Lecture II:
Descriptions of Dark Energy

Dragan Huterer

Physics Department
University of Michigan

In this Lecture*

1. How to describe DE (expansion history + growth of pert)
2. How to reconstruct/measure w(z)
3. How to choose and use Figures of Merit

*examples will be preferentially taken from my own work

Expansion History

Continuity equation:

$$\dot{\rho} + 3H(p+\rho) = 0$$

$$\Rightarrow \rho_{\rm DE}(z) = \rho_{\rm DE,0} \exp\left(3\int_0^z (1+w(z'))d\ln(1+z')\right)$$

Then can easily get expansion rate for a general w(z):

$$\begin{aligned} H^2(z) &= \frac{8\pi G}{3}\left[\rho_M(z) + \rho_{\rm DE}(z)\right] \\ &= H_0^2\left[\Omega_M(1+z)^3 + \Omega_{\rm DE}\exp\left(3\int_0^z (1+w(z'))\,d\ln(1+z')\right)\right] \end{aligned}$$

(Note: **Any** arbitrary expansion history can be described by some w(z))

Growth of density perturbations

Linear growth of density fluctuations ($\delta \equiv \delta\rho/\rho$)

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_M\delta = 0$$

Rewrite in terms of growth relative to EdS, $g(a) \equiv D(a)/a$

($\delta(a) \propto D(a)$ is 'pure' growth from Peebles book)

$$2\frac{d^2g}{d\ln a^2} + [5 - 3w(a)\Omega_{\rm DE}(a)]\,\frac{dg}{d\ln a} + 3\,[1 - w(a)]\,\Omega_{\rm DE}(a)g = 0$$

Solving this equation for **any arbitrary w(z)** (or $\Omega_{\rm DE}(z)$)
gives you linear growth g(a) (or D(a))

Beware of special closed-form solutions for growth - they are valid only for specific values of w (-1, -1/3, 0)

Philosophical but useful point

Q: given that $\rho_{\mathrm{DE}}(z)$ is related to is an integral of w(z) and is this more precisely measured than w(z), isn't it better to use it rather than w(z)?

A: Not necessarily. While w(z) indeed has larger errors, to understand dynamics of DE you need to take 'derivative by eye' of $\rho_{\mathrm{DE}}(z)$, thereby doing w(z) after all.

Too many papers written arguing about this...

Wish List

Goals:

$$w = \frac{p_{\rm DE}}{\rho_{\rm DE}}$$

$$\Omega_{\rm DE} = \frac{\rho_{\rm DE}}{\rho_{\rm crit}}$$

Measure $\Omega_{\rm DE}, w$

Measure $\rho_{\rm DE}(z)$ or $w(z)$

Measure any clustering of DE

Difficulties:

$w(z)$ enters the observables via integral relations

$$r(z) = \int_0^z \frac{dz'}{H(z')}$$

$$H^2(z) = H_0^2 \left[\Omega_M (1+z)^3 + \Omega_{DE} \exp\left(3 \int_0^z (1 + w(z')) d\ln(1+z')\right)\right]$$

DE clustering affects cosmology negligibly on scales $\ll H_0^{-1}$

Two crucial questions:

1. Is dark energy the vacuum energy ($w(z) = -1$)?
2. Is $w(z) = \text{const}$?

Simplest ways to approach these questions:

$$w(z) = w_0 + w' \, z$$

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

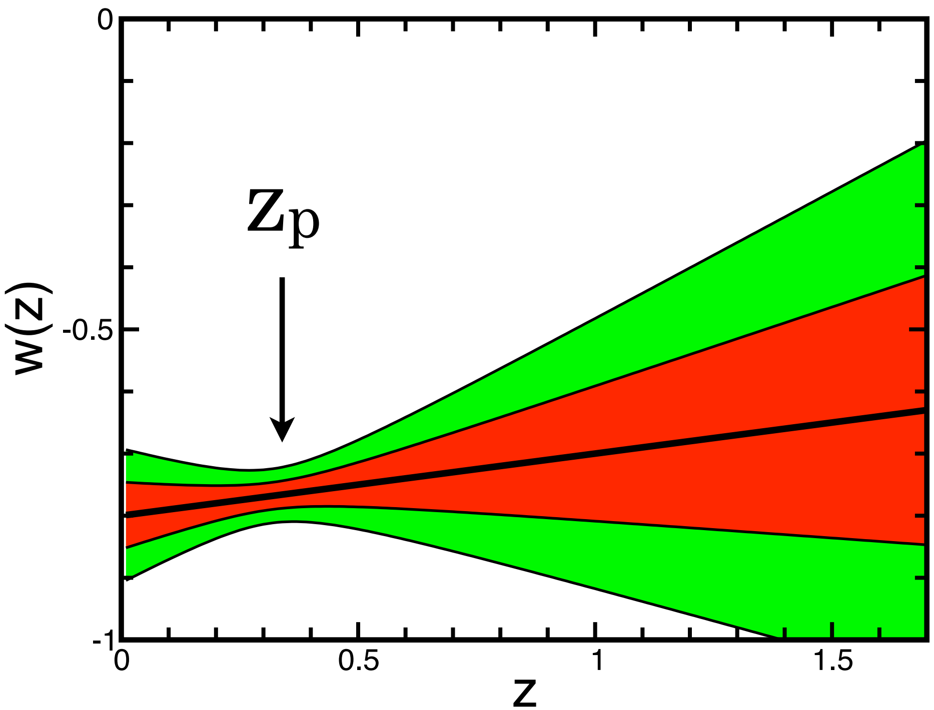
↓

then $\rho_{\rm DE}(a) = \rho_{\rm DE,0} \, a^{-3(1+w_0+w_a)} \, e^{-3(1-a)w_a}$

Pivots! (a_p, z_p and w_p)

$$
\begin{aligned}
w(a) &= w_0 + w_a(1-a) \\
&\equiv w_p + w_a(a_p - a)
\end{aligned}
$$

a_p is the pivot scale factor, at which w(a) is best determined (same for z_p)



