Non-gaussianity from the bispectrum in general multiple field inflation

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Introduction

Observations on non-Gaussianities

deviation from Gaussian distribution

- conventional parametrisation

$$\zeta = \zeta_L - \frac{3}{5} f_{NL} \zeta_L^2$$

Curvature perturbations

$$f_{NL} \sim 0$$ for almost free theories like standard inflation

- Constraints on $f_{NL}$ from WMAP 5-year

$$-9 < f_{NL} < 111$$ favoring relatively large non-Gaussianity

Komatsu and Sperbel 2001

Komatsu et al. 2008

Need to consider the early universe scenarios other than the standard inflationary scenario?
**DBI inflation: model**

Silverstein and Tong 2004

- **Set-up**
  
  Inflation is driven by a mobile D3-brane with relativistic speed

  \[ \gamma = \frac{1}{\sqrt{1 - \dot{\phi}^2/T(\phi)}} \]

- **Action**

  \[
  S = \int d^4\xi \sqrt{-g^{(-4)}} \left[ -T(\phi) \sqrt{1 + \partial_\mu \phi \partial^\mu \phi / T(\phi)} + T(\phi) - V(\phi) \right]
  \]

  **DBI part**

  \[ d\phi = T_3^{1/2} d\rho \quad T(\phi) = T_3^{1/2} h^4 \]

  \( \rho \) : radial position of the brane

  Large non-gaussianity is possible

  \[ f_{NL} \sim \frac{1}{3} \gamma^2 \sim 1/3c_s^2 \]

  sound speed
DBI inflation: present status

Baumann and Mcclister 2006, Lidsey and Huston 2007

- DBI inflation with large non-gaussianity seems inconsistent with WMAP data
- It can be consistent only in the limit when it goes back to a standard slow-roll inflation

\[
\text{Consistency relation } \quad r \geq 4(1 - n_s)/\sqrt{1 + 3f_{NL}}
\]

- large but not too large \( |f_{NL}| \) \( \rightarrow \) large \( r \)
- \( \Delta \phi / M_{pl} \)\( \rightarrow \) large \( \Delta \phi / M_{pl} \)
- \( V_6 \)\( \rightarrow \) large \( V_6 \)
- contradiction!!

- Can the situation be better for multi-field models?
  - d.o.f corresponding to the angular directions
Multi-field K-inflation

Langlois and Renaux-Petel (2008)

• Action

\[ S = \int d^4x \sqrt{-g} \left( \frac{1}{2} R + P(X, \phi^I) \right) \]

with

\[ I = 1, \ldots, N \]

\[ X = -\frac{1}{2} G_{IJ}(\phi) \nabla_{\mu} \phi^I \nabla^{\mu} \phi^J \]

field space metric

• Energy-momentum tensor

\[ T^{\mu\nu} = P g^{\mu\nu} + P_{,X} G_{IJ} \nabla^{\mu} \phi^I \nabla^{\nu} \phi^J \]

\[ \rho = 2XP_{,X} - P, \quad P = P \]

Flat FRW

Sound speed

\[ c_s^2 \equiv \frac{P_{,X}}{\rho_{,X}} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}} \]

• Propagation speed of modes

\[ \begin{cases} C_s \text{ for adiabatic mode} \\ 1 \text{ for entropy modes} \end{cases} \]

Effects of entropy perturbations are suppressed for \( c_s^2 \ll 1 \)
Multi-field DBI inflation

Langlois, Renaux-Petel, Steer and Tanaka (2008)

They carefully observed DBI action

- kinetic function

\[
\tilde{P}(\tilde{X}, \phi^I) = -\frac{1}{f(\phi^I)}(\sqrt{1 - 2f(\phi^I)\tilde{X}} - 1) - V(\phi^I)
\]

with

\[
\tilde{X} = \frac{1 - D}{2f}, \quad D = \det(\delta^I_J + f \partial^\mu \phi^I \partial_\mu \phi_J)
\]

\(X\) and \(\tilde{X}\) differ for the perturbed values in multi-field models

Multi-field DBI inflation does not belong to K-inflation

- Propagation speed of modes

  All perturbations propagate with the same speed \(C_s\)

  no suppression for the entropy modes
General multi-field inflation
SM with Arroja, Koyama (2008)

Need to analyze more general classes of multi-field models including DBI inflation

