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G-inflation

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arXiv:1008.0603

G-inflation...

What's "G"?

Galileon field

The Galileon field

$$\mathcal{L}_1 = \phi$$

$$\mathcal{L}_2 = (\nabla\phi)^2$$

$$\mathcal{L}_3 = (\nabla\phi)^2\,\Box\phi$$

$$\mathcal{L}_4 = (\nabla\phi)^2\left[2(\Box\phi)^2 - 2(\nabla_\mu\nabla_\nu\phi)^2 - \frac{R}{2}(\nabla\phi)^2\right]$$

$$\mathcal{L}_5 = (\nabla\phi)^2\left[(\Box\phi)^3 + \cdots\right]$$

Field equations are 2nd order

Galilean shift symmetry <u>in flat space</u>

$$\partial_\mu\phi \to \partial_\mu\phi + b_\mu$$

$$\mathcal{L}_n \sim \partial^{2(n-1)}\phi^n$$

Nicolis et al. '09;
Deffayet et al. '09

Our Lagrangian

$$\mathcal{L} = \frac{R}{2} + K(\phi, X) - F(\phi, X)\Box\phi$$

where $X := -\frac{1}{2}(\nabla\phi)^2$

Field equations are 2nd order

Deffayet, Pujolas, Sawicki, Vikman 1008.0048;
TK, Yamaguchi, Yokoyama 1008.0603

Simple motivation

The Galileon field has been used to explain current cosmic acceleration......

Chow, Khoury '09; Silva, Koyama '09;
TK, Tashiro, Suzuki '09; **TK** '10;
Gannouji, Sami '10;
De Felice, Tsujikawa '10; De Felice, Mukohyama, Tsujikawa '10; ...

Simple motivation

Why don't we use the Galileon field to drive inflation in the early universe?

Talk plan

I. Introduction

II. G-inflation

III. Primordial perturbations

IV. Summary & Outlook

G-inflation

Standard picture of inflation

One (or more) canonical scalar field(s) rolling slowly down a nearly flat potential