*How go get errors in w_p from errors in (w_0, w_a) is in lecture notes

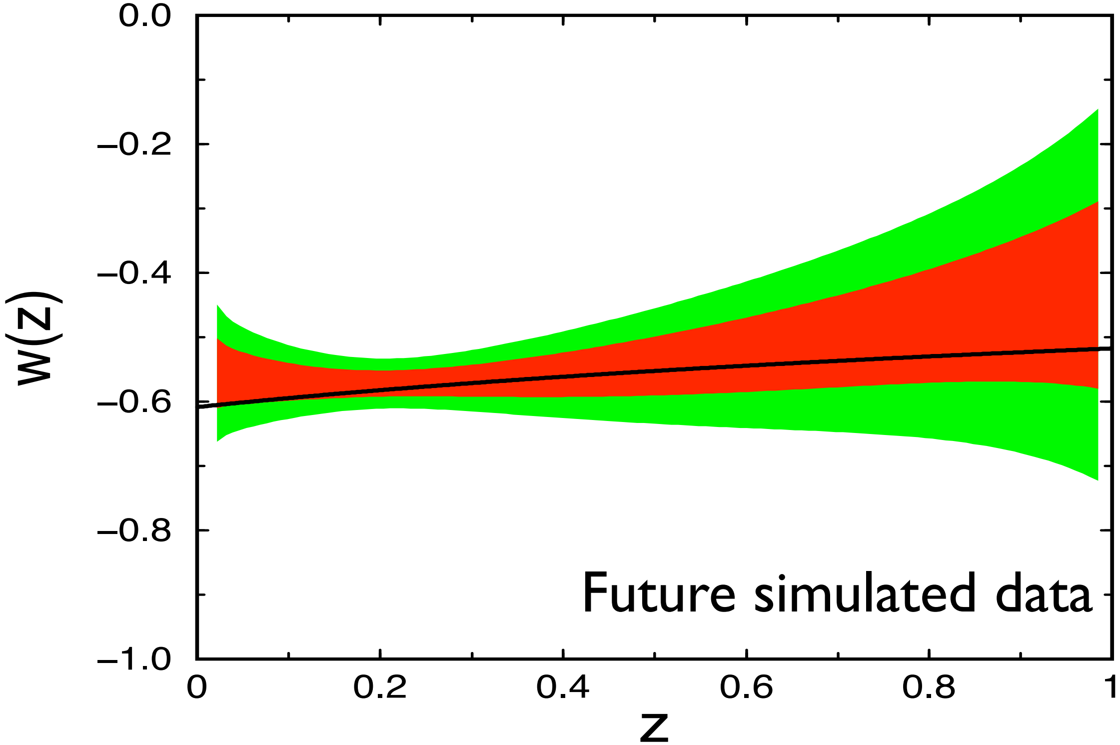
- Pivot redshift z_p tells you where you best measure the equation of state
- Error in the pivot w_p tells you how well you measure it

Direct Reconstruction of $w(z)$

$$1 + w(z) = f\left(\frac{dr}{dz}, \frac{d^2r}{dz^2}\right)$$

$$V[\phi(z)] = g\left(\frac{dr}{dz}, \frac{d^2r}{dz^2}\right)$$

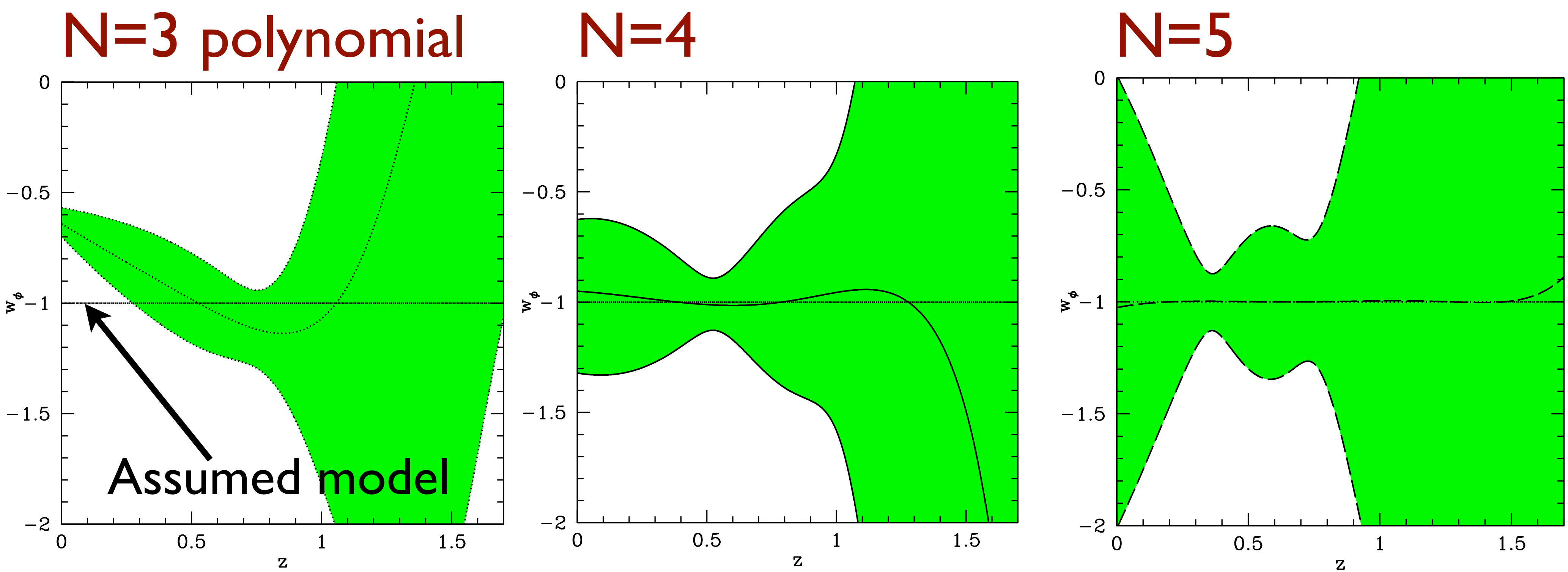
*equations are in the lecture notes



- The most general possible approach to constrain dark energy, but
- Very hard in practice: needs second derivative of (noisy) data
- Nevertheless, studied, refined and used by many authors

