University of Portsmouth

Galaxy Clustering & Observational Methods

Will Percival

ICG, University of Portsmouth, UK

Reason for my non-arrival at Hakone school 2 years ago was his early arrival

Lecture outline

- Introduction to statistics
  - Correlation function
  - Power spectrum
- Information from power spectrum shape
  - Matter density
  - Baryon Acoustic Oscillations
  - Neutrino mass
  - Inflation perturbation power spectrum
- Information from geometry
  - Galaxy clustering as a standard ruler
  - BAO and the Alcock-Paczynski effect
  - non-linear power spectrum shifts
- Information from structure growth
  - linear growth
  - peculiar velocities
  - redshift-space distortions

Lecture outline

- Introduction to statistics
  - Correlation function
  - Power spectrum
- Information from power spectrum shape
  - Matter density
  - Baryon Acoustic Oscillations
  - Neutrino mass
  - Inflation perturbation power spectrum
- Information from geometry
  - Galaxy clustering as a standard ruler
  - BAO and the Alcock-Paczynski effect
  - non-linear power spectrum shifts
- Information from structure growth
  - linear growth
  - peculiar velocities
  - redshift-space distortions

University of Portsmouth

Perturbation statistics: correlation function

overdensity field

$$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

definition of correlation function

$$\begin{aligned} \xi(\mathbf{x_1}, \mathbf{x_2}) &\equiv \langle \delta(\mathbf{x_1}) \delta(\mathbf{x_2}) \rangle \\ &= \xi(\mathbf{x_1} - \mathbf{x_2}) \\ &= \xi(|\mathbf{x_1} - \mathbf{x_2}|) \end{aligned}$$

from statistical homogeneity

from statistical isotropy

can estimate correlation function using galaxy (DD) and random (RR) pair counts at separations ~r

$$1 + \xi(r) = \frac{\langle DD \rangle_r}{\langle RR \rangle_r}$$

Perturbation statistics: power spectrum

definition of
power spectrum

$$\langle \delta(\mathbf{k_1}) \delta(\mathbf{k_2}) \rangle = (2\pi)^3 \delta_D(\mathbf{k_1} - \mathbf{k_2}) P(k_1)$$

power spectrum is the Fourier analogue of the correlation function

$$P(k) \equiv \int \xi(r) e^{i\mathbf{k}.\mathbf{r}} d^3r$$

$$\xi(r) = \int P(k) e^{-i\mathbf{k}.\mathbf{r}} \frac{d^3k}{(2\pi)^3}$$

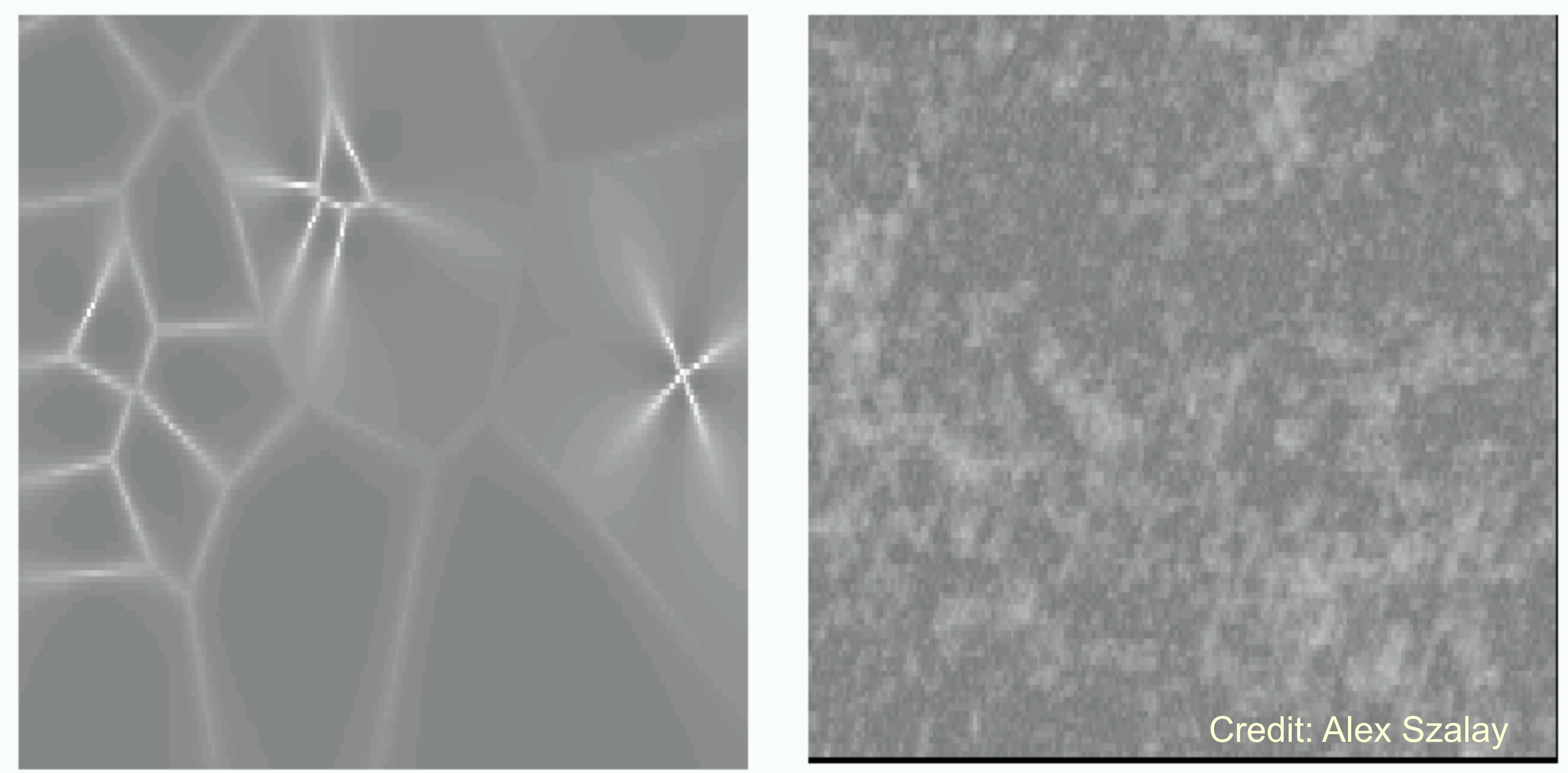
sometimes written in dimensionless form

$$\Delta^2(k) = \frac{k^3}{2\pi^2} P(k)$$

The importance of 2-pt statistics

Because the central limit theorem implies that a density distribution is asymptotically Gaussian in the limit where the density results from the average of many independent processes; and a Gaussian is completely characterised by its mean (overdensity=0) and variance (given by either the correlation function or the power spectrum)

Do 2-pt statistics tell us everything?



Same 2pt, different 3pt

Correlation function vs Power Spectrum

The power spectrum and correlation function contain the same information; accurate measurement of each give the same constraints on cosmological models.

Both power spectrum and correlation function can be measured relatively easily (and with amazing complexity)

The power spectrum has the advantage that different modes are uncorrelated (as a consequence of statistical homogeneity).

Models tend to focus on the power spectrum, so it is common for observations to do the same ...

Lecture outline

- Introduction to statistics
  - Correlation function
  - Power spectrum
- Information from power spectrum shape
  - Matter density
  - Baryon Acoustic Oscillations
  - Neutrino mass
  - Inflation perturbation power spectrum
- Information from geometry
  - Galaxy clustering as a standard ruler
  - BAO and the Alcock-Paczynski effect
  - non-linear power spectrum shifts
- Information from structure growth
  - linear growth
  - peculiar velocities
  - redshift-space distortions

Why is there structure?

Inflation (a period of rapid growth of the early Universe driven by a scalar field) was postulated to solve some serious problems with "standard" cosmology:

• why do causally disconnected regions appear to have the same properties? – they were connected in the past

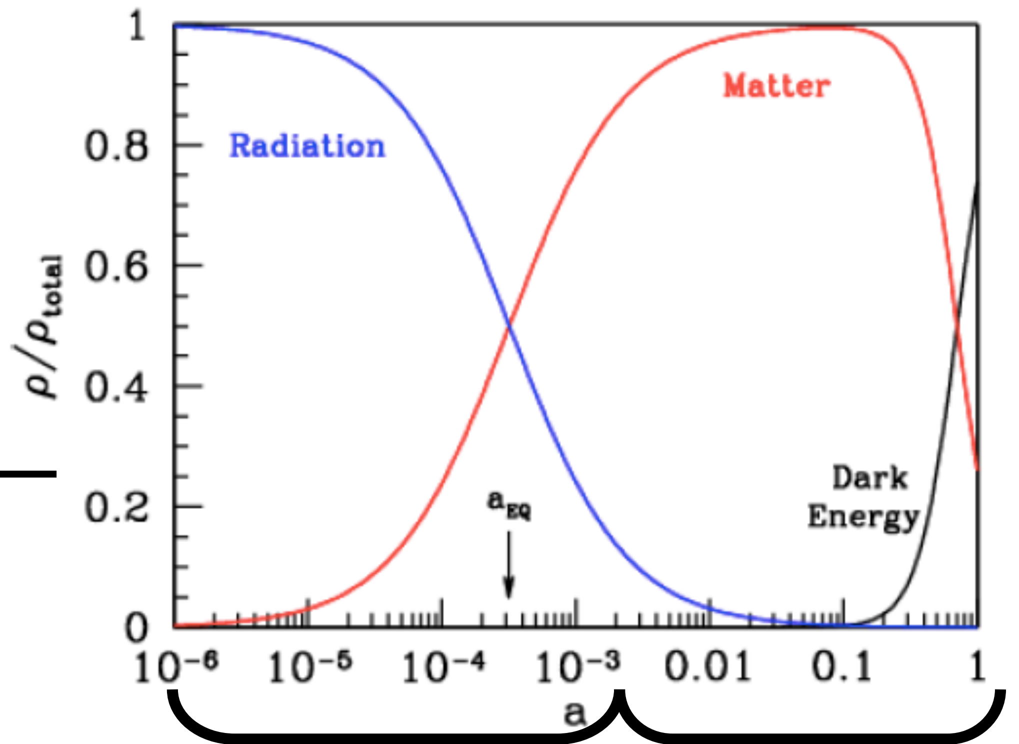
• why is the energy density of the Universe close to critical density? – driven there by inflation

•what are the seeds of present-day structure? - Quantum fluctuations in the matter density are increased to significant levels

$$P(k) = k^n$$

$$(n \approx 1)$$

Evolution of the power spectrum after inflation



Inflation

Transfer function

Matter/Dark energy domination

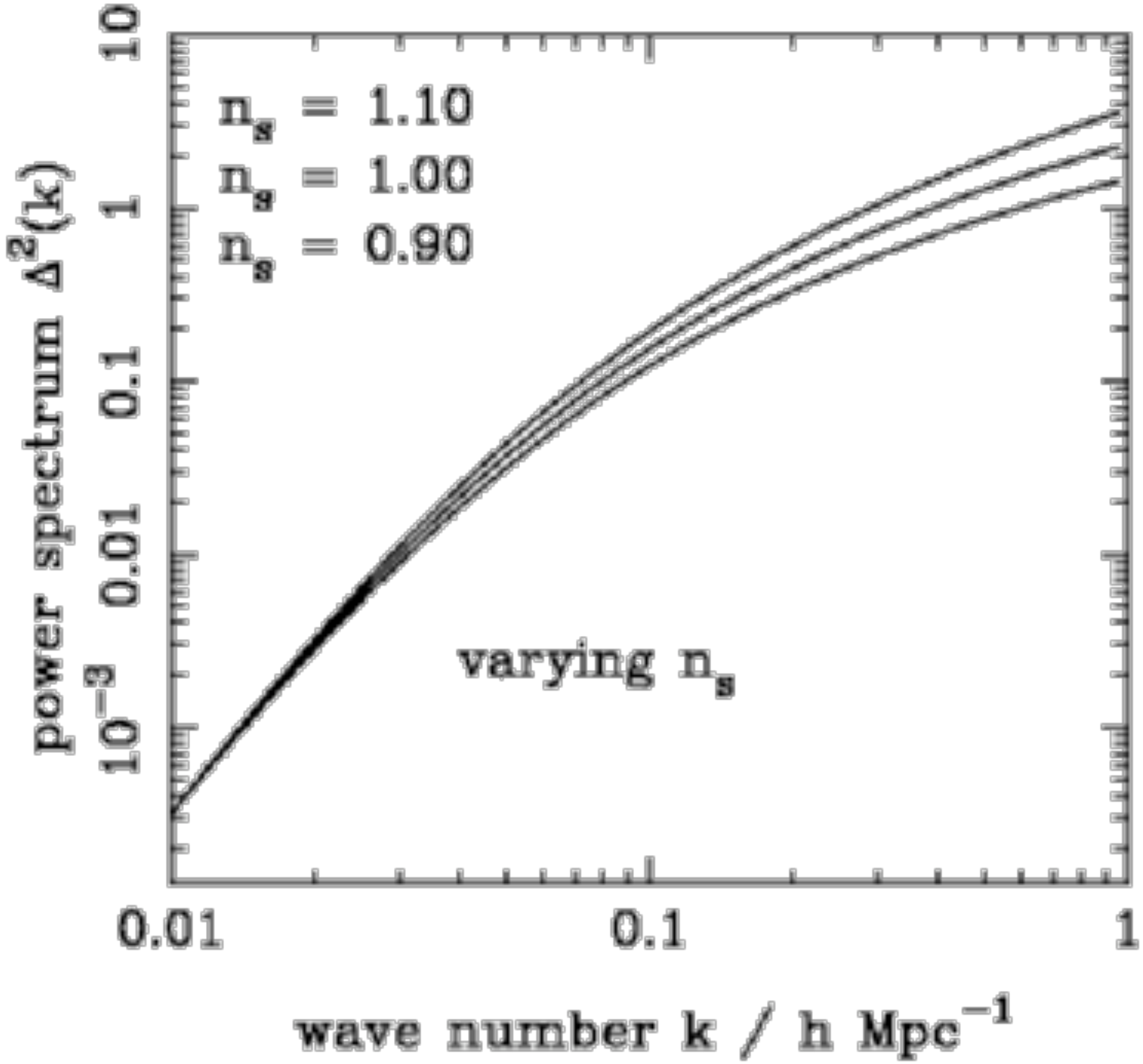
Non-linear

linear growth independent of scale

$$P_{\rm lin}(k) = k^n T^2(k) D^2(a)$$

(comoving coordinates)

Matter P(k) depends on inflation



$$P(k) = k^n$$

$$(n \approx 1)$$

University of Portsmouth

Jeans length

The transfer function depends on the composition of the matter (CDM, baryons, neutrinos, etc.)

An important scale is the Jeans Length which is the scale of fluctuation where pressure support equals gravitational collapse,

$$\lambda_J = \frac{c_s}{\sqrt{G\rho}}$$

where c_s is the sound speed of the material, and ρ is its density.

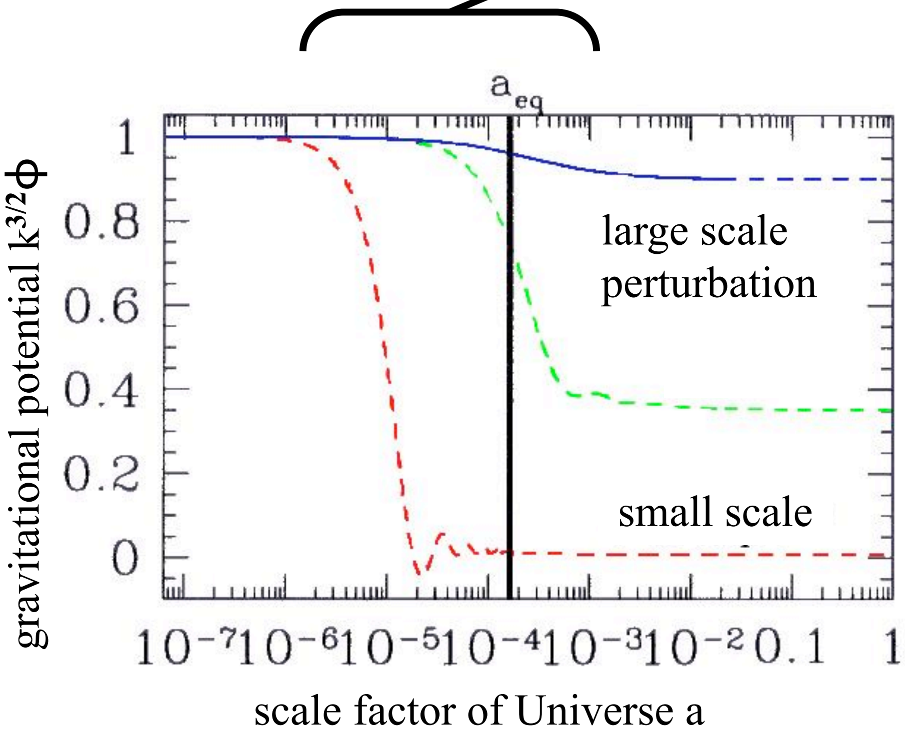
“F=ma” for perturbation growth

$$\ddot{\delta} = (\text{gravity} - \text{pressure})\delta$$

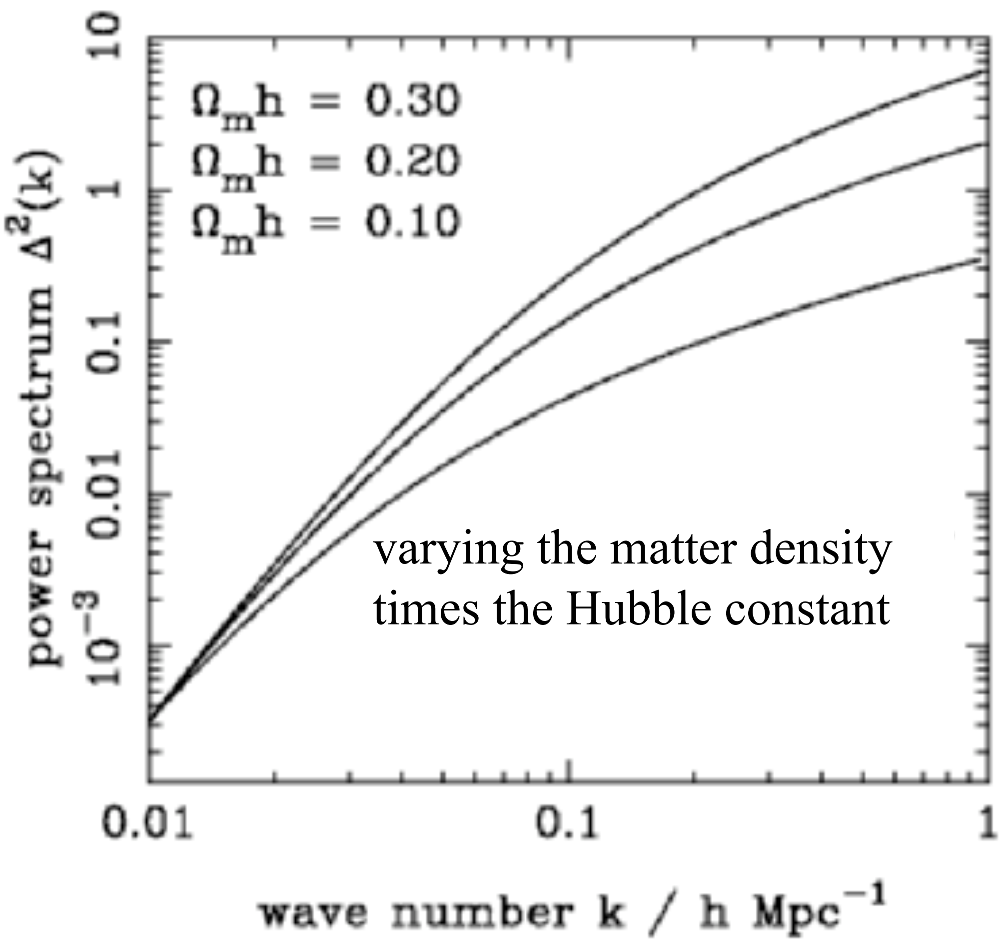
depends on Jeans scale

Transfer function evolution

in radiation dominated Universe, pressure support means that small perturbations cannot collapse (large Jeans scale). Jeans scale changes with time, leading to smooth turn-over of matter power spectrum. Cut-off dependent on matter density times the Hubble parameter $\Omega_m h$.



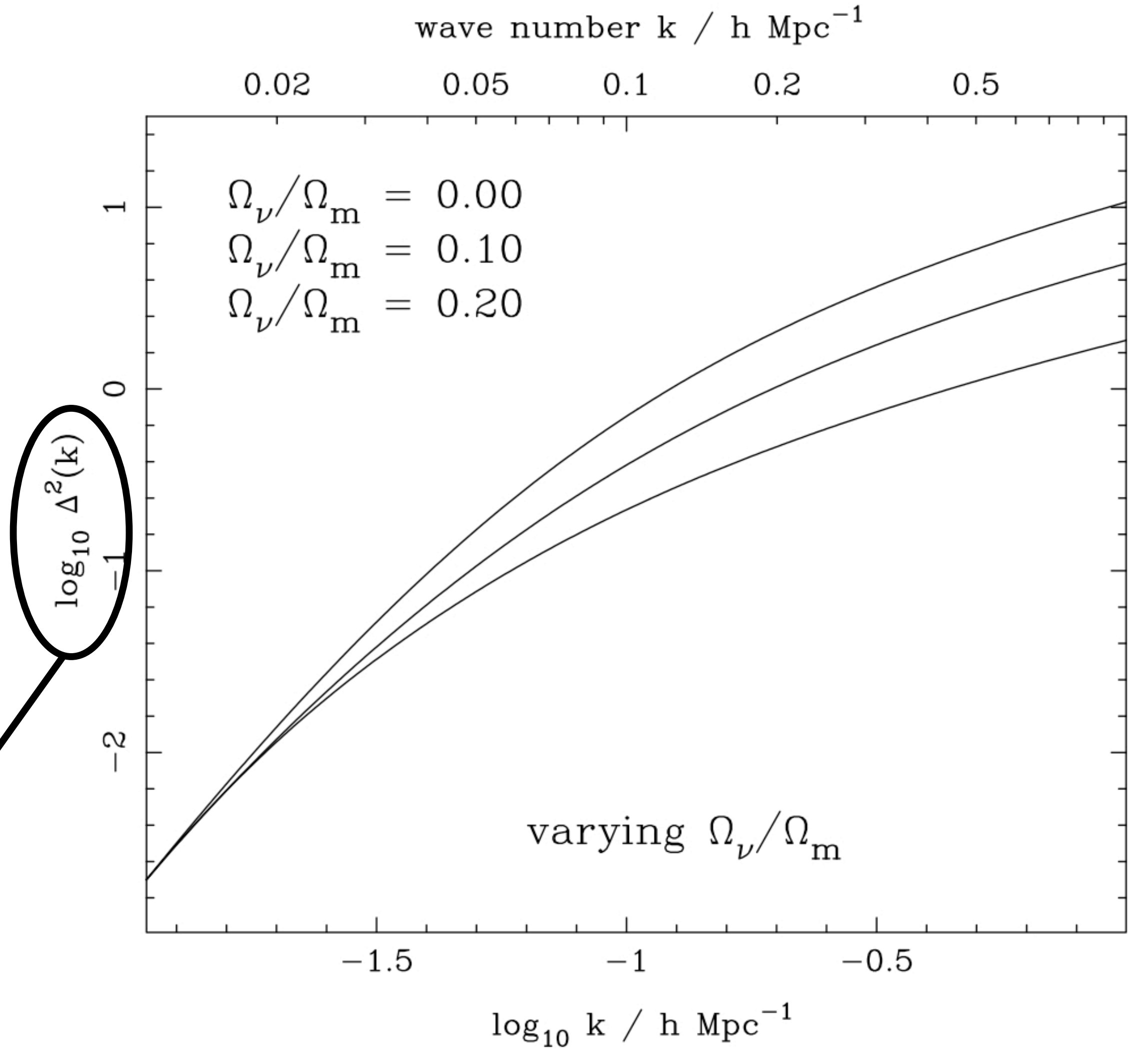
The power spectrum turn-over



Amplitude of effect depends on matter density – how long before matter-radiation equality

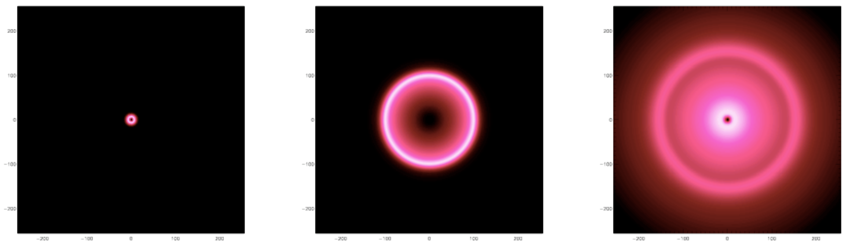
The effect of neutrinos

The existence of massive neutrinos can also introduce a suppression of T(k) on small scales relative to their Jeans length. Degenerate with the suppression caused by radiation epoch. Position depend on neutrino-mass equality scale.

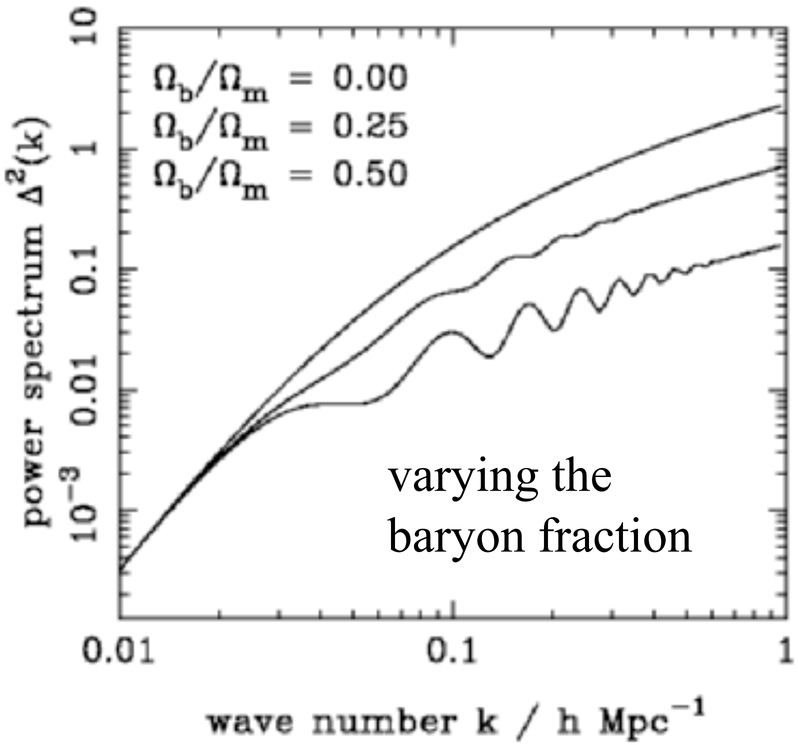


$\Delta^2(k) = k^3/2\pi^2 \, P(k)$

Baryon Acoustic Oscillations (BAO)



(images from Martin White)



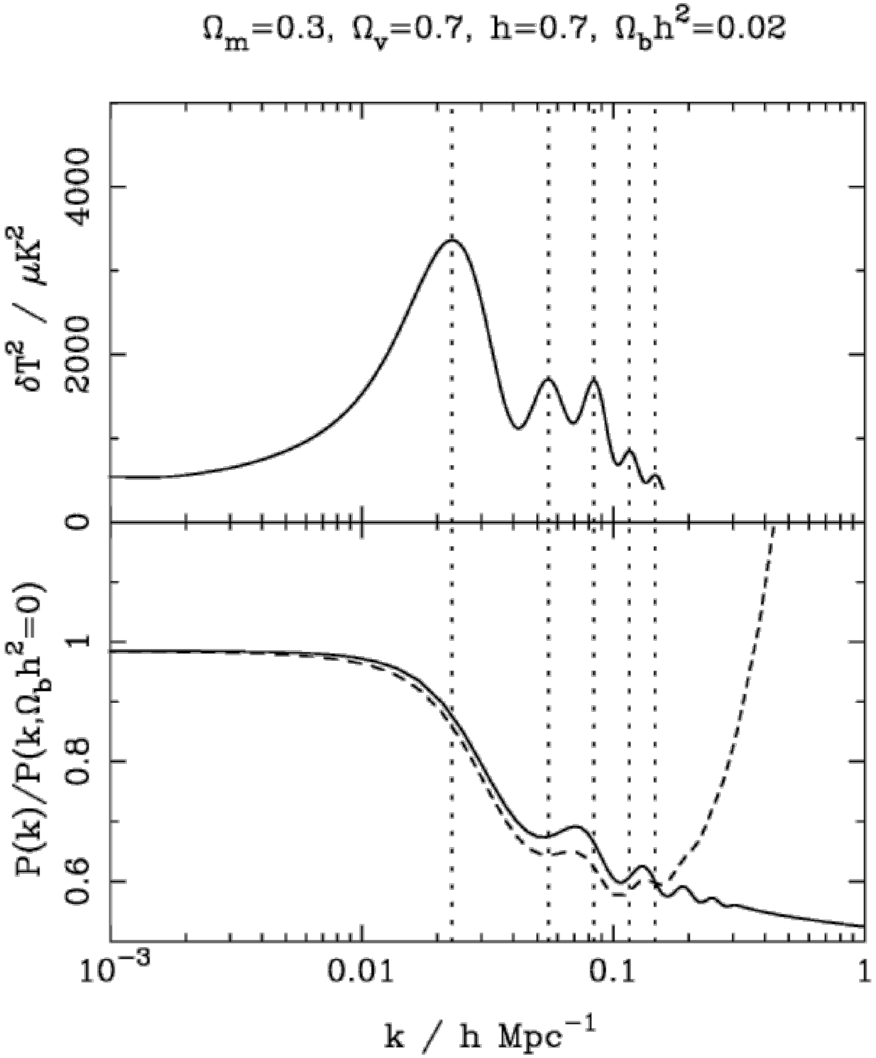
To first approximation, BAO wavelength is determined by the comoving sound horizon at recombination

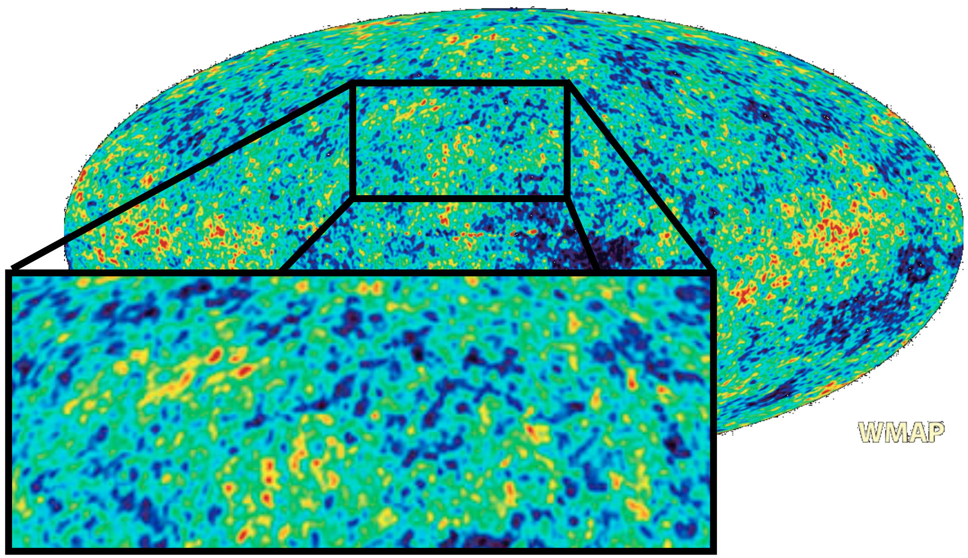
$$k_{\rm bao} = 2\pi/s$$

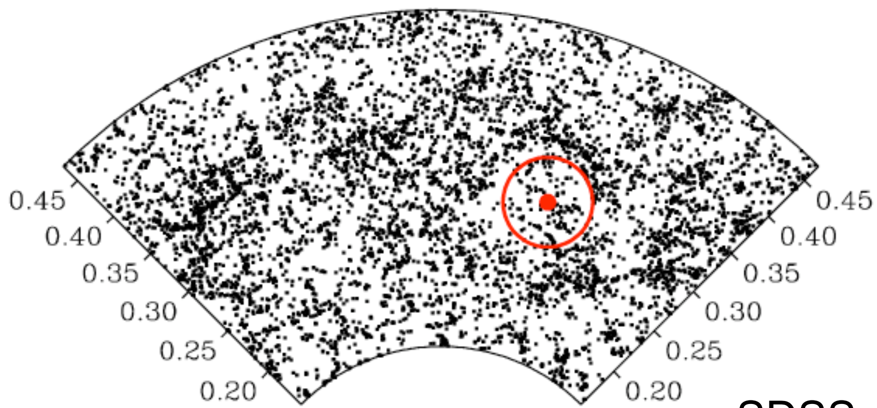
$$s = \frac{1}{H_0 \Omega_m^{1/2}} \int_0^{a_*} da \frac{c_s}{(a + a_{\rm eq})^{1/2}}$$

comoving sound horizon ~110h^{-1}Mpc, BAO wavelength 0.06hMpc^{-1}

Relationship between CMB and LSS power spectra



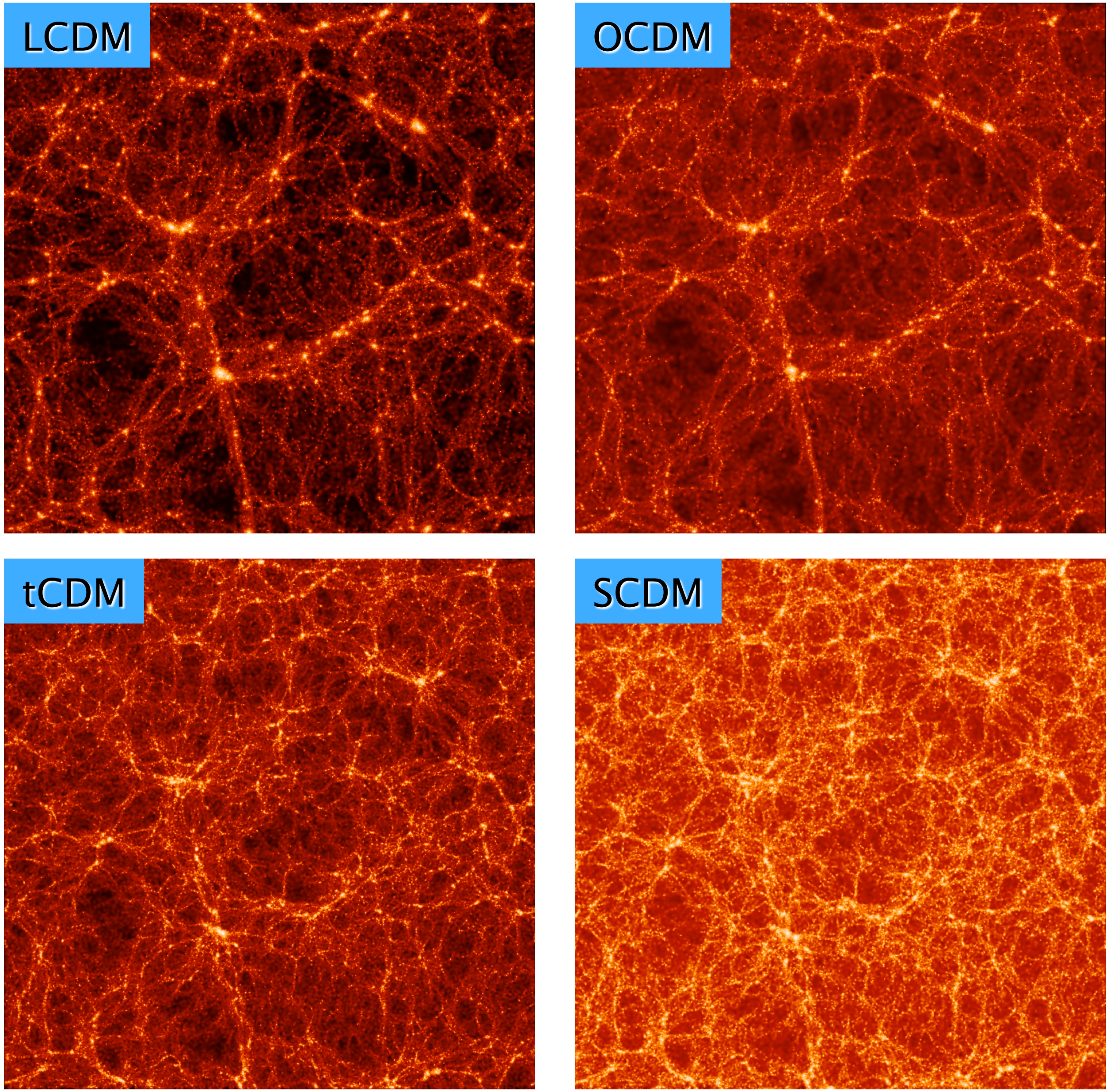




SDSS

University of Portsmouth

The shape of the power spectrum



credit: VIRGO consortium

University of Portsmouth

Lecture outline

- Introduction to statistics
  - Correlation function
  - Power spectrum
- Information from power spectrum shape
  - Matter density
  - Baryon Acoustic Oscillations
  - Neutrino mass
  - Inflation perturbation power spectrum
- Information from geometry
  - Galaxy clustering as a standard ruler
  - BAO and the Alcock-Paczynski effect
  - non-linear power spectrum shifts
- Information from structure growth
  - linear growth
  - peculiar velocities
  - redshift-space distortions

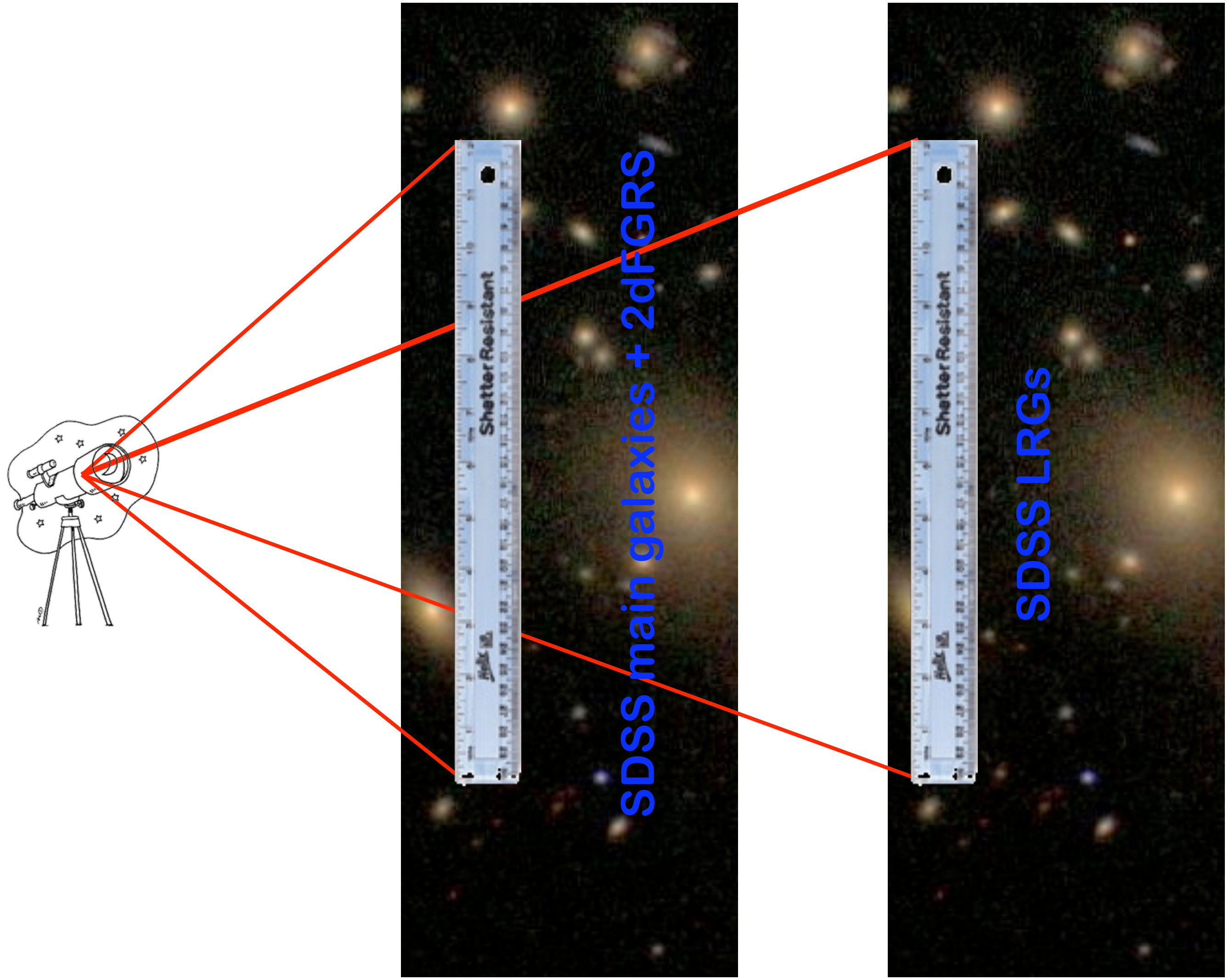
The evolution of the scale factor

If we observed the comoving power spectrum directly, we would not constrain evolution (except through linear growth – see later)

However, we measure galaxy redshifts and angles and infer distances

$$d_{\rm comov}(a) = \int_{t(a)}^{t_0} \frac{c\,dt'}{a(t')} = \int_a^1 \frac{c\,da'}{a'^2 H(a')}$$

