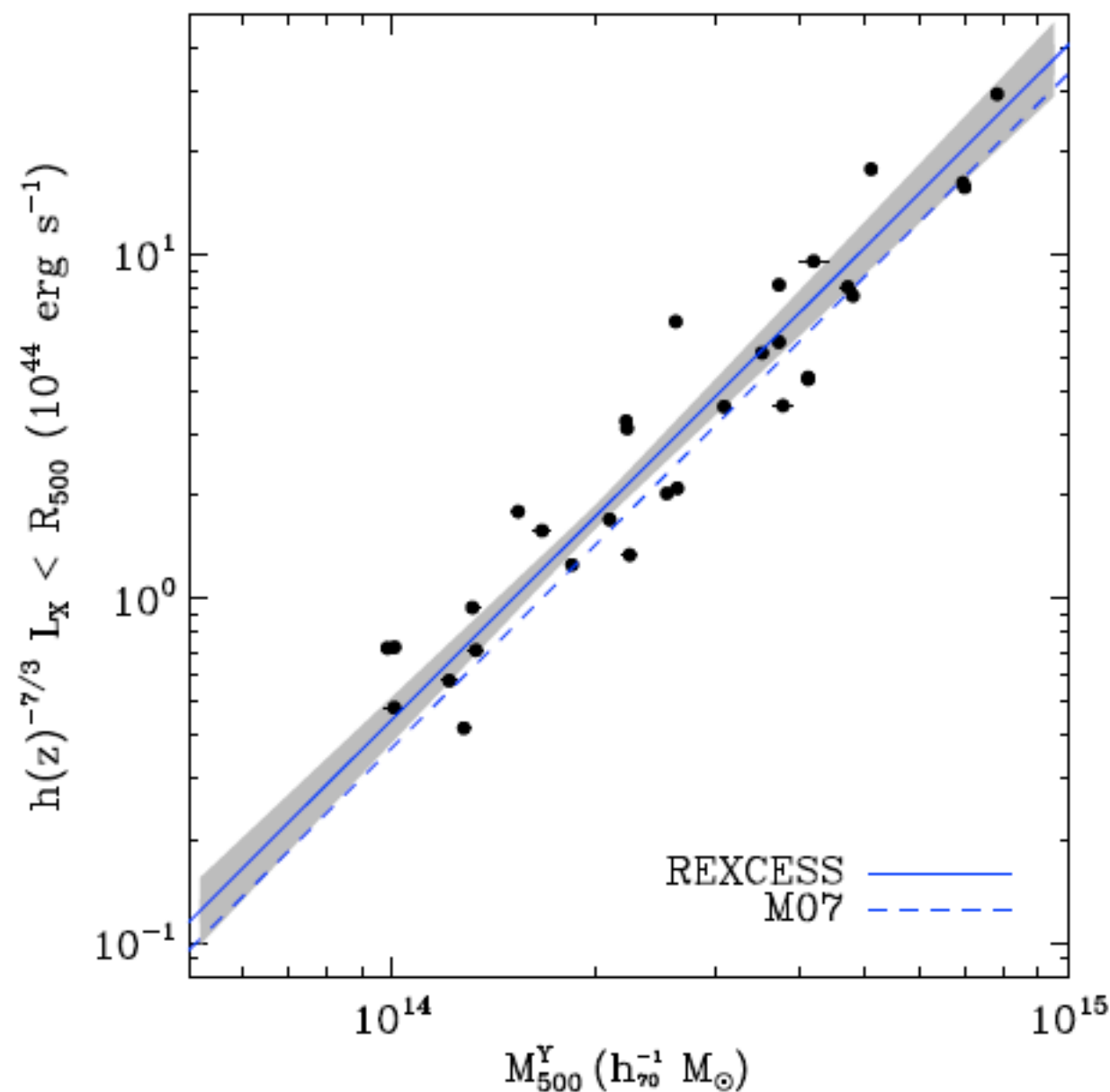
with  $\alpha=2.5-3$  (self-similar slope is 2)

Similarly, the observed L-M relation is steeper than expected from self-similarity ( $\sim 1.8-1.9$  vs 1.33)

Pratt et al. 2009: local L-T relation from the REXCESS sample

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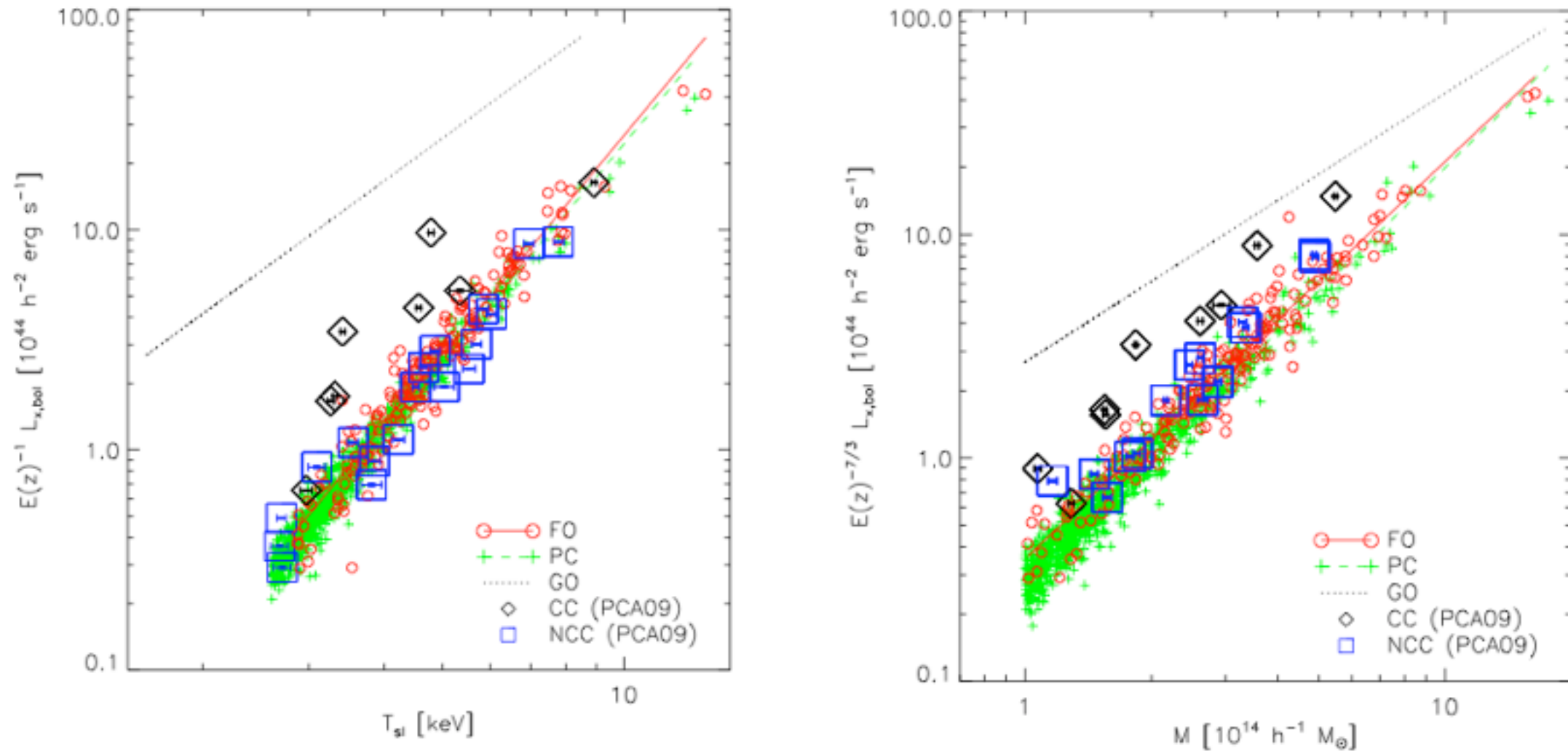
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# What is breaking the self-similarity?

