Particle Physics Models of Dark Energy

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RESCEU
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Research Center for the Early Universe

Outline

- Introduction: the acceleration of the Universe: what are the possible explanations?
- The most obvious explanation: the Cosmological Constant.
- The Cosmological Constant problem.
- A deep and difficult problem: the gravitational properties of the zero-point fluctuations.
- Other candidates for the acceleration: quintessence, scalar-tensor theories and all that …
- Conclusions

- Lecture 1:

  1- Reminder: the FLRW cosmology

  2- How to explain dark energy?

  3- What are the physical properties of dark energy?

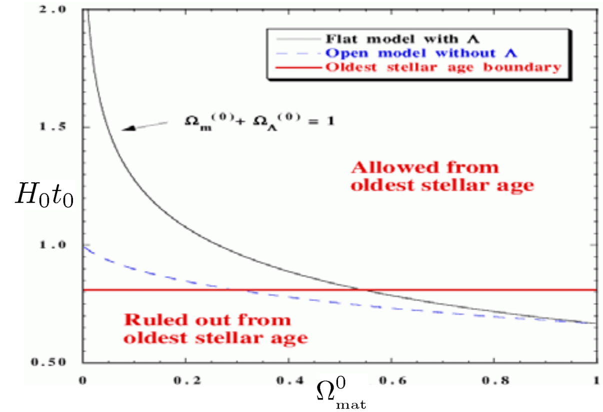
  4- The Cosmological Constant as dark energy

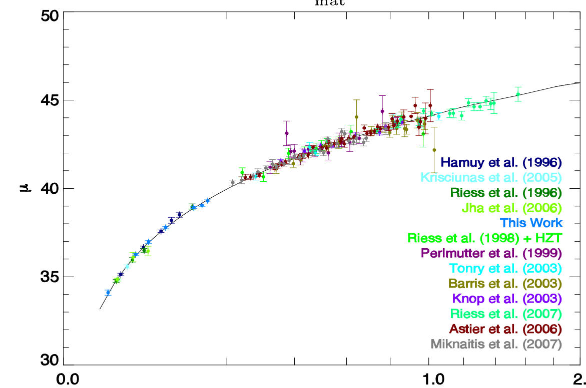
  5- Computing the zero point energy density

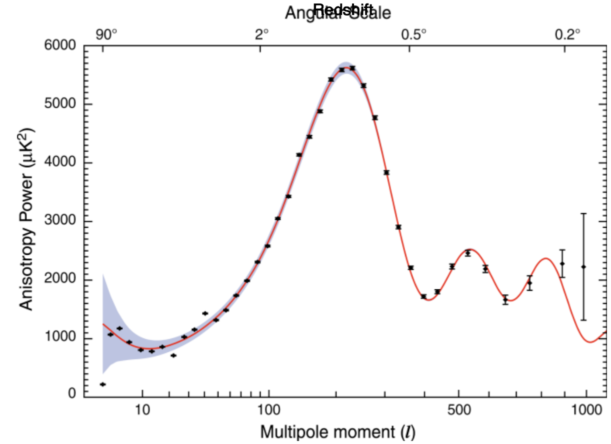
The experimental fact known as the "dark energy problem" is now supported by many independent measurements …

Strictly speaking, it means that the Friedmann equation sourced by cold dark matter does not properly describe the data

$$H^2 \neq \frac{8\pi}{3m_{\rm Pl}^2}\rho_{\rm CDM}$$







Possible explanations:

1- There is a new fluid with negative pressure dominating the present Universe

2- Gravity cannot be described by GR on large scales

3- The Cosmological principle is wrong

Gravity is described by General Relativity

"Geometry=matter"

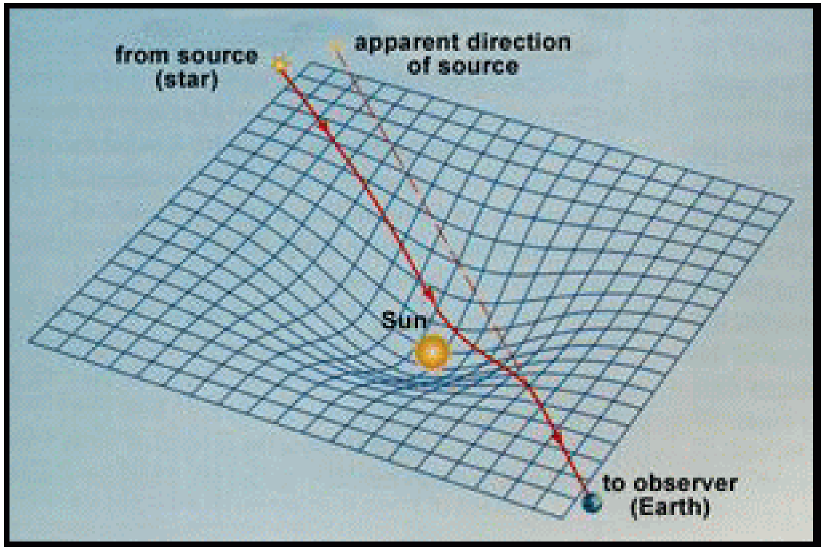
$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \kappa T_{\mu\nu}$$

Geometry

Matter

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu} = 0$$



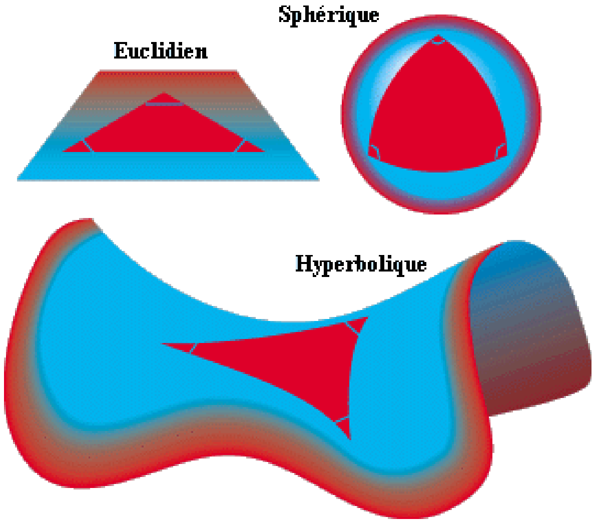
Application to Cosmology

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\varphi^2) \right]$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{\kappa}{3}\rho$$

$$-\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = \kappa p$$

$$\dot{\rho} + 3H(\rho + p) = 0$$



More on the last calculation …

$$\left.\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{\kappa}{3}\rho \\ -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) &= \kappa p \end{aligned}\right\} \qquad \boxed{\frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho + 3p)}$$

We have acceleration if the pressure is negative!

$$p < -\frac{\rho}{3}$$

The knowledge of the acceleration parameter allows us to measure the weight of each component in the Universe

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\ddot{a}}{a}\frac{1}{H^2} = \frac{1}{2}\sum_i \Omega_i(1 + 3\omega_i)$$

$$\Omega_i \equiv \frac{\rho_i}{\rho_{\rm cri}} \qquad\qquad \omega_i \equiv \frac{p_i}{\rho_i}$$

The new action with the Cosmological Constant is given by

$$S = \frac{1}{2\kappa}\int \mathrm{d}^4\mathbf{x}\sqrt{-g}\,(R - 2\Lambda_{\mathrm{B}}) + S_{\mathrm{matter}}$$

$$\boxed{R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} + \Lambda_{\mathrm{B}} g_{\mu\nu} = \kappa T_{\mu\nu}}$$

Bared cosmological constant (dimension m^{-2})

This term can be added because

$$\nabla^{\mu}\left(\Lambda_{\mathrm{B}} g_{\mu\nu}\right) = \Lambda_{\mathrm{B}} \nabla^{\mu} g_{\mu\nu} = 0$$

We still have a conserved stress-energy tensor

The equation of state of a CC is negative

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda_{\mathrm{B}} g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$T_{\mu\nu} = -\frac{\Lambda_{\mathrm{B}}}{\kappa} g_{\mu\nu}$$

$$\rho_{\Lambda} = \frac{\Lambda_{\mathrm{B}}}{\kappa}, \quad p_{\Lambda} = -\frac{\Lambda_{\mathrm{B}}}{\kappa}$$

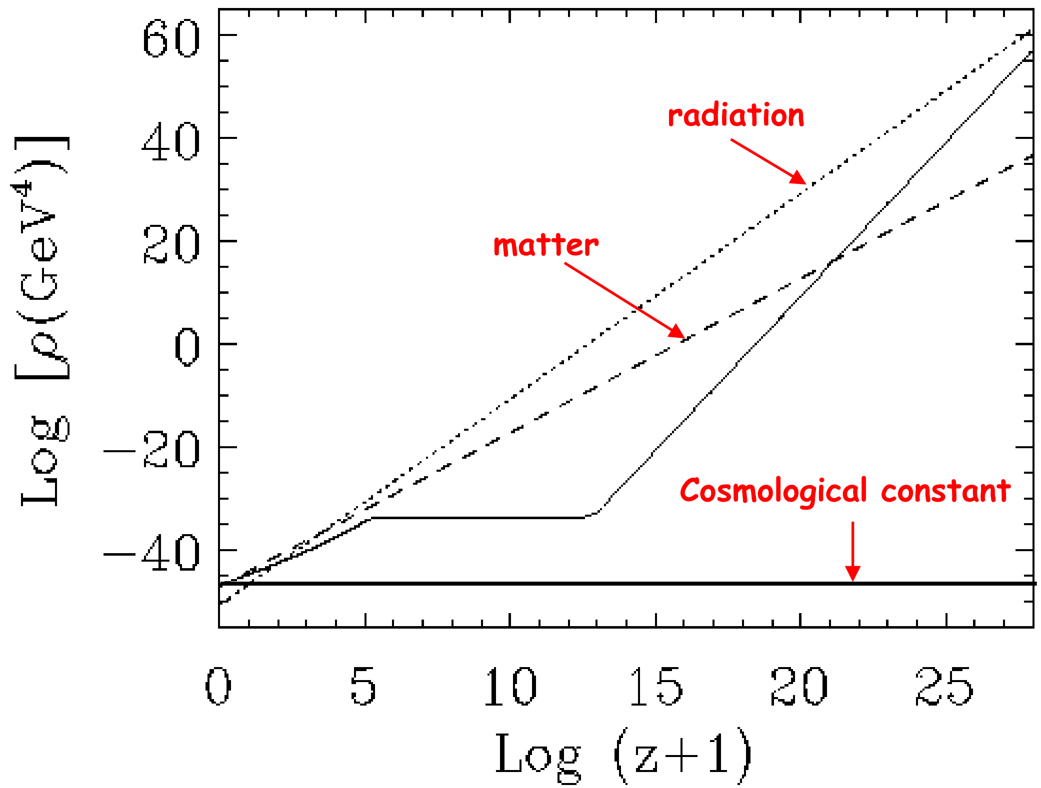
$$\omega_{\Lambda} = \frac{p_{\Lambda}}{\rho_{\Lambda}} = -1$$

Stress-energy tensor of a perfect fluid

$$T_{\mu\nu} = (\rho + p) u_{\mu} u_{\nu} + p g_{\mu\nu}$$

$$\dot{\rho} + 3H(\rho + p) = \dot{\rho} = 0$$

The energy density associated with the cosmological constant is constant with time ...!



