Quantum mechanics in 21st century

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Talk presented at RESCEU summer school "Dark energy in the universe at Tosa Yamada, Augst 31, 2010

Introduction

- Intrusion of quantum mechanics to our daily world

  Quantum information technology
  Nano fabrication technology

- Quantum metaphysics now relevant

  Quantum mechanics is mysterious
  uncertainty, entanglement, anomaly

  Counter-intuitive aspects discussed by Einstein, Bohr et al.
  --> “magical” new technologies

What is (odd about) quantum mechanics?

- Quantum interference of matter wave

- Quantum uncertainty
  Incompatible observables
  Observation changes the quantum states

- Quantum nonlocality coming from entanglement
  Bell inequality breaking; hidden variable theory disproved
  Contextuality: Kochen-Specker theorem, Conway free will

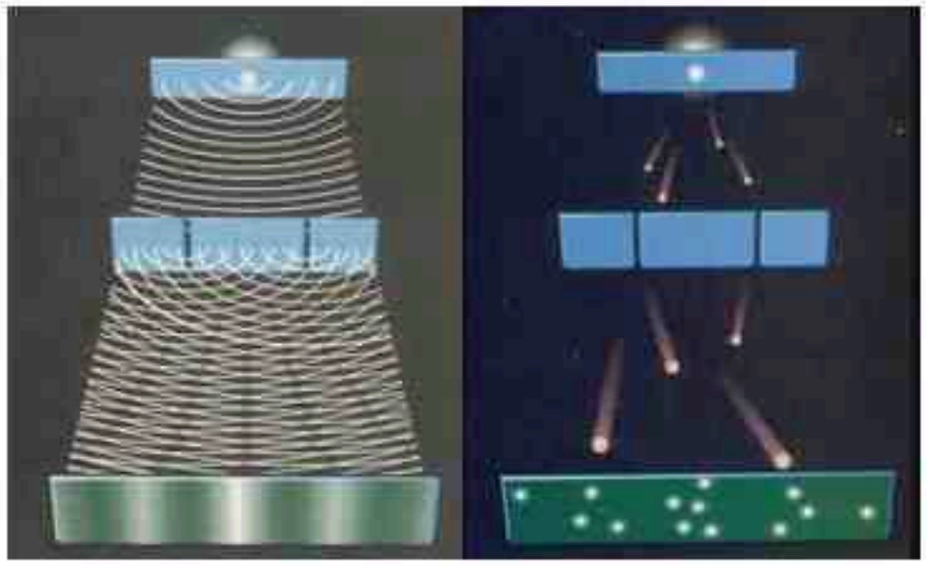
- Quantum anomaly
  scale anomaly in quantum graph; quantum device

Quantum interference

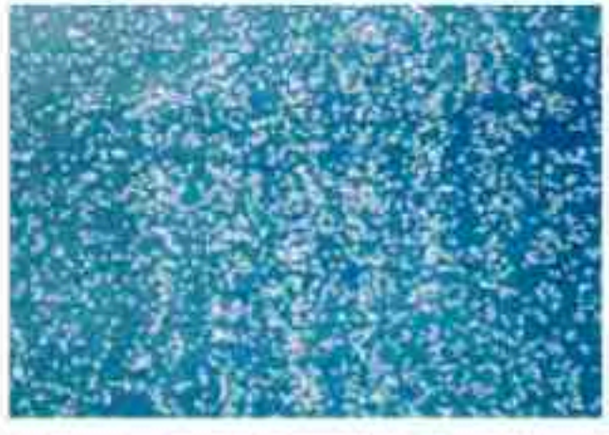
- de Broglie's matter-wave theory

$$\lambda = \frac{h}{mv}$$

- Double-slit experiment
  : particle will interfere with itself

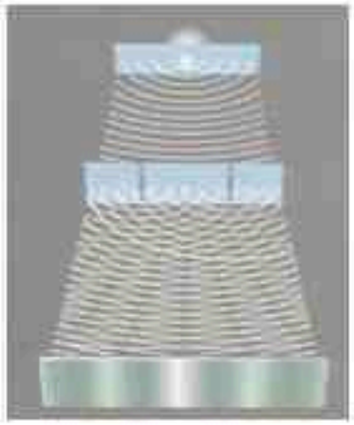


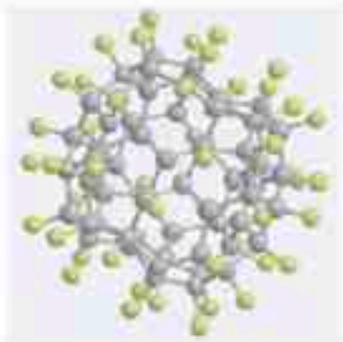
(Davidson et al. 1927)



