Lecture 2: Cluster mass reconstructions using lensing

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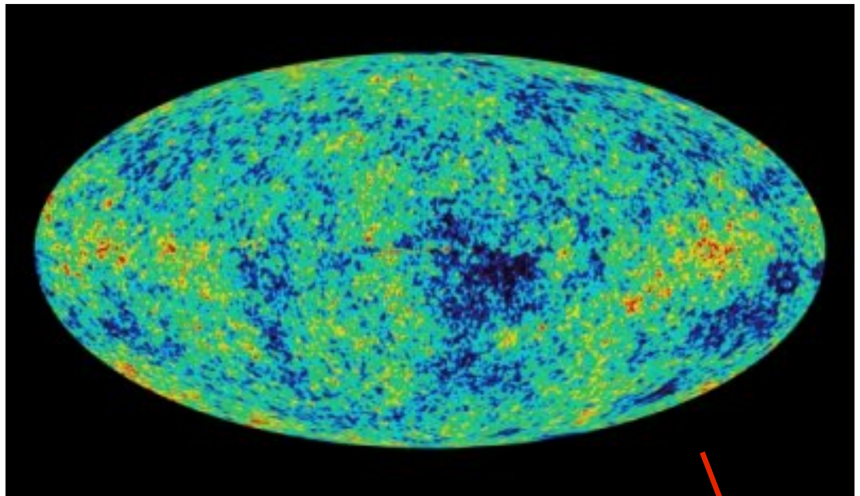
Galaxy clusters

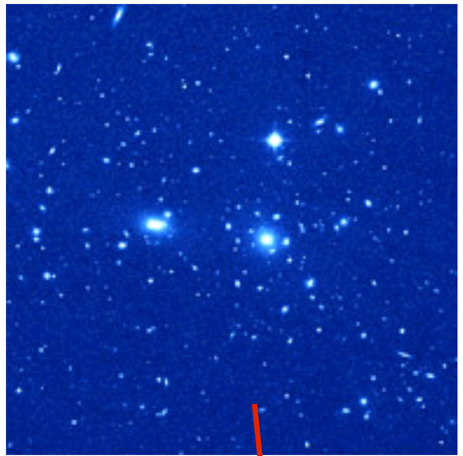
- concentrations of 100-1000 galaxies
- vel. dispersions ~1000 km/s
- Size: R~1 Mpc (t_{cross} ~ R/σ~1 Gyr < t_H~10h^{-1} Gyr)
- Mass $M \simeq \frac{R\sigma_v^2}{G} \simeq \left(\frac{R}{1}\right)\left(\frac{\sigma_v}{10^3}\right)^2 10^{15}\, h^{-1} M_\odot$
- Components: DM (~85%), BARYONS (~15%)
- Hot ICM: T_X~3-10 keV, n_{gas}~10^{-3} atoms/cm^3, Z~0.3 solar: fully ionized gas emitting via thermal Bremsstrahlung + line emission

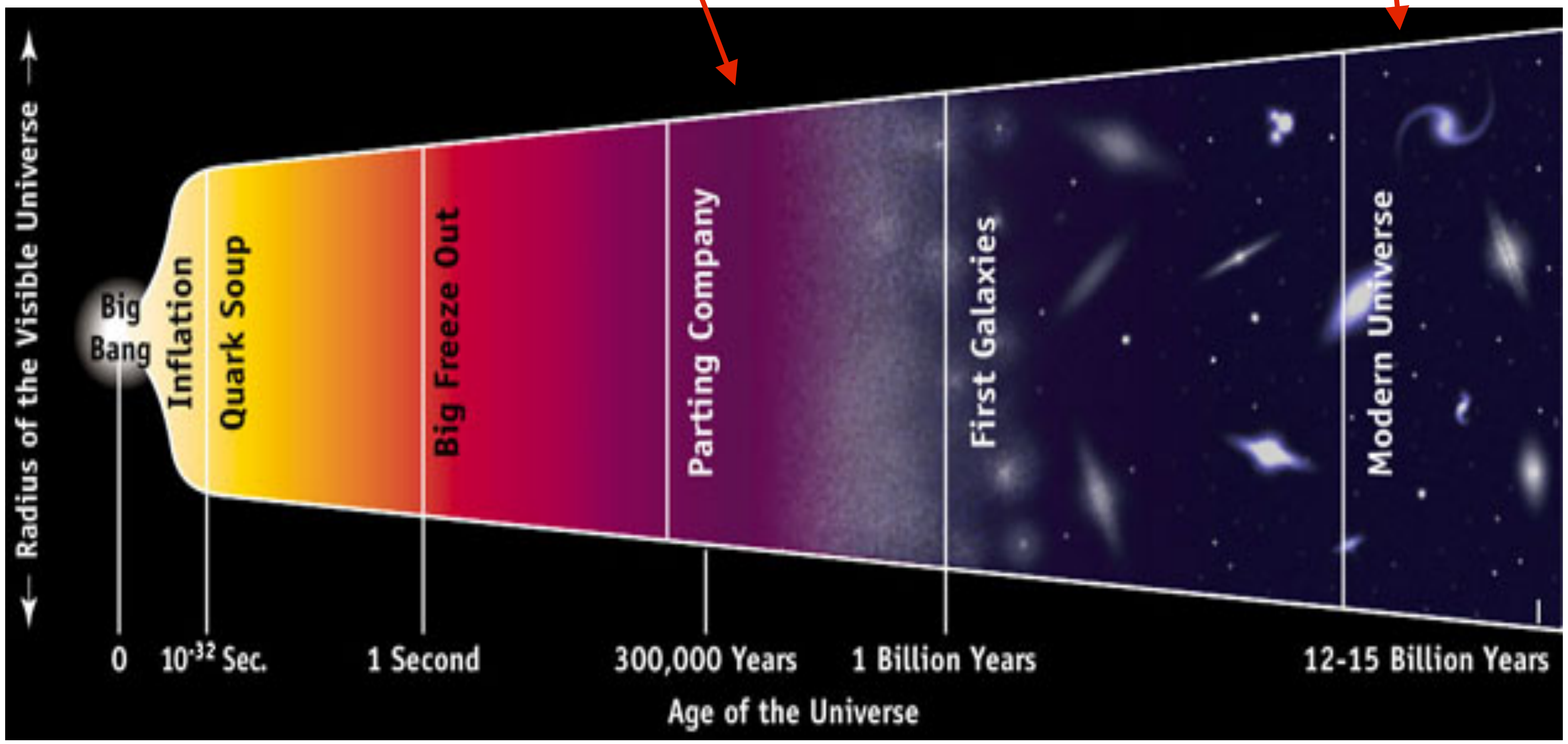
$L_X \sim n_{gas}^2(T)\ \Lambda(T) \sim 10^{43}$-$10^{45}$ erg/s

Good reasons to study galaxy clusters

- powerful cosmological tools
- ideal laboratories for testing the predictions of CDM on small scales







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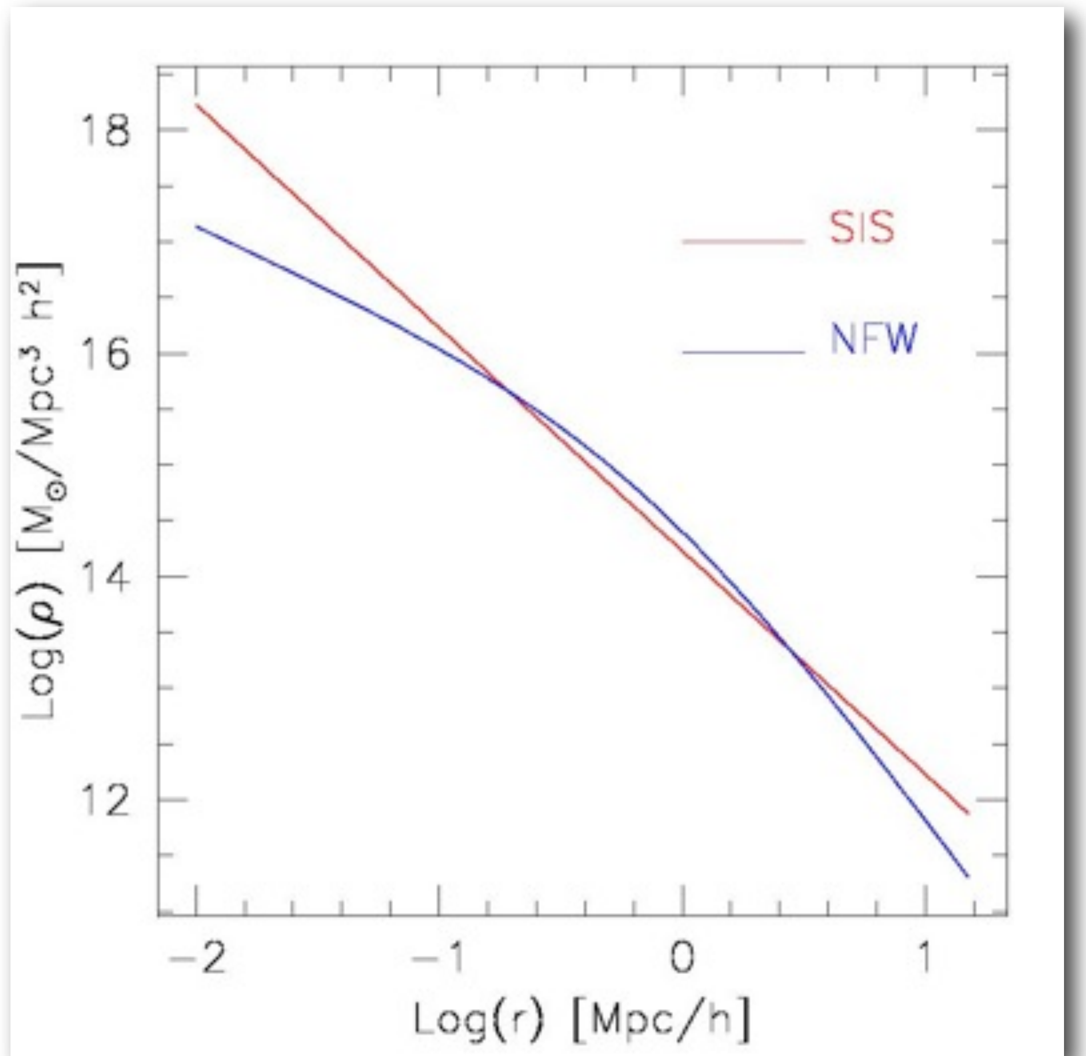

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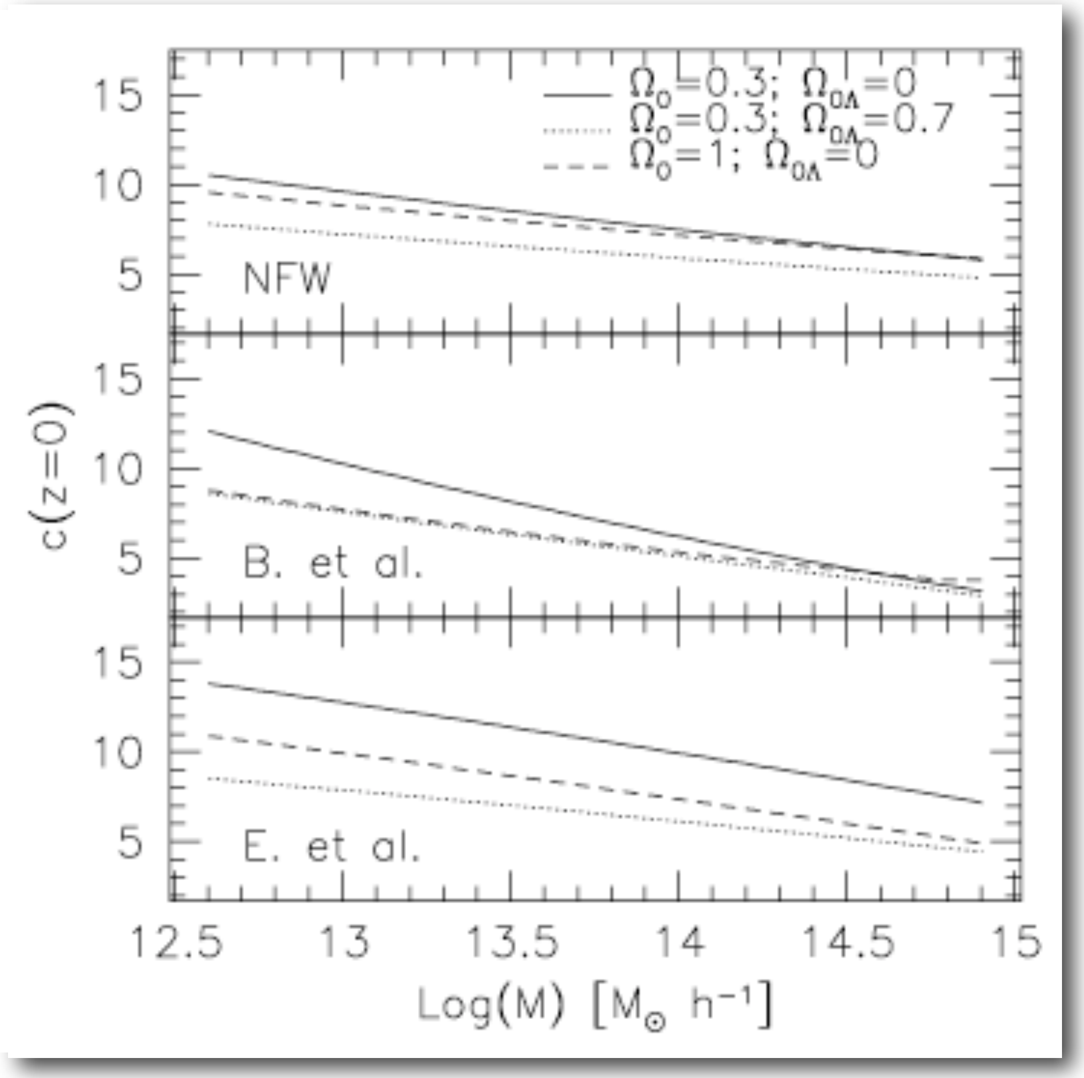
$$c = \frac{r_s}{r_{200}}$$



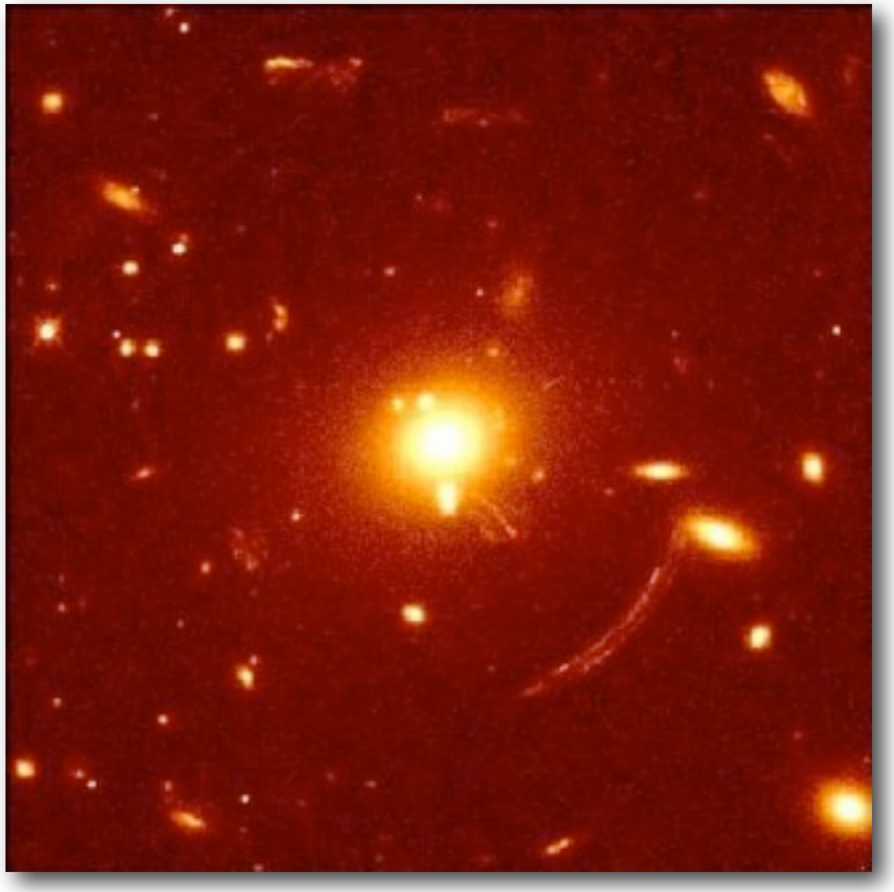
Studying galaxy clusters with lensing

- multiple images
  - positions
  - numbers
- magnifications
  - distortions
  - relative magnitudes

SL Parametric mass models

Approach:

1. combine several mass components
2. assume that galaxies trace the matter
3. model each component using a) a density profile; b) an ellipticity; c) an orientation
4. cluster galaxies often described through scaling relations
5. find parameters such that a) the model yield predicted multiple images and arcs; b) it reproduces the correct # of sources; c) it gives reasonably source sizes.



$$\sigma_0 = \sigma_0^\star \left(\frac{L}{L^\star}\right)^{1/4}$$

$$r_{core} = r_{core}^\star (L/L^\star)^{1/2} \ ,$$
$$r_{cut} = r_{cut}^\star (L/L^\star)^{\alpha} \ ;$$

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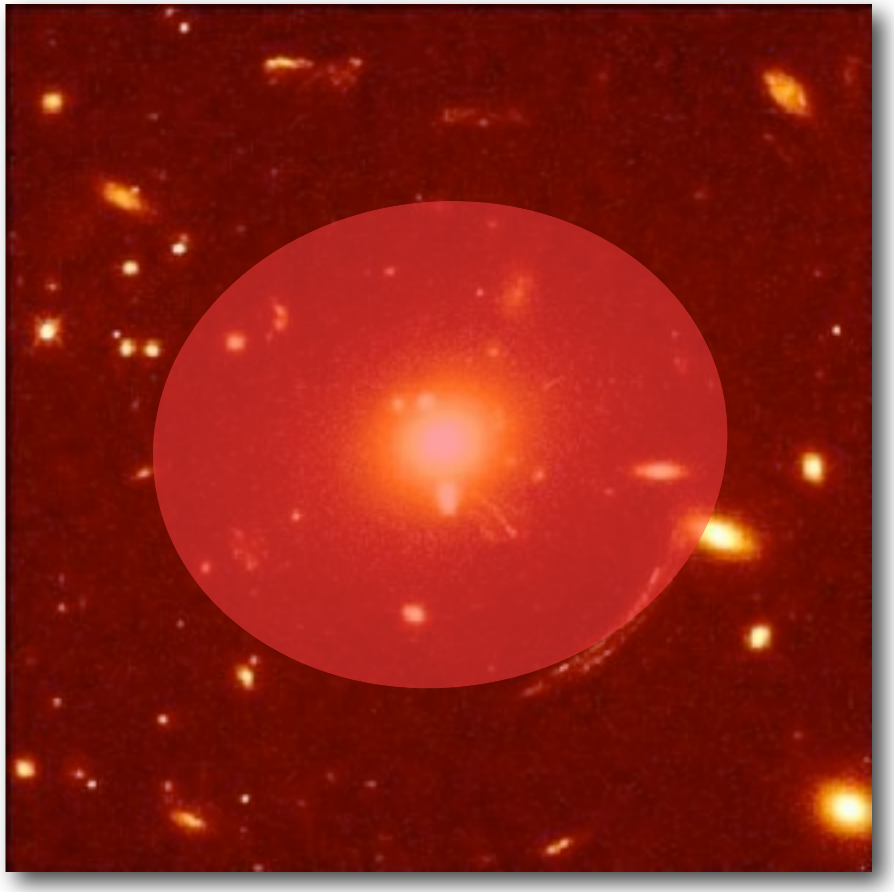

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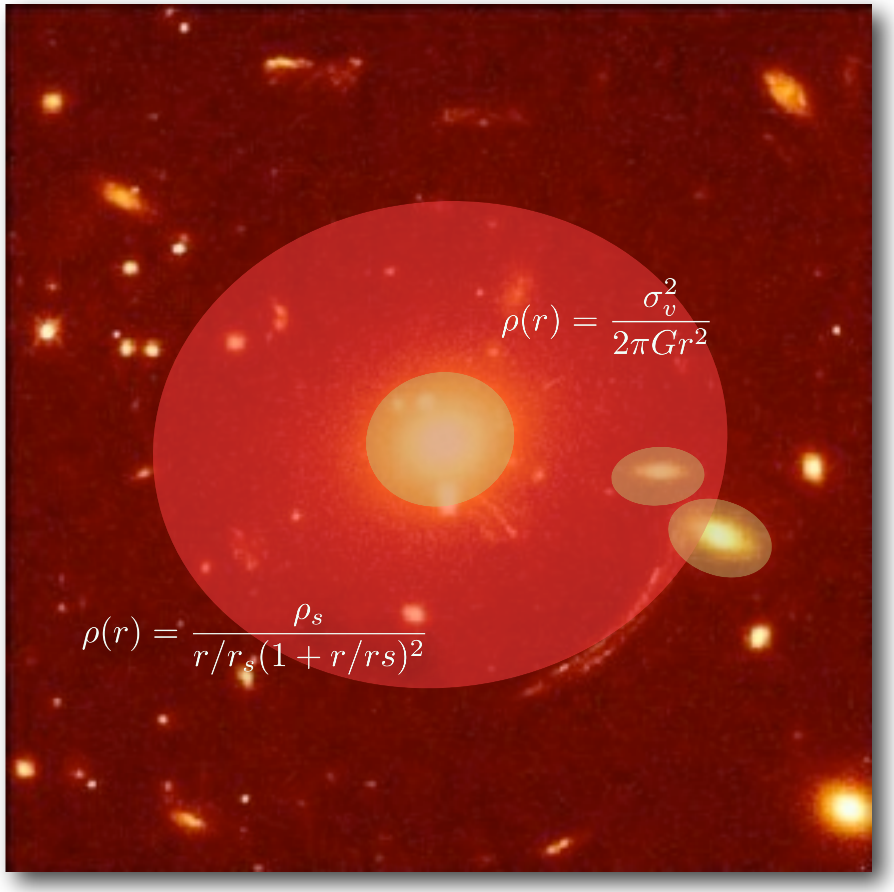

