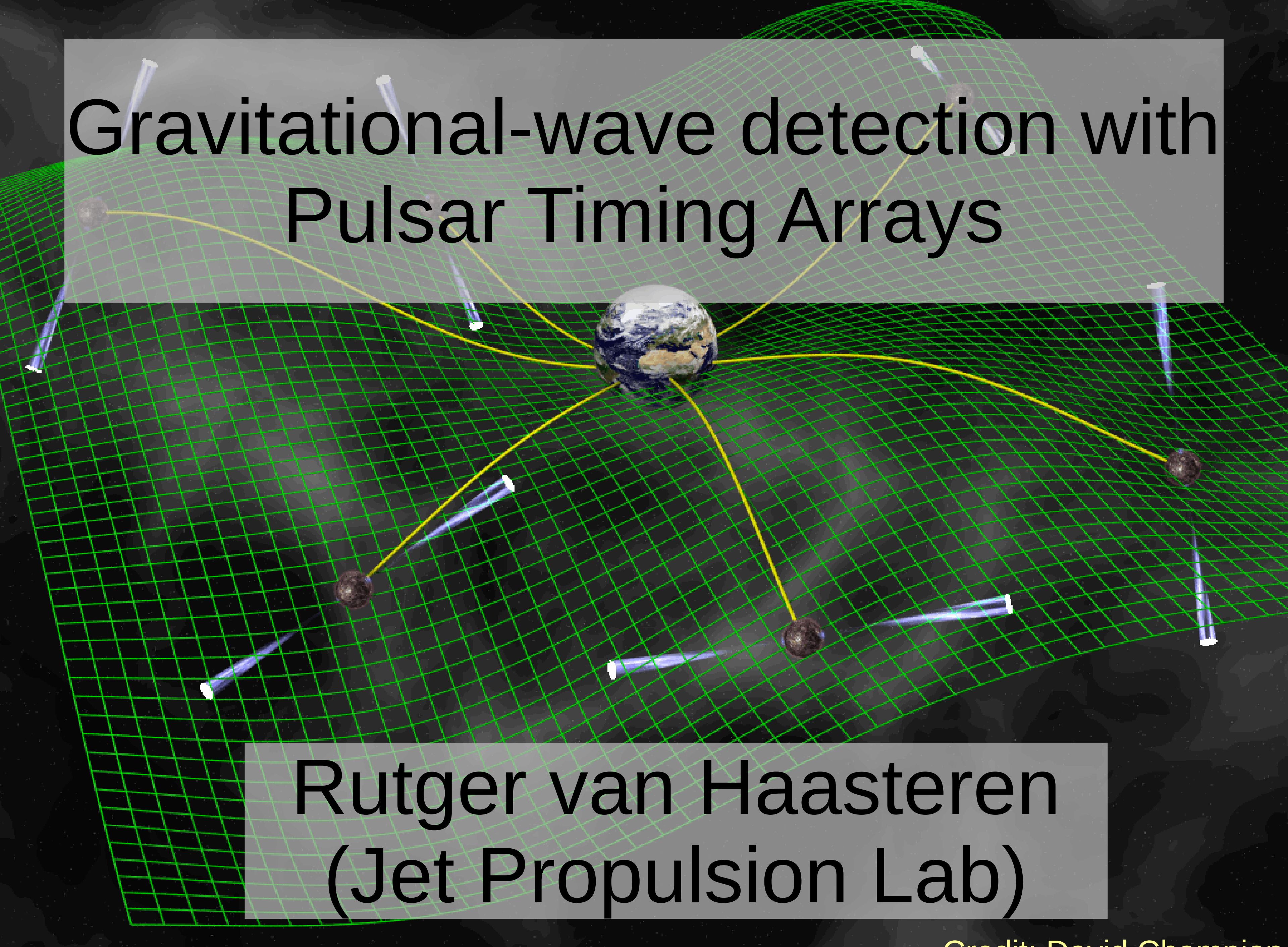
Gravitational-wave detection with Pulsar Timing Arrays

Rutger van Haasteren
(Jet Propulsion Lab)



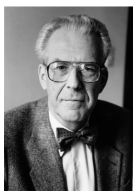
Credit: David Champion

Outline

1. Gravitational wave sources
2. Gravitational wave detection
3. Pulsar timing and examples
4. Gravitational-wave searches and analysis
5. The IPTA mock data challenges
6. Outlook

GW sources for PTAs: SMBHBs

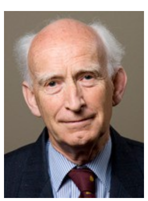
1915. Karl Schwarzschild finds an analytical solution for the Einstein field equations, predicting black holes



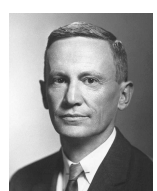
1962. Quasars discovered at billions of light years by Schmidt.



1964. Zeldovich & Novikov and Salpeter argue that Quasars are powered by the accretion of gas onto supermassive black holes



1969. Lynden-Bell argues that supermassive black holes should exist at the centers of many galaxies.



1996+. Hubble Telescope observations, analyzed using Martin Schwarzschild's method, establish that supermassive black holes exist in the large majority of galaxies with a central bulge.

Evolution of galaxies and their massive black holes

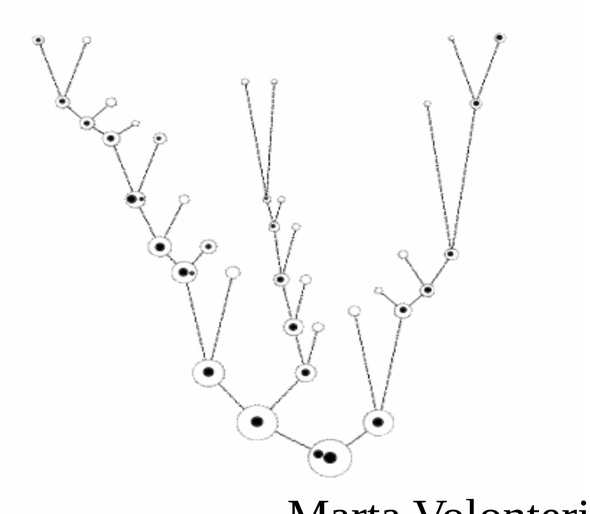
Question: how do black holes evolve?

Galaxy formation

Universe becomes matter-dominated at z=10000. Gravitational instability becomes effective.

Small halos collapse first, small galaxies form first

Smaller galaxies merge to form large spirals and ellipticals.



Marta Volonteri (2003)

White & Rees 78

Supermassive BH binaries

Begelman, Blandford, &Rees 1982:

10 kpc → 2pc → 1pc → 0.01pc → merger

dynamical friction

scattering

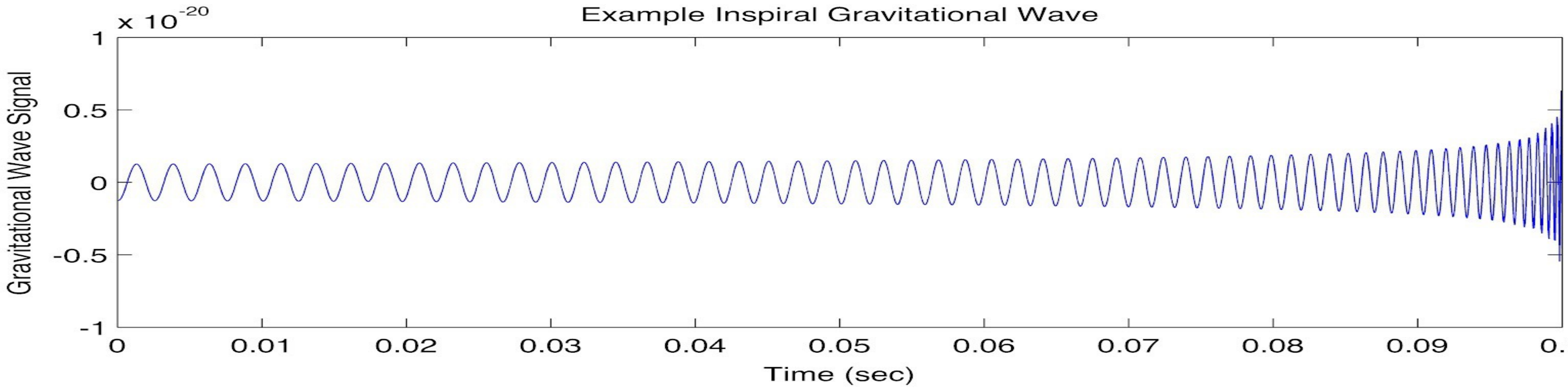
wishful thinking

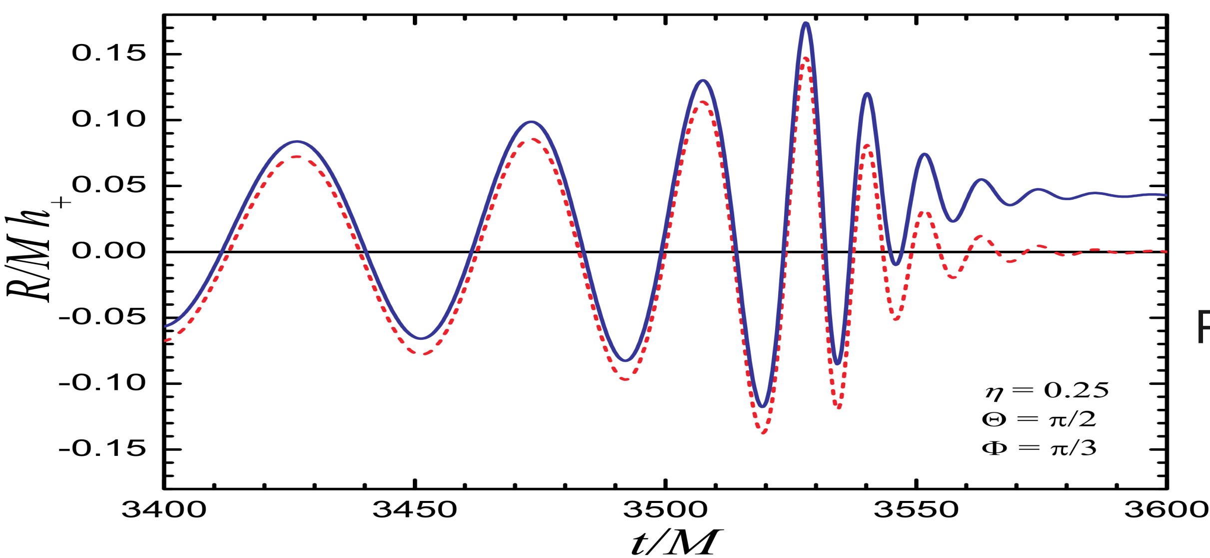
gravitational waves

wishful thinking → gas / non-spherical potential / another black hole

"last-parsec problem", considered mostly solved now

Types of waveforms of interest





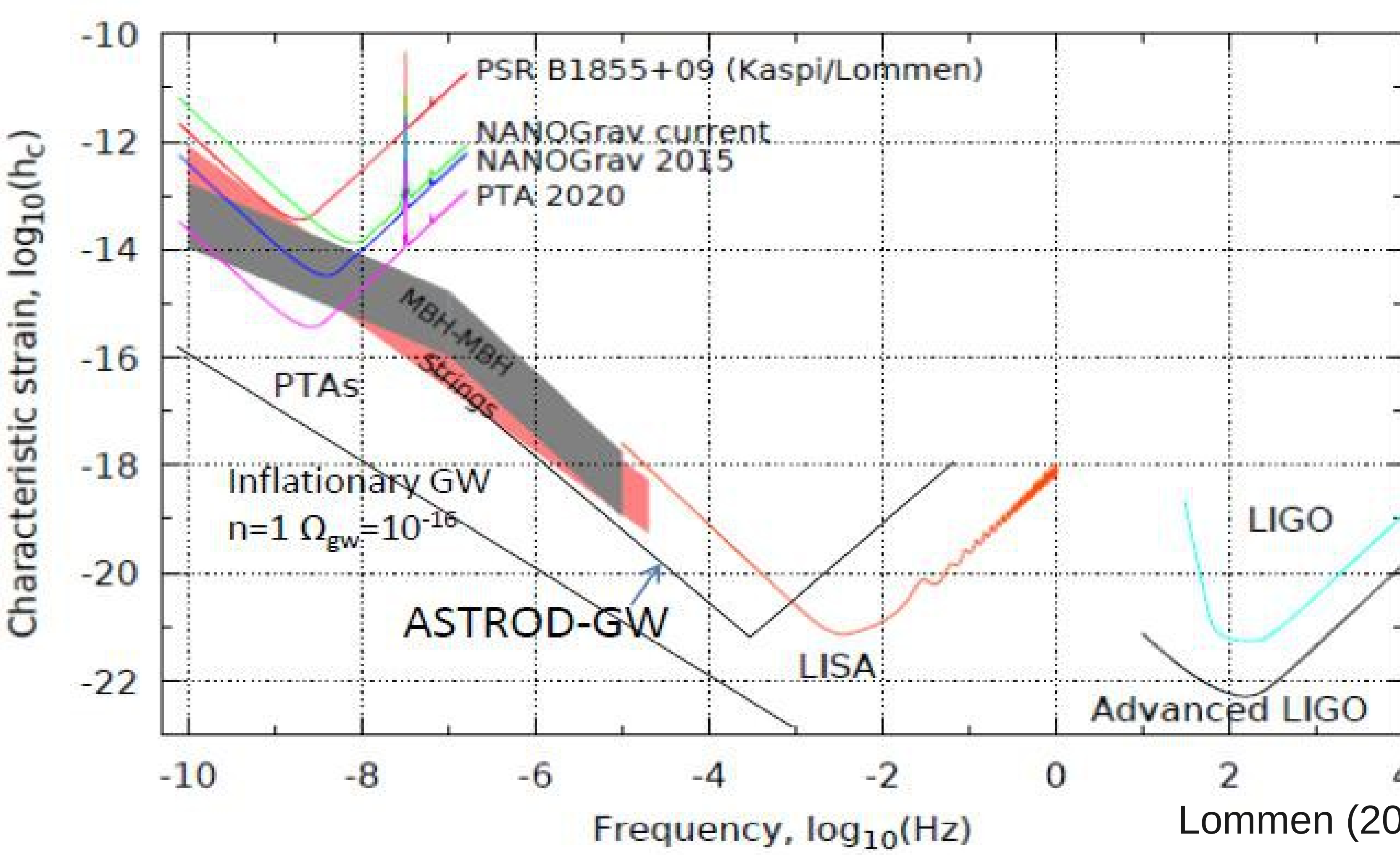
Inspiral – merger – ringdown

Inspiral: continuous wave
Merger: unresolvable.
Ringdown: unresolvable... but:

The memory effect is permanent!

Marc Favata (2010)

Frequency bands GW detectors



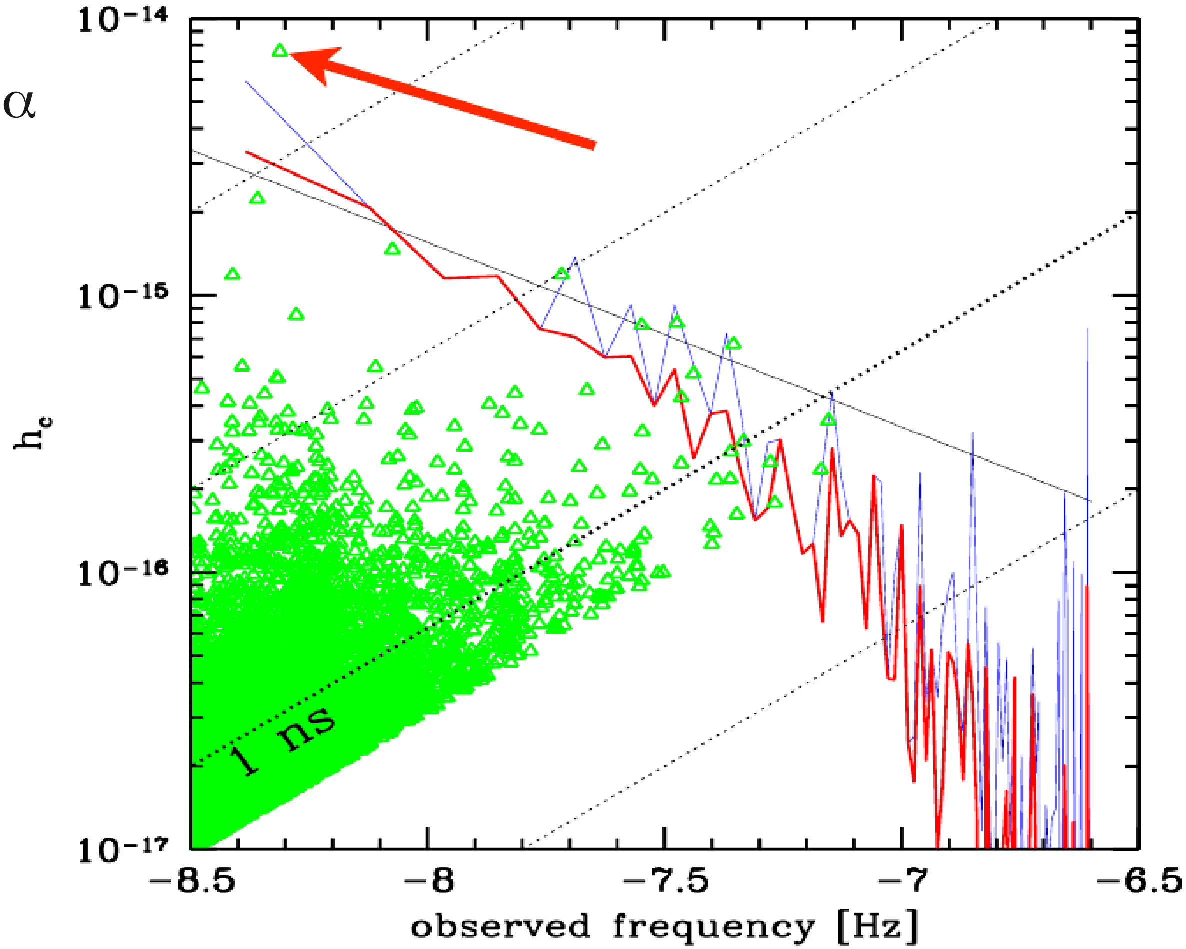
Lommen (2012)

At low frequencies: background

$$h_c(f) = h_c \times (f/f_0)^{\alpha}$$

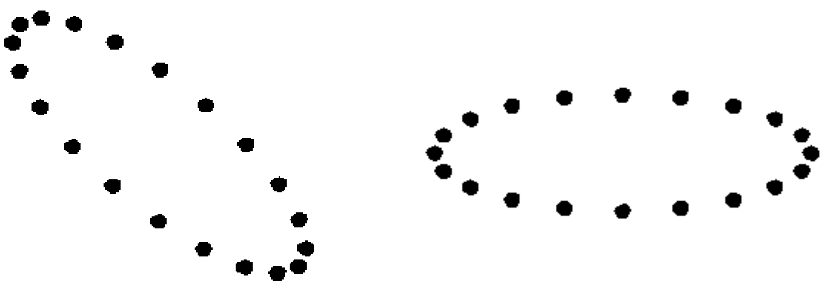
$$\alpha = -2/3$$

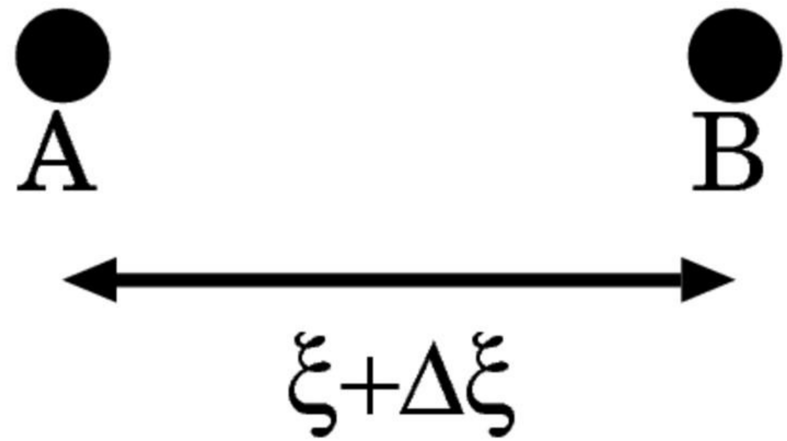
Phinney 01
Jaffe & Backer 03
Wyithe & Loeb 03
Sesana et al. 07, 09



Sesana et al. (2008), Ravi et al. (2012): Theory and simulations suggest there is a non-zero probability that individual sources have SNR above the background.

Why pulsars?





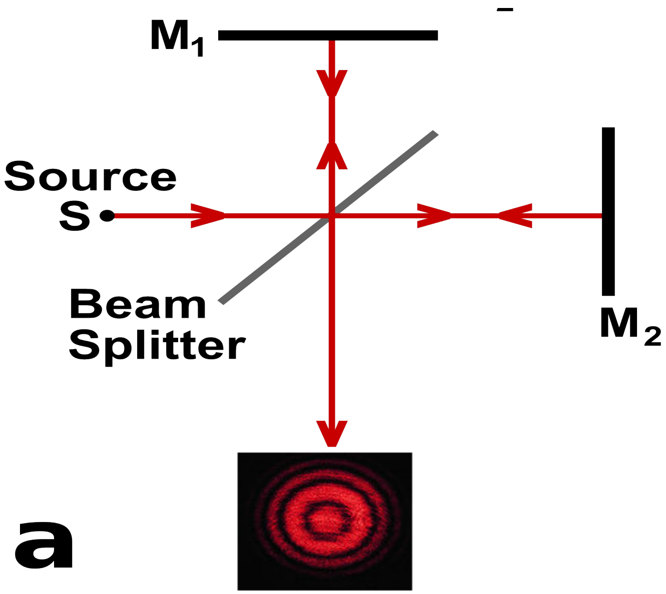
Effect of GWs is an oscillating Riemann curvature tensor, possible in two polarisations.
→ Measure propagation length!

Speed of light is constant.

Measure time, not distance.