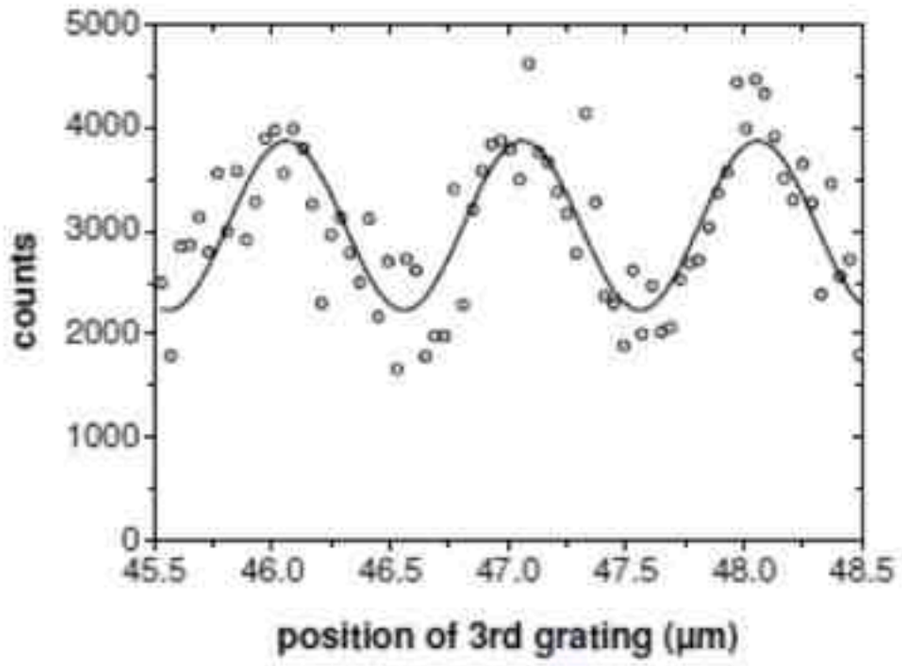
Quantum interference of large molecule

- Double-slit experiment with organic molecules

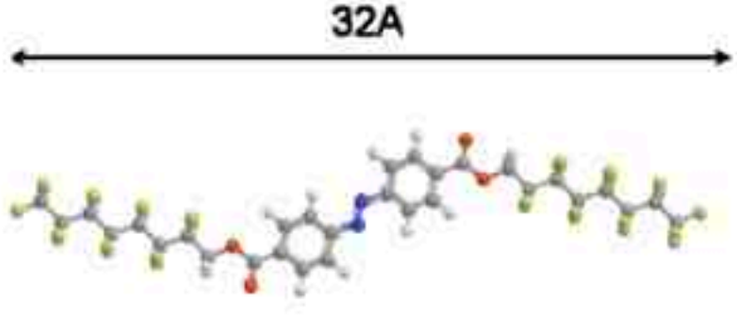




Heaviest: $C_{60}F_{48}$
fluorinated fullerene ↑
(Hackermüller et al. 2003)



Largest: azobenzene →
(Gerlich et al. 2007)



Interference in quantum theory of mind

- Breaking of Sure-thing principle (Tversky and Shafir 1985)

  choose from risky/safe bets

  Known conditions

  Favorable : risky 30 %, safe 70 %

  Unfavorable : risky 10 %, safe 90 %

  Unknown condition

  F or U (1:1) : risky 45 %, safe 55 %

- Quantum decision theory

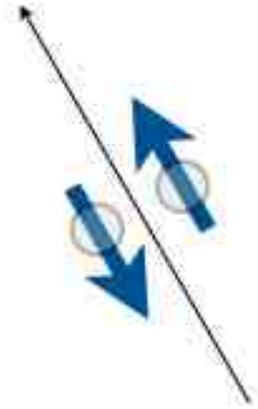
interference

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|F)\left(\sqrt{0.3}\,|r\rangle + \sqrt{0.7}e^{i\theta}\,|s\rangle\right) + \frac{1}{\sqrt{2}}|U)\left(\sqrt{0.1}\,|r\rangle + \sqrt{0.9}e^{i\phi}\,|s\rangle\right)$$

$$P(U|r) \approx \frac{0.3+0.1}{2} + \sqrt{0.3\times 0.1}\cos(\theta-\phi)$$ (Busemeyer *et al.* 2006)

Quantum uncertainty in spin states

- Direction of spin 1/2 (electron) : two states system



  * Either + or – along measurement axis
  * Measurement axis is chosen *at will* by observer