Short et al. 2010

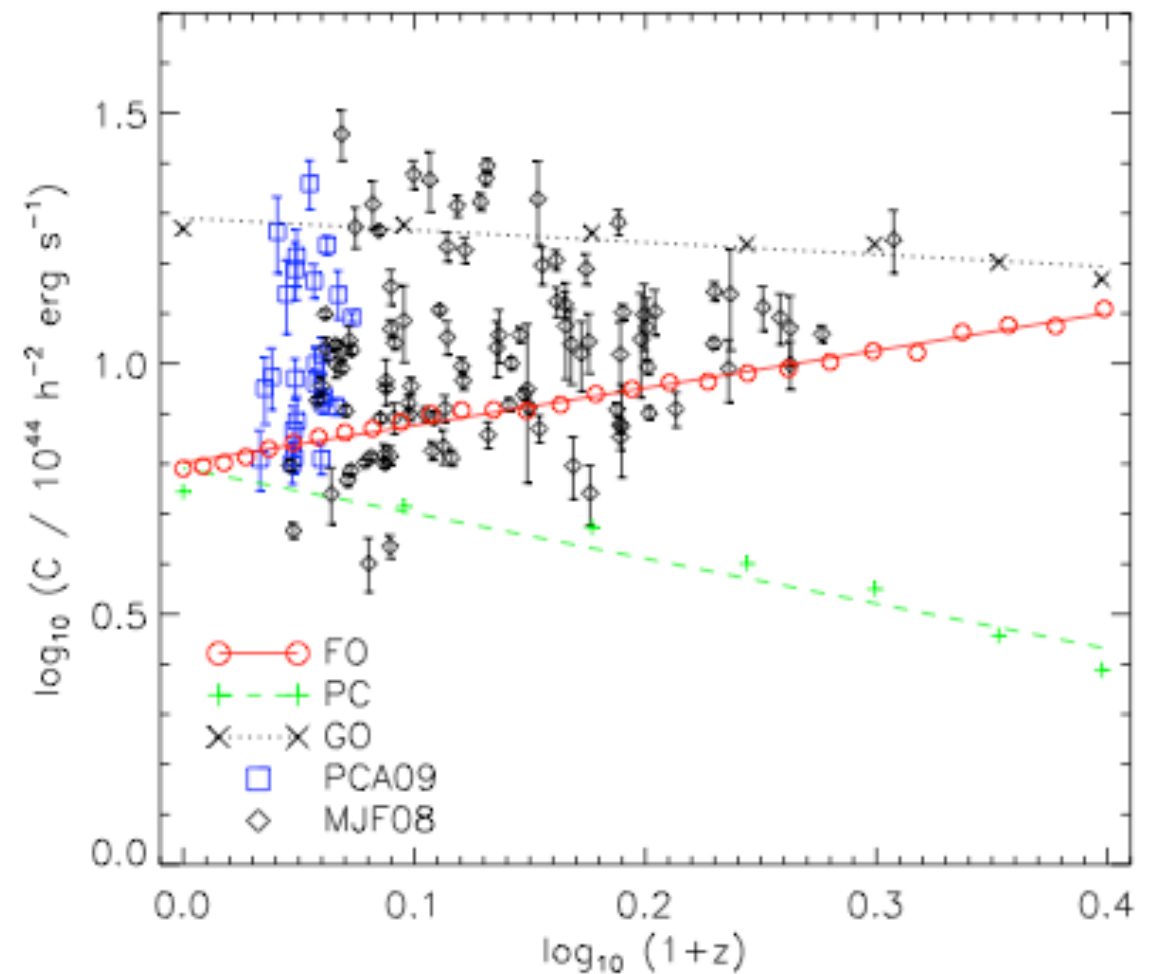
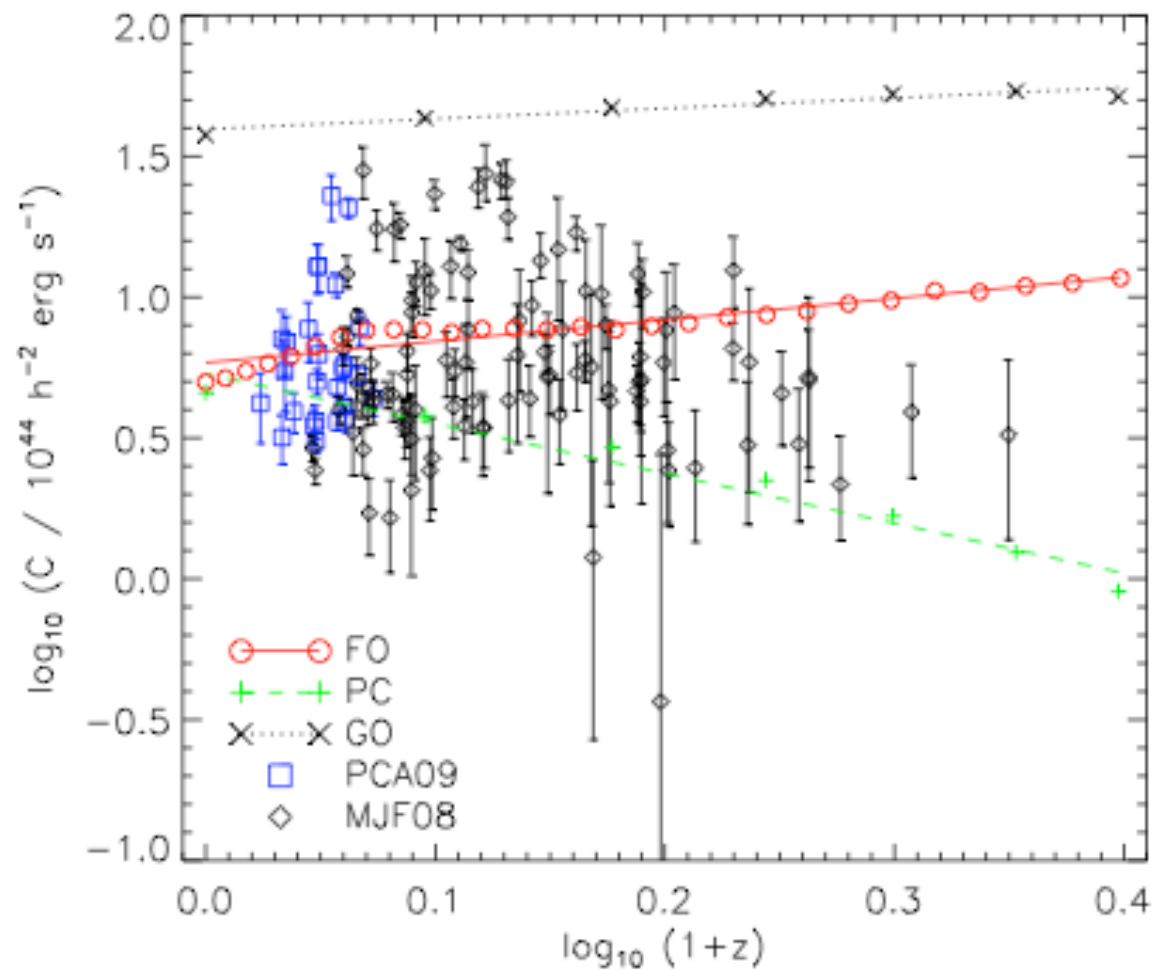


Departure from self-similarity points toward the presence of some mechanism that significantly affects the ICM thermodynamics (cooling, heating, feedback processes). See review by Borgani et al. 2008.



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Short et al. 2010



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# Can we use the scaling relations?

**The positive news:** well defined relations exist that can be used for obtaining mass estimates from easily accessible quantities!

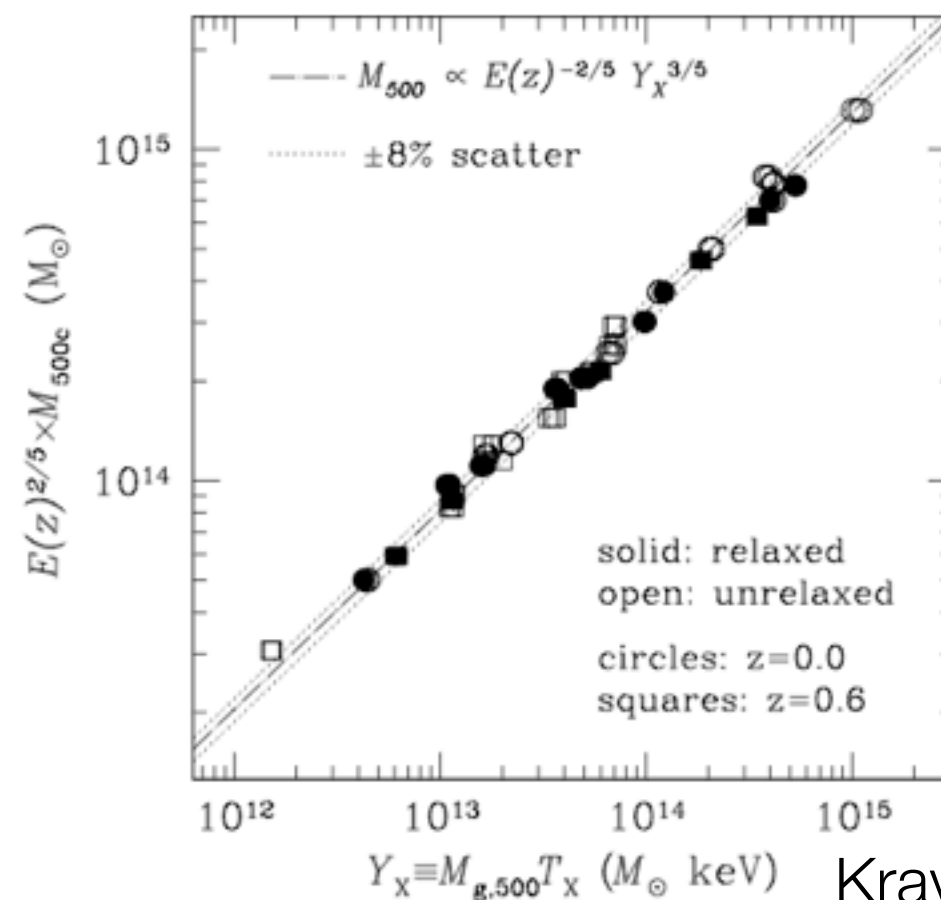
Some of these relations are supposed to have a smaller scatter, and thus to be preferable. For example the relations:

$$M_{\Delta_c} \propto Y_{SZ} = M_{gas} \times T$$

$$M_{\Delta_c} \propto Y_X = M_{gas} \times T_X$$

$$M_{\Delta_c} \propto M_{gas}$$

$$M_{\Delta_c} \propto T$$

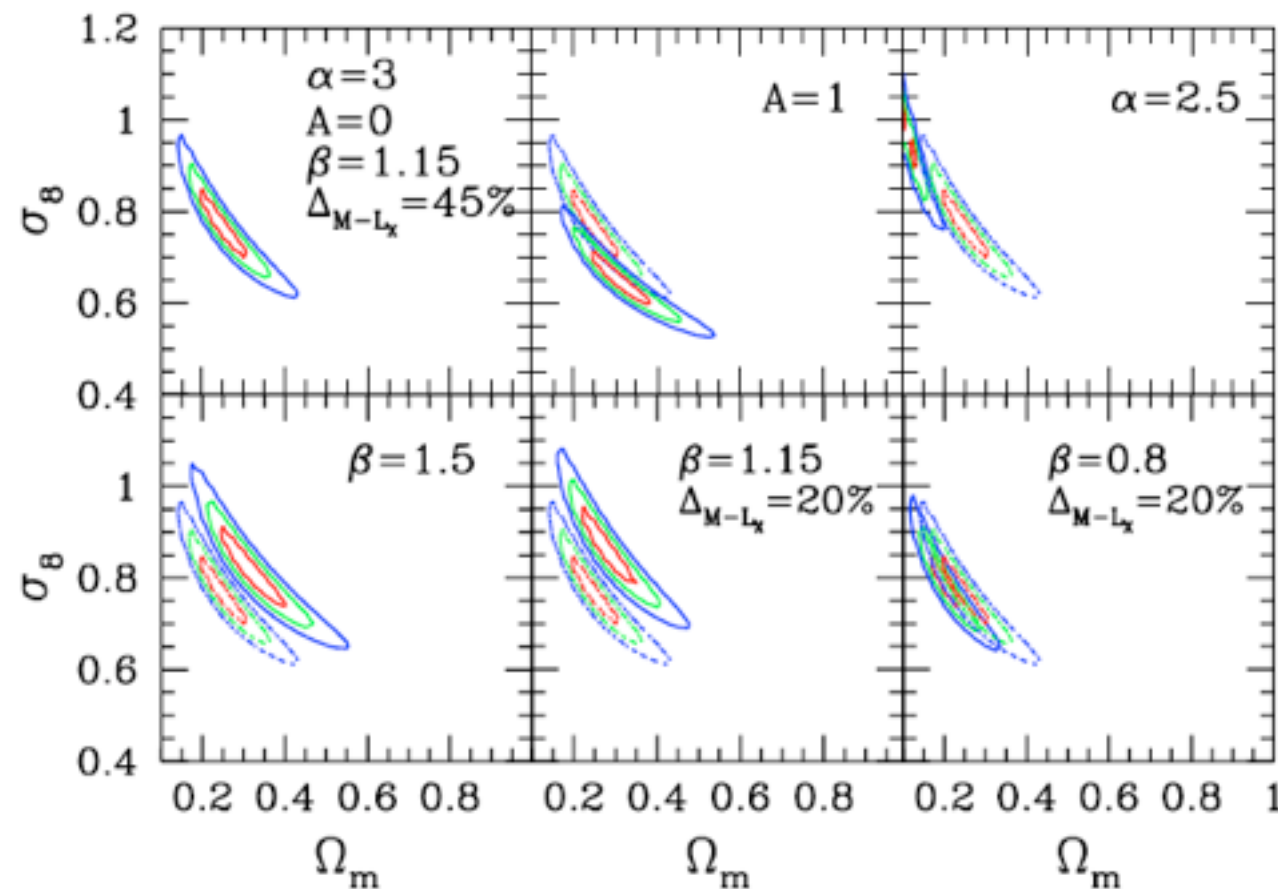


Kravstov et al. 2006

# Can we use the scaling relations?

**The negative news:** the scaling relations need to be calibrated!

Thus, it is fundamental to use robust methods to accurately measure the masses of control samples of galaxy clusters and to use these measurements for the calibrations.



Borgani et al. 2001

# Which mass to use?

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Method\Scale	Core	$R_{2500}$	$R_{500}$	$R_{200}$
<b>Galaxy dynamics</b>		X	X	
<b>X-ray</b>		X	X	
<b>Strong lensing</b>	X			
<b>Weak lensing</b>		X	X	X
<b>WL+SL</b>	X	X	X	X

**Require dynamical equilibrium**

**No equilibrium required but measure 2D masses**

# Is the assumption of equilibrium valid?

---

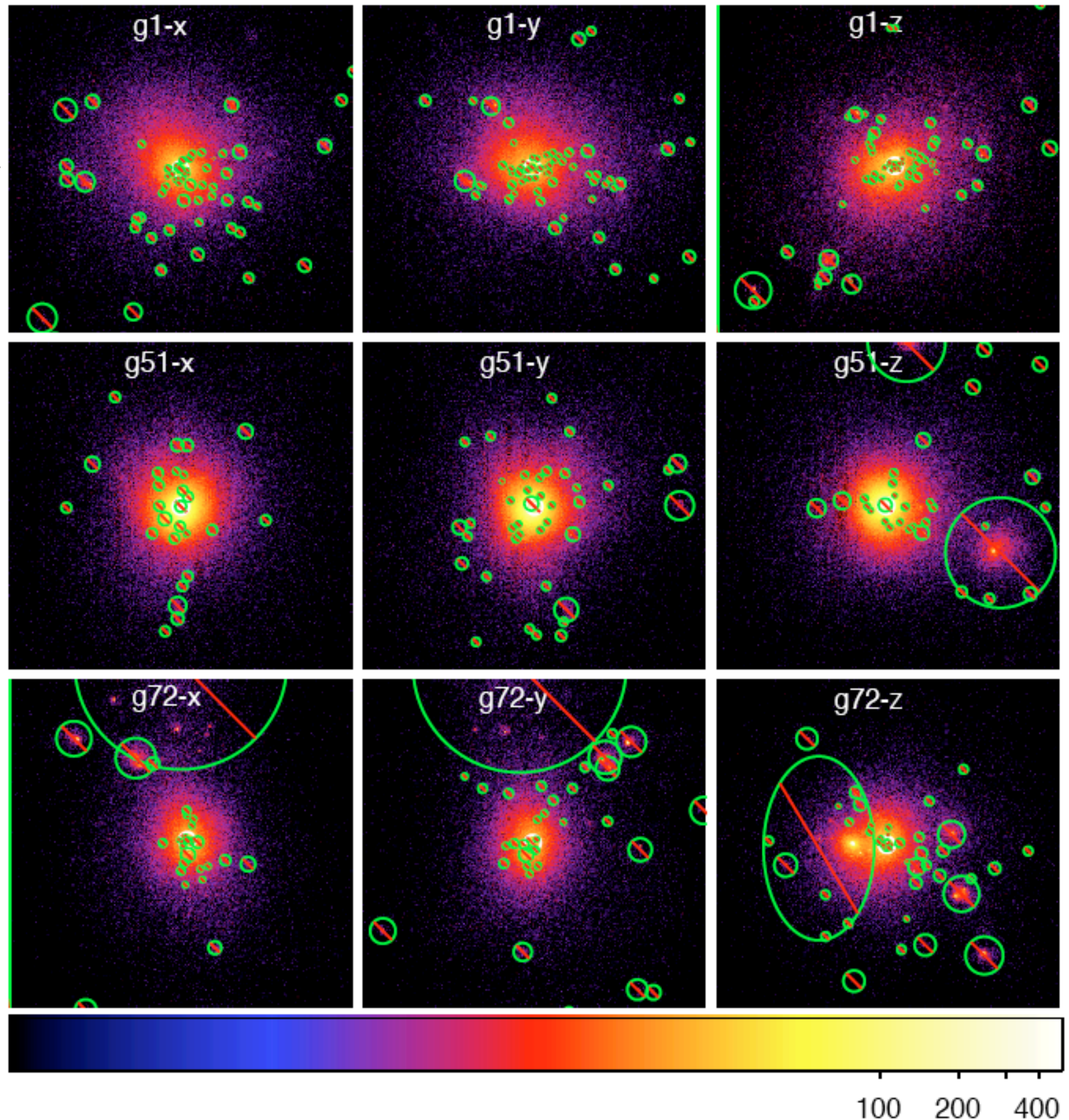
Let's try check it applying X-ray techniques to the analysis of simulated clusters...