The power spectrum as a standard ruler

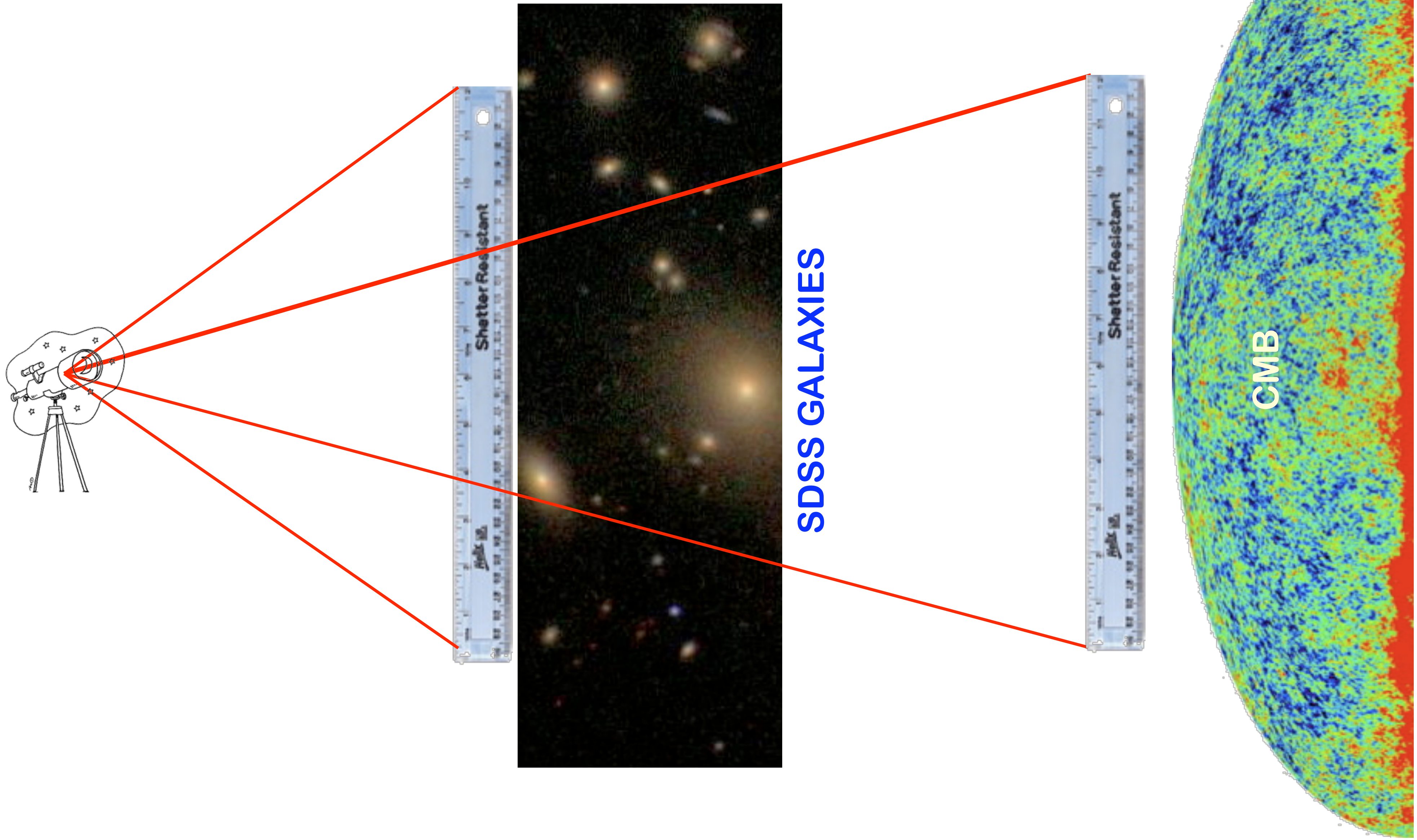


z=0.2

z=0.35

CREDIT: WMAP & SDSS websites

The power spectrum as a standard ruler



CREDIT: WMAP & SDSS websites

BAO as a standard ruler

Changes in cosmological model alter measured BAO scale (Δd_{comov}) by:

Radial direction $$\frac{c}{H(z)}\Delta z$$

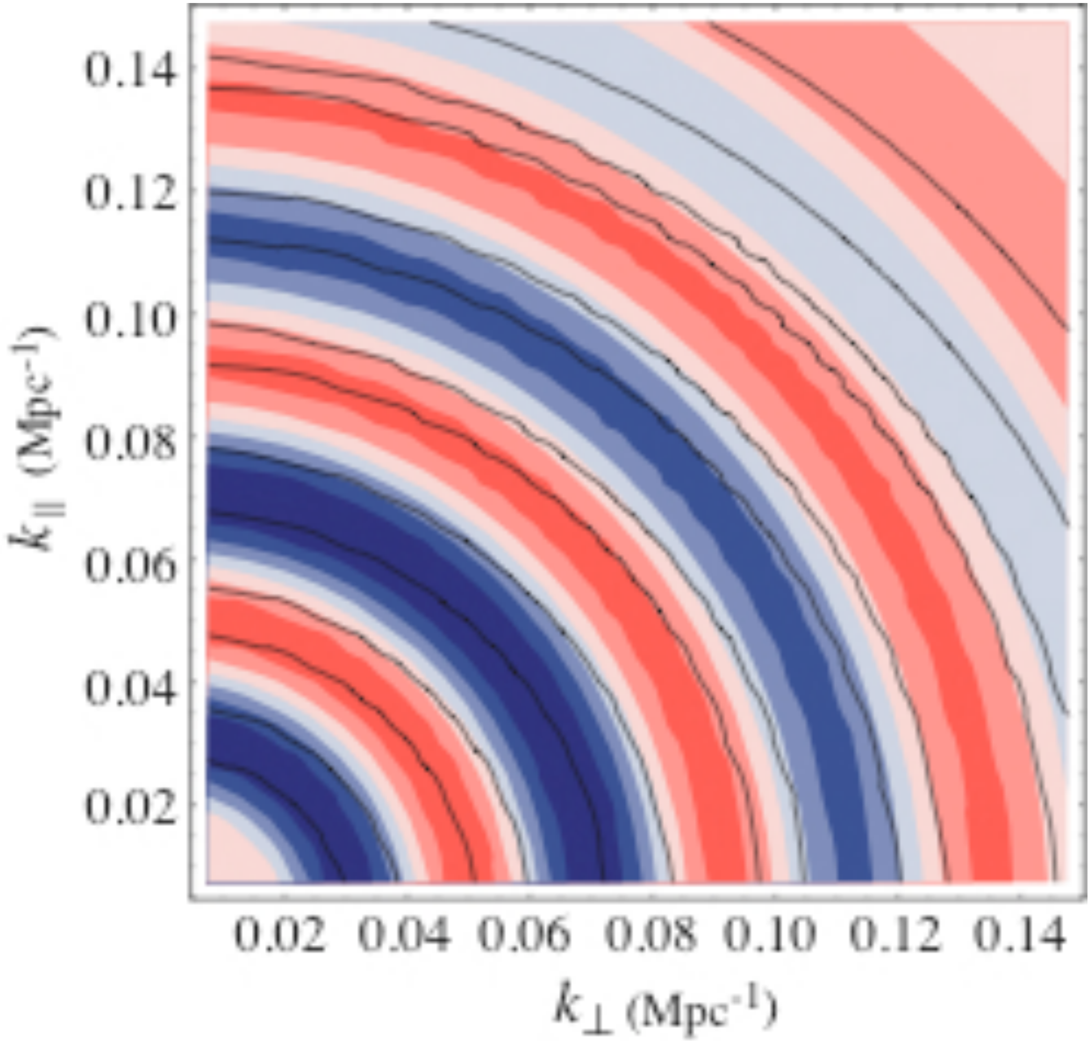
(evolution of Universe)

Angular direction $$(1+z)D_A\Delta\theta$$

(line of sight)

This is just the Alcock-Paczinski effect for small-scale perturbations

Gives rise to the "rings of power"



Hu & Haiman 2003, astro-ph/0306053

BAO as a standard ruler

Changes in cosmological model alter measured BAO scale (Δd_{comov}) by:

Radial direction $$\frac{c}{H(z)}\Delta z$$

(evolution of Universe)

Angular direction

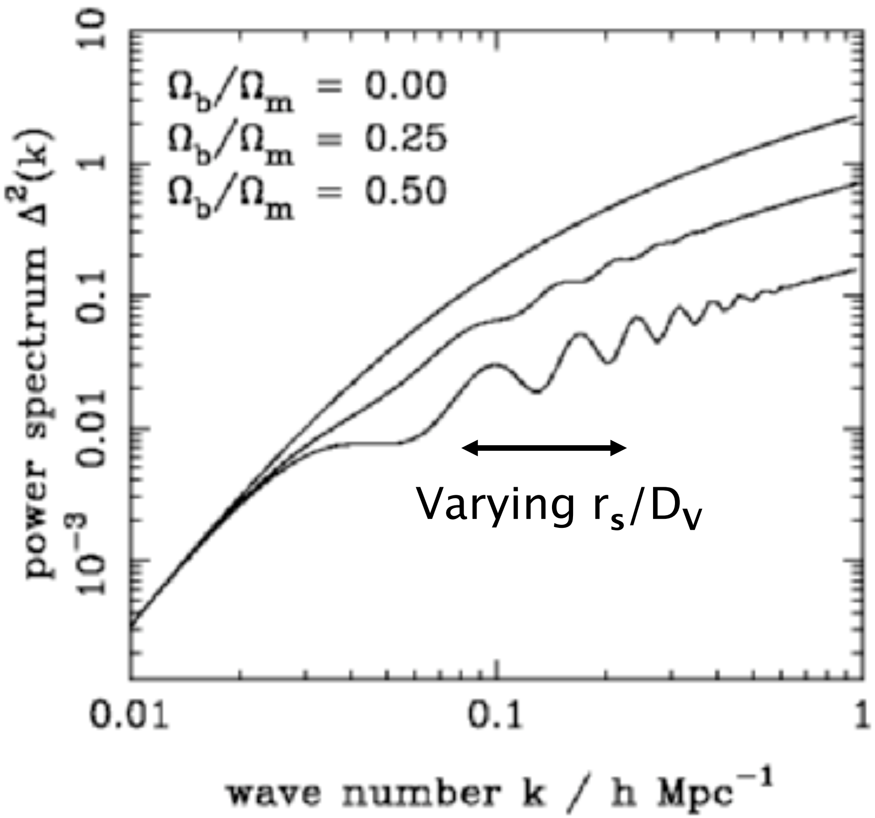
$$(1+z)D_A\Delta\theta$$

(line of sight)

If we are considering radial and angular directions using randomly orientated galaxy pairs, we constrain (to 1st order)

$$D_V = \left[(1+z)^2 D_A^2(z)\frac{cz}{H(z)}\right]^{1/3}$$

BAO position (in a redshift slice) therefore constrains some multiple of $$\frac{r_s}{D_V}$$



Why BAO are a good ruler

Linear BAO

$$B_{\rm lin} = \frac{P(k)_{\rm lin}}{\bar{P}(k)_{\rm lin}}$$

Observed power

$$B_{\rm obs} = \frac{P(k)_{\rm obs}}{\bar{P}(k)_{\rm obs}} = g(k)B_{\rm lin} + [1 - g(k)]$$

Observed BAO

$$P(k)_{\rm obs} = b^2(k)P(k)_{\rm lin} + P(k)_{\rm extra}$$

(no change in position)

Damping factor

$$g(k) = \frac{b^2(k)\bar{P}(k)_{\rm lin}}{\bar{P}(k)_{\rm obs}}$$

Need sharp features in P(k) or correlations to change BAO position

Eisenstein, Seo & White 2006, astro-ph/0604361
Percival et al. 2007, astro-ph/0705.3323

University of Portsmouth

Do BAO evolve?

Can make a perturbative treatment of (CDM+baryon) fluid system

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots$$

revise this term

$$P_{\rm lin}(k) = k^n T^2(k) D^2(a)$$

(e.g., Suto & Sasaki 1991)

New approach to 2nd order evolution

Based on field-theoretical approach,

Standard PT calculation can be improved by re-summing an infinite class of perturbative corrections at all orders.

"**Renormalized Perturbation Theory (RPT)**"

Crocce & Scoccimarro (2006ab,2007)

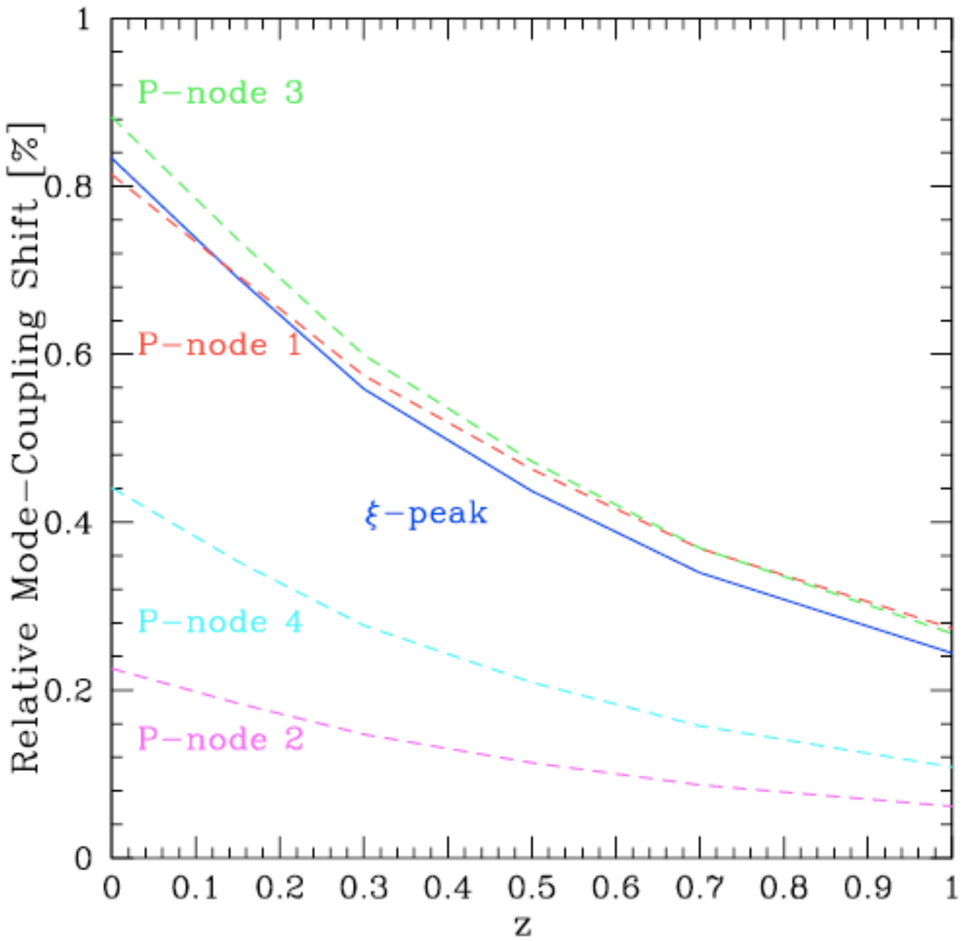
Related works: McDonald, Matarrese & Pietroni, Valageas, Matsubara ('07)

Going to 2nd order …

At second order we get mode mixing, which causes shifts in the power spectrum BAO peaks

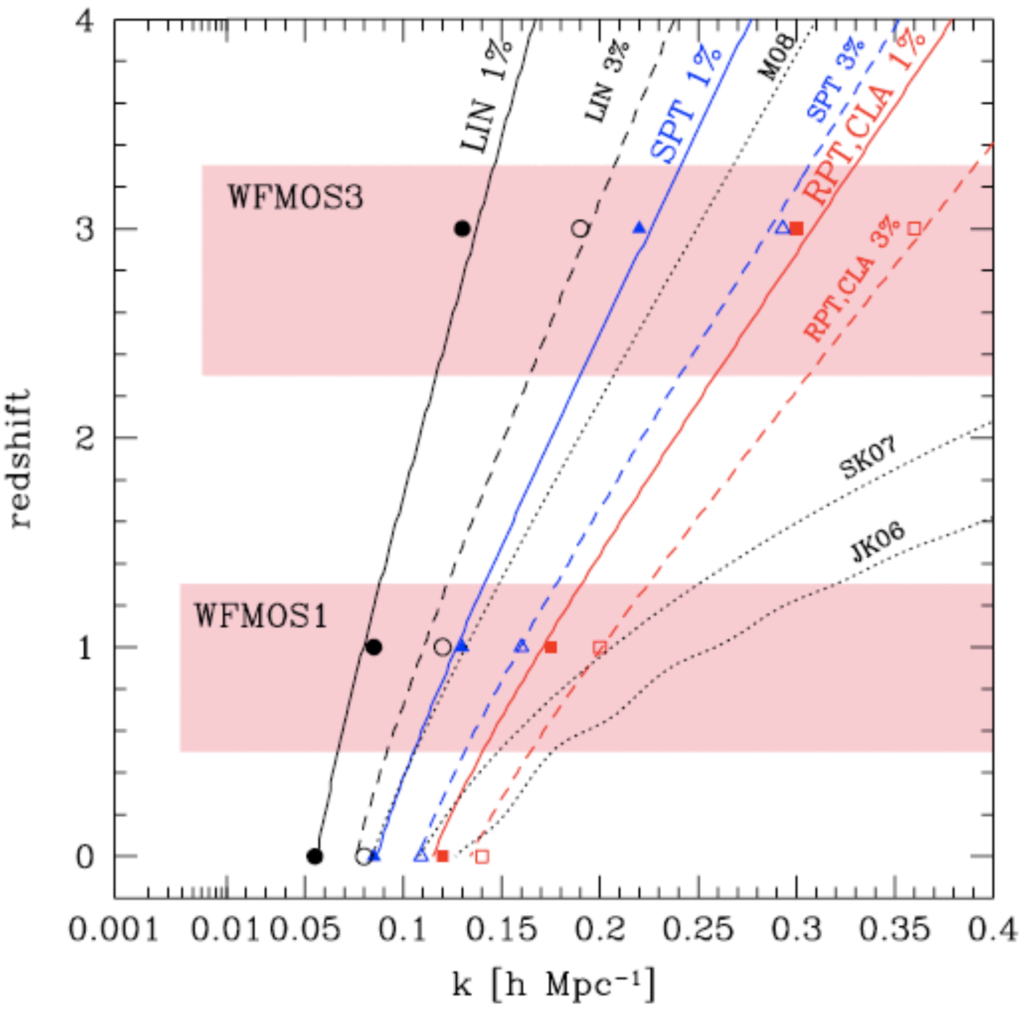
Shifts are <1%, and can be calculated

Not important for current data, but need to be included for future analyses



Crocce & Scoccimarro 2007; astro-ph/0704.2783

Using the full power spectrum as a ruler



Nishimichi et al. 2008; astro-ph/0810.0813

Lecture outline

- Introduction to statistics
  - Correlation function
  - Power spectrum
- Information from power spectrum shape
  - Matter density
  - Baryon Acoustic Oscillations
  - Neutrino mass
  - Inflation perturbation power spectrum
- Information from geometry
  - Galaxy clustering as a standard ruler
  - BAO and the Alcock-Paczynski effect
  - non-linear power spectrum shifts
- Information from structure growth
  - linear growth
  - peculiar velocities
  - redshift-space distortions

University of Portsmouth

Spherical perturbation leading to linear growth

cosmology equation

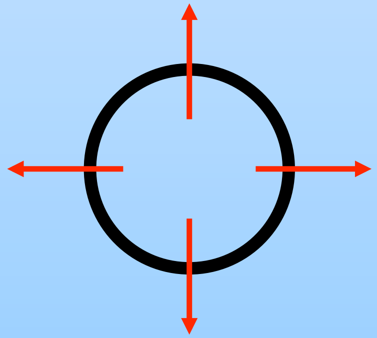
$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{H_0^2}{2}\left[\Omega_M a^{-3} + [1+3w(a)]\Omega_X a^{f(a)}\right]$$

$$\frac{1}{a_p}\frac{d^2a_p}{dt^2} = -\frac{H_0^2}{2}\left[\Omega_M a_p^{-3} + [1+3w(a)]\Omega_X a^{f(a)}\right]$$

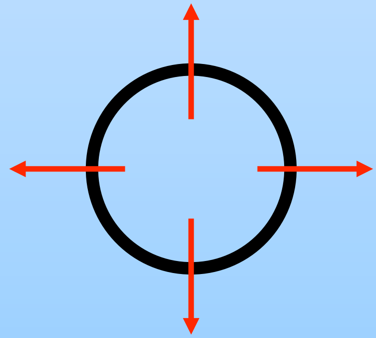
homogeneous dark energy means that this term depends on scale factor of background

Consider homogeneous spherical perturbation – evolution is "same" as "mini-universe"

"perfectly" clustering dark energy – replace a with a_p



Overdense perturbation
Radius a_p



Background
Radius a

Spherical perturbation leading to linear growth

cosmology equation

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{H_0^2}{2}\left[\Omega_M a^{-3} + [1+3w(a)]\Omega_X a^{f(a)}\right]$$

$$\frac{1}{a_p}\frac{d^2a_p}{dt^2} = -\frac{H_0^2}{2}\left[\Omega_M a_p^{-3} + [1+3w(a)]\Omega_X a^{f(a)}\right]$$

definition of δ

$$\delta = \frac{a^3}{a_p^3} - 1$$

to first order in perturbation radius (linear approximation)

$$a_p = a(1-\delta/3)$$

gives

$$\frac{d^2\delta}{dt^2} + \frac{2}{a}\frac{da}{dt}\frac{d\delta}{dt} - \frac{3}{2}H_0^2\Omega_m a^{-3}\delta = 0$$

can also be derived using the Jeans equation

only has this form if the dark energy does not cluster – derivation of equation relies on cancellation in dark energy terms in perturbation and background

Linear growth: EdS model

$$\frac{d^2\delta}{dt^2} + \frac{2}{a}\frac{da}{dt}\frac{d\delta}{dt} - \frac{3}{2}H_0^2\Omega_m a^{-3}\delta = 0$$

For flat matter dominated model,
this has solution $\delta \propto a$

Remember that the gravitational potential and the overdensity are related by Poisson's equation

$$\nabla^2\Phi = 4\pi G\bar{\rho}\delta \qquad -\frac{k^2}{a^2}\tilde{\Phi} \propto \frac{\tilde{\delta}}{a^3}$$

Then the potential is constant: there is a delicate balance between structure growth and expansion

Not true if dark energy or neutrinos

Linear growth: general models

$$\frac{d^2\delta}{dt^2} + \frac{2}{a}\frac{da}{dt}\frac{d\delta}{dt} - \frac{3}{2}H_0^2\Omega_m a^{-3}\delta = 0$$

For general models, denote linear growth parameter (solution to this differential equation)

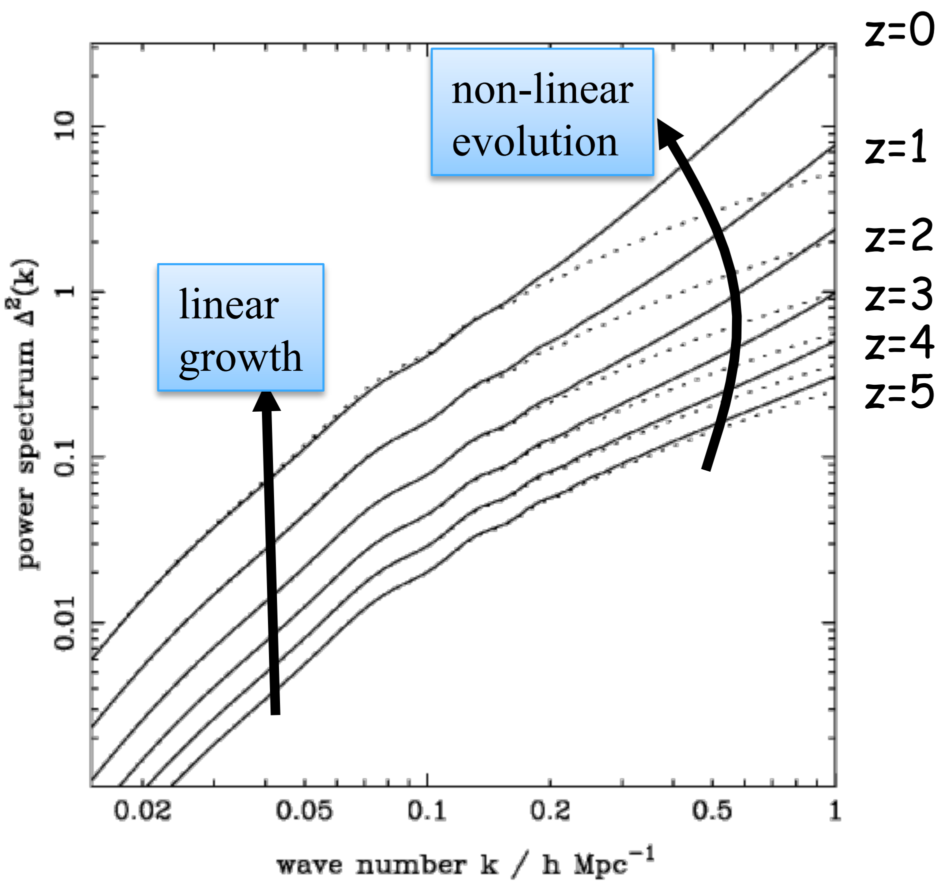
$$\delta = D(t)$$

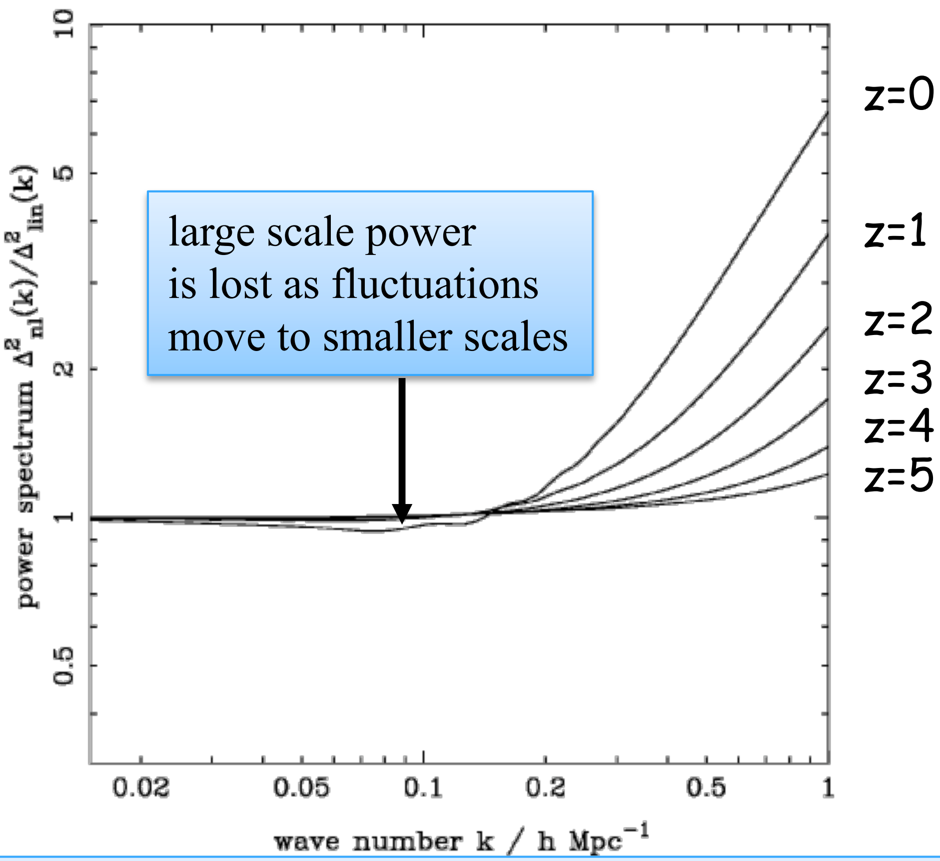
For lambda models, can use the approximation of Carroll, Press & Turner (1992)

$$D(a) \simeq \frac{5\Omega_M(a)a}{2}\left[\Omega_M(a)^{4/7} - \Omega_\Lambda(a) + \left(1 + \frac{\Omega_M(a)}{2}\right)\left(1 + \frac{\Omega_\Lambda(a)}{70}\right)\right]^{-1}$$

For general dark energy models, need to solve the differential equation numerically

Linear vs Non-linear behaviour





Cannot easily measure growth directly from galaxy surveys as degenerate with galaxy bias

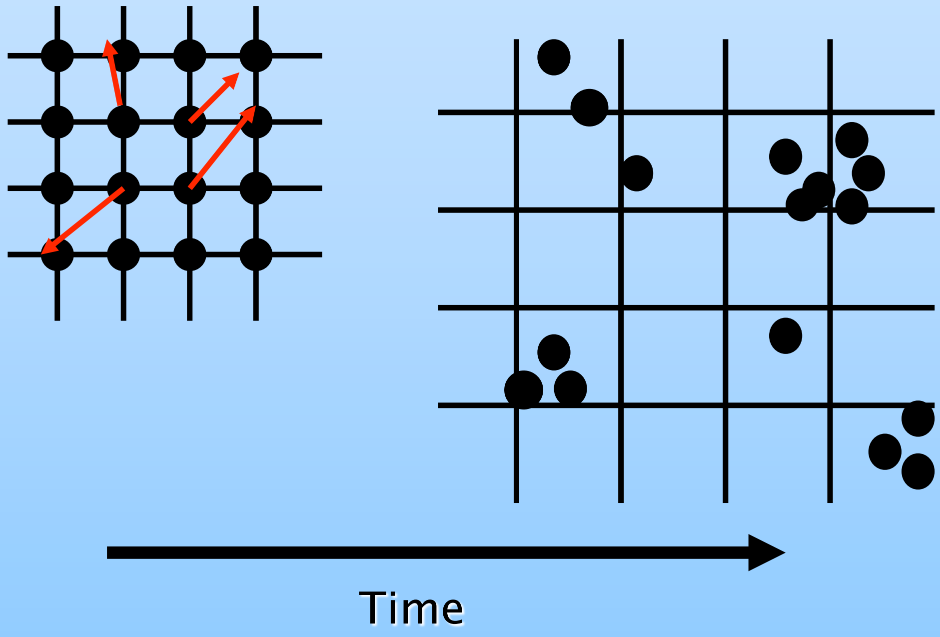
P(k) calculated from Smith et al. 2003, MNRAS, 341,1311 fitting formulae

Peculiar velocities

All of structure growth happens because of peculiar velocities

Initially distribution of matter is approximately homogeneous (δ is small)

Final distribution is clustered



Linear peculiar velocities

Consider galaxy with true spatial position ***x(t)=a(t)r(t)***, then differentiating twice and splitting the acceleration $\boldsymbol{d^2x/dt^2=g_0+g}$ into expansion ($\boldsymbol{g_0}$)and peculiar (***g***) components, gives that the peculiar velocity ***u(t)*** defined by ***a(t)u(t)=dx/dt*** satisfies

$$\dot{\mathbf{u}} + \frac{2\dot{a}}{a}\mathbf{u} = -\mathbf{g}$$

The peculiar gravitational acceleration is $\mathbf{g} = \nabla\Phi/a$

So, for linearly evolving potential, ***u*** and ***g*** are in same direction

In conformal units, the continuity and Poisson equations are

$$\frac{\partial\delta}{\partial\tau} + \nabla\cdot[(1+\delta)\mathbf{u}] \simeq \frac{\partial\delta}{\partial\tau} + \nabla\cdot\mathbf{u} = 0 \qquad \nabla\cdot\mathbf{g} = \frac{3}{2}\Omega H^2 a^2 \delta$$

Look for solutions of the continuity and Poisson equations of the form ***u****=F(a)****g***

Linear peculiar velocities

Solution is given by

$$\mathbf{u} = \frac{2f(\Omega)}{3H\Omega}\mathbf{g}$$

where

$$f(\Omega) \equiv \frac{a\dot{D}}{\dot{a}D} \simeq \Omega^{0.6}$$

Zeld'ovich approximation: mass simply propagates along straight lines given by these vectors

The continuity equation can be rewritten

$$\delta(\mathbf{x}) = -\frac{\nabla \cdot \mathbf{u}}{aHf}$$

So the power spectrum of each component of ***u*** is given by

$$P_V(k) = (aHf)^2 P(k) k^{-2}$$

k^{-1} factor shows that velocities come from larger-scale perturbations than density field

Peculiar velocity observations

So peculiar velocities constrain ***f*** : can we measure these directly?

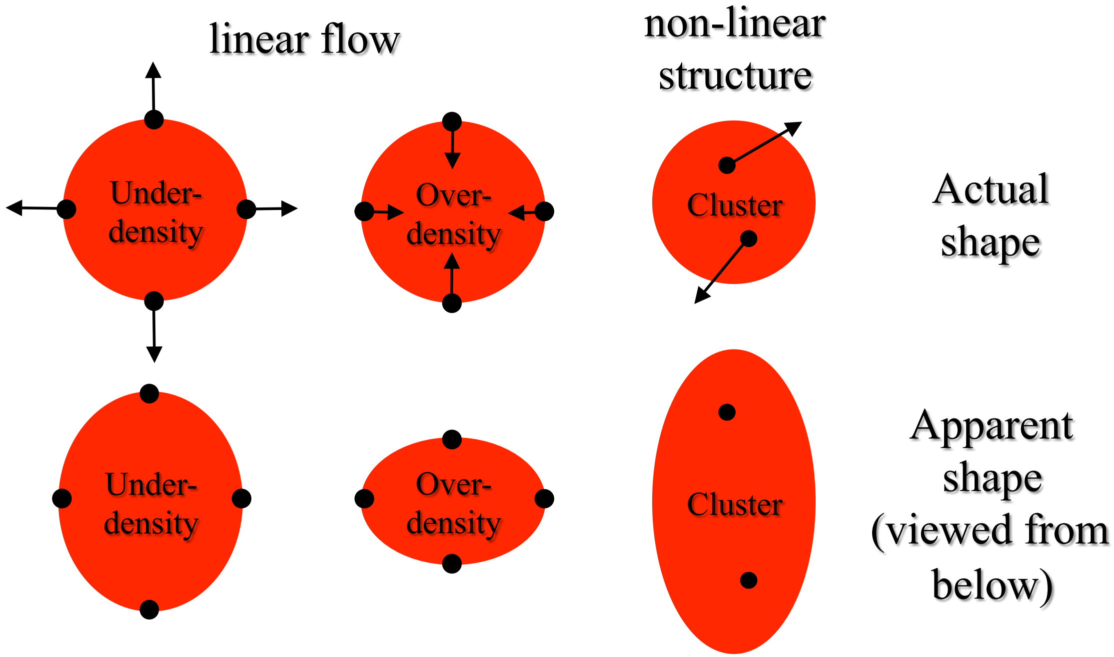
Obviously, can only hope to measure radial component of peculiar velocities

To do this, we need the redshift, and an independent measure of the distance (e.g. if galaxy lies on fundamental plane). Can then attempt to reconstruct the matter power spectrum

The 1/k term means that the velocity field probes large scales, but does directly test the matter field. However, current constraints are poor in comparison with those provided by other cosmological observations

redshift-space distortions

- Estimate distances from redshifts (i.e. velocities)
- Peculiar velocities (velocities in addition to Hubble flow) misinterpreted as distance shifts
- Coherent shifts can affect 2-pt functions



Motion of galaxies is independent of galaxy properties – galaxies act as test particles in flow of matter

what do linear z-space distortions measure?

linear scales,

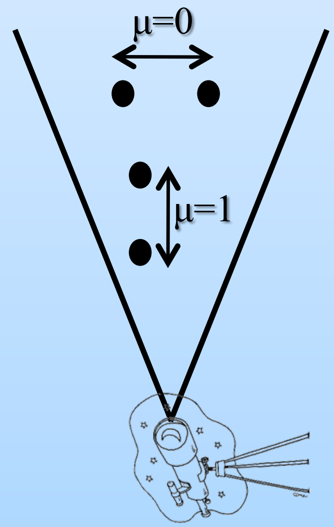
$$\begin{aligned} \delta_g^s(\mu) &= \delta_g + \mu^2\theta \\ P_g^s(\mu) &= \langle|\delta_g + \mu^2\theta|^2\rangle \\ &= P_{gg} + 2\mu^2 P_{g\theta} + \mu^4 P_{\theta\theta} \end{aligned}$$

Galaxy-galaxy power

Velocity-velocity power

Galaxy-velocity divergence cross power

$$\begin{aligned} \mu &= cos(\alpha) \\ \theta &= \nabla \cdot \mathbf{u} \end{aligned}$$



In linear regime,

$$\theta = -f\delta(\text{mass}), \quad f \equiv \frac{d\ln G}{d\ln a}$$

Linear growth rate

so amplitude of power spectrum constrains

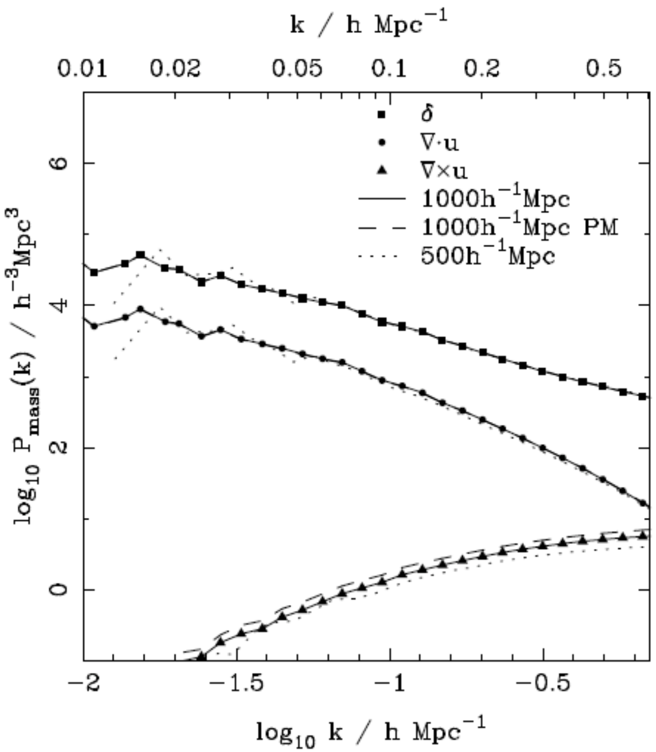
$$(\sigma_8^s)^2 = [b\sigma_8(\text{mass}) + \mu^2 f\sigma_8(\text{mass})]^2$$

Kaiser 1987, MNRAS, 227, 1

The break-down of the linear model

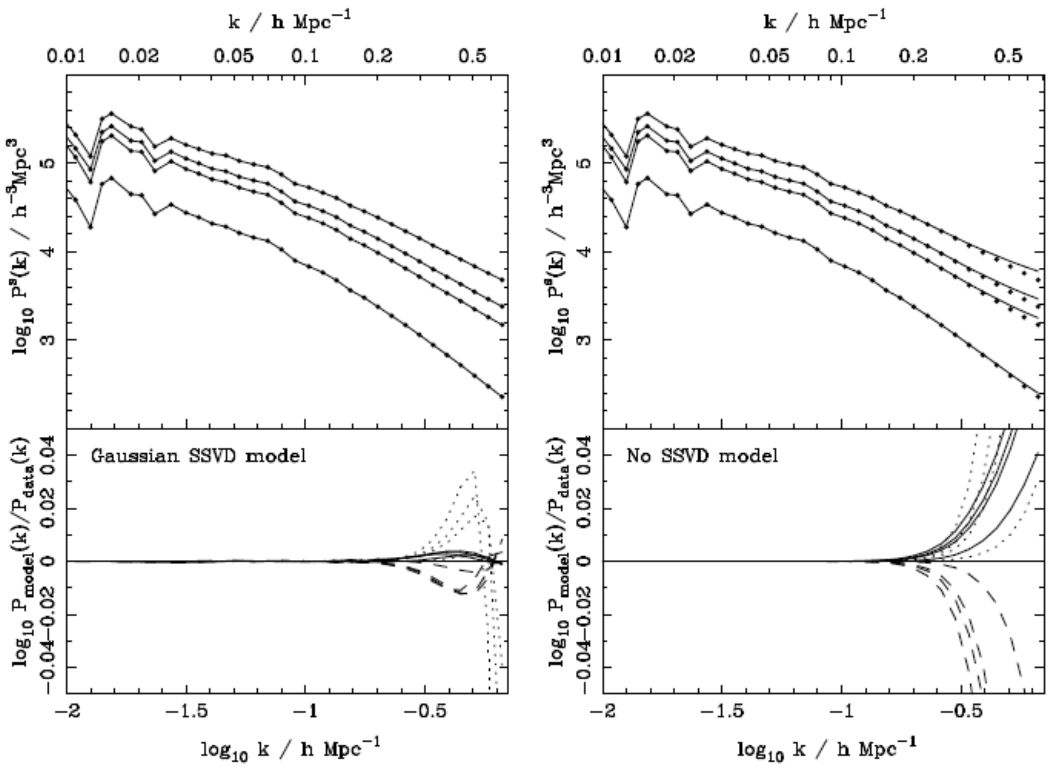
- Real-Redshift space mapping
  - Kaiser formula first order in δ and θ
  - on small scales, we need 2^{nd} and 3^{rd} order (δ, θ cross) terms
  - assumes irrotational velocity field
- Non-linear density field evolution
  - P_{gg} breaks from linear behaviour (small scale, late time)
- Non-linear velocity field evolution
  - $P_{\theta\theta}$ breaks from linear behaviour (small scale, late time)
  - Fingers-of-God
- Assumes local, deterministic density bias

The break-down of the linear model



relative amplitude of density and velocity fields in simulations

the breakdown of the Kaiser model



Percival, White, 2008, astro-ph/0808.0003

Further reading

- Dodelson, SLAC lecture notes. Available online at
  - http://www-conf.slac.stanford.edu/ssi/2007/lateReg/program.htm
- Dodelson, “Modern Cosmology”, Academic Press
- Peacock, “Cosmological Physics”, Cambridge University Press
- Liddle & Lyth, “Cosmological Inflation and Large-Scale Structure”, Cambridge University Press
- Coles & Lucchin, “Cosmology: the origin and evolution of cosmic structure”, Wiley
- Eisenstein et al. 2006, astro-ph/0604361 (configuration space description of perturbation evolution)
- Percival 2005, astro-ph/0508156 (linear growth in general dark energy models)
- Hamilton 1997, astro-ph/9708102, redshift-space distortions review

University of Portsmouth

Measuring Galaxy Clustering and Making Predictions

Will Percival
ICG, University of Portsmouth, UK

Lecture outline

- Measuring the over-density field
  - Angular & radial mask
  - Random catalogues
- Measuring 2-pt statistics
  - the power spectrum
  - the correlation function
  - redshift-space distortions
- Fisher matrices and predictions
  - background
  - future surveys
- Putting it all together
  - parameters choices
  - the MCMC technique