Huterer & Turner 1999; Chiba & Nakamura 1999

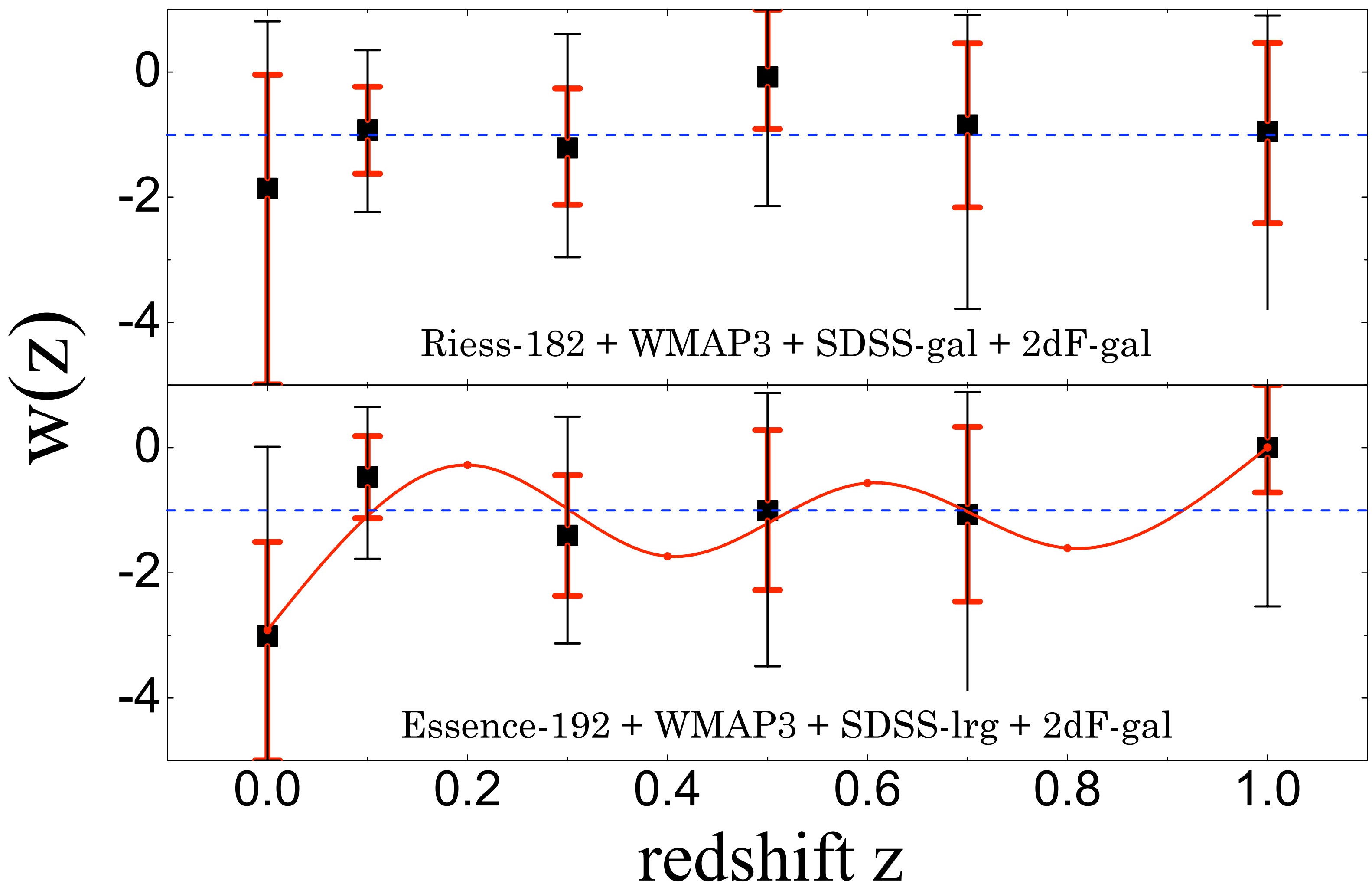
Direct Reconstruction: (parametric) example



Direct reconstruction of the equation of state leads to biases, or large errors, or both ⇒ IS NEVER ROBUST

HT 1999; Weller & Albrecht 2002; ...