Effect and detectability of GWs

Credit: Advanced Technology Center, NAOJ



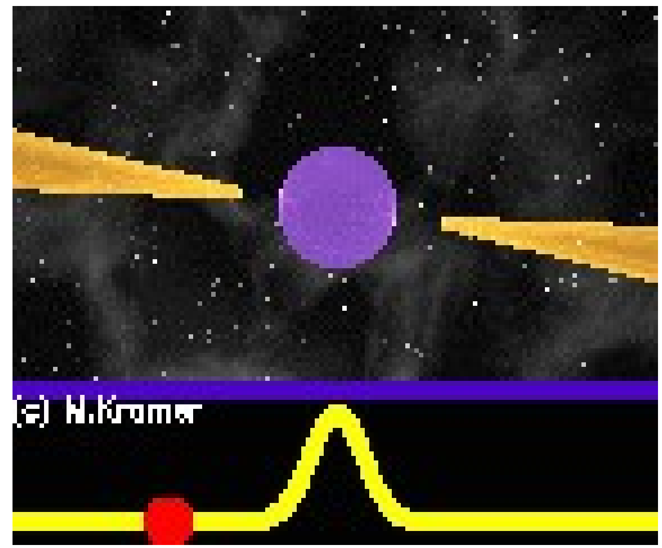
Emit light, and reflect back

Now it is truly a 'timing experiment'

LASER has precise frequency
→ equivalent to clock

Interferometry for detection

Need precise frequency/clock



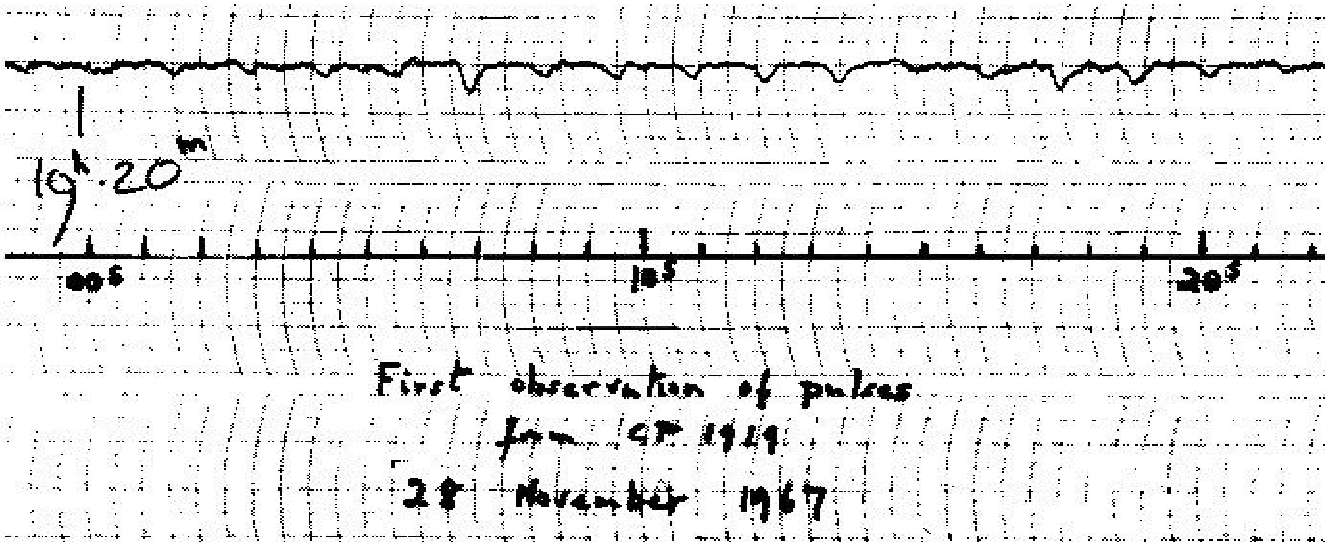
Could say that KAGRA uses a LASER as an accurate frequency standard

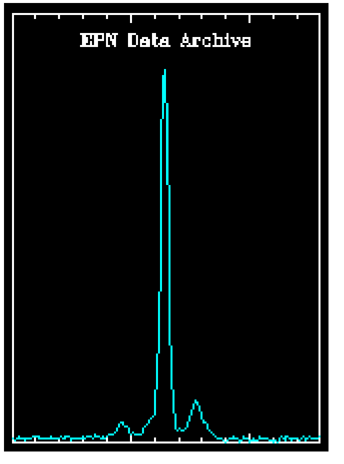
What about pulsar's spin frequency?

Period of PSR B1937+21:
T = 0.0015578064487275 s

Pulsars

Discovery: LGM1

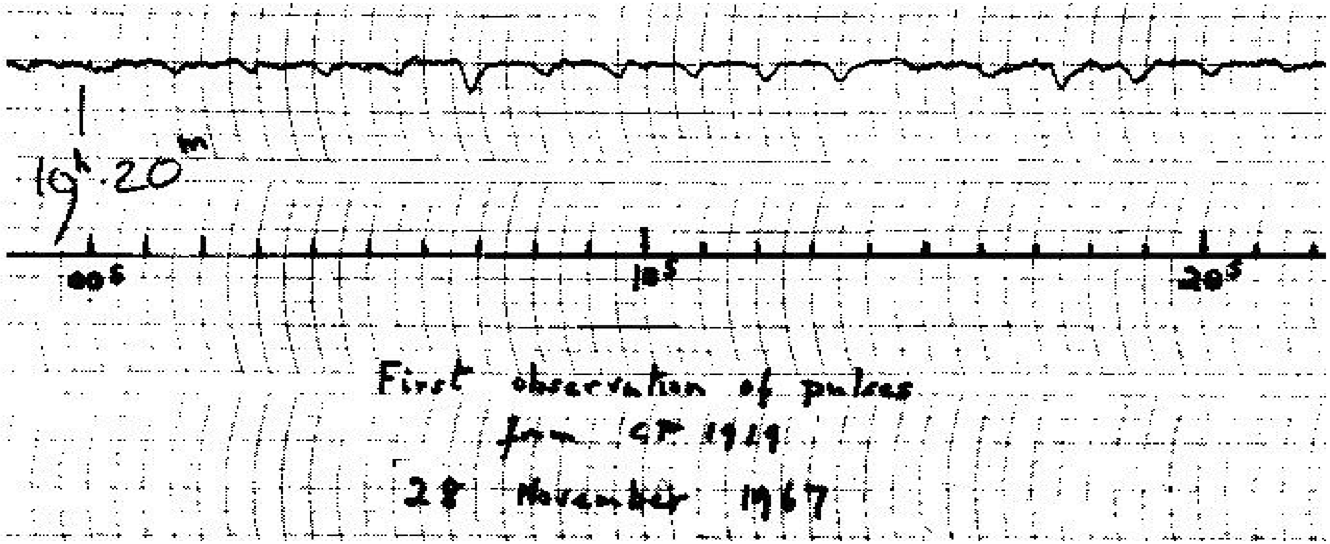




Pulsar discovery in 1967: LGM1
(= PSR B1919+21)

'Knocking sound'

Discovery: LGM1



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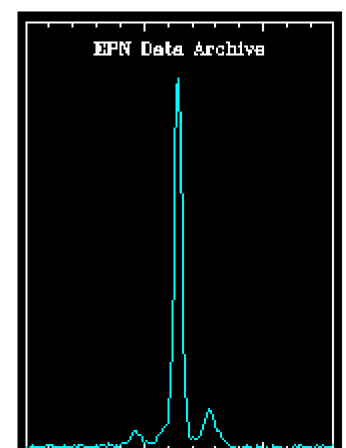
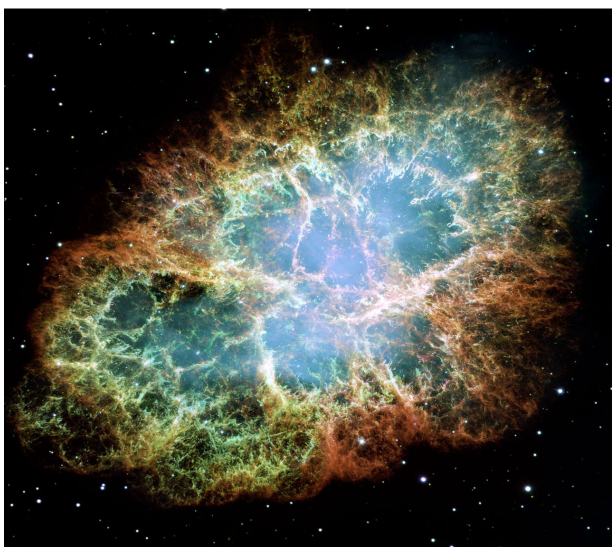


Explanation: neutron star

Baade & Zwicky in 1934: "With all reserve we advance the view that a supernova represents the transition of an ordinary star into a new form of star, the neutron star, which would be the end point of stellar evolution. Such a star may possess a very small radius and an extremely high density."



Crab Nebula. Remnant of 1054 AD supernova, seen by Chinese astronomers ('guest star').

Pulse profile of the Crab.

Associated supernova: the Crab

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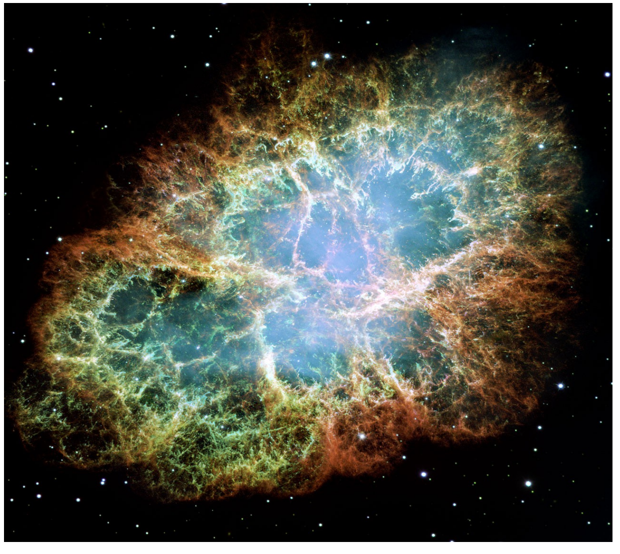