Why a cosmological constant can cause acceleration??

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3}\rho = \frac{\Lambda_{\rm B}}{3} = \text{constant}$$

$$\downarrow$$

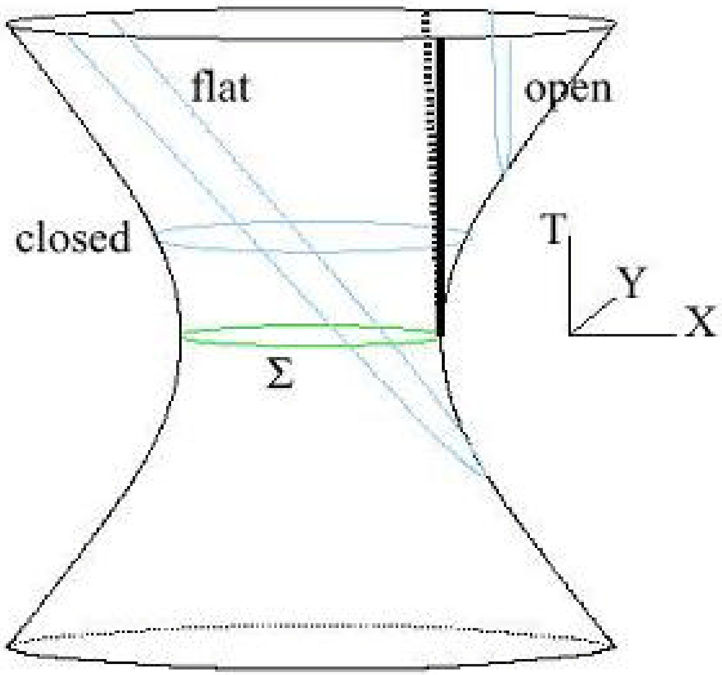
$$a(t) \propto \mathrm{e}^{Ht}$$

$$\downarrow$$

$$\frac{\ddot{a}}{a} = H^2 > 0$$

Obvious since the pressure is negative …

The corresponding spacetime is the de Sitter spacetime (as for inflation)



What is the observed value of the Cosmological Constant (if it is responsible for the acceleration!)?

assumes GR, ie the equation $\frac{\ddot{a}}{a} = -\frac{\kappa}{6}\sum_{i=1}^{N}(\rho_i + 3p_i)$

$$q_0 \equiv -\frac{a_0 \ddot{a}_0}{(\dot{a}_0)^2} = \frac{1}{2}\sum_{i=1}^{N}\Omega_i\,(1+3\omega_i) = \frac{1}{2}\Omega_{\rm m} + \frac{1}{2}\Omega_{\rm x}\,(1+3\omega_{\rm x}) \sim -0.67$$

$$\Omega_i = \frac{\rho_i}{\rho_{\rm cri}} \qquad \omega_i = \frac{p_i}{\rho_i}$$

$$\omega_{\Lambda} = -1 \longrightarrow \quad q_0 = -\Omega_{\Lambda} + \frac{1}{2}\Omega_{\rm m} \sim -0.67$$

$$\Omega_{\Lambda} + \Omega_{\rm m} = 1$$

$$\downarrow$$

$$\Omega_{\Lambda} = \frac{2}{3}\left(\frac{1}{2} - q_0\right) \sim 0.78$$

New energy scale

$$\boxed{\begin{aligned} \rho_{\Lambda} &\sim 10^{-47}{\rm GeV}^4 \sim \rho_{\rm cri} \\ &\sim \left(10^{-3}{\rm eV}\right)^4 \end{aligned}}$$

The problem comes from the fact that there are other contributions to the cosmological constant term ...

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda_{\rm B} g_{\mu\nu} = \kappa T_{\mu\nu}$$

(Quantum) vacuum states of fields (scalars, fermions …) mimic (or contribute to) the CC terms

$$\boxed{T^{\rm vac}_{\mu\nu} = - \langle \rho \rangle \, g_{\mu\nu}}$$

In GR, the vacuum gravitates and cannot be subtracted as in ordinary QFT … "weigh of the vacuum" problem

- Summary of Lecture 1: where do we stand?

- Lecture 2:

  1- How to properly regularize the vacuum energy?

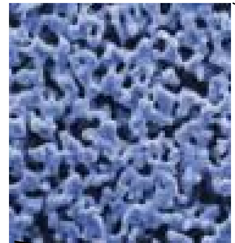
  2- Are the zero point fluctuations real?

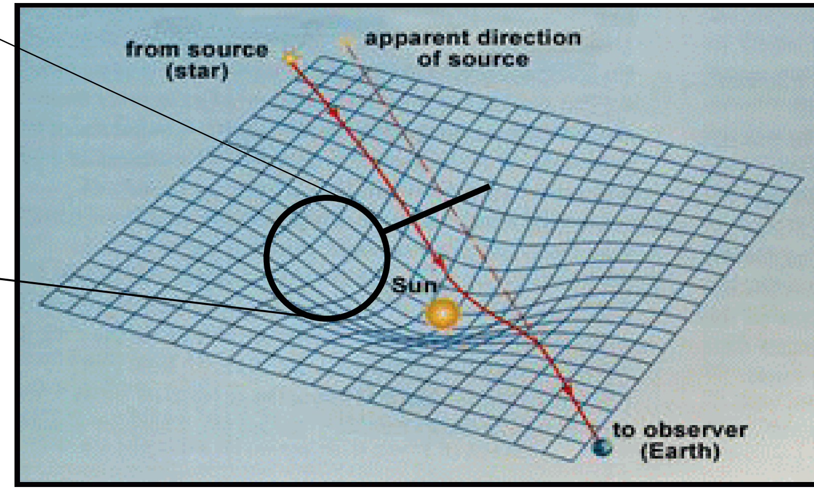
  3- The Casimir effect

  4- The Lamb shift effect

  5- Can supersymmetry save us?

“The weigh of the Vacuum”





$$T_{\mu\nu} = -\langle\rho\rangle\, g_{\mu\nu}$$

The effective cosmological constant is

$$\boxed{\Lambda_{\rm eff} = \Lambda_{\rm B} + \kappa\langle\rho\rangle}$$

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda_{\rm B} g_{\mu\nu} = \kappa T_{\mu\nu} - \kappa\langle\rho\rangle g_{\mu\nu}$$

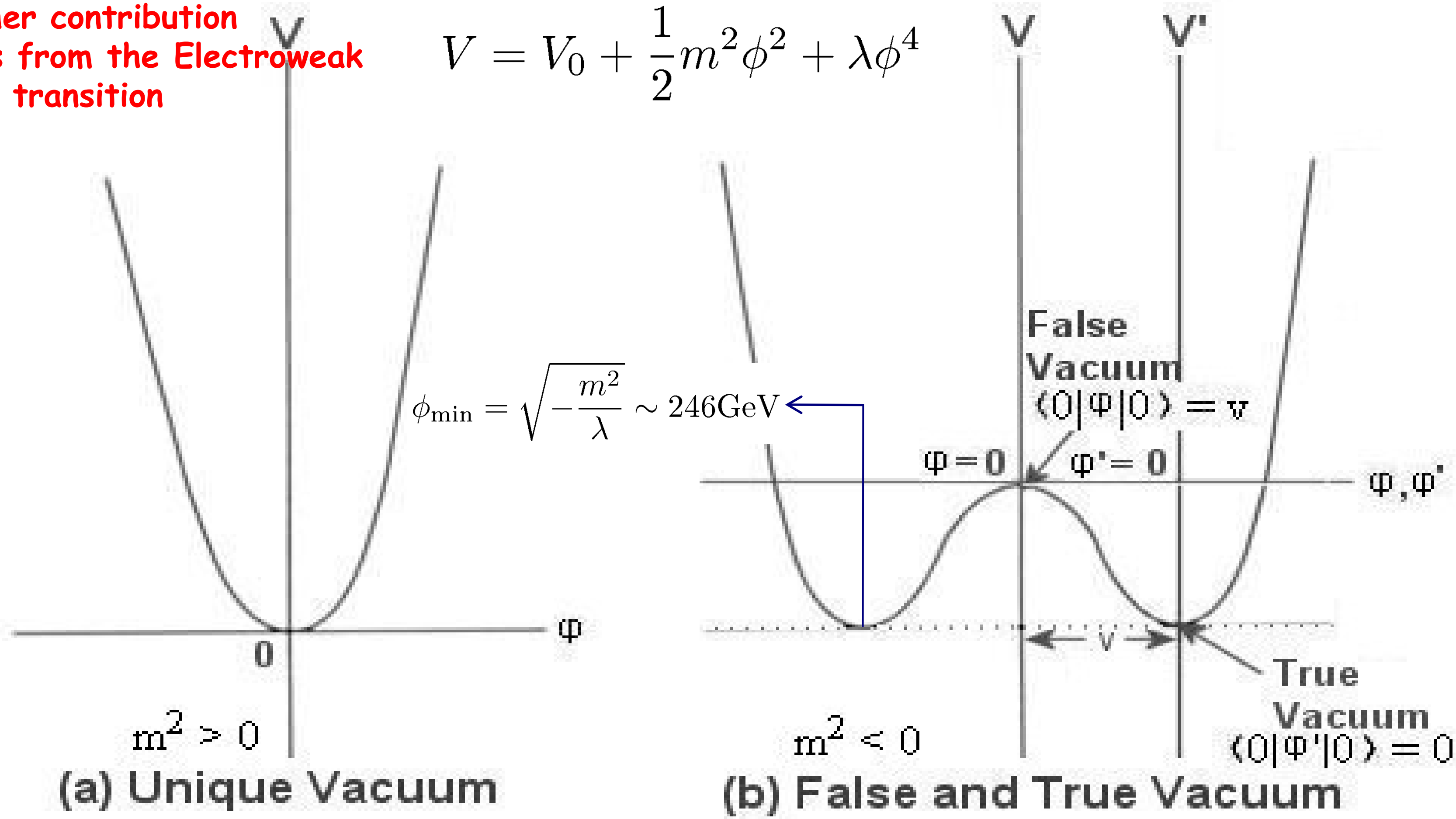
A “naïve” calculation gives $(M_{\rm C} = M_{\rm Pl})$

$$\langle\rho\rangle = \frac{1}{2}\frac{1}{(2\pi)^3}\int {\rm d}^3\mathbf{k}\sqrt{\mathbf{k}^2+m^2} = \frac{M_{\rm C}^4}{16\pi^2}\left(1+\frac{m^2}{M_{\rm C}^2}+\cdots\right) \sim 10^{122}\rho_{\rm cri}$$