Dark Energy constraints: current status

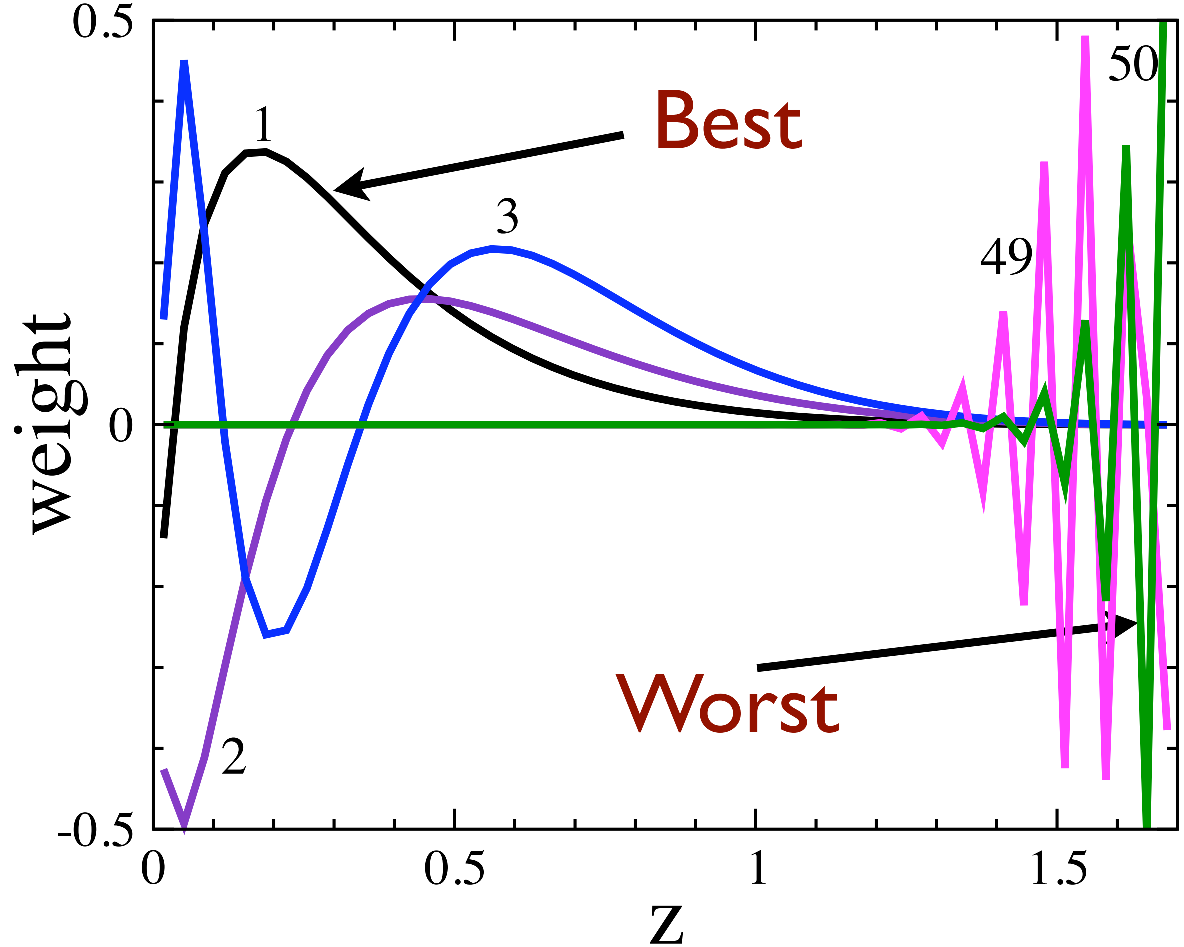


Zhao, Huterer & Zhang, arXiv:0712.2277

Principal Components of $w(z)$

These are best-to-worst measured linear combinations of w(z)