  -> observation changes the state probabilistically

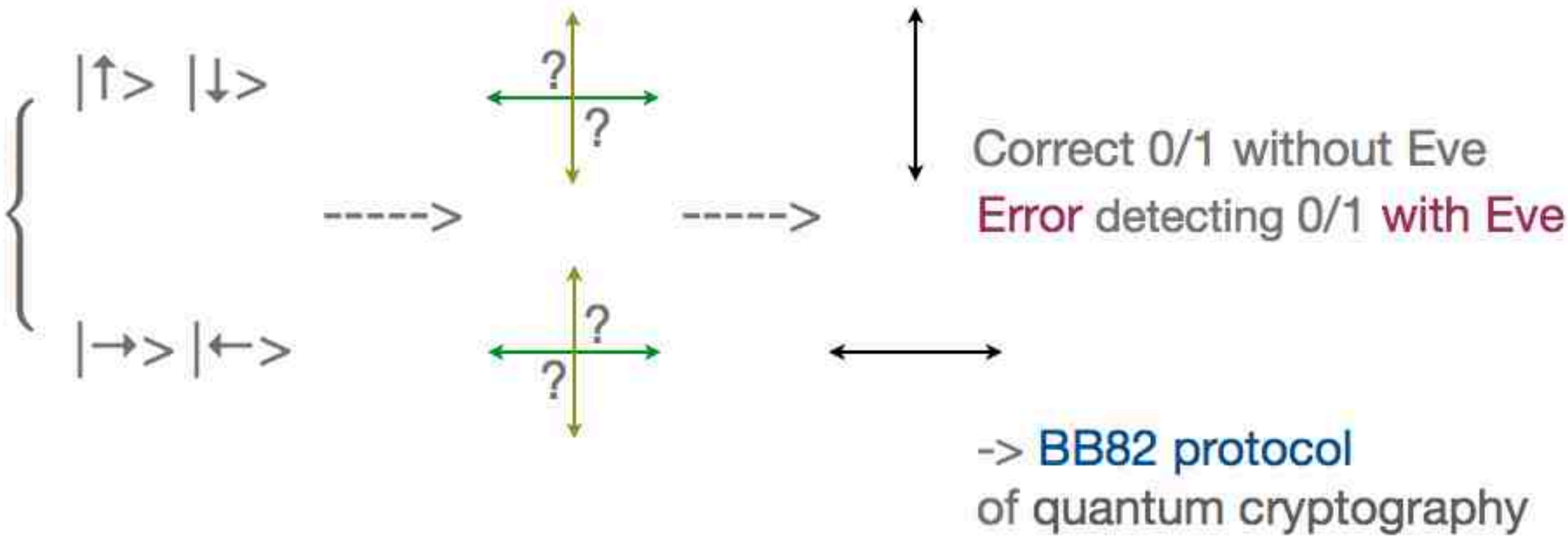
- Measurement along two axes incompatible
  direction uncertain

$$|\rightarrow\rangle = 1/\sqrt{2}\,|\uparrow\rangle + 1/\sqrt{2}\,|\downarrow\rangle$$

$$|\leftarrow\rangle = 1/\sqrt{2}\,|\uparrow\rangle + 1/\sqrt{2}\,|\downarrow\rangle$$