$$\mathcal{L} = X - V(\phi), \quad X = -\frac{1}{2}(\partial\phi)^2$$

$$3M_{\rm Pl}^2 H^2 \simeq V(\phi)$$

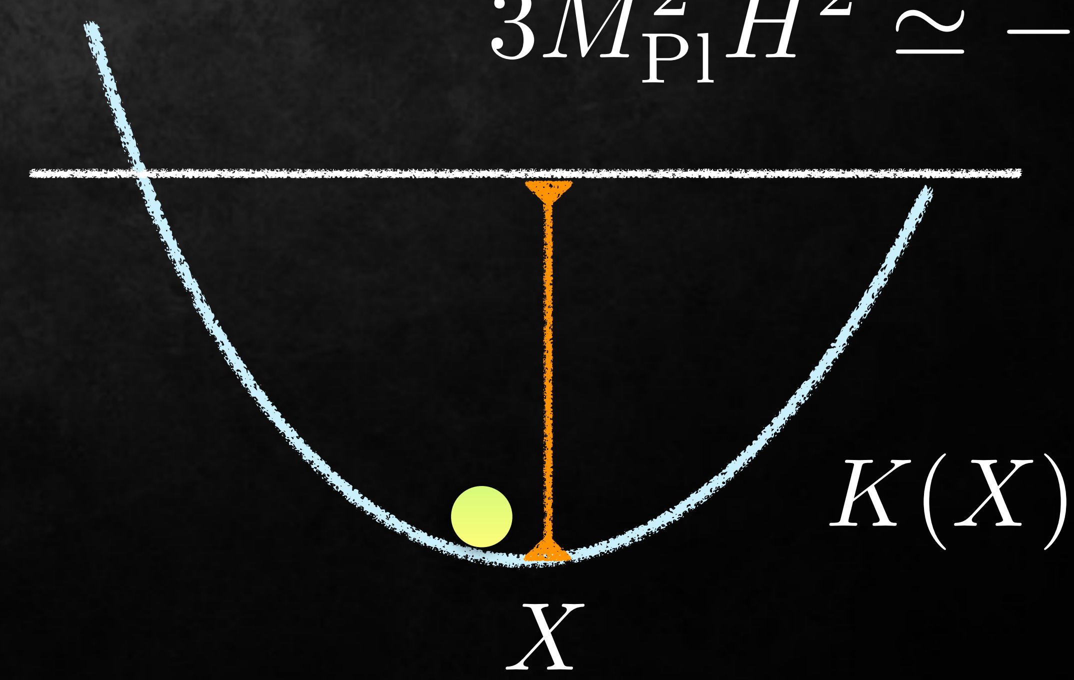
Kinematically driven inflation

$$\mathcal{L} = K(\phi, X)$$

$$\longrightarrow \quad \frac{d}{dt}\left(a^3 K_X \dot{\phi}\right) = 0$$

$$K = K(X)$$

$$3M_{\rm Pl}^2 H^2 \simeq -K$$



“k-inflation”

Armendariz-Picon et al. ’99;

“Ghost condensate”

Arkani-Hamed et al. ’04

G-inflation: background

$$\mathcal{L}_\phi = K(\phi, X) - F(\phi, X)\Box\phi$$

$$\downarrow$$

$$\begin{aligned} 3H^2 &= \rho \\ -3H^2 - 2\dot{H} &= p \end{aligned}$$

+ Scalar field EOM is automatically satisfied

$$\begin{aligned} \rho &= 2XK_X - K + 3F_X H\dot{\phi}^3 - 2F_\phi X \\ p &= K - 2\left(F_\phi + F_X\ddot{\phi}\right)X \end{aligned}$$

de Sitter G-inflation

$$K = K(X), \quad F = fX, \quad f = \mathrm{const}$$

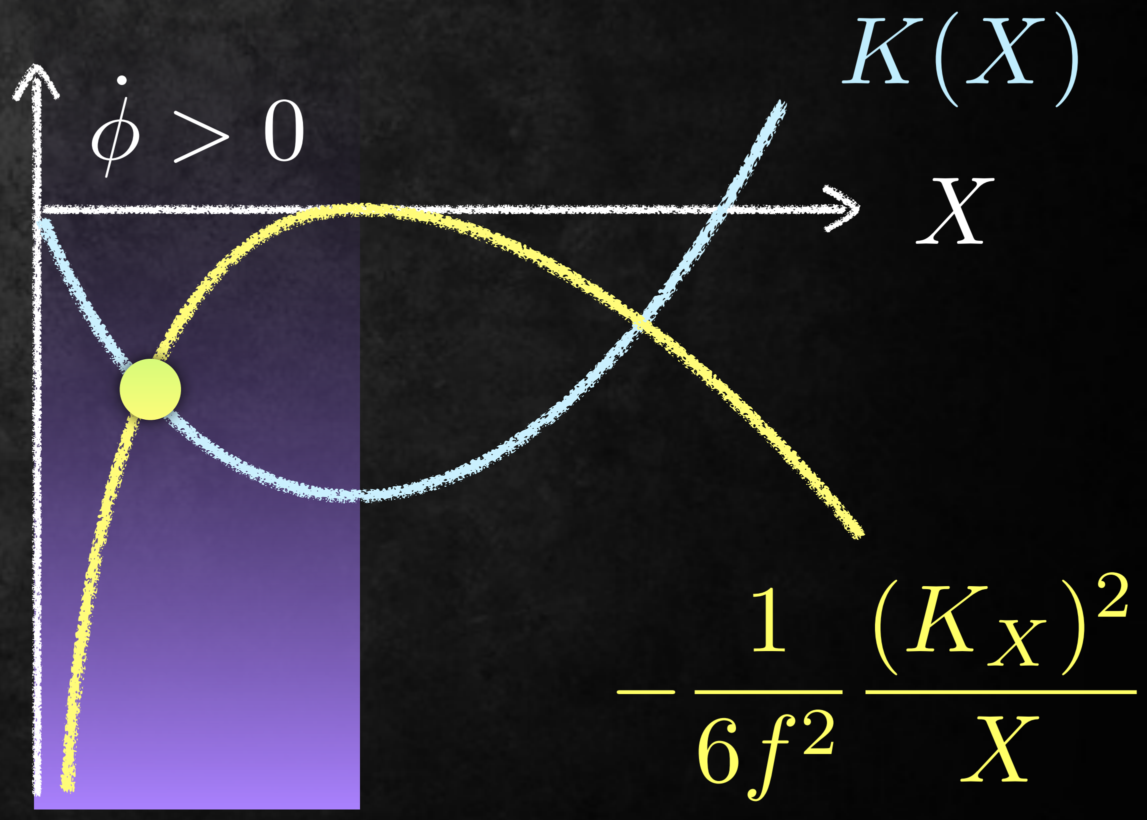
Look for exactly de Sitter solution:

$$H = \mathrm{const}$$
$$\dot{\phi} = \mathrm{const}$$

satisfying:

$$3H^2 = -K$$
$$K_X = -3fH\dot{\phi}$$

$$\longrightarrow \quad K = -\frac{1}{6f^2}\frac{(K_X)^2}{X}$$



Quasi-dS G-inflation

$$K = K(X), \quad F = f(\phi) X$$

Quasi-de Sitter solution:

Required to get $n_s - 1 \neq 0$

$$H = H(t), \quad \dot{\phi} = \dot{\phi}(t)$$

Small rate of change

satisfying:

$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1$$

$$H^2 \simeq -K(X)$$

$$K_X \simeq -3 f(\phi) H \dot{\phi}$$

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

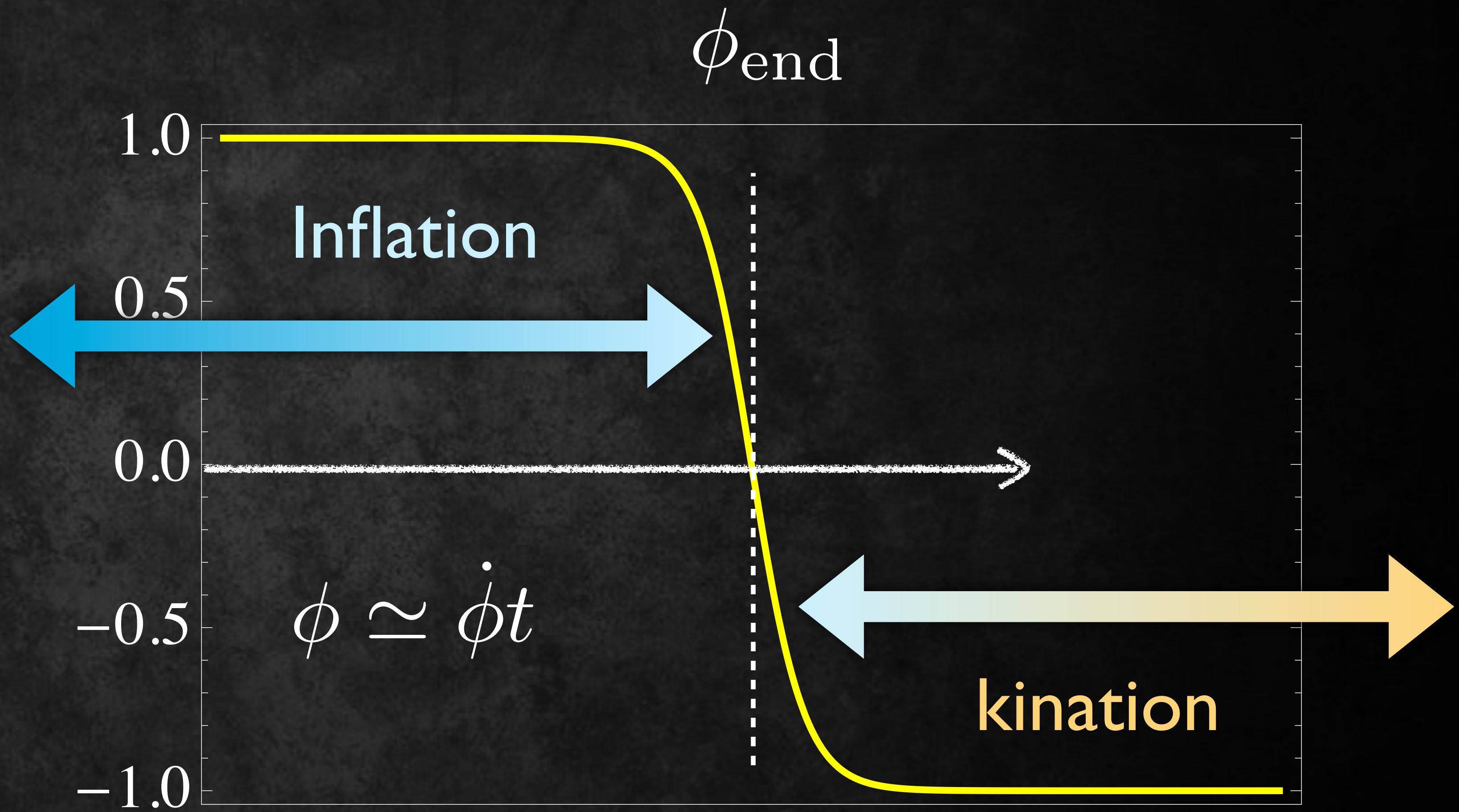
Graceful exit & Reheating

<u>Basic idea</u>

$$K = -A(\phi)X + \cdots$$

Example:

$$A = \tanh\left[\lambda(\phi_{\rm end} - \phi)\right]$$



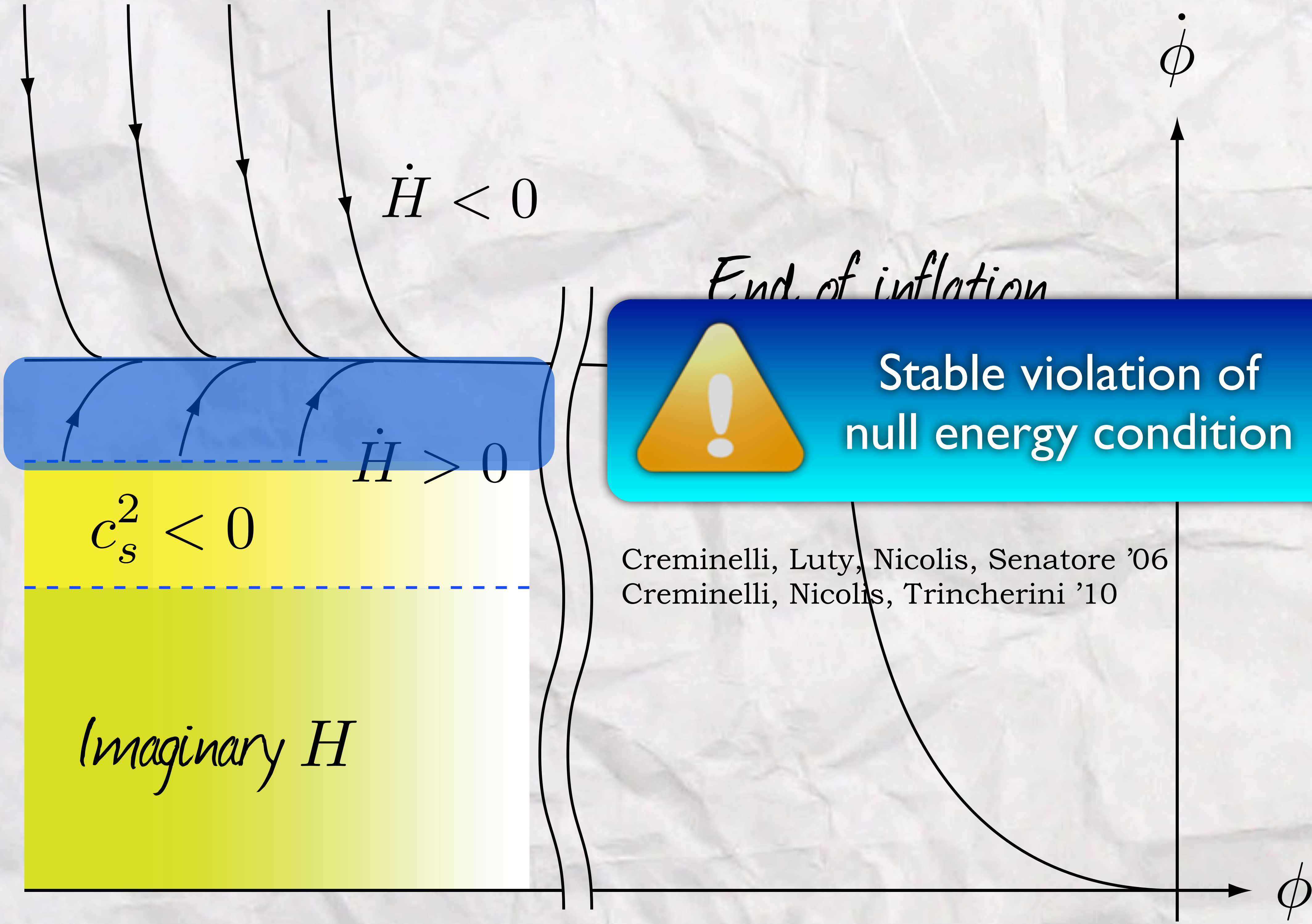
$$\rho \simeq p \simeq X \propto a^{-6}$$