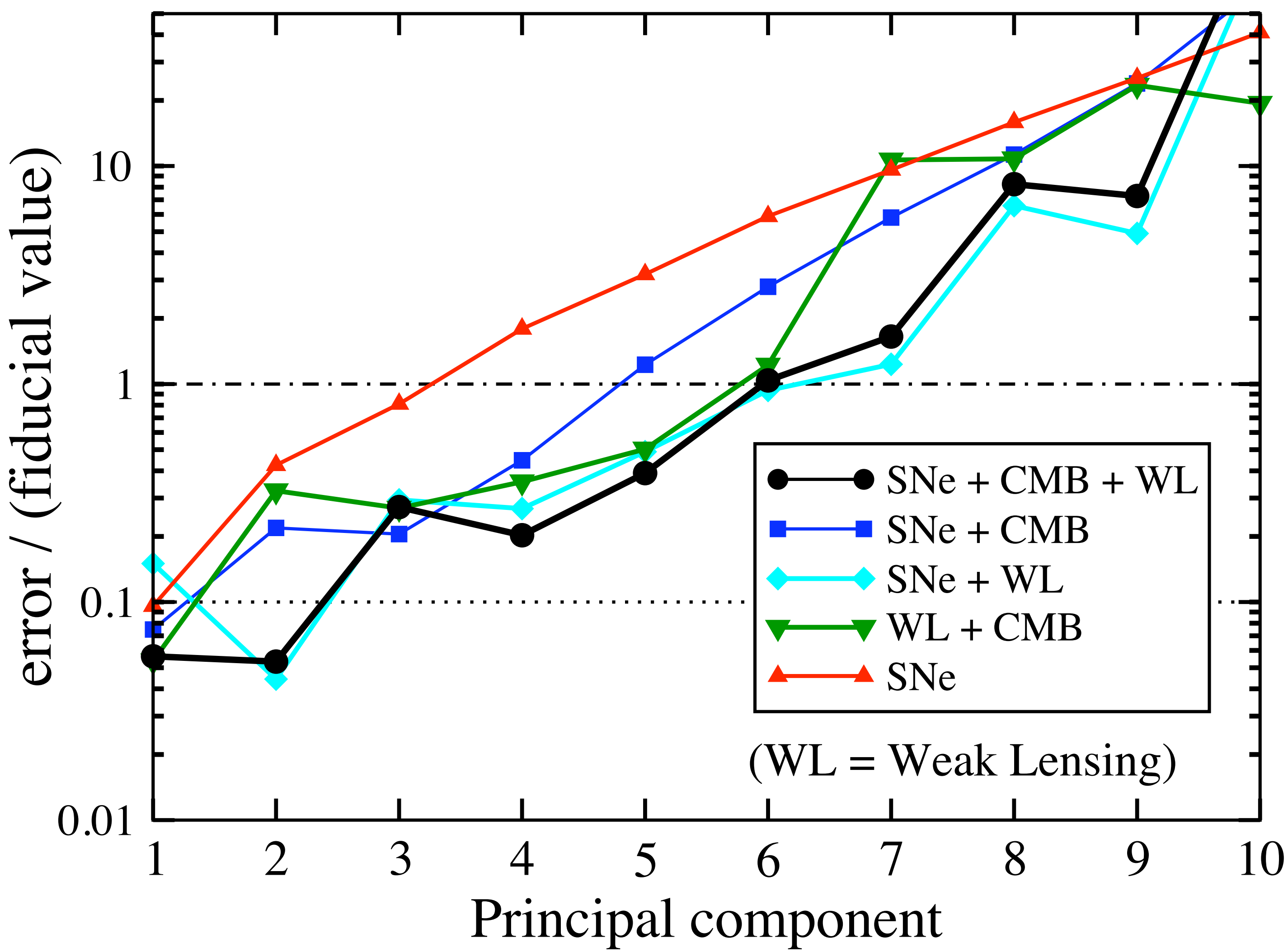
Uncorrelated by construction



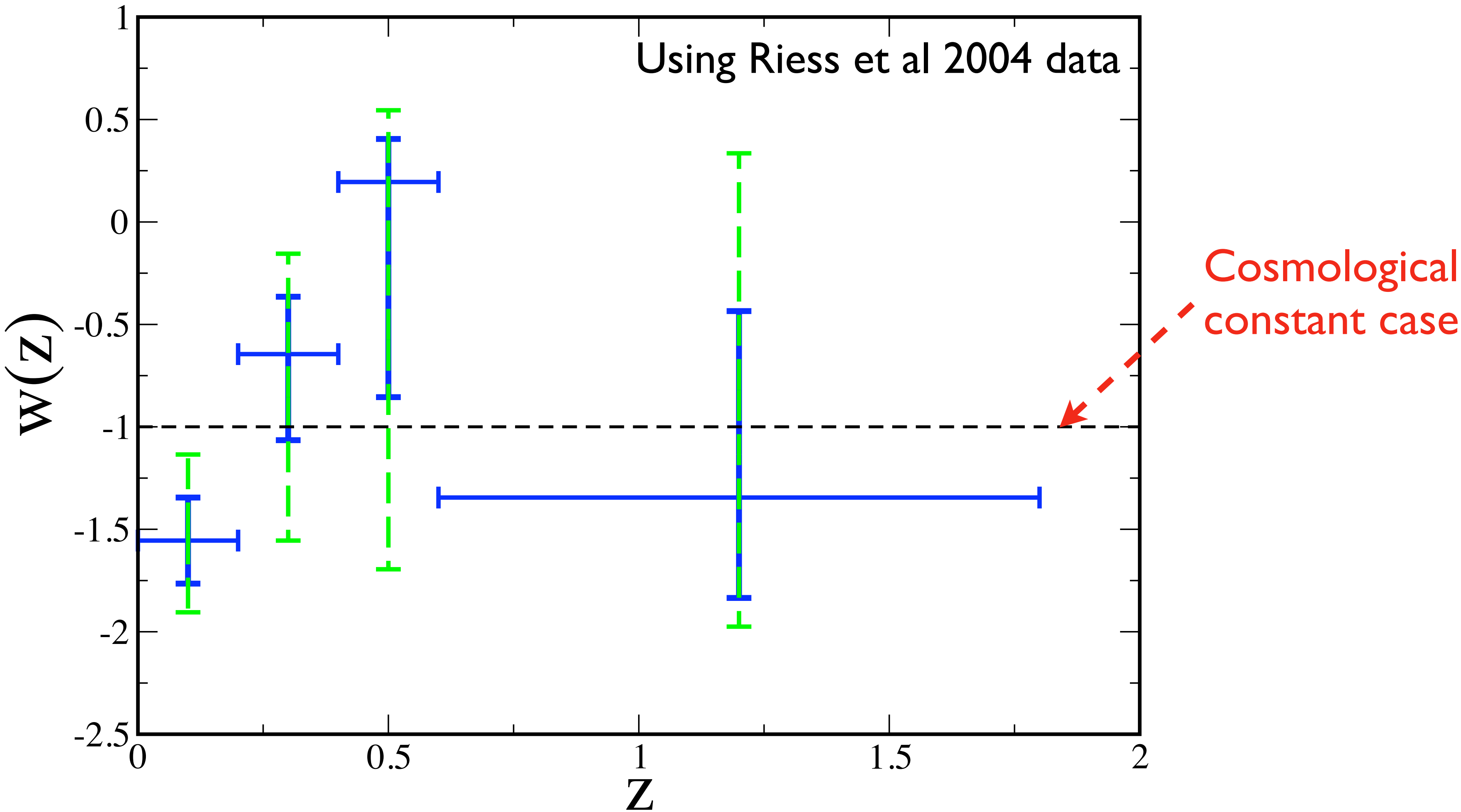
- Shows where sensitivity of any given survey is greatest
- Used by various authors to study optimization of surveys
- Used to make model-(in)dependent statements about DE