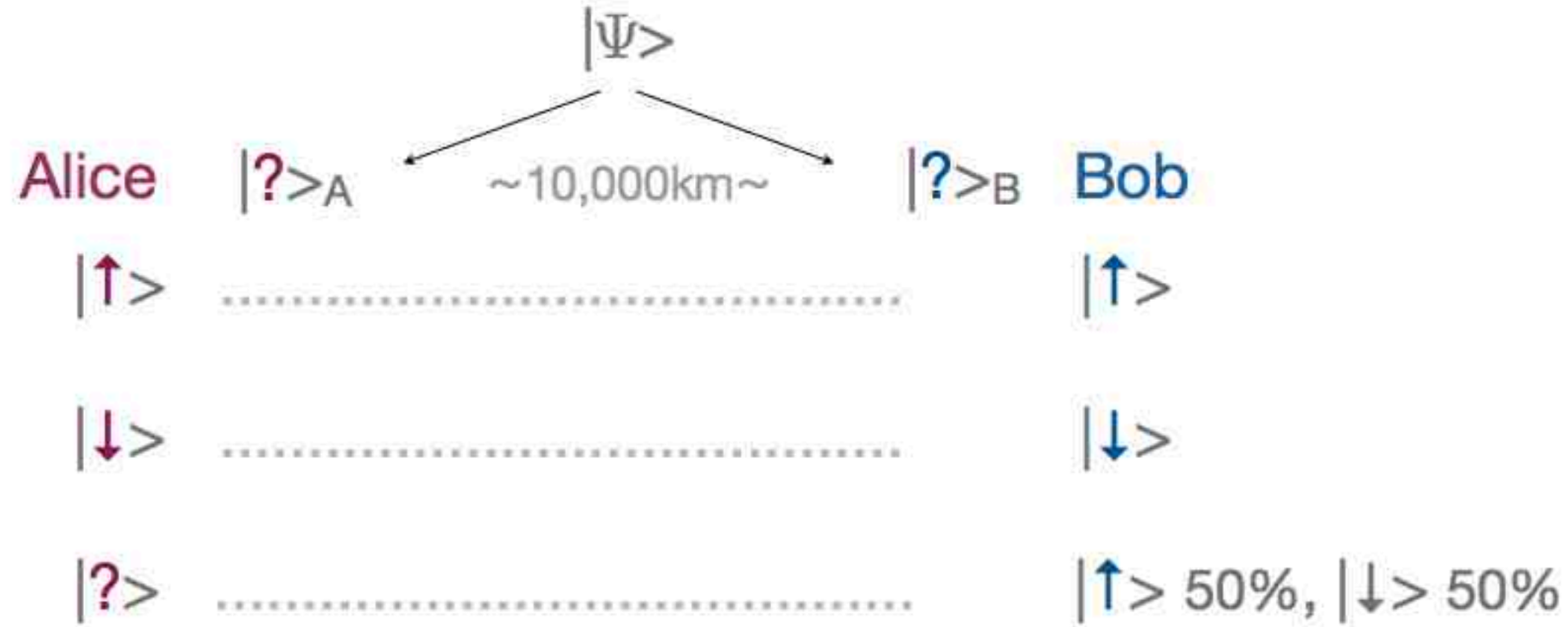
Quantum detection of eavesdropper

- Alice communicate with Bob by spin direction while protecting security by detecting Eve



Einstein-Podolski-Rosen paradox

- Two particle entangled state $|\Psi\rangle = 1/\sqrt{2}\,|\uparrow\rangle_A\,|\uparrow\rangle_B + 1/\sqrt{2}\,|\downarrow\rangle_A\,|\downarrow\rangle_B$



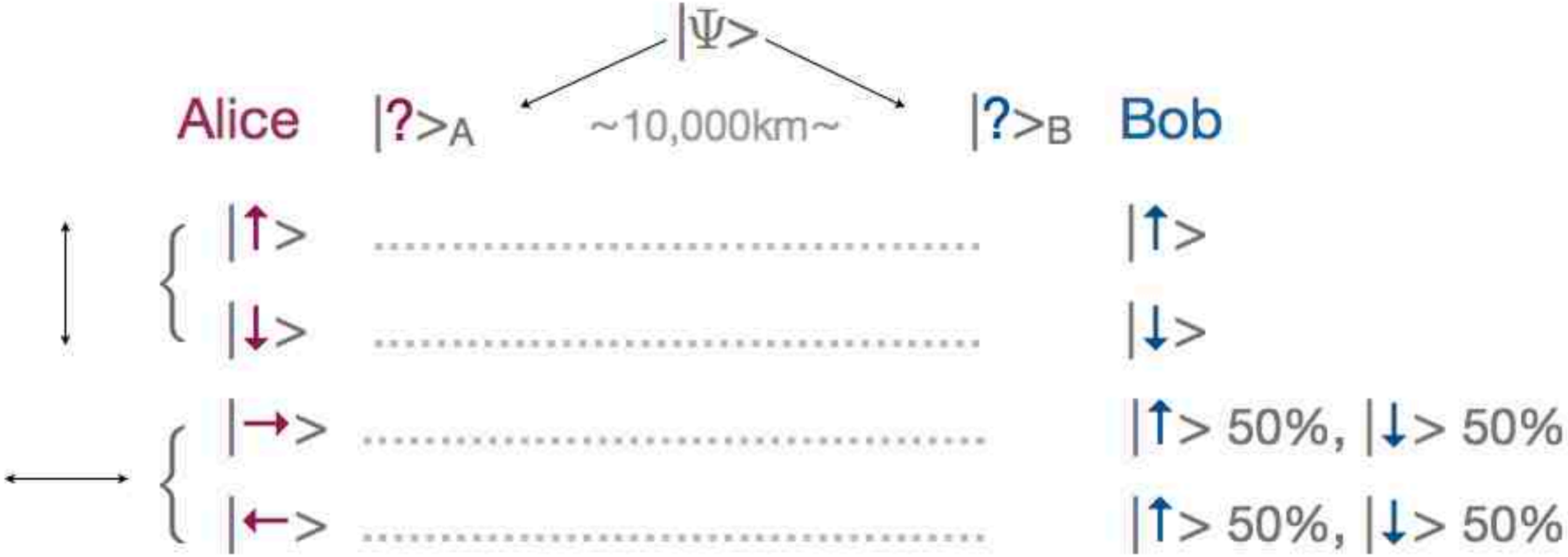
such nonlocality *impossible -> quantum mechanics wrong!*
(EPR paradox)

EPR paradox (cont'd)

- Entangled state

$$|\Psi\rangle = 1/\sqrt{2}\,|\uparrow\rangle_A\,|\uparrow\rangle_B + 1/\sqrt{2}\,|\downarrow\rangle_A\,|\downarrow\rangle_B$$

$$= 1/\sqrt{2}\,|\rightarrow\rangle_A\,|\rightarrow\rangle_B + 1/\sqrt{2}\,|\leftarrow\rangle_A\,|\leftarrow\rangle_B$$



Set up of Alice's device affects Bob's state
Nonlocality & Contextuality of observables