~ massless, canonical field (normal sign)

Reheating through gravitational particle production

Ford '87

Phase diagram



Stable violation of null energy condition

Creminelli, Luty, Nicolis, Senatore '06
Creminelli, Nicolis, Trincherini '10

Primordial perturbations

Cosmological perturbations

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\beta_{,i}dtdx^i + a^2(1+2\mathcal{R})\delta_{ij}dx^i dx^j$$

$$\phi = \phi(t)$$

Unitary gauge: $\delta\phi = 0$

1. Expand the action to 2nd order
2. Eliminate α and β using constraint eqs
3. Quadratic action for $\mathcal{R}$

$$\delta T_i{}^0 = -F_X \dot{\phi}^3 \alpha_{,i}$$

Uniform ϕ hypersurfaces
≠ comoving hypersurfaces

Quadratic action

$$S^{(2)} = \frac{1}{2}\int d\tau d^3x\, z^2 \left[\mathcal{G}(\mathcal{R}')^2 - \mathcal{F}(\vec{\nabla}\mathcal{R})^2\right]$$

where

No ghost and gradient instabilities if

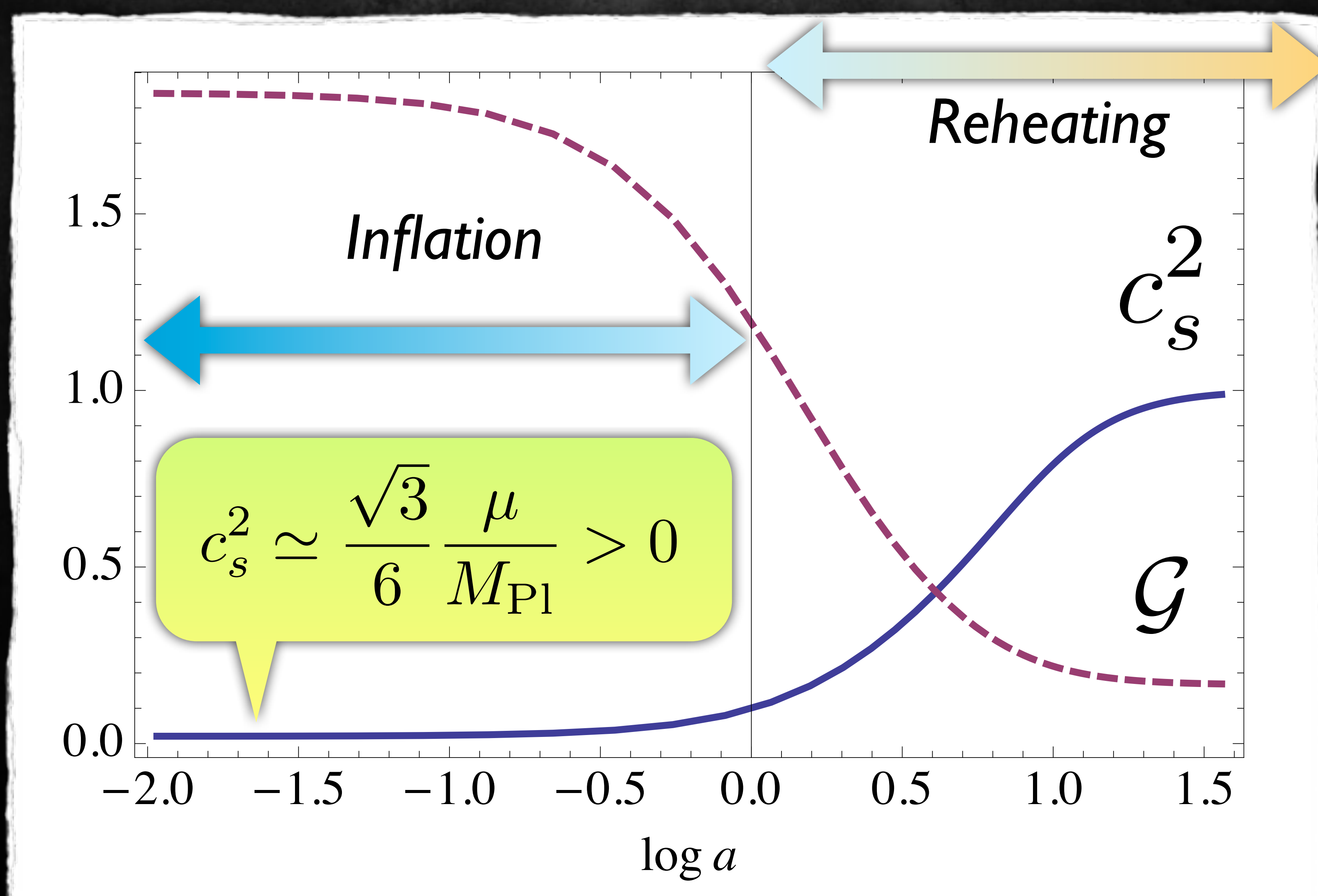
$$\mathcal{G} > 0, \quad c_s^2 = \mathcal{F}/\mathcal{G} > 0$$