Huterer & Starkman 2003

Principal Components of $w(z)$

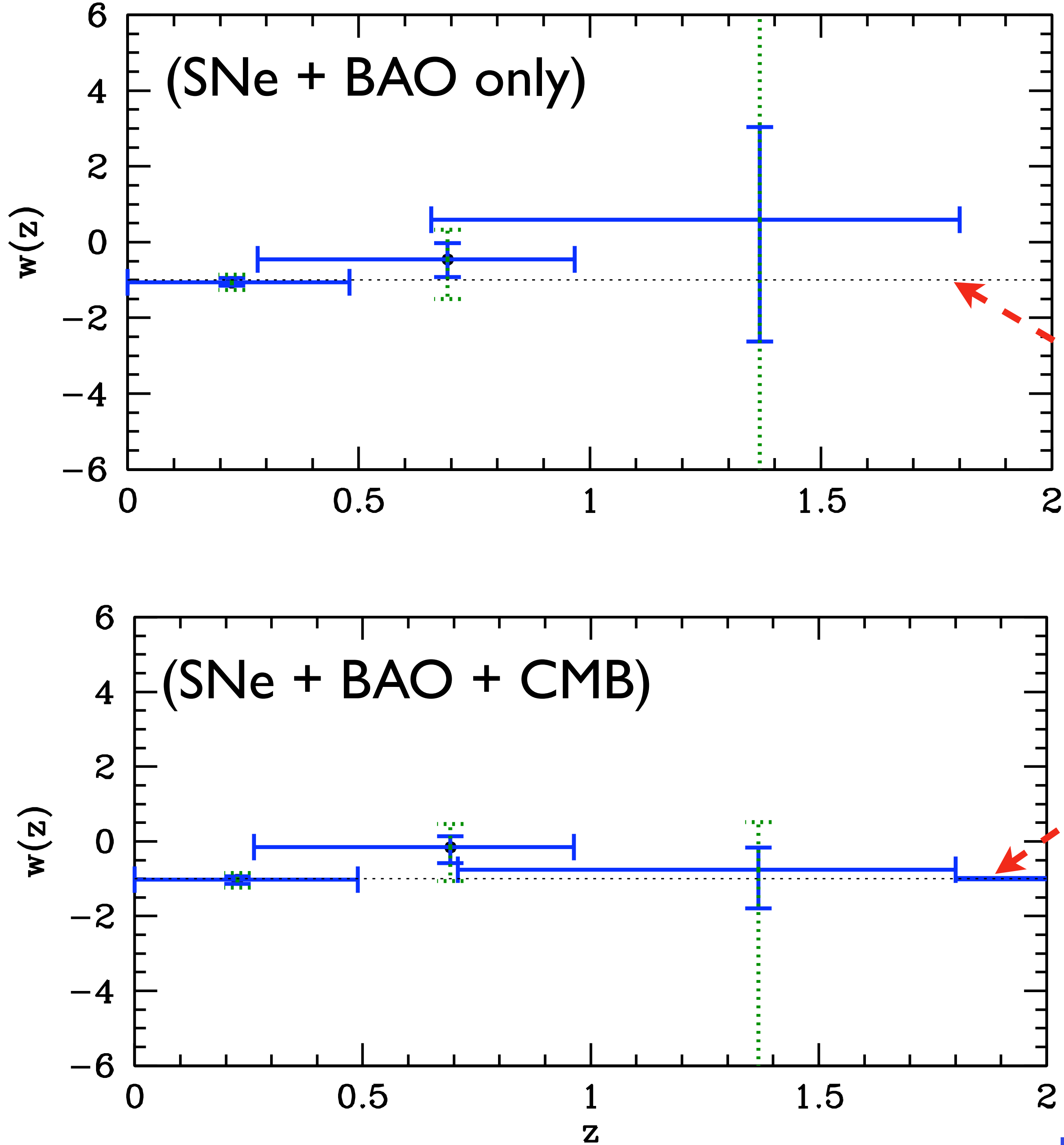


Uncorrelated measurements of Dark Energy evolution



Huterer & Cooray 2005

...and with more recent HST data



Cosmological Constant case

Riess et al, astro-ph/0611572

Modeling of Early DE

Early DE = non-negligible DE in early universe (e.g. around recombination)

$$\rho_{\rm DE}(z > z_{\rm max}) = \rho_{\rm DE}(z_{\rm max}) \left(\frac{1+z}{1+z_{\rm max}}\right)^{3(1+w_\infty)}$$

Other parametrizations of Early DE are possible too...

Early DE - current constraints

- $\Omega_{DE}(z_{rec}) < 0.03$ (CMB peaks; Doran, Robbers & Wetterich 2007)
- $\Omega_{DE}(z_{BBN}) < 0.05$ (BBN; Bean, Hansen & Melchiorri 2001)

Modeling Growth with 1 parameter: 'growth index' γ

$$g(a) \equiv \frac{\delta}{a} = \exp\left[\int_0^a d\ln a[\Omega_M(a)^{\gamma} - 1]\right]$$

Excellent fit to standard DE cosmology with

$$\gamma = 0.55 + 0.05[1 + w(z = 1)]$$

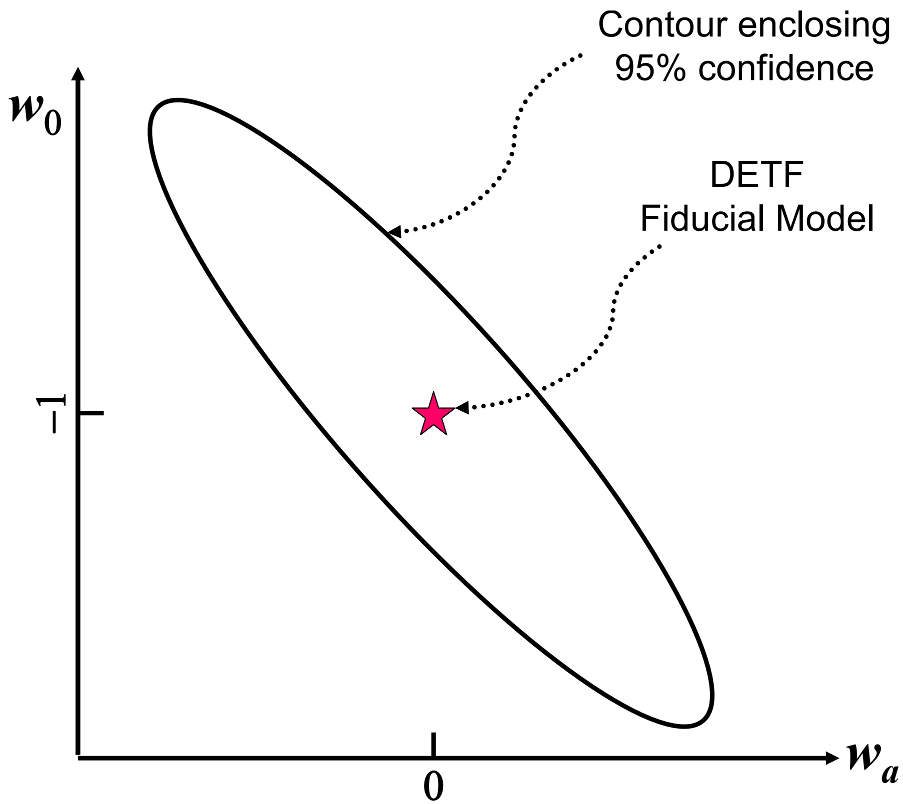
Linder 2005

- Gamma is a new parameter - the growth index - and we should measure it!
- Fits standard LCDM growth to extremely good accuracy
- Also fits e.g. DGP with value different from GR by $\Delta\gamma$=0.13

Figures of merit for DE

FoM =
a number, typically a function of cosmological parameter errors...
that serves as simple and quantifiable metrics by which to evaluate...
the accuracy constraints on dark energy from current and future experiments.