Bell inequality

- More elegant formulation: Bell 1964
  2 particle-2 setup experiment
  16 *joint probabilities P**(**)*

$$C^{\updownarrow\updownarrow} = P^{\updownarrow\updownarrow}(\uparrow\uparrow) - P^{\updownarrow\updownarrow}(\uparrow\downarrow) - P^{\updownarrow\updownarrow}(\downarrow\uparrow) + P^{\updownarrow\updownarrow}(\downarrow\downarrow)$$
$$C^{\leftrightarrow\updownarrow} = P^{\leftrightarrow\updownarrow}(\rightarrow\uparrow) - P^{\leftrightarrow\updownarrow}(\rightarrow\downarrow) - P^{\leftrightarrow\updownarrow}(\leftarrow\uparrow) + P^{\leftrightarrow\updownarrow}(\leftarrow\downarrow)$$
$$C^{\updownarrow\leftrightarrow} = P^{\updownarrow\leftrightarrow}(\uparrow\rightarrow) - P^{\updownarrow\leftrightarrow}(\uparrow\leftarrow) - P^{\updownarrow\leftrightarrow}(\downarrow\rightarrow) + P^{\updownarrow\leftrightarrow}(\downarrow\leftarrow)$$
$$C^{\leftrightarrow\leftrightarrow} = P^{\leftrightarrow\leftrightarrow}(\rightarrow\rightarrow) - P^{\leftrightarrow\leftrightarrow}(\rightarrow\leftarrow) - P^{\leftrightarrow\leftrightarrow}(\leftarrow\rightarrow) + P^{\leftrightarrow\leftrightarrow}(\leftarrow\leftarrow)$$

correlations

- CHSH inequality (Best known of Bell inequalities)

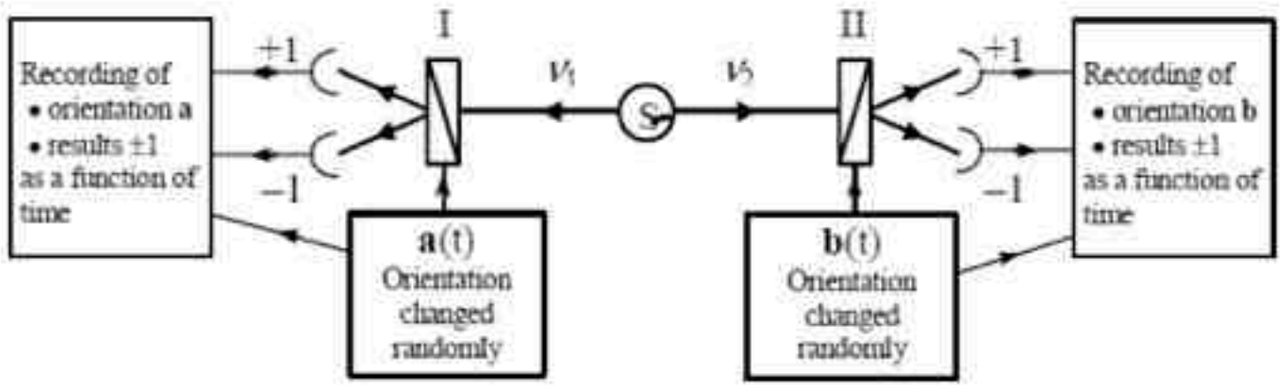
$$-2 \leq C^{\updownarrow\updownarrow} - C^{\updownarrow\leftrightarrow} + C^{\leftrightarrow\updownarrow} + C^{\leftrightarrow\leftrightarrow} \leq 2$$

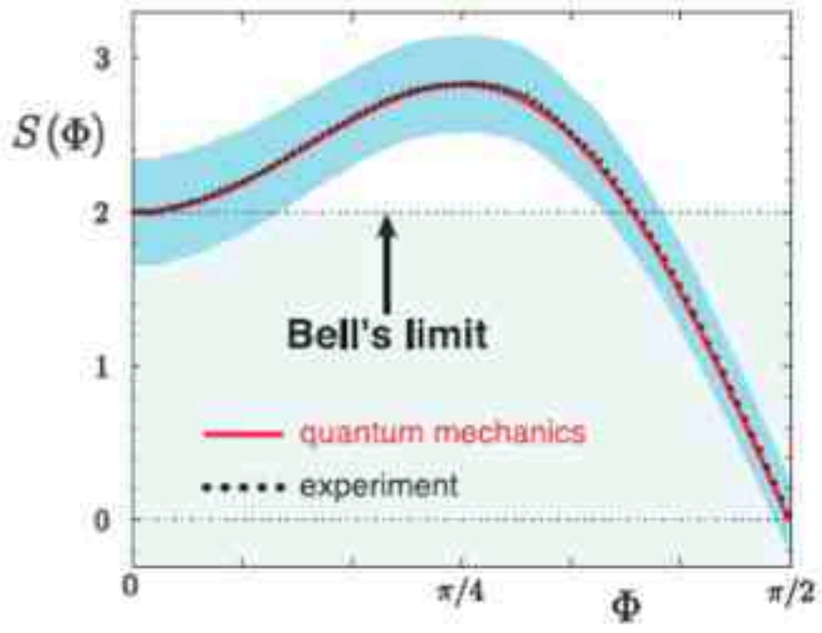
must be satisfied if local realism holds