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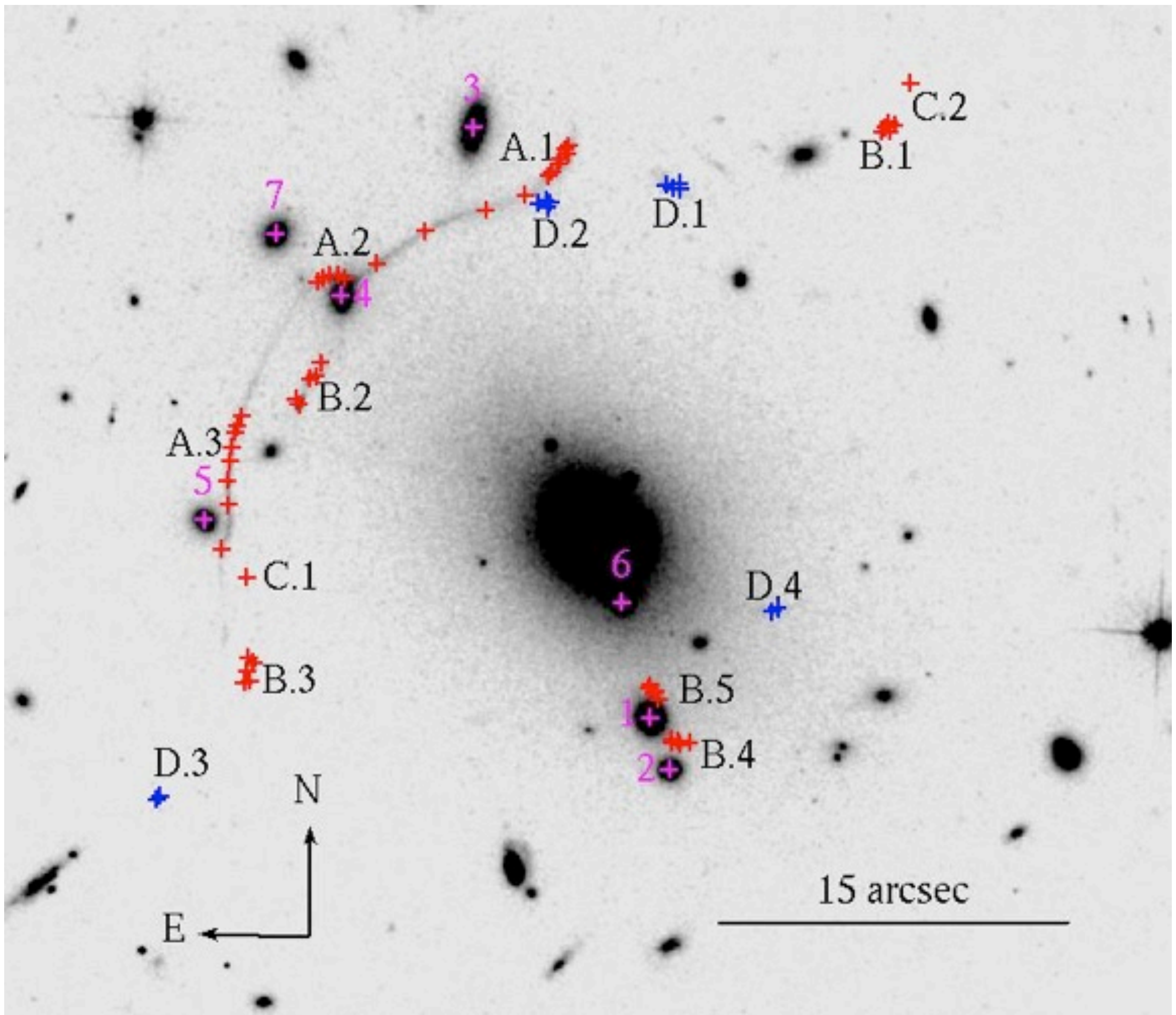
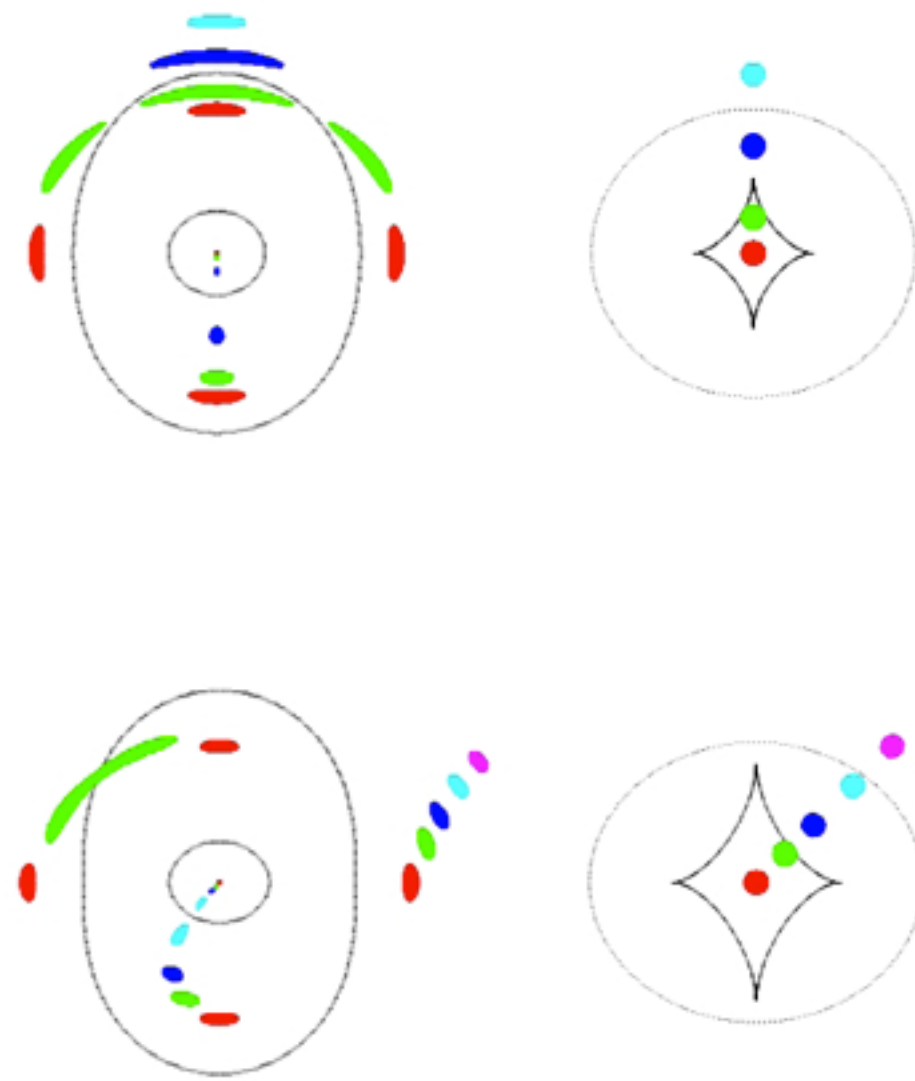
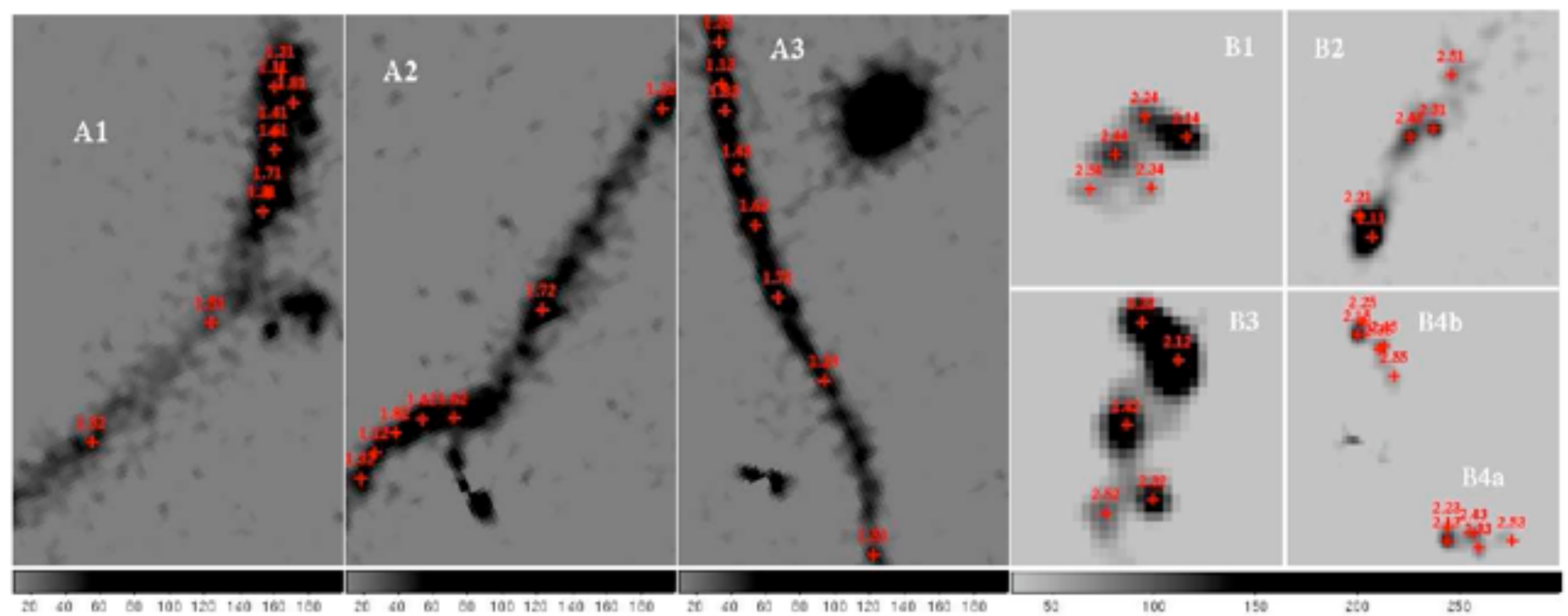
Donnarumma et al. 2010
see also: Richard et al. 2009, Newman et al. 2009





χ^2 minimization

$$\vec{\beta}_i = \vec{\theta}_i - \vec{\alpha}(\vec{\theta}_i, \mathbf{p})$$



$$\chi^2_{src} = \sum_i \left(\frac{\vec{\beta} - \vec{\beta}_i}{\sigma_i} \right)^2$$

$$\chi^2_{img} = \sum_i \left(\frac{\vec{\theta}_i(\vec{\beta}) - \vec{\theta}_i}{\sigma_i} \right)^2$$

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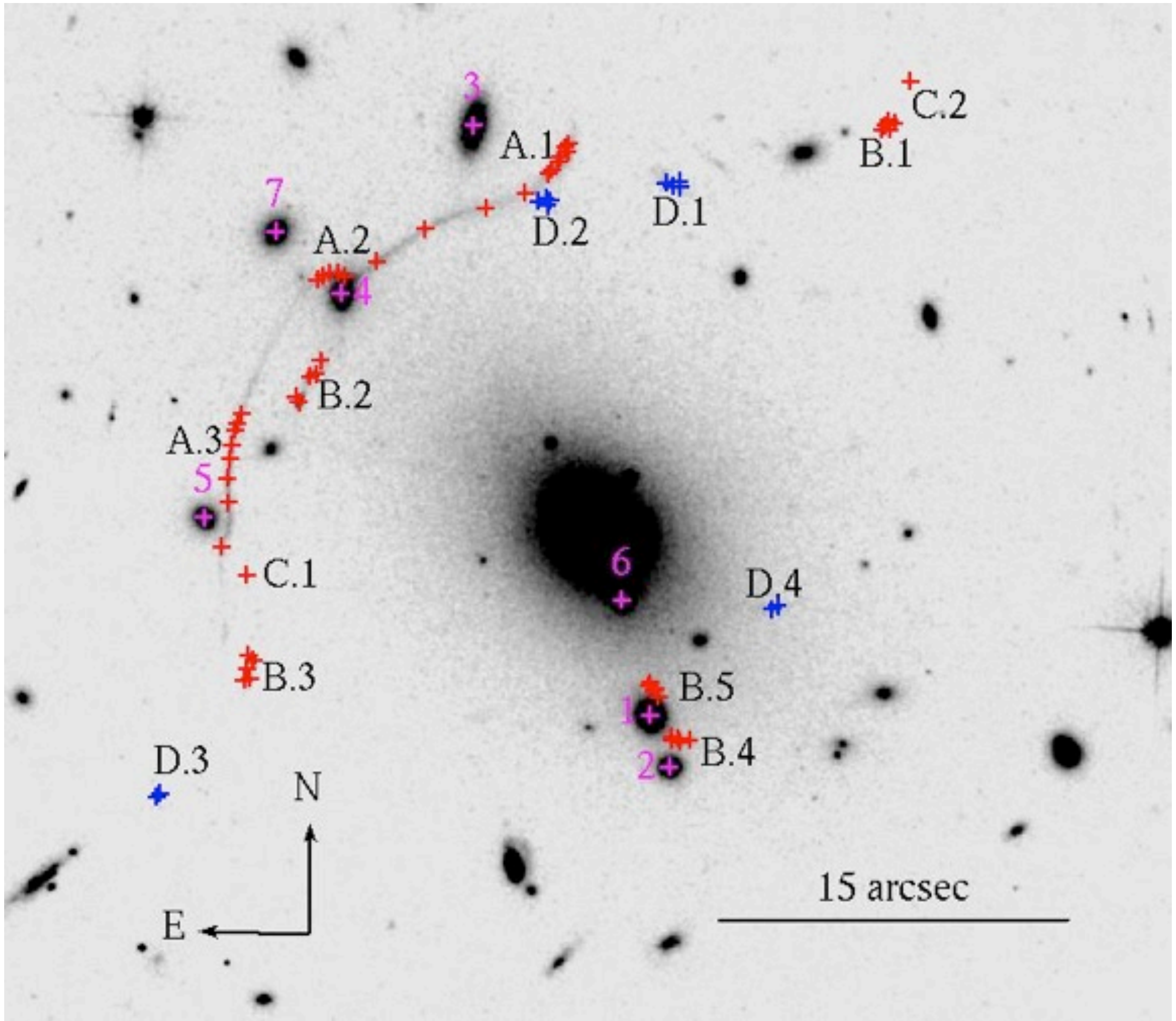

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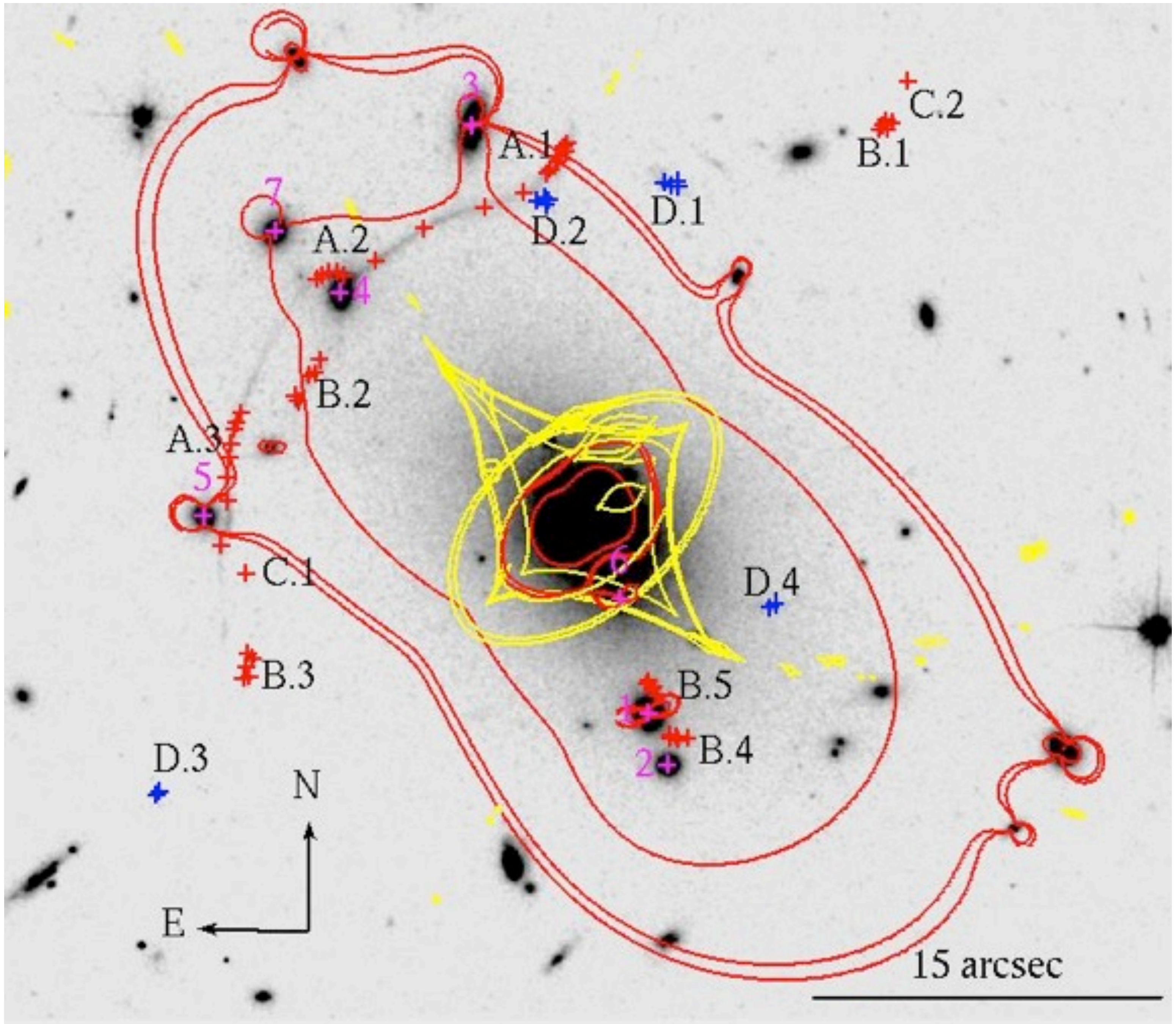
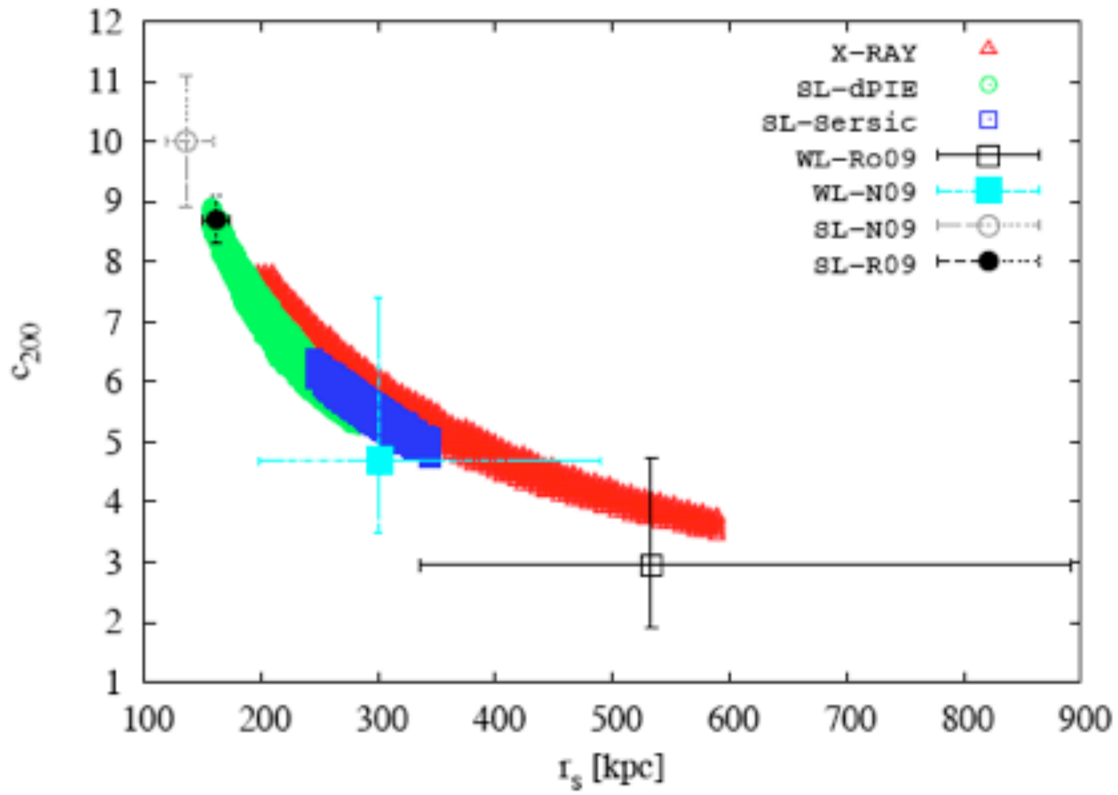
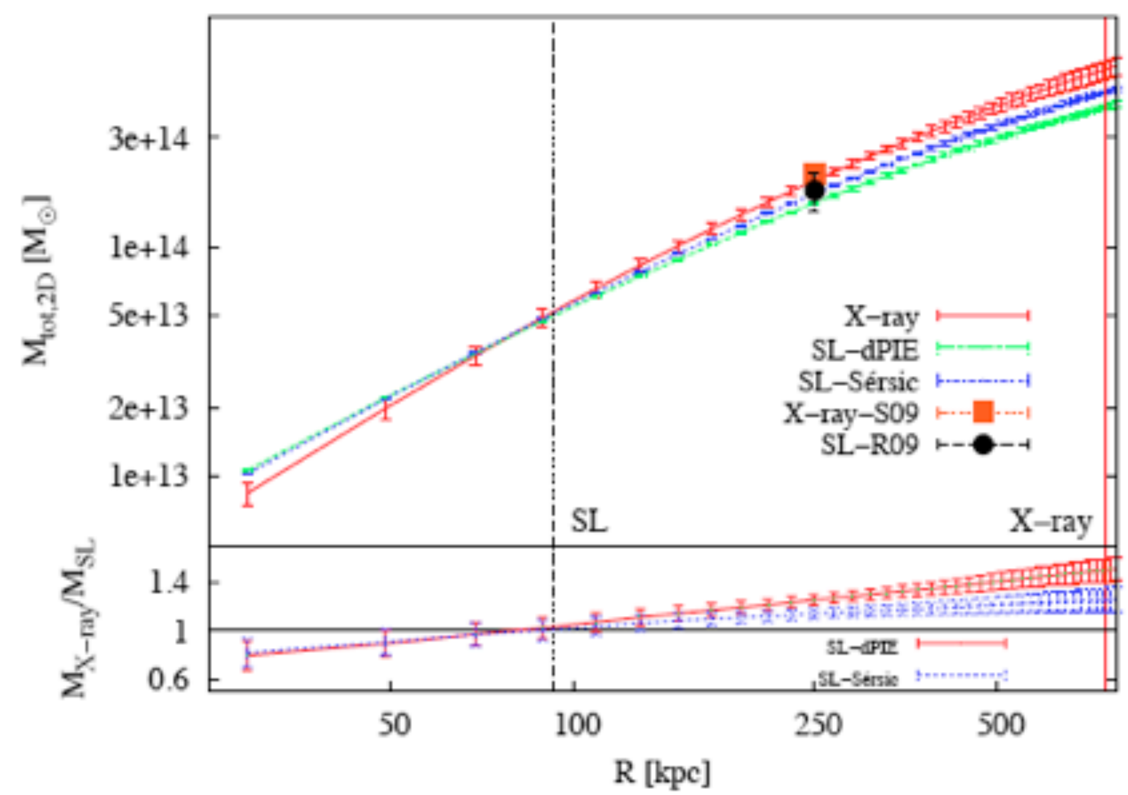