Gravity (G and c) and the quantum theory (~) points toward a very high energy scale in comparison to what we see …

Another contribution comes from the Electroweak phase transition

$$V = V_0 + \frac{1}{2} m^2 \phi^2 + \lambda \phi^4$$



$$\phi_{\min} = \sqrt{-\frac{m^2}{\lambda}} \sim 246\mathrm{GeV}$$

$$\langle \rho \rangle_{\mathrm{PT}} = V_0 - \frac{m^4}{4\lambda} = V_0 - \frac{\lambda}{4} (246\,\mathrm{GeV})^4$$

Huge in comparison with the critical energy density

So the cosmological constant problem consists in the following …

- In General Relativity, the vacuum gravitates

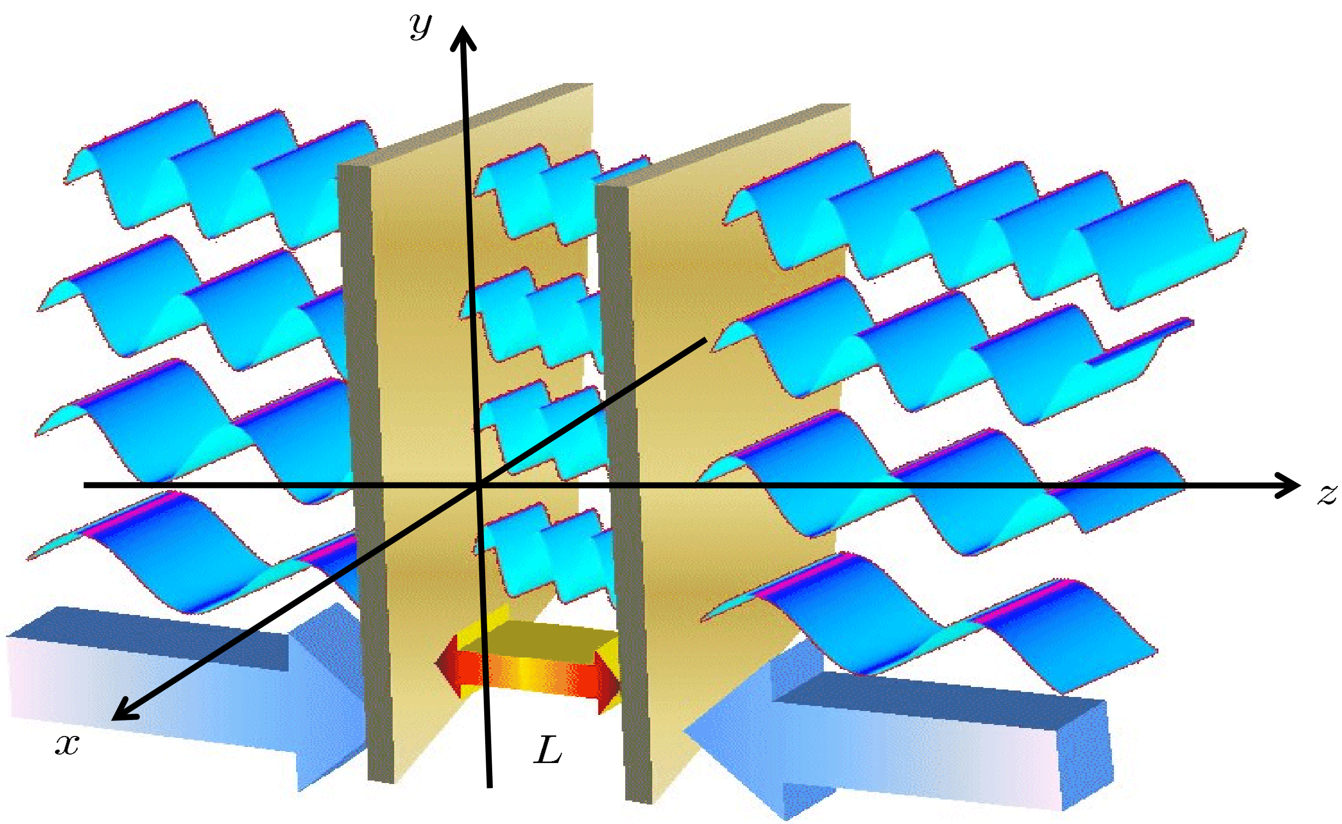
- The vacuum energy density is made of huge disconnected pieces while the observed value is tiny

$$\underbrace{\Lambda_{\rm eff} = \Lambda_{\rm B}}_{\text{Small or vanishing}} + \underbrace{\kappa\langle\rho\rangle_{\rm vev}}_{\text{huge}} + \underbrace{\kappa\langle\rho\rangle_{\rm PT}}_{\text{huge}} + \cdots$$

- Miraculous cancellation?

- Landscape (ie eternal inflation+string theory)?

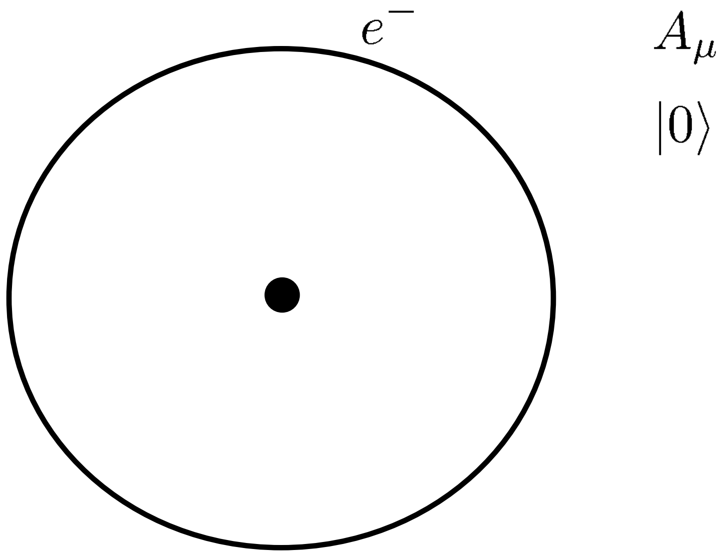
Casimir effect



$$E = \frac{1}{2}\sum_{\alpha}\hbar\omega_{\alpha} = \left(\frac{D}{2\pi}\right)^2 \int \mathrm{d}k_x \int \mathrm{d}k_y \sum_{n=1}^{\infty} \frac{1}{2}\sqrt{k_x^2 + k_y^2 + n^2\frac{\pi^2}{L^2}}$$

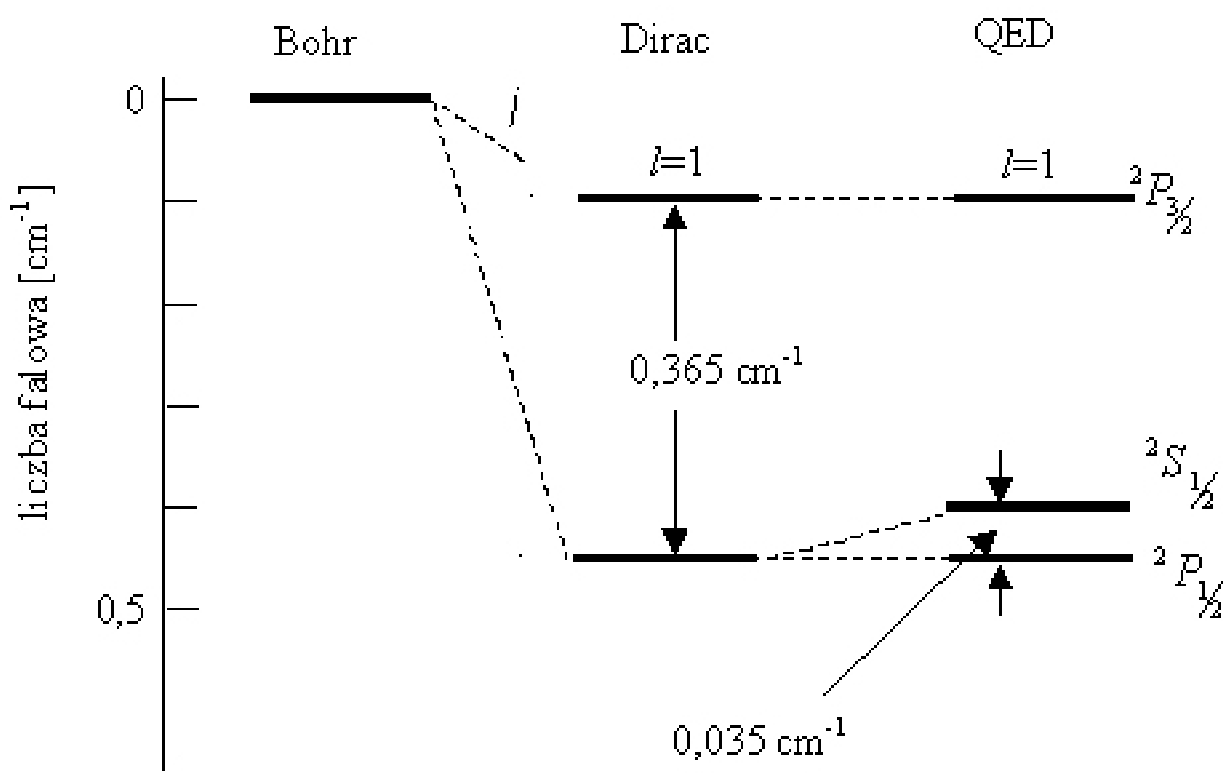
Do the zero-point fluctuations exist?