z

$$\begin{aligned}\mathcal{F} &= K_X + 2F_X\left(\ddot{\phi} + 2H\dot{\phi}\right) - 2F_X^2X^2 \\ &\quad +2F_{XX}X\ddot{\phi} - 2\left(F_\phi - XF_{\phi X}\right) \\ \mathcal{G} &= K_X + 2XK_{XX} + 6F_XH\dot{\phi} + 6F_X^2X^2 \\ &\quad -2\left(F_\phi + XF_{\phi X}\right) + 6F_{XX}HX\dot{\phi}\end{aligned}$$

Stable example

$$K = -A(\phi)X + \frac{X^2}{2M^3\mu}, \quad F = \frac{X}{M^3}$$



Primordial spectrum

Consider G-inflation with:

$$K = K(X), \quad F = f(\phi)X$$

New variables:

$$\begin{aligned} dy &= c_s d\tau \\ \tilde{z} &= (\mathcal{F}\mathcal{G})^{1/4} z \\ u &= \tilde{z}\mathcal{R} \end{aligned}$$

$$\frac{d^2u}{dy^2} + \left(k^2 - \frac{\tilde{z}_{,yy}}{\tilde{z}}\right) u = 0$$

$$\frac{\tilde{z}_{,yy}}{\tilde{z}} \simeq \frac{1}{(-y)^2}\left[2 + 3\epsilon\mathcal{C}(X)\right]$$

$$\mathcal{C}(X) = \frac{K}{K_X}\frac{Q_X}{Q}$$

$$Q(X) = \frac{(K - XK_X)^2}{18Xc_s^2\sqrt{\mathcal{F}\mathcal{G}}}$$

Primordial spectrum

Normalized mode: $$u = \frac{\sqrt{\pi}}{2}\sqrt{-y}H^{(1)}_{3/2+\epsilon\mathcal{C}}(-ky)$$

$$\mathcal{P}_{\mathcal{R}} = \frac{Q}{4\pi^2}\bigg|_{c_s k = 1/(-\tau)}, \quad n_s - 1 = -2\epsilon\mathcal{C}$$

$\propto f_{,\phi}$

$\mathcal{R}$ can be generated even from exact de Sitter

where $$Q(X) = \frac{(K - XK_X)^2}{18M_{\rm Pl}^4 X c_s^2\sqrt{\mathcal{F}\mathcal{G}}}$$

* Tensor mode dynamics: unchanged

Tensor-to-scalar ratio

$$K = -X + \frac{X^2}{2M^3\mu}, \quad F = fX \quad \longrightarrow \quad H^2 \sim \frac{\mu M^3}{M_{\rm Pl}^2}$$

$$r \simeq \frac{16\sqrt{6}}{3}\left(\frac{\sqrt{3}\mu}{M_{\rm Pl}}\right)^{3/2}$$

Standard consistency relation is violated

$$r \neq 16\epsilon$$

$\epsilon \propto f_{,\phi}$

$$M = 0.00435 \times M_{\rm Pl}, \ \mu = 0.032 \times M_{\rm Pl}$$

$$\longrightarrow \quad \mathcal{P}_{\mathcal{R}} = 2.4 \times 10^{-9}, \ r = 0.17$$

r can be large!