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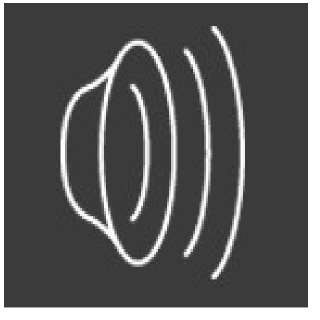
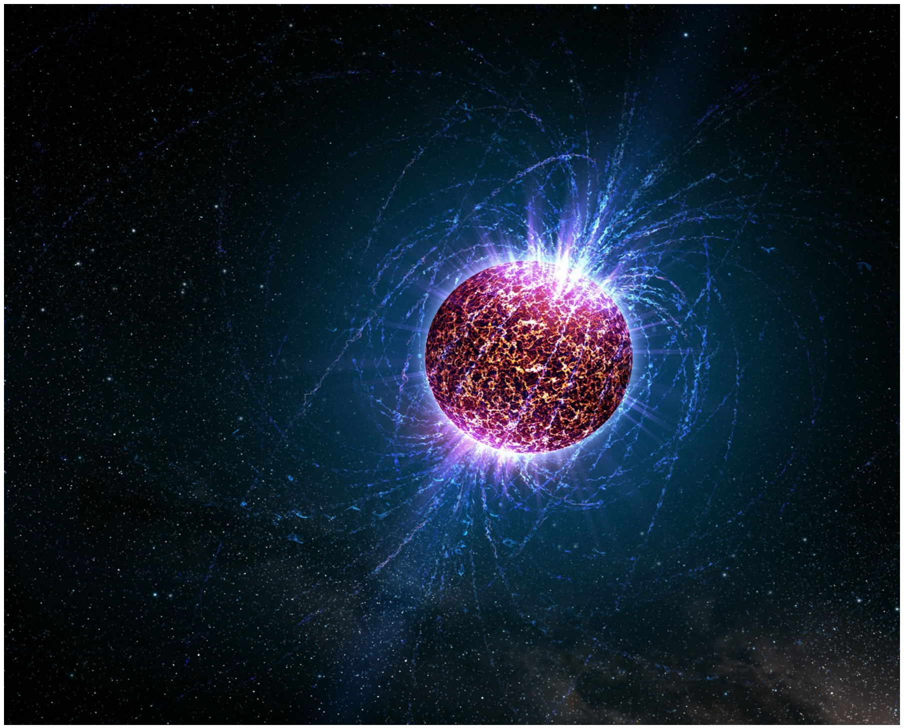
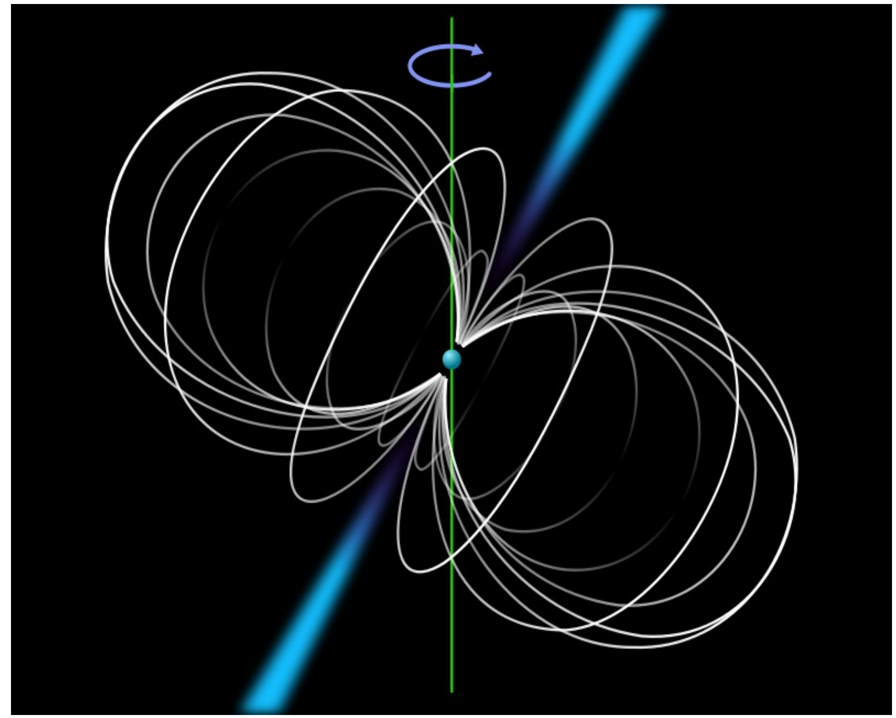


Pulse profile of the Crab.

Pulsars





Star dies → core pressure gone
Star collapses → compact object

Neutron star for heavy stars

Conserved from star:
- Angular momentum
- Magnetic field

→ Dynamo!

Period of 1.5 ms???

Don Backer et al. (1982), found a pulsar with a spin frequency of 716 Hz (P = 1.5 ms). This was the first millisecond pulsar. Can this still be a rotating neutron star?

Arecibo Observatory

Pulse profile of B1937

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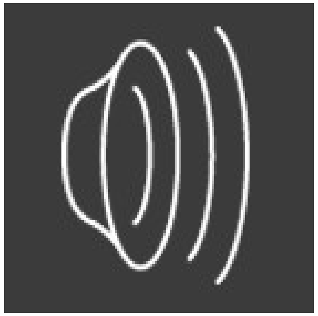
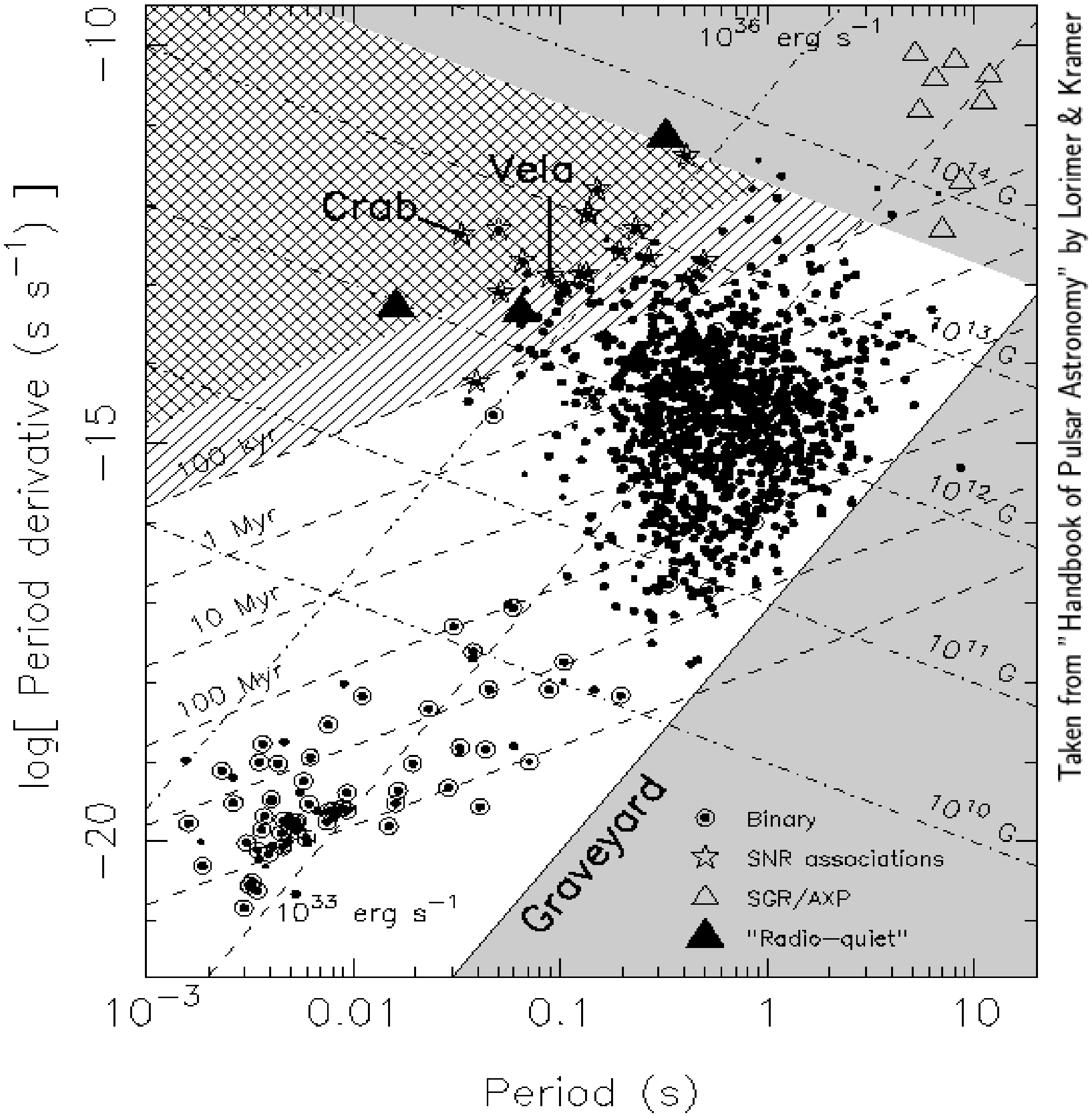


Arecibo Observatory

Radius less than 16km. At equator, spin velocity > 70,000 km/s (= 24% speed of light)