# XMAS2

(Gardini et al. 2004, Rasia et al. 2007)

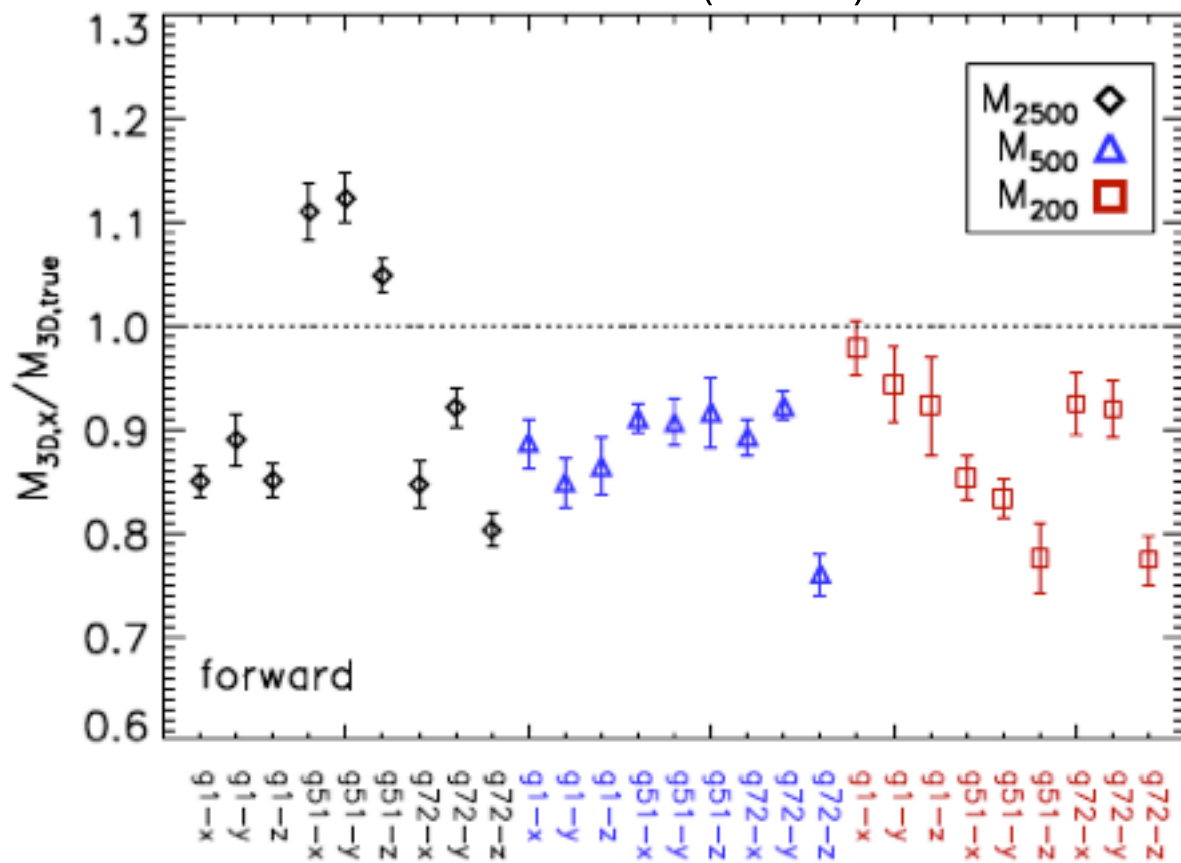
X-ray simulator  
Reads input hydro sim.  
and produces Chandra and XMM images of clusters



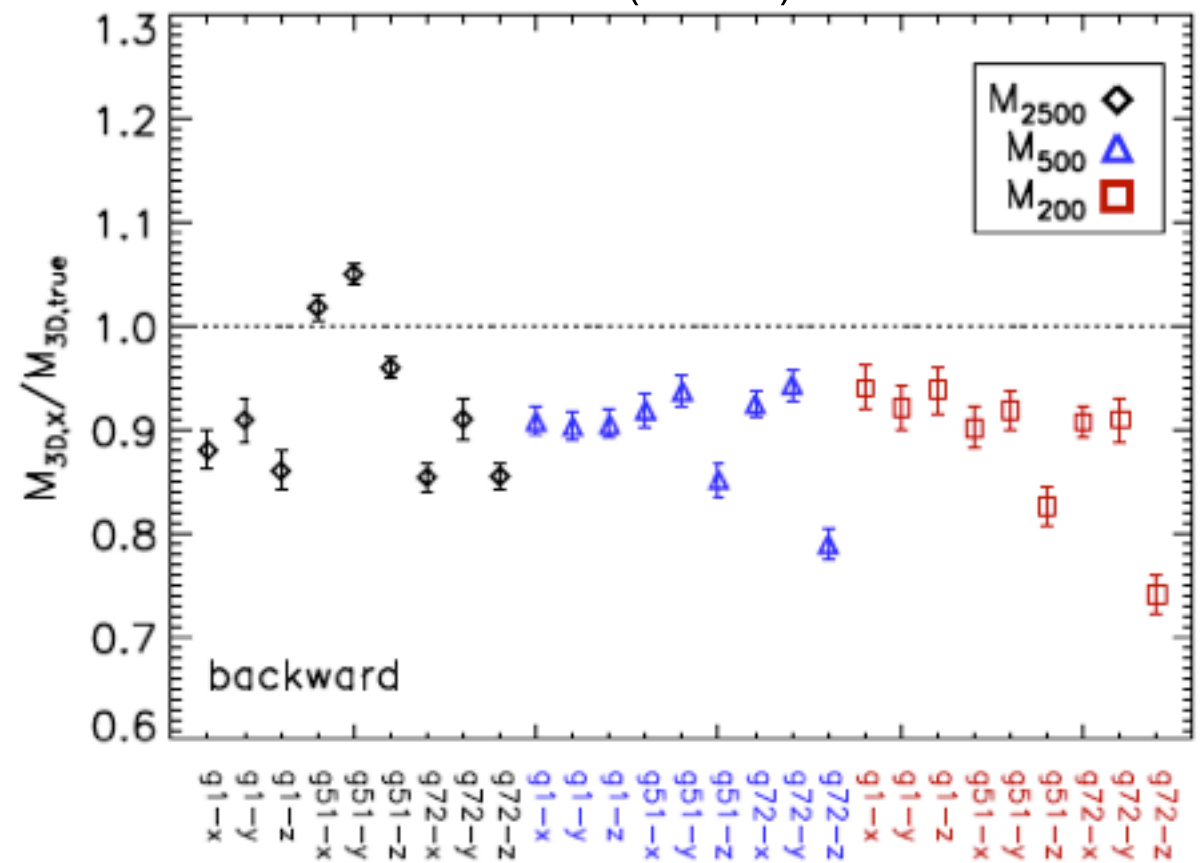


# X-ray (total) masses

à la Vikhlinin et al. (2006)



à la Ettori et al. (2004)

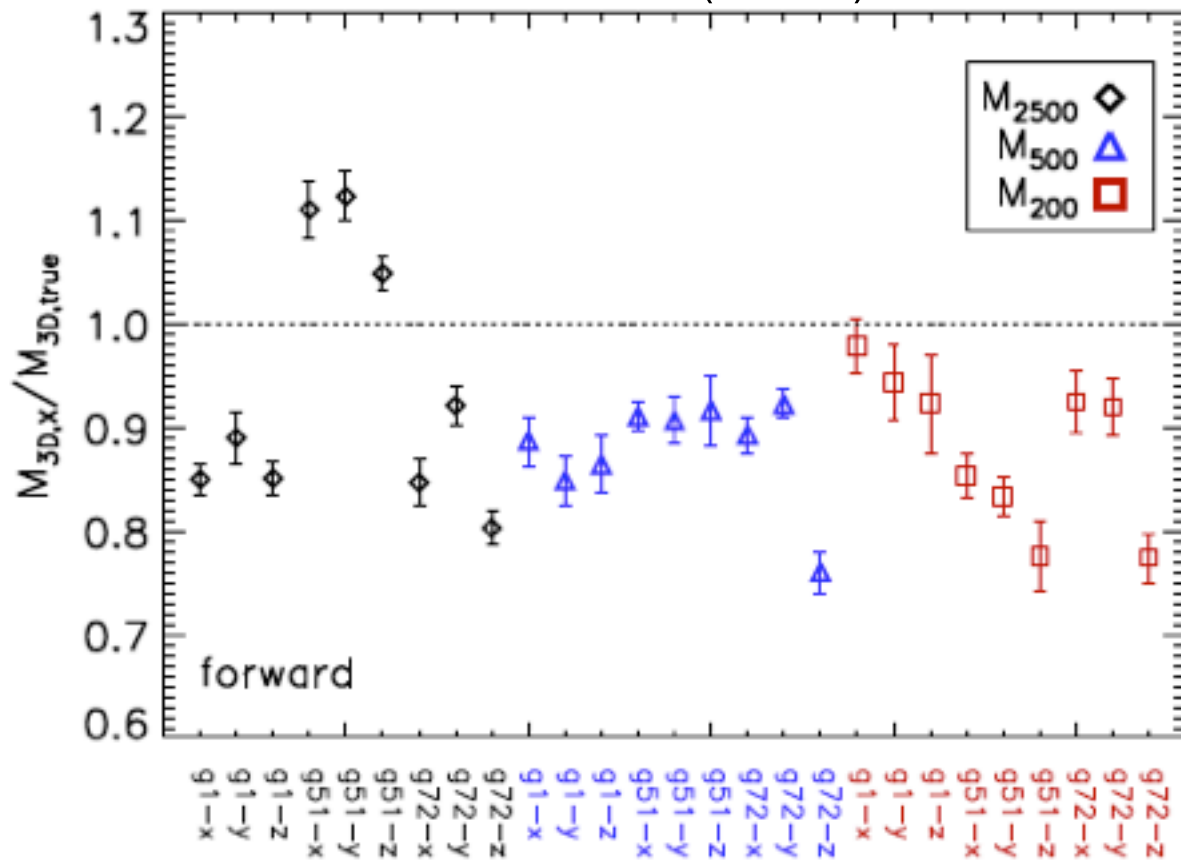


The X-ray total mass is under-estimated by 10-20%: this is in agreement with several other numerical studies, where it has been shown that gas bulk motions provide non-thermal pressure support (e.g. Rasia et al. 2004, 2007; Nagai et al. 2007; Piffaretti & Valdarnini 2008; Ameglio et al. 2009)