X-RAY
SL-dPIE
SL-Sersic
WL-Ro09
WL-N09
SL-N09
SL-R09
c_{200}
r_s [kpc]



$M_{tot,2D}$ [$M_\odot$]
X-ray
SL-dPIE
SL-Sérsic
X-ray-S09
SL-R09
SL
X-ray
M_{X-ray}/M_{SL}
SL-dPIE
SL-Sérsic
R [kpc]

Test with simulations: SkyLens

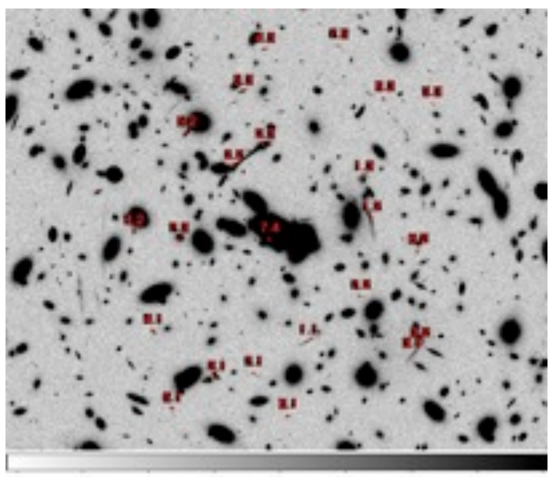
(Meneghetti et al. 2006, 2010)

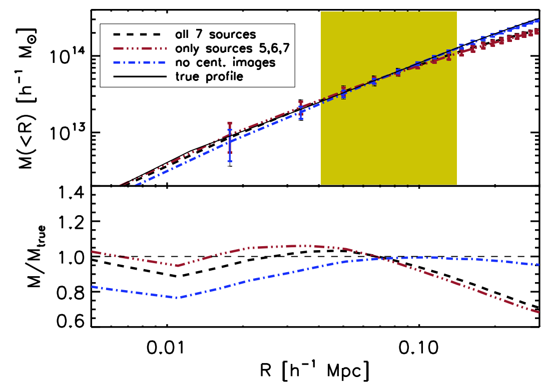
optical simulator which produces maps of the sky

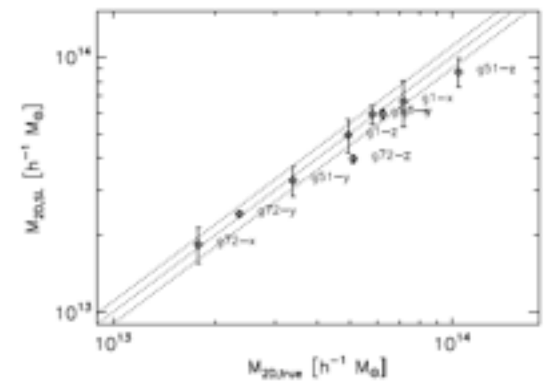
- UDF galaxies as templates (shapelets)
- including lensing
- including colors

HST ACS images of the cluster centers

Strong Lensing







- Multiple images detected in the HST images are used construct a parametric lens model using the Lenstool public software (Kneib et al., 1993; Jullo et al. 2007)
- The model consists of
  - Main halo, modeled using NFW
  - Additional mass components associated to star-groups, modeled using PIEMDs

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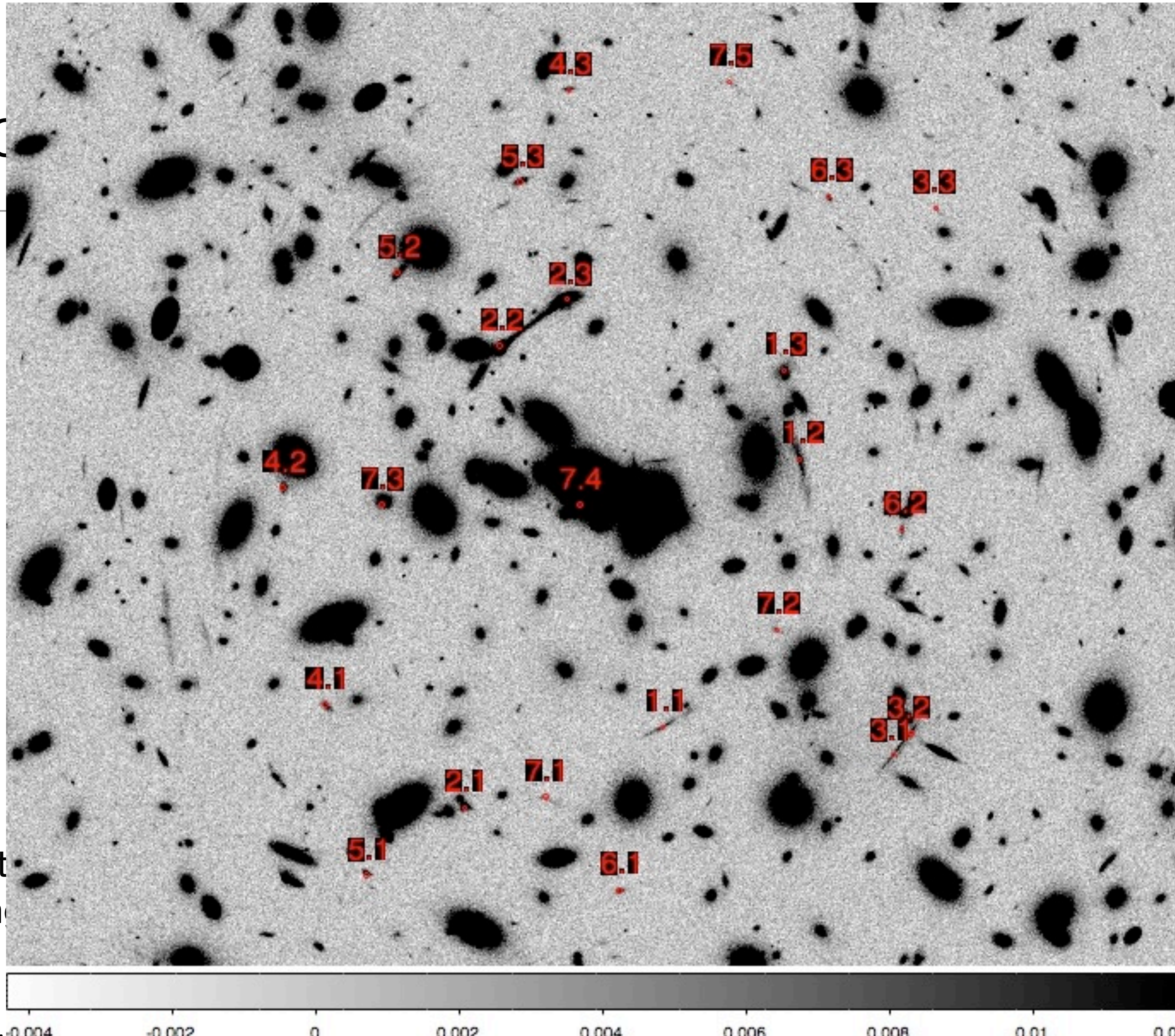

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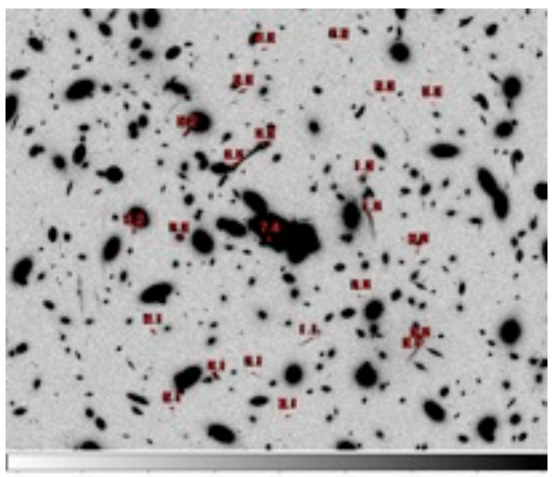

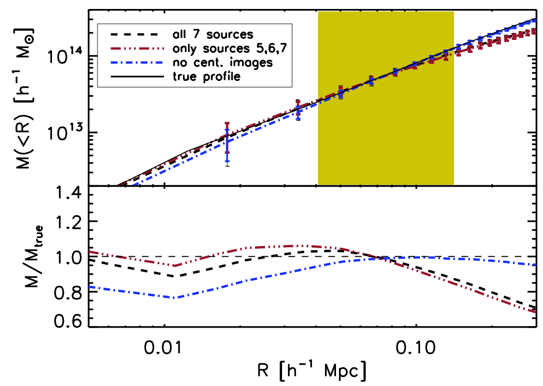

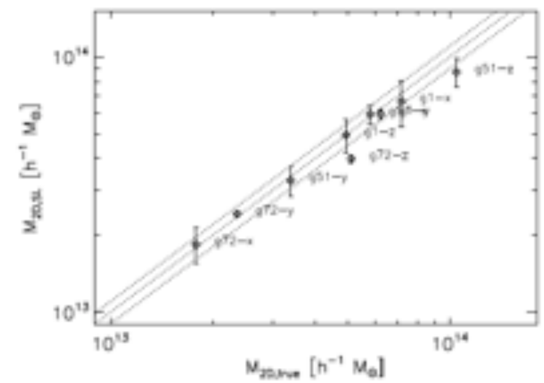

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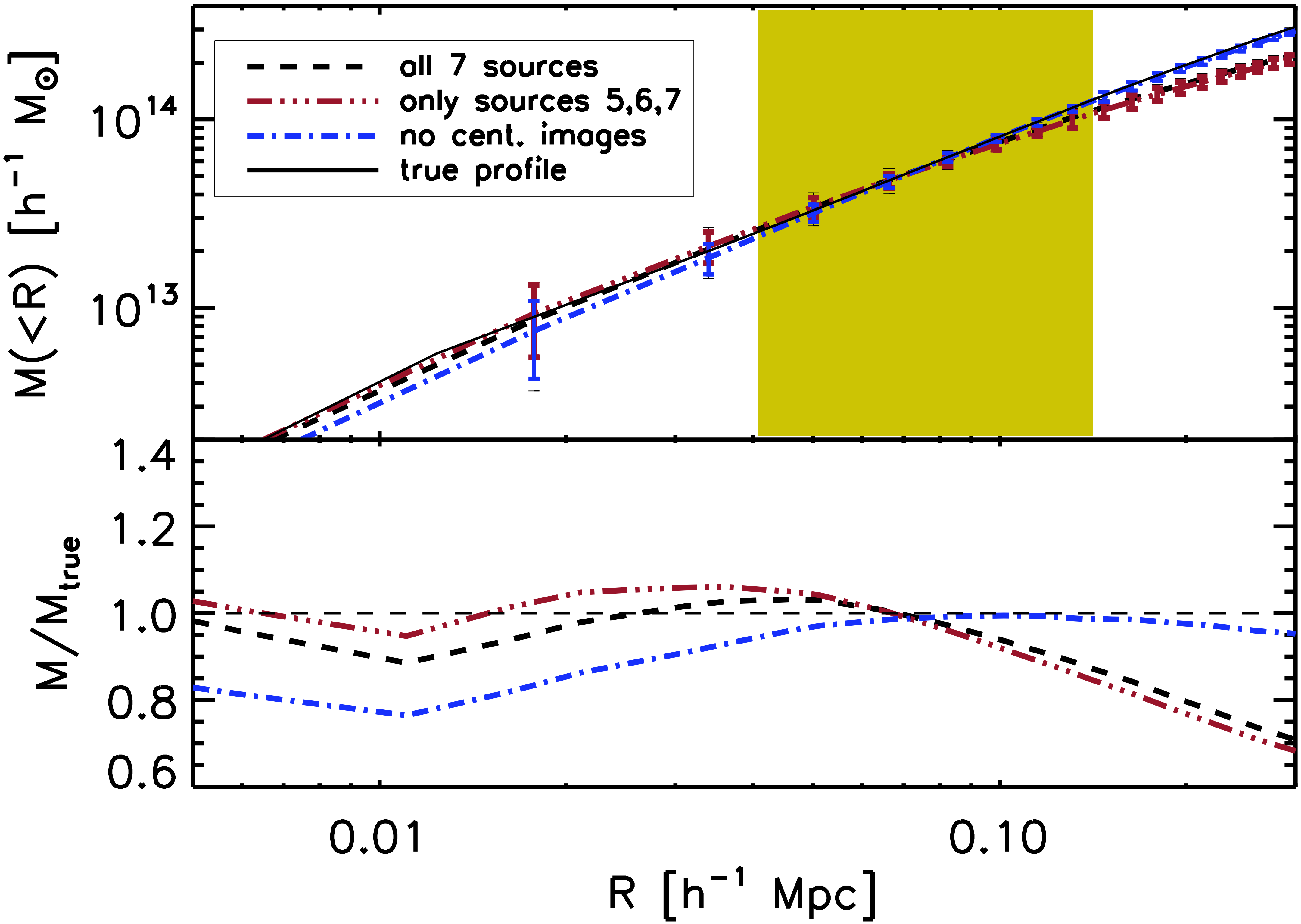

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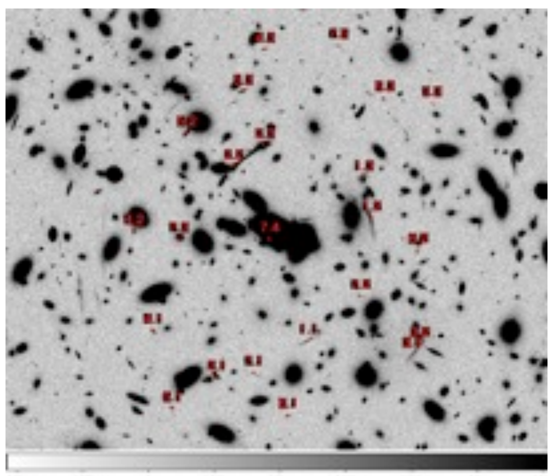

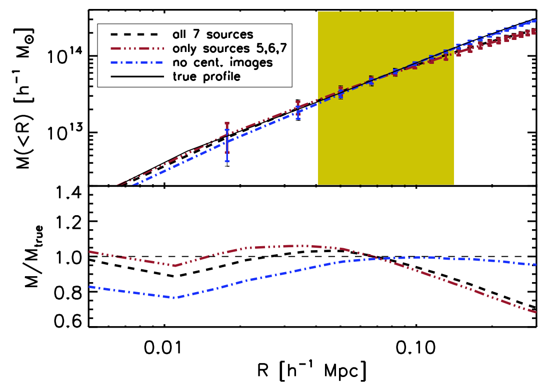

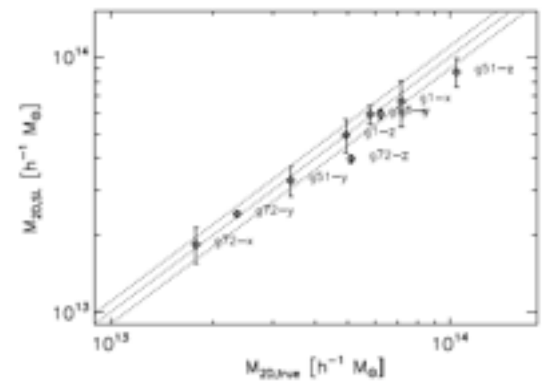

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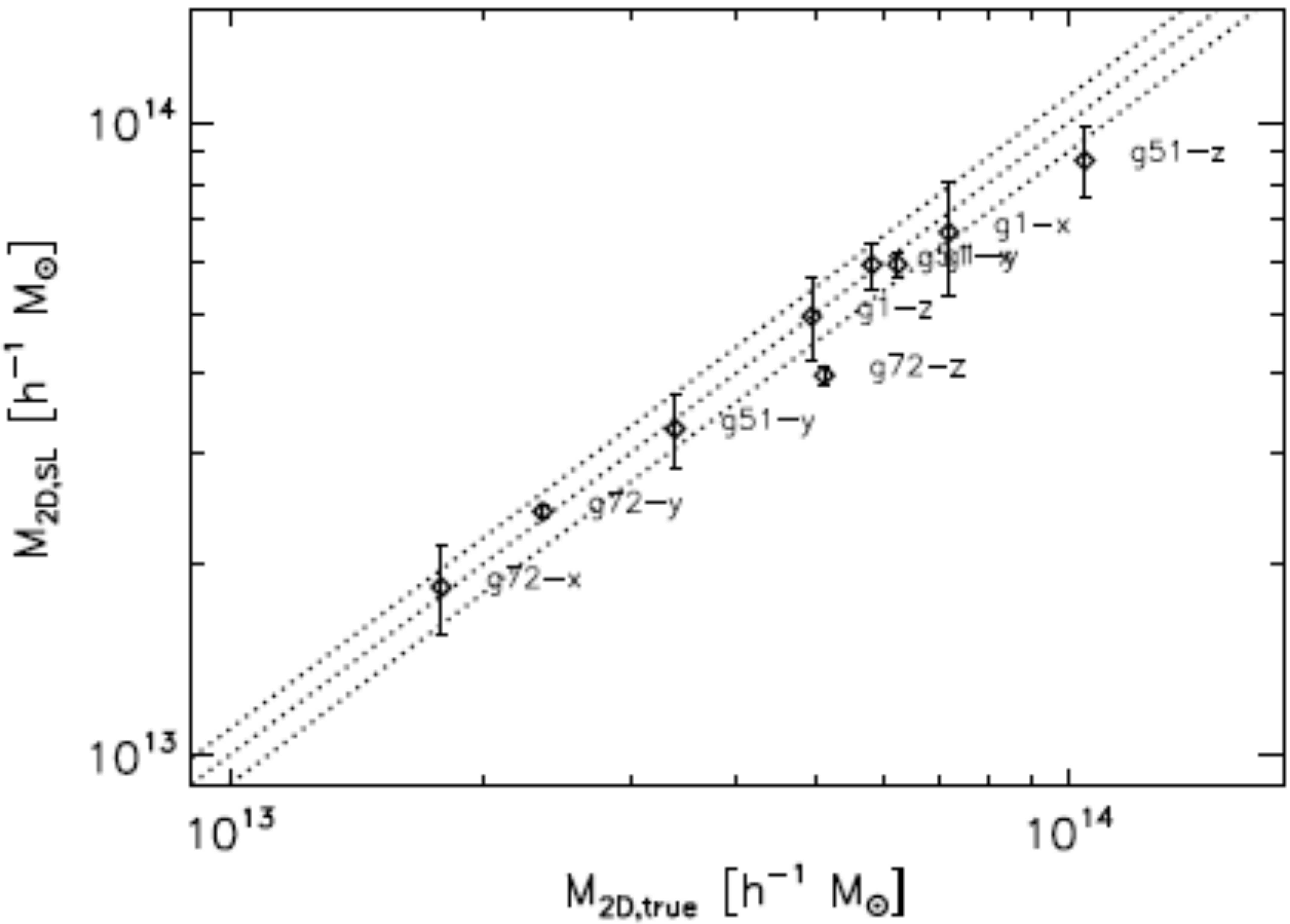