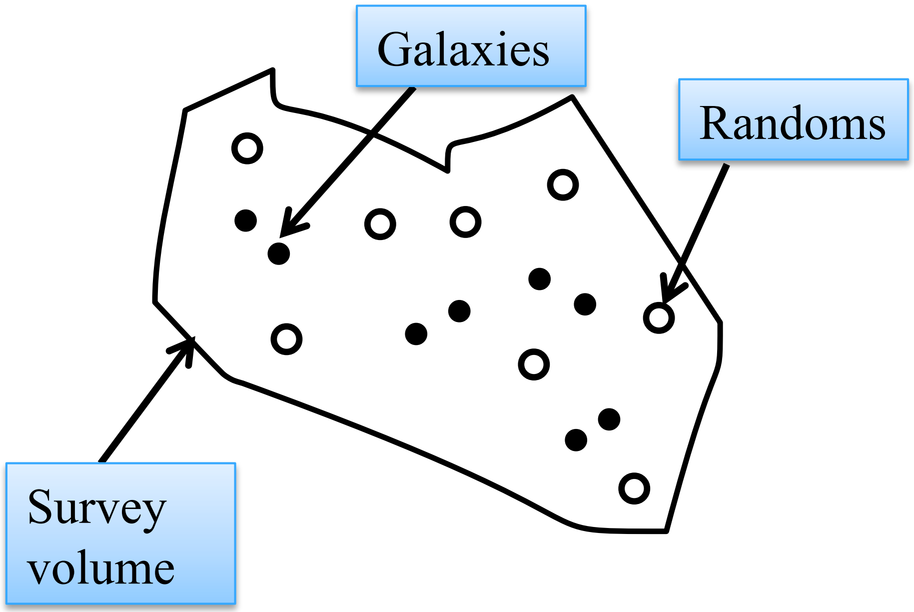
Lecture outline

- Measuring the over-density field
  - Angular & radial mask
  - Random catalogues
- Measuring 2-pt statistics
  - the power spectrum
  - the correlation function
  - redshift-space distortions
- Fisher matrices and predictions
  - background
  - future surveys
- Putting it all together
  - parameters choices
  - the MCMC technique

translating to an overdensity field

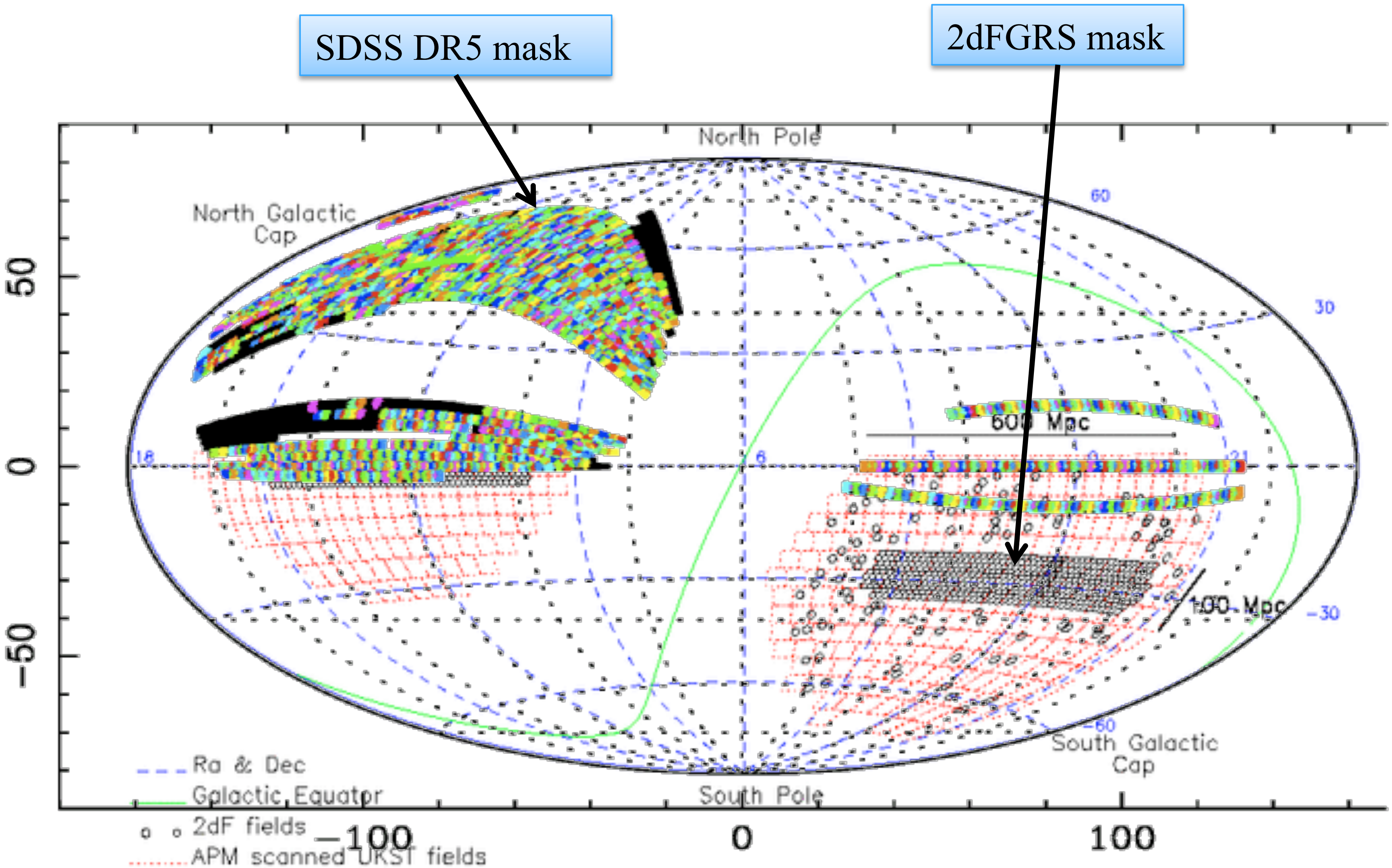
overdensity field

$$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

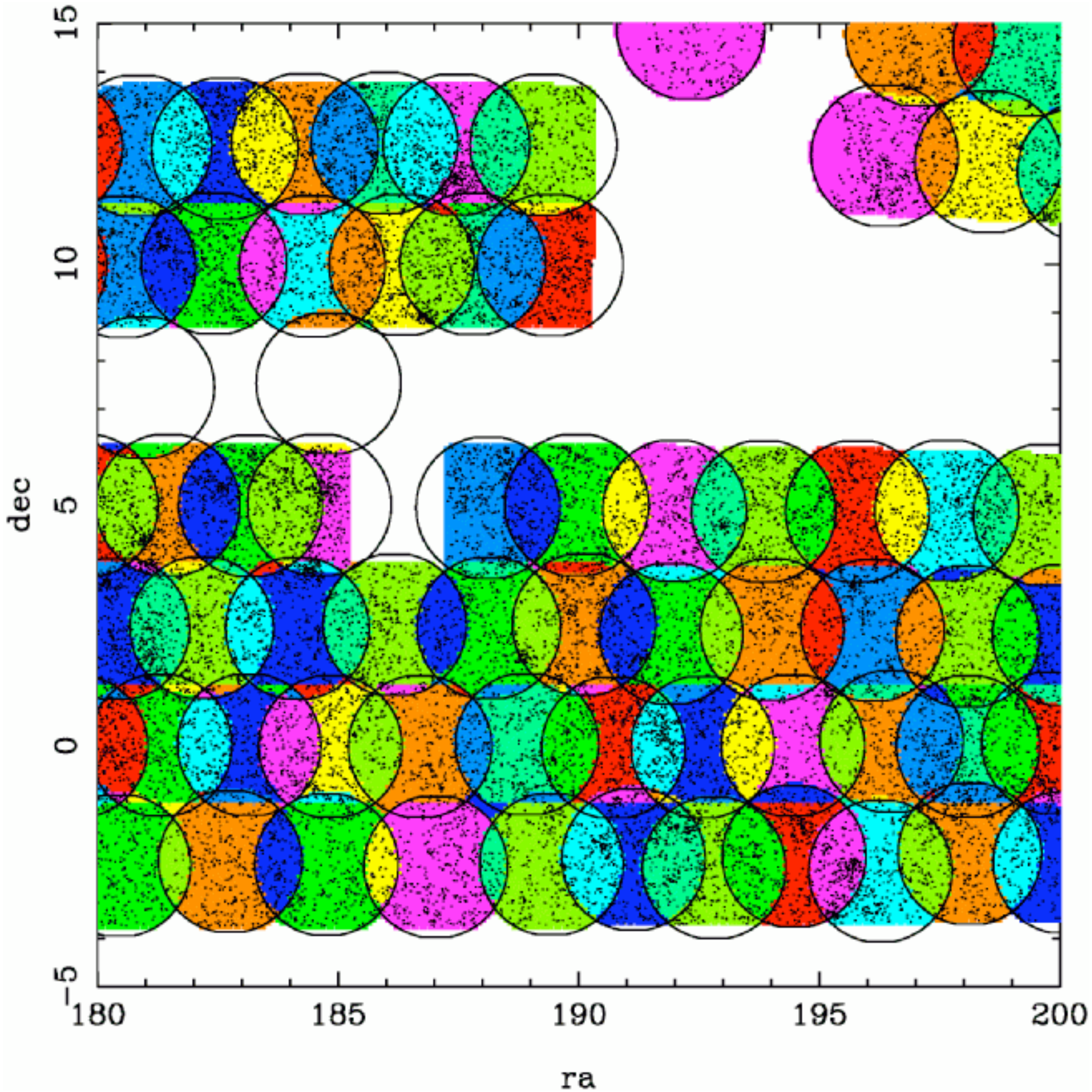


Create random catalogue with same spatial sampling as galaxies, but no clustering. α times as many objects. Use this to define the mean density

Modeling the angular galaxy mask



Modeling the angular galaxy mask

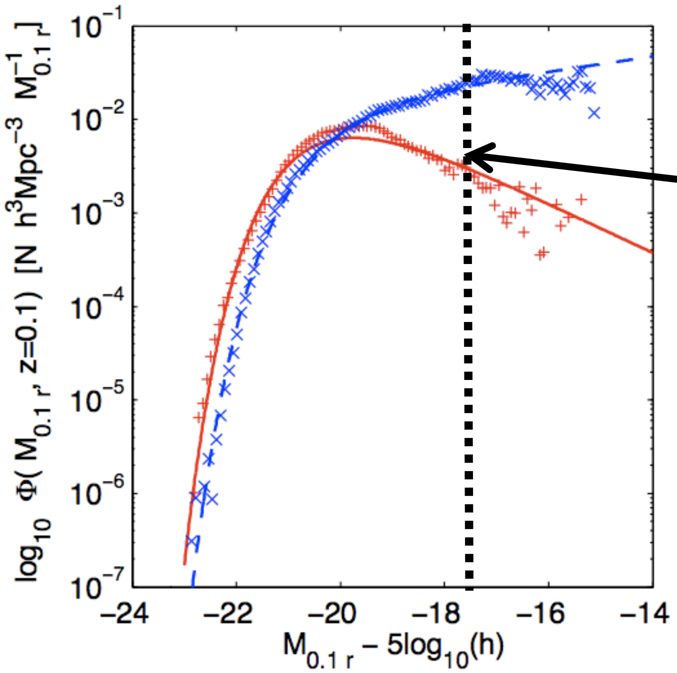


- completeness varies between plate overlap regions
- also need to consider region covered by parent catalogue
- both SDSS and 2dFGRS use an adaptive tiling strategy
- BOSS will not use an adaptive system, but a fixed set of pointings

Modeling the radial galaxy distribution

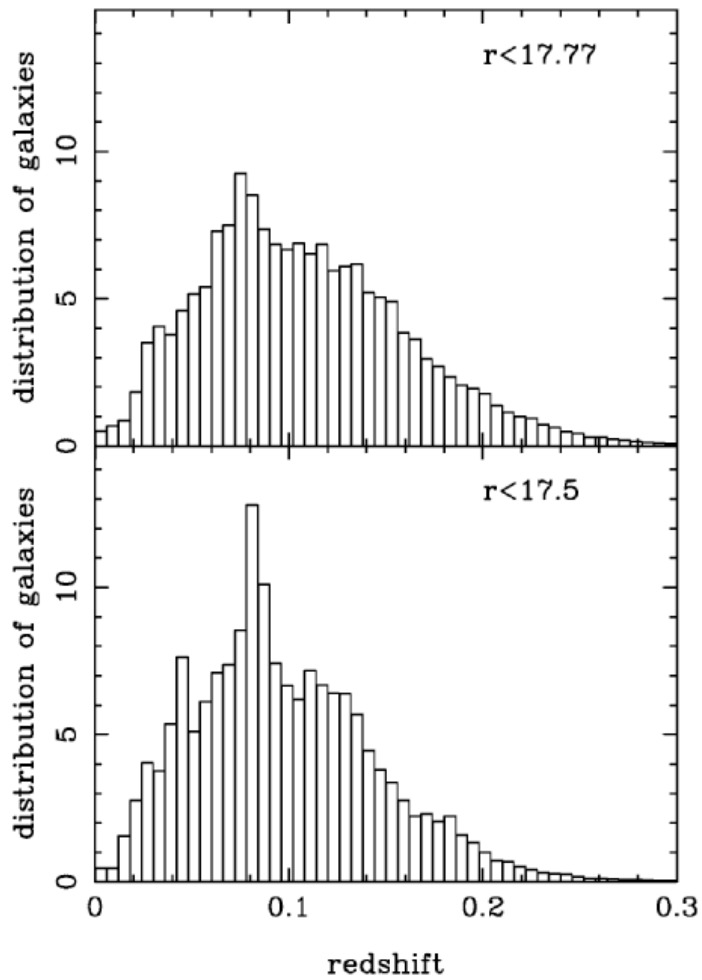
- for both the 2dFGRS and SDSS the magnitude limit changes with angular position, so the radial distribution of galaxies also changes
- Best approach – fit luminosity function (allowing for K+E corrections)

- possible to also just fit to redshift distribution in bins



each angular position will have different magnitude limit

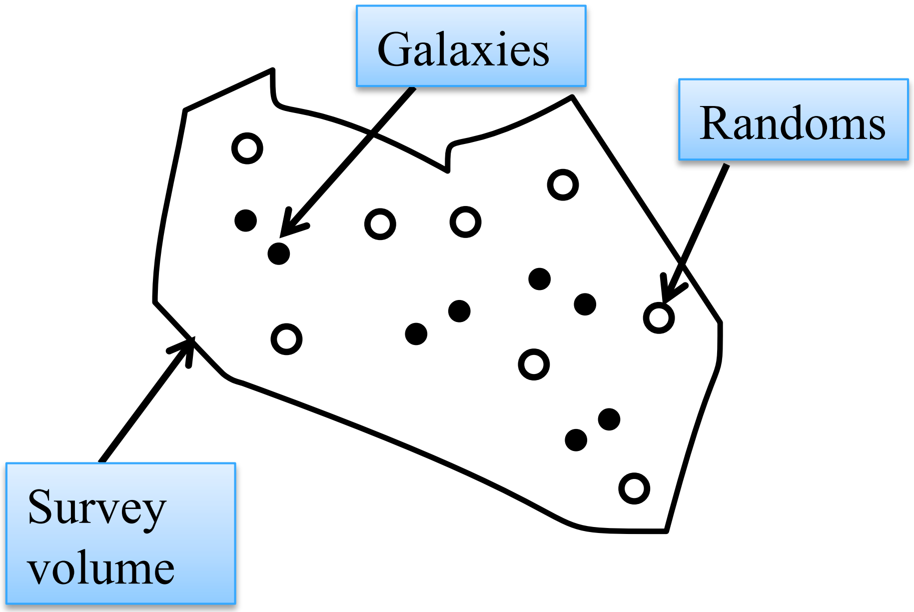
can then convert to redshift distribution



translating to an overdensity field

Can define an overdensity field

$$F(\mathbf{r}) = n_g(\mathbf{r}) - n_s(\mathbf{r})/\alpha$$



BUT: NEED TO TRANSLATE REDSHIFTS TO DISTANCES

There are a number of ways of dealing with this:

1. assume one model. Check no significant change for nearby models
   P(k) shape only (Percival et al. 2001)
2. change the model used for each model to be tested (Percival et al. 2007)
3. allow for the effect of getting the cosmology wrong (Percival et al. 2009)

Lecture outline

- Measuring the over-density field
  - Angular & radial mask
  - Random catalogues
- Measuring 2-pt statistics
  - the power spectrum
  - the correlation function
  - redshift-space distortions
- Fisher matrices and predictions
  - background
  - future surveys
- Putting it all together
  - parameters choices
  - the MCMC technique

Measuring the power spectrum

Translate over-density field onto grid, take Fourier transform, and measure the spherically averaged amplitude of the Fourier modes squared (ie. the power spectrum)

Effect of grid assignment is convolution in configuration-space, so multiplication in Fourier space easily included

$$\left\langle |F(\mathbf{k})|^2 \right\rangle = \int \frac{d^3k'}{(2\pi)^3} [P(\mathbf{k}') - P(0)\delta_D(\mathbf{k})] |G(\mathbf{k}-\mathbf{k}')|^2$$

$$+ (1 + \frac{1}{\alpha}) \int d^3r \bar{n}(\mathbf{r})$$

convolution with window function

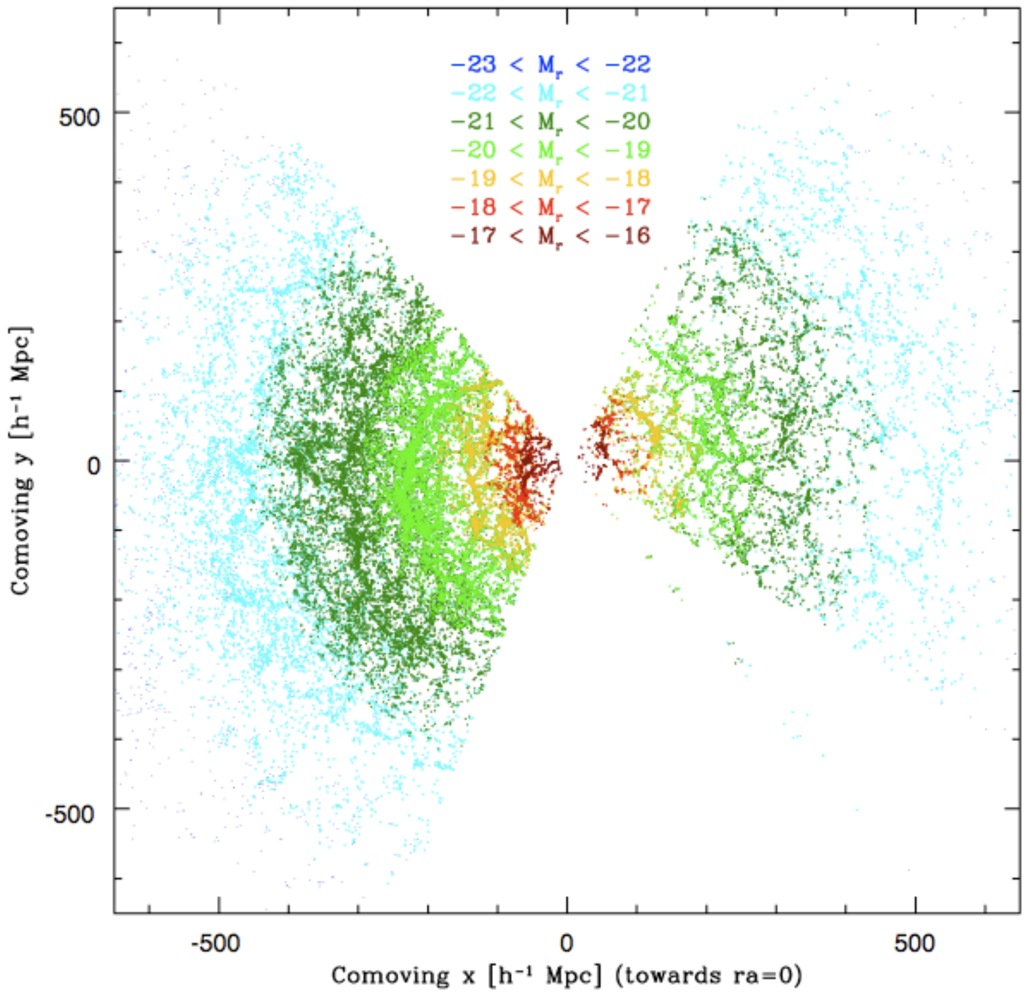
correction for not knowing true mean galaxy density

shot noise term – can be subtracted

Feldman et al. (1994; ApJ 426, 23)

Correcting for galaxy bias

In general, galaxy surveys do not produce homogeneous samples of galaxies



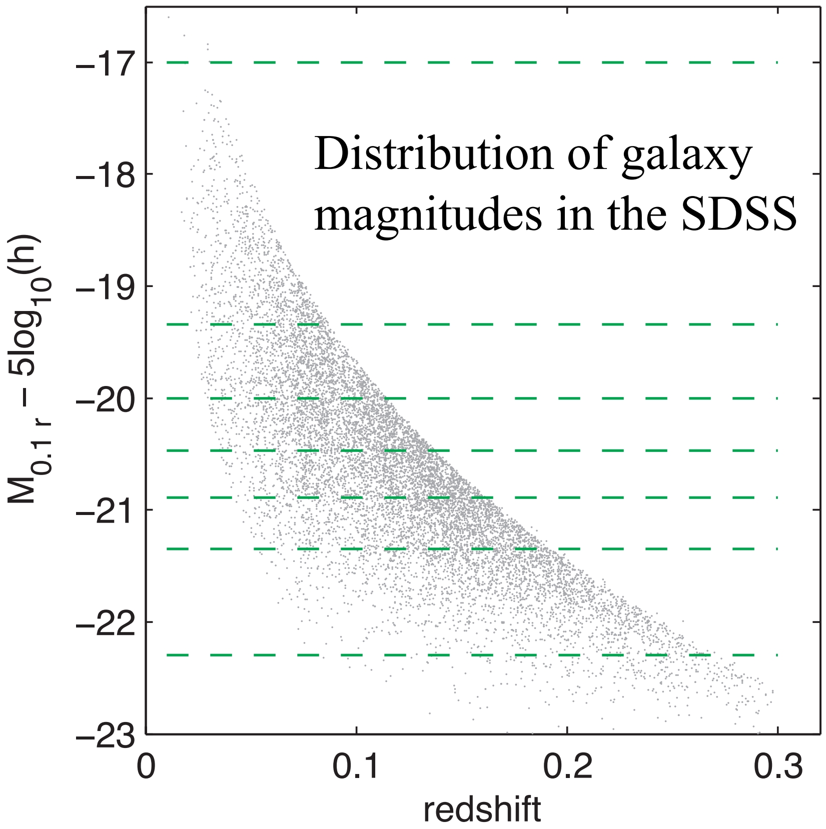
Galaxy luminosity varies systematically

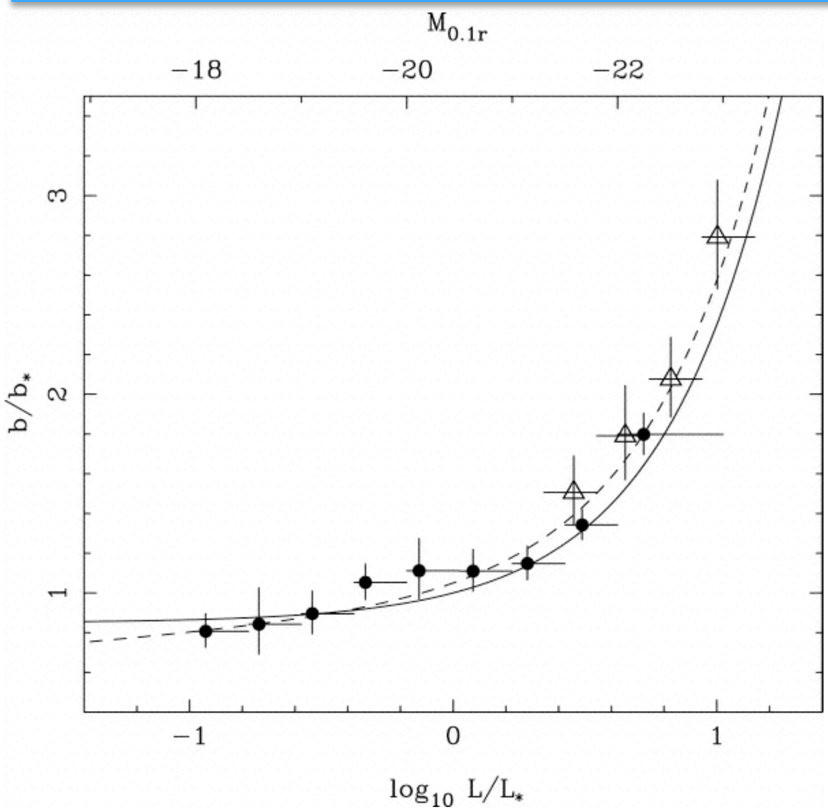
Galaxy density varies systematically

Correcting for galaxy bias

In general, galaxy surveys do not produce homogeneous samples of galaxies

Large-scales are only traced by more distant, intrinsically brighter galaxies, which are more clustered





With a bias model,

1. we can weight galaxies to remove such effects (Percival et al. 2004)
2. We can weight a measured power spectrum by an estimated median bias (Tegmark et al. 2006)

Weighting the galaxies for number density and bias

University of Portsmouth

$$w_i = \frac{1}{1 + \bar{n}(\mathbf{r_i})\widehat{P}(k)}$$

Depends on P(k) prior

Low density – weight by galaxy
High density – weight by volume

Change occurs when statistical and volume errors are equal, when nP=1

Now suppose each galaxy has a linear bias b_i, the optimal weight is

$$w_i = \frac{b_i^2}{1 + \sum_j \bar{n}(\mathbf{r_i}, L_j) b_j^2 \widehat{P}(k)}$$

up-weights very biased galaxies, containing the most signal

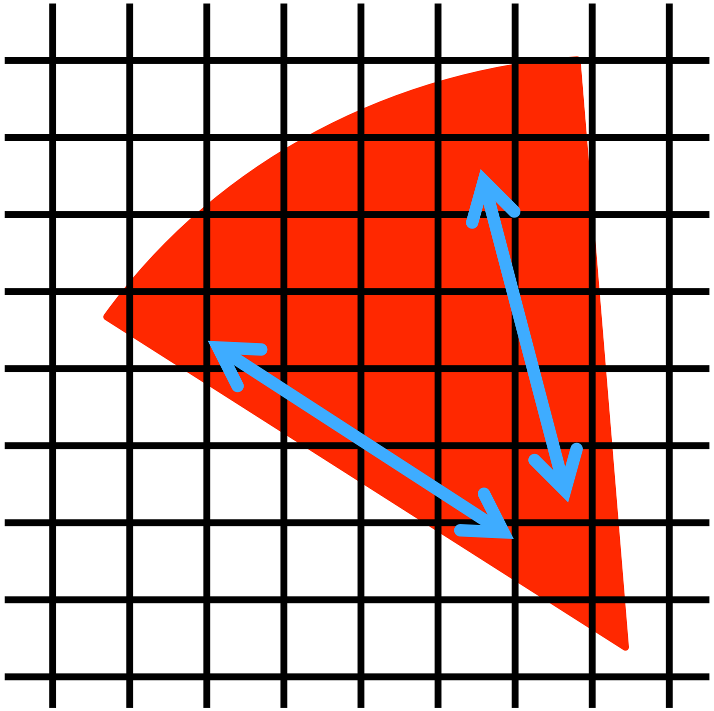
normalisation changes to match

Feldman et al. (1994; ApJ 426, 23), Percival et al (2004; MNRAS)

Anisotropic power spectrum

Redshift-space distortions mean that the power spectrum is anisotropic

$$
\begin{aligned}
P^s_{\rm lin}(k) &= \langle|\delta_g + \mu^2\theta|^2\rangle \\
&= P_{gg} + 2\mu^2 P_{g\theta} + \mu^4 P_{\theta\theta} \\
&= (1+\beta\mu^2)^2 P_{gg} \\
&= (b + f\mu^2)^2 P_{mass}
\end{aligned}
$$

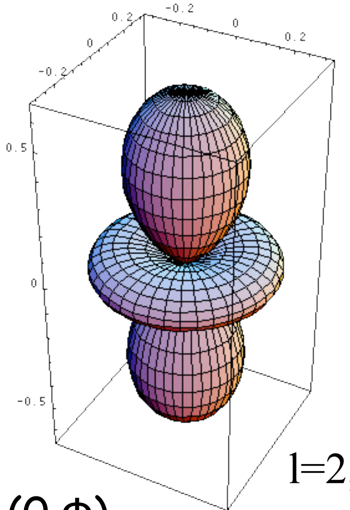
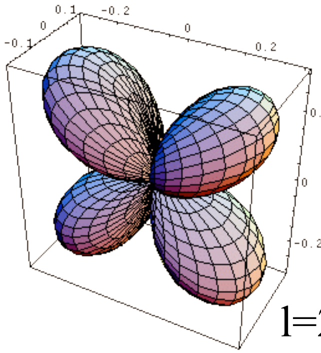


Cannot easily measure P(k,μ) using a Fourier basis (except in the distant observer approximation). μ does not line up with cartesian grid

Additionally, redshift-space distortions

1. Couple modes
2. Cause a complicated dependence on the anisotropic model

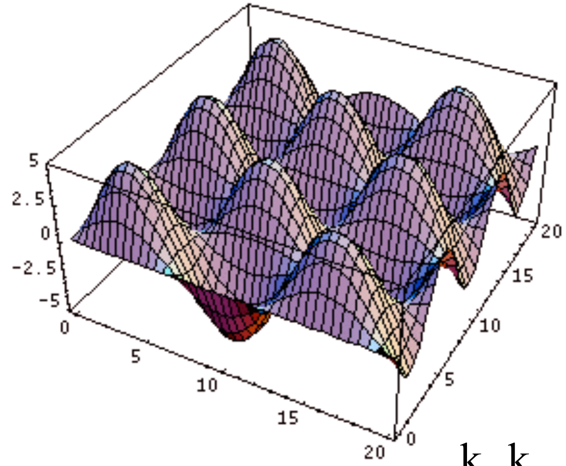
Modeling an anisotropic power spectrum

l=2,m=0

l=2,m=1

Spherical Harmonics (θ,Φ)

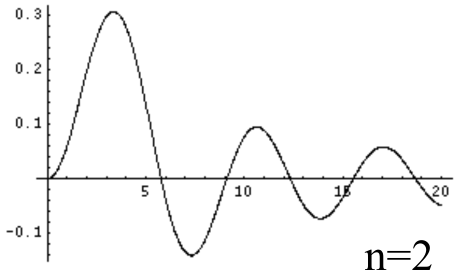


2d Fourier basis (x,y)

k_x, k_y

+

Spherical Bessel function (r)

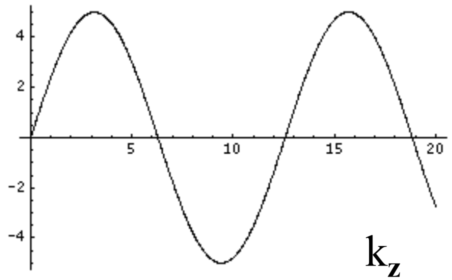


r

n=2

+

1d Fourier basis (z)



z

k_z

advantage: radial/angular split – more matched to survey geometry, easily model redshift space distortions

advantage: simplicity, speed

e.g. Heavens & Taylor (1995: MNRAS, 275, 483)

Power spectrum errors

Following Feldman et al. (1994), the error on the power spectrum is

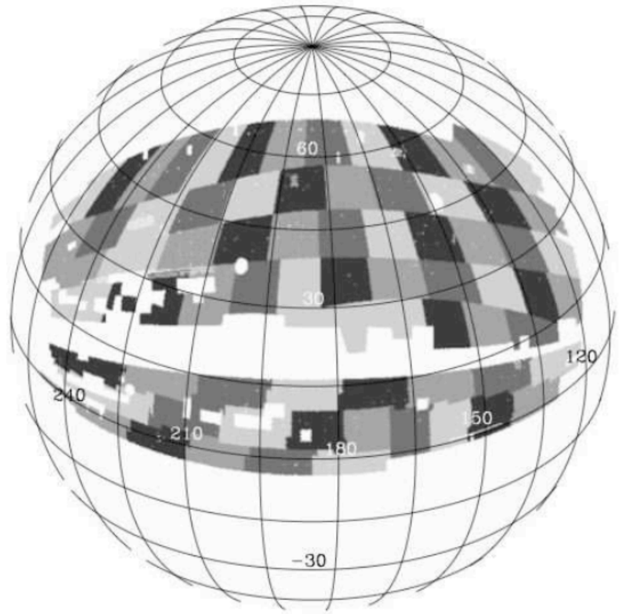
$$\langle \delta\hat{P}(\mathbf{k})\delta\hat{P}(\mathbf{k}+\delta\mathbf{k})\rangle = |P(\mathbf{k})Q(\delta\mathbf{k}) + S(\delta\mathbf{k})|^2$$

$$S(\mathbf{k}) \equiv \frac{(1+\alpha)\int d^3r\,\bar{n}(\mathbf{r})w^2(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}}{\int d^3r\,\bar{n}^2(\mathbf{r})w^2(\mathbf{r})} \qquad Q(\mathbf{k}) \equiv \frac{\int d^3r\,\bar{n}^2(\mathbf{r})w^2(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}}{\int d^3r\,\bar{n}^2(\mathbf{r})w^2(\mathbf{r})}$$

This is often difficult to calculate analytically, so mock catalogues, or jack-knife errors are usually used

For a sample with constant number density within a volume V, the error on the average power spectrum within a *k*-volume V_n reduces to

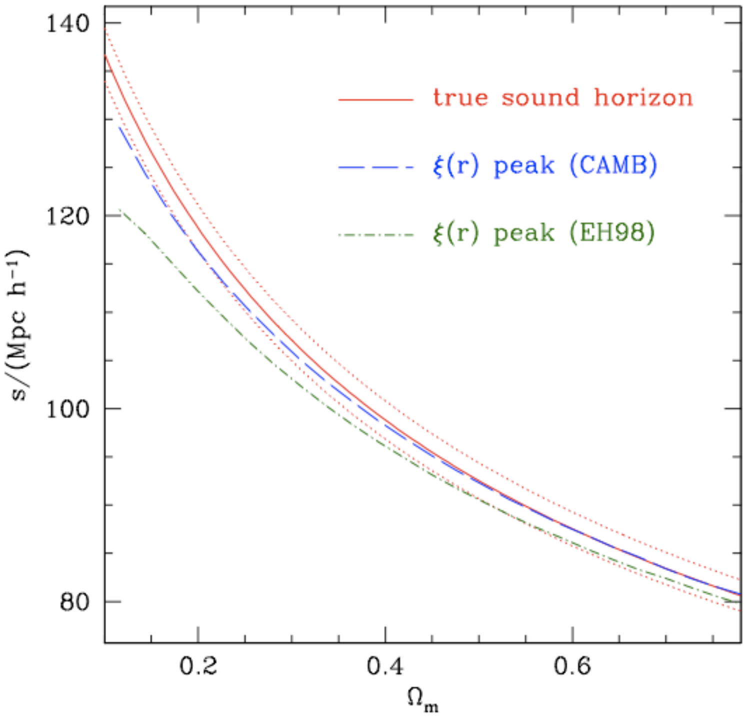
$$\langle P(\mathbf{k})P(\mathbf{k}')\rangle = \frac{1}{V_n V}\left[P(k)\delta_D(\mathbf{k}-\mathbf{k}') + 1/\bar{n}\right]^2$$



JK zones in the mock catalogue

Feldman et al. (1994; ApJ 426, 23), Cabre & Gaztanaga (2008; arXiv:0807.2460)

Fitting the power spectrum



Sanchez et al. (2008: arXiv:0804.0233)

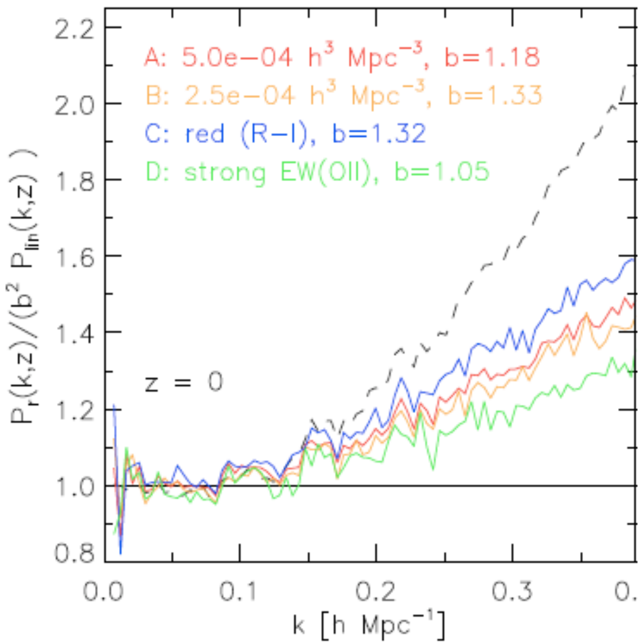
Linear model power spectrum calculated by CMBfast or CAMB

Eisenstein & Hu (1998) fitting formulae do not give a P(k) with sufficient accuracy for current measurements

Focus on modeling

1. non-linear matter evolution
2. galaxy bias
3. redshift-space effects

Currently need simulations & analytic models to test these effects



Angulo et al. (2008: arXiv:0702543)

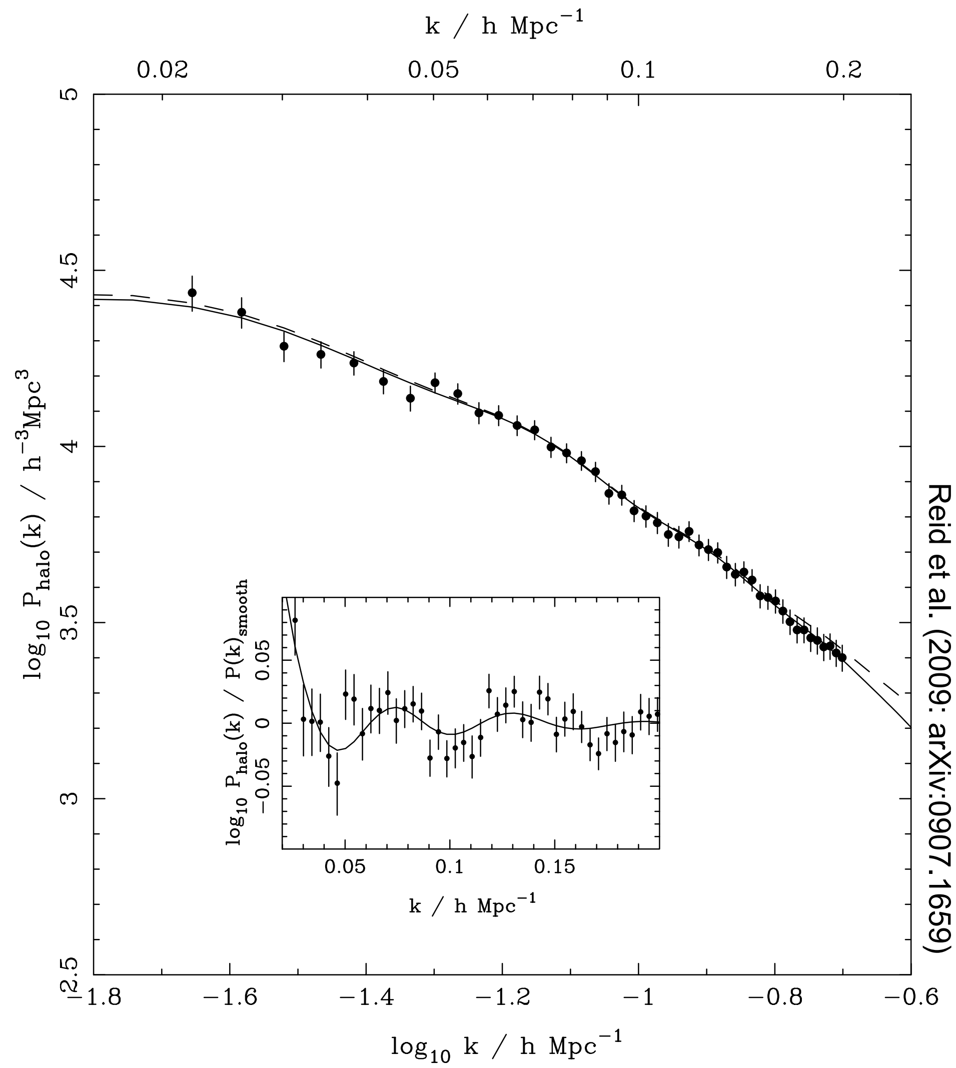
Fitting to just the BAO

Need to pull out the BAO from the power spectrum

Model shape of power spectrum with smooth fit

1. polynomial
2. spline
3. ???

Less risk of systematic if allow P(k) shape to vary with BAO model. ie. fit power with varying shape model combined with BAO, rather than just the BAO.