• Action

\[
S = \int d^4 x \sqrt{-g} \left( \frac{1}{2} R + P(X^{IJ}, \phi^I) \right)
\]

\[
X^{IJ} \equiv -\frac{1}{2} \partial^\mu \phi^I \partial_\mu \phi^J
\]

\[
\begin{align*}
\text{(ex.) } D &= 1 - 2fX + 4f^2 X_I^I X_J^J \\
& \quad - 8f^3 X_I^I X_J^J X_K^K + 16f^4 X_I^I X_J^J X_K^K X_L^L
\end{align*}
\]

\[
P(X^{IJ}, \phi^I) = \tilde{P}(X, \phi^I) \quad \longrightarrow \quad K\text{-inflation}
\]

\[
P(X^{IJ}, \phi^I) = \tilde{P}(\tilde{X}, \phi^I) = -\frac{1}{f} \left( \sqrt{1 - 2f\tilde{X}} - 1 \right) - V(\phi^I), \quad \tilde{X} = \frac{1 - D}{2f}
\]

\[
\longrightarrow \quad \text{DBI inflation}
\]
Linear perturbations

- Scalar fields on the flat hypersurface
  \[ \phi^I = \phi_0^I + Q^I \] linear perturbation

- Second order action for \( Q \)
  \[ S(2) = \frac{1}{2} \int dt d^3 x a^3 \left[ (P, X^I J + P, X^I K X^J \dot{\phi}_0^K \dot{\phi}_0^L) \dot{Q}^I \dot{Q}^J \right. \]
  \[ \left. - \frac{1}{a^2} P, X^I J \partial_i Q^I \partial_i Q^J - \mathcal{M}_{IJ} Q^I Q^J + \mathcal{N}_{IJ} Q^I \dot{Q}^J \right] \]

  To go further, we impose the following assumption

- Kinetic function
  \[ P(X^I J, \phi^I) = \tilde{P}(Y, \phi^I), \quad Y = G_{IJ}(\phi) X^I J + \frac{b(\phi)}{2} (X^2 - X^I J X^J) \]

  \[ b = 0 \quad \longrightarrow \quad \text{K-inflation} \]
  \[ b = -2f \quad \longrightarrow \quad \text{DBI inflation} \]
Adiabatic and entropy perturbations

• Decomposition of the perturbations

\[
Q^I = Q_n e^I_n
\]

new basis

\[
e^I_1 \equiv \frac{\phi^I}{\sqrt{P_{,XJK} \phi^J \phi^K}}
\]

adiabatic vector

\[
P_{,XIJ} e^I_m e^J_n = \delta_{mn}
\]

orthonormality condition

• Second order action  

\[
S_{(2)} = \frac{1}{2} \int dt d^3 x a^3 \left[ K_{mn} D_t Q^m D_t Q^n - \frac{1}{a^2} \delta_{mn} \partial_i Q^m \partial^i Q^n + \cdots \right]
\]

\[
K_{mn} = \begin{cases} 
\frac{1}{c_{ad}^2} & (m = n = 1) \\
\frac{1}{c_{en}^2} & (m = n \neq 1)
\end{cases}
\]

\[
c_{ad}^2 = \frac{\tilde{P}_{,Y}}{\tilde{P}_{,Y} + 2X \tilde{P}_{,YY}}
\]

\[
c_{en}^2 = 1 + bX
\]

In general, propagation speeds are determined independently !!
Leading order three-point function

• Assumptions
  
i) Coupling between adiabatic mode and entropy mode is negligible (usual quantization is possible)
  
ii) The following parameters are small (slow-roll)

\[ \epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H}, \quad \chi_{ad} \equiv \frac{\dot{c}_{ad}}{c_{ad}H}, \quad \chi_{en} \equiv \frac{\dot{c}_{en}}{c_{en}H}, \quad \ell \equiv \frac{\lambda}{\lambda H} \]

\[ \lambda \equiv 2/3X^3\tilde{P}_{YYY} + X^2\tilde{P}_{YY} \]

• Third order action adiabatic and entropy fields

\[ S_{(3)} = \int dx^3 dt a^3 \left[ \frac{1}{2} \Xi_{nml} \dot{Q}_n \dot{Q}_m \dot{Q}_l - \frac{1}{2a^2} \Xi_{nml} \dot{Q}_n (\partial_i Q_m) (\partial^i Q_l) \right] \]

\[ \Xi_{nml} = (2X\tilde{P}_Y)^{-\frac{1}{2}} \left[ \frac{(1 - c_{ad}^2)}{c_{ad}^2 c_{en}^2} \delta_{1(n}\delta_{ml)} + \left( \frac{4}{3} \frac{X^2\tilde{P}_{YYY}}{\tilde{P}_Y} - \frac{(1 - c_{ad}^2)(1 - c_{en}^2)}{c_{ad}^2 c_{en}^2} \right) \delta_{n1}\delta_{m1}\delta_{l1} \right] \]

\[ \Upsilon_{nml} = (2X\tilde{P}_Y)^{-\frac{1}{2}} \left( \frac{1 - c_{ad}^2}{c_{ad}^2} \delta_{n1}\delta_{ml} - \frac{2(1 - c_{en}^2)}{c_{en}^2} (\delta_{n1}\delta_{ml} - \delta_{n(m}\delta_{l1)}) \right) \]
Three-point function of the fields