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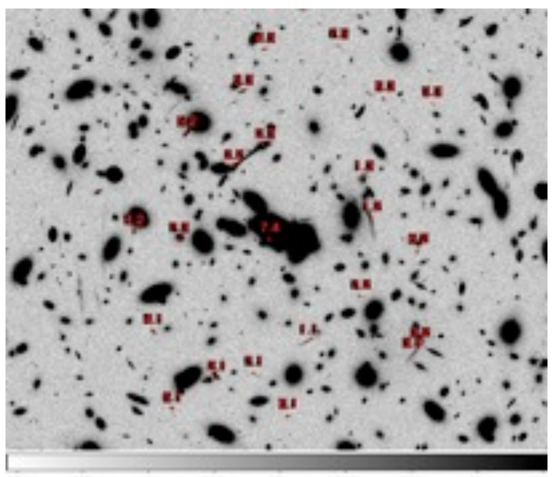

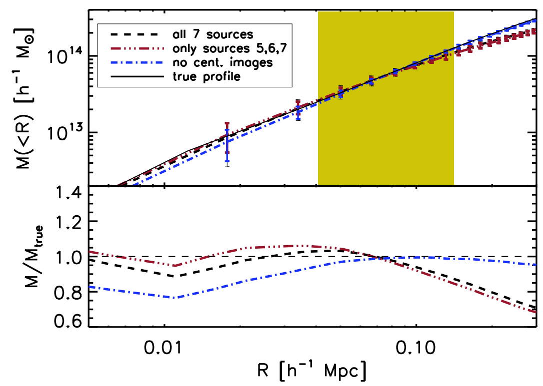

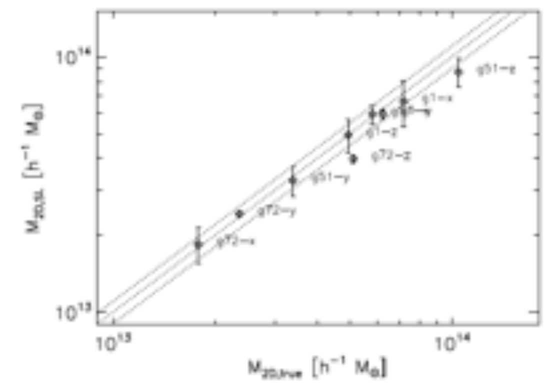

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Distortion of faint galaxies

- Let consider a particular lensing regime where we have small deflections, small distortions, no multiple images (Weak Lensing)
- As we have seen earlier in the course, in the limit of small deflections, the lens equation can be linearized and the lens mapping is described by the Jacobian matrix
- The conservation of surface brightness in combination with the lens equation, allows to derive the distortion of the isophotes

$$\boldsymbol{\beta} - \boldsymbol{\beta}_0 = \mathcal{A}(\boldsymbol{\theta}_0) \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}_0)$$

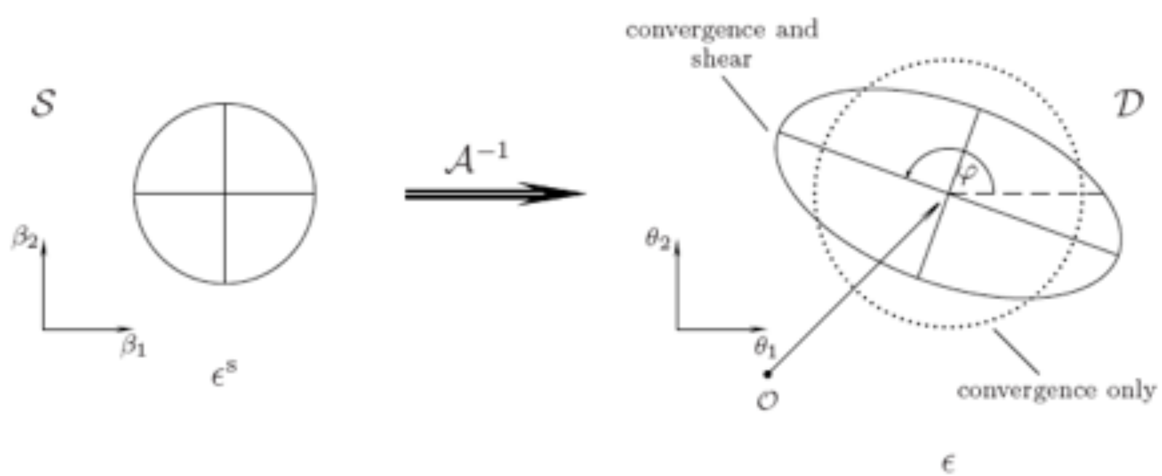
$$\mathcal{A}(\boldsymbol{\theta}) = (1 - \kappa) \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}$$

$$g(\boldsymbol{\theta}) = \frac{\gamma(\boldsymbol{\theta})}{[1 - \kappa(\boldsymbol{\theta})]}$$

$$I(\boldsymbol{\theta}) = I^{(\mathrm{s})}[\boldsymbol{\beta}_0 + \mathcal{A}(\boldsymbol{\theta}_0) \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}_0)]$$

Distortion of faint galaxies

- A circular source is mapped into an ellipse by first-order lensing
- the major and the minor axes of the ellipse are given by combinations of shear and convergence
- the ellipticity can also be written in terms of convergence and shear
- choosing the right definition, one finds that the ellipticity is actually the reduced shear!
- thus: I measure the ellipticity, I measure the reduced shear
- in the very weak lensing regime, when the convergence is small, the ellipticity is an estimate of the shear



$$a = \frac{r}{1-\kappa-\gamma} \quad , \quad b = \frac{r}{1-\kappa+\gamma}$$

$$\epsilon = \frac{a-b}{a+b} = \frac{2\gamma}{2(1-\kappa)} = \frac{\gamma}{1-\kappa}$$

$$|g| = \frac{1-b/a}{1+b/a} \quad \Leftrightarrow \quad \frac{b}{a} = \frac{1-|g|}{1+|g|}$$

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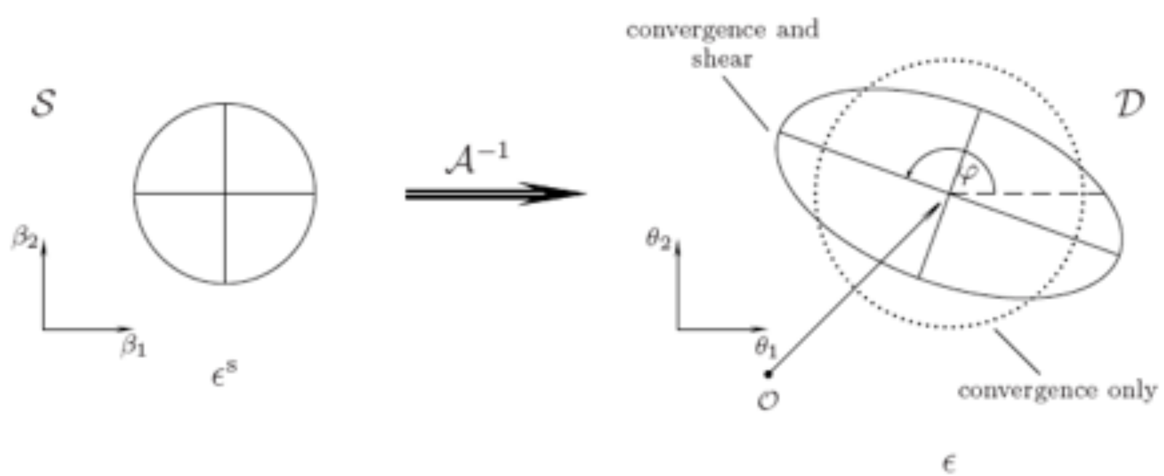

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$$\epsilon = \frac{a-b}{a+b} = \frac{2\gamma}{2(1-\kappa)} = \frac{\gamma}{1-\kappa} \approx \gamma$$

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Measurements of shapes and shear

- Unfortunately sources are not circular: they have their intrinsic ellipticity
- moreover, sources have their own surface brightness profile
- the center of the galaxy is the "center of light" of the galaxy
- the ellipticity is defined through the second moments of the brightness distribution
- It is very common to work with the complex notation

image centroid: $$\bar{\boldsymbol{\theta}} \equiv \frac{\int d^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})] \, \boldsymbol{\theta}}{\int d^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})]}$$

$$q_I(I) = \mathrm{H}(I - I_{\mathrm{th}})$$

$$Q_{ij} = \frac{\int d^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})] \, (\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})]} , \quad i, j \in \{1, 2\}$$

$$\epsilon \equiv \frac{Q_{11} - Q_{22} + 2\mathrm{i}Q_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

$$\epsilon = |\epsilon| \exp(2i\varphi) = \epsilon_1 + i\epsilon_2$$

$$\gamma = |\gamma| \exp(2i\varphi) = \gamma_1 + i\gamma_2$$

$$g = |g| \exp(2i\varphi) = g_1 + ig_2$$

From source to image ellipticities

As done for the lensed source, we can define the source intrinsic ellipticity in terms of the second moments of the unlensed brightness distribution

$$Q_{ij}^{(\mathrm{s})} = \frac{\int \mathrm{d}^2\beta \, I^{(\mathrm{s})}(\boldsymbol{\theta}) \, q_I[I^{(\mathrm{s})}(\boldsymbol{\beta})] \, (\beta_i - \bar{\beta}_i) \, (\beta_j - \bar{\beta}_j)}{\int \mathrm{d}^2\beta \, I^{(\mathrm{s})}(\boldsymbol{\theta}) \, q_I[I^{(\mathrm{s})}(\boldsymbol{\beta})]}$$

Using the fact that

$$\mathrm{d}^2\beta = \det \mathcal{A} \, \mathrm{d}^2\theta,$$

$$\boldsymbol{\beta} - \bar{\boldsymbol{\beta}} = \mathcal{A} \left(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}} \right)$$

$$Q^{(\mathrm{s})} = \mathcal{A} \, Q \, \mathcal{A}^T = \mathcal{A} \, Q \, \mathcal{A}$$

$$\epsilon^{(\mathrm{s})} = \begin{cases} \dfrac{\epsilon - g}{1 - g^* \epsilon} & \text{if } |g| \le 1 \, ; \\[2ex] \dfrac{1 - g \epsilon^*}{\epsilon^* - g^*} & \text{if } |g| > 1 \, . \end{cases}$$

The inverse transformations are found by changing the source and the image ellipticities and g with -g

Expectation value for the source ellipticities

Remember that ellipticities are complex numbers characterized by a phase.

Suppose that sources have intrinsically random phases. 

In this case, averaging over a number of sources, the expectation value of the ellipticity is...

$$E(\epsilon_s) = 0$$

Averaging

$$\epsilon^{(\mathrm{s})} = \begin{cases} \dfrac{\epsilon - g}{1 - g^*\epsilon} & \text{if } |g| \leq 1 \,; \\[2ex] \dfrac{1 - g\epsilon^*}{\epsilon^* - g^*} & \text{if } |g| > 1 \,. \end{cases}$$

we get

$$\mathrm{E}(\epsilon) = \begin{cases} g & \text{if } |g| \leq 1 \\ 1/g^* & \text{if } |g| > 1 \end{cases} \qquad \gamma \approx g \approx \langle \epsilon \rangle$$

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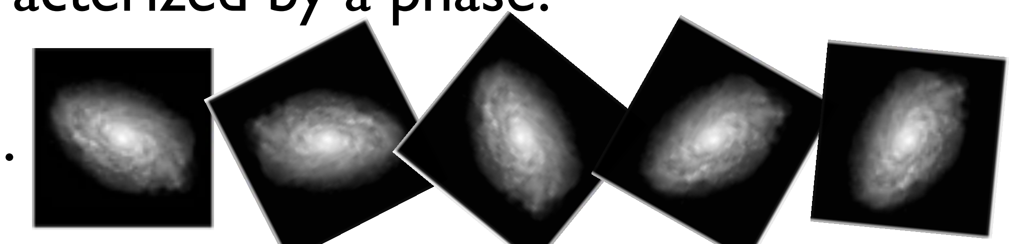

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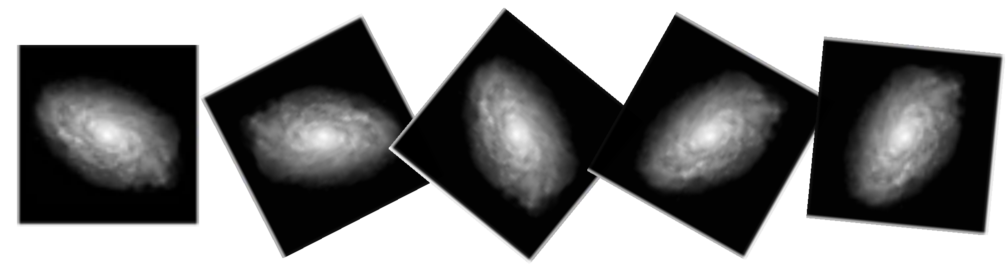
Noise



The noise is given by the dispersion in the intrinsic ellipticity distribution

$$\sigma_\epsilon = \sqrt{\left\langle \epsilon^{(\mathrm{s})} \epsilon^{(\mathrm{s})*} \right\rangle}$$

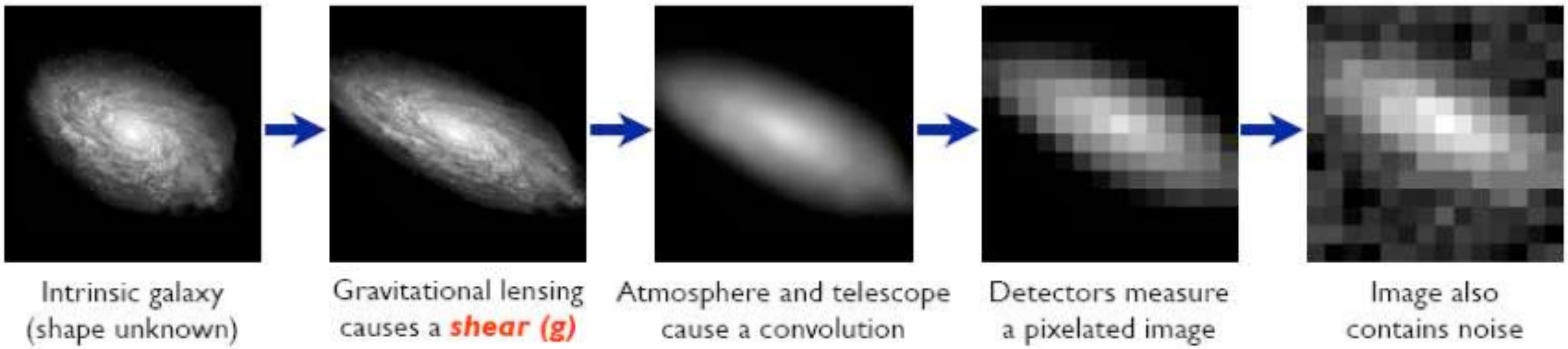
Averaging over N galaxies, the 1-σ deviation from the mean ellipticity is

$$\sigma_\epsilon / \sqrt{N}$$

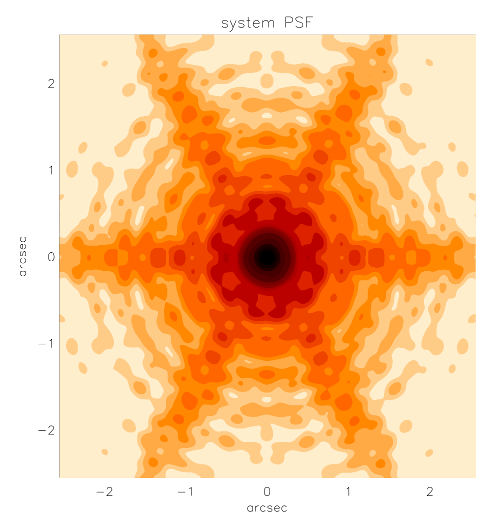
Thus, we can beat the noise by averaging over many galaxies!

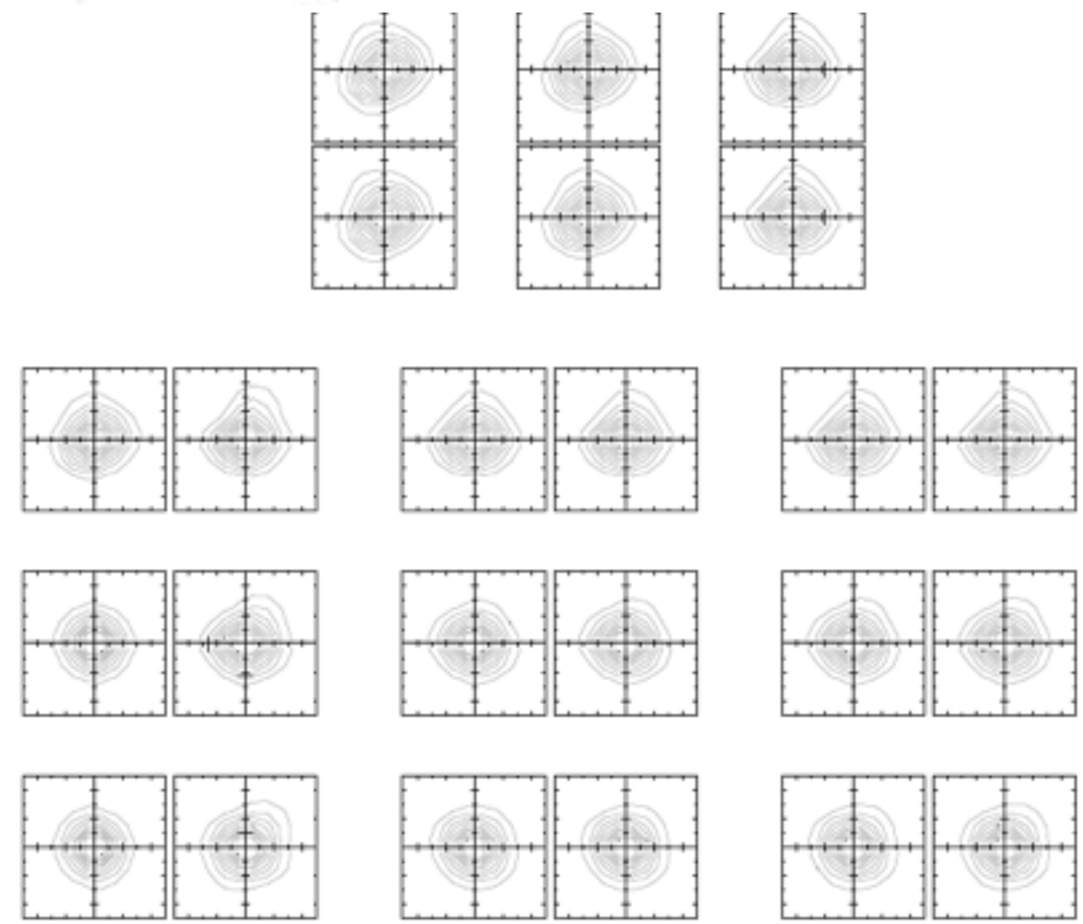
- ☐ select a number of galaxies in a region and assume that the shear is constant within the region
- ☐ if the region is too large, the shear is smoothed
- ☐ increase the number density of galaxies

Point spread function



$$I^{\mathrm{obs}}(\boldsymbol{\theta}) = \int \mathrm{d}^2\vartheta \; I(\boldsymbol{\vartheta}) \, P(\boldsymbol{\theta} - \boldsymbol{\vartheta})$$

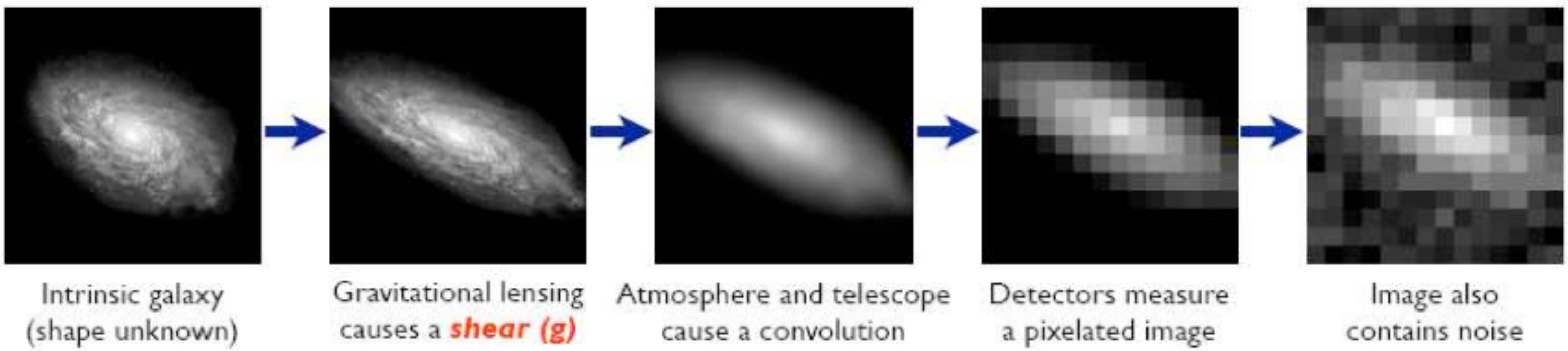




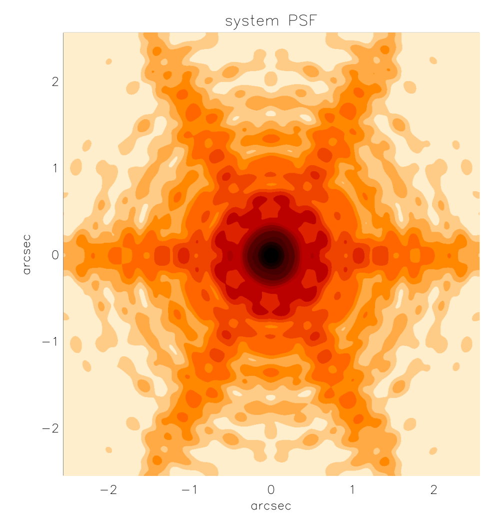
PSF has several contributors: telescope (airy disk), atmosphere, AOCS,...