Pulse profile of B1937

P-Pdot diagram



Most stable 'clocks' are in the bottom left

Note: almost all binaries there

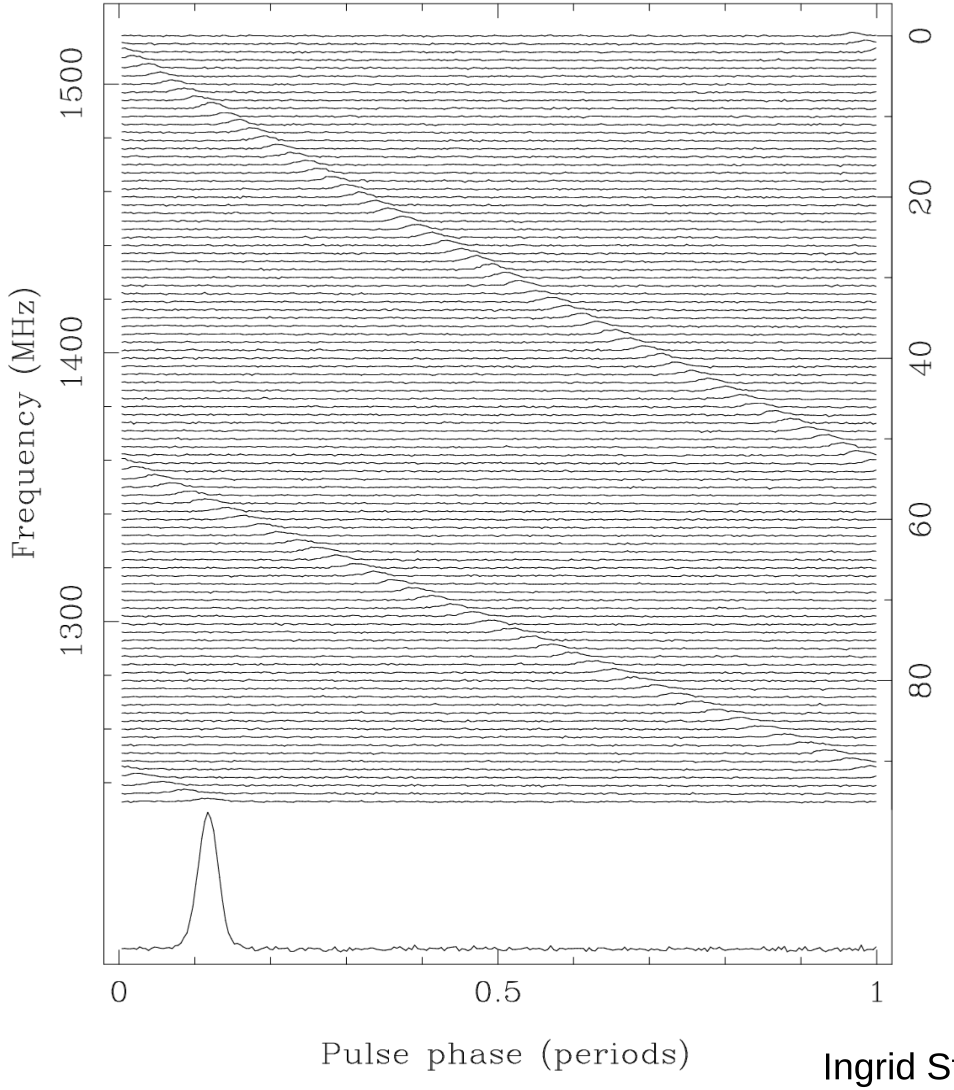
Millisecond pulsars

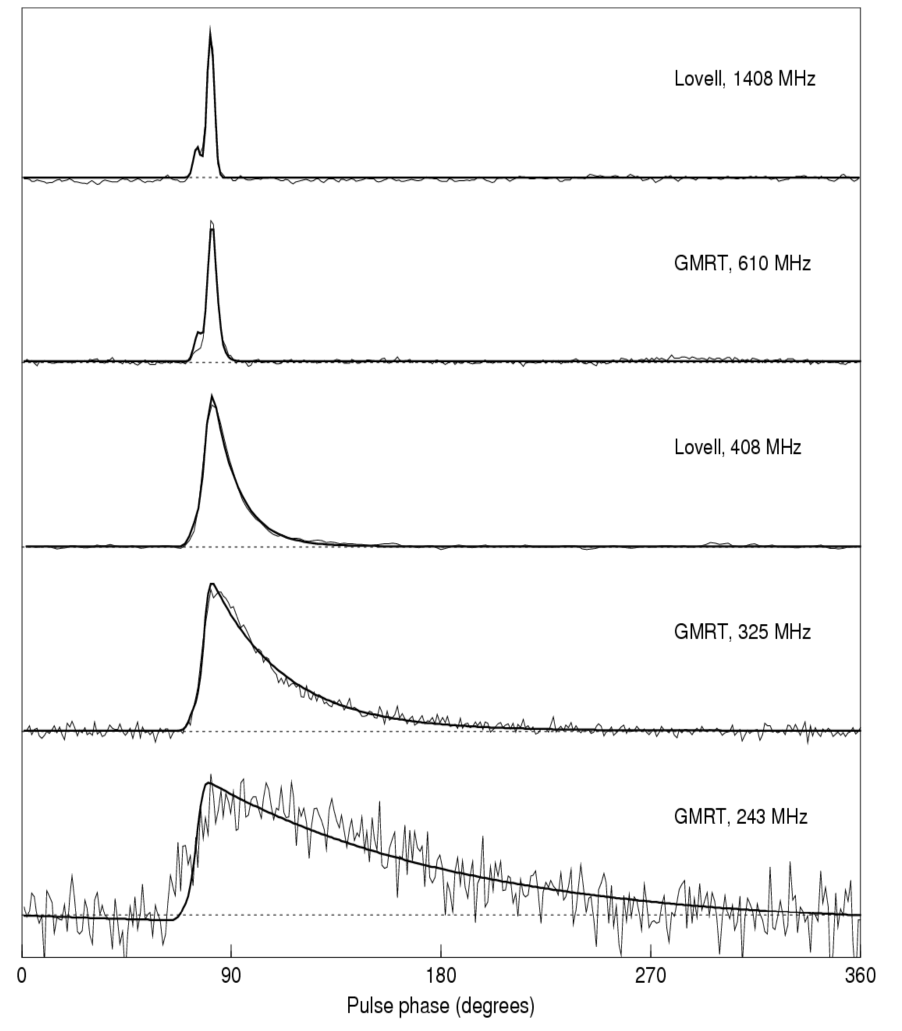
Credit: NASA animations

Pulsar Timing

Parkes Radio Telescope

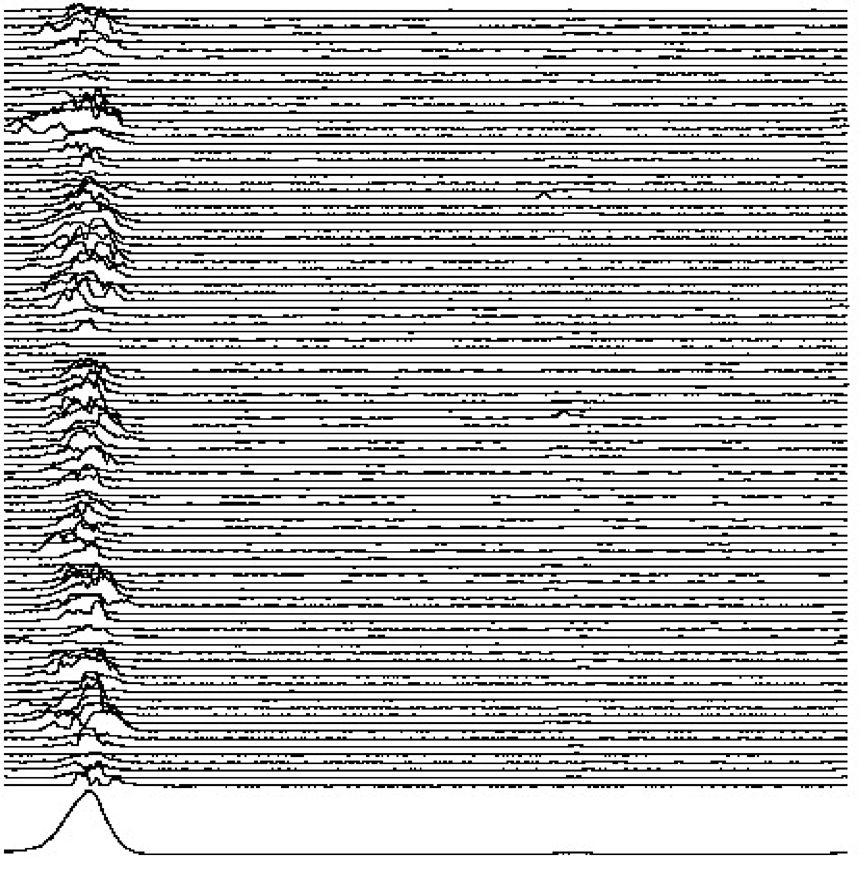
Interstellar medium

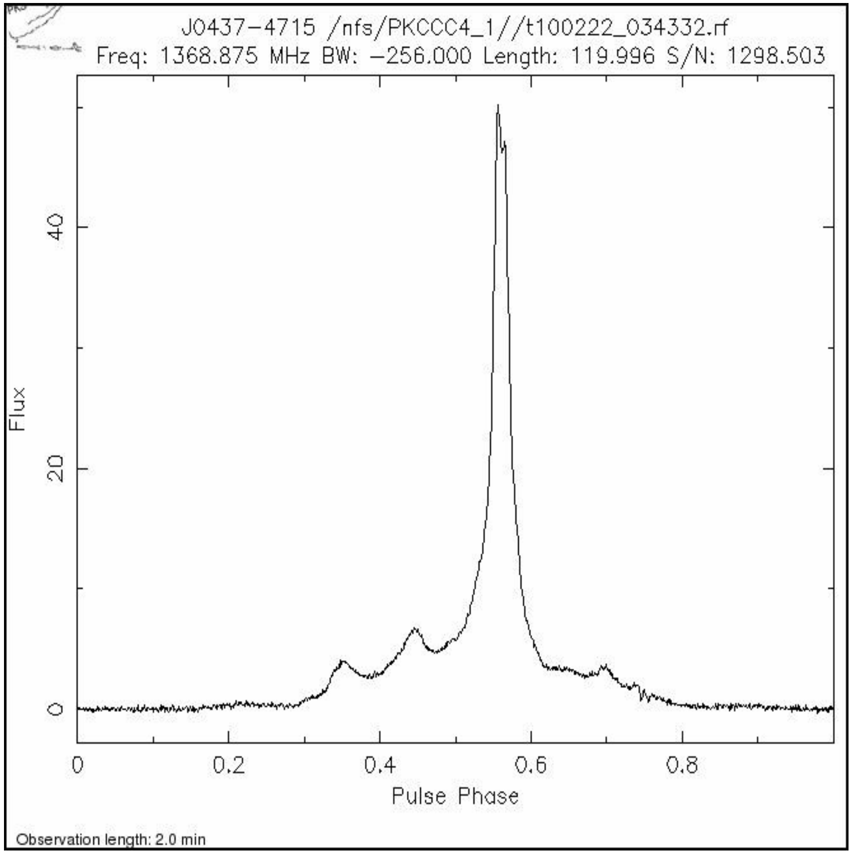




Ingrid Stairs (2001)

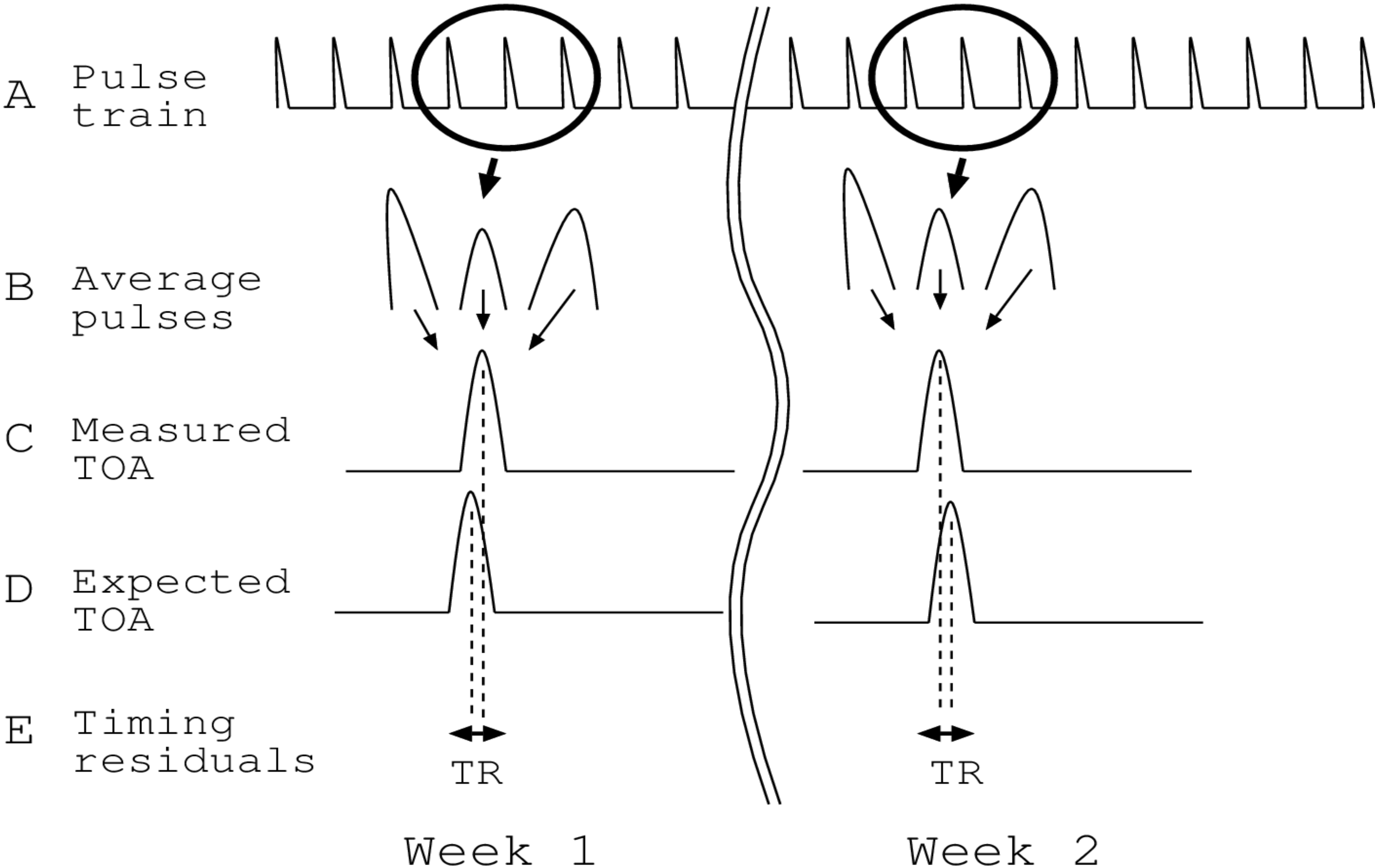
Pulse profiles





Ingrid Stairs (2001)