Lamb shift effect



The electron is going to fluctuate because of the presence of zero-point fluctuations

Lamb shift effect



- Summary of Lecture 2:

  1- The zero-point fluctuations give a hudge contribution to the CC (even if they are usually not properly computed; free case scales as $m^2 M_{\mathrm{C}}^2$ and not as M_{C}^4)

  2- The zero-point fluctuations are real as proven experimentally by the Casimir and the Lamb shift effects!

  3- The CC problem is therefore a disturbing mystery! Even more problematic if this is indeed the dark energy!

- Lecture 3:

  1- Can supersymmetry save us?

  2- What if dark energy is not the CC?

So the cosmological constant problem consists in the following …

- In General Relativity, the vacuum gravitates

- The vacuum energy density is made of huge disconnected pieces while the observed value is tiny

$$\Lambda_{\rm eff} = \Lambda_{\rm B} + \kappa\langle\rho\rangle_{\rm vev} + \kappa\langle\rho\rangle_{\rm PT} + \cdots$$

Small or vanishing   huge   huge

- Miraculous cancellation?

- Landscape (ie eternal inflation+string theory)?

The previous difficulties has led to the search of alternatives models of dark energy

1- This does not solve the CC problem!

2- This means that the CC is not the dark energy … even if everything is compatible with a CC (so far!)

3- It may be argued that this makes the CC problem easier since we are back to the old CC problem, ie why is it zero (seems easier than why is it tiny but not vanishing)

4- Either we modify the stress-energy tensor and we say that we have forgotten some form of energy in the energy budget of the Universe (quintessence, quintessence with non-canonical kinetic terms, Chapligyn gaz, …)

5- Either gravity is not described by GR on large scales and we attempt to modify it (scalar tensor theories, DGP models, …)

6- Or, locally, there is violation of the cosmological principle

7- Something else?

Can the dark energy be a (scalar) field?

$$\rho_Q = \frac{\dot{Q}^2}{2} + V(Q)$$

$$p_Q = \frac{\dot{Q}^2}{2} - V(Q)$$

If the potential energy dominates, one can have negative pressure (as for inflation)

1- This is not a simple "reverse-engineering" problem, ie give me the equation of state and I will give you the potential because we require additional properties, to be discussed in the following.

2- This allows us to study dark energy with time-dependent equation of state: the scenario is falsifiable (ie different from the CC case)

3- Since we have a microscopic model, we can consistently computed the cosmological perturbations

4- This allows us to discuss the link with high-energy physics and to play the game of model building.

5- This does not solve the CC problem. Instead of explaining $\Omega_\Lambda = 0.7$ of the critical energy density we are just back to $\Lambda = 0$

Quintessence (II)

- <u>Interesting potential to have a good model of dark energy from the cosmological point of view?</u>

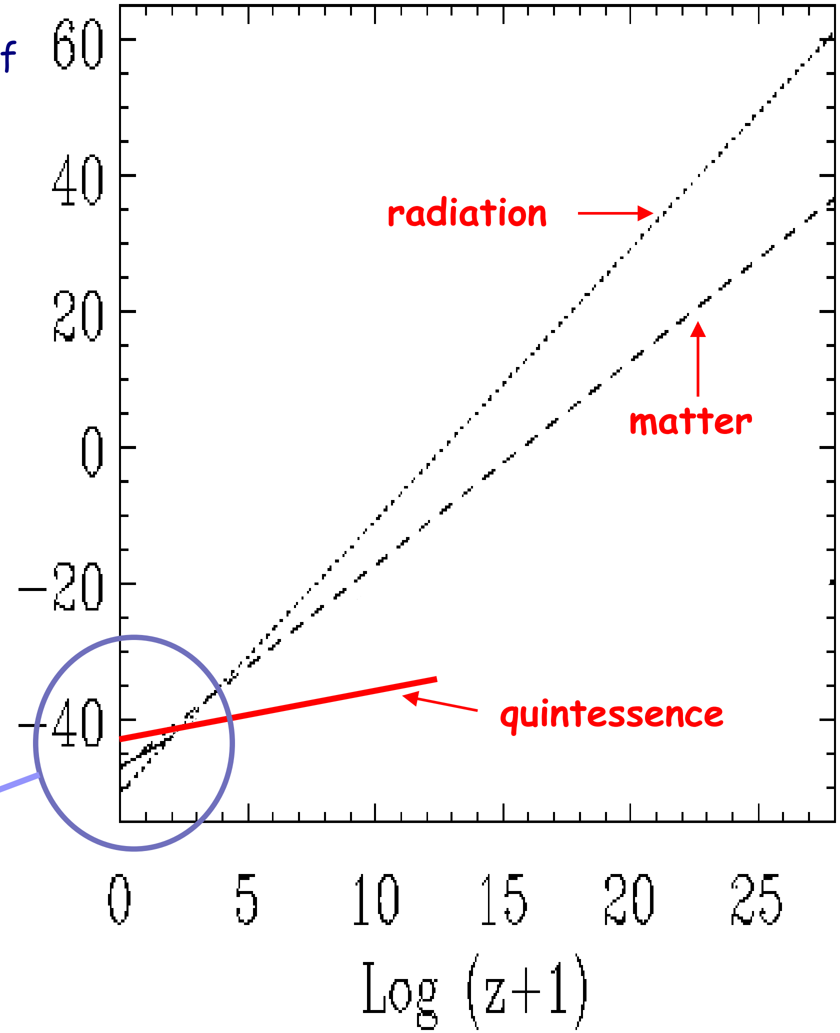
  - The scalar field must dominate the matter content of the Universe today and have a negative equation of state.

  - The domination must have started quite recently

  - Before that, the scalar field must be hidden, ie subdominant (a test field) in order not to spoil structure formation, BBN etc …

  - This means that the scalar field must scale less rapidly than the background

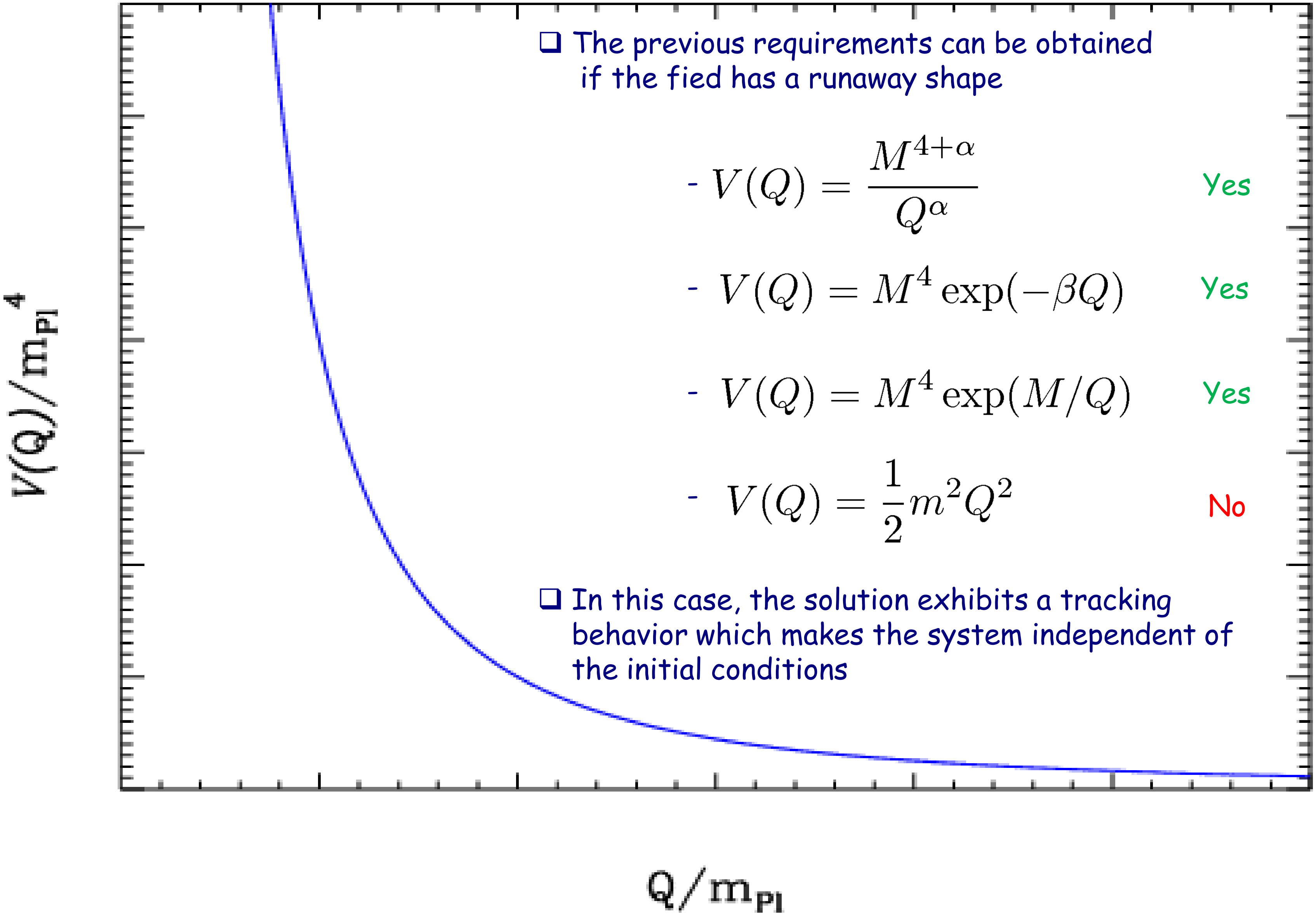
  - The initial conditions must be « generic », ie no fine tuning.



$$\Omega_Q \simeq 0.7$$

$$\omega_Q = \frac{\dot{Q}^2/2 - V(Q)}{\dot{Q}^2/2 + V(Q)} < 0$$

Quintessence (III)