PSF can have weird shapes (anisotropy caused by coma, jitter, defocus, astigmatism, ecc.) and change across the field!

Point spread function



$$I^{\mathrm{obs}}(\boldsymbol{\theta}) = \int \mathrm{d}^2\vartheta \; I(\boldsymbol{\vartheta}) \, P(\boldsymbol{\theta} - \boldsymbol{\vartheta})$$



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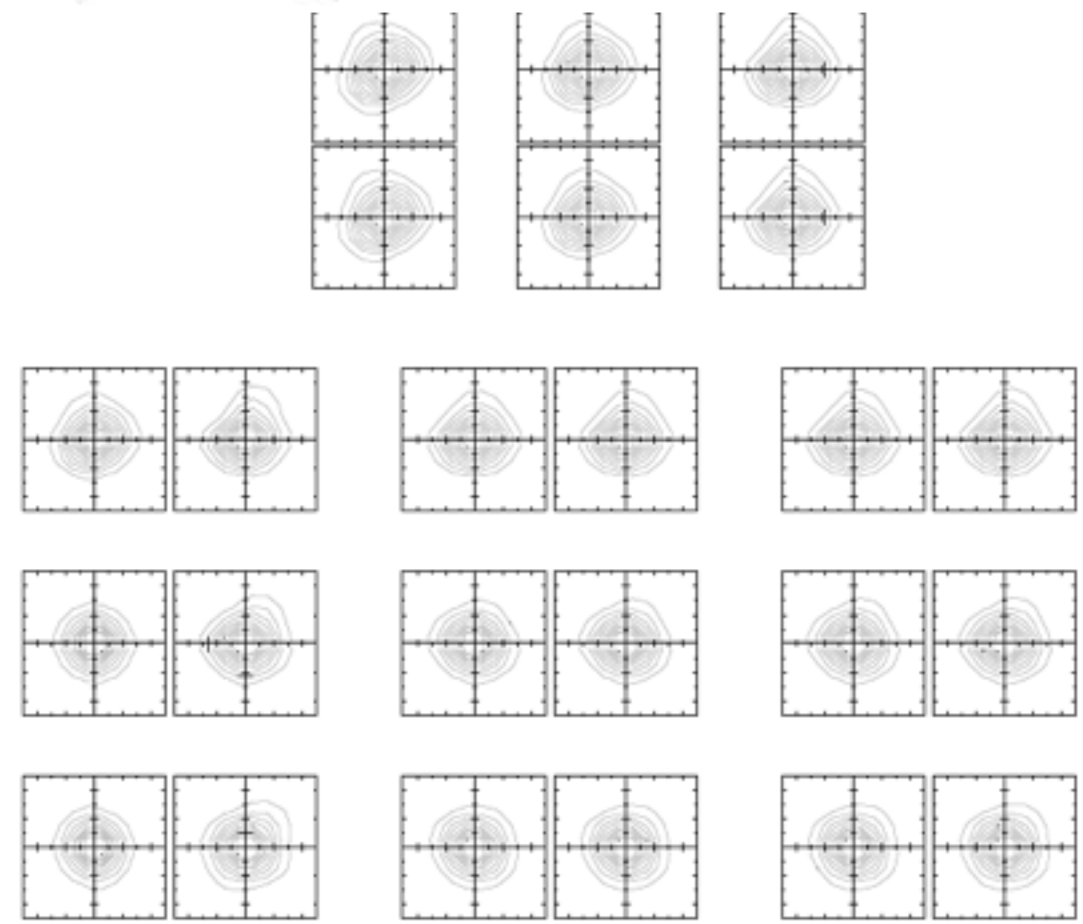
LBT →



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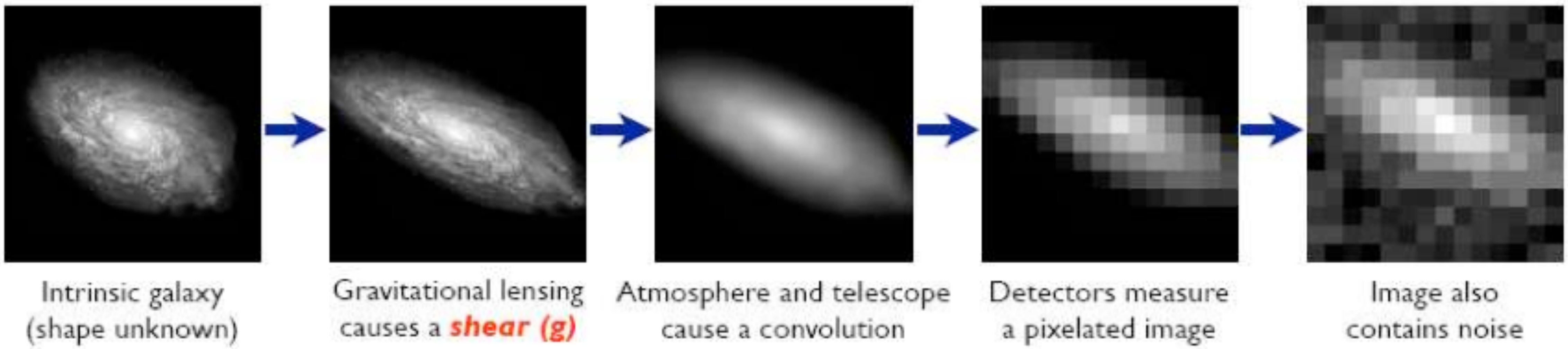

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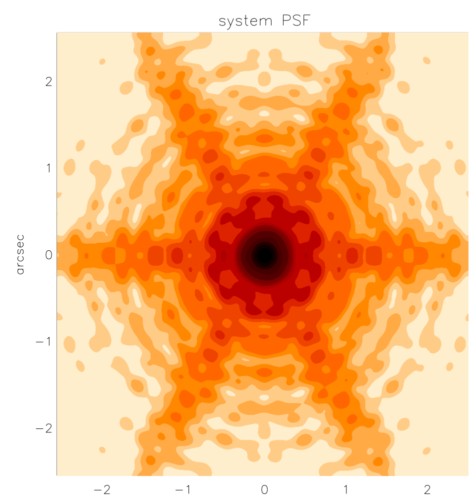

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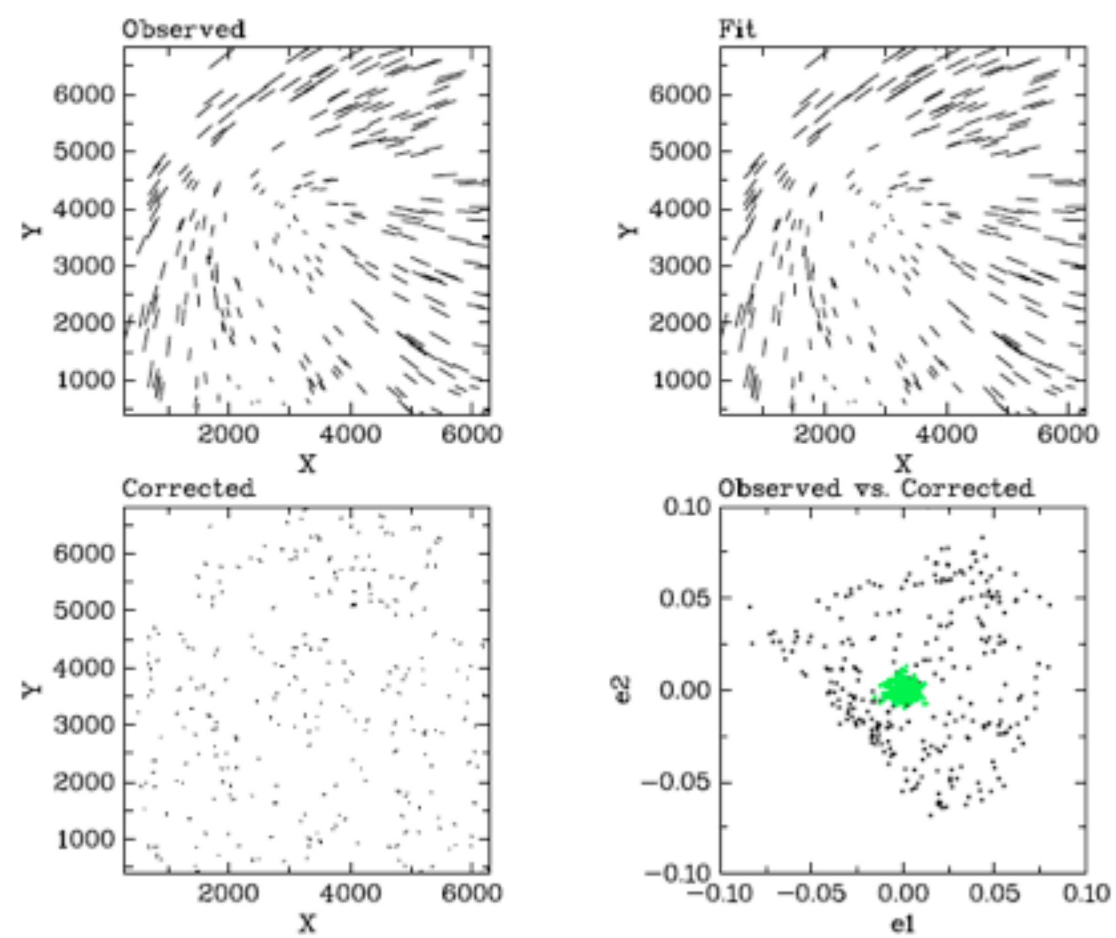
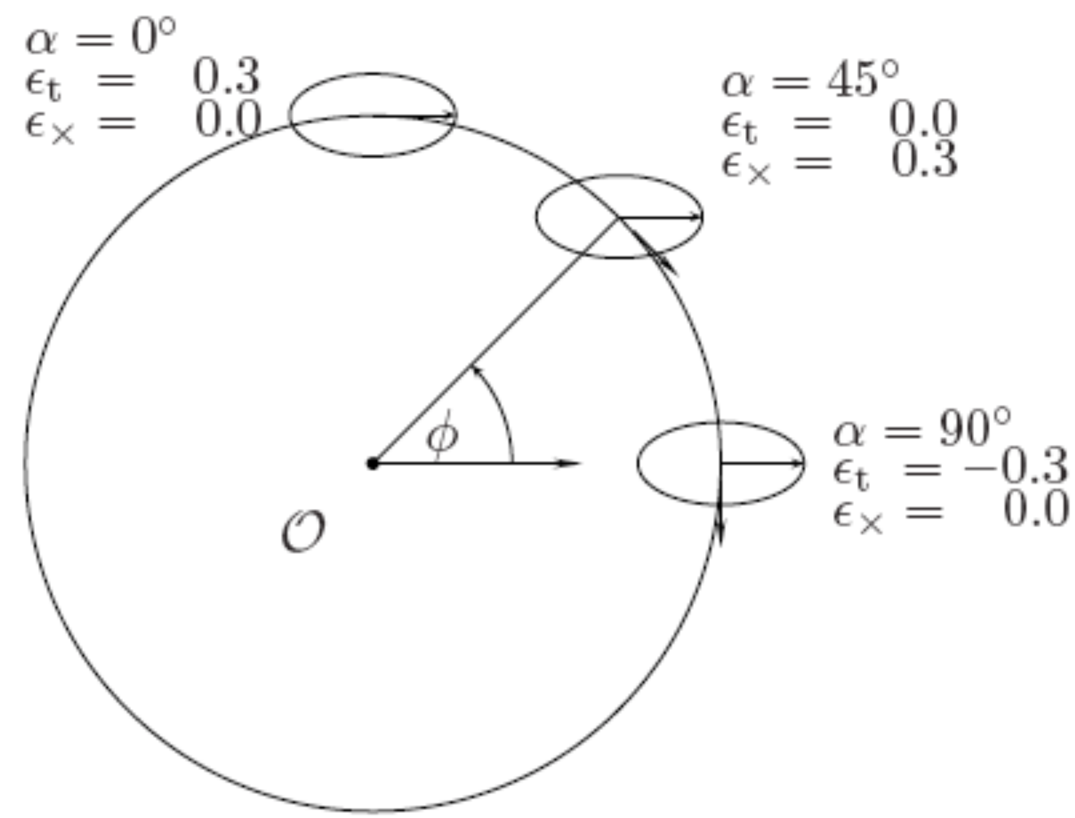


Tangential and cross component of the shear

Given a direction ϕ we can define a tangential and a cross component of the shear relative to this direction.

$$\gamma_{\rm t} = -\mathcal{R}{\rm e}\left[\gamma\,{\rm e}^{-2{\rm i}\phi}\right] \quad , \quad \gamma_\times = -\mathcal{I}{\rm m}\left[\gamma\,{\rm e}^{-2{\rm i}\phi}\right]$$



Note that, under this convention, "tangential" means both tangentially and radially oriented shears

With this we want to emphasize that lensing, being caused by a scalar potential is curl-free

The signs are chosen such that the tangential component is positive for tangentially distorted images, and it is negative for radially distorted images.

Fit of the tangential shear profile

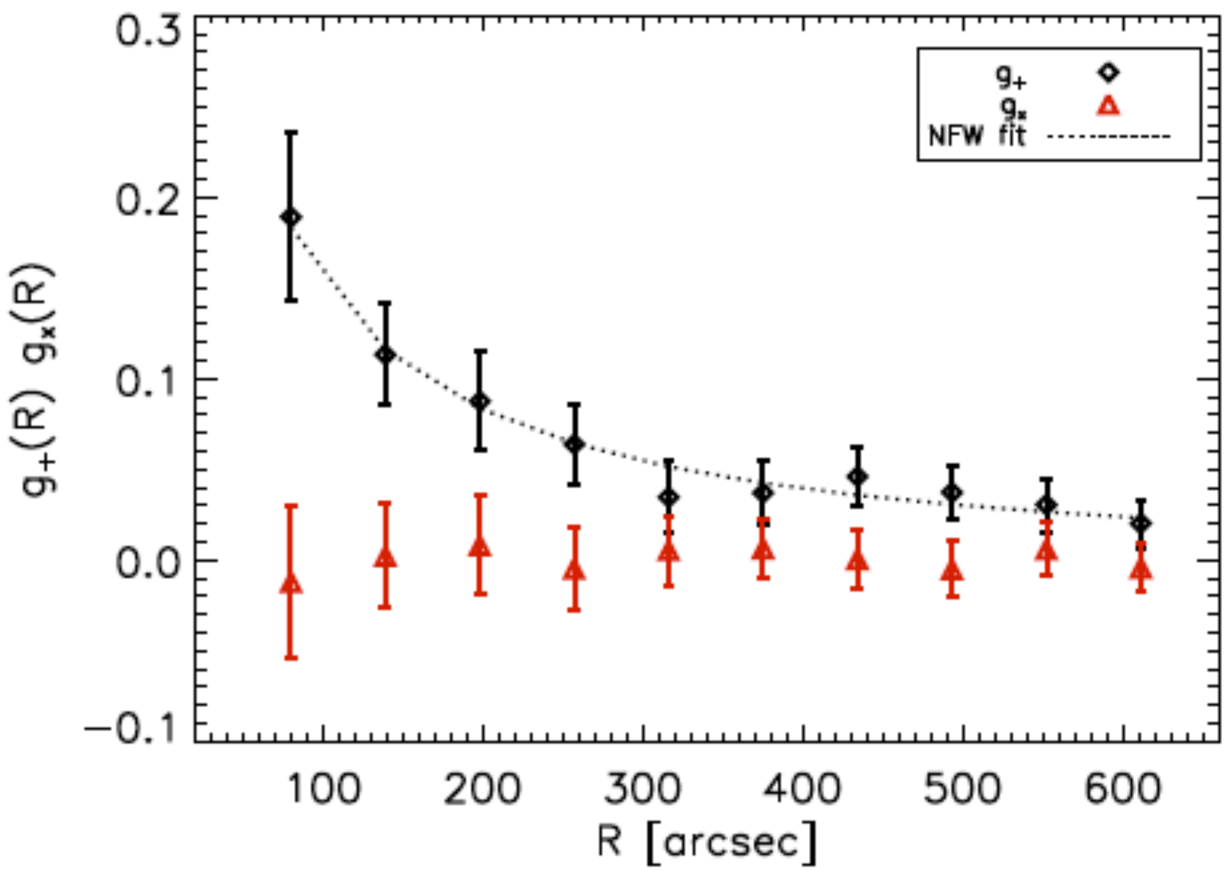
Having measured the tangential shear profile, we can fit it with some parametric model

SIS $\gamma(x) = (\gamma_1^2 + \gamma_2^2)^{1/2} = \frac{1}{2x} = \kappa(x)$

NFW $\kappa(x) = \frac{\Sigma(\xi_0 x)}{\Sigma_{cr}} = 2\kappa_s \frac{f(x)}{x^2 - 1}$

$$f(x) = \begin{cases} 1 - \frac{2}{\sqrt{x^2-1}} \text{arctan}\sqrt{\frac{x-1}{x+1}} & (x > 1) \\ 1 - \frac{2}{\sqrt{1-x^2}} \text{arctanh}\sqrt{\frac{1-x}{1+x}} & (x < 1) \\ 0 & (x = 1) \end{cases}$$

$$\gamma(x) = \overline{\kappa}(x) - \kappa(x)$$



$$l_\gamma = \sum_{i=1}^{N_\gamma} \left[\frac{|\epsilon_i - g(\theta_i)|^2}{\sigma^2[g(\theta_i)]} + 2\ln\sigma[g(\theta_i)] \right]$$

Aperture densitometry

$$\zeta(\theta_1) = \overline{\kappa}(\theta_1) - \overline{\kappa}(\theta_1 < \theta < \theta_{max}) = \frac{2}{1 - \theta_1^2/\theta_{max}^2} \int_{\theta_1}^{\theta_{max}} \langle \gamma_t \rangle d \ln \theta$$

$$\zeta_c(\theta_1) = \overline{\kappa}(\theta_1) - \overline{\kappa}(\theta_2 < \theta < \theta_{max}) = 2 \int_{\theta_1}^{\theta_2} \langle \gamma_t \rangle d \ln \theta + \frac{2}{1 - \theta_2^2/\theta_{max}^2} \int_{\theta_1}^{\theta_{max}} \langle \gamma_t \rangle d \ln \theta$$

$$m_\zeta(\theta) = \theta^2 \zeta_c(\theta) = m(\theta) - m(\theta_2 < \theta < \theta_{max})$$

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Using the aperture densitometry one can estimate a lower limit to the mass within a given radius

$$m_\zeta(\theta) = \theta^2 \zeta_c(\theta) = m(\theta) - m(\theta_2 < \theta < \theta_{max})$$

Kaiser & Squires

$$\kappa(\vec{\theta}) = \frac{1}{2}\left(\frac{\partial^2\psi(\vec{\theta})}{\partial\theta_1^2} + \frac{\partial^2\psi(\vec{\theta})}{\partial\theta_2^2}\right)$$
$$\gamma_1(\vec{\theta}) = \frac{1}{2}\left(\frac{\partial^2\psi(\vec{\theta})}{\partial\theta_1^2} - \frac{\partial^2\psi(\vec{\theta})}{\partial\theta_2^2}\right)$$
$$\gamma_2(\vec{\theta}) = \frac{\partial^2\psi(\vec{\theta})}{\partial\theta_1\partial\theta_2} .$$

$$\hat{\kappa}(\vec{k}) = -\frac{1}{2}(k_1^2 + k_2^2)\hat{\psi}(\vec{k}) ,$$
$$\hat{\gamma}_1(\vec{k}) = -\frac{1}{2}(k_1^2 - k_2^2)\hat{\psi}(\vec{k}) ,$$
$$\hat{\gamma}_2(\vec{k}) = -k_1 k_2 \hat{\psi}(\vec{k}) ,$$

$$\begin{pmatrix}\hat{\gamma}_1\\ \hat{\gamma}_2\end{pmatrix} = k^{-2}\begin{pmatrix}(k_1^2 - k_2^2)\\ 2k_1k_2\end{pmatrix}\hat{\kappa} ,$$
$$\hat{\kappa} = k^{-2}\left[(k_1^2 - k_2^2), (2k_1k_2)\right]\begin{pmatrix}\hat{\gamma}_1\\ \hat{\gamma}_2\end{pmatrix}$$

$$\kappa(\vec{\theta}) = \frac{1}{\pi}\int d^2\theta' \,\mathrm{Re}\left[\mathcal{D}^*(\vec{\theta} - \vec{\theta}')\,\gamma(\vec{\theta}')\right]$$

$$\mathcal{D}(\vec{\theta}) = \frac{(\theta_2^2 - \theta_1^2) - 2\mathrm{i}\theta_1\theta_2}{\theta^4}$$