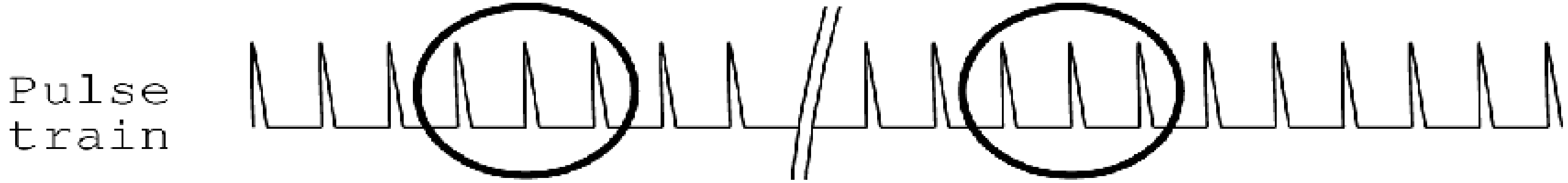
Timing residuals



Some typical numbers

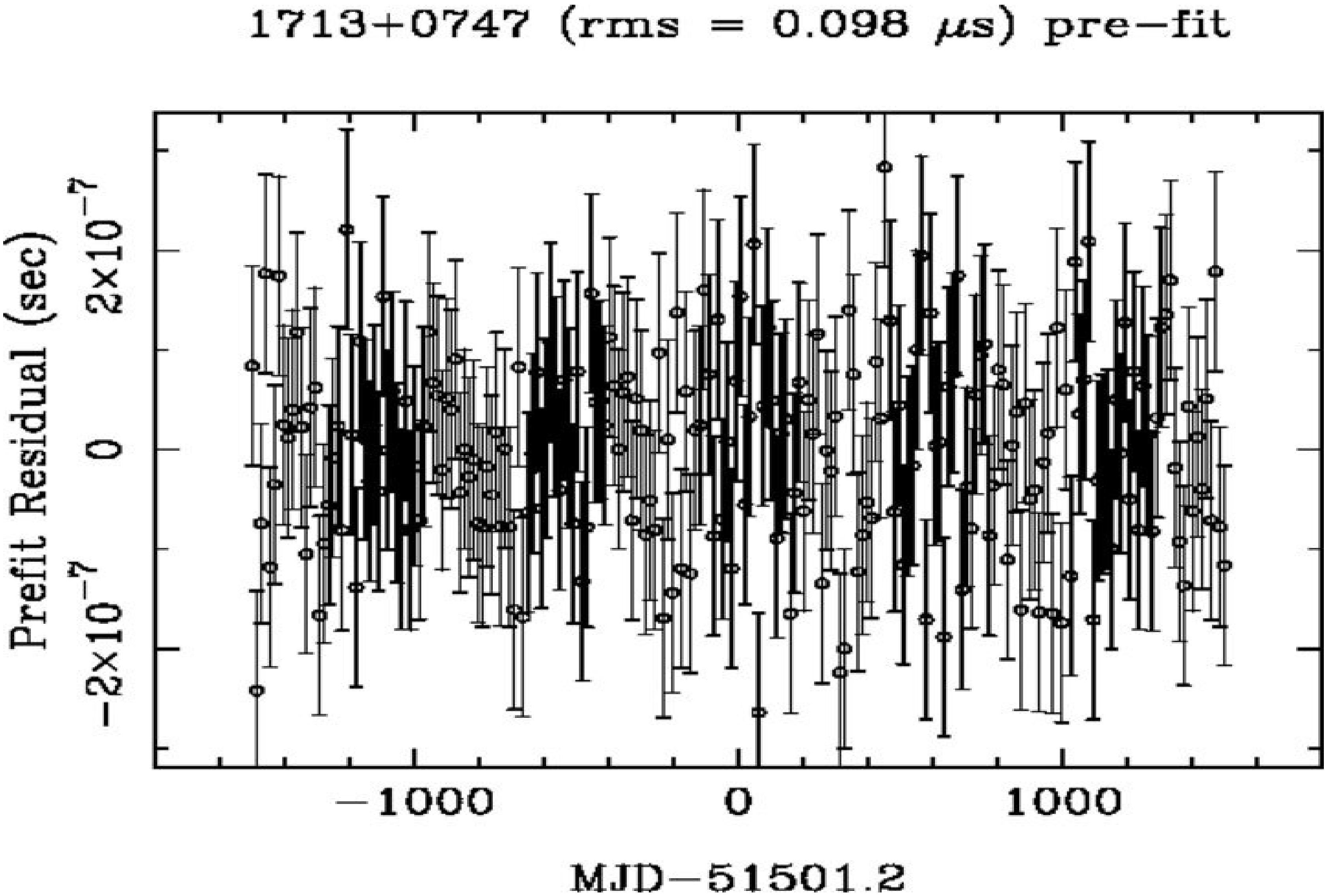
- **Pulse period:** 5 ms
- **Pulse width:** 0.5 ms (~10% of period)
- **Timing accuracy:** 100 ns
- **Pulsar distance:** several kpc (3 * 10^19 m)

→ sensitivity to distance variations of 30 m (< 1 part in 10^18)



Can account for every not-observed rotation!

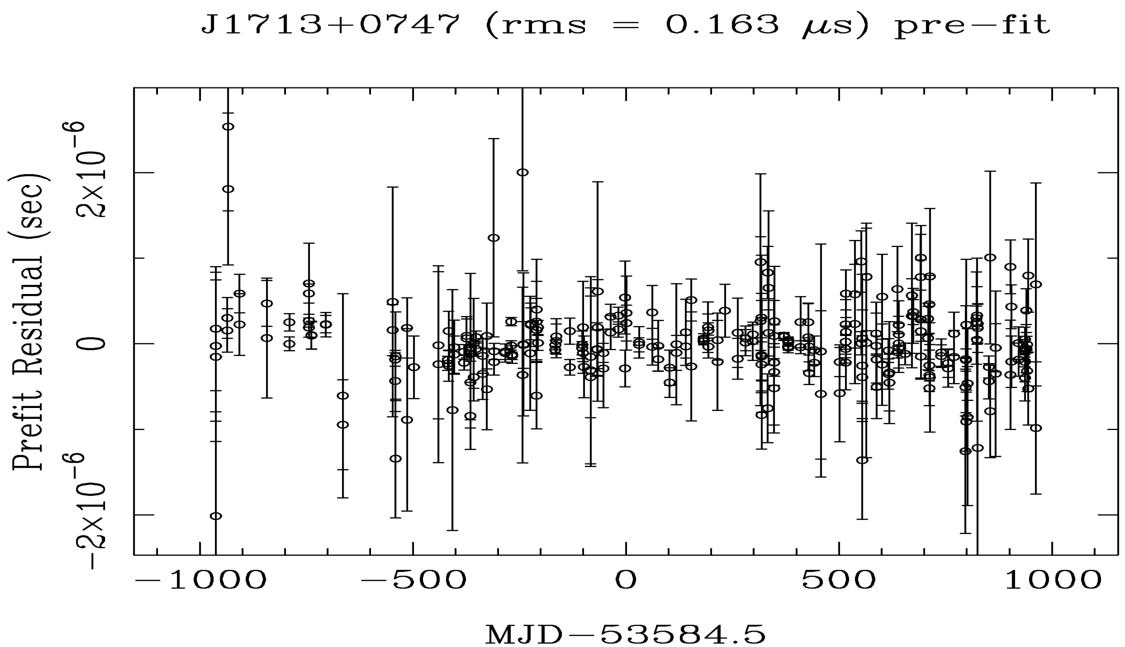
Timing residuals



Standard procedure

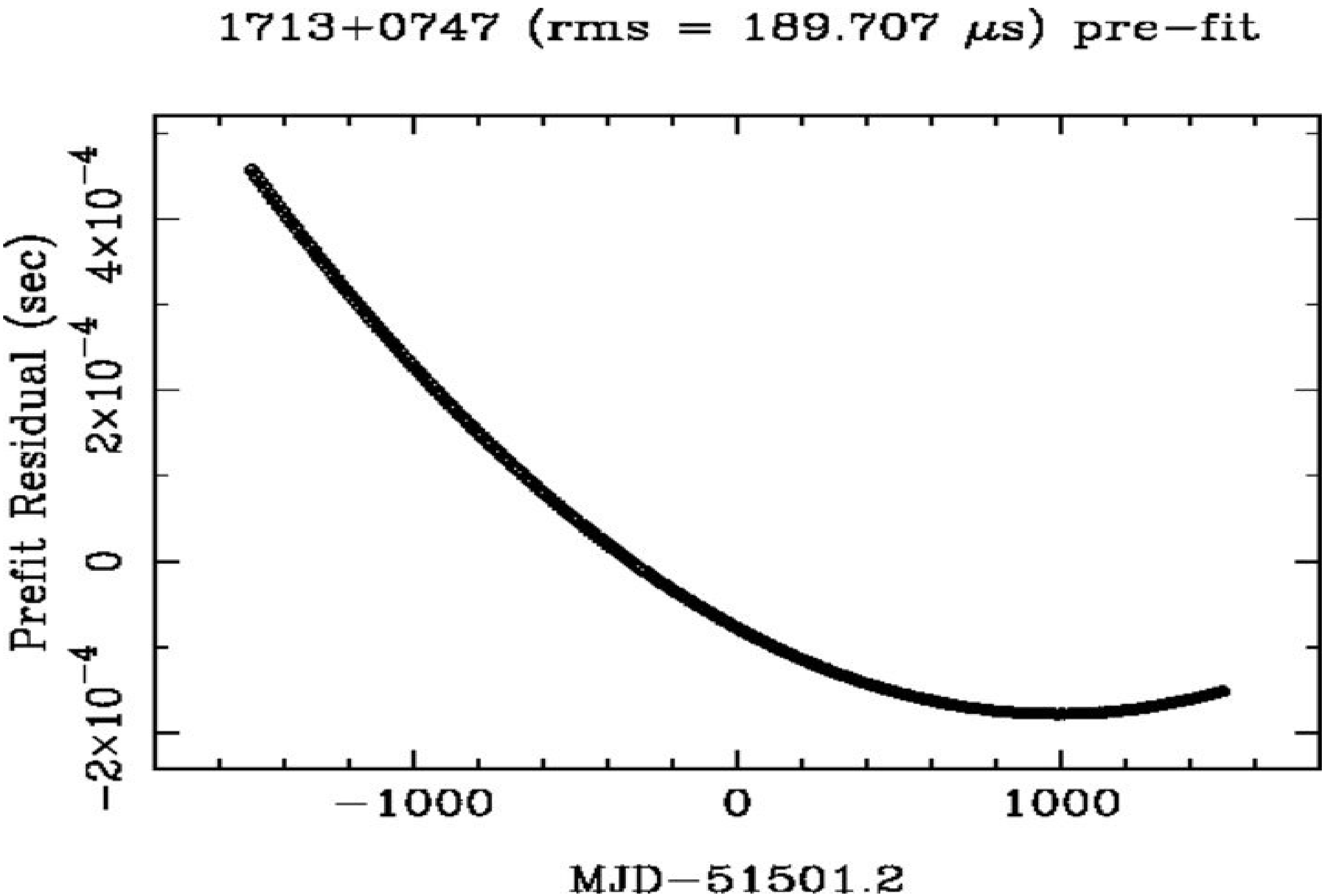
- Problem: model of pulsar motion obtained from timing
- But to produce residuals, we need the timing model