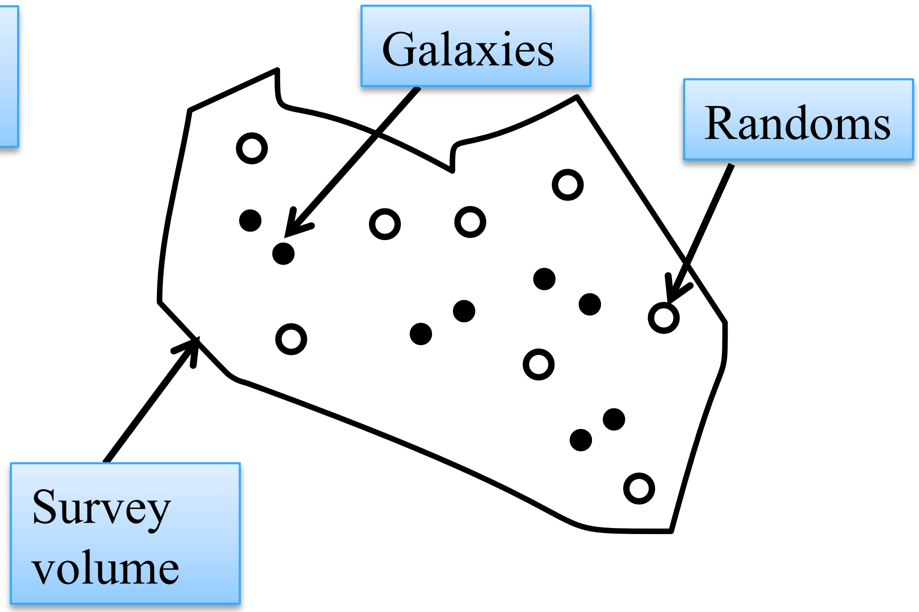


Reid et al. (2009: arXiv:0907.1659)

Measuring the correlation function

Create random catalogue with same spatial sampling as galaxies, but no clustering

DD = number of galaxy-galaxy pairs
DR = number of galaxy-random pairs
RR = number of random-random pairs



$$\xi = \frac{DD}{RR} - 1$$

$$\xi = \frac{DD}{DR} - 1$$

$$\xi = \frac{DD}{DR^2} - 1$$

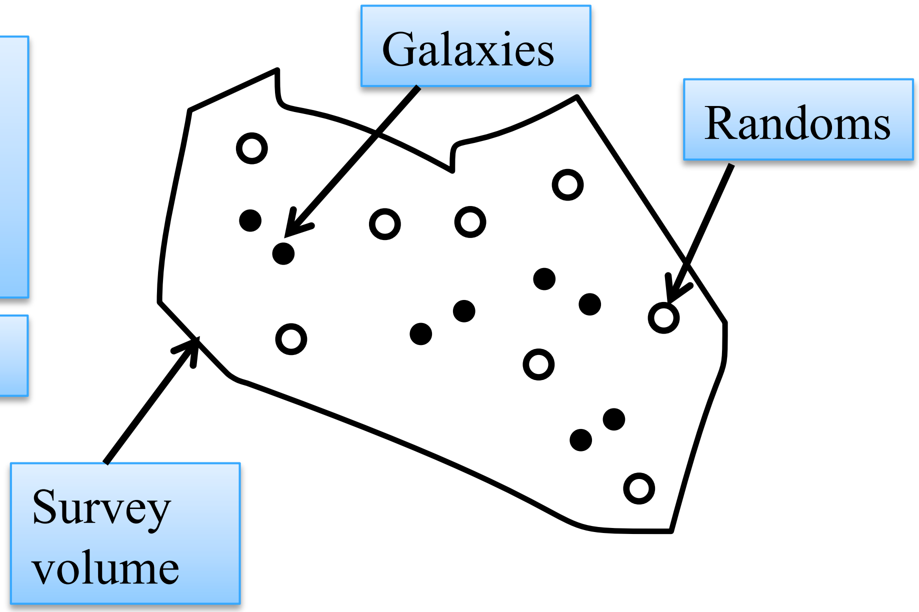
$$\xi = \frac{DD - 2DR + RR}{RR}$$

Landy & Szalay (1993) considered noise from these estimators, and showed that this has the best noise properties

Landy & Szalay (1993; ApJ 412, 64)

Window functions and the integral constraint

Correlation function calculation automatically takes care of window function: multiplicative contribution is removed by random catalogue



Integral constraint is important:

$$\langle DD \rangle = \langle RR \rangle \frac{1+\xi}{1+w_V}$$

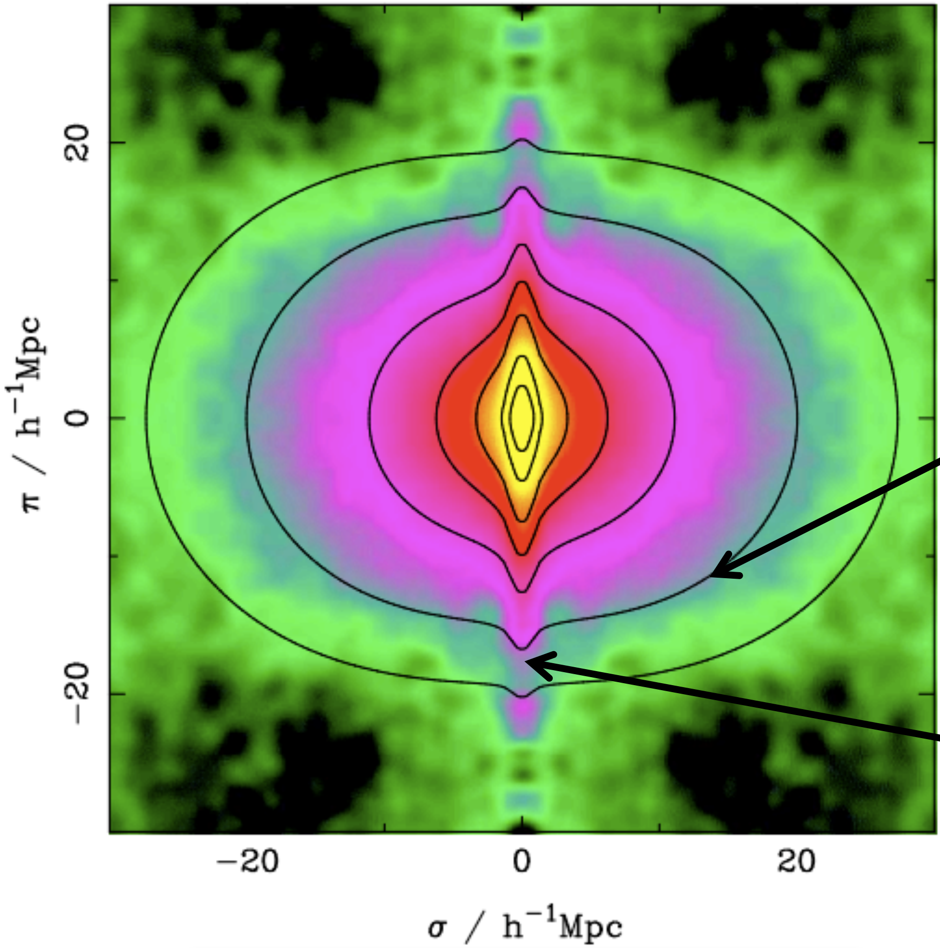
This term is called the integral constraint and includes the fact that we estimate the mean galaxy density from the sample itself

$$w_V = \int_V \xi(r, r')\, dr\, dr'$$

Landy & Szalay (1993; ApJ 412, 64)

The anisotropic correlation function

Radial component of distance



Angular component of distance

Only signal in one quadrant, but often copied to cover full circle

Quadrupole component caused by linear redshift-space distortions

Fingers-Of-God increase pair count in radial direction

Peacock & 2dFGRS team (2001; Nature 410, 169)

Modeling the correlation function

Power spectrum and correlation function form a Fourier pair

$$
\begin{aligned}
P(\mathbf{k}) &= \int \xi(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} d^3r \\
\xi(\mathbf{r}) &= \int P(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{r}} \frac{d^3k}{(2\pi)^3}
\end{aligned}
$$

In absence of window function, power spectrum of different Fourier modes are uncorrelated. Not true for correlation function

To solve this integral, we could adopt a spherical basis with z-axis (θ=0) along **r**, and line-of-sight in (y,z)-plane (i.e. $\Phi_{\mathbf{los}}$=0).

$$
\begin{aligned}
P^s(\mathbf{k}) &= (1+\beta\mu^2)^2 P^r(k) \\
e^{-i\mathbf{k}\cdot\mathbf{r}} &= rk\cos(\theta) \\
\mu &= \cos(\phi)\sin(\theta)\sin(\theta_{\rm los}) + \cos(\theta)\cos(\theta_{\rm los})
\end{aligned}
$$

Modeling the correlation function

Inverse Laplacian

Legendre polynomial expansion

$$\xi^s(\mathbf{r}) = [b + f(\partial/\partial z)^2(\nabla^2)^{-1}]^2\xi(r)$$

$$\xi^s(\mathbf{r}) = \xi_0(r)L_0(\mu) + \xi_2(r)L_2(\mu) + \xi_4(r)L_4(\mu)$$

$$L_0 = 1$$
$$L_2 = (3\mu^2 - 1)/2$$
$$L_4 = (35\mu^4 - 30\mu^2 + 3)/8$$

$$\xi_0(r) = (b^2 + \frac{2}{3}bf + \frac{1}{5}f^2)\xi(r)$$

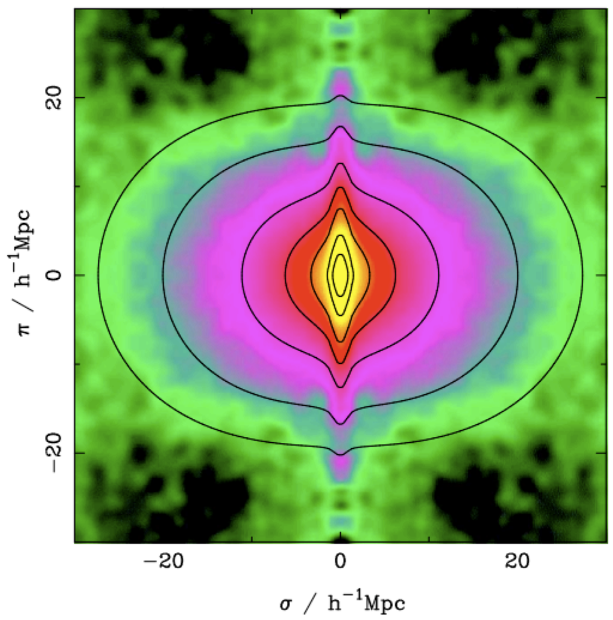
$$\xi_2(r) = (\frac{4}{3}bf + \frac{4}{7}f^2)[\xi(r) - \bar{\xi}(r)]$$

$$\xi_4(r) = \frac{8}{35}f^2[\xi(r) + \frac{5}{2}\bar{\xi}(r) - \frac{7}{2}\bar{\bar{\xi}}(r)]$$

Correlation function moments

$$\bar{\xi}(r) = \frac{3}{r^3}\int_0^r \xi(r')r'^2\,dr'$$

$$\bar{\bar{\xi}}(r) = \frac{5}{r^5}\int_0^r \xi(r')r'^4\,dr'$$



We see that the primary effect is the addition of a quadrupole moment

Correlation function even changes when μ=0, so not just flattening

Hamilton (1992; ApJ 412, 64)

University of Portsmouth

modeling redshift-space distortions in ξ

METHOD 1: Measure Q, and relate to β

$$Q(s) = \frac{\xi_2(s)}{\xi_0(s) - (3/s^2)\int_0^s \xi_0(s')s'^2\,ds'}$$

$$Q(s) = \frac{\frac{4}{3}\beta + \frac{4}{7}\beta^2}{1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2}$$

METHOD 2: Estimate real-space ξ from projected ξ,
and relate to average redshift-space ξ

$$\Xi = 2\int_\sigma^\infty \frac{r\xi(r)\,dr}{(r^2-\sigma^2)^{\frac{1}{2}}}$$

$$\xi = -\frac{1}{\pi}\int_r^\infty \frac{(d\Xi(\sigma)/d\sigma)d\sigma}{(r^2-\sigma^2)^{\frac{1}{2}}}$$

$$\frac{\xi(s)}{\xi(r)} = 1 + \frac{2\beta}{3} + \frac{\beta^2}{5}$$

METHOD 3:Model full anisotropic correlation function

University of Portsmouth

modeling redshift-space distortions in P(k)

METHOD 1: Perform Legendre polynomial expansion of anisotropic power spectrum, and calculate quadrupole

$$P_\ell \equiv \frac{2\ell+1}{2}\int_{-1}^{+1} d\mu\, P^s(k,\mu)L_\ell(\mu)$$

$$\frac{P_2(k)}{P_0(k)} = \frac{(4/3)\beta + (4/7)\beta^2}{1+(2/3)\beta+(1/5)\beta^2}$$

METHOD 2: calculate mix of monopole and quadrupole. This removes the bias dependence and measure the matter velocity power spectrum (on large scales)

$$\hat{P}(k) = \frac{7}{48}\left[5(7P_0+P_2) - \sqrt{35}(35P_0^2+10P_0P_2-7P_2^2)^{1/2}\right]$$

Percival & White, 2008, astro-ph/0808.0003

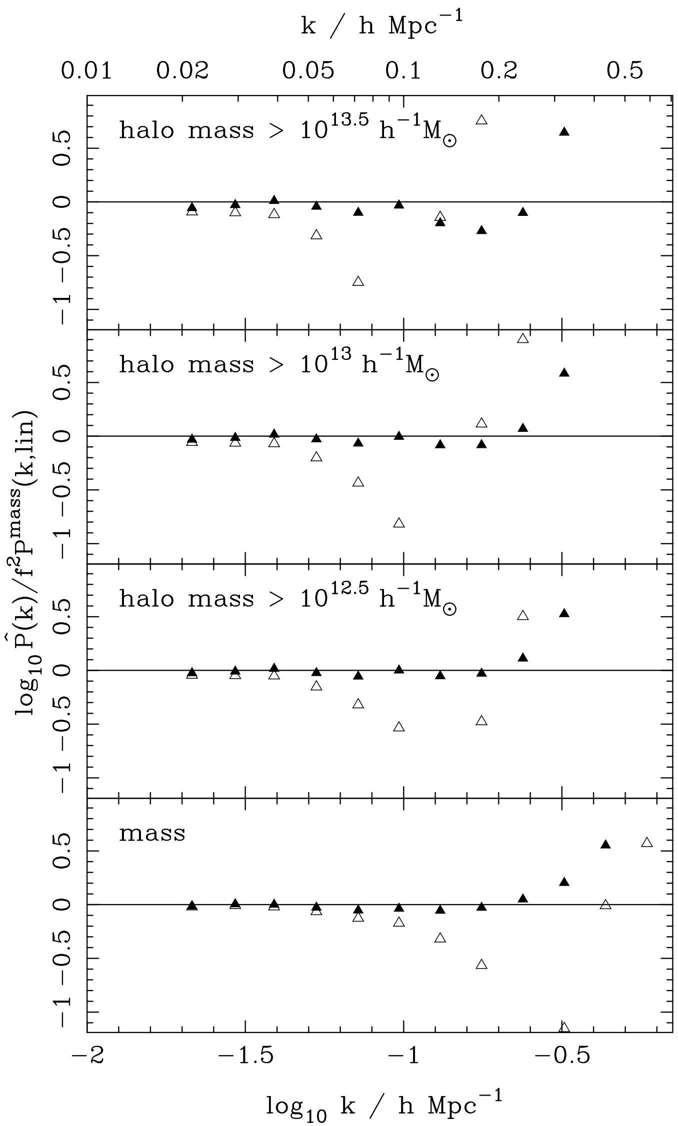
Measuring the velocity power spectrum

$$\hat{P}(k) = \frac{7}{48}\left[5(7P_0 + P_2) - \sqrt{35}(35P_0^2 + 10P_0P_2 - 7P_2^2)^{1/2}\right]$$

This plane-parallel approximation works on scales $k < 0.1 h\mathrm{Mpc}^{-1}$

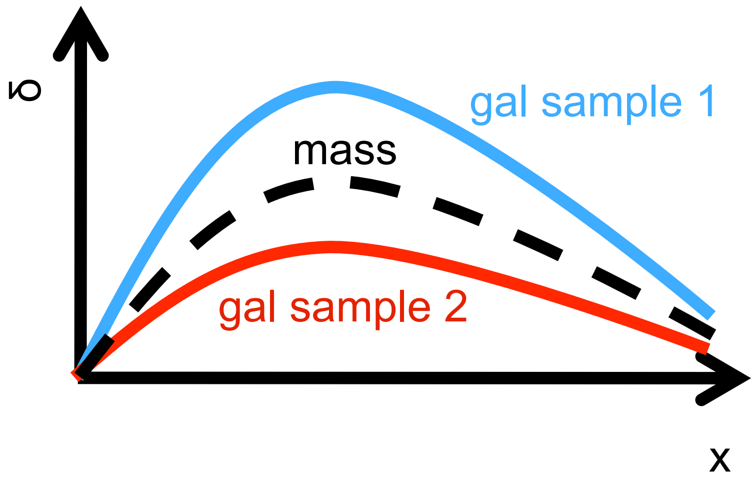
On large-scales, the primary systematic is a possible velocity-bias …

Although galaxy velocities will trace those of the mass, the distribution does not have to
(e.g. Percival & Schafer 2008, MNRAS 385, L78)



Percival & White, 2008, astro-ph/0808.0003

Beating the cosmic variance limit



If we have 2 samples of galaxies (in real space) with different deterministic biases,

$$\delta_1 = b_1 \delta_{\rm mass},\ \ \delta_2 = b_2 \delta_{\rm mass}$$

accuracy of b_1/b_2 measurement only depends on shot noise

In redshift-space this result generalizes to

$$\delta_1 = (b_1 + f\mu^2)\delta_{\rm mass},\ \ \delta_2 = (b_2 + f\mu^2)\delta_{\rm mass}$$

giving b_1/b_2, f/b_1, and f/b_2 limited by only shot noise

Allows us to use non-radial modes to "extract" information about $f^2P(k)_{\rm mass}$

Does not affect errors on the power spectrum shape.

McDonald & Seljak, astro-ph/0808.1101

redshift-space distortions vs weak lensing

redshift-space distortions	weak lensing
velocity & distortions depends on mass	lensing potential depends on mass
two galaxies at same location have same redshift distortion	two galaxies at same location have same lensing distortion
tests temporal metric fluctuations	tests temporal and spatial metric fluctuations
velocity-bias - galaxy velocities not Poisson sampling of mass	intrinsic alignments - original galaxy shapes depend on lensing potential
science vs money? - need spectra - with deterministic bias, direct probe of fluctuations in all directions - for high galaxy density, limited by cosmic variance	science vs money? - need careful imaging - 1-2% projection effect, but tests fluctuations in all directions - for (very) high galaxy density, limited by cosmic variance

Lecture outline

- Measuring the over-density field
  - Angular & radial mask
  - Random catalogues
- Measuring 2-pt statistics
  - the power spectrum
  - the correlation function
  - redshift-space distortions
- Fisher matrices and predictions
  - background
  - future surveys
- Putting it all together
  - parameters choices
  - the MCMC technique

Fisher matrix introduction

Suppose we have a model with parameters $\Theta = (\theta_1, \theta_2, ..., \theta_n)$

That we wish to constrain with data $X = (x_1, x_2, ..., x_n)$

The Likelihood of the data given a model is $L(X|\Theta)$

Sir Ronald Aylmer Fisher (1890-1962)

Fisher matrix is given by $F_{ij} \equiv \left\langle \frac{\partial^2 \mathcal{L}}{\partial\theta_i \partial\theta_j} \right\rangle$ where $\mathcal{L} \equiv - \ln L$

The Cramer-Rao inequality shows that the Fisher matrix gives the best model error we can hope to achieve

$$\Delta\theta_i \geq (F_{ii})^{-1/2}$$

Gaussian Fisher matrix for P(k)

Suppose that the Likelihood has a multi-variate Gaussian distribution, with <x>=μ, and covariance matrix C

$$F_{ij} = \frac{1}{2}\left[C^{-1}\frac{\partial C}{\partial \theta_i}C^{-1}\frac{\partial C}{\partial \theta_j}\right] + \frac{\partial \mu^T}{\partial \theta_i}C^{-1}\frac{\partial \mu}{\partial \theta_j}$$

For the galaxy power spectrum, we can use the Feldman et al (1994) result, to approximate the power spectrum covariance in band powers with *k*-space volume $V_{\mathbf{n}}$ as

$$\langle P(k_n)P(k_m)\rangle = 2\frac{P(k_n)P(k_n)}{V_n V_{\rm eff}(k_n)}\delta_D(k_m - k_n)$$

$$V_{\rm eff}(k) \equiv \int \left[\frac{\bar{n}(\mathbf{r})P(k)}{1+\bar{n}(\mathbf{r})P(k)}\right]^2 d^3r$$

Which gives a Fisher matrix, that can be written (if we now integrate over many small shells $V_{\mathbf{n}}$)

$$F_{ij} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left(\frac{\partial \ln P}{\partial \theta_i}\right)\left(\frac{\partial \ln P}{\partial \theta_j}\right)V_{\rm eff}(\mathbf{k})$$

Tegmark (1997; PRL, 79, 3806)

Gaussian Fisher matrix for δ(k)

As an alternative to the power spectrum, we can work with δ(k) as our "data", and covariance (as before)

$$\langle\delta(\mathbf{k})\delta(\mathbf{k}')\rangle = \frac{1}{V_n V}\left[P(k)\delta_D(\mathbf{k}-\mathbf{k}') + 1/\bar{n}\right]$$

This gives exactly the same Fisher matrix as we recovered using P(k) in band powers as our data (exercise!)

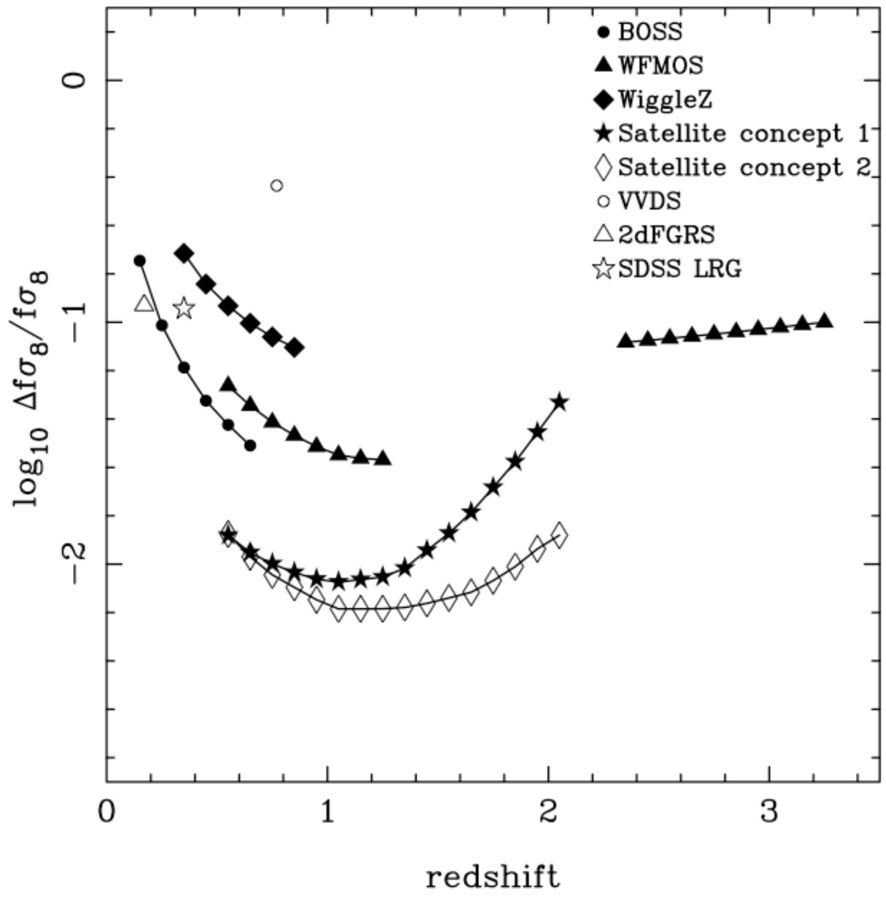
Redshift-space distortion Fisher matrix

For our set of parameters to test, we have freedom to choose any of the physical processes that contribute to the clustering

For example, we can single out the parameters controlling the anisotropic normalization of the power spectrum within a background cosmology, $f\sigma_8$ and $b\sigma_8$

Code to estimate errors on $f\sigma_8$ from the Fisher matrix formalism is available from:

http://mwhite.berkeley.edu/Redshift



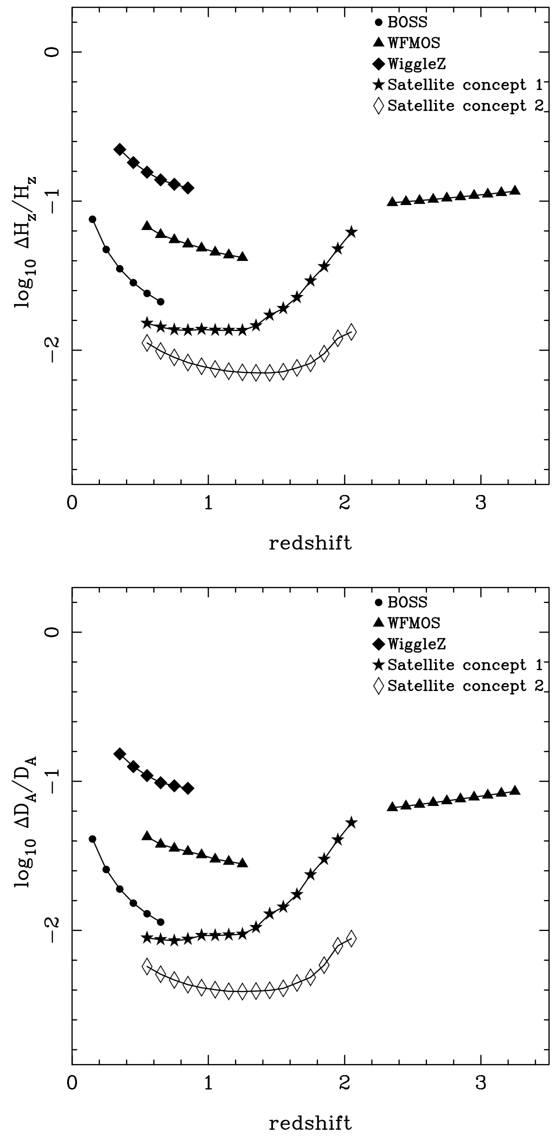
White, Song & Percival, 2009, MNRAS, 397, 1348

BAO Fisher matrix

For our set of parameters to test, we have freedom to choose any of the physical processes that contribute to the clustering

It is also possible to isolate the BAO, and fit for $H(z)$ and $D_A(z)$, which cause radial and angular distortions

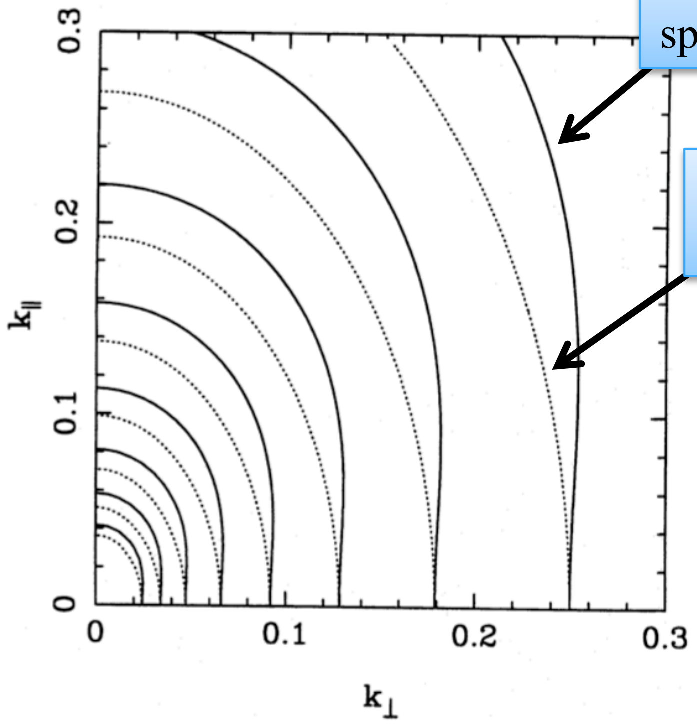
Code to estimate these errors from the Fisher matrix formalism is available from:
http://cmb.as.arizona.edu/~eisenste/acousticpeak/bao_forecast.html



Seo & Eisenstein, 2007, astro-ph/0701079

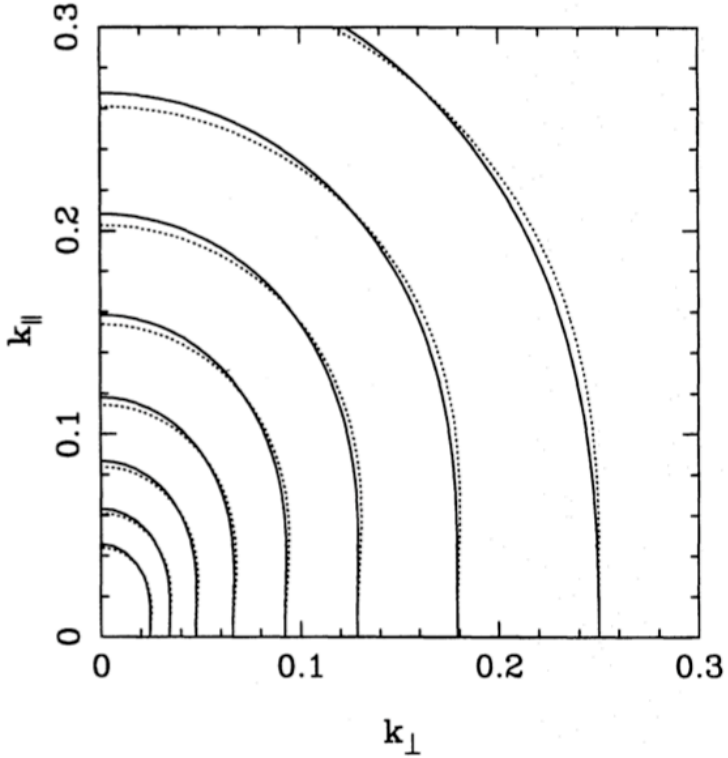
Linking z-space distortions and BAO?

We should allow for the coupling between the redshift-space distortions and the geometrical squashing caused by getting the geometry wrong. Effects are not perfectly degenerate



Linear redshift-space distortions

Geometric squashing

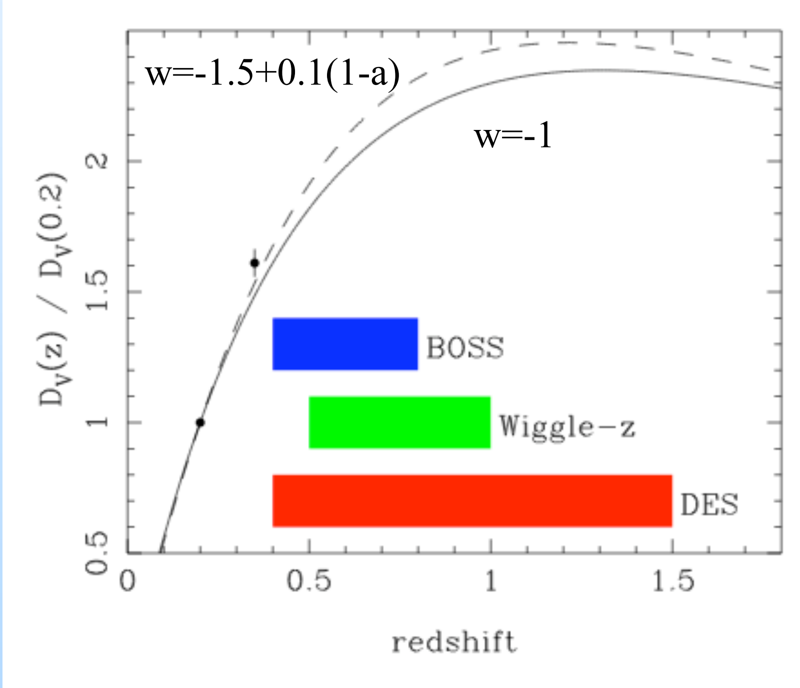


Fit to redshift-space distortions cannot mimic geometric squashing

Ballinger, Peacock & Heavens 1999, MNRAS, 282, 877

Present/Future BAO Surveys

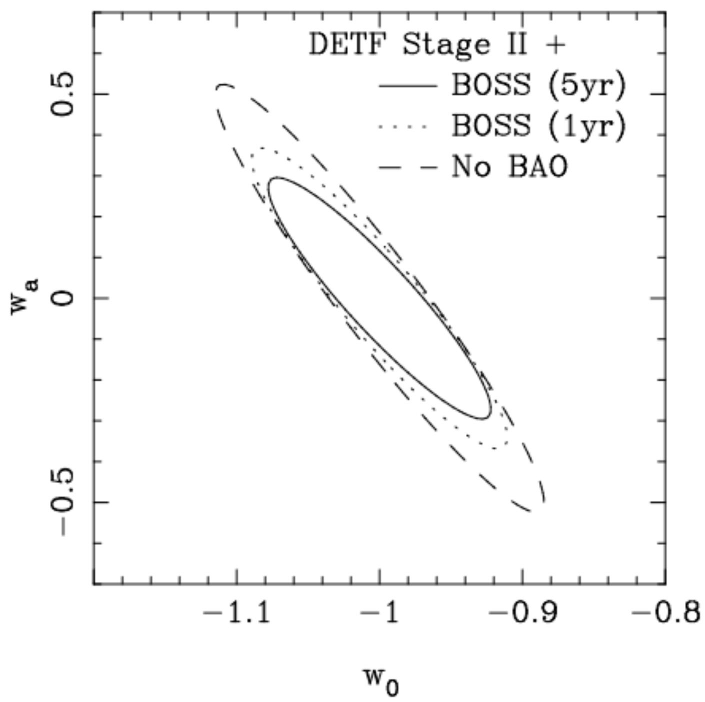
- SDSS-II (present - 2008)
  - 800,000 z<0.5 spectra over $7500deg^2$
- Wiggle-Z (present - 2010)
  - 400,000 z~0.75 spectra over $1000deg^2$
- Baryon Oscillation Spectroscopic Survey (BOSS: 2009-2014)
  - 1,500,000 z~0.6 spectra over $10{,}000deg^2$ +QSOs
- Dark Energy Survey (DES: 2010-2015)
  - $5000deg^2$ multi-colour imaging survey on Blanco 4m + VISTA
  - photo-z for 300,000,000 galaxies
- Plus: VST-Atlas, SUMIRE , SKA, HETDEX, LAMOST, LSST, Pan-STARRS, PAU, Euclid/JDEM, + other MOS plans?



Predicted Dark Energy Constraints

DETF Figure-of-merit is area of 1σ confidence region for 2-parameter DE model, with equation of state:

$$\begin{aligned} w(z) &= w_0 + (1-a)w_a \\ &= w_p + w_a(a_p - a) \end{aligned}$$



Survey	DETF figure of merit
Current + Planck	53.7
+ BOSS (1 year)	79.7
+ BOSS (5 year)	109.9
+ DES (BAO only)	75.1
+ Wiggle-Z	71.5

BOSS: A next generation BAO experiment

- SDSS finished its original, legacy survey in 2008
- Spectroscopic survey role identified as remaining world-class (7.5deg^2 field-of-view, even though 2.5m telescope)
- SDSS Imaging detects and can distinguish luminous red galaxies out to z<0.7 (e.g. AGES, 2SLAQ)
- Obvious next step: large survey for LRGs to measure BAO for z<0.7

- Can also pick up high-z BAO signal in Ly-α forest in QSO spectra
- Very efficient: each spectra gives skewer though density field, rather than single point

BOSS: Summary

- $\Omega = 10{,}000 deg^2$
- Selected from 11,000deg^2 of imaging
  - – 8,500deg^2 in North
  - – 2,500deg^2 in South (fill in SDSS-II Southern stripes)
- LRGs : 150/deg^2, $z \sim 0.1 - 0.7$ (direct BAO)
- 1% d_A, 1.8% H at $z \sim 0.35, 0.6$
- QSOs : 20/deg^2, $z \sim 2.1 - 3.0$ (BAO from Ly-α forest)
- 1.5% d_A, 1.2% H at $z \sim 2.5$
- Cosmic variance limited to $z \sim 0.6$: as good as LSS mapping will get with a single ground based telescope
- Leverage existing SDSS hardware & software where possible
- Sufficient funding is in place and project is underway
- www.sdss3.org/boss

University of Portsmouth

Lecture outline

- Measuring the over-density field
  - Angular & radial mask
  - Random catalogues
- Measuring 2-pt statistics
  - the power spectrum
  - the correlation function
  - redshift-space distortions
- Fisher matrices and predictions
  - background
  - future surveys
- Putting it all together
  - parameters choices
  - the MCMC technique

Model parameters (describing LSS & CMB)

content of the Universe

total energy density
Ω_{tot} (=1?)
matter density
Ω_m
baryon density
Ω_b
neutrino density
Ω_n (=0?)
Neutrino species
f_n
dark energy eq^n of state
w(a) (=-1?)
or w_0, w_1

perturbations after inflation

scalar spectral index
n_s (=1?)
normalisation
σ_8
running
$a = dn_s/dk$ (=0?)
tensor spectral index
n_t (=0?)
tensor/scalar ratio
r (=0?)

evolution to present day

Hubble parameter
h
Optical depth to CMB
τ

parameters usually marginalised and ignored

galaxy bias model
b(k) (=cst?)
or b,Q
CMB beam error
B
CMB calibration error
C

Assume Gaussian, adiabatic fluctuations

WMAP3 parameters used

Cosmological Parameters Used in the Analysis

Parameter	Description	Definition
H_0	Hubble expansion factor	$H_0 = 100h\ \mathrm{Mpc^{-1}\ km\ s^{-1}}$
ω_b	Baryon density	$\omega_b = \Omega_b h^2 = \rho_b/1.88 \times 10^{-26}\ \mathrm{kg\ m^{-3}}$
ω_c	Cold dark matter density	$\omega_c = \Omega_c h^2 = \rho_c/18.8$ yoctograms $\mathrm{m^{-3}}$
f_ν	Massive neutrino fraction	$f_\nu = \Omega_\nu/\Omega_c$
$\sum m_\nu$	Total neutrino mass (eV)	$\sum m_\nu = 94\Omega_\nu h^2$
N_ν	Effective number of relativistic neutrino species	
Ω_k	Spatial curvature	
Ω_{DE}	Dark energy density	For $w = -1$, $\Omega_\Lambda = \Omega_{\mathrm{DE}}$
Ω_m	Matter energy density	$\Omega_m = \Omega_b + \Omega_c + \Omega_\nu$
w	Dark energy equation of state	$w = p_{\mathrm{DE}}/\rho_{\mathrm{DE}}$
$\Delta^2_{\mathcal{R}}$	Amplitude of curvature perturbations $\mathcal{R}$	$\Delta^2_{\mathcal{R}}(k = 0.002\ \mathrm{Mpc^{-1}}) \approx 29.5 \times 10^{-10} A$
A	Amplitude of density fluctuations ($k = 0.002\ \mathrm{Mpc^{-1}}$)	See Spergel et al. (2003)
n_s	Scalar spectral index at 0.002 $\mathrm{Mpc^{-1}}$	
α	Running in scalar spectral index	$\alpha = dn_s/d\ln k$ (assume constant)
r	Ratio of the amplitude of tensor fluctuations to scalar potential fluctuations at $k = 0.002\ \mathrm{Mpc^{-1}}$	
n_t	Tensor spectral index	Assume $n_t = -r/8$
τ	Reionization optical depth	
σ_8	Linear theory amplitude of matter fluctuations on 8 h^{-1} Mpc	
Θ_s	Acoustic peak scale (deg)	See Kosowsky et al. (2002)
A_{SZ}	SZ marginalization factor	See Appendix A
b_{SDSS}	Galaxy bias factor for SDSS sample	$b = [P_{\mathrm{SDSS}}(k, z = 0)/P(k)]^{1/2}$ (constant)
C_{220}^{TT}	Amplitude of the TT temperature power spectrum at $l = 220$	
z_s	Weak lensing source redshift	

University of Portsmouth

Multi-parameter fits to multiple data sets

- Given WMAP3 data, other data are used to break CMB degeneracies and understand dark energy
- Main problem is keeping a handle on what is being constrained and why
  - difficult to allow for systematics
  - you have to believe all of the data!
- Have two sets of parameters
  - those you fix (part of the prior)
  - those you vary
- Need to define a prior
  - what set of models
  - what prior assumptions to make on them (usual to use uniform priors on physically motivated variables)
- Most analyses use the Monte-Carlo Markov-Chain technique

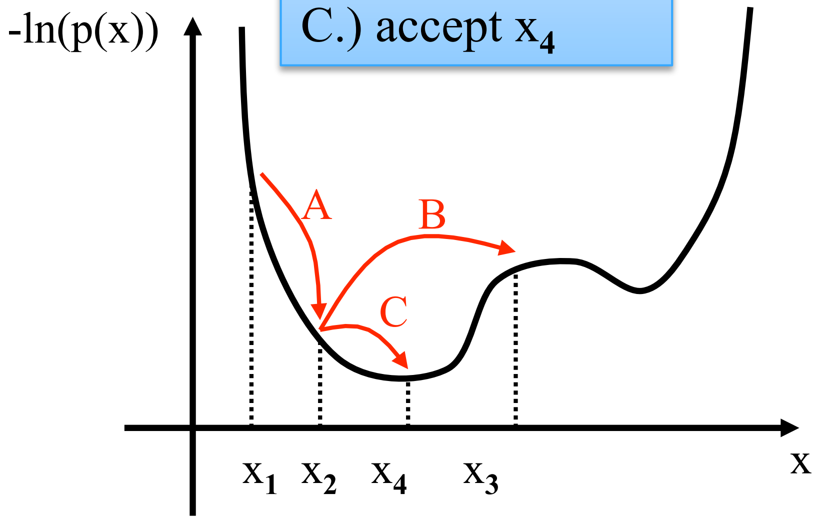
Markov-Chain Monte-Carlo method

MCMC method maps the likelihood surface by building a chain of parameter values whose density at any location is proportional to the likelihood at that location p(x)

given a chain at parameter x, and a candidate for the next step x', then x' is accepted with probability

$$\begin{cases} 1 & p(x') > p(x) \\ p(x')/p(x) & \text{otherwise} \end{cases}$$

an example chain starting at x_1
A.) accept x_2
B.) reject x_3
C.) accept x_4



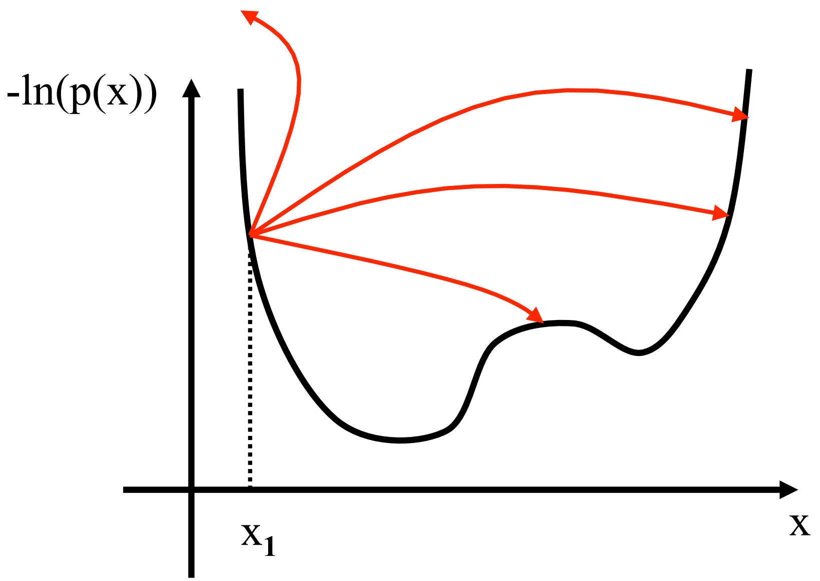
CHAIN: $x_1, x_2, x_2, x_4, \ldots$

for any symmetric proposal distribution $q(x|x') = q(x'|x)$, then an infinite number of steps leads to a chain in which the density of samples is proportional to p(x).