Experimental test of Bell inequality

- CHSH-Bell experiment --> *nonlocal correlation exists!*

Aspect *et al.* 1982	12 m
Weihs *et al.* 1998	400 m
Branchard *et al.* 2007	10 km





Quantum magic with entanglement

- Quantum cryptography E91 ← experimentally observed
  eavesdropper breaks the entanglement : detectable

- Quantum parallel computation ←
  computation as unitary operation $U\ |a,b\rangle = |a',b'\rangle$
  $U\ (\ |ab\rangle + |cd\rangle\ ) = |a'b'\rangle + |c'd'\rangle$
  n-spin state (n qubits) : 2^n parallel operations

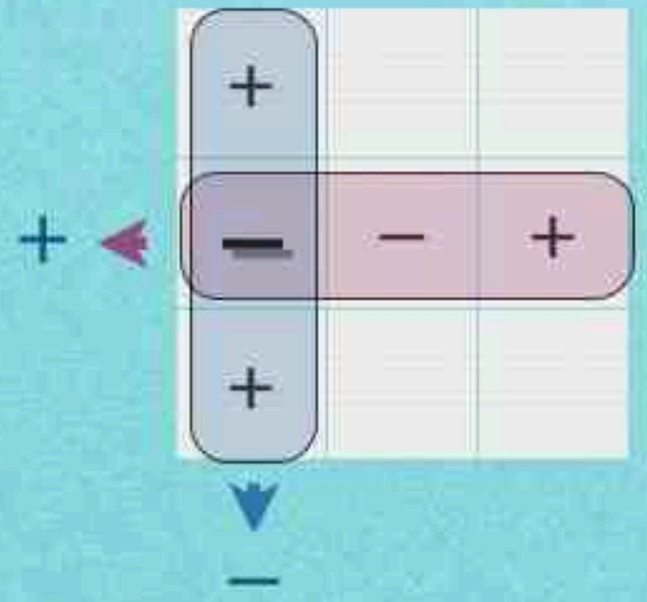
  Striking example: *Shor's prime factorization*

- Quantum teleportation ←
  entanglement swapping $|a\ (bc)\rangle$ ... observe first two ... $|(a'b')\ a\rangle$

- Kochen-Specker contextuality --> Conway free will theorem

Quiz : Arvind's magic square

- Fill +1/−1 in the cells of 3x3 square such that
  1) *Alice reads out a raw, multiply three numbers and gets +1*
  2) *Bob reads out a column, multiply three numbers and gets −1*
  3) *The crossings of Alice's raw & Bob's column is checked*



+	+	+
−	−	+
+	+	?

impossible

$\sigma_x \otimes I$	$\sigma_x \otimes \sigma_x$	$I \otimes \sigma_x$
$-\sigma_x \otimes \sigma_z$	$\sigma_y \otimes \sigma_y$	$-\sigma_z \otimes \sigma_x$
$I \otimes \sigma_z$	$\sigma_z \otimes \sigma_z$	$\sigma_z \otimes I$

Arvind's square

Enabled by *contextuality*

$$|\Psi\rangle = \frac{1}{2}\left(|\uparrow\rangle_1^a |\uparrow\rangle_1^b + |\downarrow\rangle_1^a |\downarrow\rangle_1^b\right)\left(|\uparrow\rangle_2^a |\uparrow\rangle_2^b + |\downarrow\rangle_2^a |\downarrow\rangle_2^b\right)$$

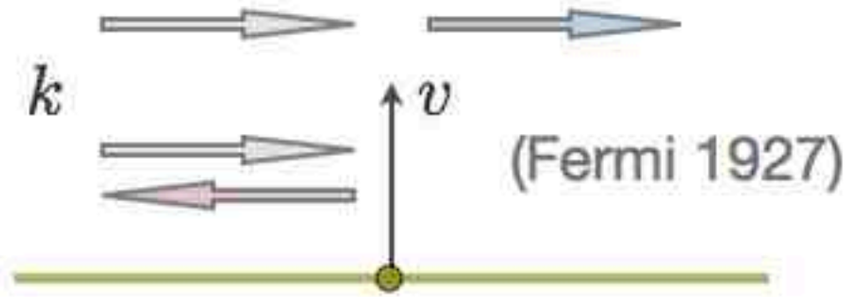
Scale anomaly in 1D quantum mechanics

- Thin obstacle in one-dimensional line

Classical : *bouncing wall* or *nothing* <- no scale

- Quantum : delta potential; **v** [1/L]