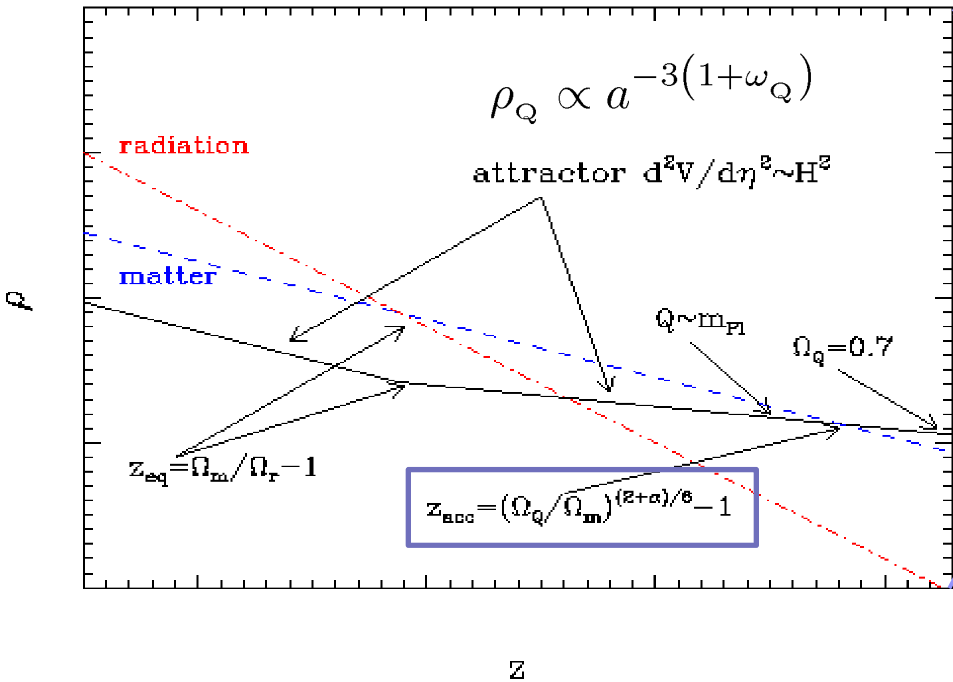
- The previous requirements can be obtained if the fied has a runaway shape

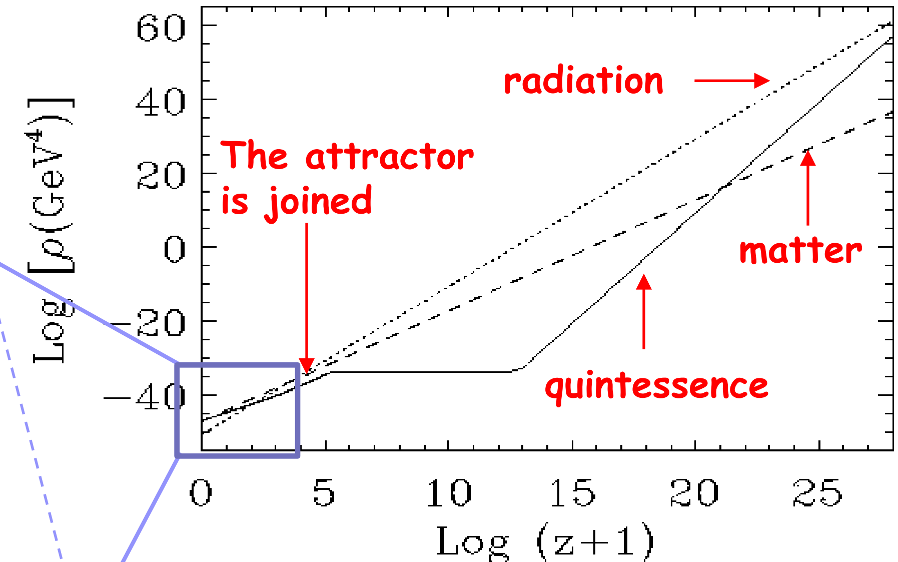
  - $V(Q) = \frac{M^{4+\alpha}}{Q^{\alpha}}$ Yes
  - $V(Q) = M^4 \exp(-\beta Q)$ Yes
  - $V(Q) = M^4 \exp(M/Q)$ Yes
  - $V(Q) = \frac{1}{2} m^2 Q^2$ No

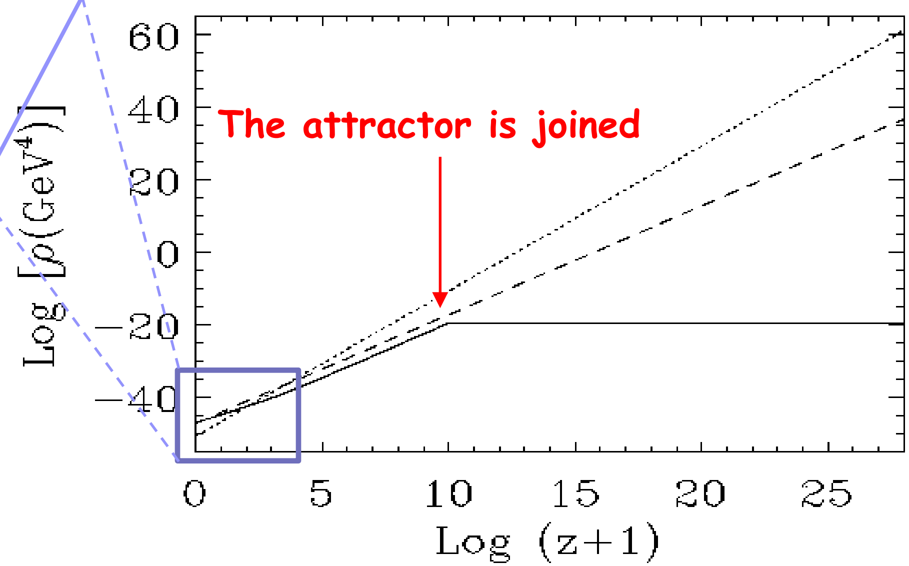
- In this case, the solution exhibits a tracking behavior which makes the system independent of the initial conditions

Example: the Ratra-Peebles potential

$$V(Q) = M^{4+\alpha} Q^{-\alpha}$$







The quintessence field tracks the background fluid

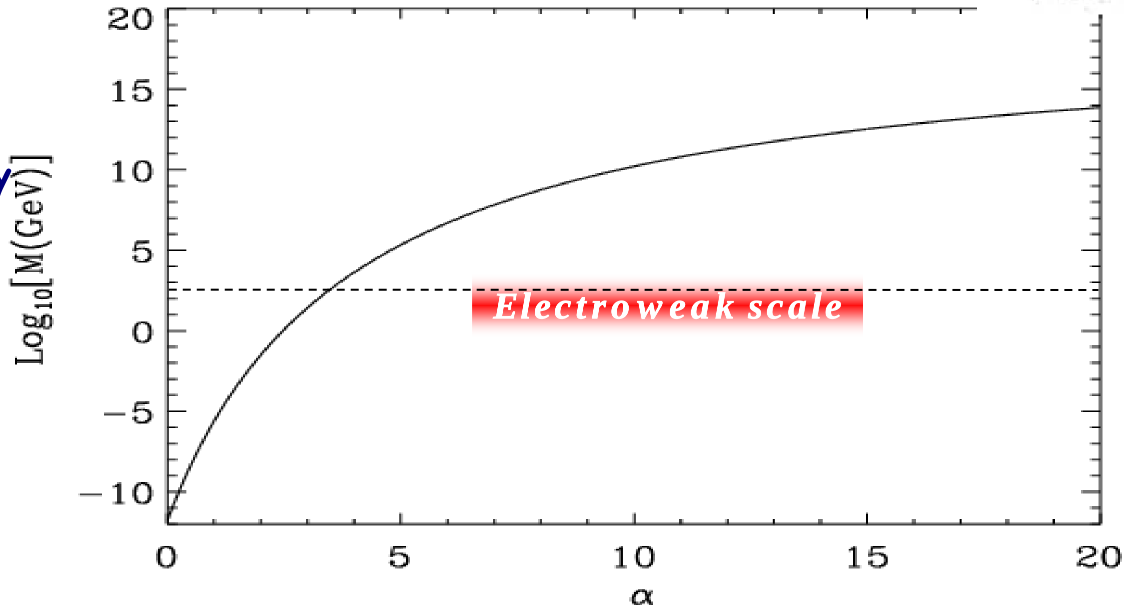
$$\omega_Q = \frac{-2 + \alpha\omega_B}{2 + \alpha}$$

Quintessence (V)

- The energy scale M of the potential is fixed by the requirement that the quintessence energy density today represents 70% of the critical energy density

$$\frac{M^{4+\alpha}}{m_{\rm Pl}^{\alpha}} \simeq \rho_{\rm cri} \Rightarrow$$

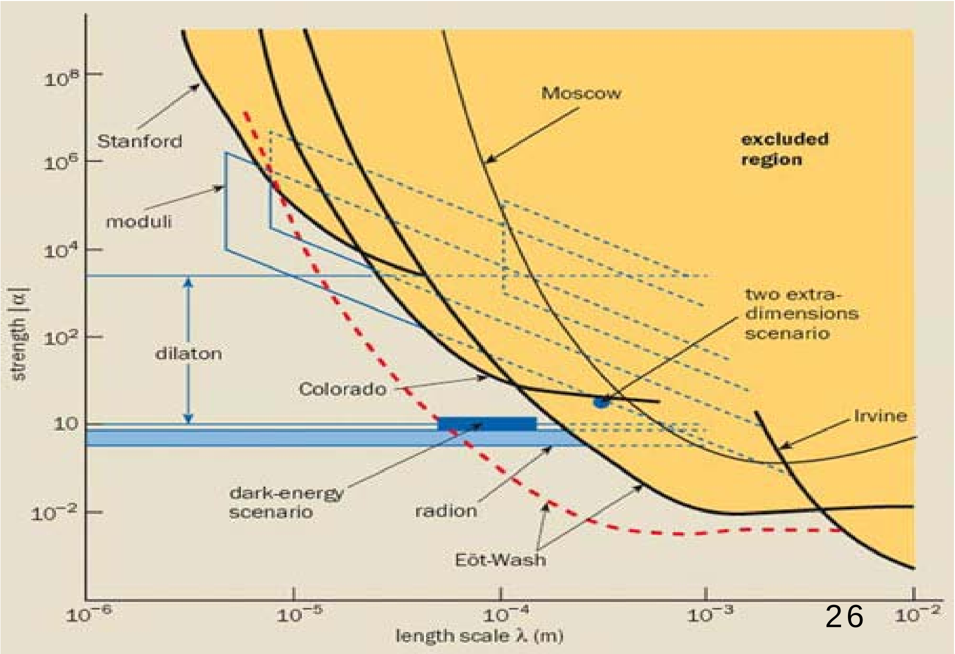
$$\log_{10}\left[M\,({\rm GeV})\right] \simeq \frac{19\alpha-47}{\alpha+4}$$



- The mass of the field is tiny

$$m_Q^2 = V'' = \frac{\rho_{\rm cri}}{m_{\rm Pl}^2} \sim \left(10^{-33}{\rm eV}\right)^2$$

ie very long range force: danger because already well constrained by various experiments



- … but the vev is huge

$$\langle Q \rangle_{\rm today} \sim m_{\rm Pl}$$

- Can we find a candidate for the quintessence scalar field in particle physics?