MCMC problems: jump sizes

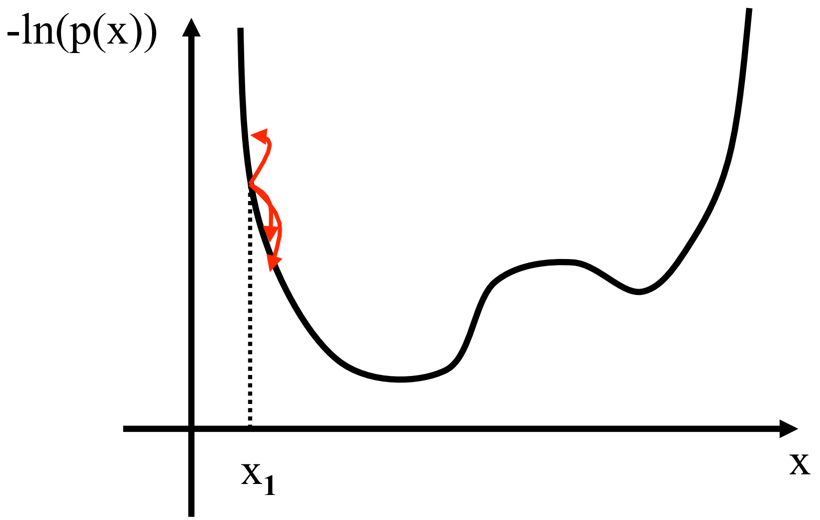
q(x) too broad

chain lacks mobility as all candidates are unlikely



q(x) too narrow

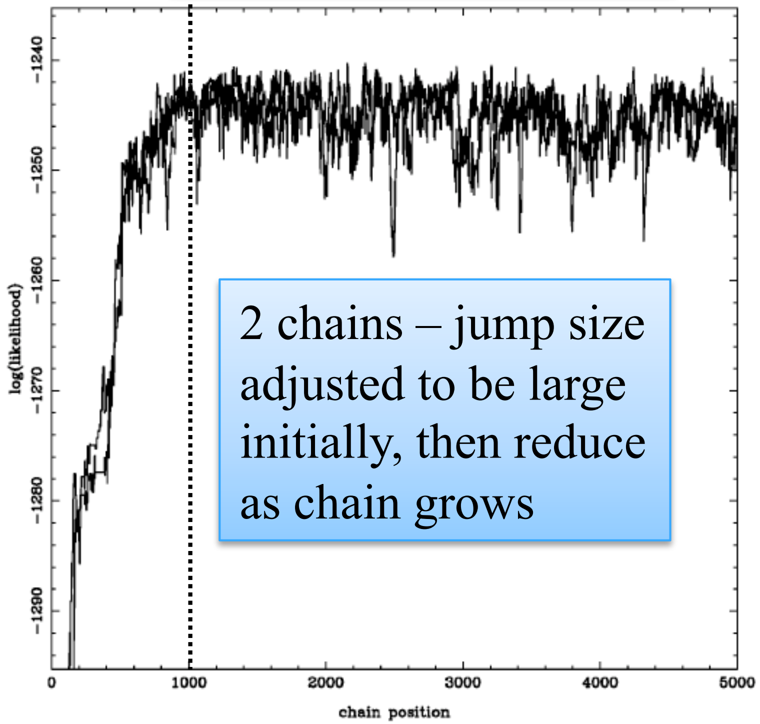
chain only moves slowly to sample all of parameter space



MCMC problems: burn in

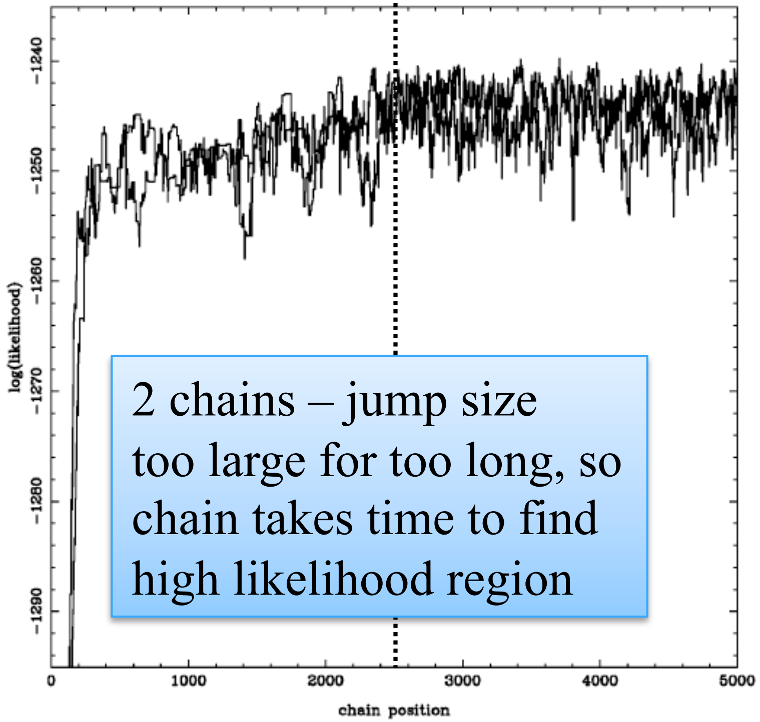
Chain takes some time to reach a point where the initial position chosen has no influence on the statistics of the chain (very dependent on the proposal distribution q(x))

Approx. end of burn-in



2 chains – jump size adjusted to be large initially, then reduce as chain grows

Approx. end of burn-in



2 chains – jump size too large for too long, so chain takes time to find high likelihood region

MCMC problems: convergence

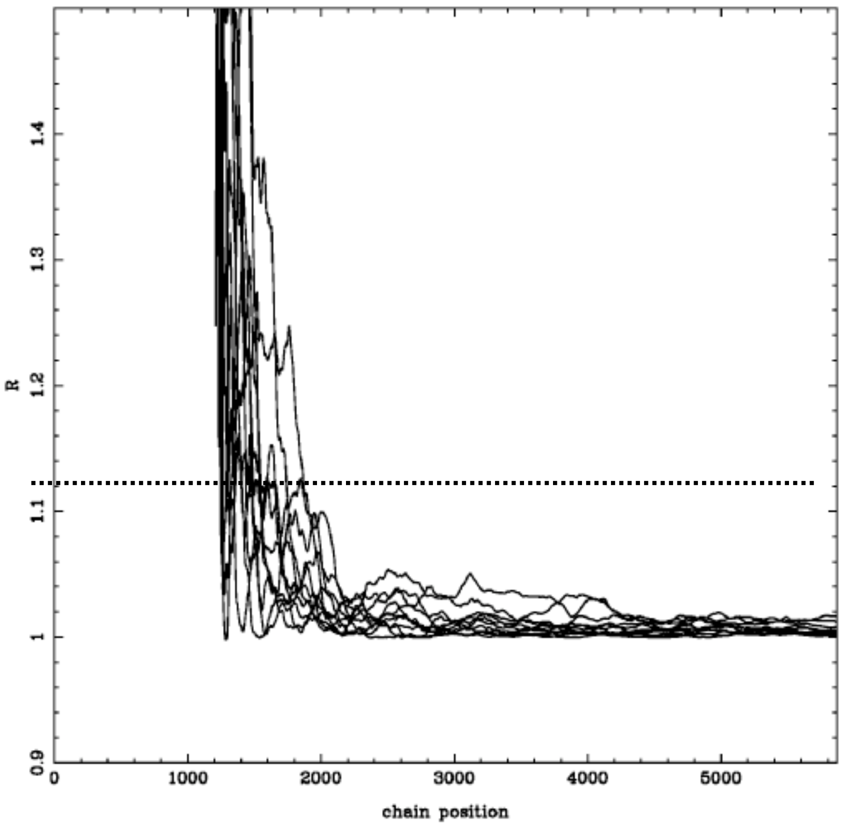
How do we know when the chain has sampled the likelihood surface sufficiently well, that the mean & std deviation for each parameter are well constrained?

Gelman & Rubin (1992) convergence test:

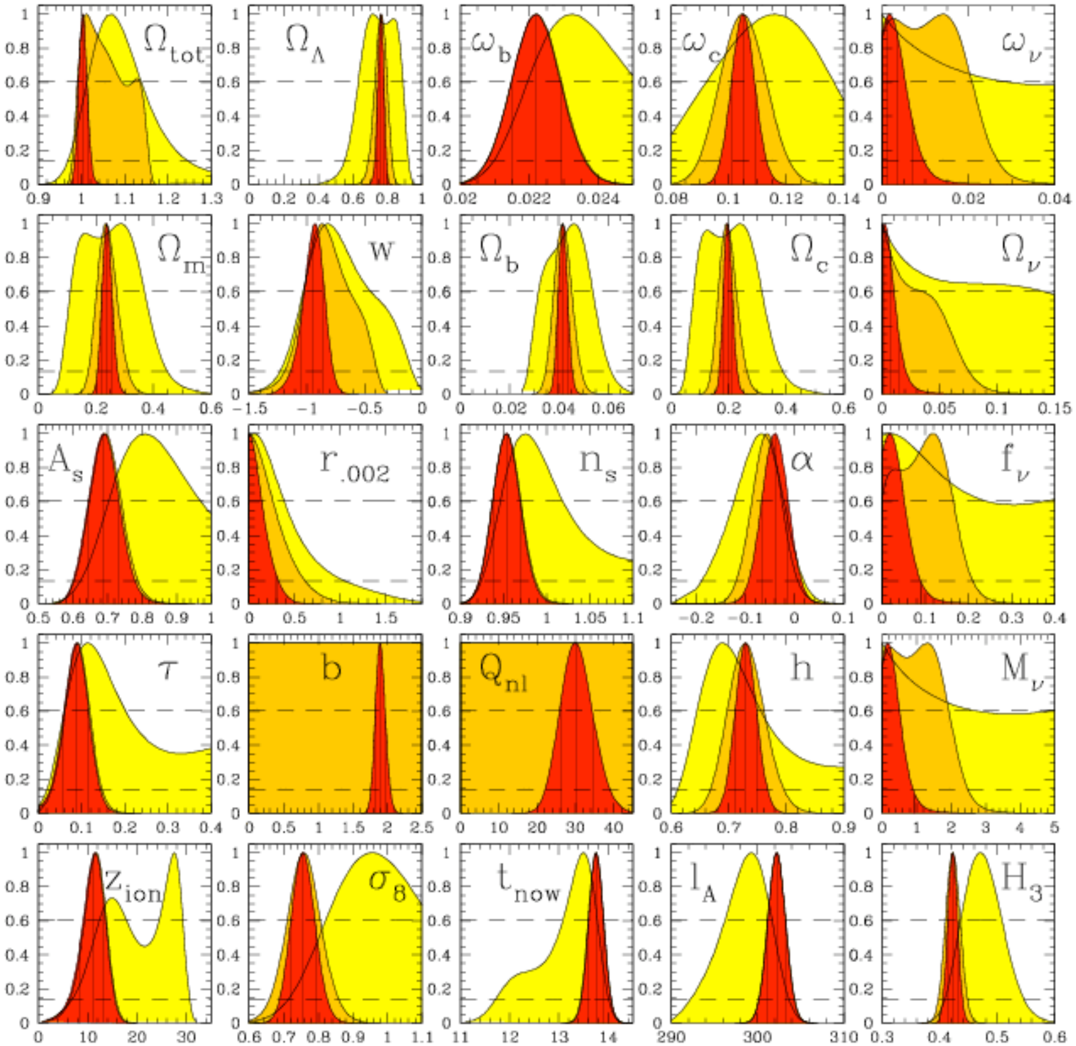
Given M chains (or sections of chain) of length N, Let W be the average variance calculated from individual chains, and B be the variance in the mean recovered from the M chains. Define

$$R = \frac{N-1}{N} + \frac{1}{W}\left(1 + \frac{B}{N}\right)$$

Then R is the ratio of two estimates of the variance. The numerator is unbiased if the chains fully sample the target, otherwise it is an overestimate. The denominator is an underestimate if the chains have not converged.
Test: set a limit R<1.1



Resulting constraints



Tegmark et al, 2006, arXiv:0608632

Further reading

- Power spectrum measurement
  - Feldman, Kaiser & Peacock (1994), ApJ 426, 23
  - Hamilton (1997), astro-ph/9708102
- Correlation function measurement
  - Landy & Szalay (1993), ApJ 412, 64
- Fisher matrix
  - BAO: Seo & Eisenstein (2003), ApJ, 598, 720
  - z-space distortions: White, Song & Percival (2009), MNRAS, 397, 1348
- Combined constraints (for example)
  - Sanchez et al. (2005), astro-ph/0507538
  - Tegmark et al. (2006), astro-ph/0608632
  - Spergel et al. (2007), ApJSS, 170, 3777

Current Galaxy Clustering Measurements and Cosmological Constraints

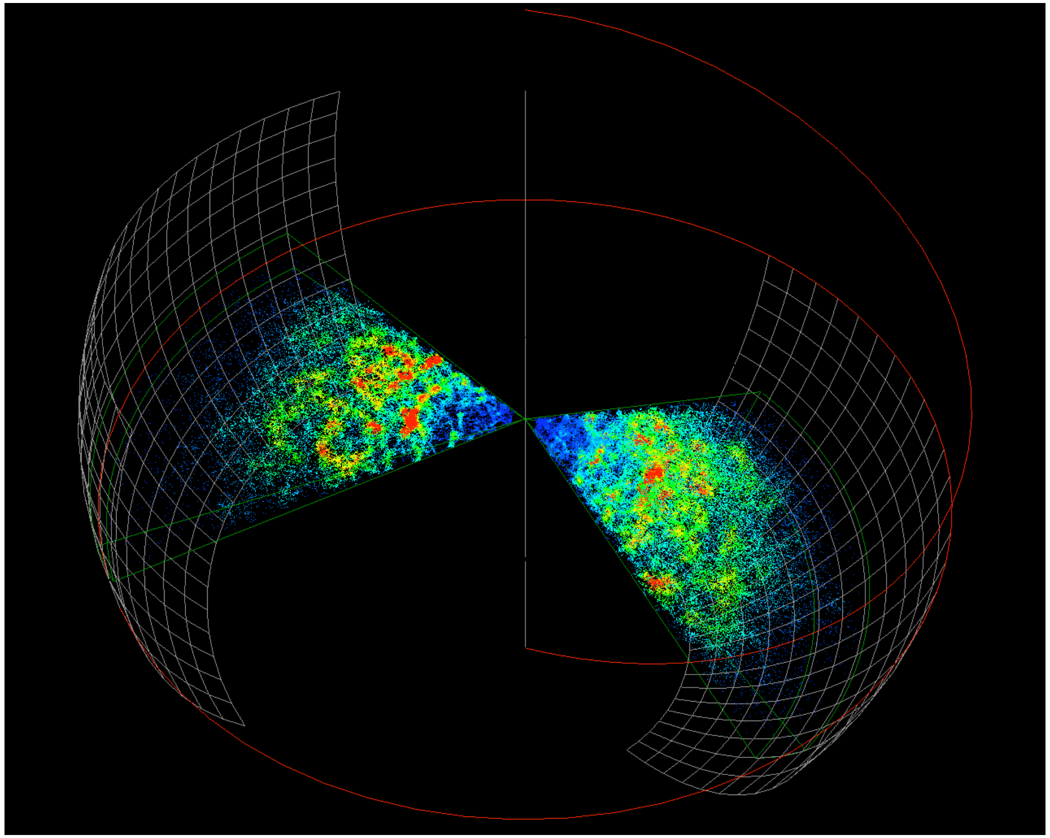
Will Percival

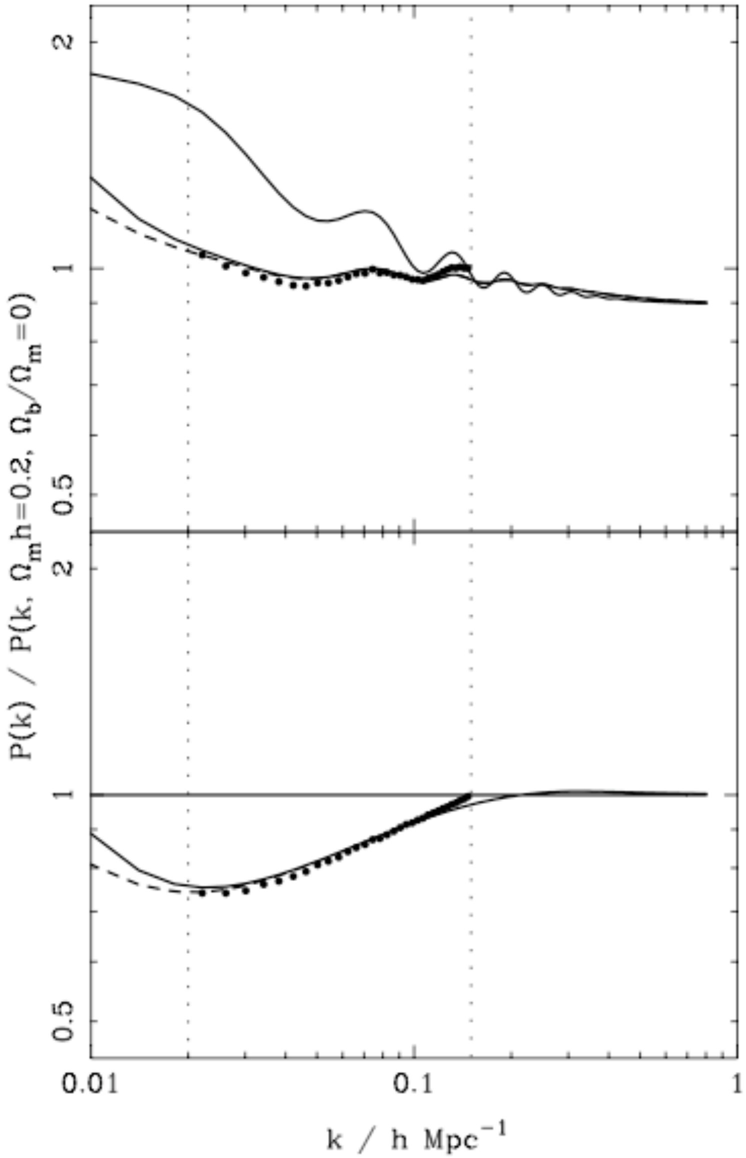
ICG, University of Portsmouth, UK

Lecture outline

- review of 2-pt analyses from SDSS and 2dFGRS (apologies for incomplete & often biased nature)
  - $\Omega_m h$ constraints from P(k) shape
  - BAO detection
- Latest SDSS DR7 analyses (by myself and collaborators)
  - LRG P(k)
  - BAO observations
  - combination with WMAP data

Analysis of the part-complete 2dFGRS

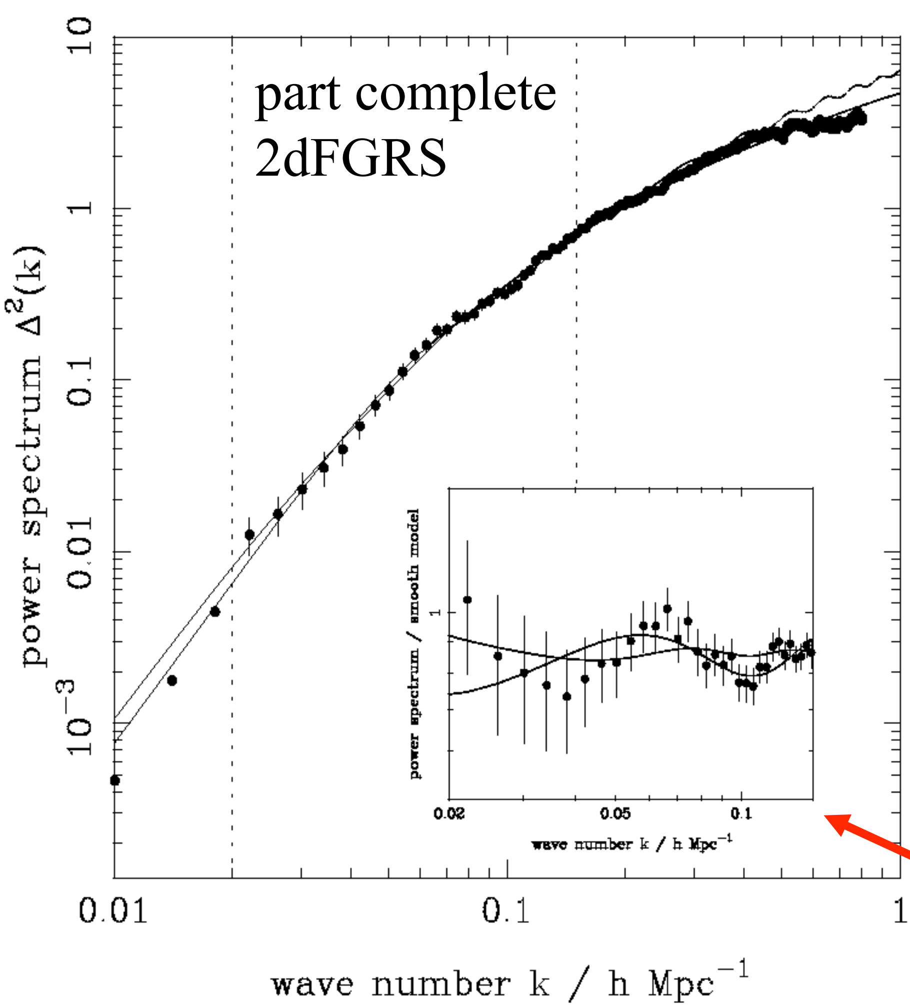


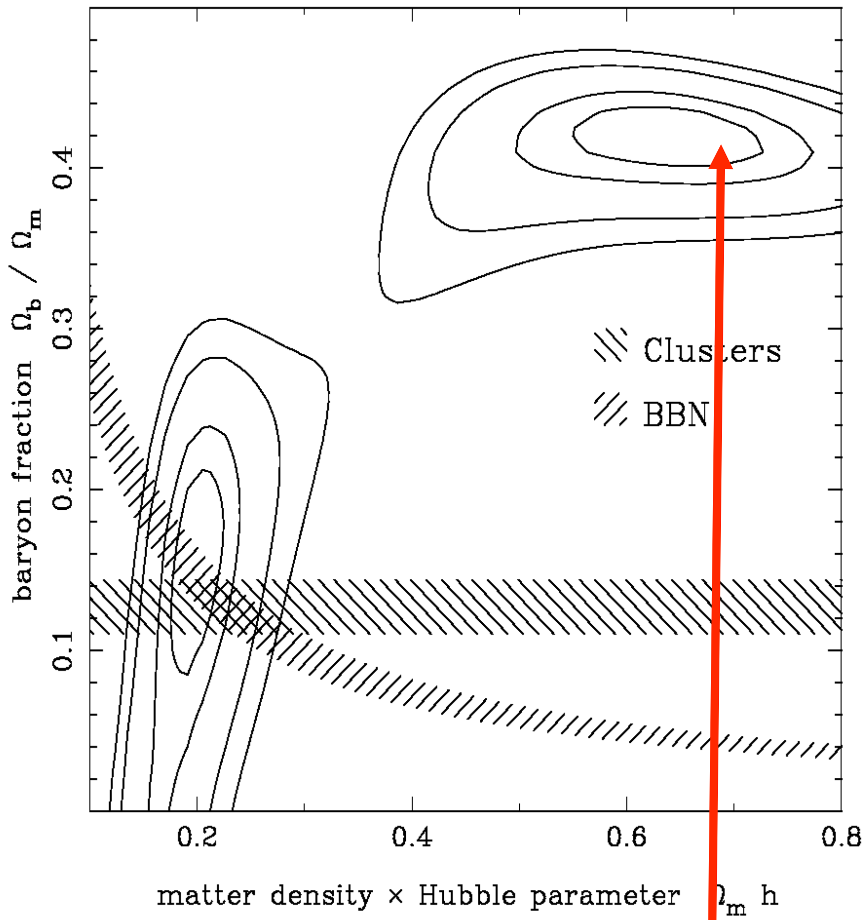


Analysed the part-complete 2dFGRS (147000 galaxies)

Percival et al. 2001, MNRAS, 327, 1297

Analysis of the part-complete 2dFGRS

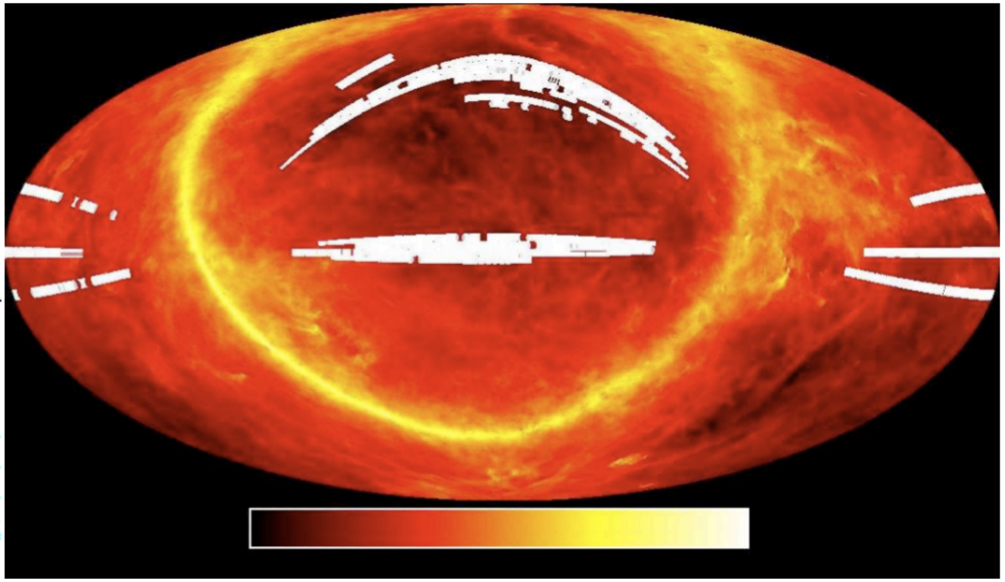


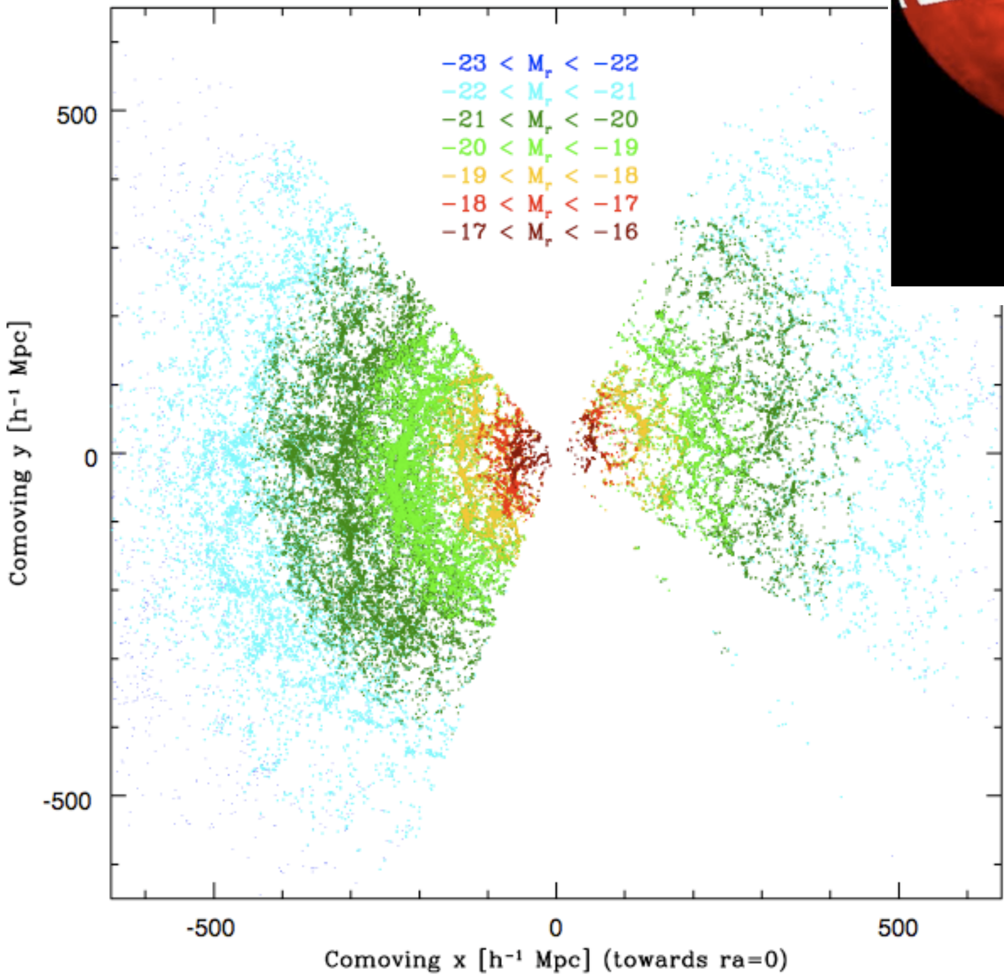


cosmological interpretation of wiggles requires high baryon fraction

Percival et al. 2001, MNRAS, 327, 1297

Analysis of the SDSS DR2 main galaxies

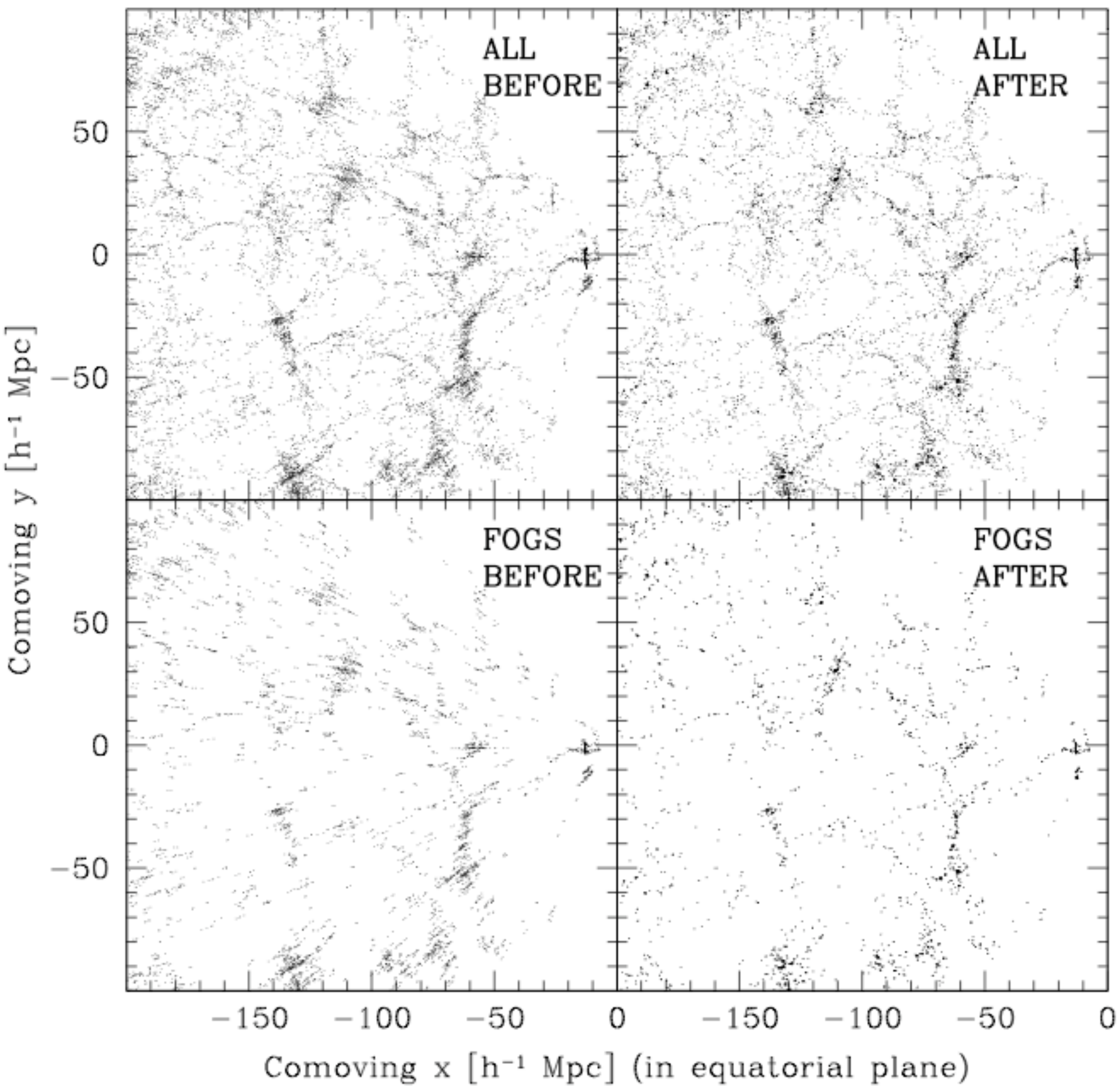




Analysed the SDSS ~DR2 main galaxy sample

Tegmark et al. 2004, arXiv:0310725

Analysis of the SDSS DR2 main galaxies

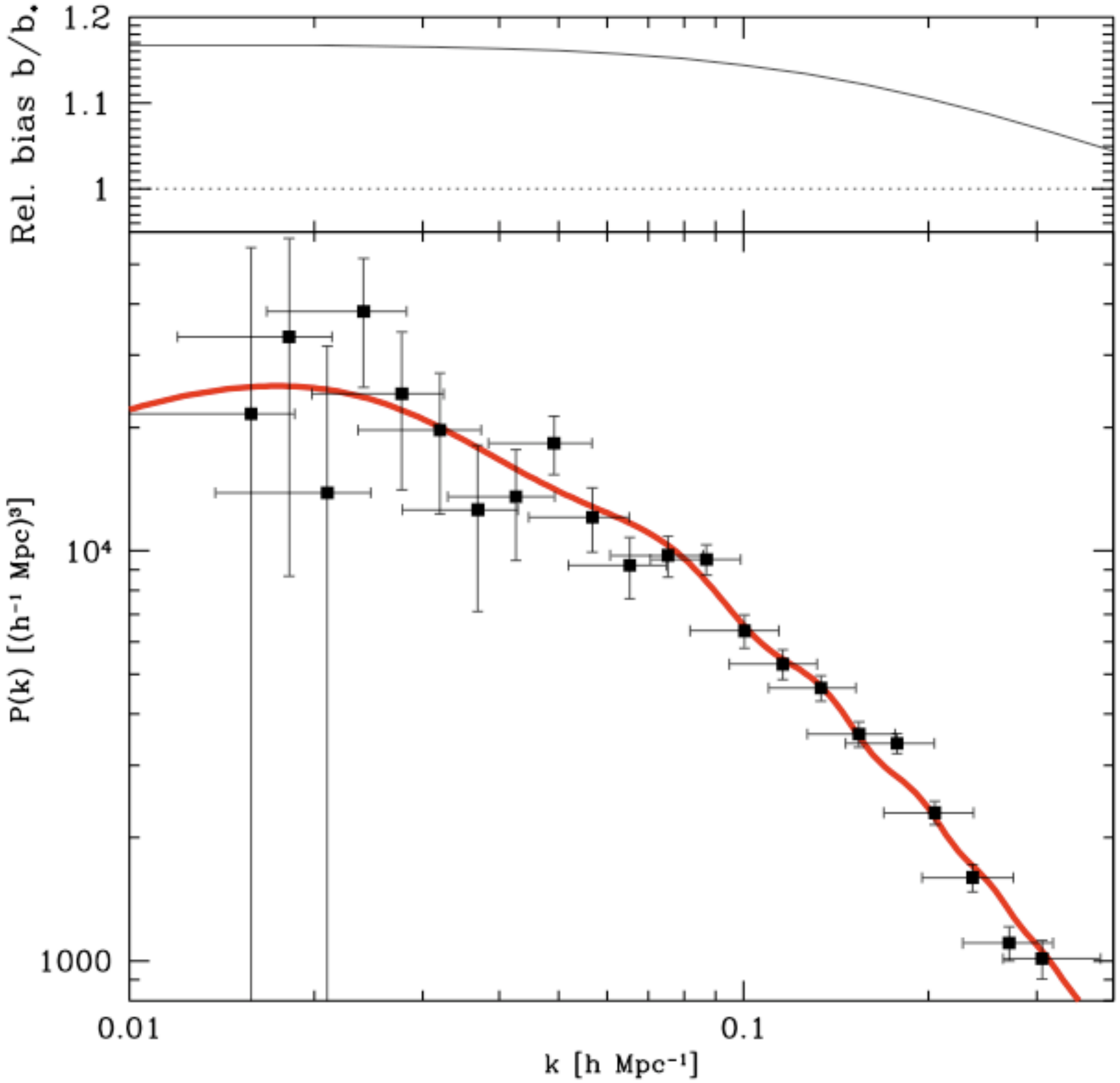


Cluster collapsing, can be used to remove FOG

Need to be very careful: power spectrum shape is quite sensitive to this

Tegmark et al. 2004, arXiv:0310725

Analysis of the SDSS DR2 main galaxies

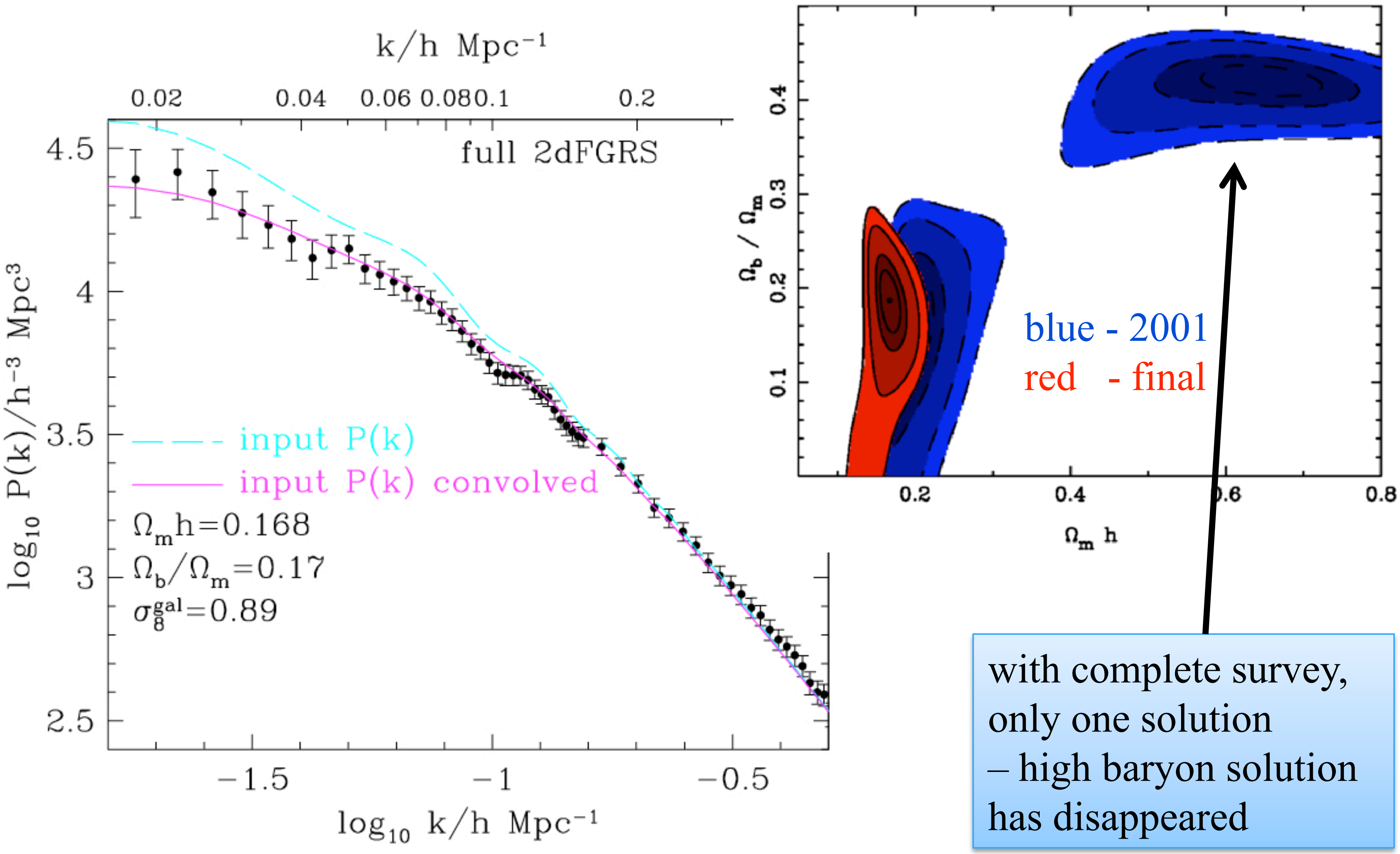


Need to remove the effect of choosing different galaxies to measure different scales

Here, this was done by correcting the power spectrum after it was measured

Tegmark et al. 2004, arXiv:0310725

Analysis of the final 2dFGRS sample



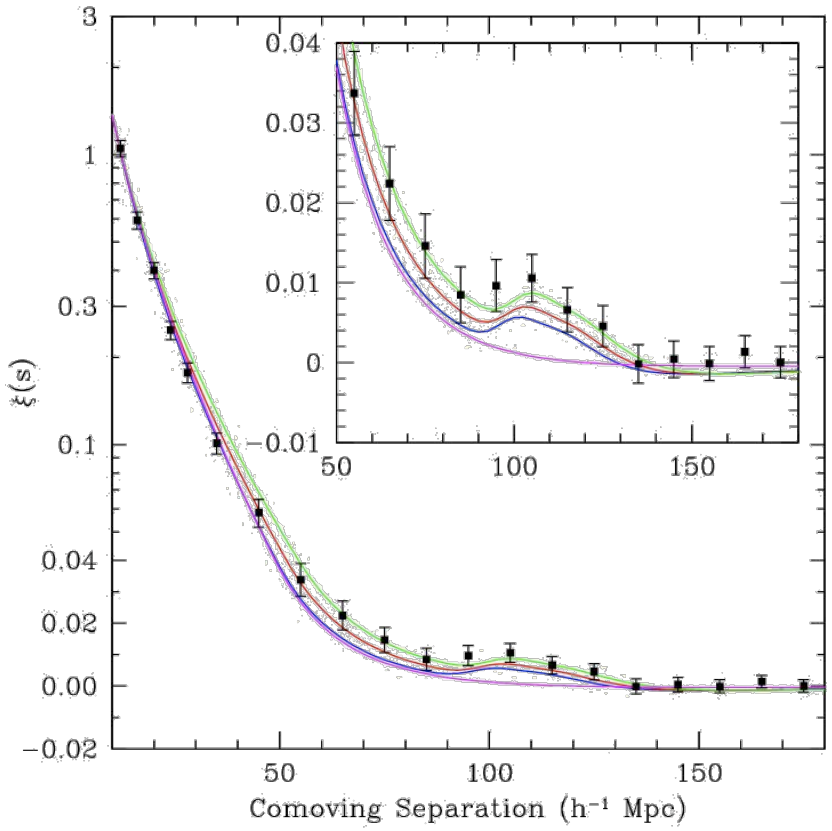
with complete survey, only one solution
– high baryon solution has disappeared

Cole et al. 2005, MNRAS, 362, 505

SDSS DR3 LRG Correlation Function analysis

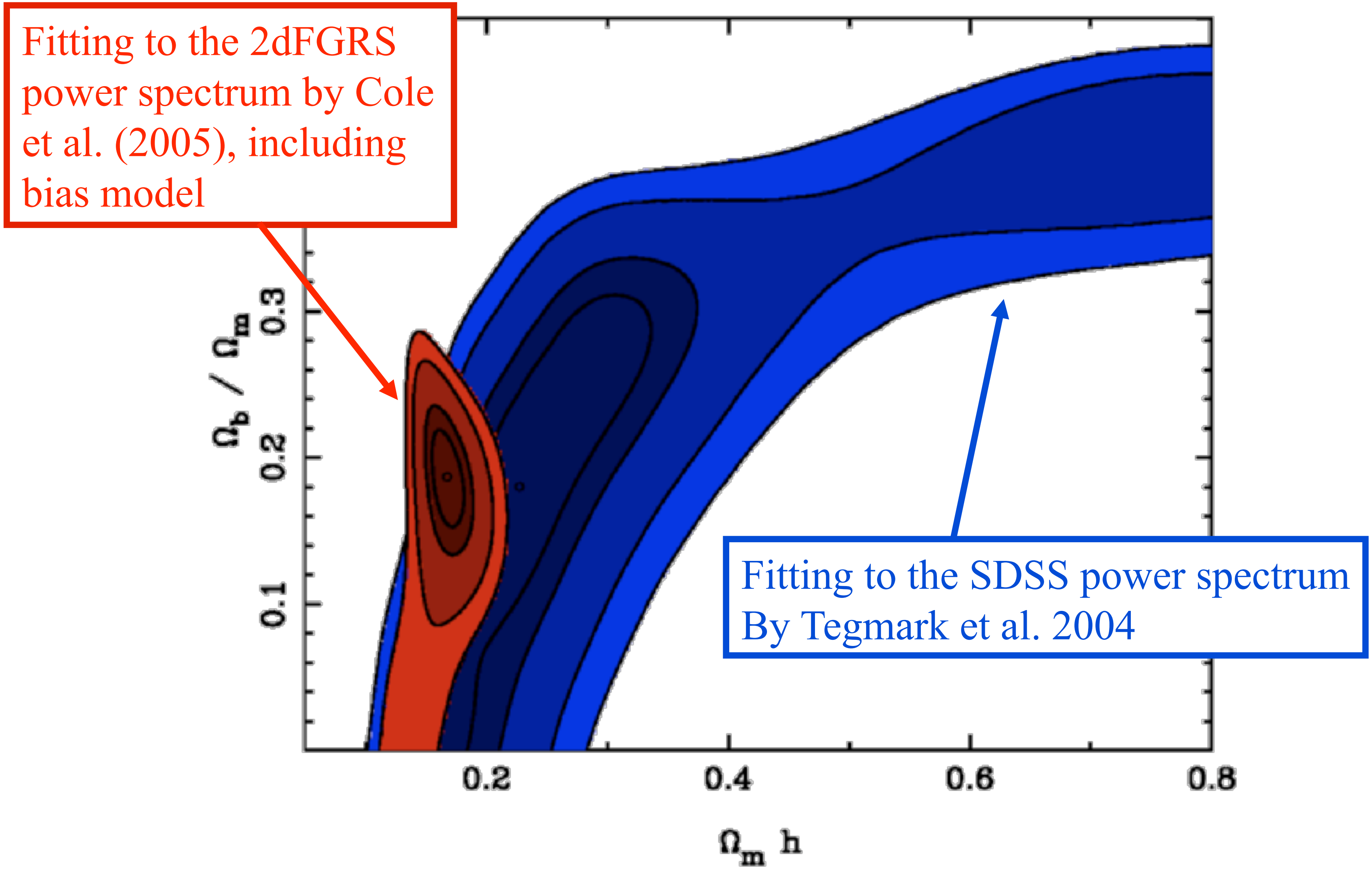
Again, CDM models fit the correlation function adequately well (although peak height is slightly too large) with (assuming n_s=1, h=0.72)

assuming $\Omega_b h^2$ =0.024,
$\Omega_m h^2$ =0.133±0.011,
Giving Ω_b/Ω_m= 0.18