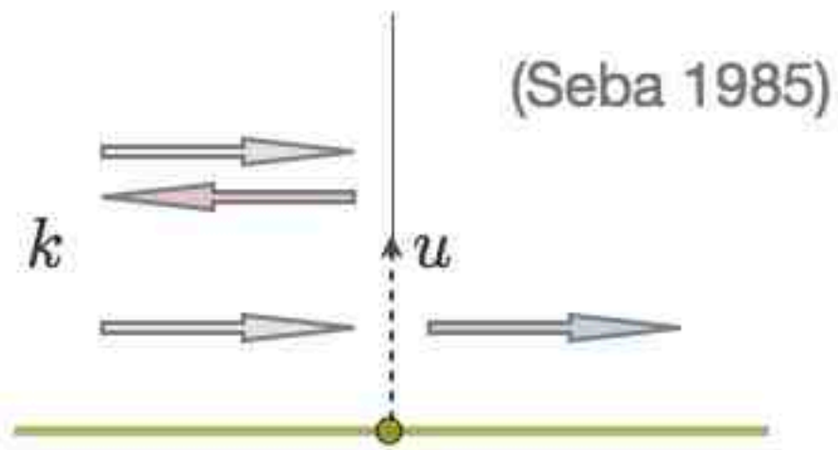
$$\psi'_+ - \psi'_- = v\psi$$
$$\psi_+ = \psi_- = \psi$$
$$\mathcal{T} = \frac{k}{k + iv/2}$$



(Fermi 1927)

- Quantum : delta' potential; **u** [L]

$$\psi_+ - \psi_- = u\psi'$$
$$\psi'_+ = \psi'_- = \psi'$$
$$\mathcal{T} = -\frac{1}{1 - iku/2}$$



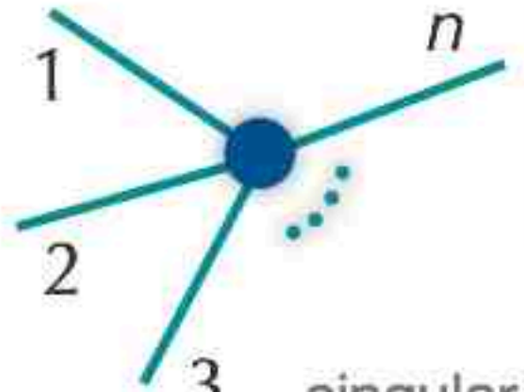
(Seba 1985)

scale anomaly -> dual partners δ & δ'

Quantum graph

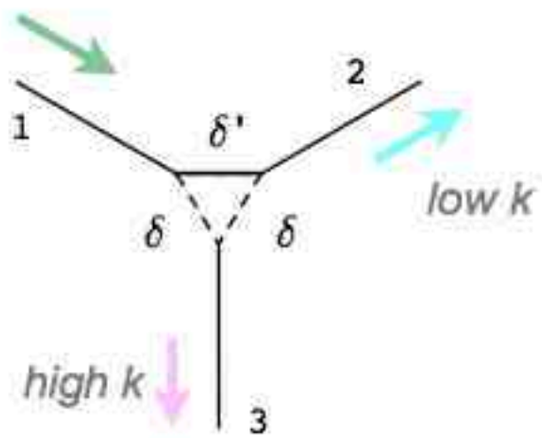
- Quantum graph :
  quantum particle on 1D lines joined at nodes