- Can we derive the Ratra-Peebles potential in a consistent way from particle physics? Can we predict the shape of the potential?

- What is the influence of the quantum corrections on the shape of this potential?

- If the dark energy is just a field, does it interact with the rest of the world? Can we compute this interaction? What are the consequences?

Quintessence and particle physics

- There is no known candidate in the standard model of particle physics. Hence, one must consider the extension of the standard model, ie (or eg) extensions based on SUSY.

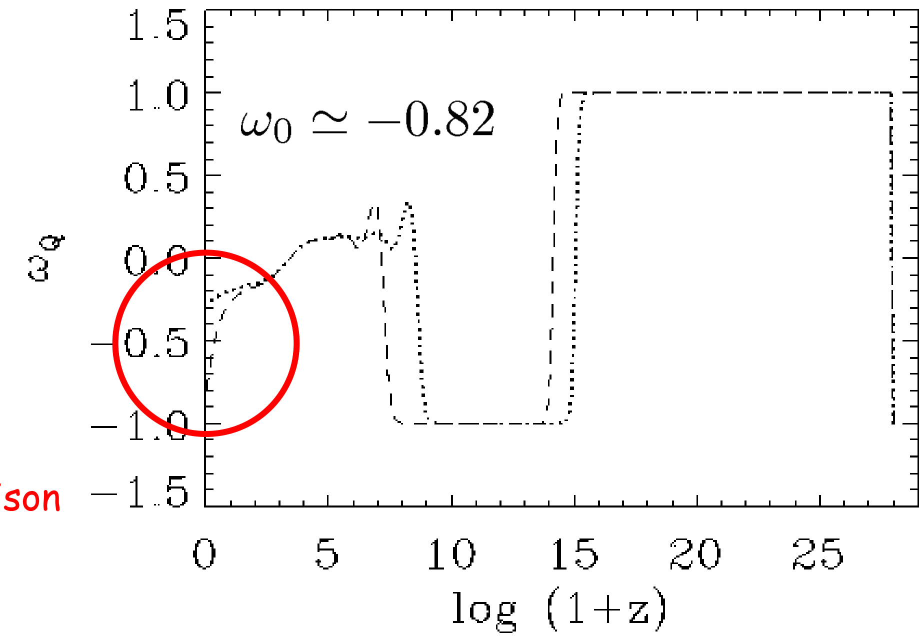
- If the quintessence field is considered as isolated, it is possible to contruct interesting models of dark energy, stable against quantum corrections.

- However, the interaction with other field seems unavoidable. It is a source of of very serious (and unsolved) problems

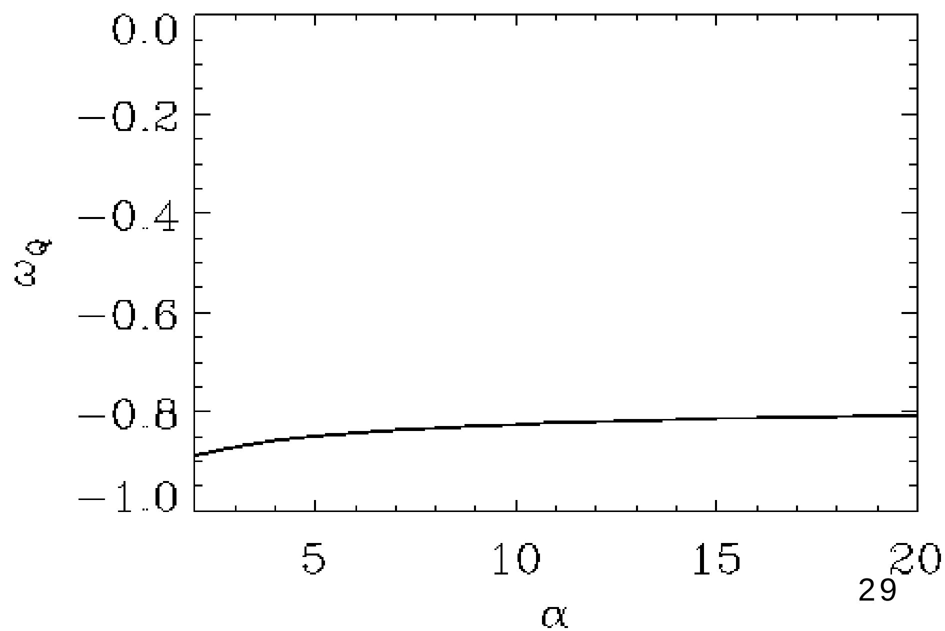
  - Modifications of the runaway shape of the potential
  - presence of a fifth force
  - Violation of the weak equivalence principle
  - Chameleon effect
  - Apparent equation of state less than -1
  - Variation of constants (fine structure constant, etc …)

Example of a successful potential derived from Particle Physics

$$V_{\rm quint}(Q) = e^{\kappa Q^2/2} \frac{M^{4+\alpha}}{Q^\alpha}$$



- The attractor solution still exists since, for large redshifts, the vev of Q is small in comparison with the Planck mass
- The exponential corrections pushes the equation of state towards -1 at small redshifts
- The present value of the equation of state becomes "universal", i.e. does not depend on α
- The model is "relatively" stable against quantum corrections
- However, the model is artificially decoupled from the rest of the world



- Taking into account the coupling with the other fields;

  - generically, the potential will receive corrections

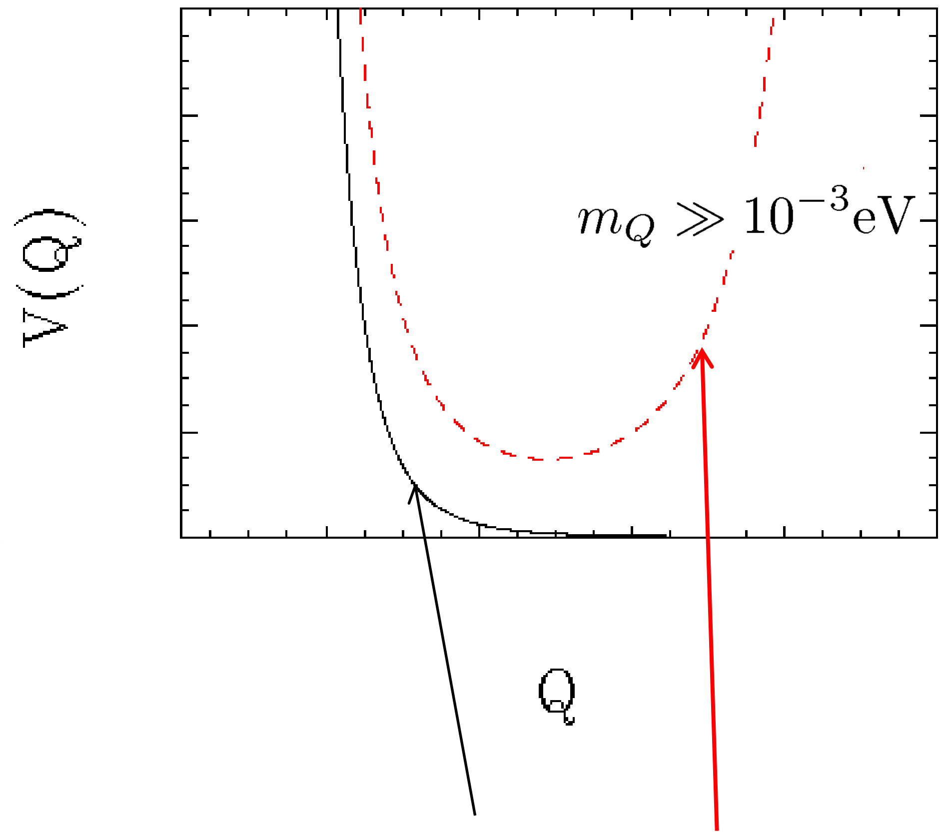
  - The shape of the corrections is model dependent

- The corrections can induce a minimum; the smallness of the mass is destroyed.

$$V(Q) = V_{\rm quint} + \Delta V$$

- Quantum corrections
- Soft terms (Susy breaking)

- The minimum is located at smal vev which means that the field will settle down very early in the history of the universe

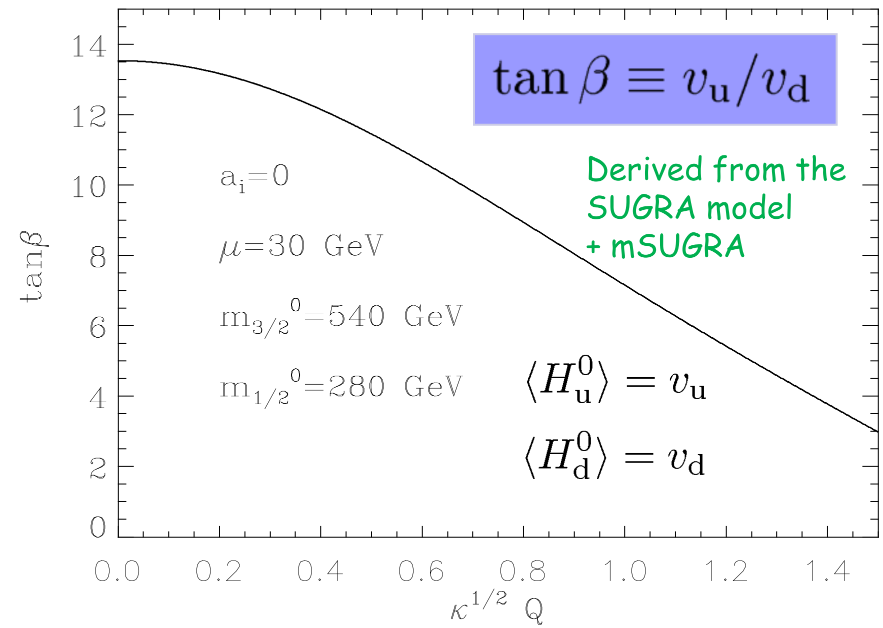


$$V_{\rm DE} = e^{\kappa Q^2}\left[\frac{M^{4+2p}}{Q^{2p}} + \left(m^0_{3/2}\right)^2 Q^2 + \cdots\right]$$