Eisenstein et al., 2005, ApJ, 633, 560

power spectrum shape constraints



Pre-2006 constraints from P(k)

SURVEY	publication	redshifts	method	$W_m h$	f_b	$W_m h$ f_b=0.17
2dFGRS	Percival et al. 2001	166,490	Fourier analysis	0.20 ± 0.03	0.15 ± 0.07	0.206 ± 0.023
2dFGRS	Percival et al. 2004	142,756	Spherical Harmonics			0.215 ± 0.035
2dFGRS	Cole et al. 2005	221,414	Fourier analysis	0.168 ± 0.016	0.185 ± 0.046	0.172 ± 0.014
SDSS	Pope et al. 2004	205,484	KL analysis	0.264 ± 0.043	0.286 ± 0.065	0.207 ± 0.030
SDSS	Tegmark et al. 2004	205,443	Spherical Harmonics			0.225 ± 0.040
SDSS LRGs	Eisenstein et al 2005	46,748	correlation function			0.185 ± 0.015*

*uses $W_b h^2$=0.024, rather than f_b=0.17

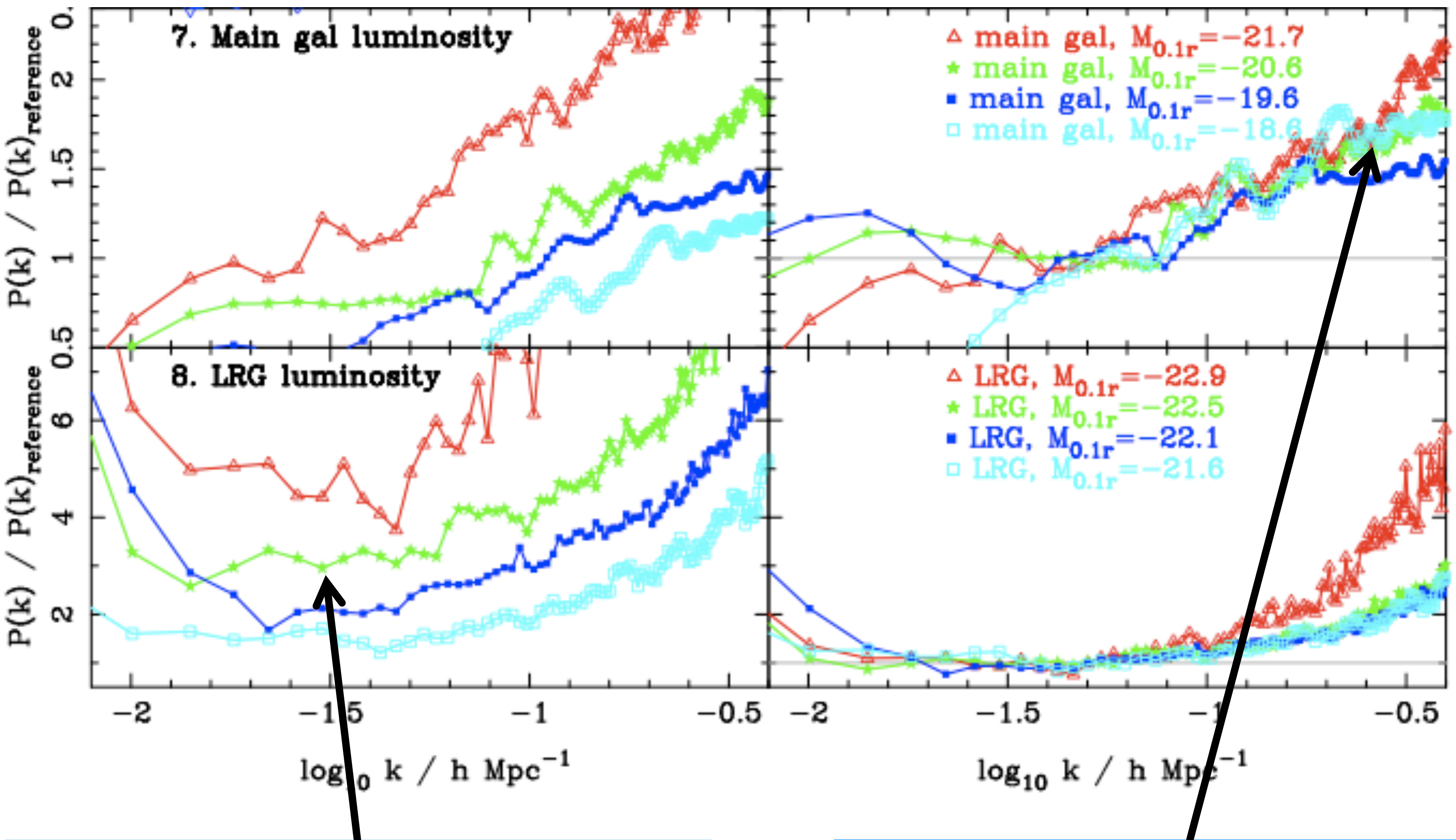
WMAP 3-year analysis found a discrepancy

Parameter	WMAP Only	WMAP +CBI+VSA	WMAP+ACBAR +BOOMERanG	WMAP + 2dFGRS
$100\Omega_b h^2$	$2.233^{+0.072}_{-0.091}$	$2.203^{+0.072}_{-0.090}$	$2.228^{+0.066}_{-0.082}$	$2.223^{+0.066}_{-0.083}$
$\Omega_m h^2$	$0.1268^{+0.0073}_{-0.0128}$	$0.1238^{+0.0066}_{-0.0118}$	$0.1271^{+0.0070}_{-0.0128}$	$0.1262^{+0.0050}_{-0.0103}$
h	$0.734^{+0.028}_{-0.038}$	$0.738^{+0.028}_{-0.037}$	$0.733^{+0.030}_{-0.038}$	$0.732^{+0.018}_{-0.025}$
A	$0.801^{+0.043}_{-0.054}$	$0.798^{+0.047}_{-0.057}$	$0.801^{+0.048}_{-0.056}$	$0.799^{+0.042}_{-0.051}$
τ	$0.088^{+0.028}_{-0.034}$	$0.084^{+0.031}_{-0.038}$	$0.084^{+0.027}_{-0.034}$	$0.083^{+0.027}_{-0.031}$
n_s	$0.951^{+0.015}_{-0.019}$	$0.945^{+0.015}_{-0.019}$	$0.949^{+0.015}_{-0.019}$	$0.948^{+0.014}_{-0.018}$
σ_8	$0.744^{+0.050}_{-0.060}$	$0.722^{+0.044}_{-0.056}$	$0.742^{+0.045}_{-0.057}$	$0.737^{+0.033}_{-0.045}$
Ω_m	$0.238^{+0.027}_{-0.045}$	$0.229^{+0.026}_{-0.042}$	$0.239^{+0.025}_{-0.046}$	$0.236^{+0.016}_{-0.029}$

Parameter	WMAP+ SDSS	WMAP+ LRG	WMAP+ SNLS	WMAP + SN Gold	WMAP+ CFHTLS
$100\Omega_b h^2$	$2.233^{+0.062}_{-0.086}$	$2.242^{+0.062}_{-0.084}$	$2.233^{+0.069}_{-0.088}$	$2.227^{+0.065}_{-0.082}$	$2.247^{+0.064}_{-0.082}$
$\Omega_m h^2$	$0.1329^{+0.0057}_{-0.0109}$	$0.1337^{+0.0047}_{-0.0098}$	$0.1295^{+0.0055}_{-0.0106}$	$0.1349^{+0.0054}_{-0.0106}$	$0.1410^{+0.0042}_{-0.0094}$
h	$0.709^{+0.024}_{-0.032}$	$0.709^{+0.016}_{-0.023}$	$0.723^{+0.021}_{-0.030}$	$0.701^{+0.020}_{-0.026}$	$0.686^{+0.017}_{-0.024}$
A	$0.813^{+0.042}_{-0.052}$	$0.816^{+0.042}_{-0.049}$	$0.808^{+0.044}_{-0.051}$	$0.827^{+0.045}_{-0.053}$	$0.852^{+0.036}_{-0.047}$
τ	$0.079^{+0.029}_{-0.032}$	$0.082^{+0.028}_{-0.033}$	$0.085^{+0.028}_{-0.032}$	$0.079^{+0.028}_{-0.034}$	$0.088^{+0.021}_{-0.031}$
n_s	$0.948^{+0.015}_{-0.018}$	$0.951^{+0.014}_{-0.018}$	$0.950^{+0.015}_{-0.019}$	$0.946^{+0.015}_{-0.019}$	$0.950^{+0.015}_{-0.019}$
σ_8	$0.772^{+0.036}_{-0.048}$	$0.781^{+0.032}_{-0.045}$	$0.758^{+0.038}_{-0.052}$	$0.784^{+0.035}_{-0.049}$	$0.826^{+0.023}_{-0.035}$
Ω_m	$0.266^{+0.025}_{-0.040}$	$0.267^{+0.017}_{-0.029}$	$0.249^{+0.023}_{-0.034}$	$0.276^{+0.022}_{-0.036}$	$0.301^{+0.018}_{-0.031}$

Spergel et al. 2006, astro-ph/0603449

the problem is scale-dependent bias

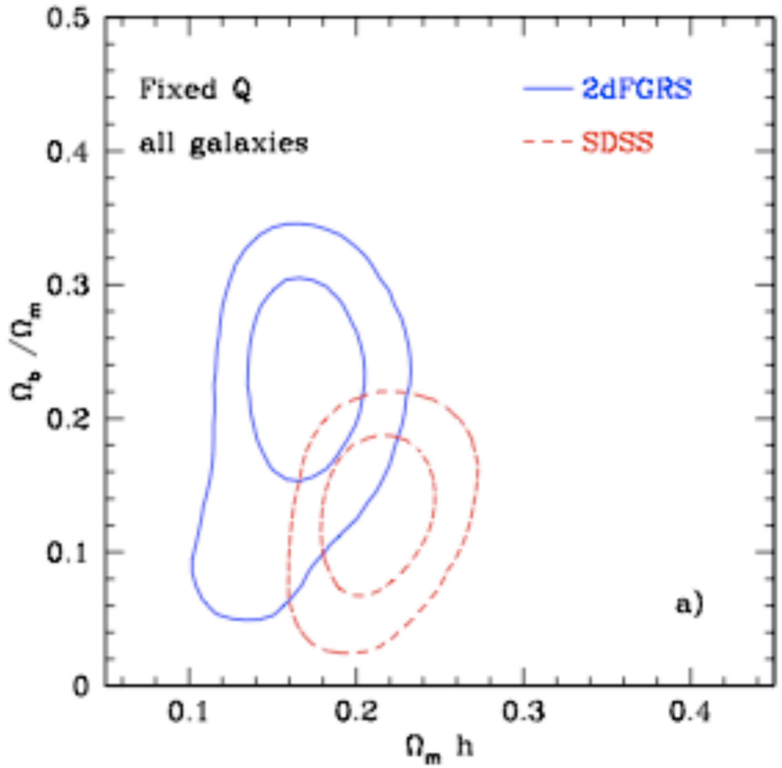


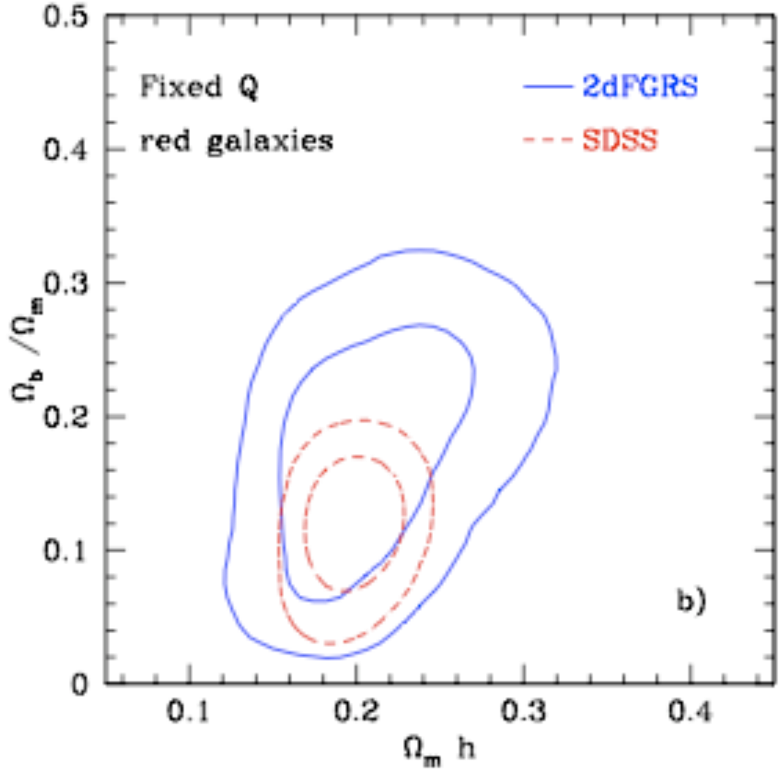
linear behaviour strongly depends on luminosity

non-linear behaviour depends on luminosity

Percival et al. 2007, astro-ph/0608636

the problem is scale-dependent bias





By subdividing 2dFGRS into red and blue galaxies, Sanchez & Cole also concluded that differences with SDSS were caused by scale-dependent galaxy bias

Sanchez & Cole 2007, arXiv:0708.1517

Modeling scale-dependent galaxy bias

Bias in this context means relation between P(k)$_{\text{lin}}$ & P(k)$_{\text{obs}}$

shot noise change (Seljak 2001)

$$P(k)_{\text{obs}} = b^2 P(k)_{\text{lin}} + A$$

Quadratic (Seo & Eisenstein 2005)

$$P(k)_{\text{obs}} = b^2 P(k)_{\text{lin}} + A_0 + A_1 k + A_2 k^2$$

Q model (Cole 2005)

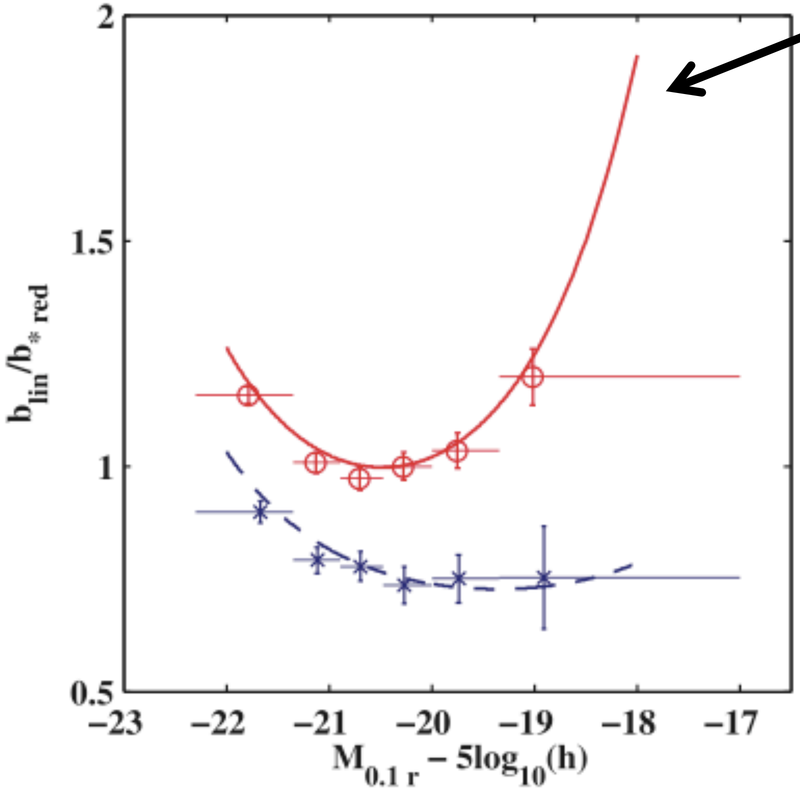
$$P(k)_{\text{obs}} = b^2 P(k)_{\text{lin}} \frac{1 + Qk^2}{1 + Ak}$$

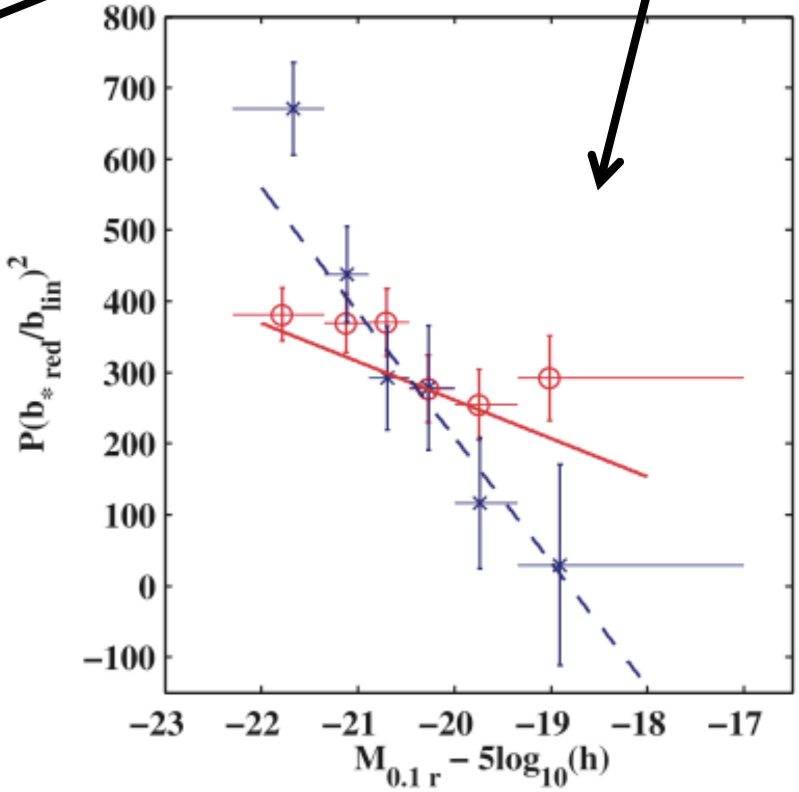
We really want a full model for galaxy formation

see review by Smith et al. 2007, astro-ph/0609547

Measuring scale-dependent galaxy bias

shot noise change (Seljak 2001) $P_g(k) = b_{\rm lin}^2 P_{\rm lin}(k) + P$





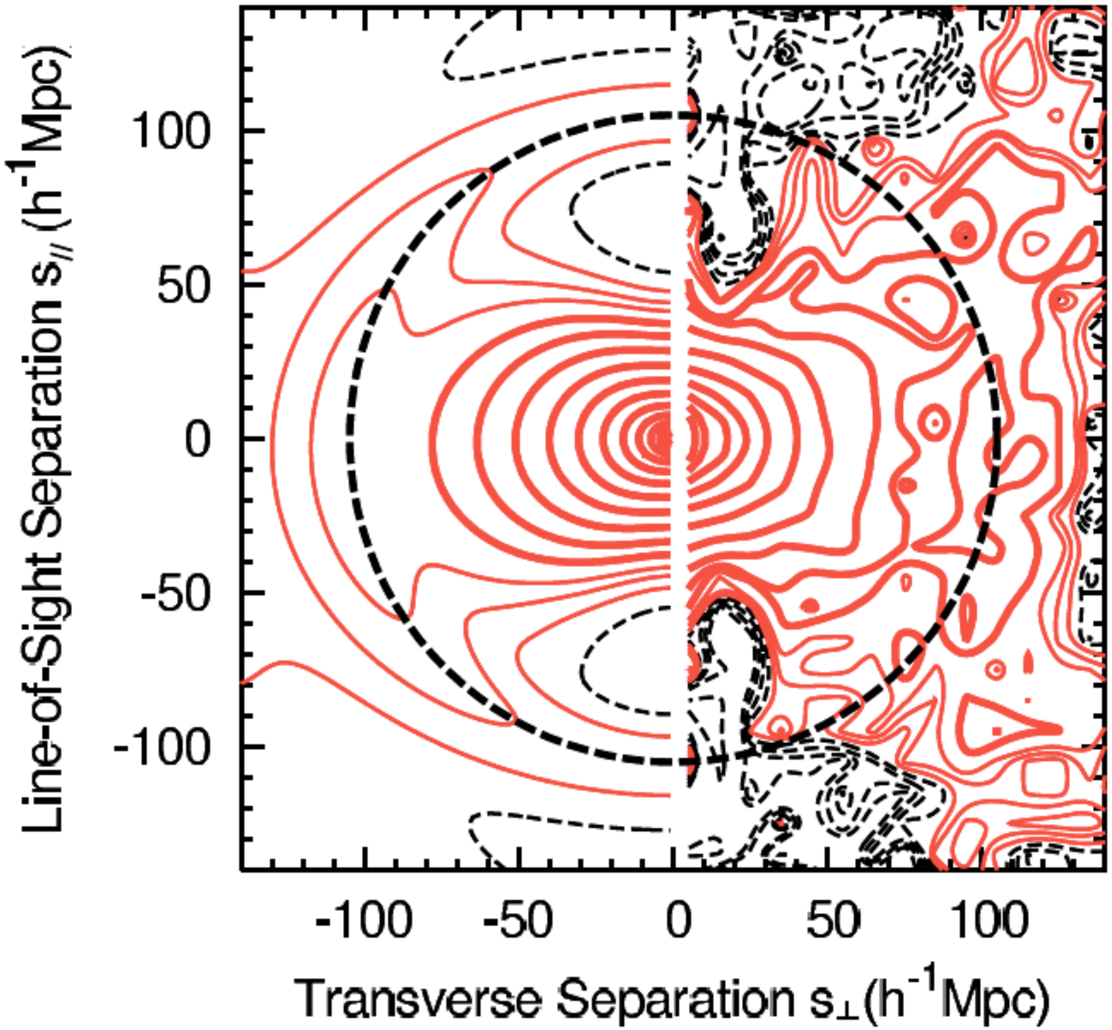
linear behaviour strongly depends on luminosity and colour

As does the non-linear departure from the linear power spectrum

Cresswell & Percival 2009, MNRAS 392, 682

Anisotropic ξ analysis of the SDSS DR3 LRGs

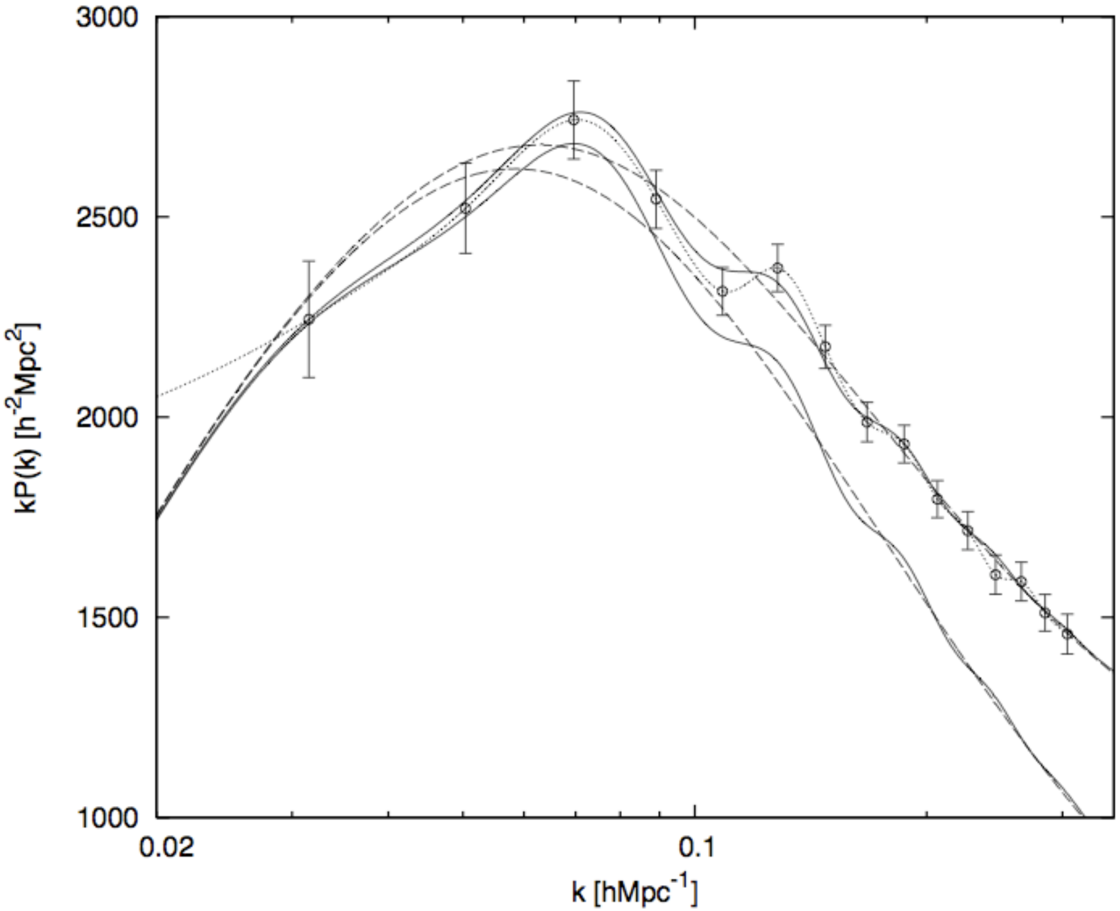
Linear theory prediction



Contours of SDSS LRG correlation function ~DR3 sample

Okumura et al., 2007, arXiv:0711.3640

power spectrum analysis of SDSS DR4 LRGs

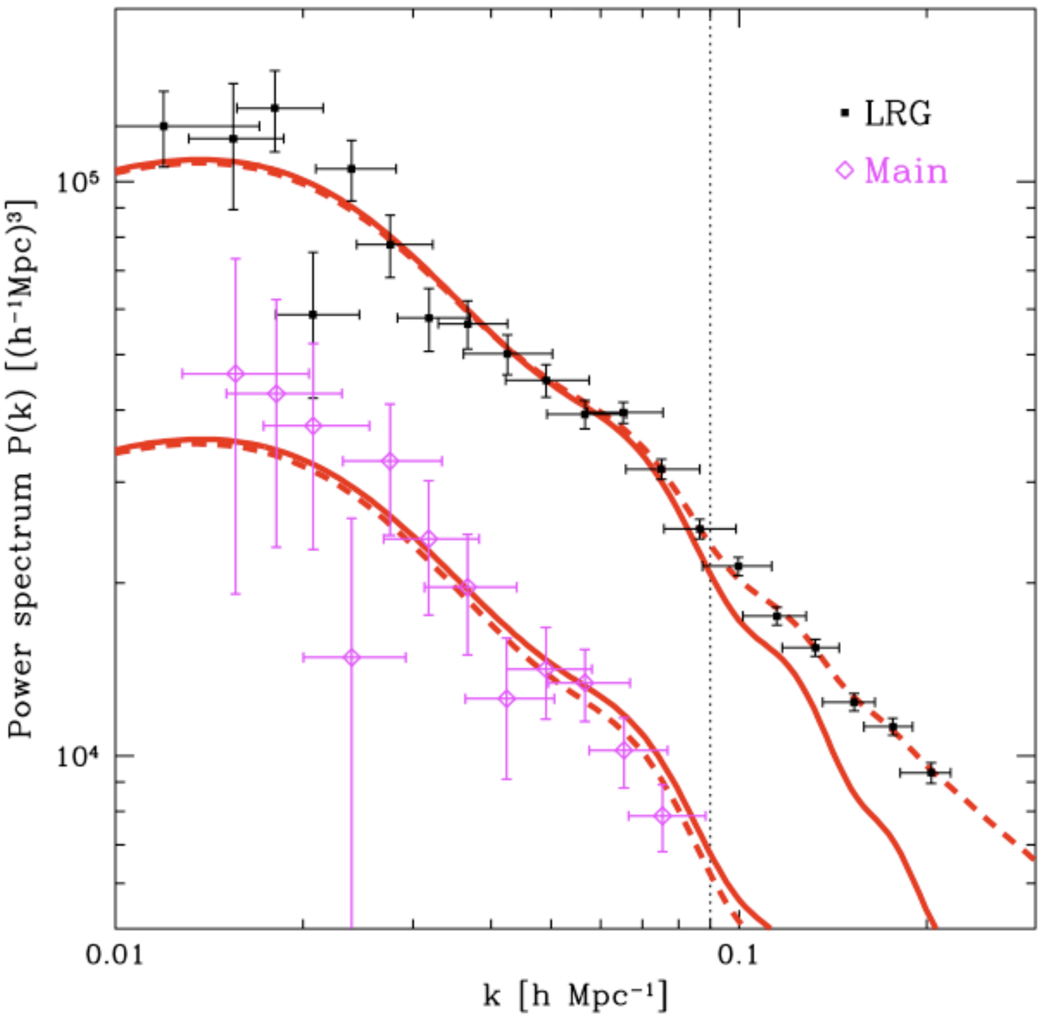


Analysis of SDSS DR4 LRG sample (approx ½ of final DR7 sample) shows strong oscillations, detected at 3.3σ

BAO location is consistent with Eisenstein et al. analysis of DR3

Huetsi, 2005, arXiv:0512201

power spectrum analysis of SDSS DR4 LRGs

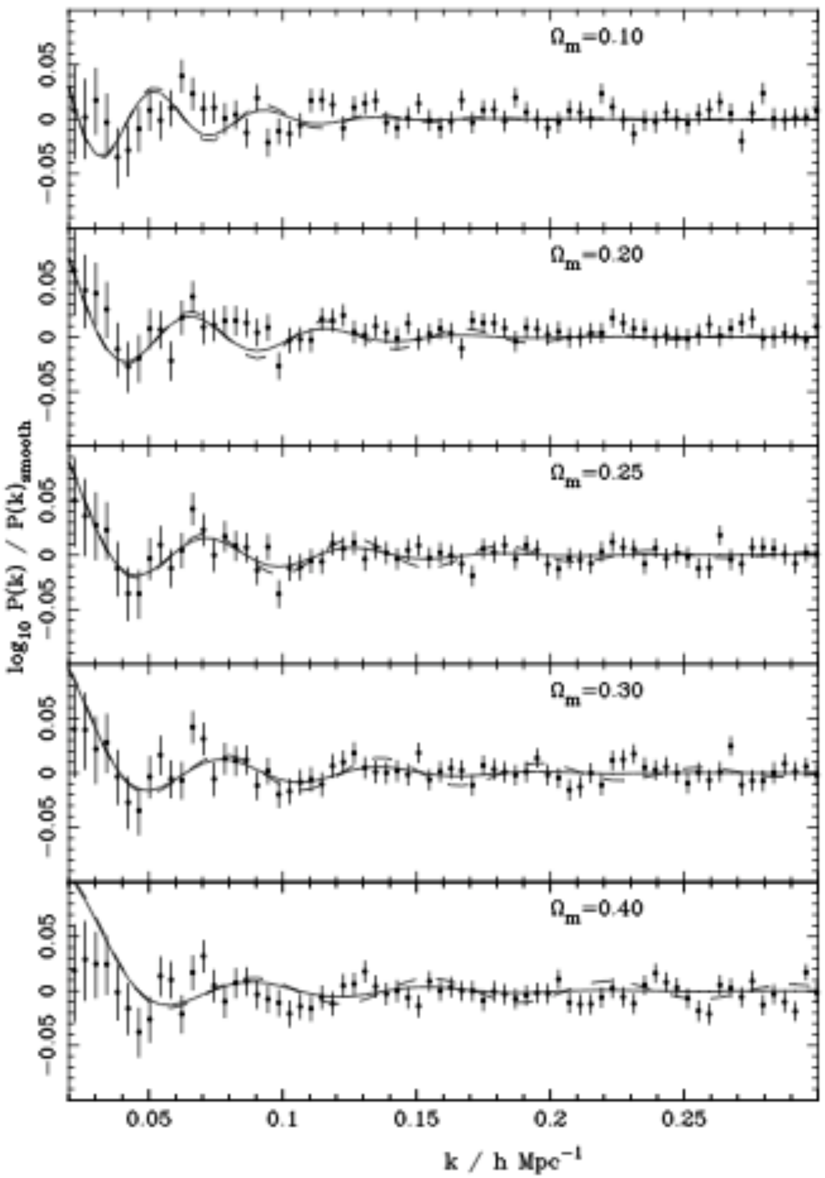


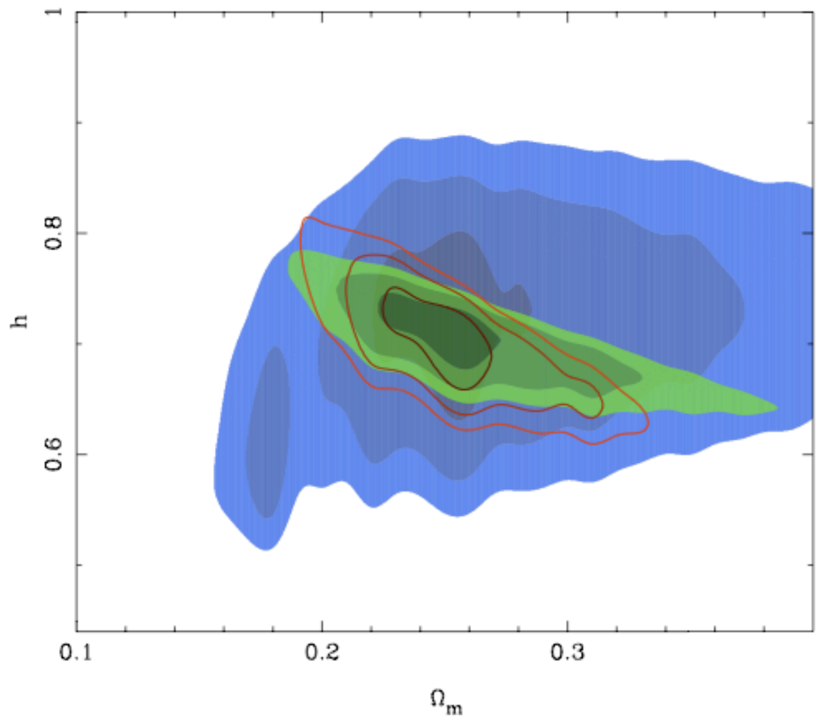
Full analysis of SDSS DR4 LRG and main galaxy samples

Analysis included FOG compression, but shown by Reid et al. (2008; arXiv: 0811.1025) to be too agressive

Tegmark et al, 2006, arXiv:0608632

BAO of SDSS DR5 galaxies

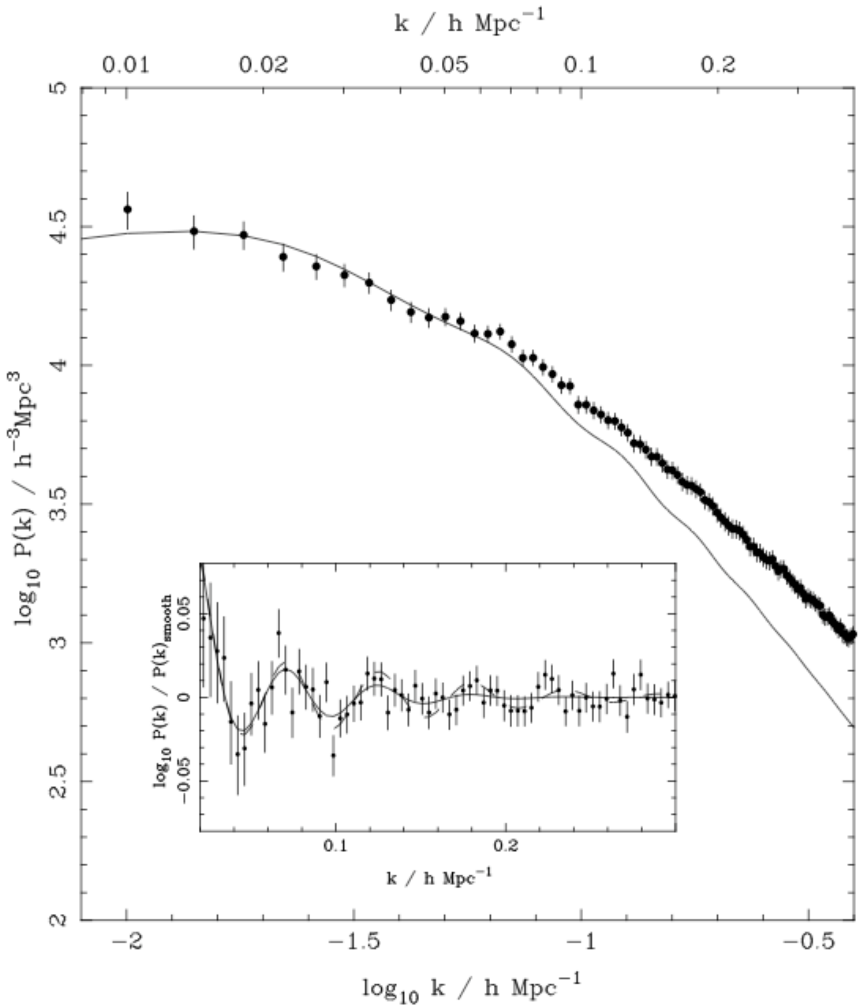


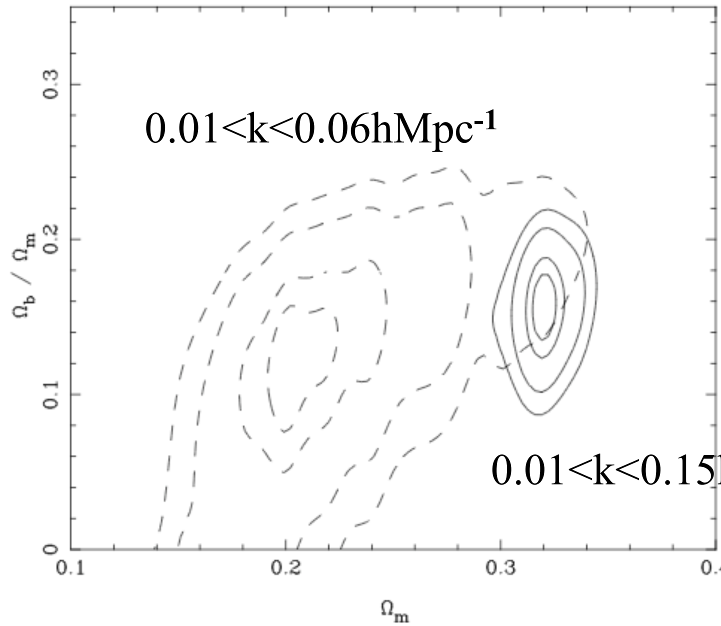


Isolated BAO in power spectrum by dividing by a spline fit. Measured Ω_m=0.256±0.027 for flat ΛCDM models

Percival et al, 2007a, arXiv:0608635

Power spectrum of SDSS DR5 galaxies

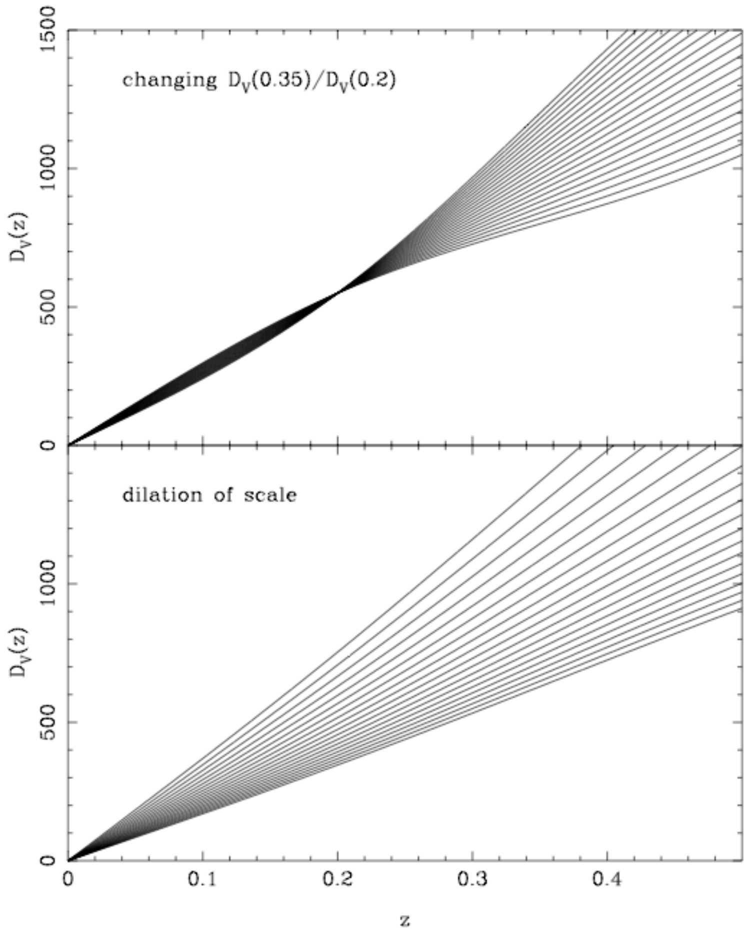




Measured P(k) for SDSS DR5 and showed that the shape is strongly dependent on the range of scales fitted. Any derived shape constraints are almost completely degenerate with the galaxy bias model assumed

Percival et al, 2007a, arXiv:0608636

modeling the distance-redshift relation



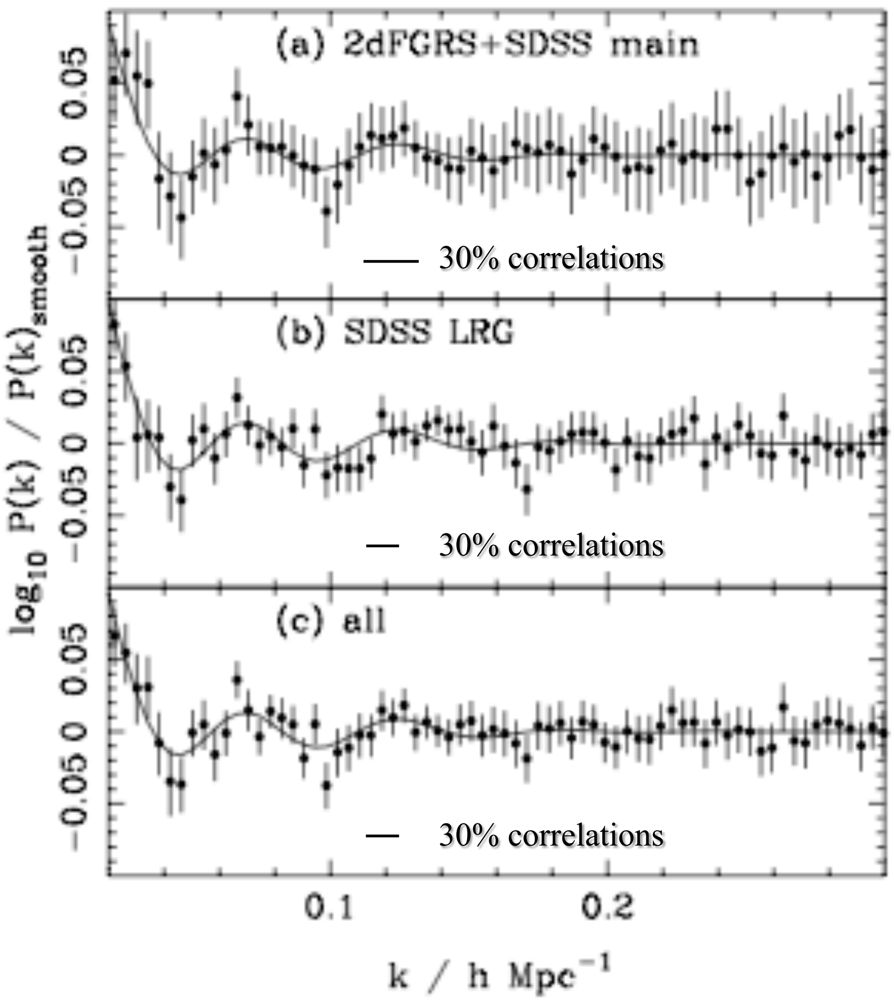
Galaxy redshifts need to be converted to distances before BAO can be measured

Not a problem for small sets of models (1-2 parameters), but time consuming for more

Solve problem by parametrising distance-redshift relation by smooth fit: can then be used to constrain multiple sets of models

For SDSS+2dFGRS analysis, choose two modes at z=0.2 and z=0.35, for fit to D_V

BAO from the 2dFGRS + SDSS DR5



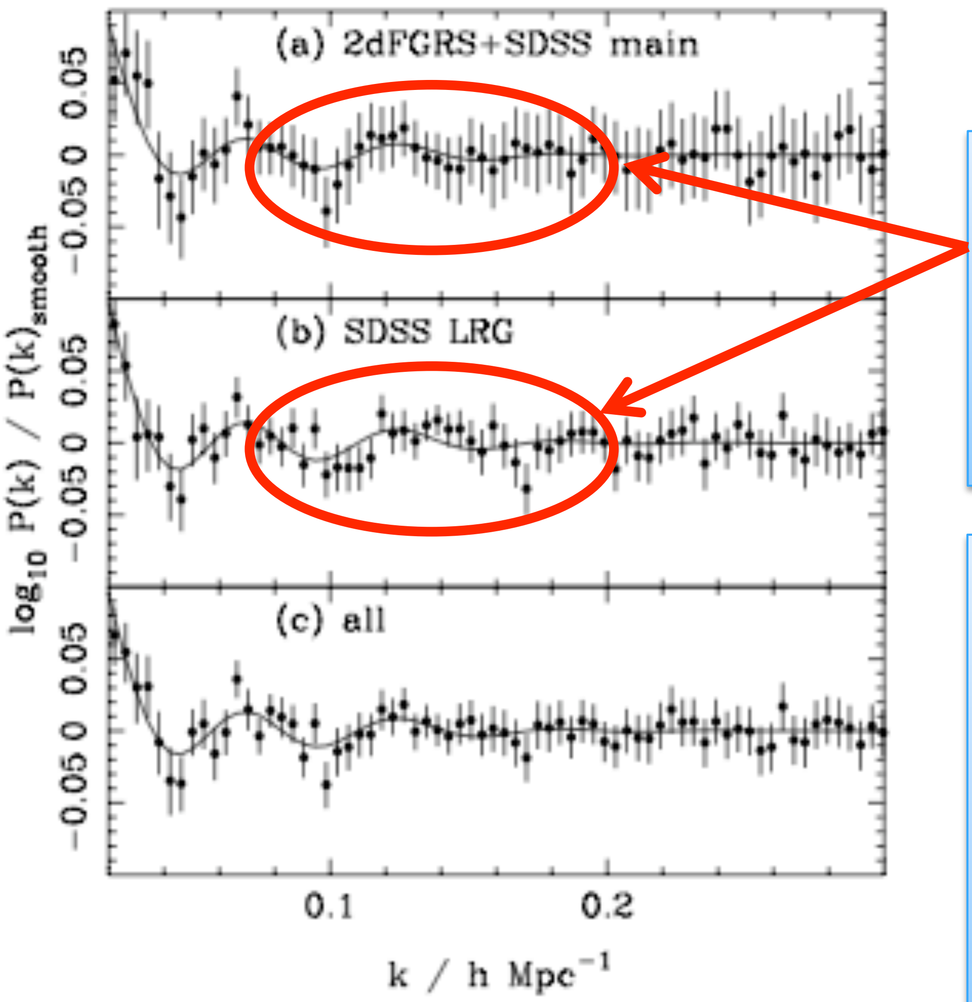
BAO detected at z~0.2

BAO detected at z~0.35

BAO from combined sample

Percival et al., 2007, MNRAS, 381, 1053

Discrepancy with ΛCDM?