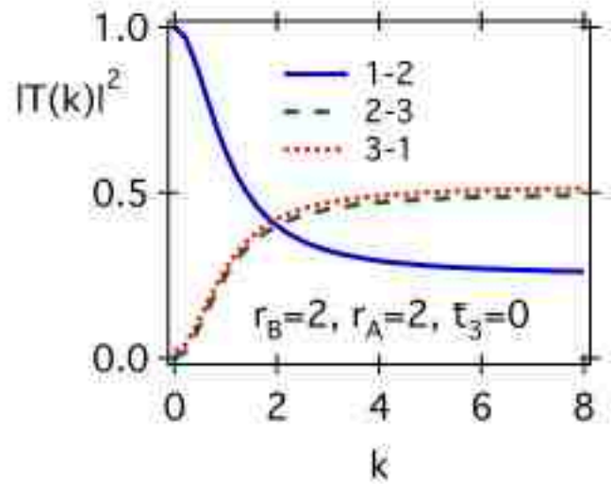
- Line with a defect (δ & δ') : N=2

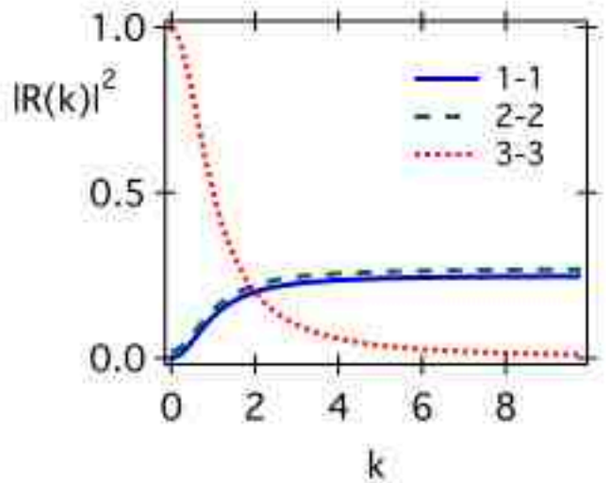


singular vertex of order N

- Spectral branching filter
  as generalization of δ & δ' couplings





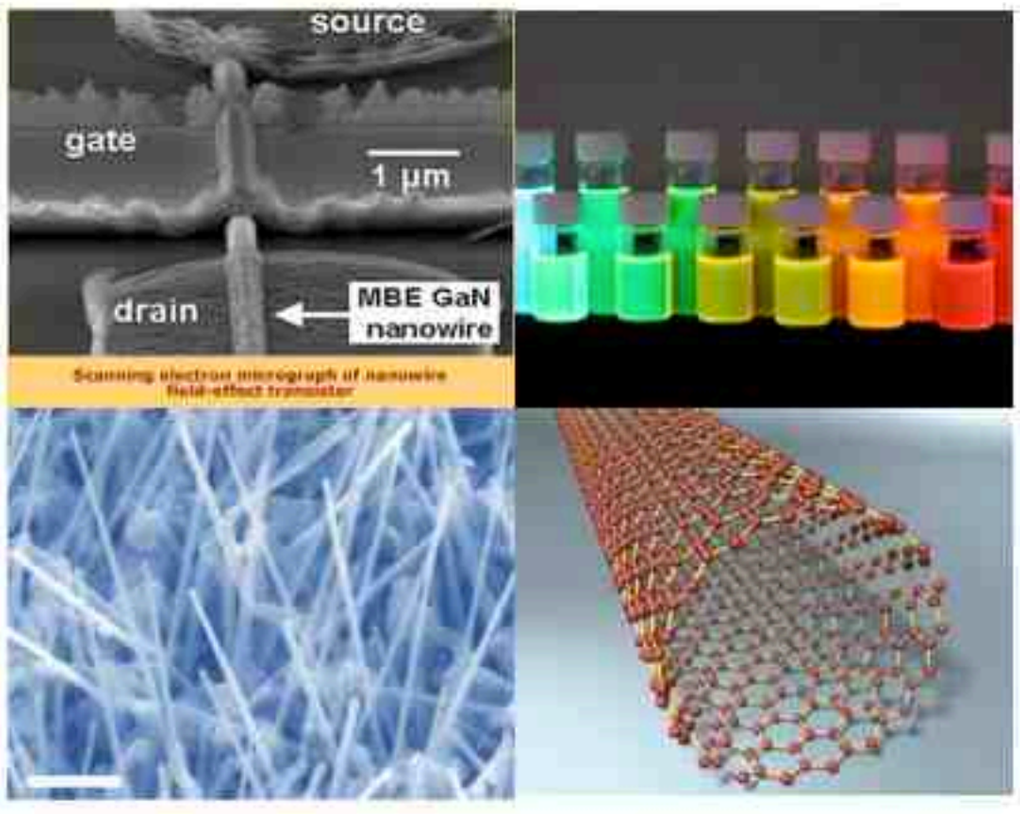


Quantum device

- Semiconductor device ~ 30nm
  bellow 1nm, quantum noise non-suppressible
  --> quantum device

- Single-electron device
  elements
  -- quantum dot
  -- quantum nanowire

- Quantum filter & transistor
  with dot and wire in the future

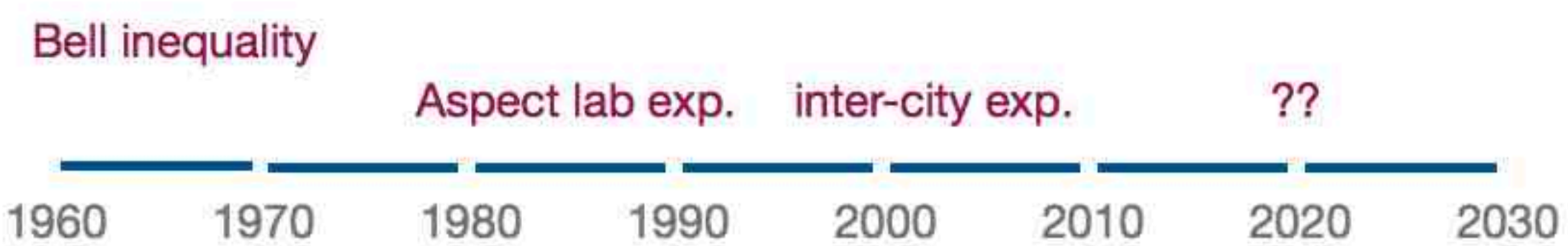


Prospects

- Historical parallel

- electromagnetic wave

- quantum entanglement



Prospects

- Dark energy in the universe summer school at Tosa Yamada

- Aug. 30

DENET schedule

regular session　lecture session　night at Kochi

08:00　10:00　12:00　14:00　16:00　18:00　20:00　22:00

- Aug. 31

DENET schedule

lecture session　regular session　quantum interlude

08:00　10:00　12:00　14:00　16:00　18:00　20:00　22:00

you are here