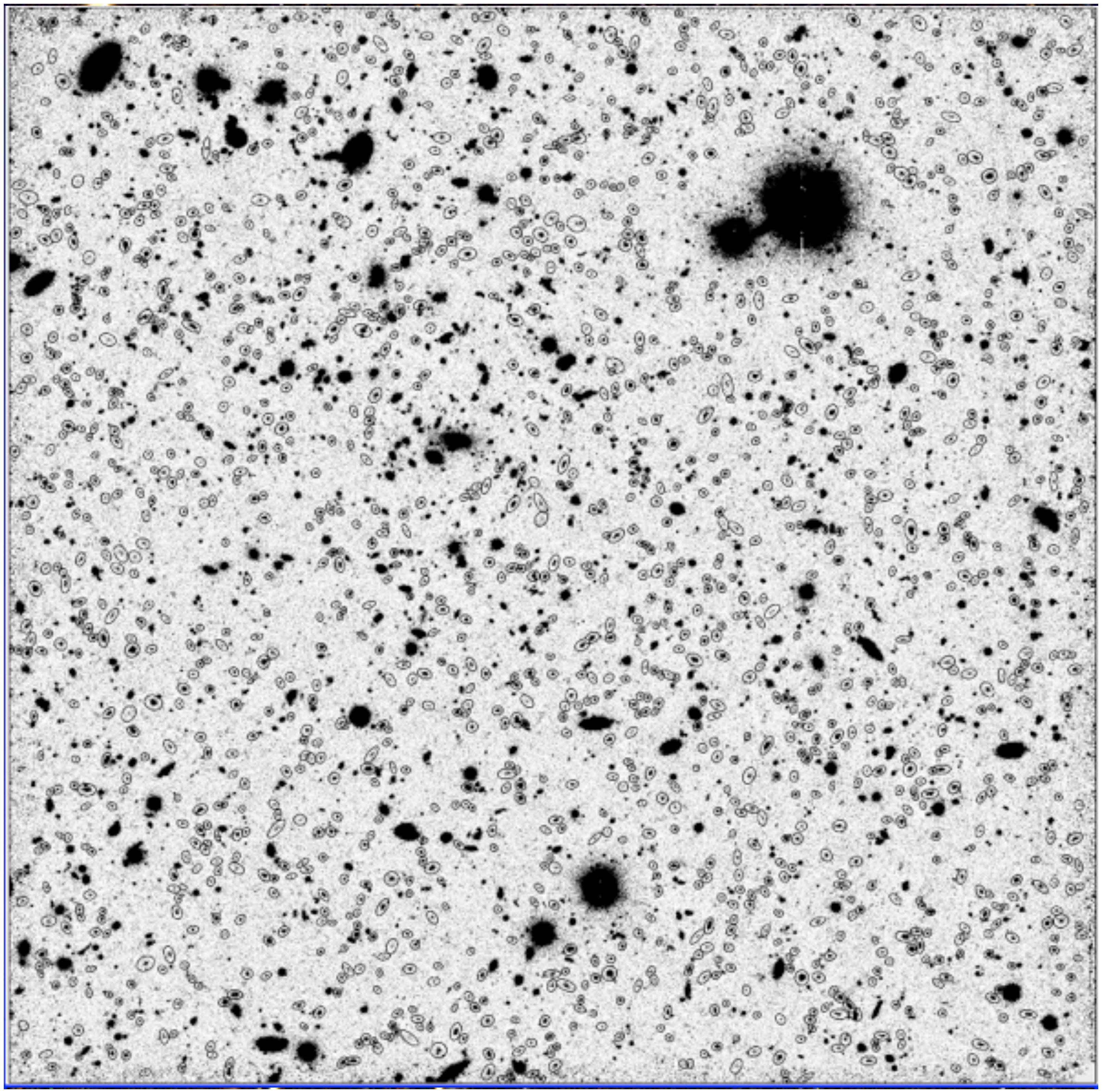
Tracing the mass with weak lensing

Kaiser & Squires inversion

CL1232-1250
(Clowe et al.)

Tracing the mass with weak lensing

Kaiser & Squires inversion



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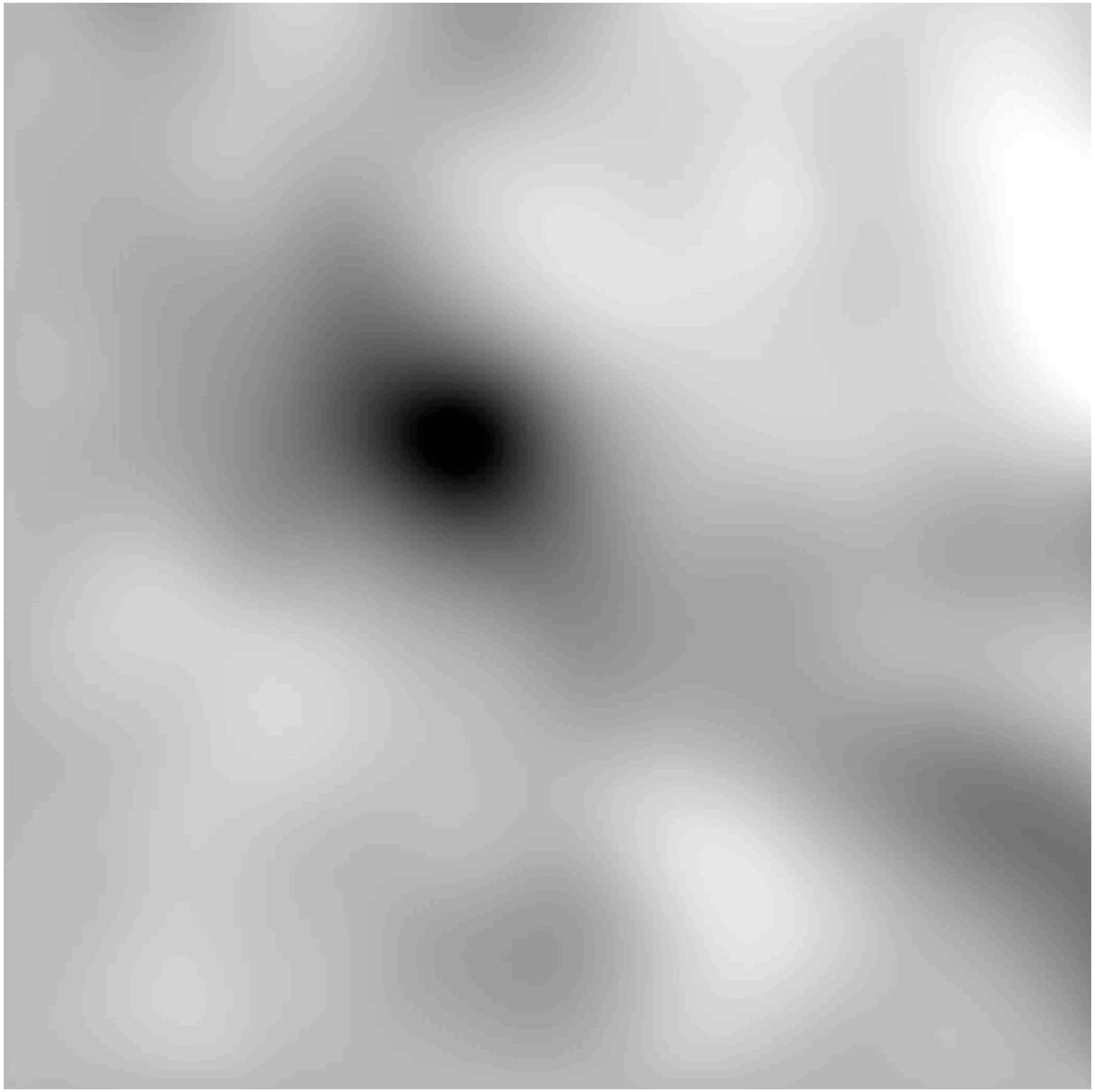

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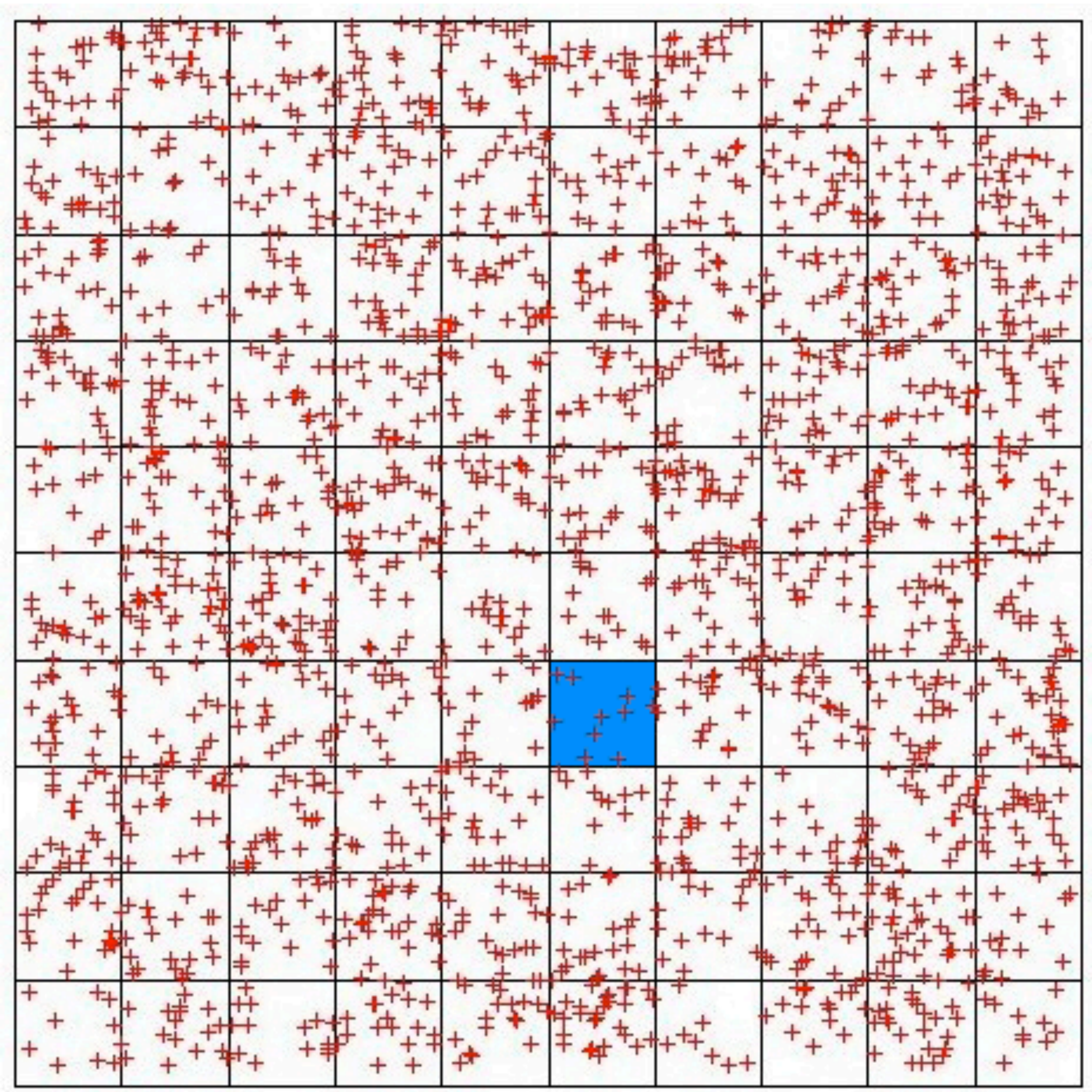
Maximum-likelihood approach



$$\gamma_1 = \frac{1}{2}\left(\psi_{,11} - \psi_{,22}\right) \qquad \gamma_2 = \psi_{,12}$$

$$\kappa = \frac{1}{2}\left(\psi_{,11} + \psi_{,22}\right)$$

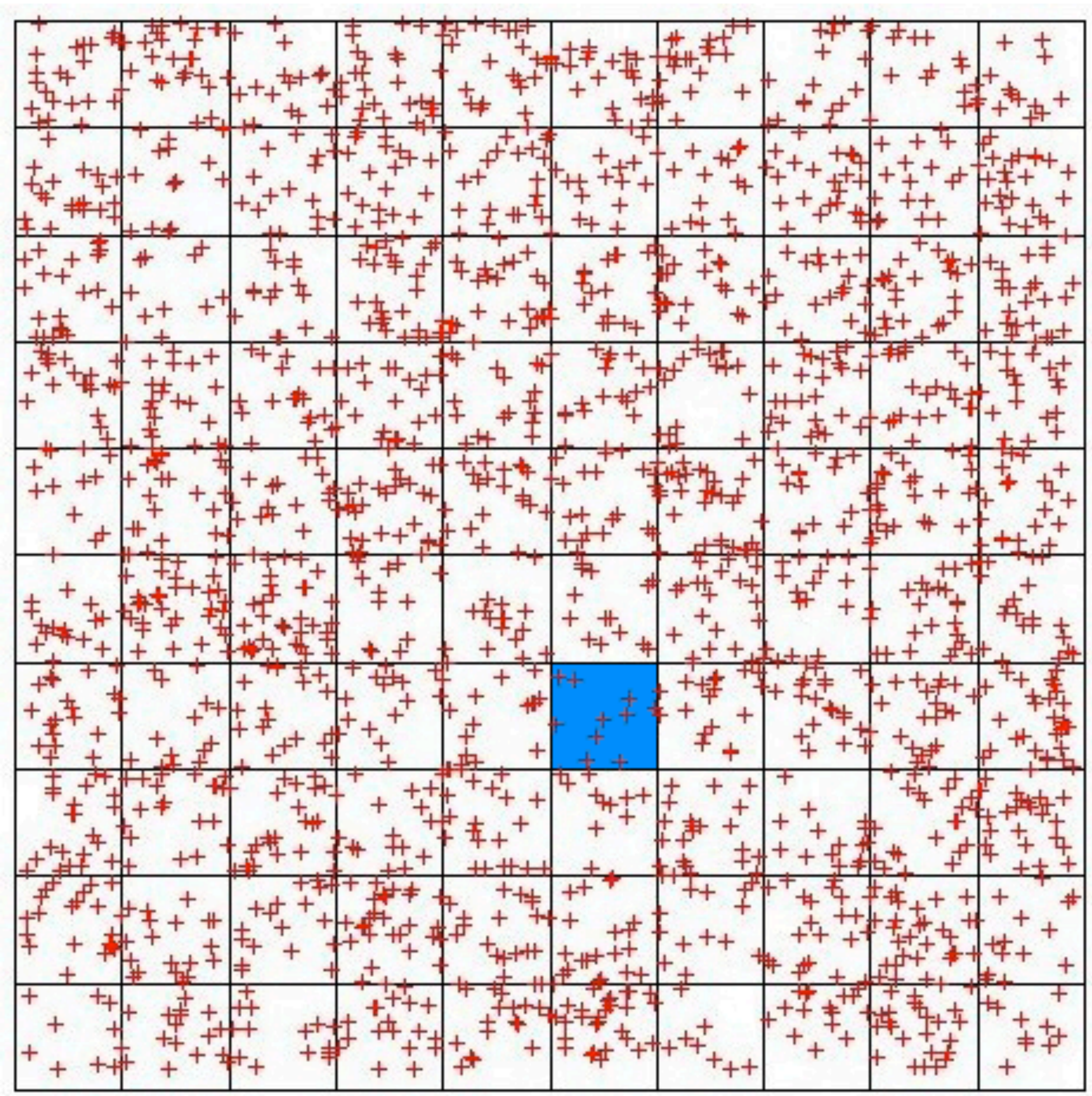
$$\langle \varepsilon \rangle = \frac{\gamma}{1-\kappa}$$

$$\frac{\partial \chi^2(\psi_\mathrm{k})}{\partial \psi_\mathrm{l}} \overset{!}{=} 0$$

$$\Rightarrow \mathcal{B}_{lk}\psi_k = \mathcal{V}_l$$

Bartelmann et al. 1996, Bradac et al. 2005,
Merten et al. 2009

Maximum-likelihood approach



Bartelmann et al. 1996, Bradac et al. 2005,
Merten et al. 2009

Having seen the KS inversion method, we consider now a "maximum likelihood" method.

$$\gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22}) \qquad \gamma_2 = \psi_{,12}$$

$$\kappa = \frac{1}{2}(\psi_{,11} + \psi_{,22})$$

$$\langle \varepsilon \rangle = \frac{\gamma}{1-\kappa}$$

$$\frac{\partial \chi^2(\psi_{\mathrm{k}})}{\partial \psi_{\mathrm{l}}} \stackrel{!}{=} 0$$

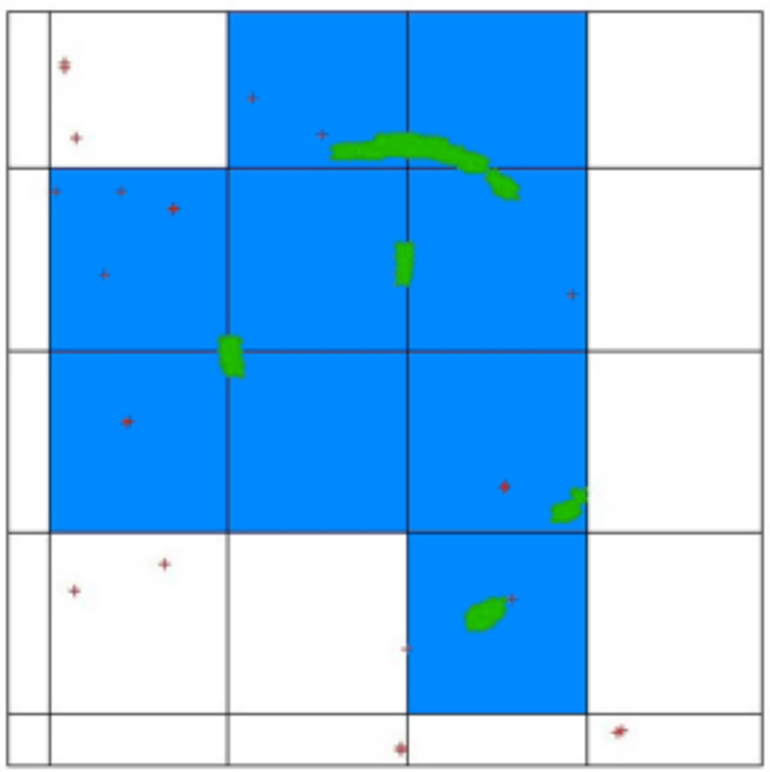
$$\Rightarrow \mathcal{B}_{lk}\psi_k = \mathcal{V}_l$$