LRG BAO on too small scales: further away than expected, so more acceleration between z=0.2 and 0.35

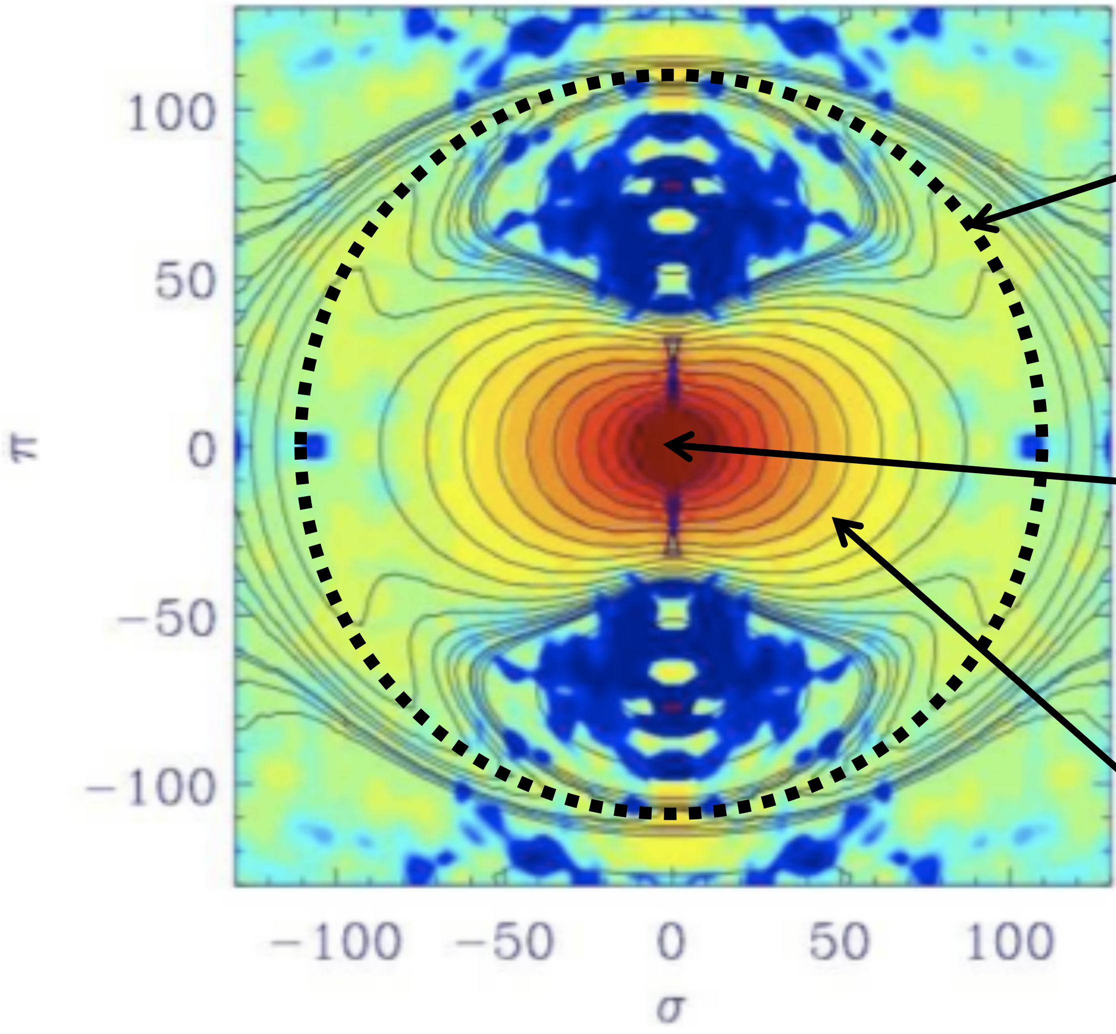
Distance ratio found is $D_V(0.35)/D_V(0.2) = 1.812 \pm 0.060$

CDM expects $D_V(0.35)/D_V(0.2) = 1.67$

Discrepancy is 2.4σ

Percival et al., 2007, MNRAS, 381, 1053

Anisotropic ξ analysis of the SDSS DR6 LRGs



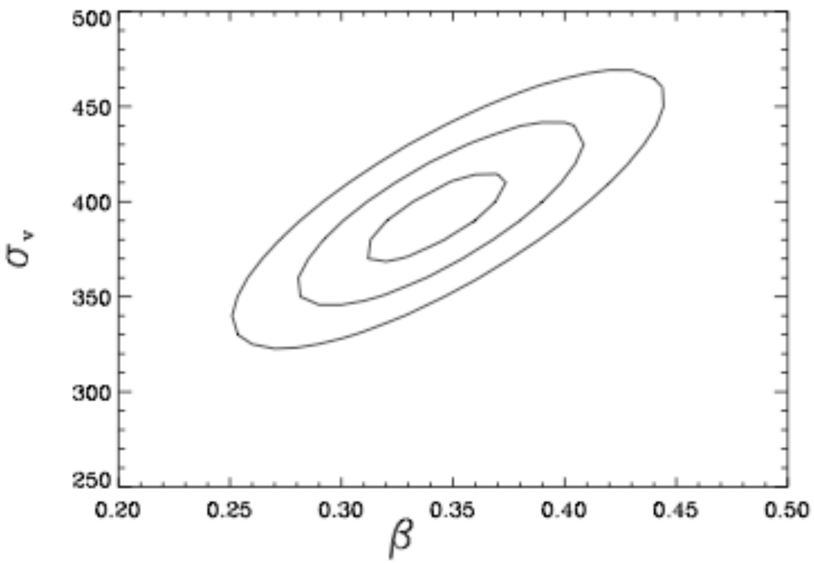
$\xi(\pi = r_{parallel}, \sigma = r_{perp})$ for LRG sample showing BAO ring

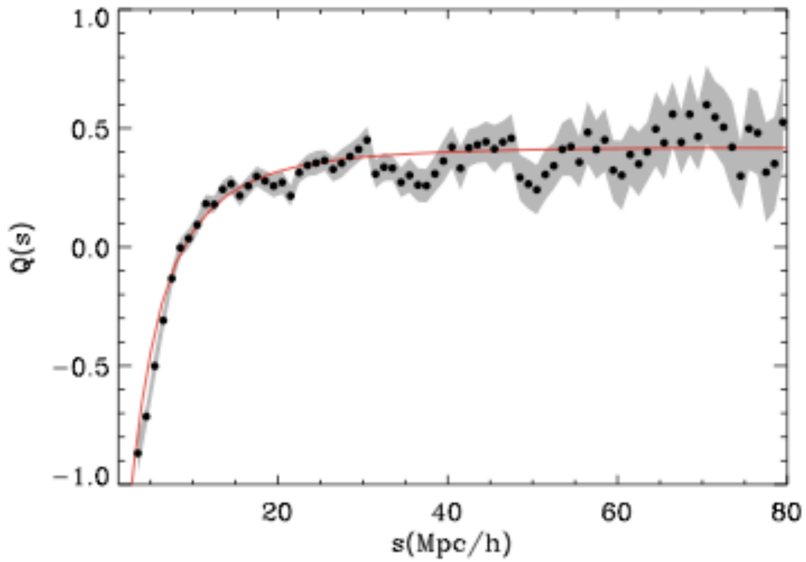
Pixel size in this analysis means that the FOG signal is washed out

The quadrupole signal from linear redshift-space distortions is clearly visible

Cabre & Gaztanaga, 2008, arXiv:0807.2460
Gaztanaga et al., 2008, arXiv:0807.3551

z-space distortions in the SDSS DR6 LRGs





Use multipoles of ξ, and define the quadrupole Q(s)

$$\xi_\ell(s) = \frac{2\ell+1}{2}\int_{-1}^{+1} \xi(s,\mu) P_\ell(\mu)\, d\mu$$

$$Q(s) = \frac{\xi_2(s)}{\xi_0(s) - (3/s^2)\int_0^s \xi_0(s') s'^2\, ds'}$$

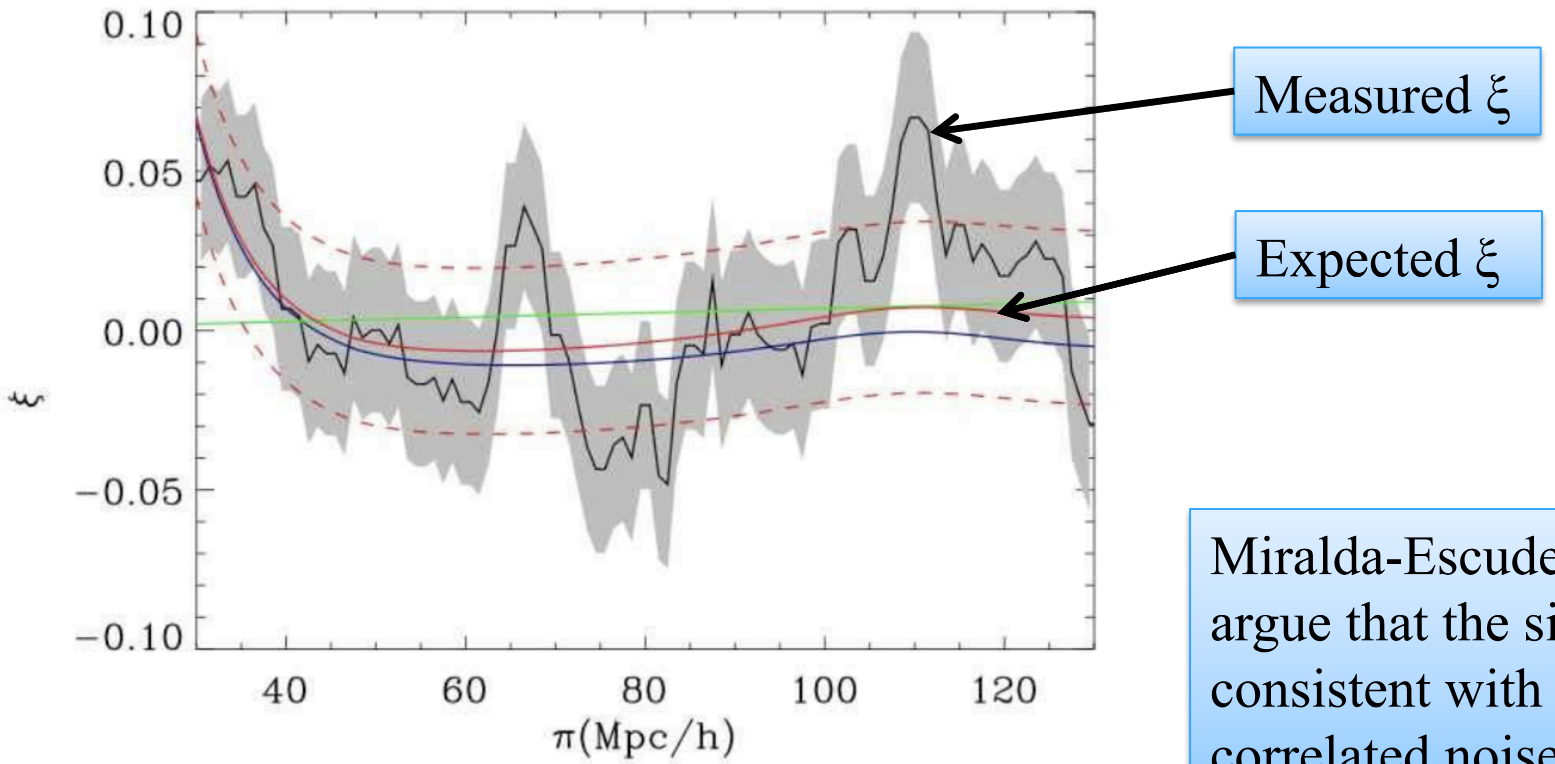
Using linear theory, and in the distant-observer approximation, these can be related to

$$Q(s) = \frac{\frac{4}{3}\beta + \frac{4}{7}\beta^2}{1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2}$$

Giving β=0.34±0.03

Cabre & Gaztanaga, 2008, arXiv:0807.2460

Radial BAO in the SDSS DR6 LRGs?

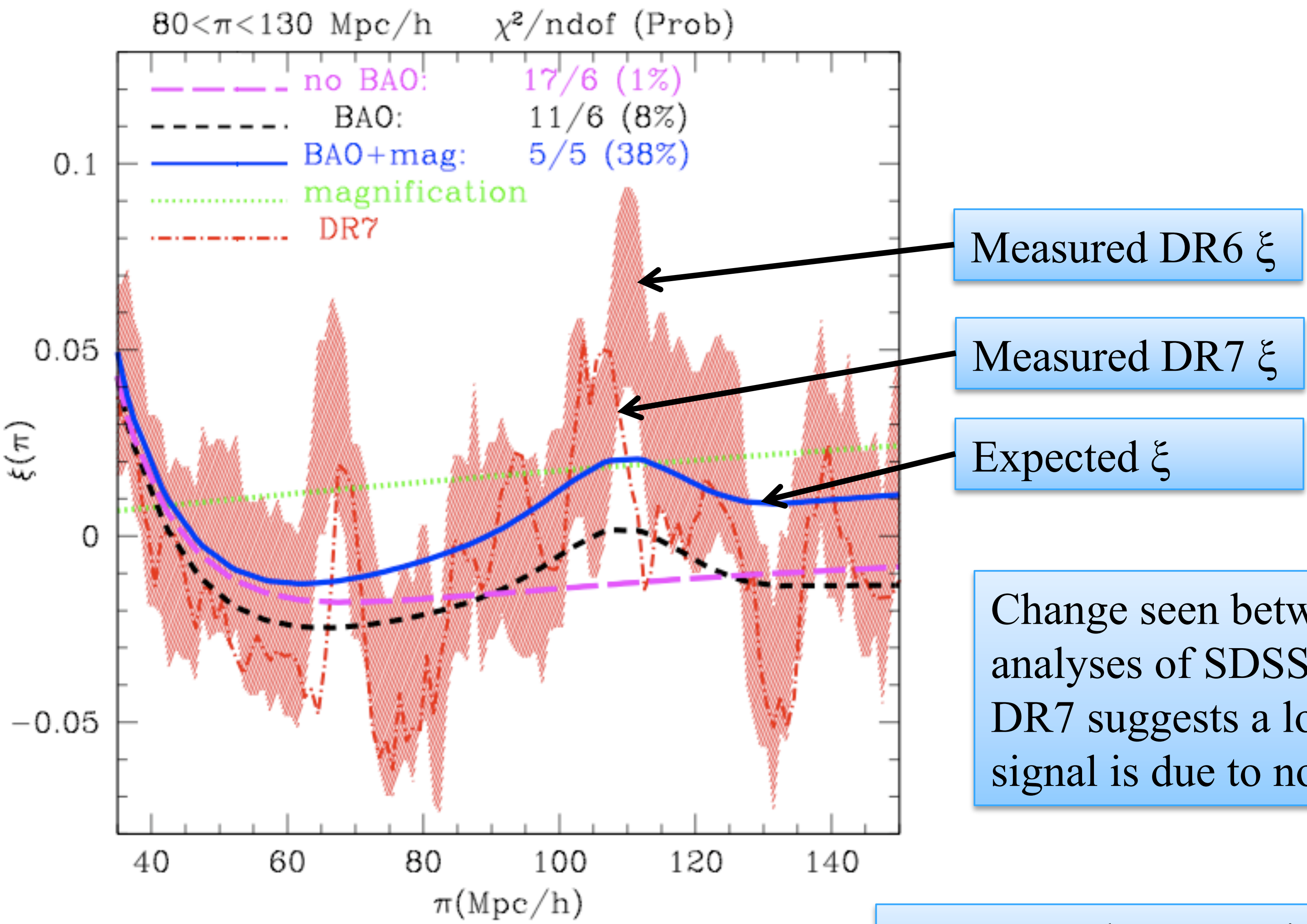


radial slice through $\xi(\pi=r_{parallel},\sigma=r_{perp})$ showing the "BAO feature"

Miralda-Escude (2009) argue that the signal is consistent with correlated noise

Gaztanaga et al., 2008, arXiv:0807.3551
Miralda-Escude, 2009, arXiv:0901.1219

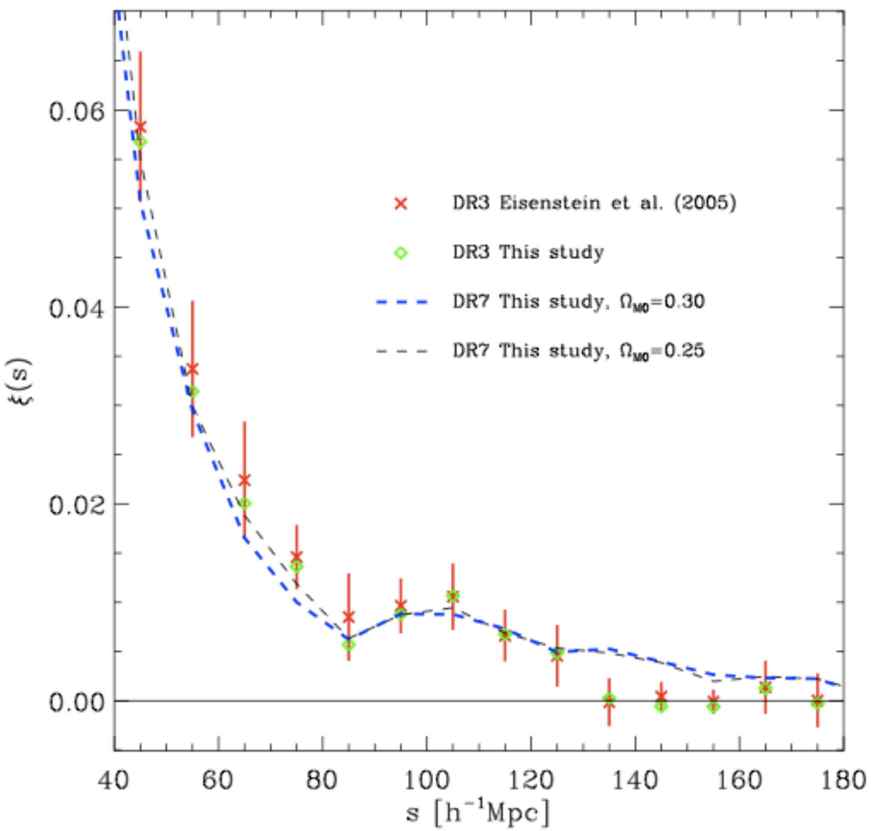
DR7 update on radial BAO signal



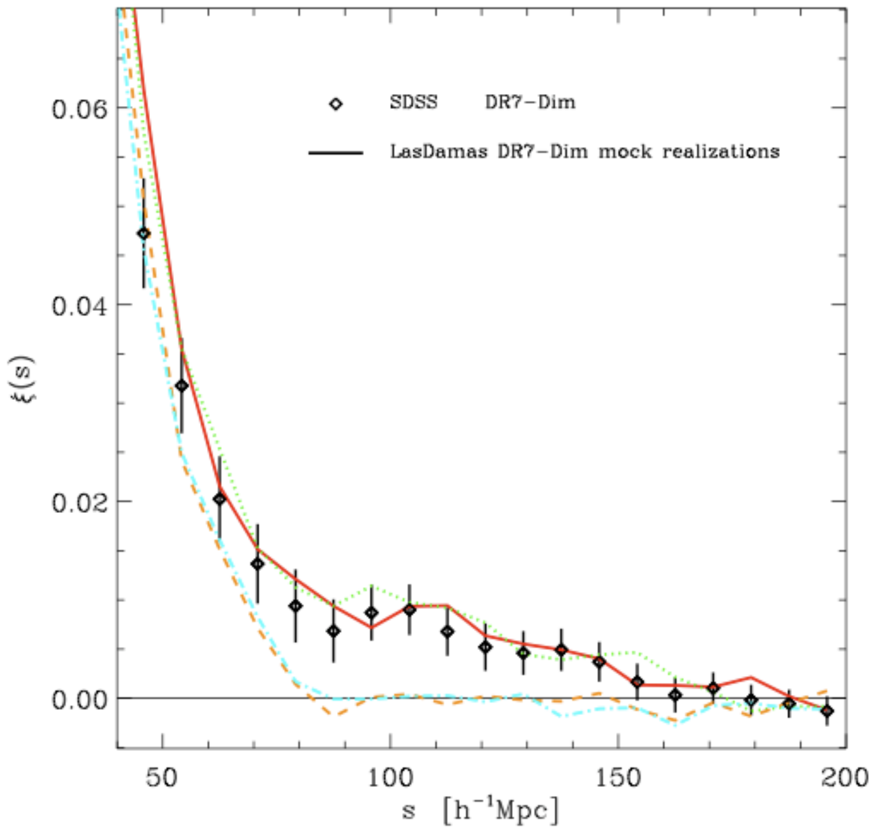
Change seen between analyses of SDSS DR6 and DR7 suggests a lot of the signal is due to noise

Gaztanaga et al., 2008, arXiv:0807.3551
Miralda-Escude, 2009, arXiv:0901.1219

SDSS DR7 LRG correlation function analysis



From analysis of mock catalogues, show that there is a 10% chance that we would not see a peak

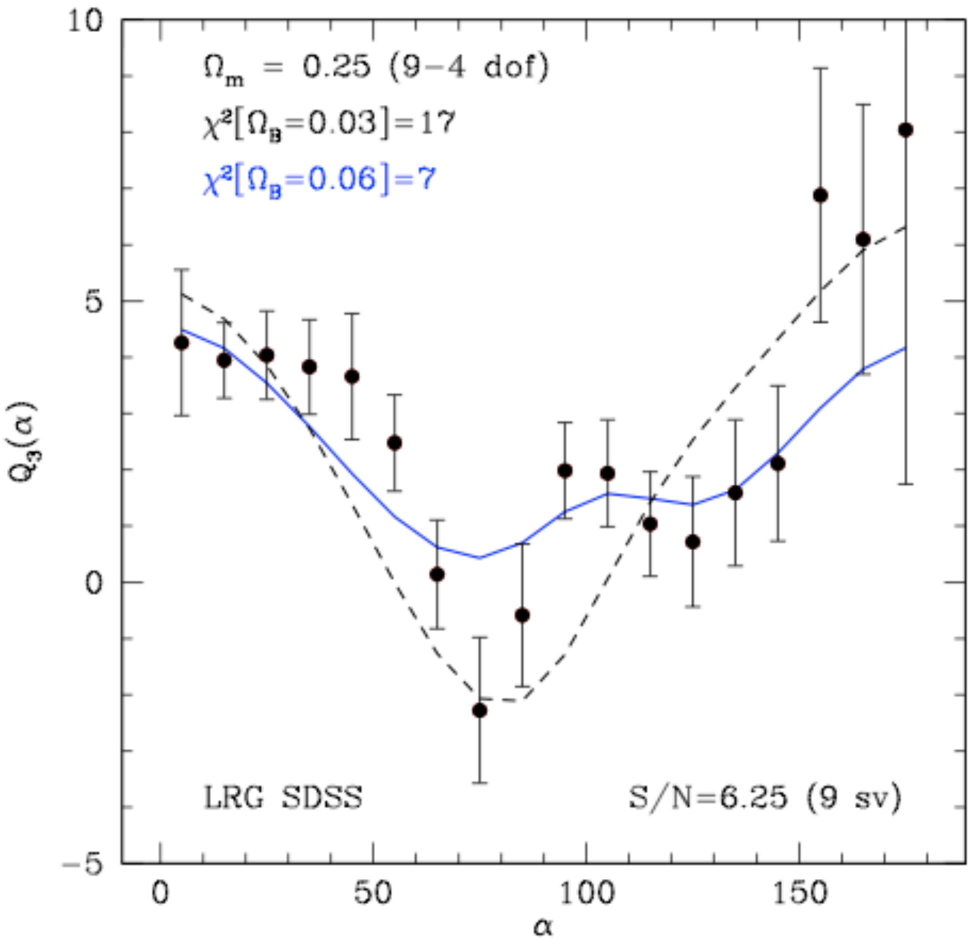


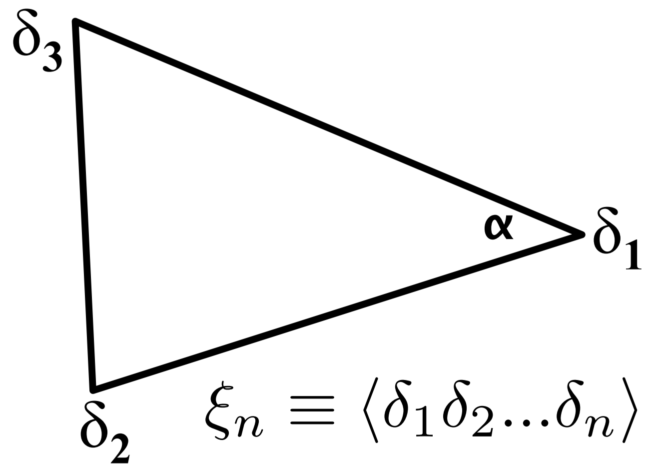
Find results consistent with DR3 analysis, although with higher amplitude in the large-scale tail, also seen by Sanchez et al. (2009)

Kazin et al., 2009, arXiv:0908.2598

BAO in the 3-pt

$$Q_3 = \frac{\xi_3(r_{12}, r_{23}, r_{13})}{\xi_2(r_{12})\xi_2(r_{23}) + \xi_2(r_{12})\xi_2(r_{13}) + \xi_2(r_{23})\xi_2(r_{13})}$$





$$\xi_n \equiv \langle \delta_1 \delta_2 \ldots \delta_n \rangle$$

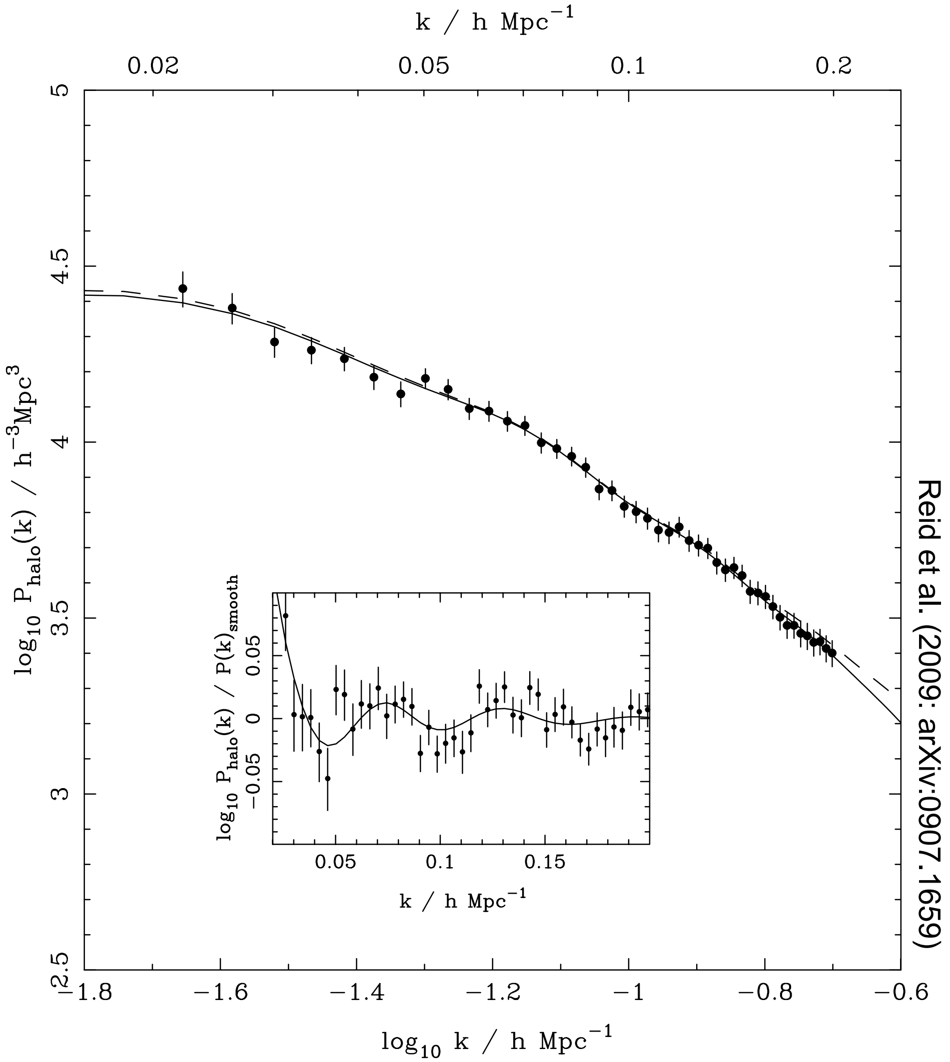
Perturbation theory predicts that the 3-pt function, created by non-linear gravitational infall, will be affected by the BAO feature

Gaztanaga et al., 2008, arXiv:0807.2448

Lecture outline

- review of 2-pt analyses from SDSS and 2dFGRS (apologies for incomplete & often biased nature)
  - Ω_mh constraints from P(k) shape
  - BAO detection
- Latest SDSS DR7 analyses (by myself and collaborators)
  - LRG P(k)
  - BAO observations
  - combination with WMAP data

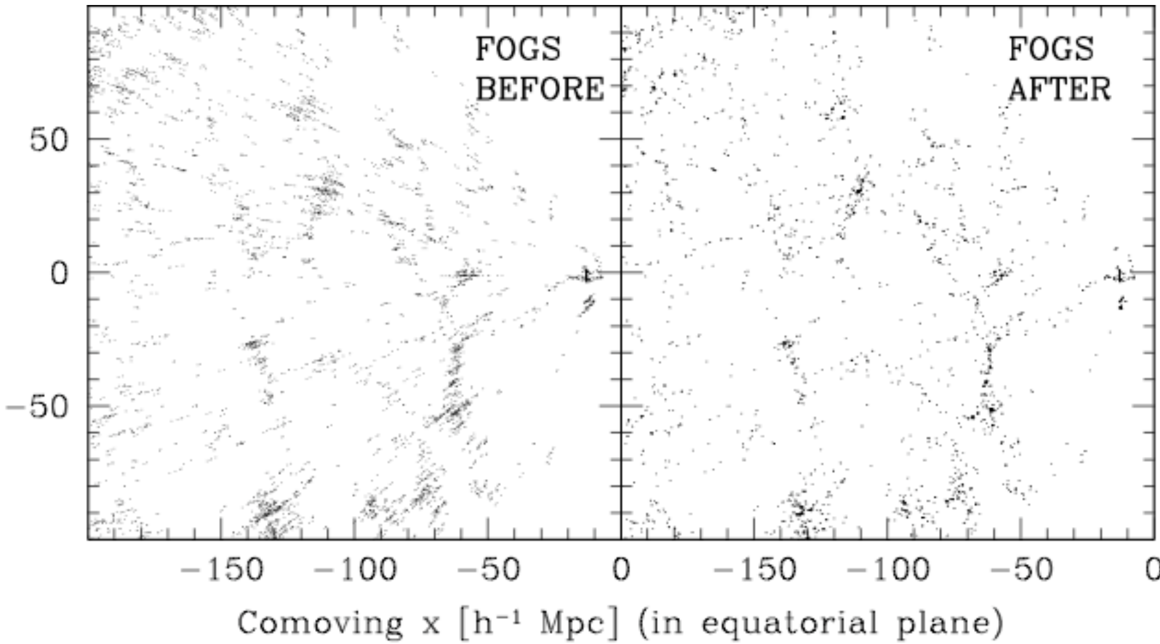
Cosmology from the SDSS DR7 LRG Clustering



Plan to use all of our new knowledge about the effects and model the shape of the LRG power spectrum to constrain cosmological models

Reid et al. (2009, arXiv:0907.1659)

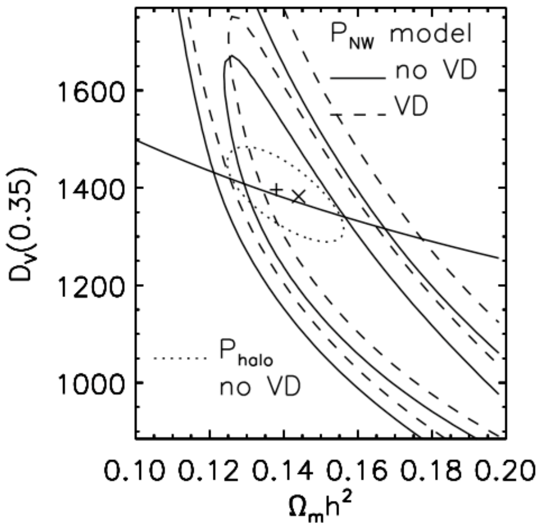
step 1: FOG compression



Tegmark et al. (2004, astro-ph/0310725)

Use an anisotropic FOF group finder with parameters derived from simulations to find halo centres.