The DETF Figure of Merit



$$
\begin{aligned}
w(z) &= w_0 + w_a(1-a) \\
&= w_p + w_a(a_p - a) \\
\mathrm{FoM} &\equiv \frac{1}{\sigma(w_p) \times \sigma(w_a)}
\end{aligned}
$$

Huterer & Turner 2001; Albrecht et al 2006 (DETF report)

DETF FoM - advantages and disadvantages

Advantages:

- Captures not only w=const but also variation in w(z)
- (w_0, w_a) parametrization reasonable yet simple
- Easy to compute and intuitive

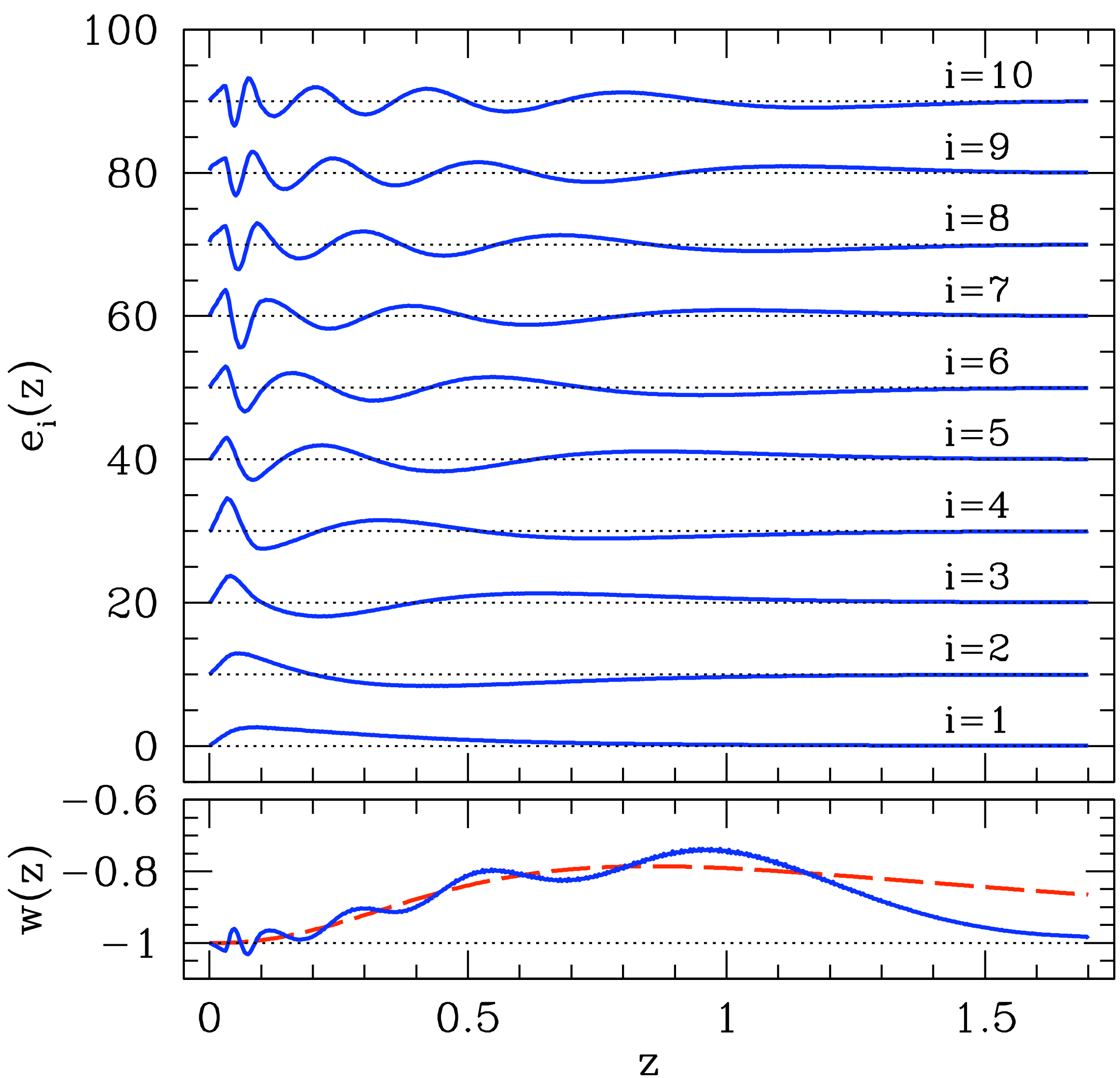
Disadvantages:

- **Captures only two numbers of DE; more will be measured**
- It definitely fails to capture success at measuring early DE
- It does not address anything about modified gravity vs. DE
- It doesn't account for clustering of DE
- It's not designed to measure deviations from LCDM