- In presence of dark energy, the vev's of the Higgs become Q-dependent

$$m_{\rm u}(Q) = \lambda_{\rm d} e^{\kappa K_{\rm quint}/2 + \sum_i |a_i|^2/2} v_{\rm u}(Q)$$

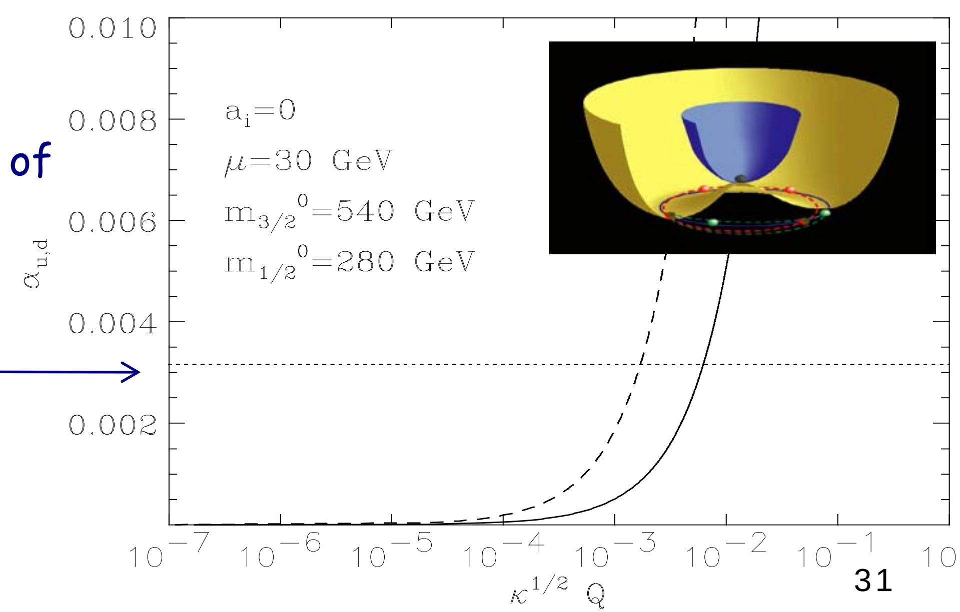
$$m_{\rm d}(Q) = \lambda_{\rm d} e^{\kappa K_{\rm quint}/2 + \sum_i |a_i|^2/2} v_{\rm d}(Q)$$



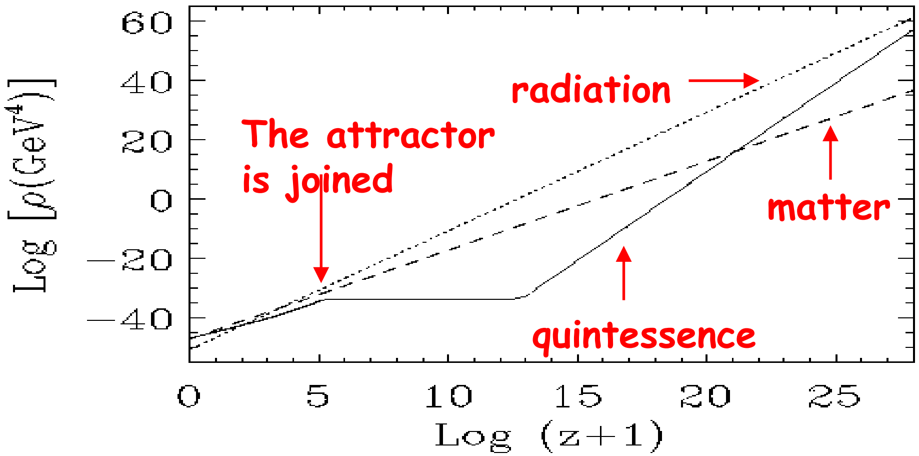
- The fermions pick up a Q-dependent mass

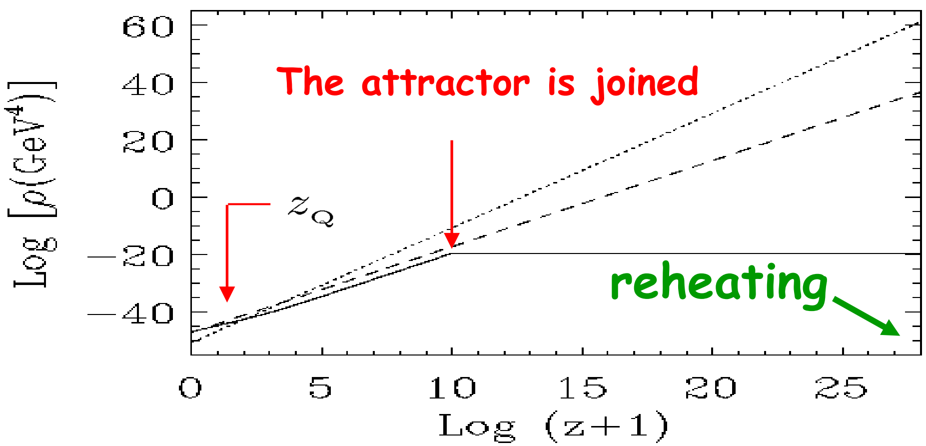
- Generically, problem with the local test of gravity

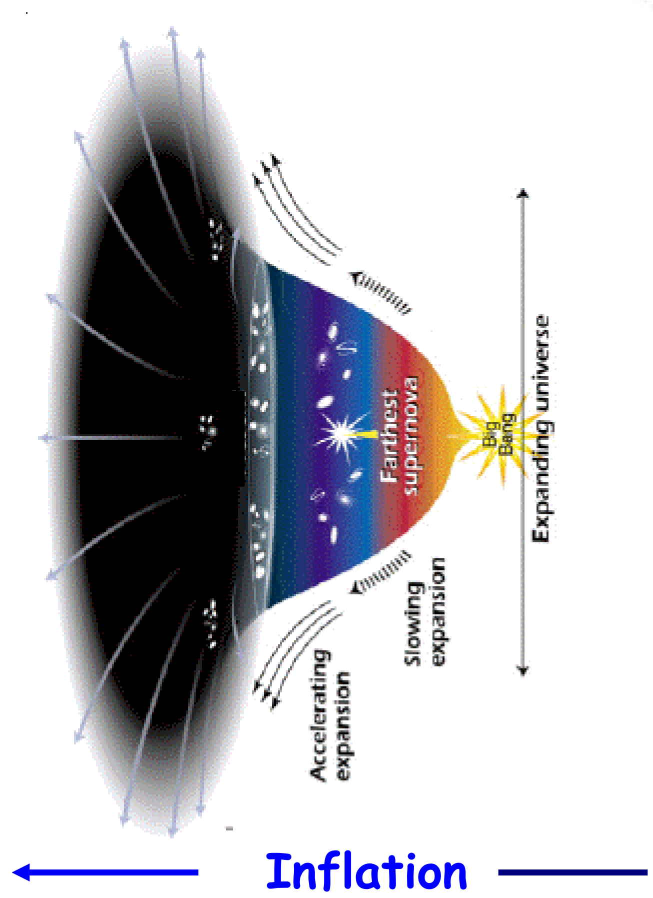
$$\alpha_{\rm u,d}(Q) = \left|\frac{1}{\kappa^{1/2}} \frac{{\rm d}\ln m_{\rm u,d}(Q)}{{\rm d}Q}\right| < 10^{-2.5}$$



The quintessence field during inflation





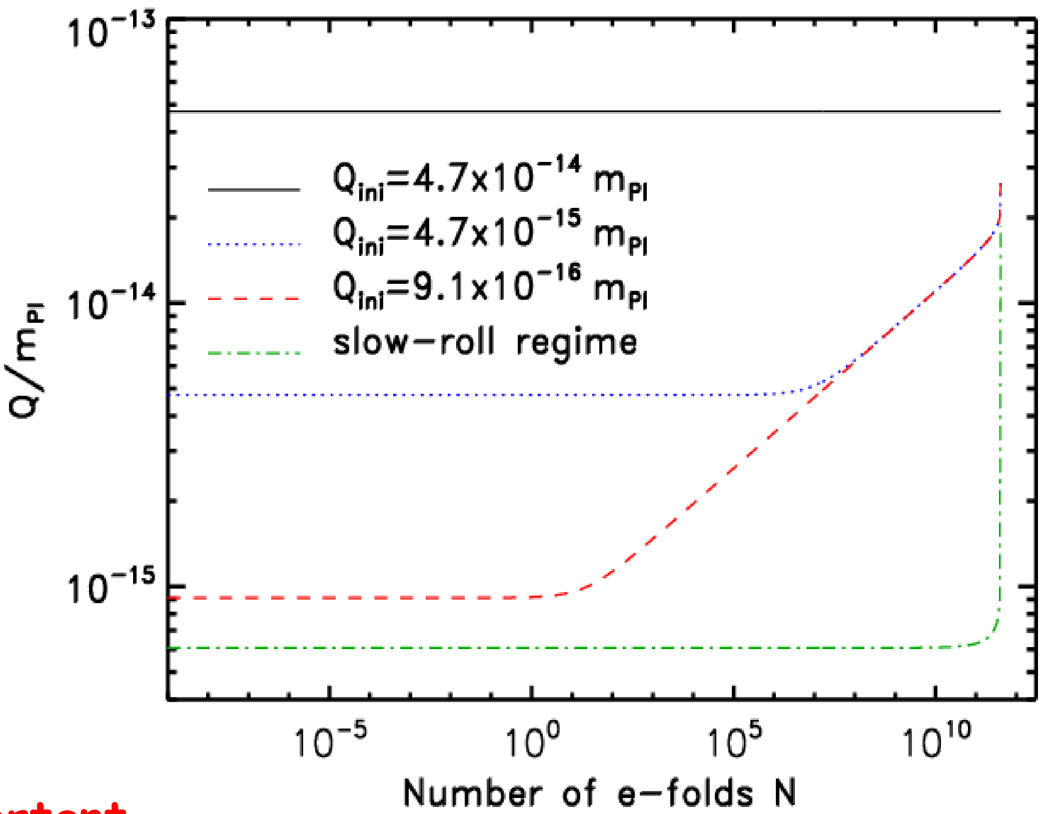


$$10^{-37}\mathrm{GeV}^4 < \rho_Q < 10^{61}\mathrm{GeV}^4$$

$$10^{-107/\alpha} < \frac{Q}{m_{\mathrm{Pl}}} < 10^{-9/\alpha}$$

- Do we have initial conditions (final conditions from the point of view of inflation) that are compatible with the attractor?
- What are the initial conditions for the quintessence field at the beginning of inflation?