Definition:

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}}, \quad \mathcal{P}_T = \frac{8}{M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2$$

Summary & Outlook

Summary

- G-inflation: A general class of single field inflation

$$\mathcal{L}_\phi = K(\phi, X) - F(\phi, X)\Box\phi$$

  - $n_s - 1 \simeq 0$
  - Large r
  - ~~Consistency relation~~

G-inflation would make gravitational wave people happy!

$$+(3\mathcal{R}-\alpha_1)(\partial_i\partial_j\beta_1)^2-(3\mathcal{R}-\alpha_1)(\partial^2\beta_1)^2-4\partial^2\beta_1(\partial\mathcal{R}\cdot\partial\beta_1)\Big].$$

Here,

TK, Yamaguchi, Yokoyama *in progress*

$$\mathcal{I} := XK_{XX}+\frac{2X^2}{3}K_{XXX}+H\dot\phi F_X+6X^2F_X^2+5HX\dot\phi F_{XX}+6X^3F_XF_{XX}+2HX^2\dot\phi F_{XXX}$$
$$-\frac{2X}{3}\left(2F_{\phi X}+XF_{\phi XX}\right).$$

Now calculating primordial non-Gaussianity from G-inflation

$$\mathcal{G} := K_X+2XK_{XX}+6F_XH\dot\phi+6F_X^2X^2-2\left(F_\phi+XF_{\phi X}\right)+6F_{XX}HX\dot\phi,$$

$$\Lambda := \frac{a^2}{\Theta^2}X\mathcal{G}\dot{\mathcal{R}}.$$

The field equation which follows from the quadratic action is

$$\left.\frac{\delta L}{\delta\mathcal{R}}\right|_1 = \frac{\mathrm{d}\Lambda}{\mathrm{d}t}+H\Lambda-\frac{X\mathcal{F}}{\Theta^2}\partial^2\mathcal{R}.$$

$\chi := \partial^{-2}\Lambda.$
constraint, $a^2\beta_1=\chi-\mathcal{R}/\Theta.$
The action corresponding to e.q. (47) of Lidsey and Seery is:

$$S_3 = \frac{1}{2}\int\mathrm{d}t\mathrm{d}xa^3\Bigg[-\frac{2}{a^2}\frac{X\mathcal{F}}{\Theta^2}\mathcal{R}(\partial\mathcal{R})^2-(2X\mathcal{G}+4X\mathcal{I})\frac{\dot{\mathcal{R}}^3}{\Theta^3}+6X\mathcal{G}\mathcal{R}\frac{\dot{\mathcal{R}}^2}{\Theta^2}$$
$$-4\dot\phi\left(XF_X+X^2F_{XX}\right)\frac{\dot{\mathcal{R}}^2}{a^2\Theta^2}\left(\Lambda-\frac{1}{\Theta}\partial^2\mathcal{R}\right)$$
$$+\frac{2}{a^4}\frac{H\Lambda}{\Theta^2}(\partial\mathcal{R})^2-\frac{4}{a^4}\frac{H}{\Theta}\Lambda(\partial\mathcal{R}\cdot\partial\chi)+\frac{1}{a^4}\frac{X\mathcal{F}}{\Theta^2}\Lambda(\partial\mathcal{R}\cdot\partial\chi)+\frac{1}{2a^4}\frac{X\mathcal{F}}{\Theta^2}\partial^2\mathcal{R}(\partial\chi)^2+\Xi$$
$$-\frac{2}{a^4}\frac{XF_X\dot\phi}{\Theta}\partial^2\mathcal{R}\left(\partial\chi-\frac{1}{\Theta}\partial\mathcal{R}\right)\cdot\left(\partial\chi-\frac{1}{\Theta}\partial\mathcal{R}\right)\Bigg].$$