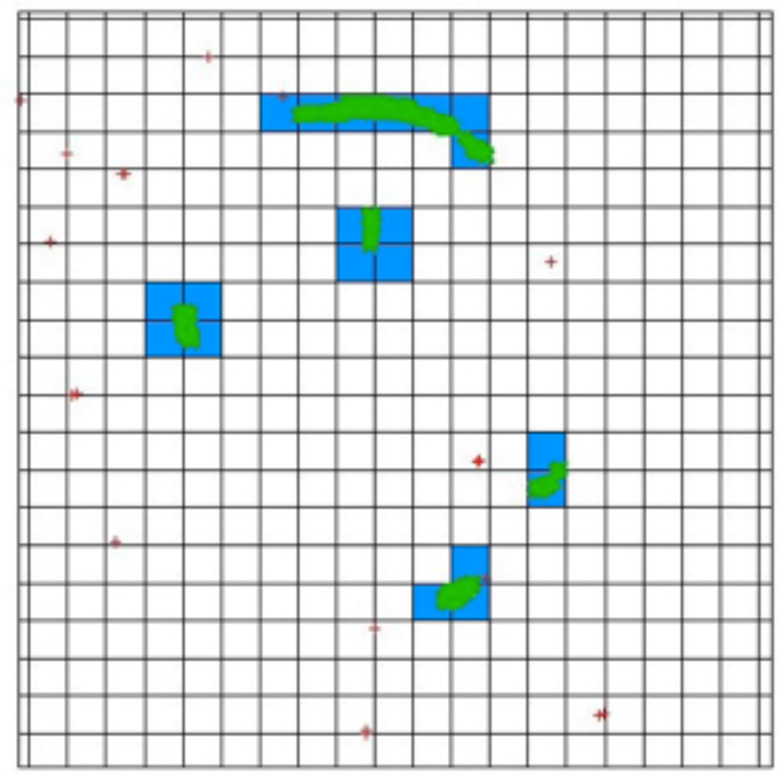
Combining WL+SL

$$\left|(1-\kappa)^2 - (\gamma)^2\right|_{\mathrm{crit}} = 0$$

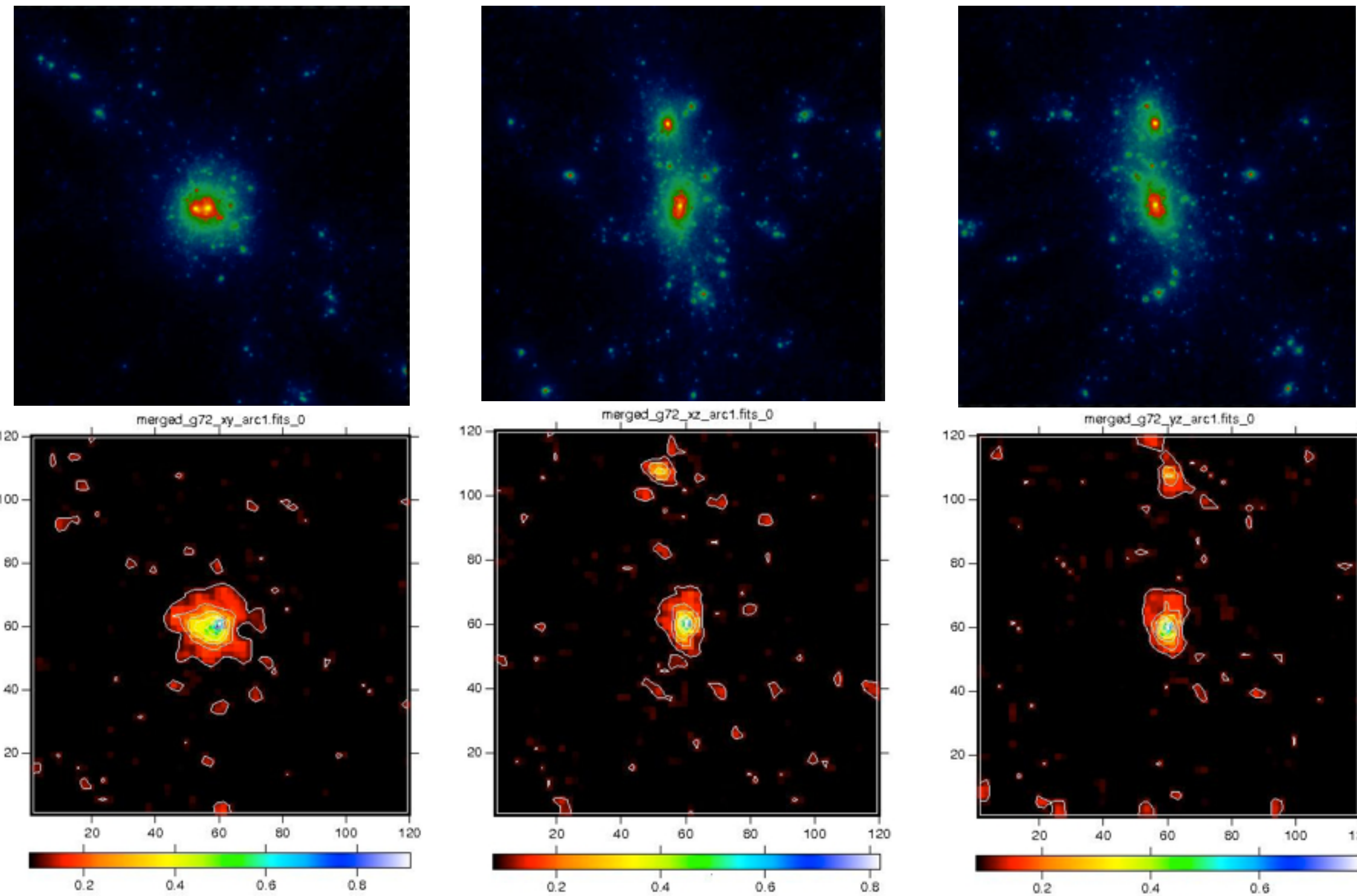
$$\chi^2(\psi) = \chi^2_{\mathrm{w}}(\psi) + \chi^2_{\mathrm{s}}(\psi)$$

$$\frac{\partial \chi^2(\psi)}{\partial \psi} \stackrel{!}{=} 0$$





Examples from simulations



Mass sheet degeneracy

A circular source is mapped by a lens with Jacobian A into an ellipse with axes:

$$a = \frac{r}{1-\kappa-\gamma} \quad , \quad b = \frac{r}{1-\kappa+\gamma}$$

The ellipticity is then:

$$\epsilon = \frac{a-b}{a+b} = \frac{2\gamma}{2(1-\kappa)} = \frac{\gamma}{1-\kappa}$$

Consider a lens whose Jacobian is $\lambda A \equiv A'$

This transformation is equivalent to changing the convergence and the shear of the lens as:

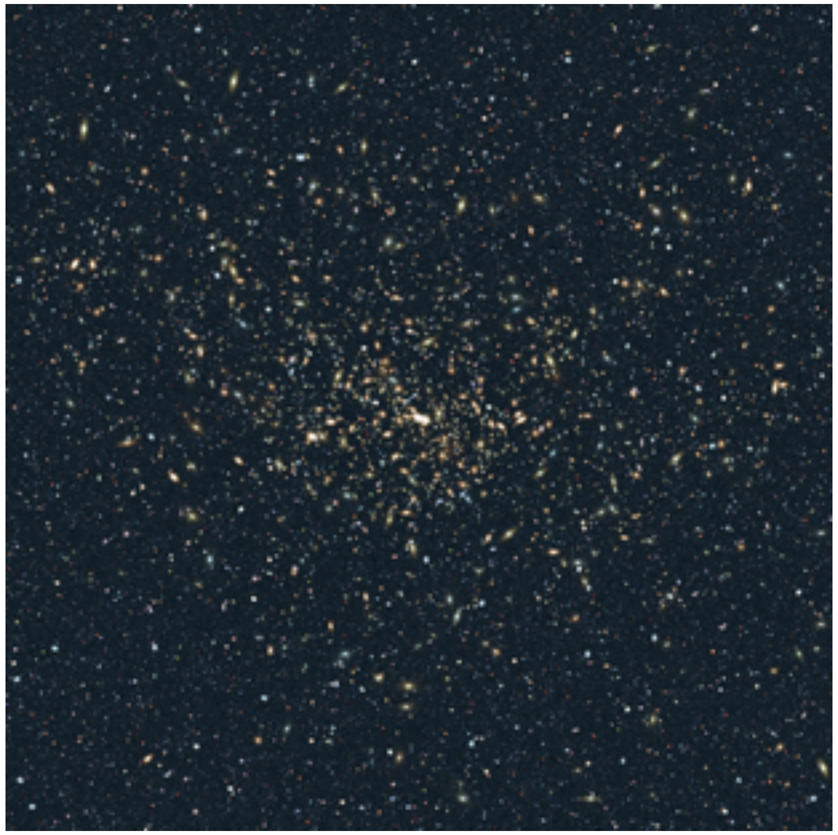
$$\gamma \to \lambda\gamma \qquad\qquad (1-\kappa) \to \lambda(1-\kappa)$$

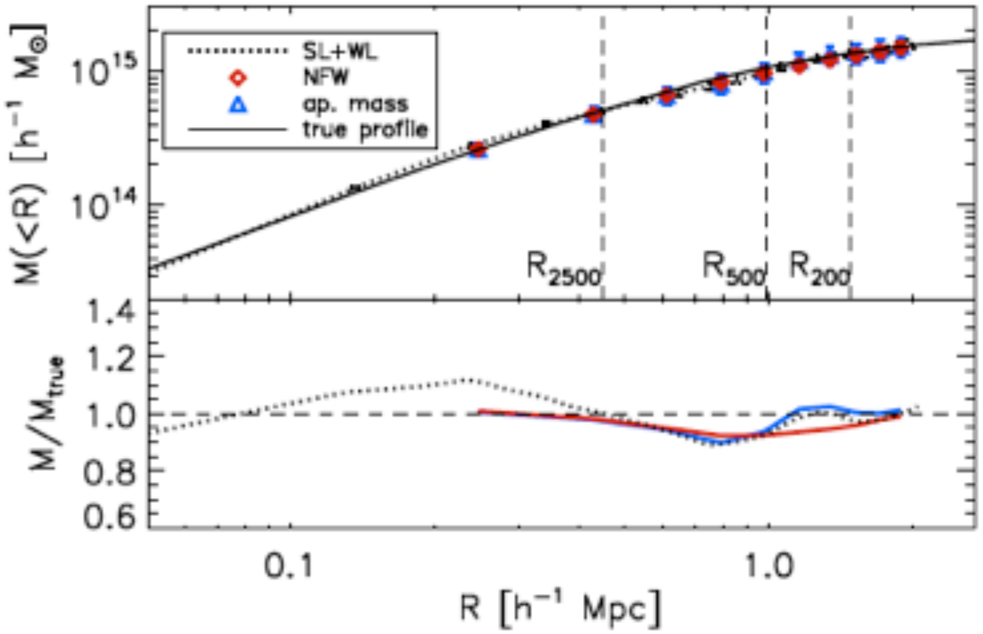
By means of this transformation the ellipticity of the lensed image would be:

$$\epsilon' = \frac{\lambda\gamma}{\lambda(1-\kappa)} = \epsilon$$

Thus, the ellipticity does not allow me to discriminate between lenses which differ by the factor λ

Performances





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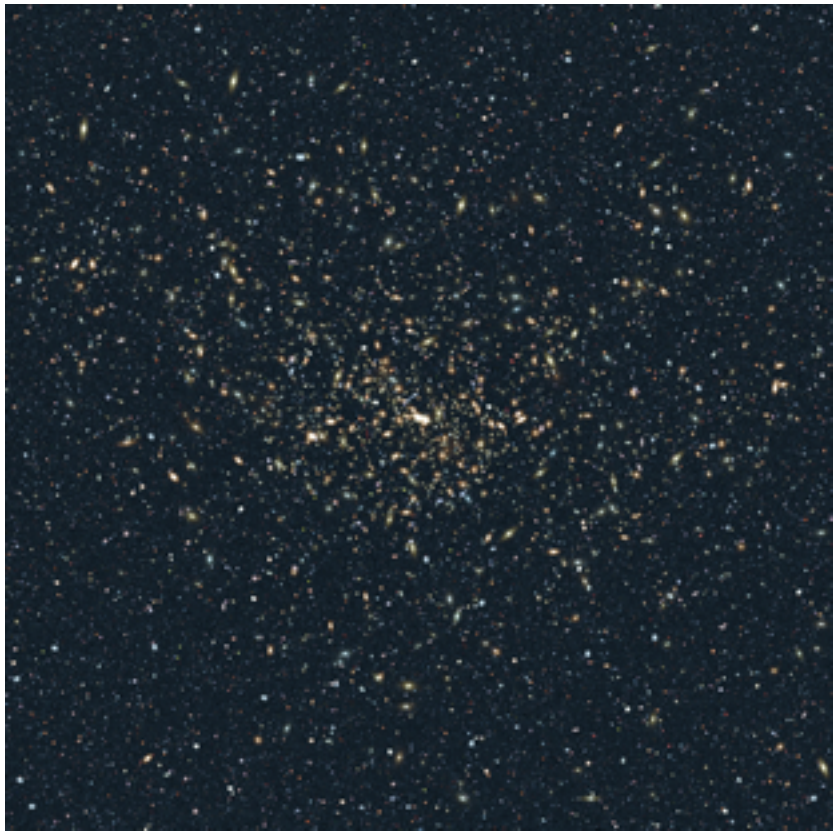

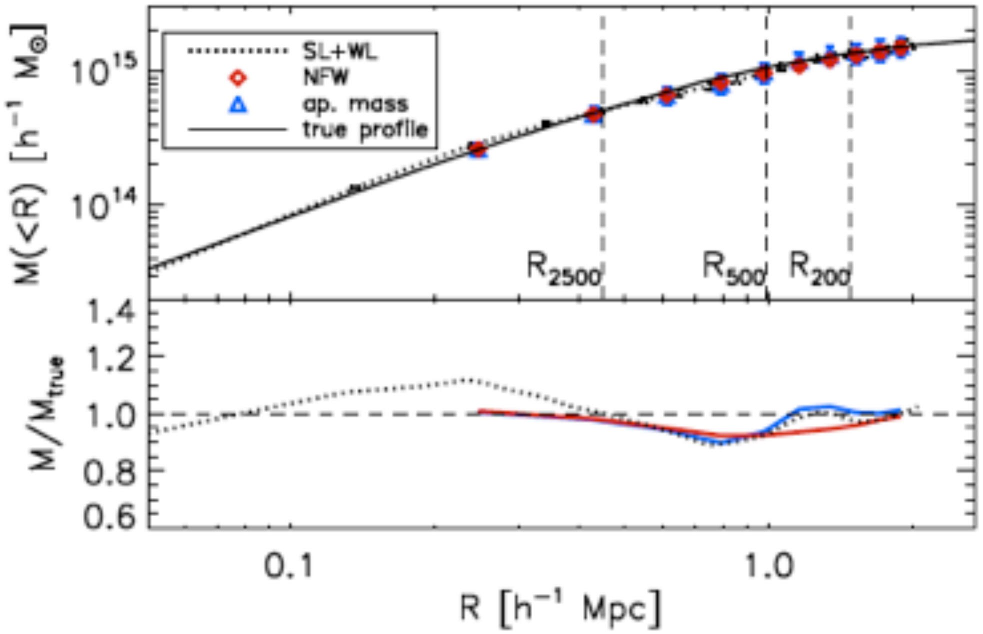

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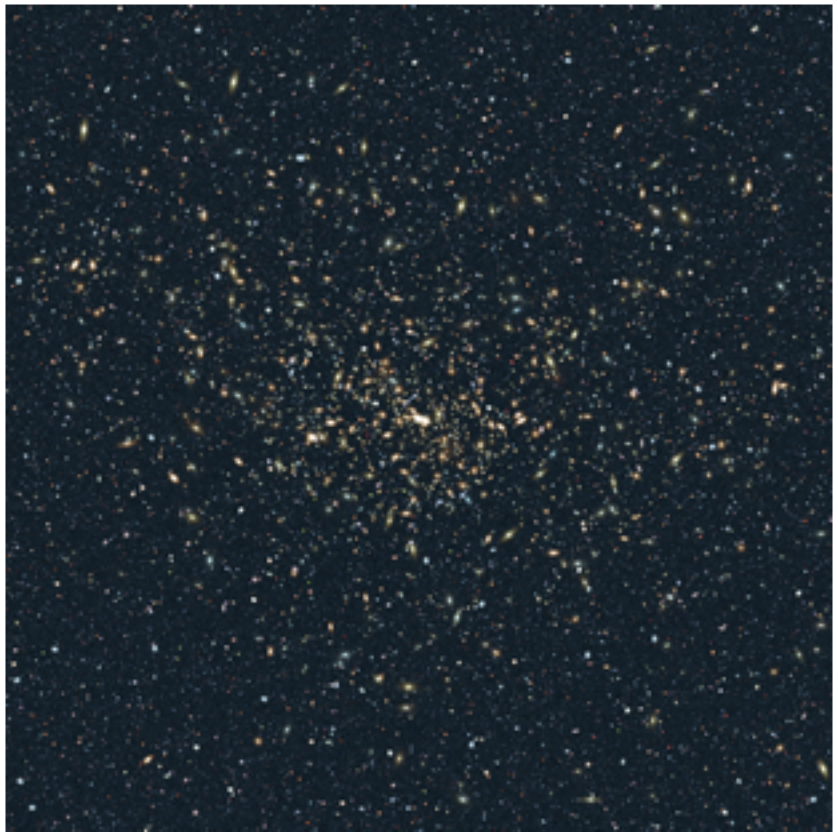
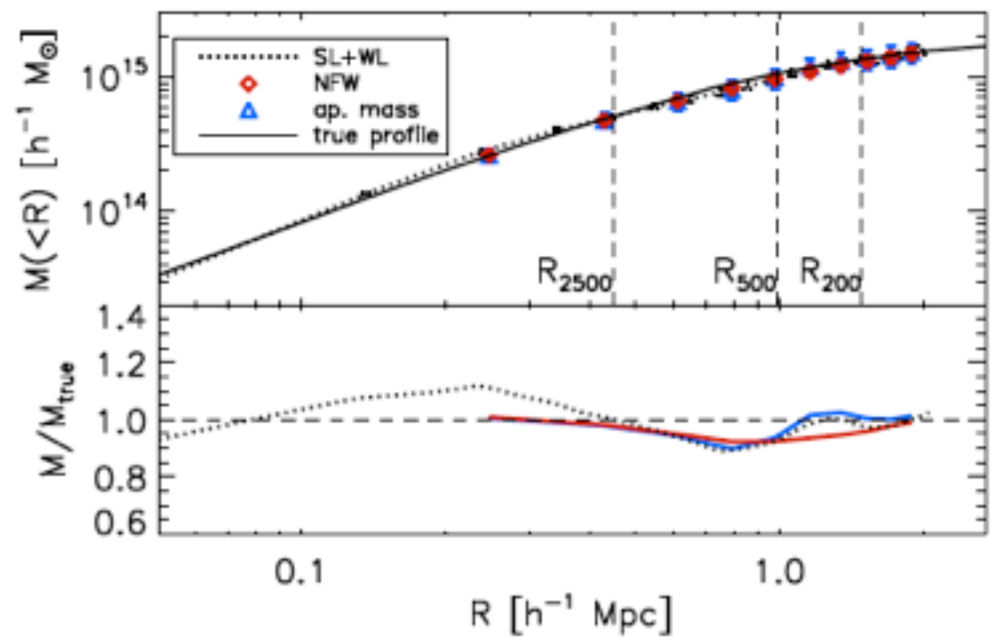
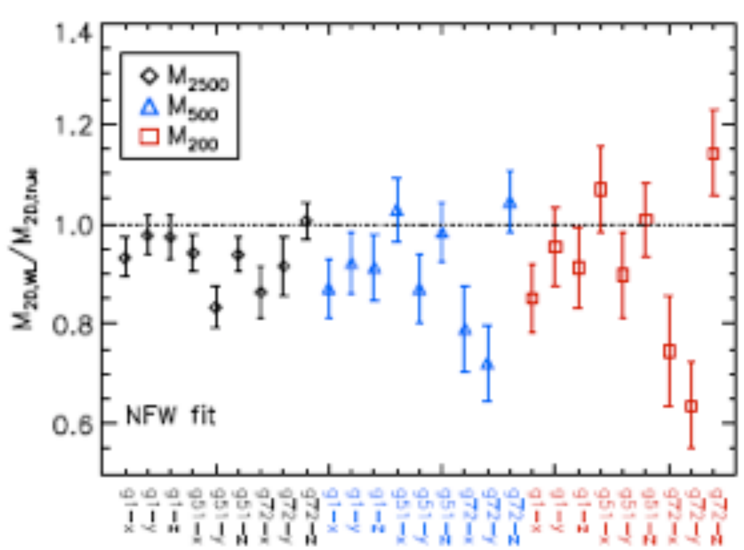
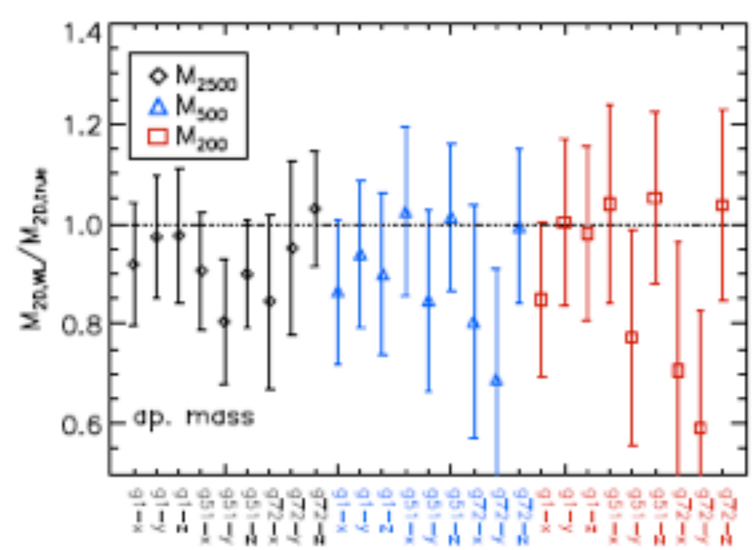
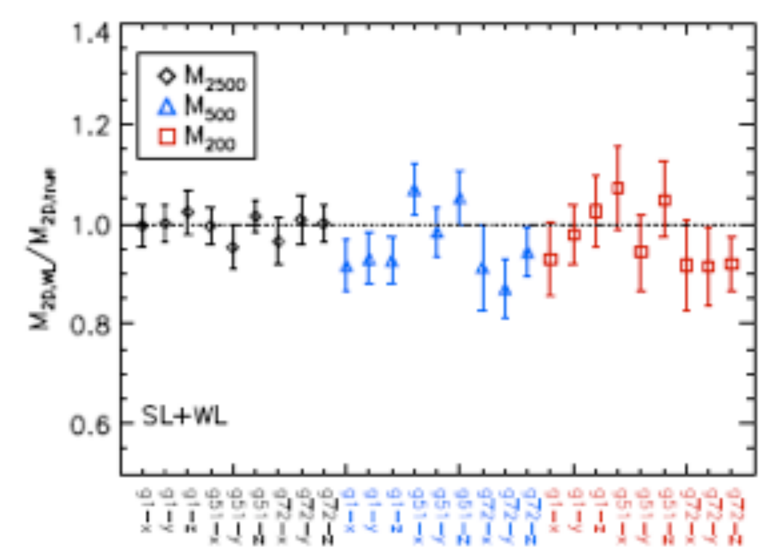