- Iterate least-squares fitting until it converges (by hand)

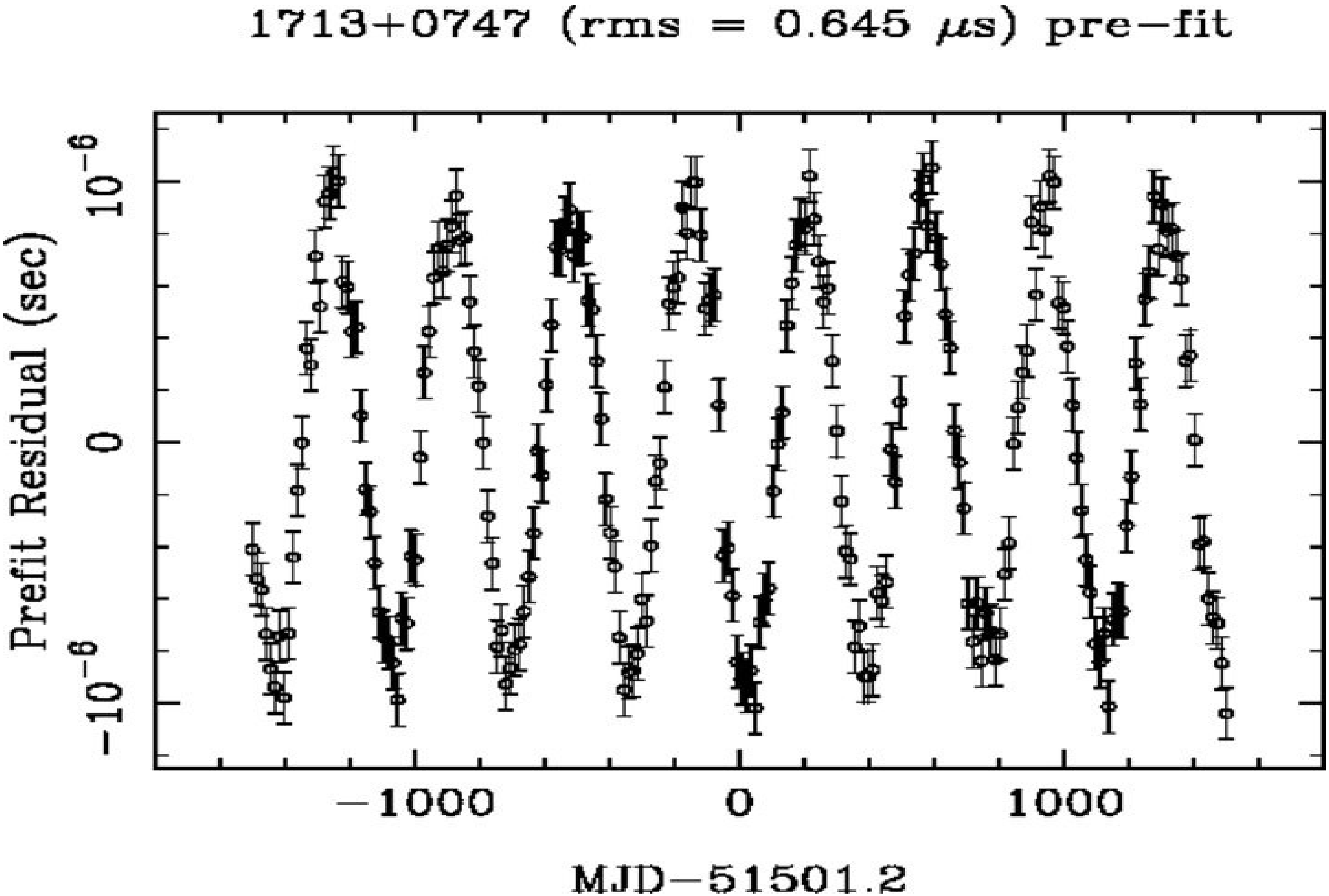


Not automated
“Pulsar timing is an art”
– G.H. Hobbs

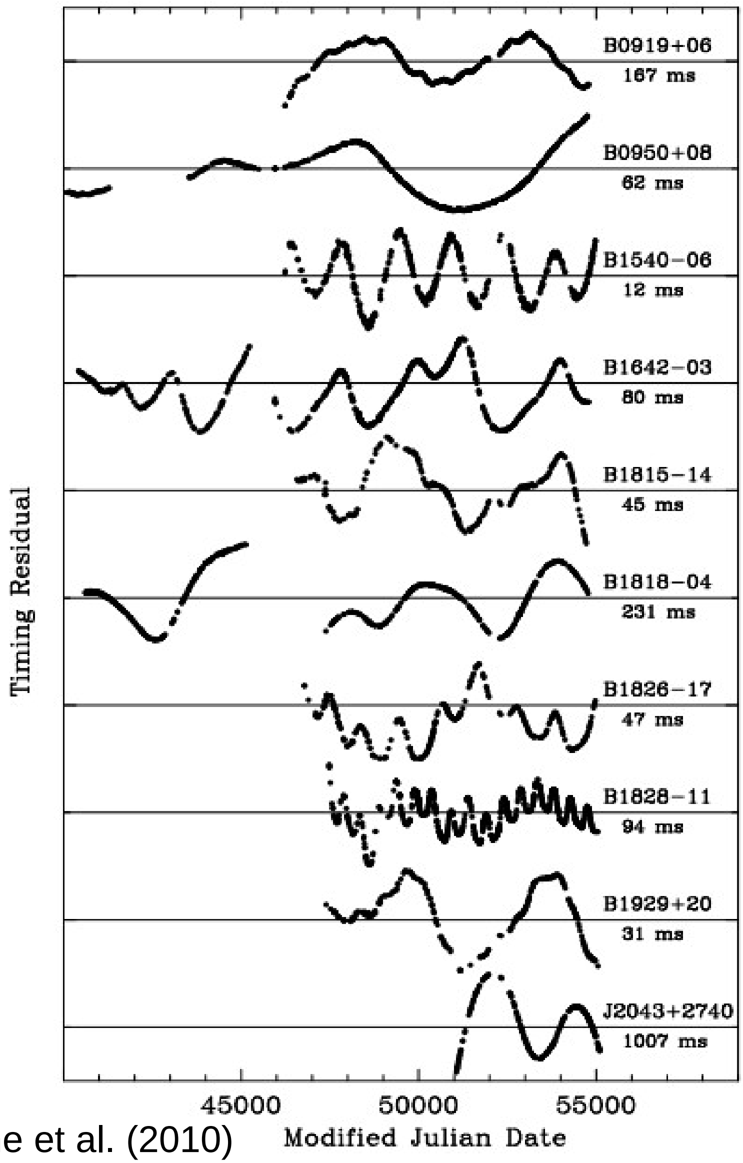
The timing model: spindown



The timing model: declination



Red timing noise



Lyne et al. (2010)

Random walk in torque plus extra effects?

Timing noise severe in canonical pulsars → use millisecond pulsars (MSPs)

Only seen in few MSPs. For now...

Examples of pulsar timing

Bottom line: we do not fully understand pulsars and pulsar beam emission. Does not matter for pulsar timing. **'It just works'**, and we use pulsars as tools.

Let's look at some applications of pulsar timing

The Hulse-Taylor binary

Hulse and Taylor found a binary pulsar in 1973. Nobel prize 1993.

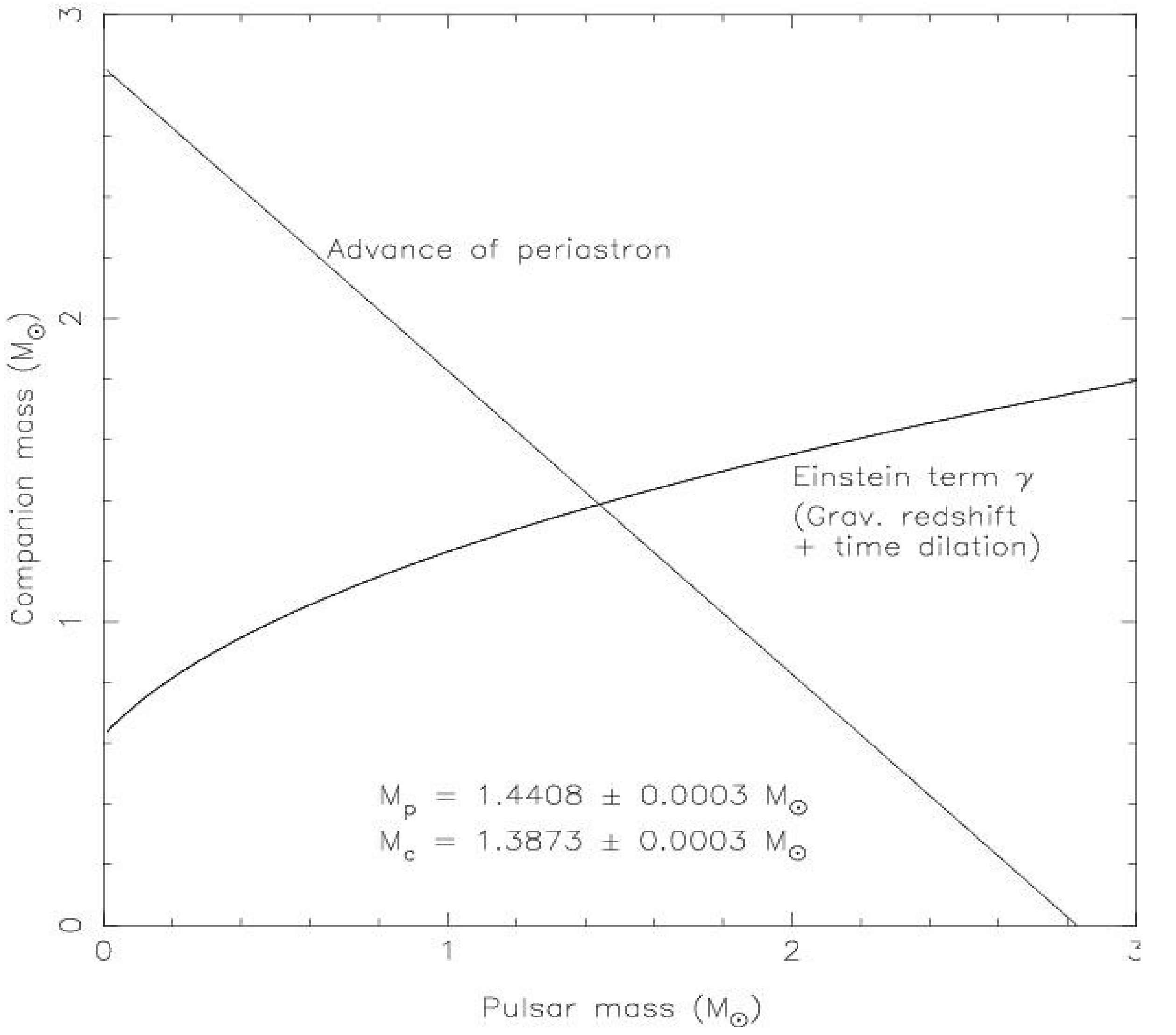
Parameter	Value
Orbital period P_b (d)	0.322997462727(5)
Projected semi-major axis x (s)	2.341774(1)
Eccentricity e	0.6171338(4)
Longitude of periastron ω (deg)	226.57518(4)
Epoch of periastron T_0 (MJD)	46443.99588317(3)
Advance of periastron $\dot{\omega}$ (deg yr^{-1})	4.226607(7)
Gravitational redshift γ (ms)	4.294(1)
Orbital period derivative $(\dot{P}_b)^{obs}$ (10^{-12})	−2.4211(14)

Table 2: *Orbital parameters for PSR B1913+16 in the DD framework, taken from [144].*

Post-keplerian parameters

The PK parameters are constructed such that only the two masses are unknown.