FoM with principal components

Modeling of low-z w(z):
Principal Components

$$w(z_j) = -1 + \sum_{i=1}^{N} \alpha_i e_i(z_j)$$



500 bins (so 500 PCs)
0.03<z<1.7

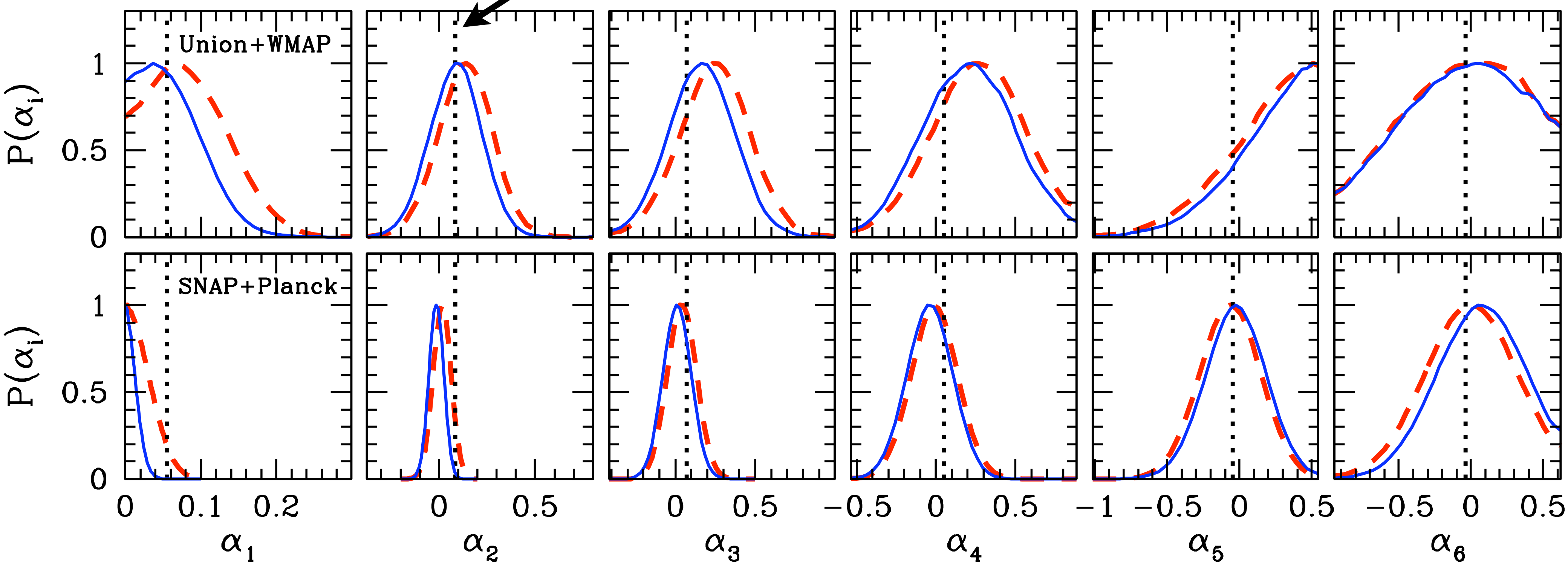
We use first ~10 PCs;
(results converge 10→15)

Fit of a quintessence model with PCs

In *principal*, constraints are good...

values for example quintessence model

Current



Future (assumes α_i=0)

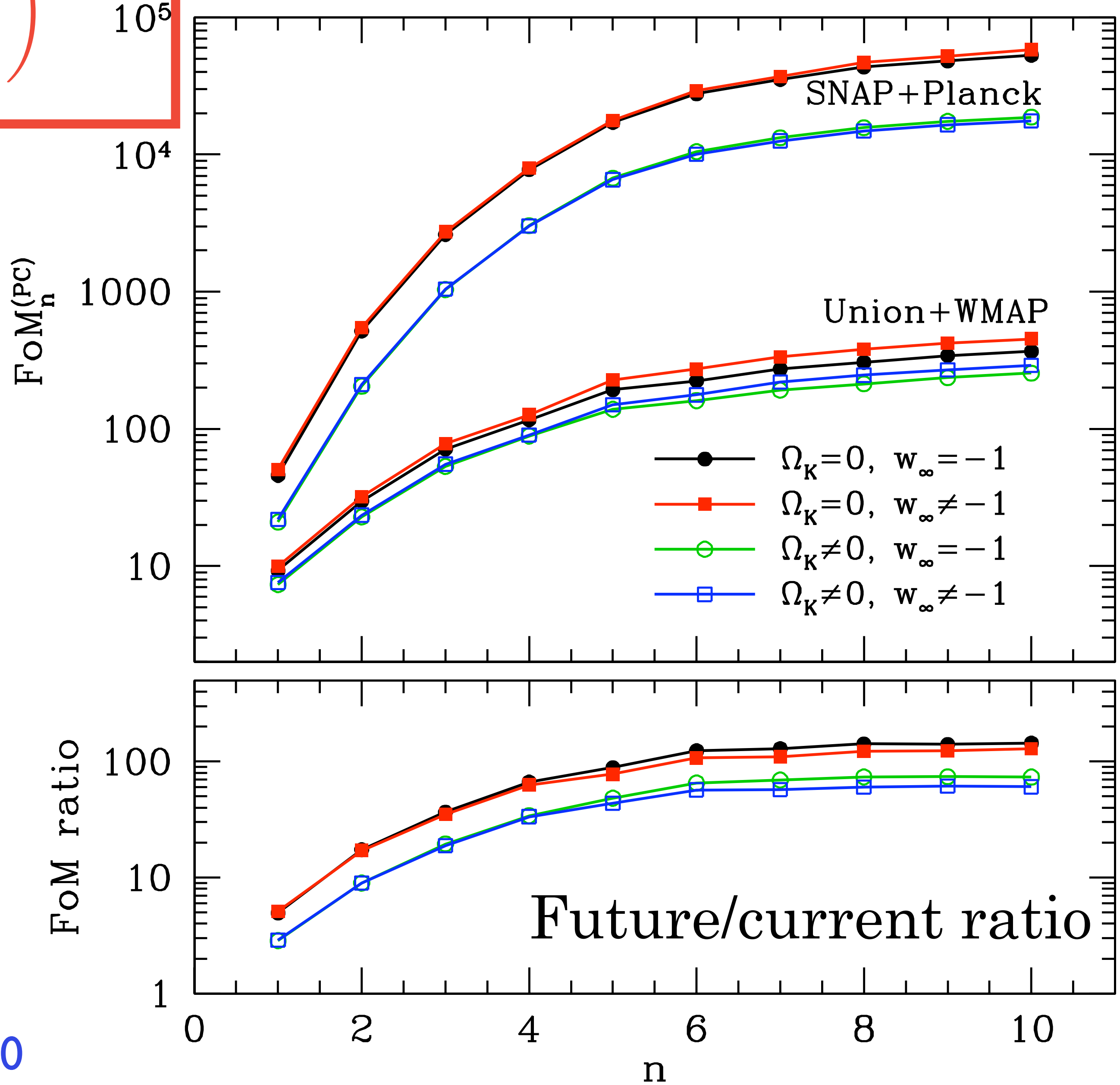
Flat

Curved

Mortonson, Huterer & Hu 2010

FoM with principal components

$$\text{FoM}_n^{(\text{PC})} \equiv \left(\frac{\det \mathbf{C}_n}{\det \mathbf{C}_n^{(\text{prior})}} \right)^{-1/2}$$



Mortonson, Huterer & Hu 2010