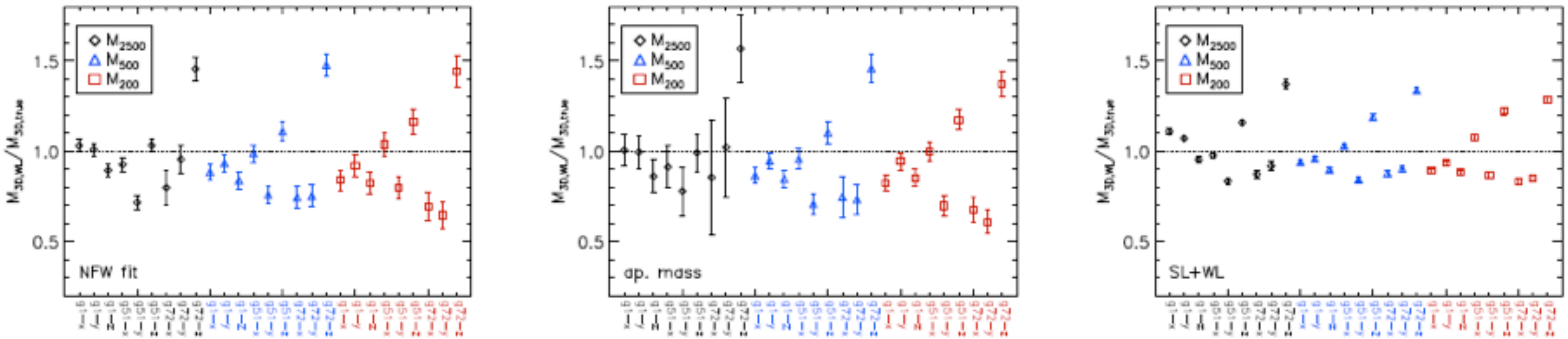






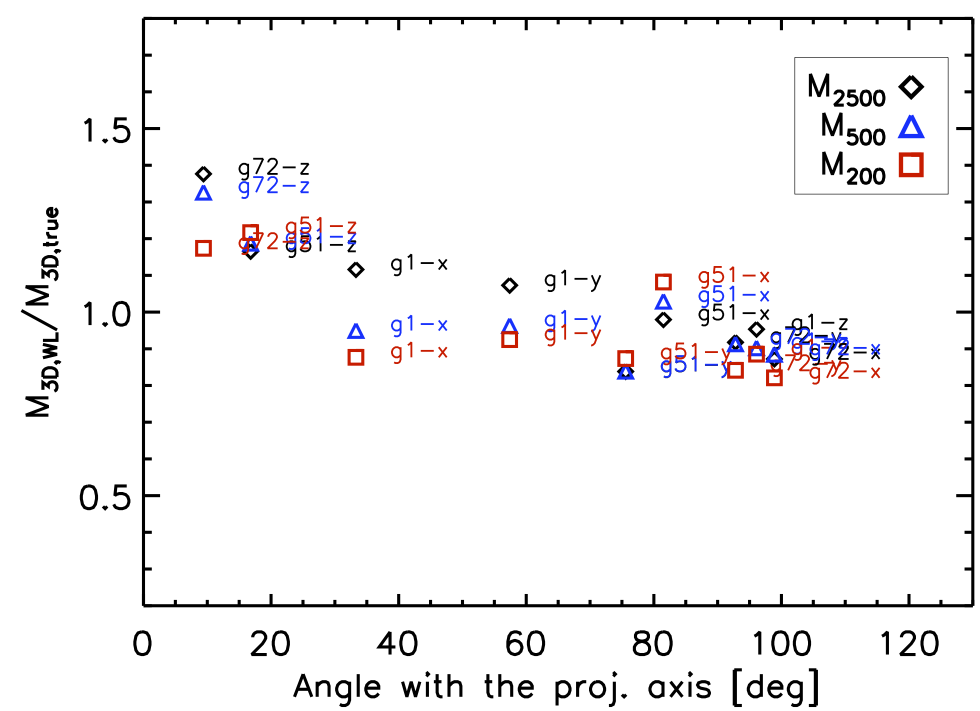


De-projection



- Lensing measures projected masses.
- 3D masses can be derived making assumptions on the 3D-shape of the clusters and on their density profiles.
- Our choices: spherical symmetry, NFW profile

cluster	z	r_{200} [h^{-1} Mpc]	M_{200} [$h^{-1}M_\odot$]	b/a	c/a	θ_x [deg]	θ_y [deg]	θ_z [deg]	c	r_s [h^{-1} Mpc]
g1	0.297	1.87	1.30×10^{15}	0.64	0.57	33.3	57.4	96.1	4.62	0.310
g51	0.2335	1.71	8.85×10^{14}	0.78	0.65	81.5	75.59	16.8	5.37	0.241
g72	0.297	1.60	8.15×10^{14}	0.31	0.29	98.9	92.8	9.4	3.99	0.299



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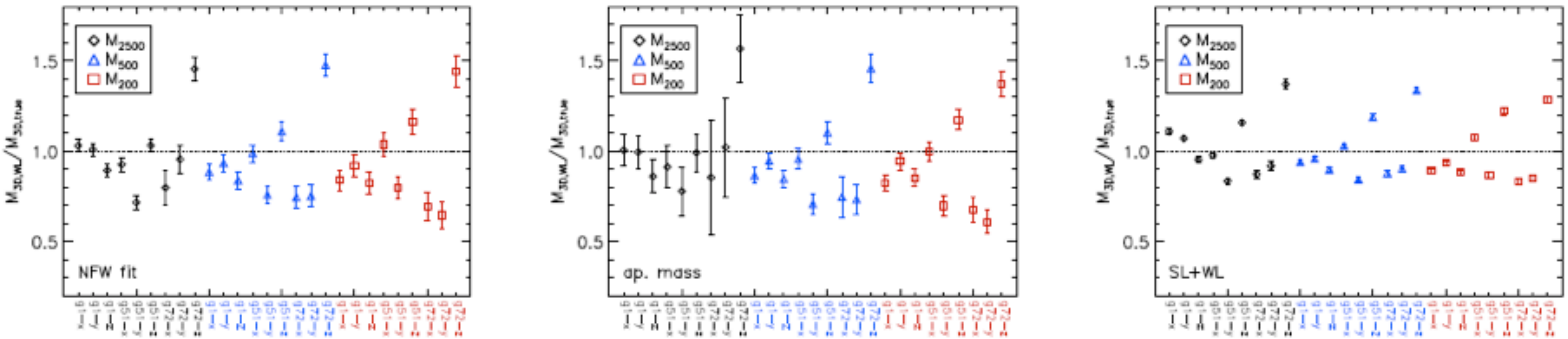

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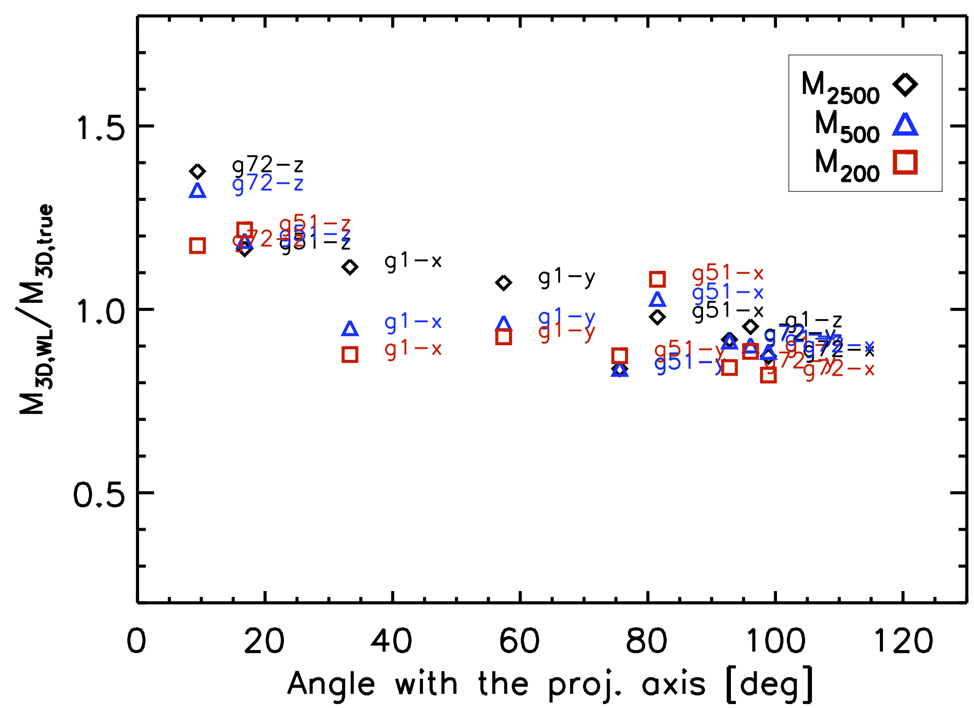
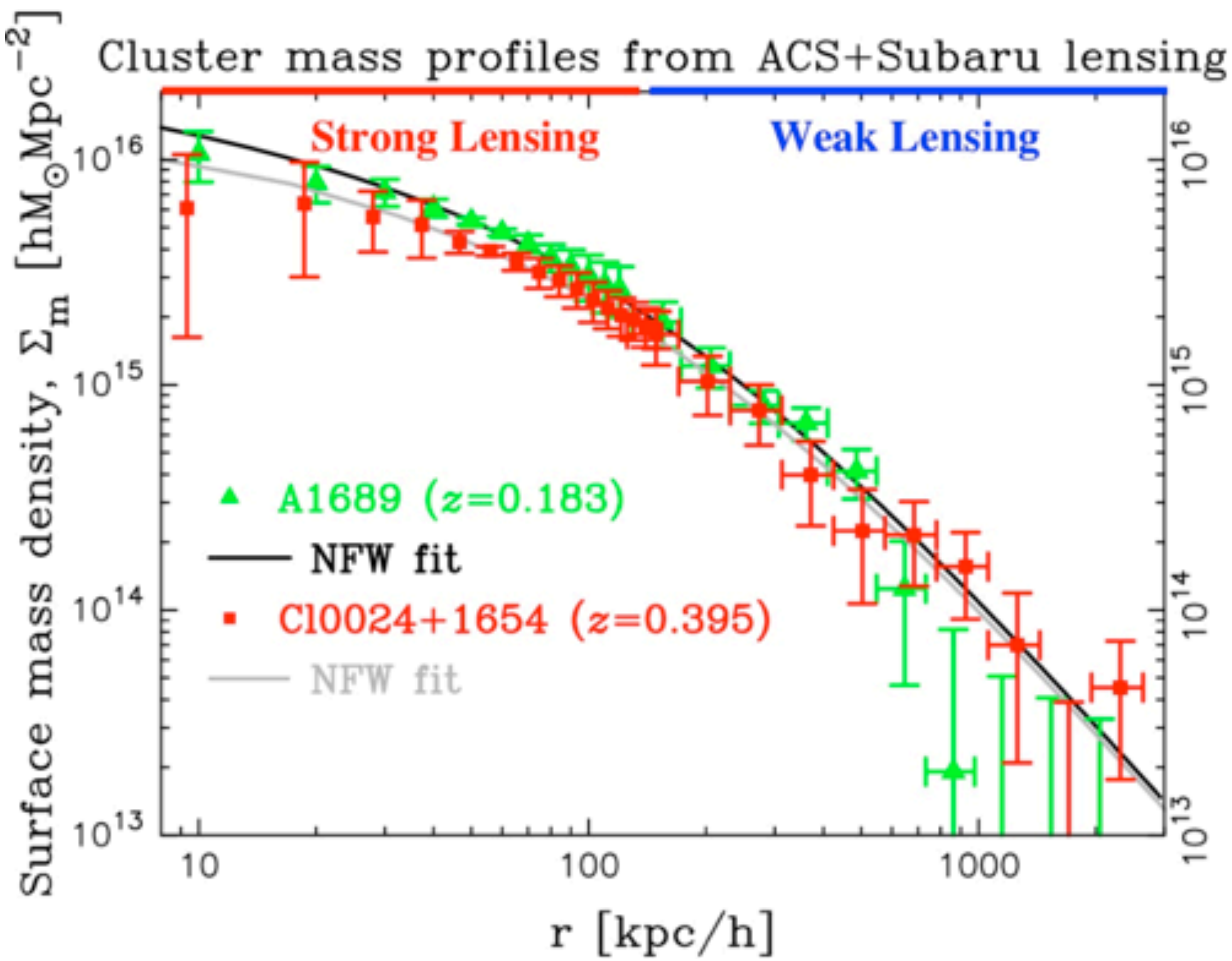
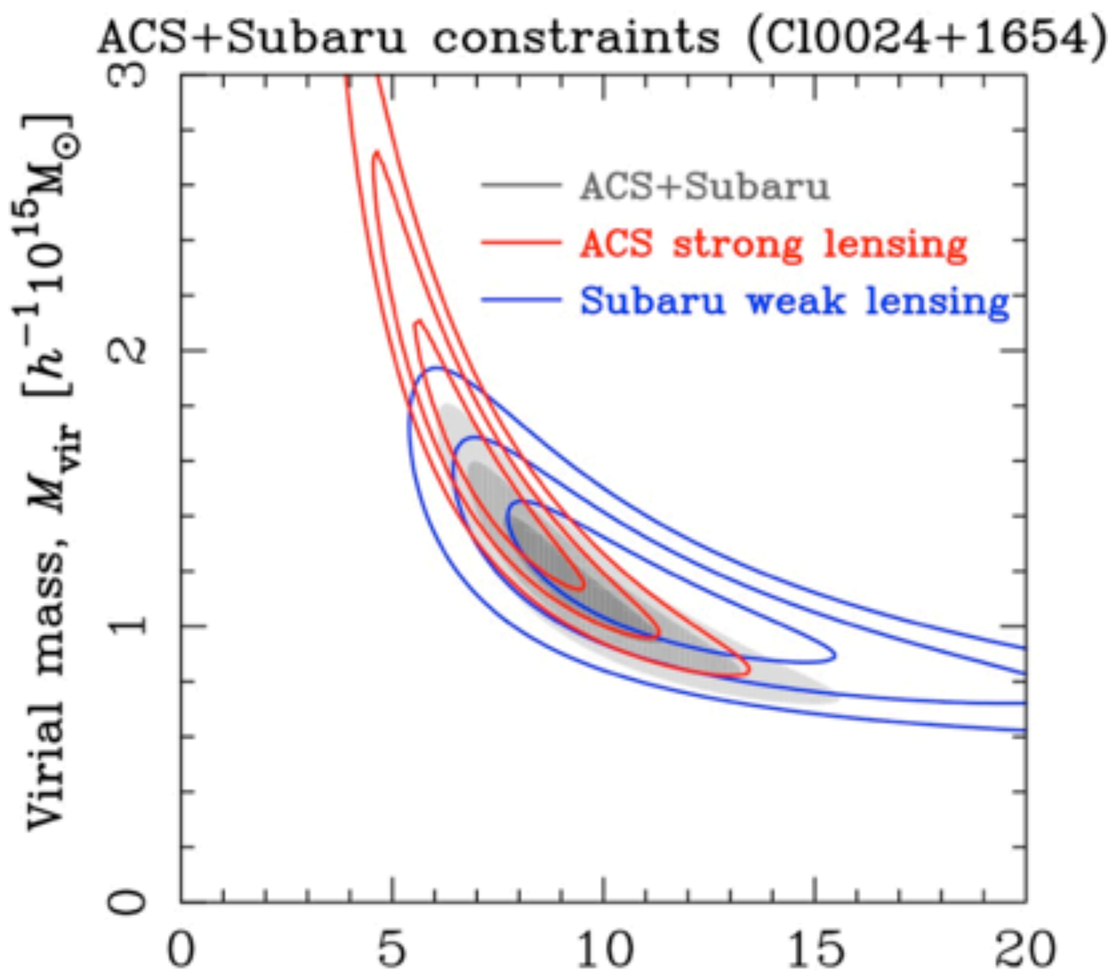
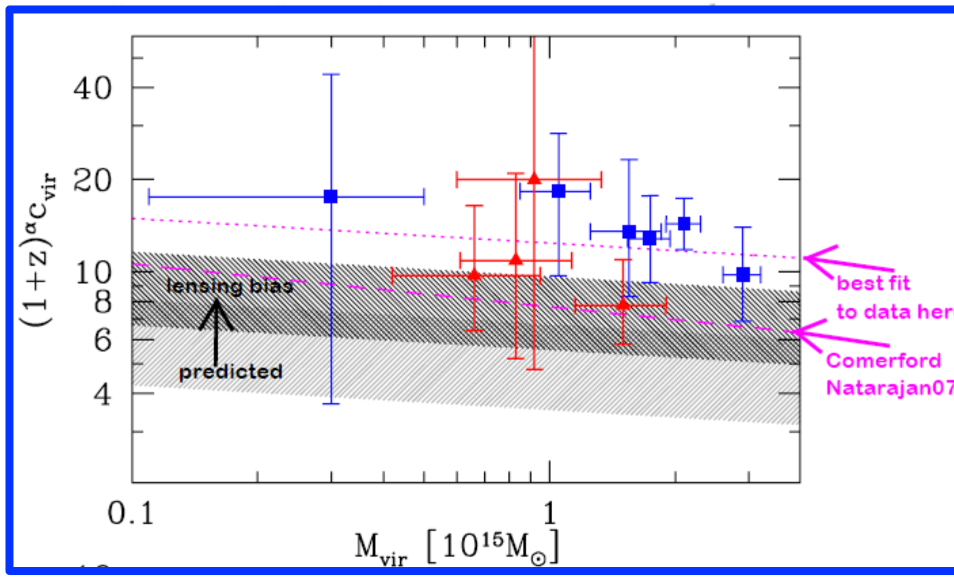


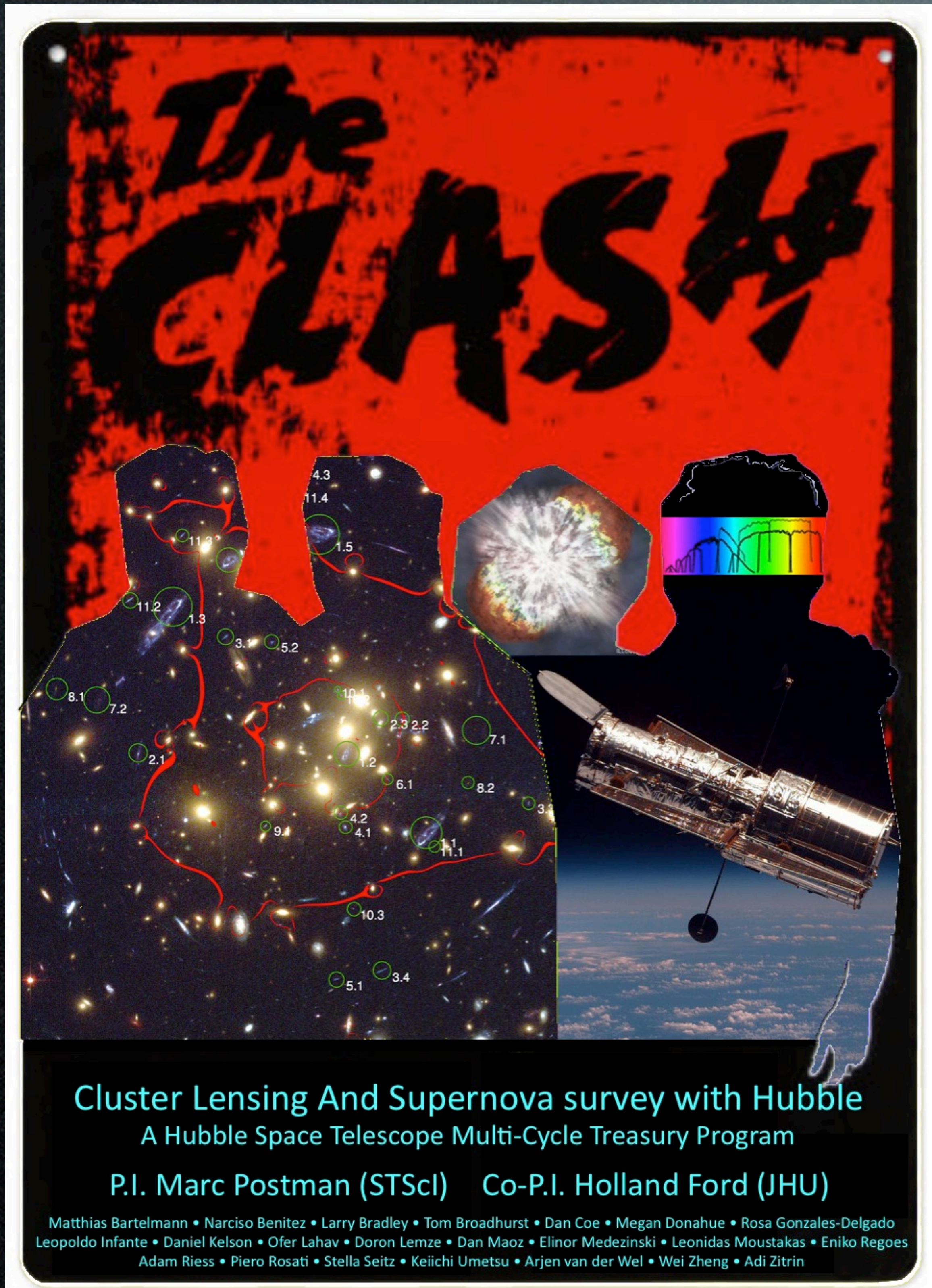
Recent measurements of cluster mass profiles





over-concentrated clusters?





- CLASH means "Cluster Lensing And Supernova survey with Hubble"
- This program has been recently approved as a Multi-Cycle Treasury program using the HST (Cycles 18-20)

Approved MCT proposals

target	P.I.	orbits
Wide field	Sandra Faber Harry Ferguson	902
Andromeda	Julianne Dalcanton	828
Galaxy Clusters	Marc Postman Holland Ford	524

http://www.stsci.edu/institute/org/spd/mctp.html/

What will CLASH do?

- will observe 25 galaxy clusters (20 orbits/cluster) in 16 ACS & WFC3 filters
- looking for strong lensing events and highly magnified sources behind clusters
    - insights into structure formation
    - mass profiles
    - cluster and lensed galaxies
    - high redshift (z>7) galaxies
- looking for SNIa in parallel fields: dark energy

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