- Definition
  \[ \langle \Omega | Q_l(t, k_1) Q_m(t, k_2) Q_n(t, k_3) | \Omega \rangle \quad \text{Maldacena (2003)} \]
  \[ = -i \int_{t_0}^{\tilde{t}} d\tilde{t} \langle 0 | [Q_l(t, k_1) Q_m(t, k_2) Q_n(t, k_3), H_I(\tilde{t})] | 0 \rangle \]

- mixed component
  \[ \langle \Omega | Q_\sigma(0, k_1) Q_s(0, k_2) Q_s(0, k_3) | \Omega \rangle \]
  \[ = (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) \frac{H^5}{8c_{ad}c_{en}^2} \prod_{i=1}^{3} k_i^3 \left\{ \begin{array}{c} \tilde{K} = c_{ad}k_1 + c_{en}(k_2 + k_3) \\ \tilde{K} = k_1 + k_2 + k_3 \end{array} \right. \]
  \[ \times \left[ C_2 c_{en}^2 k_3^2 k_1 \cdot k_2 \left( 1 + \frac{c_{ad}k_1 + c_{en}k_2}{\tilde{K}} + \frac{2c_{ad}c_{en}k_1k_2}{\tilde{K}^2} \right) + (k_2 \leftrightarrow k_3) \right. \]
  \[ + 4C_3 c_{ad}^2 c_{en}^4 \frac{k_1^2 k_2^2 k_3^2}{\tilde{K}^2} - 2(C_1 + C_2)c_{ad}^2 k_1^2 k_2 \cdot k_3 \left( 1 + c_{en} \frac{k_2 + k_3}{\tilde{K}} + 2c_{en}^2 \frac{k_2 k_3}{\tilde{K}^2} \right) \left. \right] \]

For DBI inflation, this momentum dependence is same as purely adiabatic component.
Non-gaussianities in multi-field DBI inflation (cf.) Langlois, Renax-Petel, Steer and Tanaka (2008)

- curvature perturbation Wands, Bartolo, Matarrese and Riotto (2002)

\[ \mathcal{R} = A_\sigma Q_{\sigma*} + A_s Q_{s*} \]

\[ A_\sigma = \left( \frac{H \sqrt{c_s}}{\dot{\sigma}} \right)_*, \quad A_s = T_{RS} \left( \frac{H \sqrt{c_s}}{\dot{\sigma}} \right)_* \]

transfer function

\[ \begin{aligned}
\mathcal{P}_R &= (1 + T_{RS}^2) \mathcal{P}_{\mathcal{R}_*} \\
\langle \mathcal{R}(k_1) \mathcal{R}(k_2) \mathcal{R}(k_3) \rangle &= A_\sigma^3 \langle Q_\sigma(k_1) Q_\sigma(k_2) Q_\sigma(k_3) \rangle (1 + T_{RS}^2)
\end{aligned} \]

- non-linear parameter

\[ f_{NL} \propto \frac{1}{c_s^2} \frac{1}{1 + T_{RS}^2} \]

good for stringy inflation
Conclusion

• Perturbations in general multi-field inflation model

\[ P = P(X^{IJ}, \phi^I) \]

\[ X^{IJ} \equiv -\frac{1}{2} \partial^\mu \phi^I \partial_\mu \phi^J \]

including multi-field K-inflation, multi-field DBI inflation as special cases

• Generally, the sound speeds for the adiabatic and entropy perturbations are different

• Generally, the momentum dependence of the three point function from the adiabatic and entropy modes are different

• multi-field effect could help to ease the constraints on the stringy DBI-inflation models

\[ f_{NL} \propto \frac{1}{c_s^2} \frac{1}{1 + \frac{T^2}{R_s}} \]
Discussion

• Is there some deep reason for the coincidence in the multi-field DBI inflation models?

  ref) For the four-point function, the momentum dependence from the entropy mode is different from the adiabatic mode

• Reheating mechanism in DBI-inflation

  \( T_{RS} \)

• Relaxing the assumptions

  slow-roll approximation

  coupling between the adiabatic and entropy mode