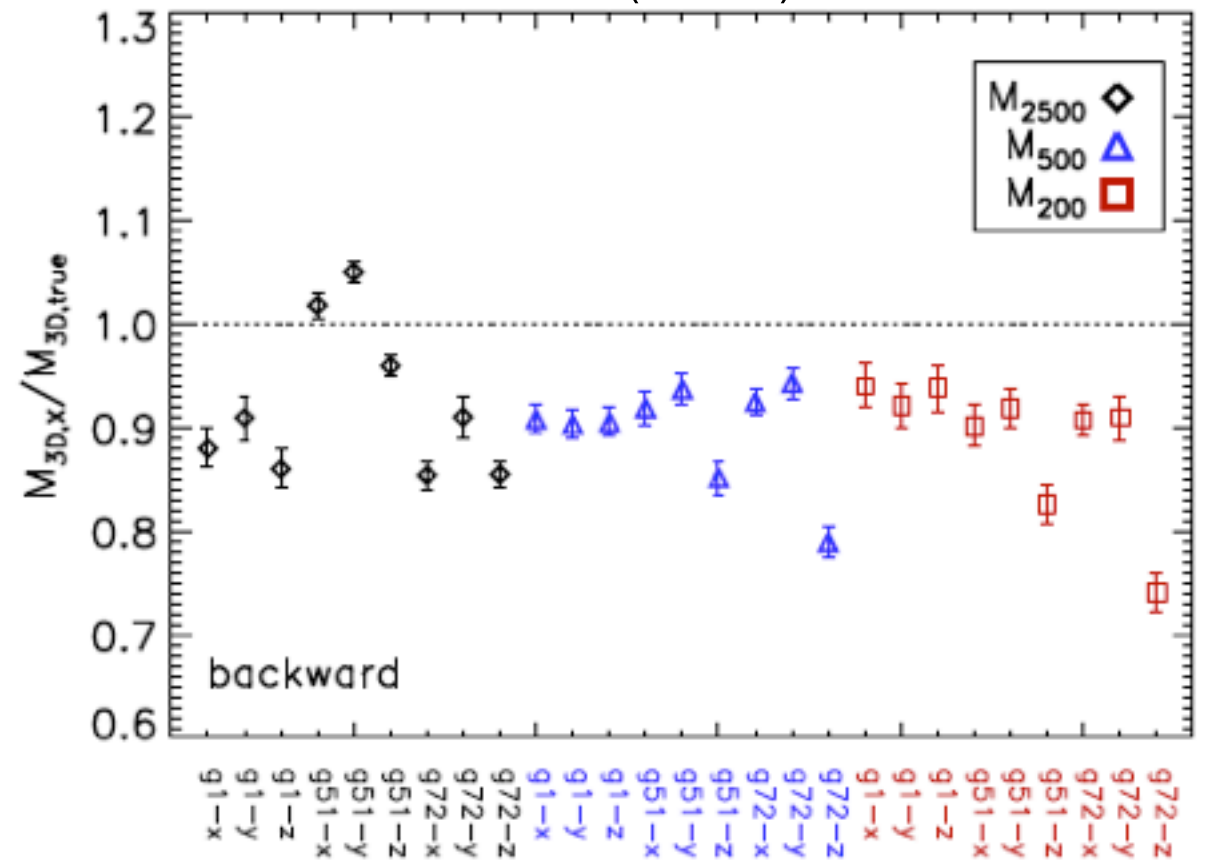


# X-ray (total) masses

à la Vikhlinin et al. (2006)



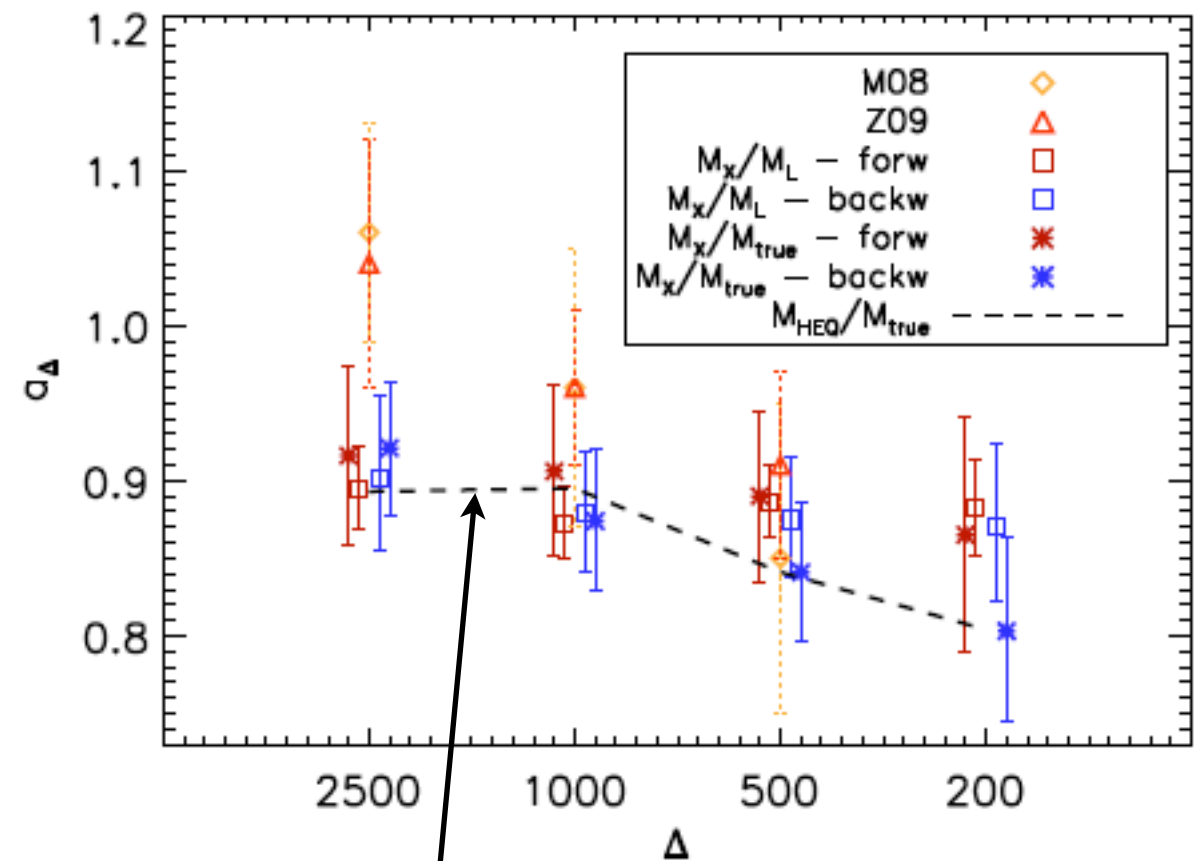
à la Ettori et al. (2004)



$M_{3D,X}^{\text{forw}} / M_{3D,\text{true}}$	2500	0.9241	0.1147
$M_{3D,X}^{\text{forw}} / M_{3D,\text{true}}$	500	0.8811	0.0538
$M_{3D,X}^{\text{forw}} / M_{3D,\text{true}}$	200	0.8947	0.0625
$M_{3D,X}^{\text{back}} / M_{3D,\text{true}}$	2500	0.9205	0.0690
$M_{3D,X}^{\text{back}} / M_{3D,\text{true}}$	500	0.8961	0.0582
$M_{3D,X}^{\text{back}} / M_{3D,\text{true}}$	200	0.8832	0.0664

# Measuring deviations from hydrostatic equilibrium

- If lensing provides an unbiased estimate of the mass, the comparison between X-ray and lensing masses can reveal deviations from hydrostatic equilibrium
- Attempted by Mahdavi et al. (2008 -CCCP) and Zhang et al. (2009-LoCuSS): both find a decrement of  $M_x/M_L$  towards large radii
- Is this trend measurable despite the scatter introduced by triaxiality and substructures?