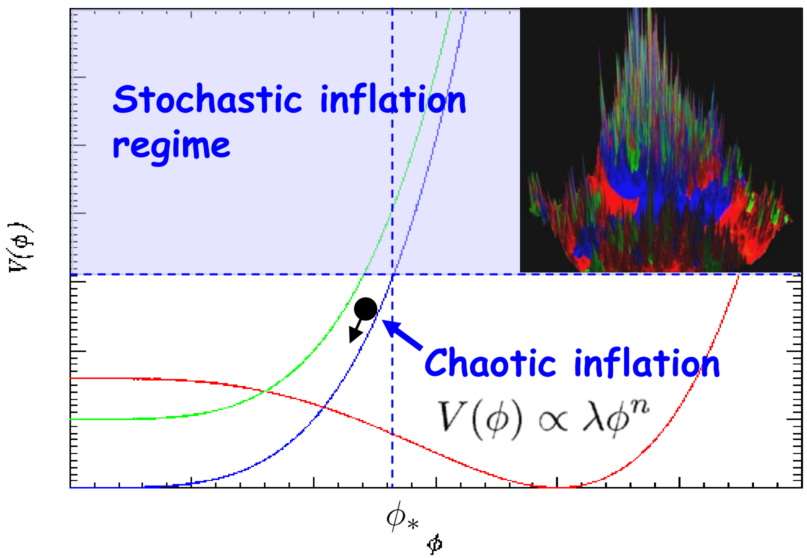
Typically, the quintessence field is frozen during inflation



However …

- **Stochastic effects are important**
- **From high energy considerations, the inflaton and the quintessence field must interact**

Quantum effects

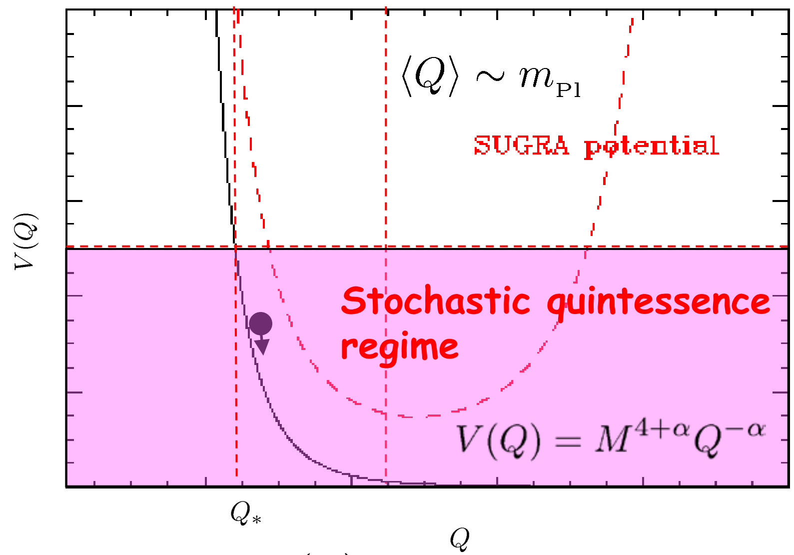


The field undergoes quantum jump H/2π every Hubble time

$$\Delta\phi_{\rm qu} \sim \frac{H}{2\pi}$$

$$\Delta\phi_{\rm cl} \sim -\frac{V_{,\phi}}{3H}\Delta t = -\frac{V_{,\phi}}{3H}\frac{1}{H}$$

$$\boxed{\Delta\phi_{\rm qu} > \Delta\phi_{\rm cl} \Rightarrow \phi > \phi_* \sim \lambda^{-\frac{1}{n+2}}}$$



$$\Delta Q_{\rm qu} \sim \frac{H(\phi)}{2\pi}$$

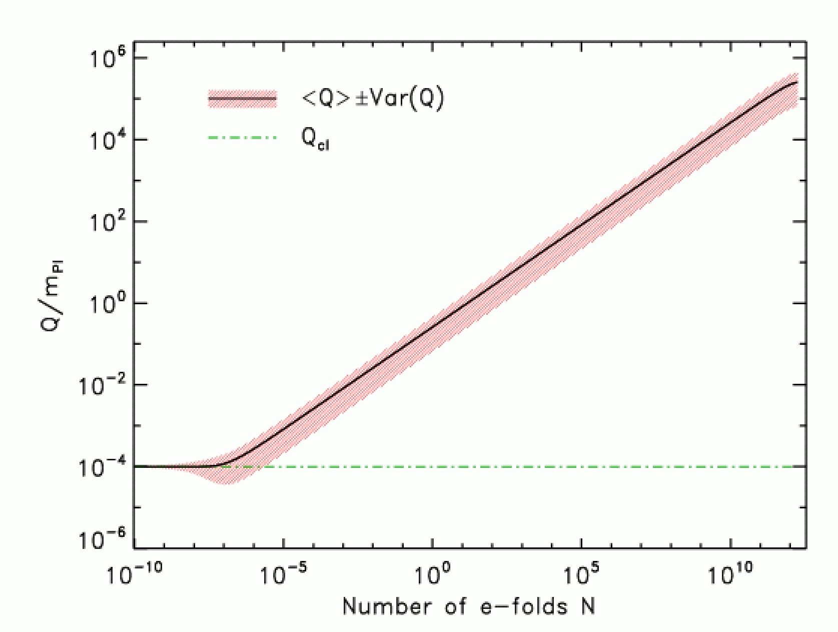
$$\Delta Q_{\rm cl} \sim -\frac{V_{,Q}}{3H(\phi)}\Delta t = -\frac{V_{,Q}}{3H(\phi)}\frac{1}{H(\phi)}$$

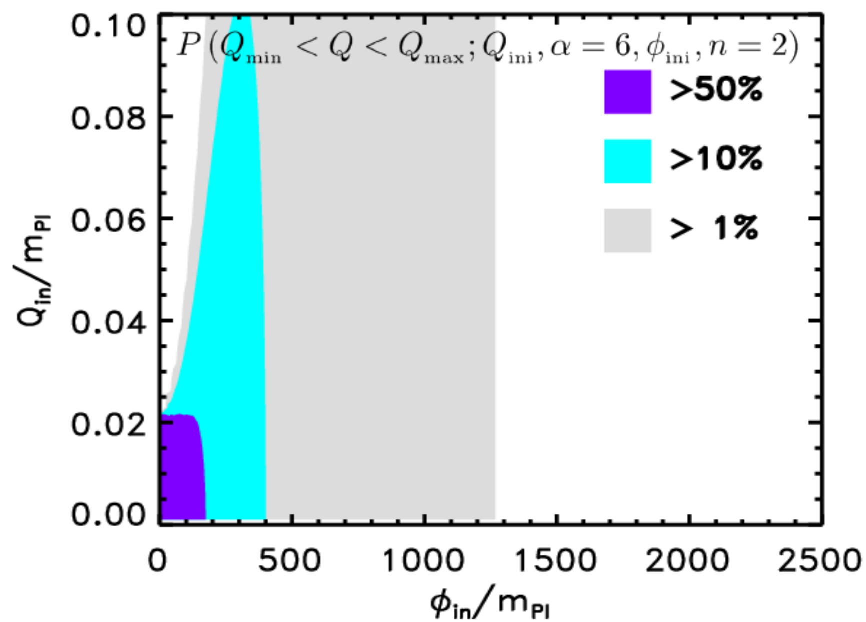
$$\boxed{Q > Q_* \sim \left(\frac{M^{4+\alpha}}{H^3}\right)^{\frac{1}{\alpha+1}}}$$

Quantum effects are important for quintessence!

M. Malquarti & A. Liddle (2002)
J. Martin & M. Musso (2004)

Quantum effects





- The confidence region enlarges with the power index α
- A "small" number of total e-foldings is favored because otherwise the Quintessence field has too much time to drift away from the allowed range of initial conditions

J. Martin & M. Musso (2004)

Another possibility is to modify the gravitational sector. For instance, one can consider Scalar-Tensor theories

$$S = \frac{1}{2\kappa_*} \int \mathrm{d}^4 x \sqrt{-g} \left[F(\varphi) R - 2\Lambda_{\rm B} - Z(\varphi) g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - U(\varphi) \right] + S_{\rm matter} \left[g_{\mu\nu}, \Psi \right]$$

- Most natural extensions of GR (weak EP is satisfied)
- In this case gravity is mediated not only by a spin 2 graviton but by a scalar field (spin 0)
- Can be justified from particle physics (low energy action of string theory)
- It is very tempting to assume that the scalar field is dark energy
- If the mass of the scalar field is very high, no deviation is present locally. But, precisely, we know that the mass is small if the scalar field represents dark energy … Therefore, one must check that the theory passes the solar system tests!

Another possibility is to modify gravity is to consider the following class of models

$$S[g_{\mu\nu}] = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}f(R)$$

But let us re-write the action as

$$S[g_{\mu\nu},\Phi] = \frac{1}{2\kappa}\int \sqrt{-g}\left[f(\Phi) + f'(\Phi)(R-\Phi)\right]$$

- Variation wrt to the scalar field leads R=Φ

- Variation wrt to the metrics leads to an equation which reduces to the original one if R=Φ

Therefore, this class of theories are just particular examples of ST theories (in particular they have to pass the local tests).

Conclusions

- The most natural explanation for the acceleration of the Universe is the Cosmological Constant. In addition all the observations are consistent with this assumption.

- However, the value of the CC is also determined by the zero point fluctuations of QFT. These ones are real as proven experimentally by the Casimir and Lamb shift effects. The resulting CC is huge in comparison to what we see and this represents a deep mystery.

- Dark energy is maybe not the CC? … but this does not solve the CC problem … ! Measuring $w \neq -1$ would be a (theoretical?) revolution!

- Constructing alternative HEP models is always possible but they are not natural from the HEP physics point of view. For all models different from the CC (quintessence, ST theories etc …), we face the question of the interaction of dark energy with the rest of the world …

- From HEP we expect local deviations: probing dark energy is not only measuring the large scale behaviour of the Universe but is also about testing gravitation locally!