We therefore calculate the halo rather than LRG power spectrum



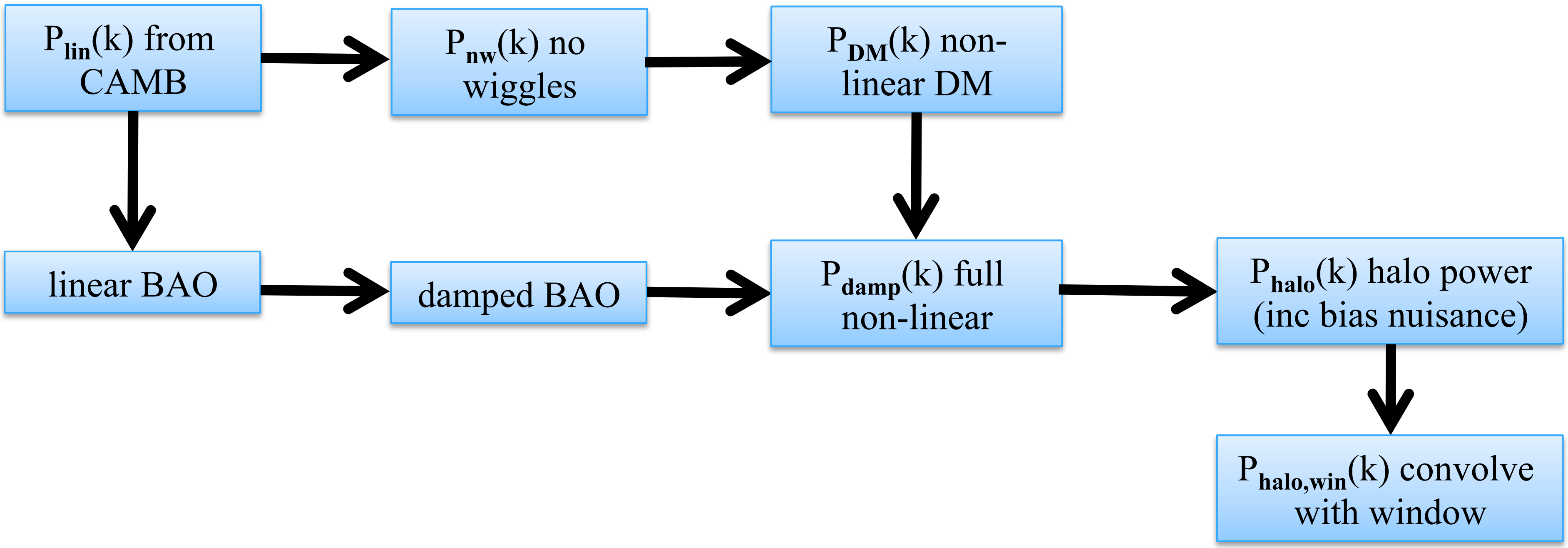
Test cosmological constraints against residual velocity dispersion from LRGs in single halos that are not central

Reid et al. (2009, arXiv:0907.1659)

University of Portsmouth

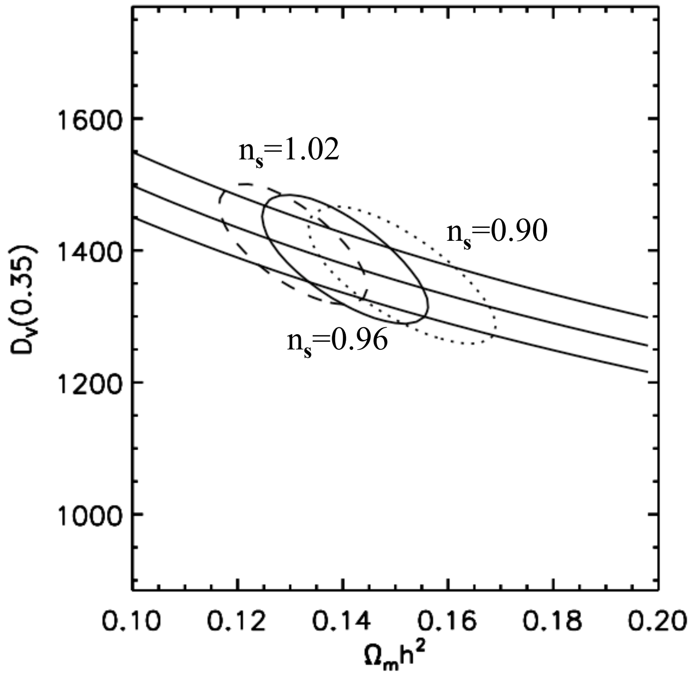
step 2: model window, non-linearities, bias

$P(k)$	Definition	Reference
$\hat{P}_{LRG}(k)$	measured angle averaged redshift-space power spectrum of the LRGs	-
$\hat{P}_{halo}(k)$	measured power spectrum of reconstructed halo density field	-
$P_{lin}(k)$	linear power spectrum computed by CAMB	Lewis et al. (2000)
$P_{DM}(k)$	theoretical real-space non-linear power spectrum of dark matter	-
$P_{nw}(k)$	theoretical linear power spectrum without BAO ("no wiggles")	Eisenstein & Hu (1998)
$P_{damp}(k)$	theoretical linear power spectrum with damped BAO (Eqn. 10)	Eisenstein et al. (2007b)
$P_{halo}(k, \mathbf{p})$	model for the reconstructed halo power spectrum for cosmological parameters $\mathbf{p}$	Reid et al. (2008)
$P_{halo,win}(k, \mathbf{p})$	$P_{halo}(k, \mathbf{p})$ convolved with survey window function (Eqn. 5) and directly compared with $\hat{P}_{halo}(k)$ in the likelihood calculation (Eqn. 6)	Percival et al. (2007)



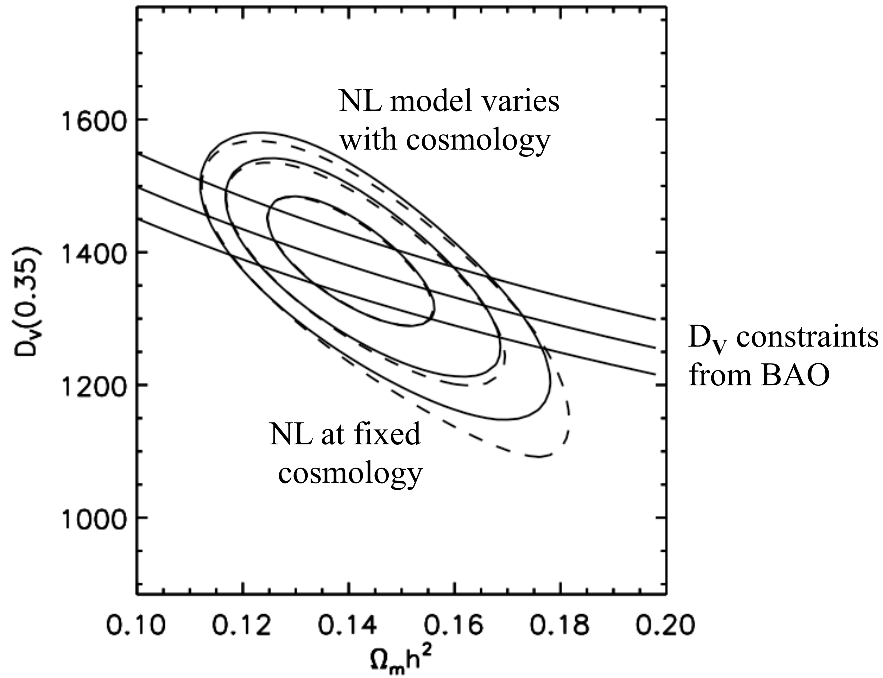
Reid et al. (2009, arXiv:0907.1659)

step 3: tests, tests, tests



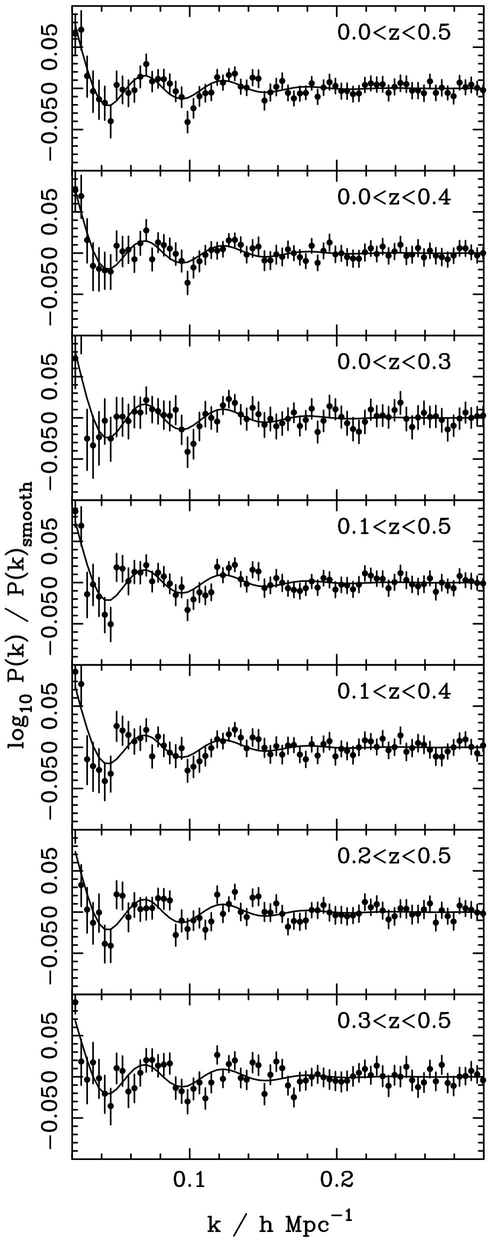
Effect of changing the scalar spectral index n_s on the primary constraints on $\Omega_m h^2$ and D_V

Effect of changing the non-linear prescription



Reid et al. (2009, arXiv:0907.1659)

BAO in SDSS DR7 + 2dFGRS power spectra



Percival et al. (2009: arXiv:0907.1660)

- Combine 2dFGRS, SDSS DR7 LRG and Main galaxy samples
- split into redshift slices and fit P(k) with model comprising smooth fit × BAO

SLICE	$z_{\min}$	$z_{\max}$	$N_{\rm gal}$	$V_{\rm eff}$	$\bar{n}$
1	0.0	0.5	895 834	0.42	128.1
2	0.0	0.4	874 330	0.38	131.2
3	0.0	0.3	827 760	0.27	138.3
4	0.1	0.5	505 355	0.40	34.5
5	0.1	0.4	483 851	0.36	35.9
6	0.2	0.5	129 045	0.27	1.92
7	0.3	0.5	68 074	0.15	0.67

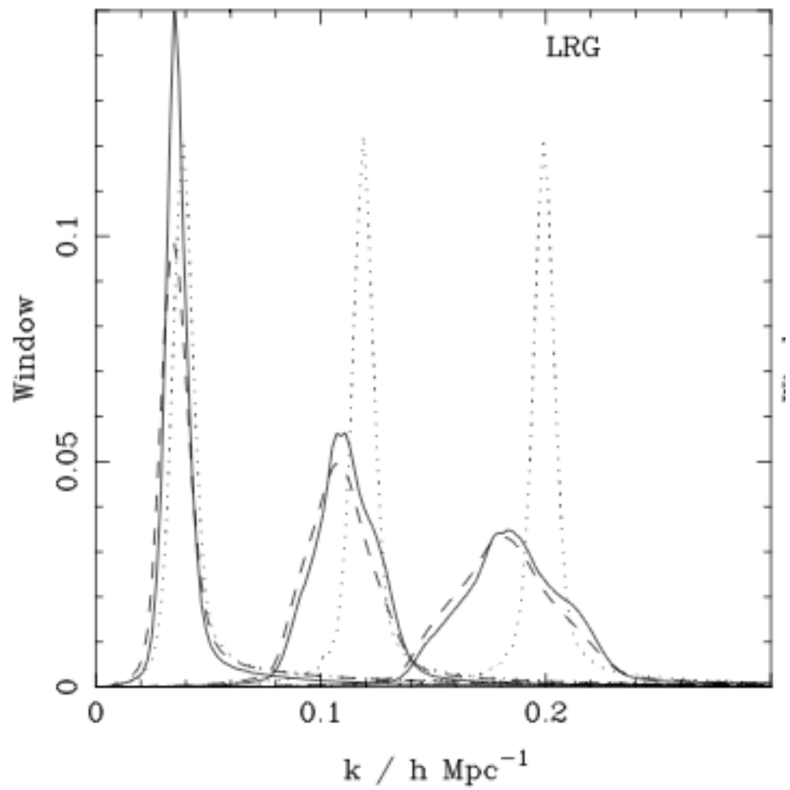
Percival et al. (2009, arXiv:0907.1660)

Dealing with cosmological dependencies

BAO position measured by fitting single measured P(k) calculated assuming a ΛCDM cosmology

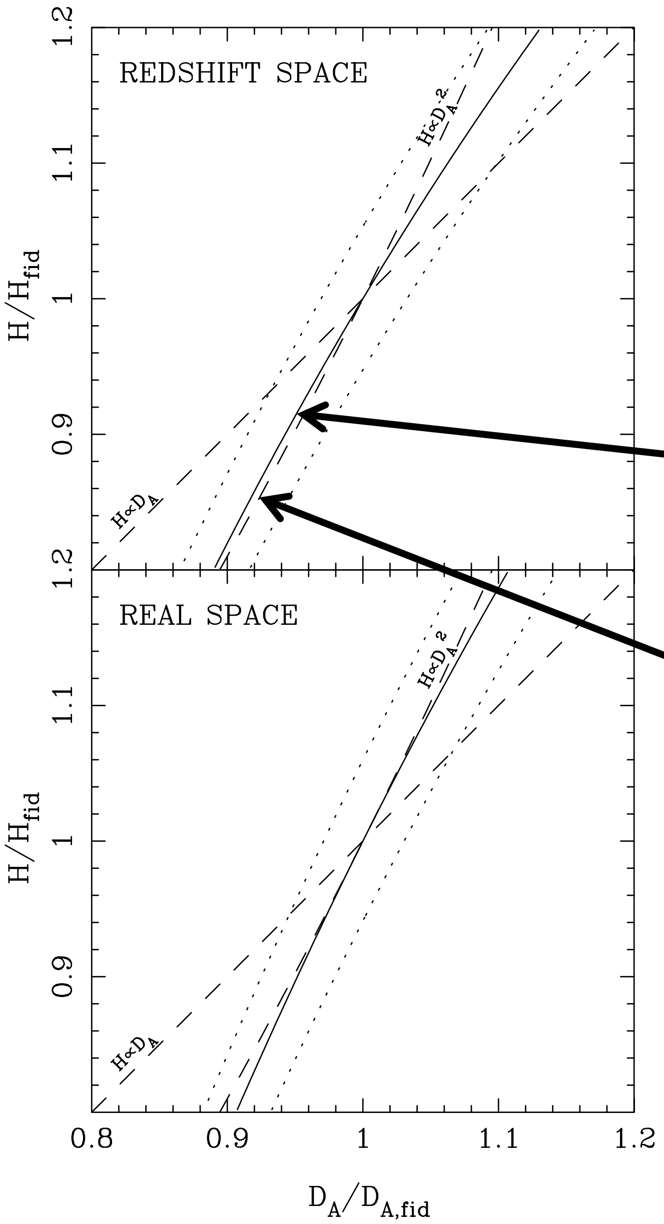
A window function which depends on the model to be tested, is used to convolve the model power spectrum before testing against the data

This includes the offset caused by the dilation of scale, and the broadening of the window caused by getting the cosmology to be tested wrong



Percival et al. (2009, arXiv:0907.1660)

Is D_V the correct parameter to constrain?



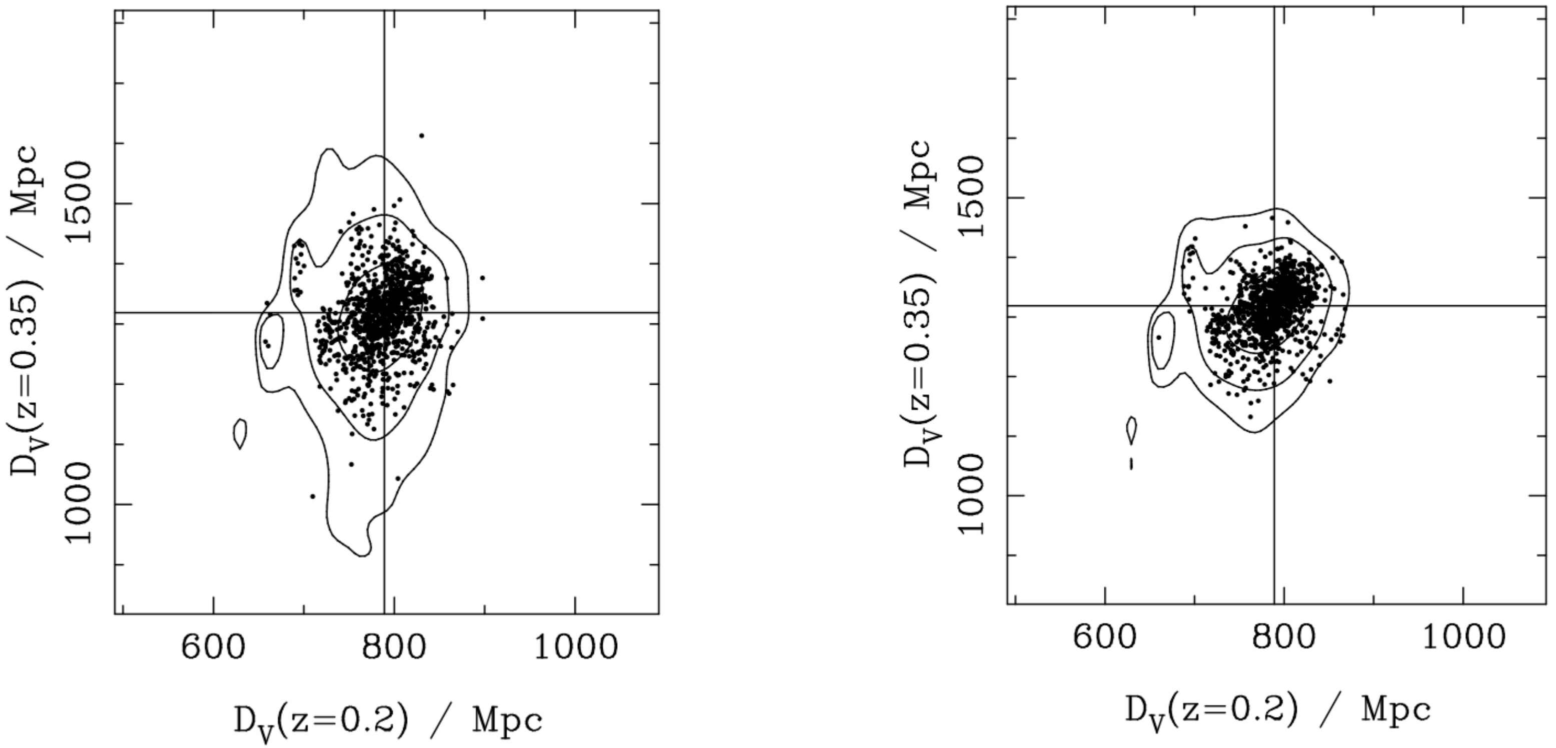
Test how well BAO position in different anisotropic cosmologies depends on D_V ?

Models with the same BAO position

Models with constant D_V

Percival et al. (2009, arXiv:0907.1660)

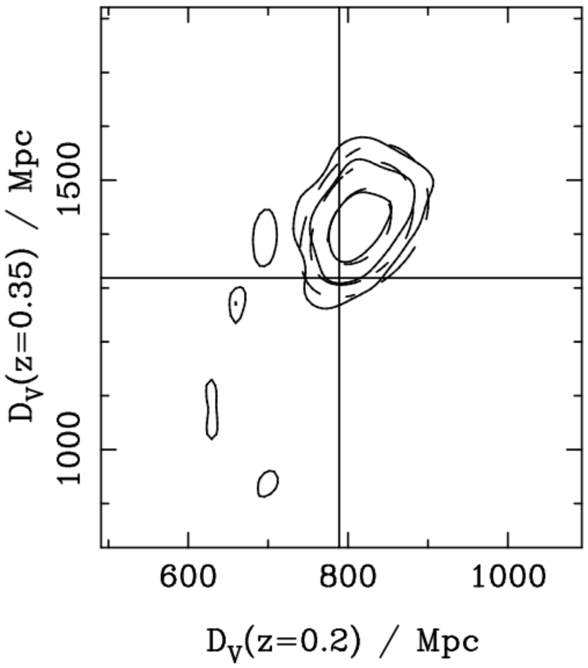
Testing the errors



Tests comparing parameters and errors recovered for mock data against the true cosmology, show we need to increase the errors. Gaussian realisations of power spectra show this is caused by the non-Gaussian nature of the Likelihood

Percival et al. (2009, arXiv:0907.1660)

BAO in SDSS DR7 + 2dFGRS power spectra



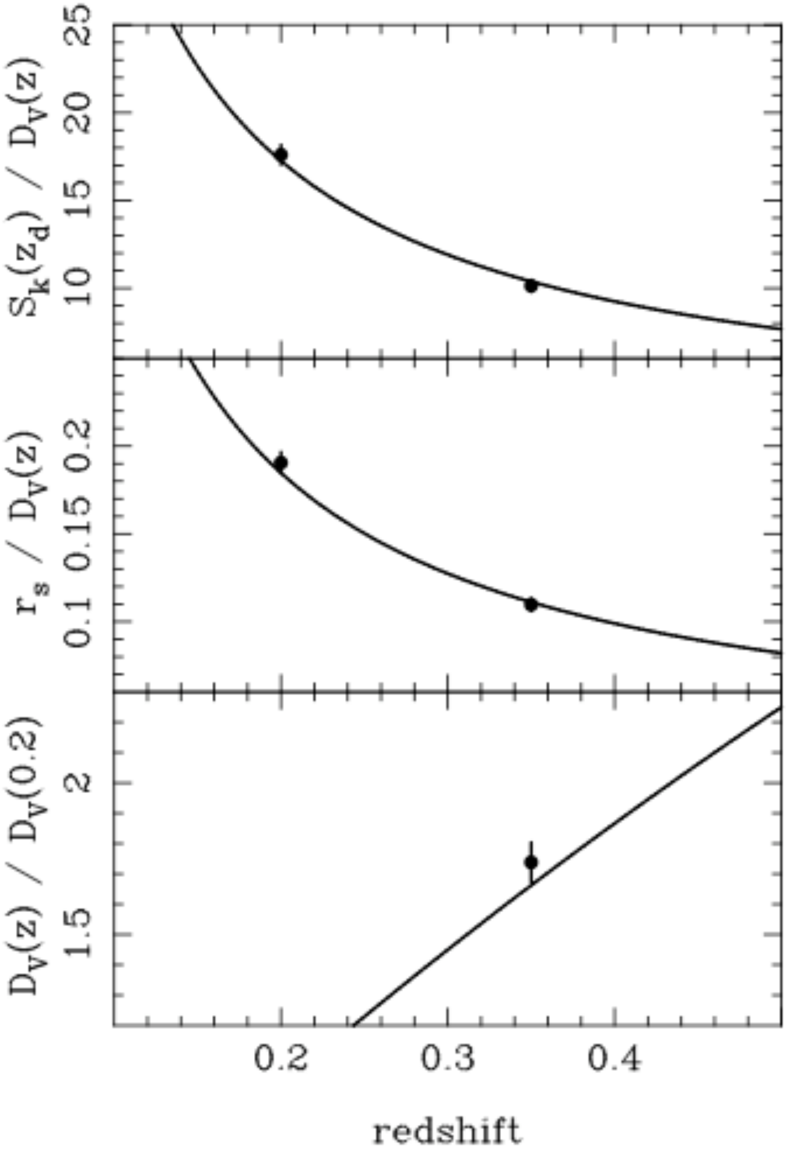
- results can be written as independent constraints on a distance measure and a tilt around this

$$r_s(z_d)/D_V(0.275) = 0.1390 \pm 0.0037 \quad (2.7\%)$$
$$D_V(0.37)/D_V(0.2) = 1.736 \pm 0.065$$

- consistent with ΛCDM models at 1.1σ when combined with WMAP5
- Reduced discrepancy compared with DR5 analysis
  - more data
  - revised error analysis (allow for non-Gaussian likelihood)
  - more redshift slices analyzed
  - improved modeling of LRG z-distribution

Percival et al. (2009, arXiv:0907.1660)

How to present BAO constraints?



Anchor BAO at high redshift using the CMB?

Include parameter dependencies from modeling the comoving sound horizon?

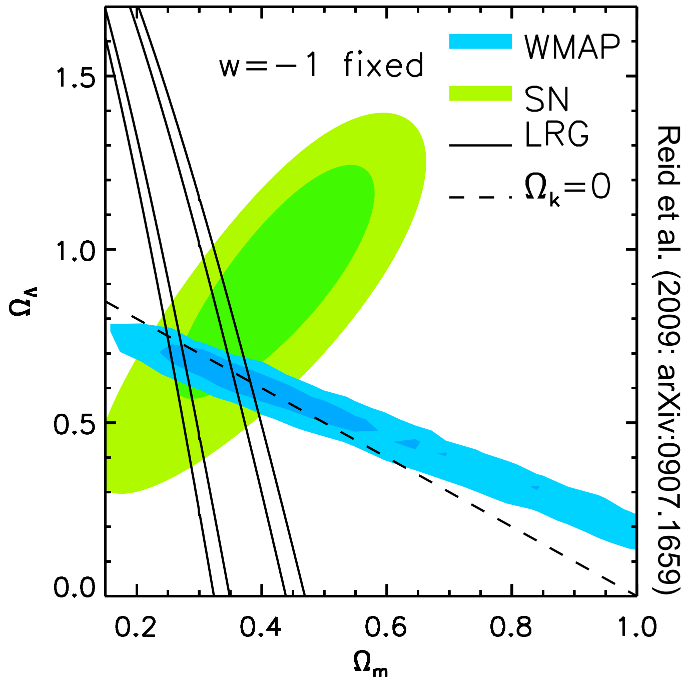
Only consider ratios of BAO measurements, effectively forcing a cosmological model that lines up the BAO

Percival et al. (2009, arXiv:0907.1660)

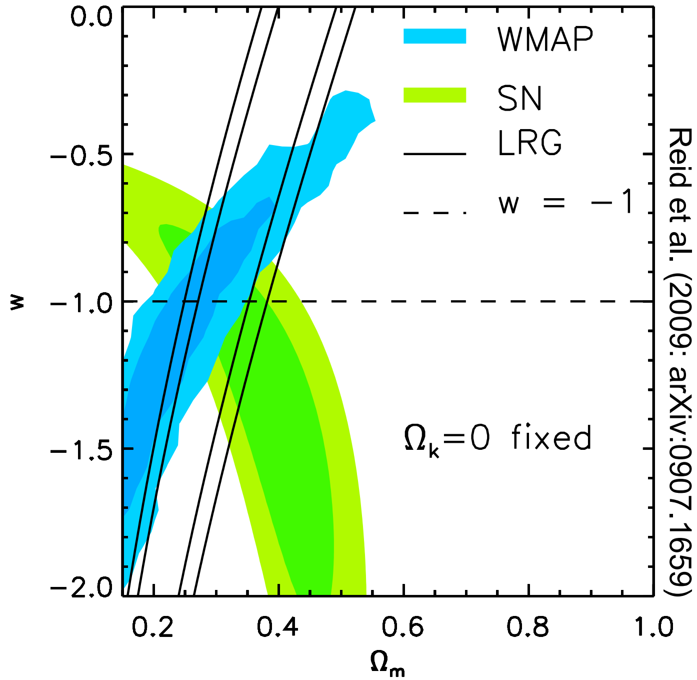
University of Portsmouth

Comparing P(k) constraints against different data

ΛCDM models with curvature



flat wCDM models



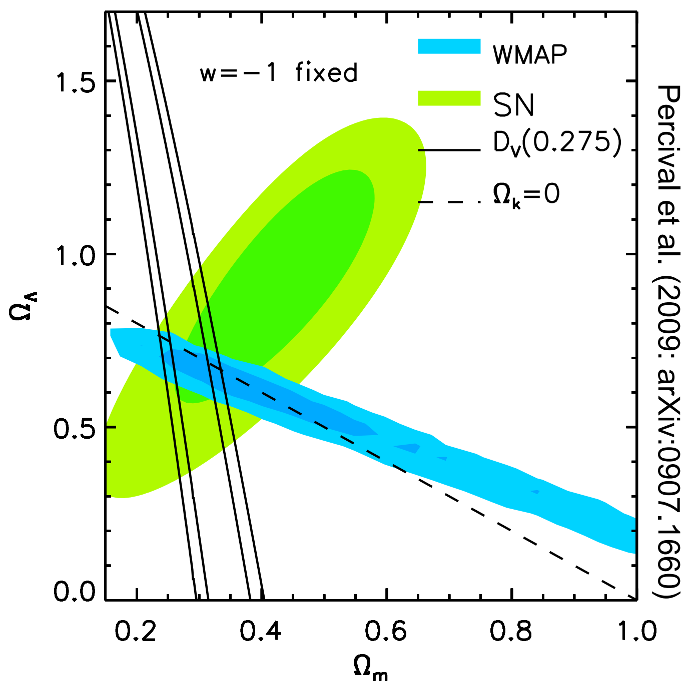
Union supernovae

WMAP 5year

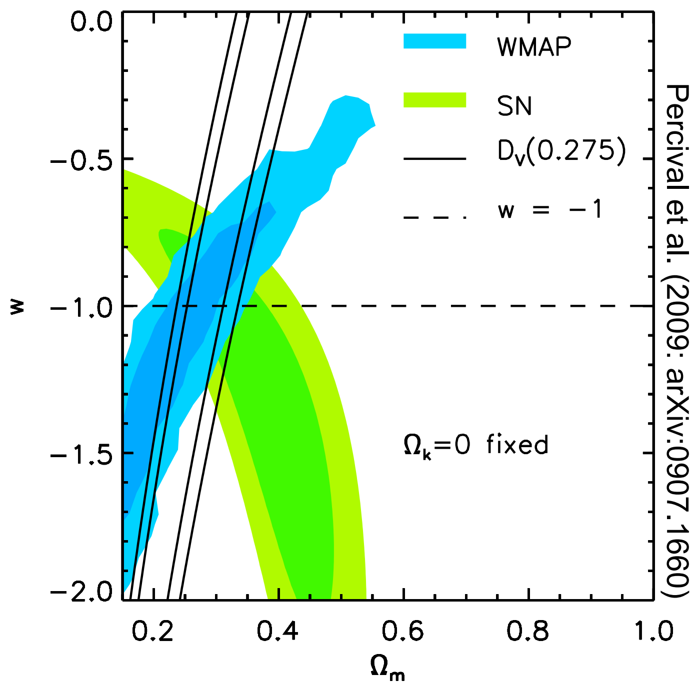
LRG halo P(k) shape + BAO

Comparing BAO constraints against different data

ΛCDM models with curvature



flat wCDM models



Union supernovae

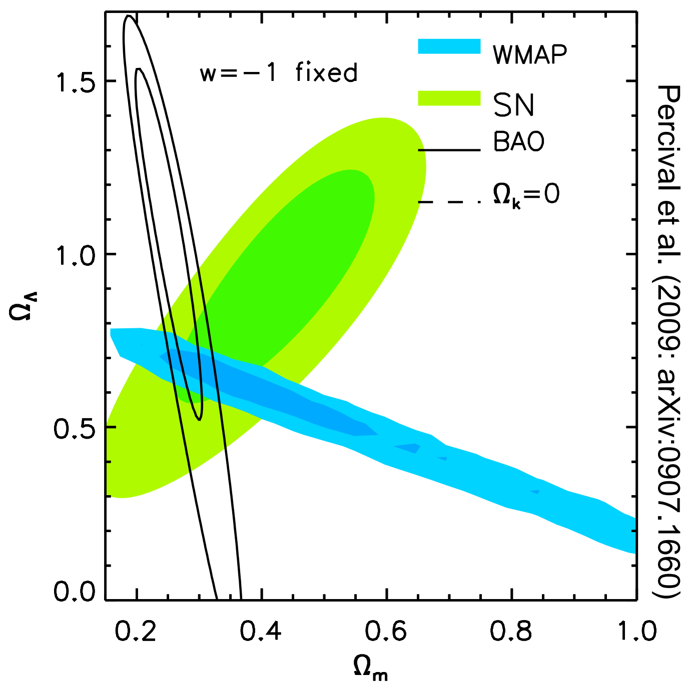
WMAP 5year

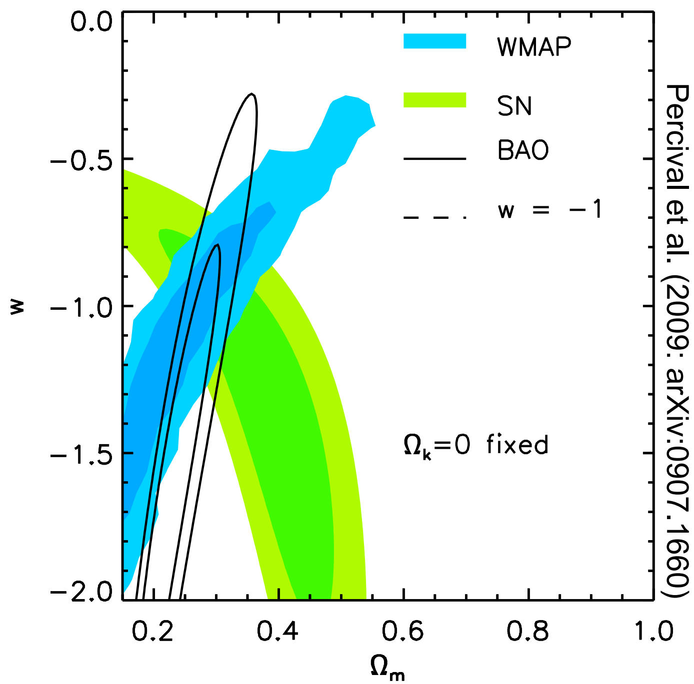
SDSS BAO Constraint on $r_s(z_d)/D_V(0.275)$

Comparing BAO constraints against different data

ΛCDM models with curvature

flat wCDM models

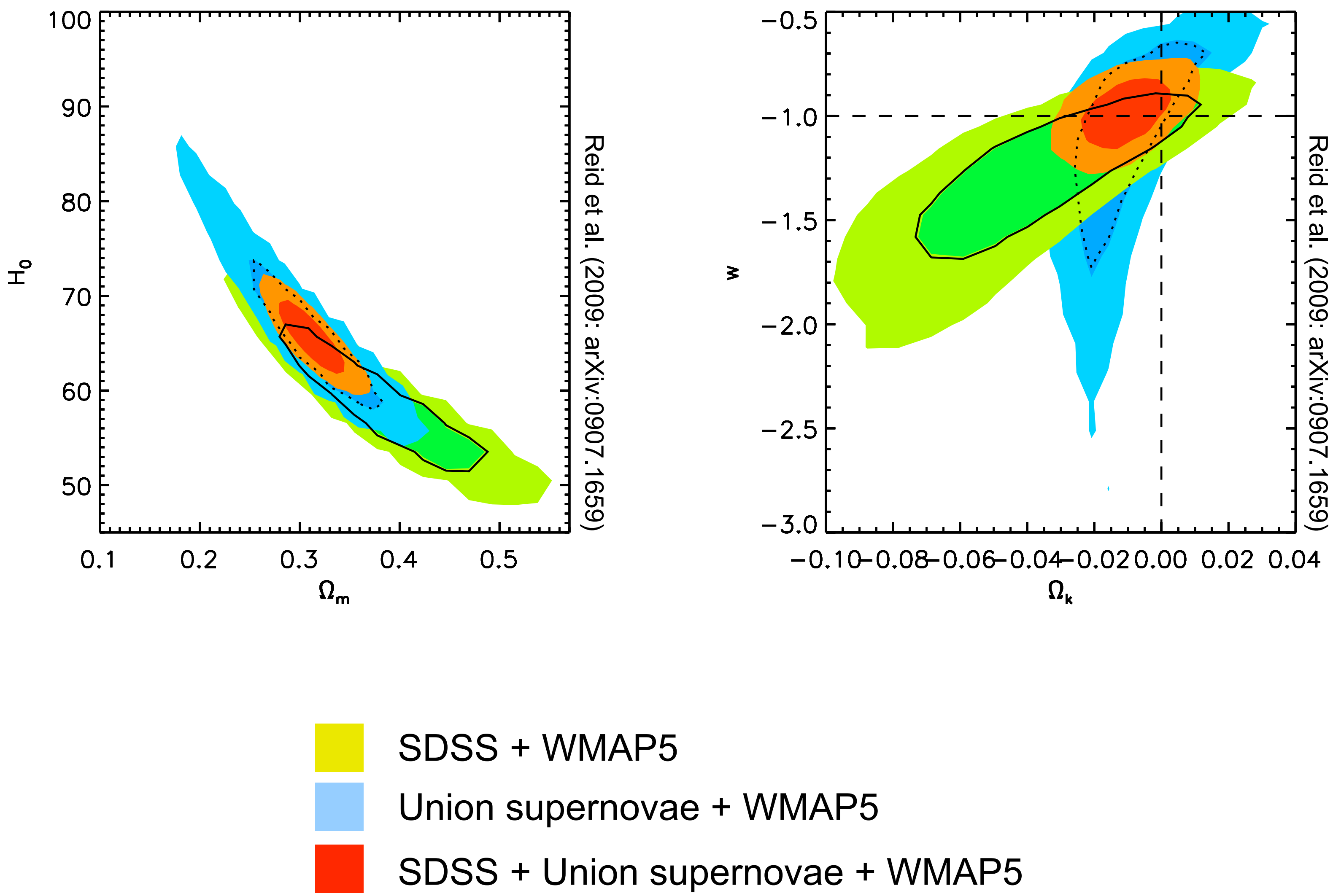




- Union supernovae
- WMAP 5year
- SDSS BAO Constraint on $r_s(z_d)/D_V(0.2)$ & $r_s(z_d)/D_V(0.35)$

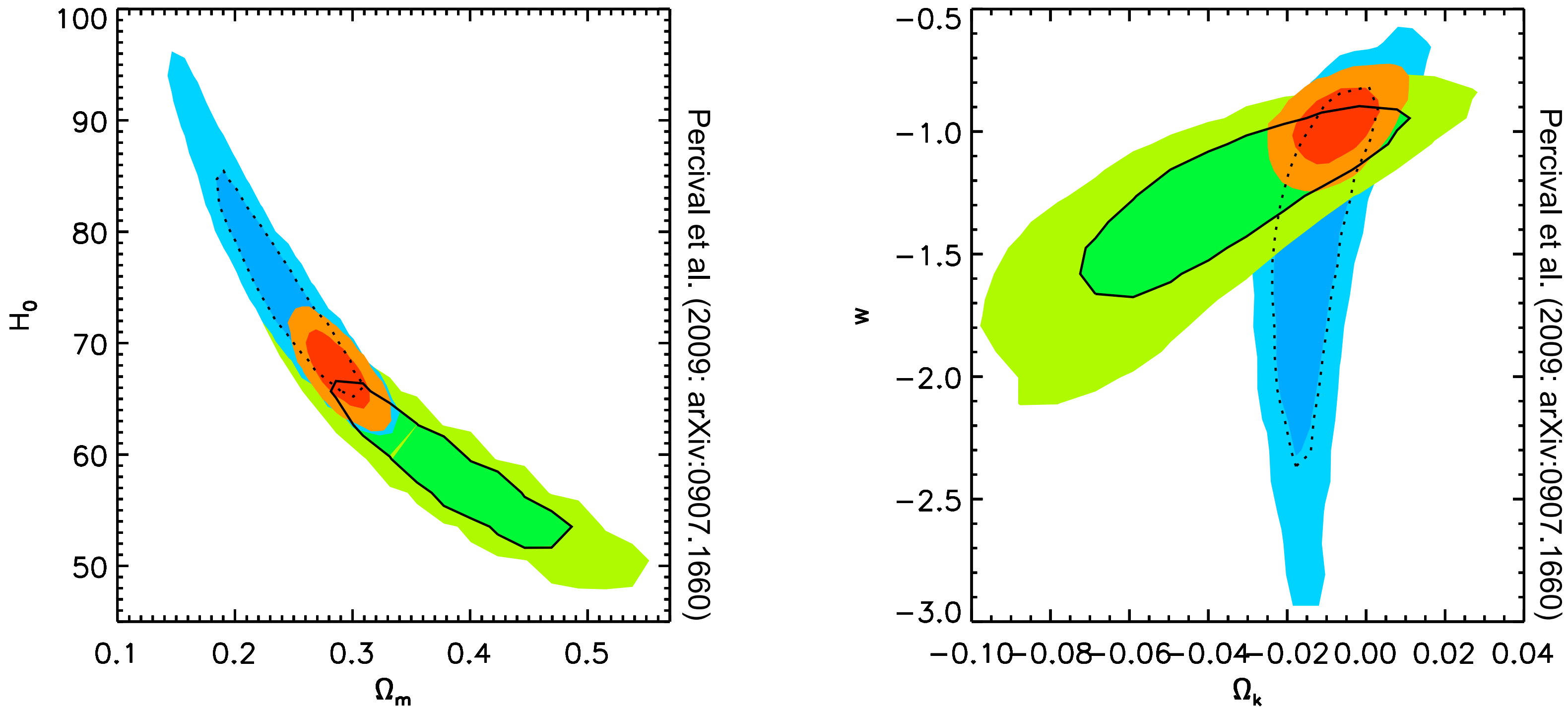
LRG halo P(k) + CMB + SN model constraints

w-CDM models with curvature



BAO + CMB + SN model constraints

w-CDM models with curvature



- SDSS + WMAP5
- Union supernovae + WMAP5
- SDSS + Union supernovae + WMAP5

Parameter constraints

parameter	ΛCDM	oΛCDM	wCDM	owCDM	owCDM+SN
Ω_m	0.289 ± 0.019	0.309 ± 0.025	0.328 ± 0.037	0.306 ± 0.050	0.312 ± 0.022
H_0	69.4 ± 1.6	66.0 ± 2.7	64.3 ± 4.1	$66.7^{+5.9}_{-5.6}$	65.6 ± 2.5
$D_V(0.35)$	1349 ± 23	1415 ± 49	1398 ± 45	1424 ± 49	1418 ± 49
$r_s/D_V(0.35)$	0.1125 ± 0.0023	0.1084 ± 0.0034	0.1094 ± 0.0032	$0.1078^{+0.0033}_{-0.0034}$	0.1081 ± 0.0034
Ω_k	-	$-0.0114^{+0.0076}_{-0.0077}$	-	-0.009 ± 0.012	-0.0109 ± 0.0088
w	-	-	-0.79 ± 0.15	-1.06 ± 0.38	-0.99 ± 0.11
Ω_Λ	0.711 ± 0.019	0.703 ± 0.021	0.672 ± 0.037	$0.703^{+0.057}_{-0.058}$	0.699 ± 0.020
Age (Gyr)	13.73 ± 0.13	14.25 ± 0.37	13.87 ± 0.17	14.27 ± 0.52	14.24 ± 0.40
Ω_{tot}	-	$1.0114^{+0.0077}_{-0.0076}$	-	1.009 ± 0.012	1.0109 ± 0.0088
$100\Omega_b h^2$	2.272 ± 0.058	2.274 ± 0.059	$2.293^{+0.062}_{-0.063}$	$2.279^{+0.066}_{-0.065}$	$2.276^{+0.060}_{-0.059}$
$\Omega_c h^2$	$0.1161^{+0.0039}_{-0.0038}$	0.1110 ± 0.0052	$0.1112^{+0.0056}_{-0.0057}$	$0.1103^{+0.0055}_{-0.0054}$	$0.1110^{+0.0051}_{-0.0052}$
τ	0.084 ± 0.016	0.089 ± 0.017	0.088 ± 0.017	0.088 ± 0.017	0.088 ± 0.017
n_s	0.961 ± 0.013	0.962 ± 0.014	0.969 ± 0.015	0.965 ± 0.016	0.964 ± 0.014
$\ln(10^{10} A_{05})$	$3.080^{+0.036}_{-0.037}$	3.068 ± 0.040	$3.071^{+0.040}_{-0.039}$	3.064 ± 0.041	3.068 ± 0.039
σ_8	0.824 ± 0.025	0.796 ± 0.032	0.735 ± 0.073	0.79 ± 0.11	$0.790^{+0.045}_{-0.046}$

Reid et al. (2009, arXiv:0907.1659)

Parameter constraints

parameter	ΛCDM	oΛCDM	wCDM	owCDM	owCDM+SN	owCDM+H_0	owCDM+SN+H_0
Ω_m	0.278 ± 0.018	0.283 ± 0.019	0.283 ± 0.026	$0.240^{+0.044}_{-0.043}$	0.290 ± 0.019	$0.240^{+0.025}_{-0.024}$	0.279 ± 0.016
H_0	70.1 ± 1.5	$68.3^{+2.2}_{-2.1}$	69.3 ± 3.9	75.3 ± 7.1	67.6 ± 2.2	74.8 ± 3.6	69.5 ± 2.0
Ω_k	-	$-0.007^{+0.006}_{-0.007}$	-	-0.013 ± 0.007	-0.006 ± 0.008	-0.014 ± 0.007	-0.003 ± 0.007
w	-	-	-0.97 ± 0.17	$-1.53^{+0.51}_{-0.50}$	-0.97 ± 0.10	$-1.49^{+0.32}_{-0.31}$	-1.00 ± 0.10
Ω_Λ	0.722 ± 0.018	0.724 ± 0.019	0.717 ± 0.026	0.772 ± 0.048	0.716 ± 0.019	0.773 ± 0.029	0.724 ± 0.018
$100\Omega_b h^2$	2.267 ± 0.058	2.269 ± 0.060	2.275 ± 0.061	$2.254^{+0.062}_{-0.061}$	2.271 ± 0.061	$2.254^{+0.061}_{-0.062}$	2.284 ± 0.061
τ	0.086 ± 0.016	0.089 ± 0.017	0.087 ± 0.017	0.088 ± 0.017	0.089 ± 0.017	0.088 ± 0.017	$0.089^{+0.017}_{-0.018}$
n_s	0.961 ± 0.013	0.963 ± 0.014	0.963 ± 0.015	0.958 ± 0.014	0.963 ± 0.014	0.957 ± 0.014	0.964 ± 0.014
$\ln(10^{10} A_{05})$	$3.074^{+0.040}_{-0.039}$	3.060 ± 0.042	3.070 ± 0.041	$3.062^{+0.042}_{-0.043}$	$3.062^{+0.041}_{-0.042}$	3.062 ± 0.042	3.072 ± 0.042
$d_{0.275}$	0.1411 ± 0.0030	0.1387 ± 0.0036	$0.1404^{+0.0036}_{-0.0035}$	0.1382 ± 0.0037	0.1379 ± 0.0036	$0.1387^{+0.0036}_{-0.0037}$	$0.1402^{+0.0033}_{-0.0034}$
$D_V(0.275)$	1080 ± 18	1110^{+32}_{-31}	1089 ± 31	1111 ± 33	1115 ± 32	1107 ± 31	1091^{+27}_{-28}
f	1.6645 ± 0.0043	1.6643 ± 0.0045	1.661 ± 0.019	1.72 ± 0.056	1.660 ± 0.011	$1.7187^{+0.0337}_{-0.0334}$	1.6645 ± 0.0107
Age (Gyr)	13.73 ± 0.12	14.08 ± 0.33	$13.76^{+0.15}_{-0.14}$	14.49 ± 0.52	14.04 ± 0.36	14.48 ± 0.48	$13.86^{+0.34}_{-0.33}$
$\Omega_c h^2$	0.1139 ± 0.0041	$0.1090^{+0.0060}_{-0.0061}$	$0.1122^{+0.0068}_{-0.0069}$	$0.1107^{+0.0063}_{-0.0062}$	$0.1096^{+0.0061}_{-0.0062}$	$0.1108^{+0.0060}_{-0.0061}$	0.1115 ± 0.0061
Ω_{tot}	-	$1.007^{+0.006}_{-0.007}$	-	1.013 ± 0.007	1.006 ± 0.008	1.014 ± 0.007	1.003 ± 0.007
σ_8	0.813 ± 0.028	0.787 ± 0.037	$0.792^{+0.081}_{-0.082}$	0.907 ± 0.117	$0.780^{+0.052}_{-0.053}$	0.904 ± 0.074	$0.801^{+0.053}_{-0.052}$

Percival et al. (2009, arXiv:0907.1660)

Further reading

- Previous analysis references listed through talk
- SDSS DR7 analyses
  - LRG P(k), Reid et al. (2009, arXiv:0907.1659)
  - BAO from combined sample, Percival et al. (2009, arXiv:0907.1660)