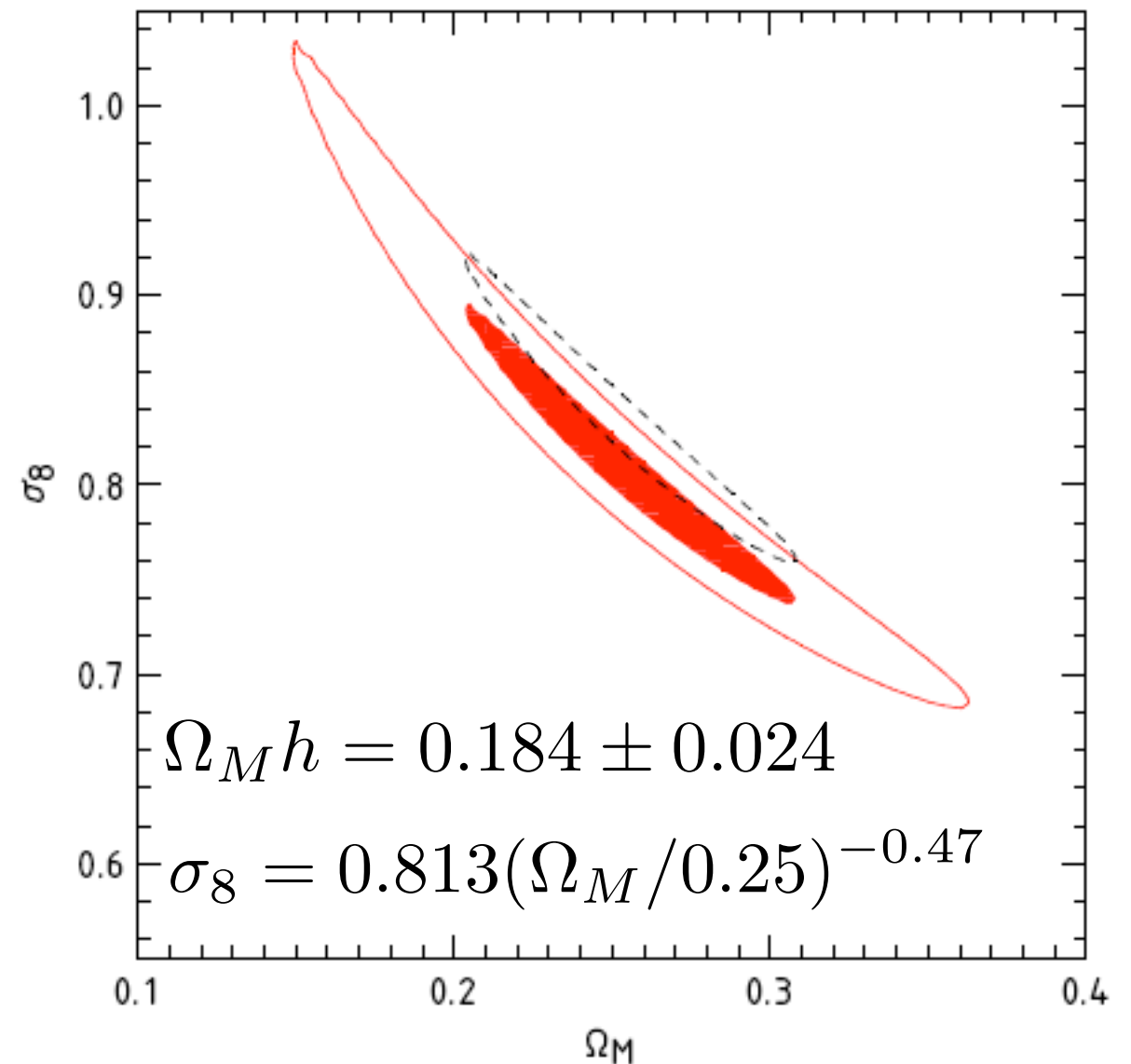
Simulation - no observational noises

$$-G\mu m_p \frac{n_{\text{gas}} M_{\text{tot}}(< r)}{r^2} = \frac{d(n_{\text{gas}} \times T_{3D})}{dr}$$

# Example: Vikhlinin et al. 2009

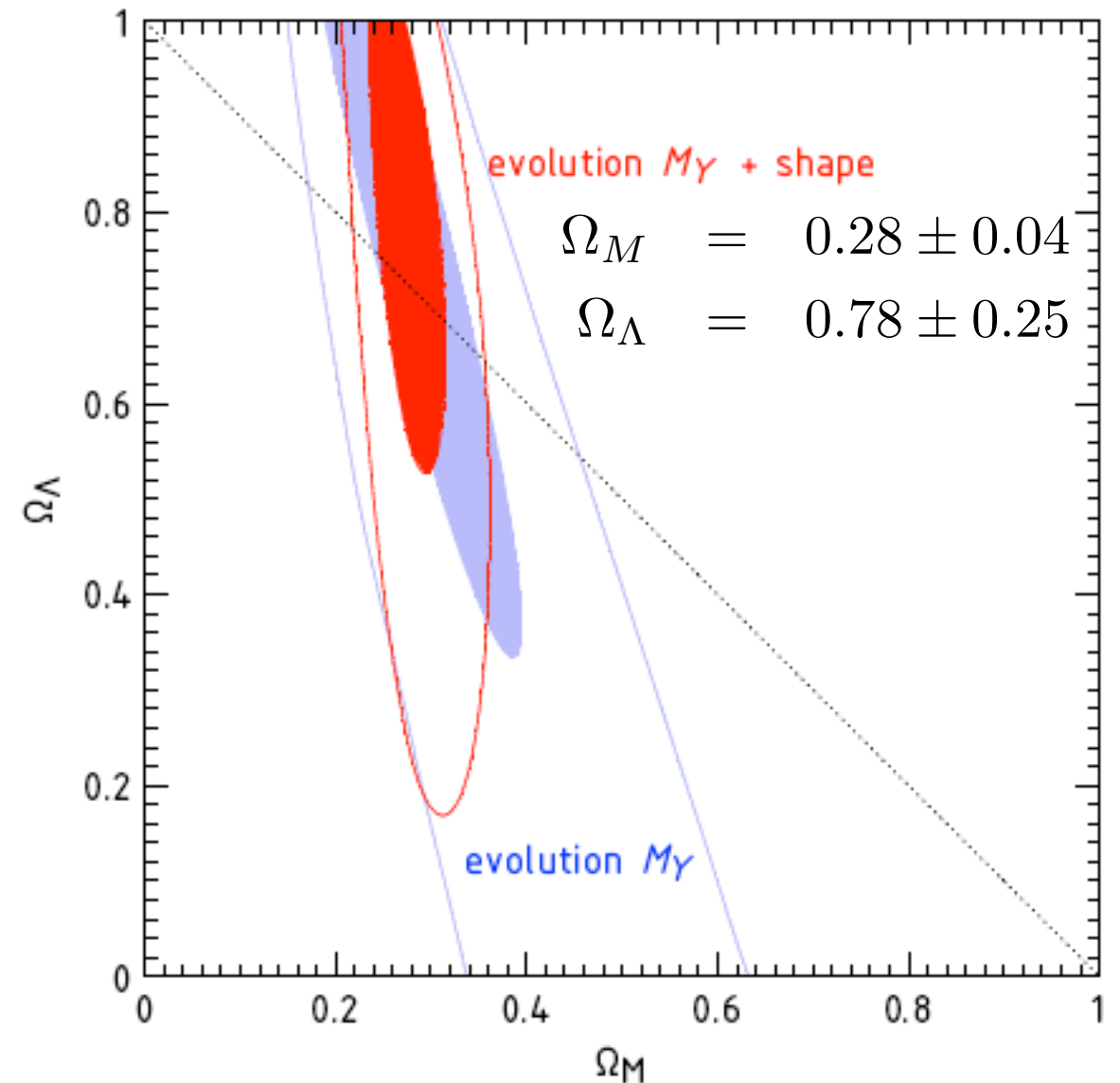
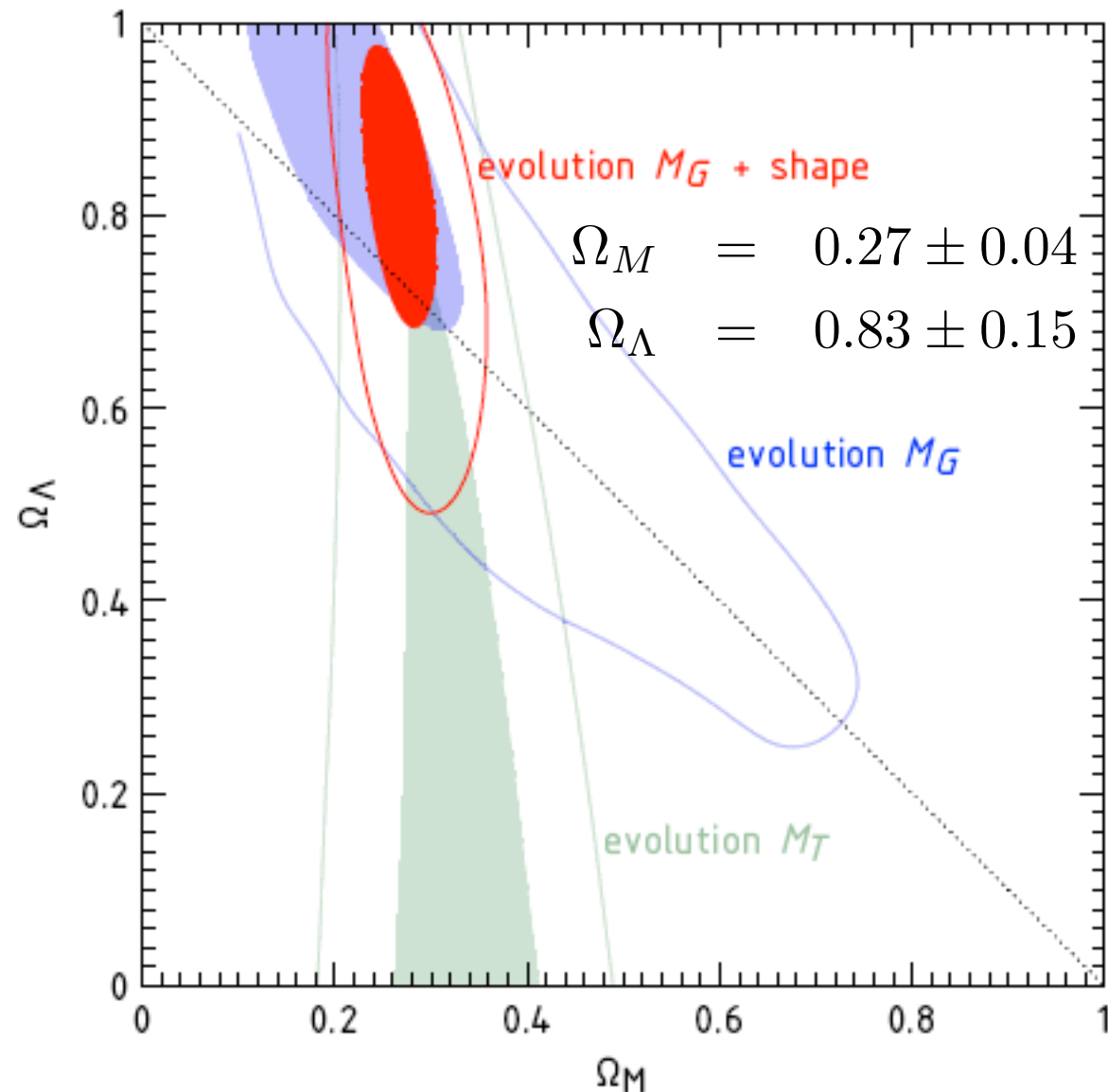
Vikhlinin et al. (2009) have recently used Chandra observations of two samples of clusters to apply the techniques discussed so far:

- one sample of 49 nearby clusters ( $z \sim 0.05$ ) detected in the RASS
- one sample of 37 clusters at  $\langle z \rangle = 0.55$  derived from the 400 deg<sup>2</sup> Rosat serendipitous survey
- using  $Y_x$ ,  $M_{\text{gas}}$ , and  $T_x$  as mass proxies to study the shape and the evolution of the MF



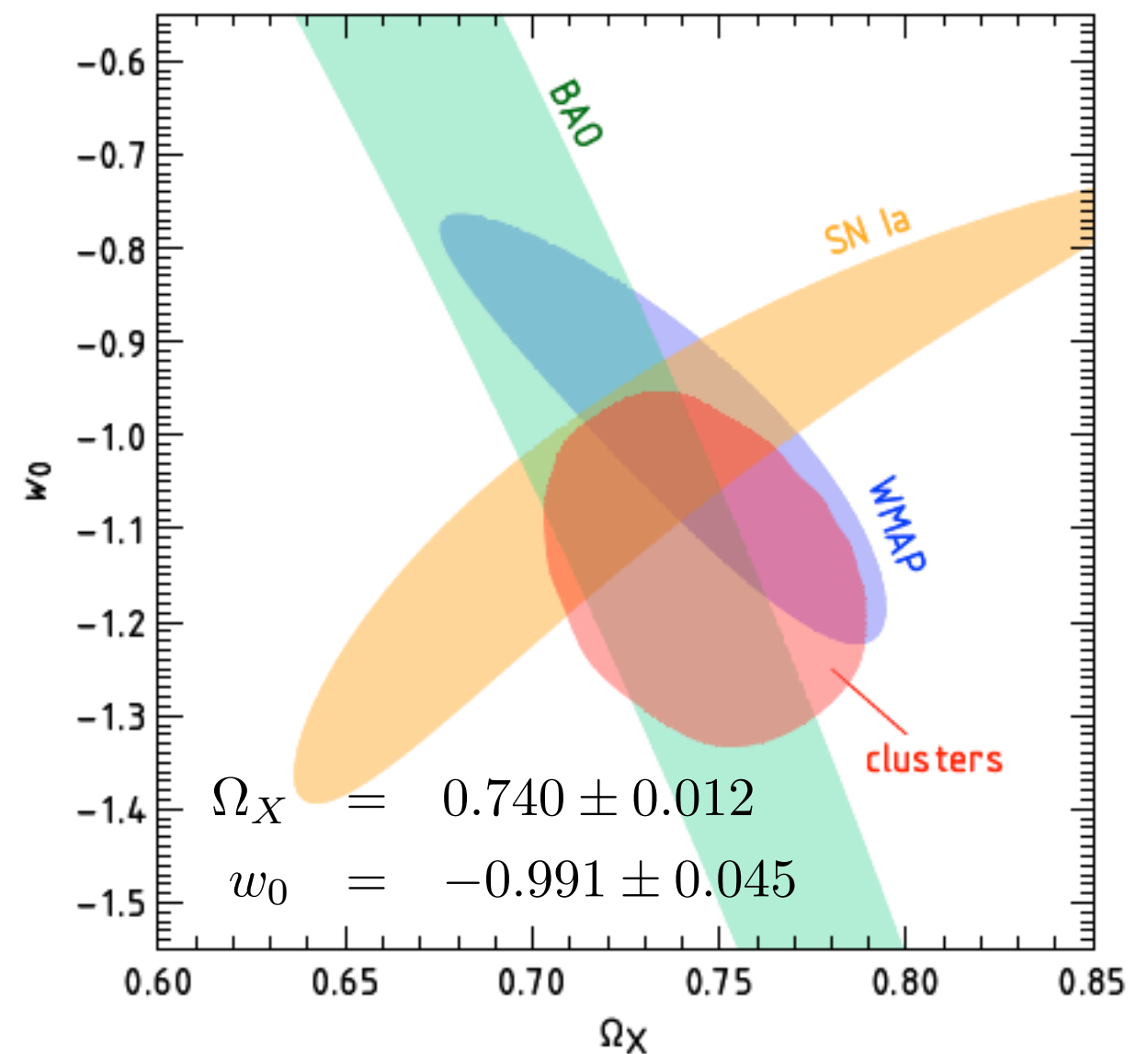
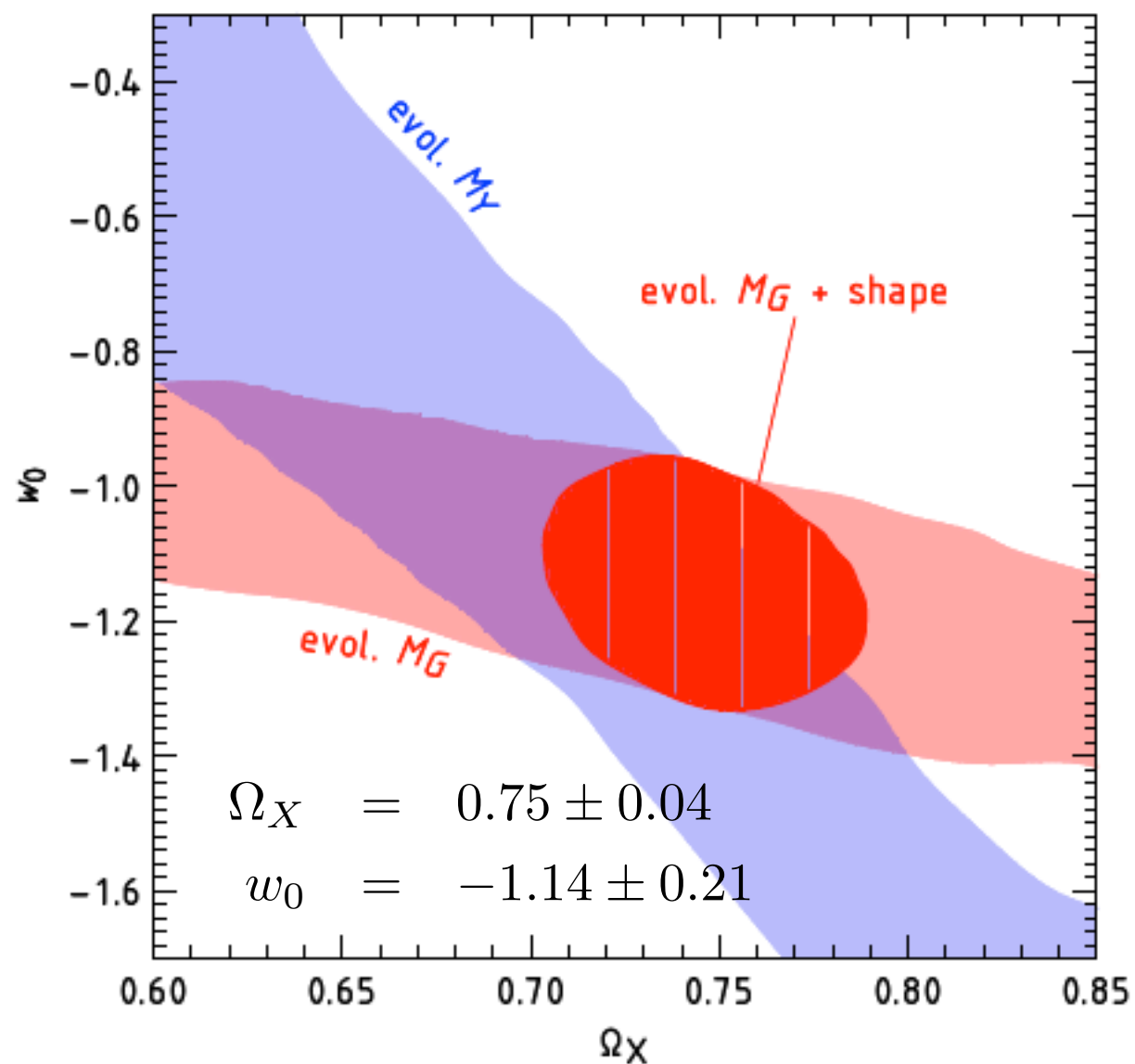
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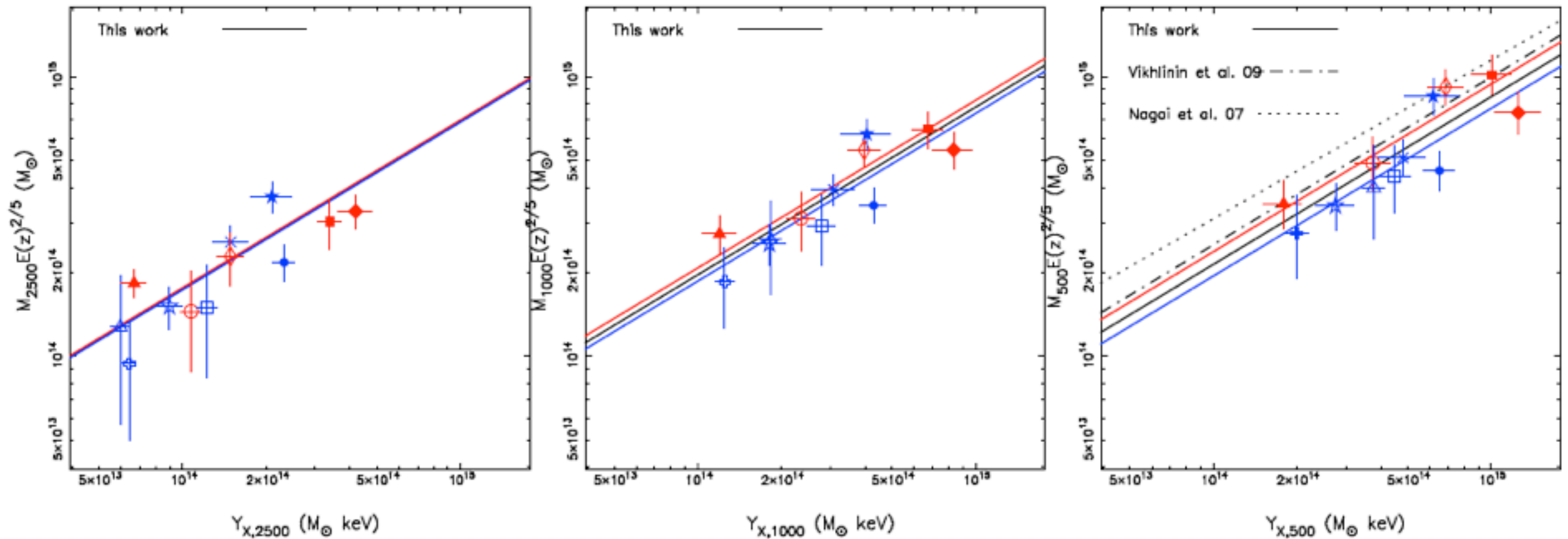


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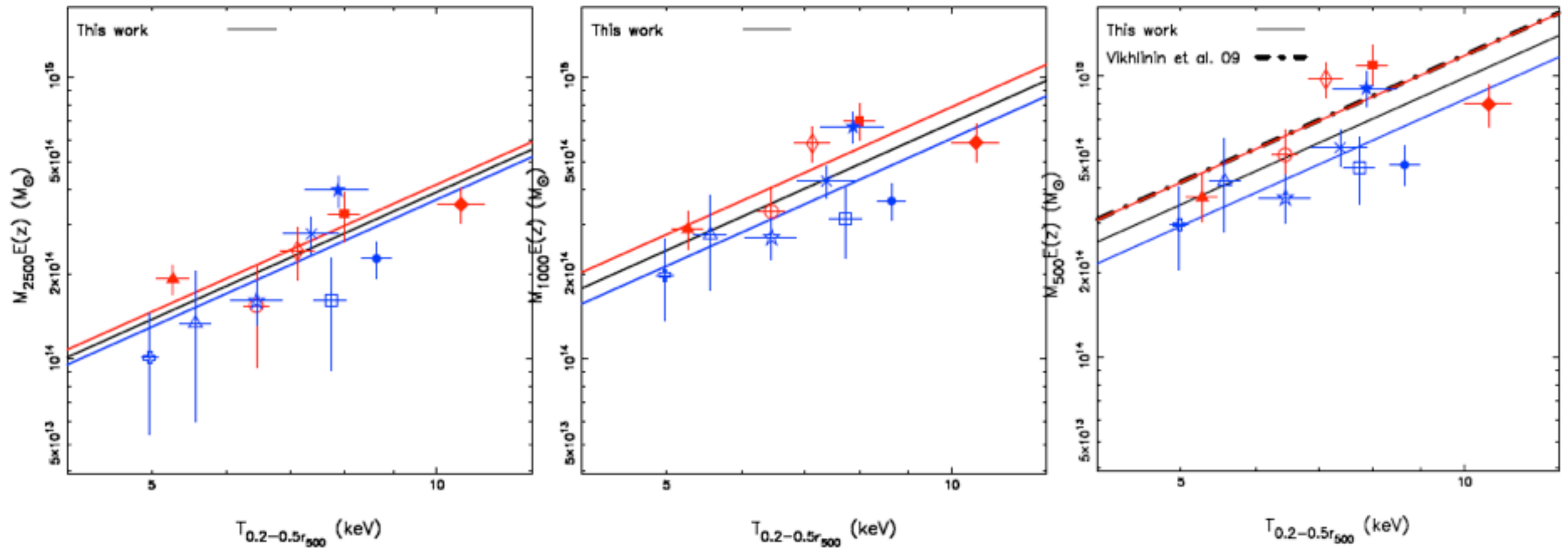
# Recent results from LOCUSS



- Structural segregation
- Importance of overdensity radius
- Differences with simulations



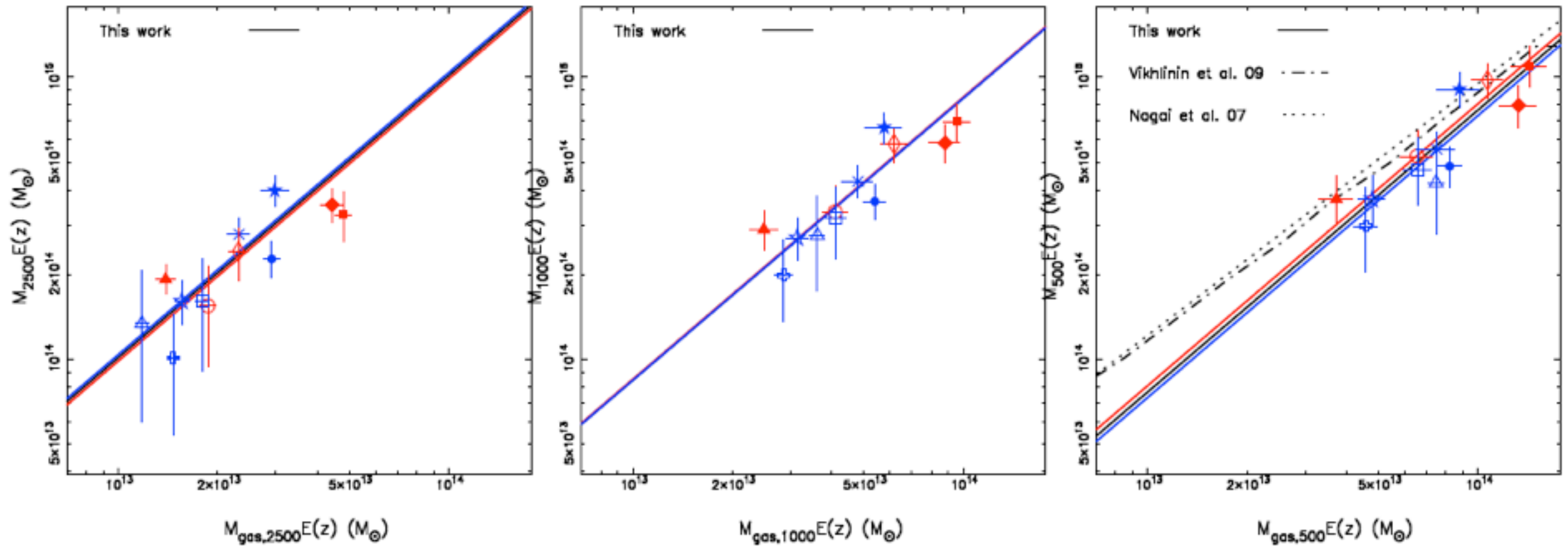
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