Thus: only two unknown parameters!



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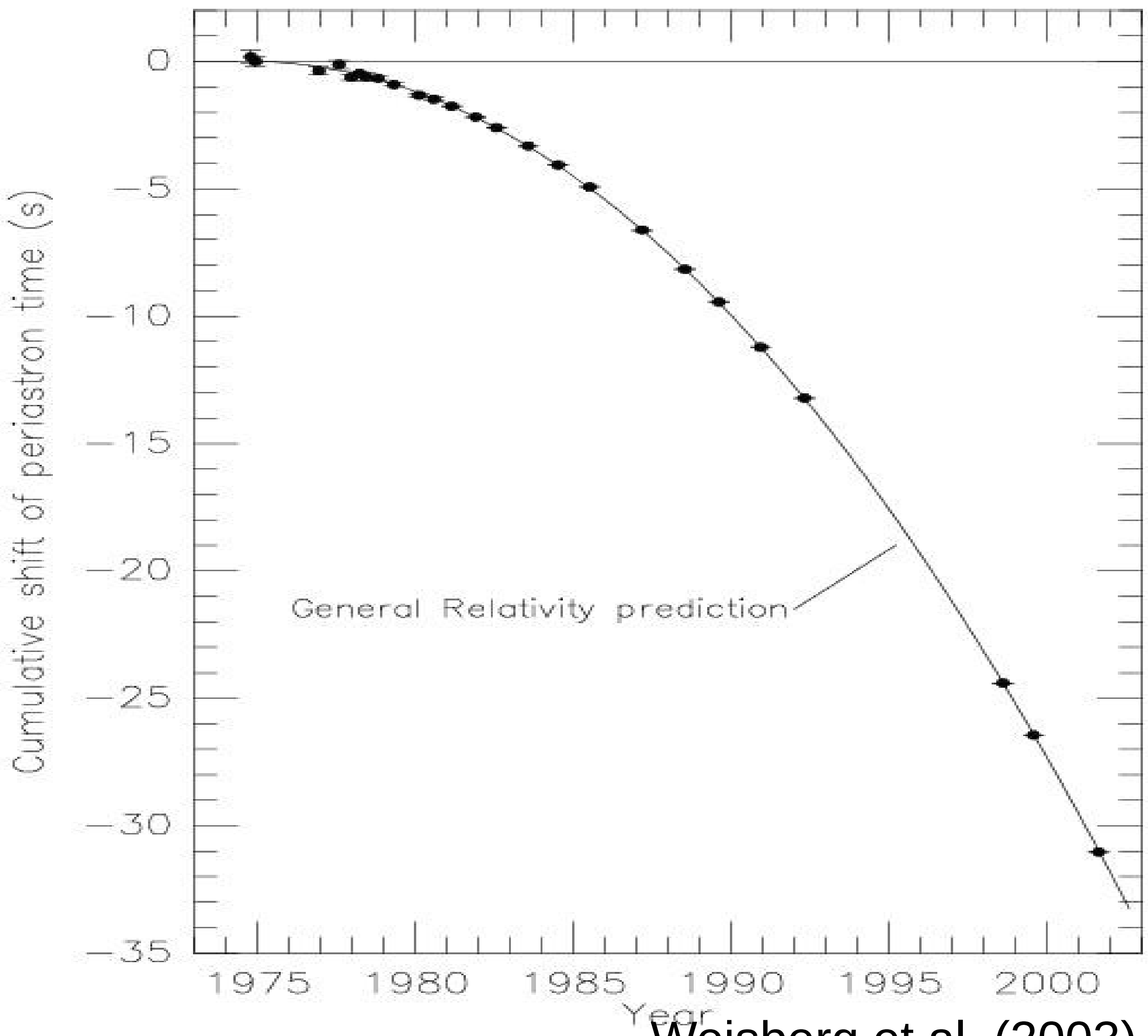
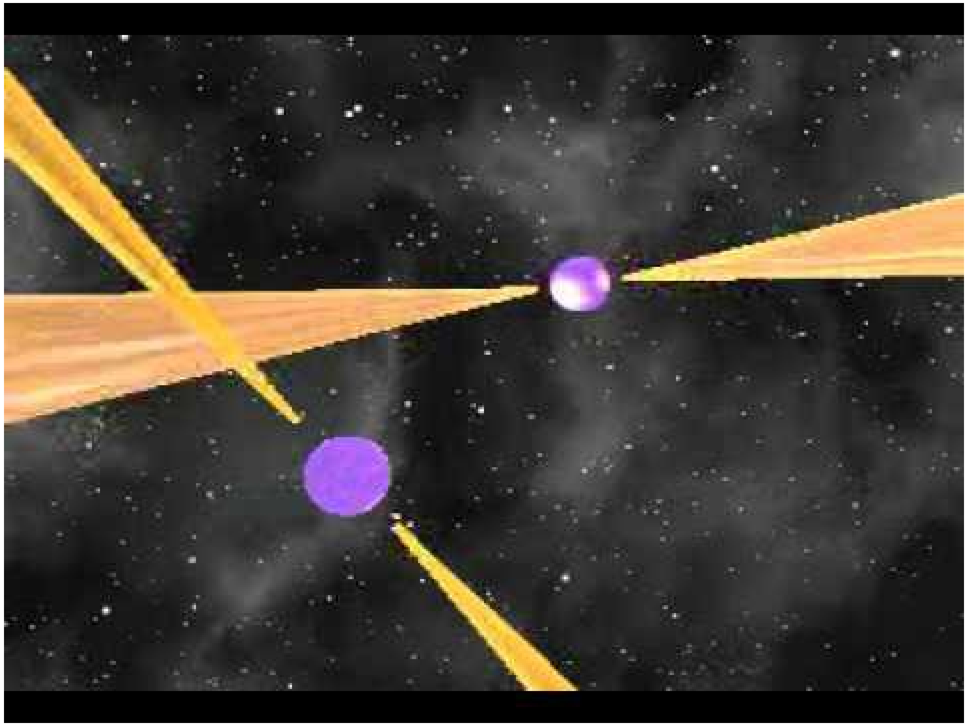


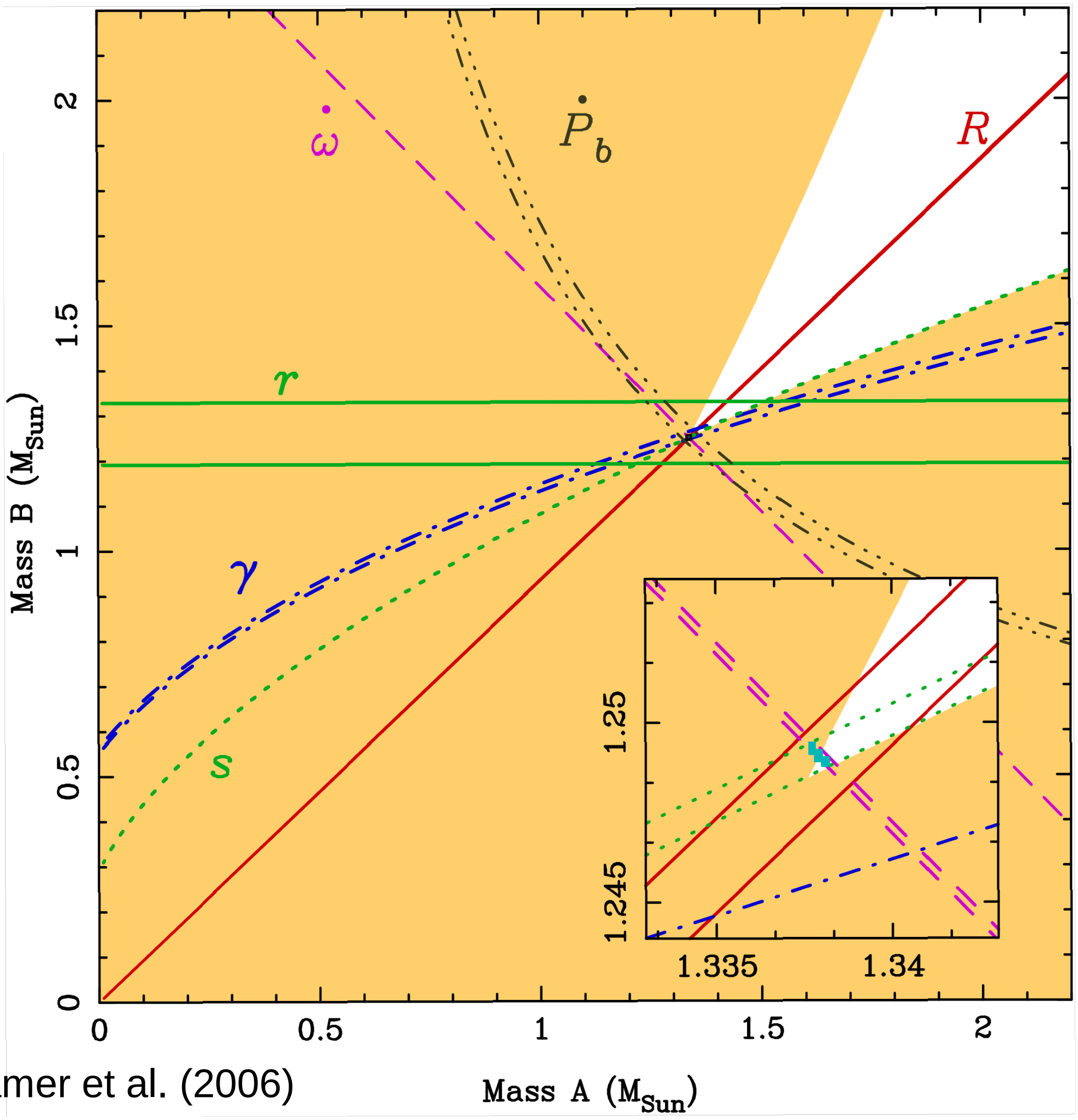
Weisberg et al. (2003)

Double pulsar

Discovered in the Parkes multibeam survey (Burgay et al. 2003). Incredibly lucky: edge-on system. Eclipses probe pulsar magnetosphere



Double pulsar GR tests



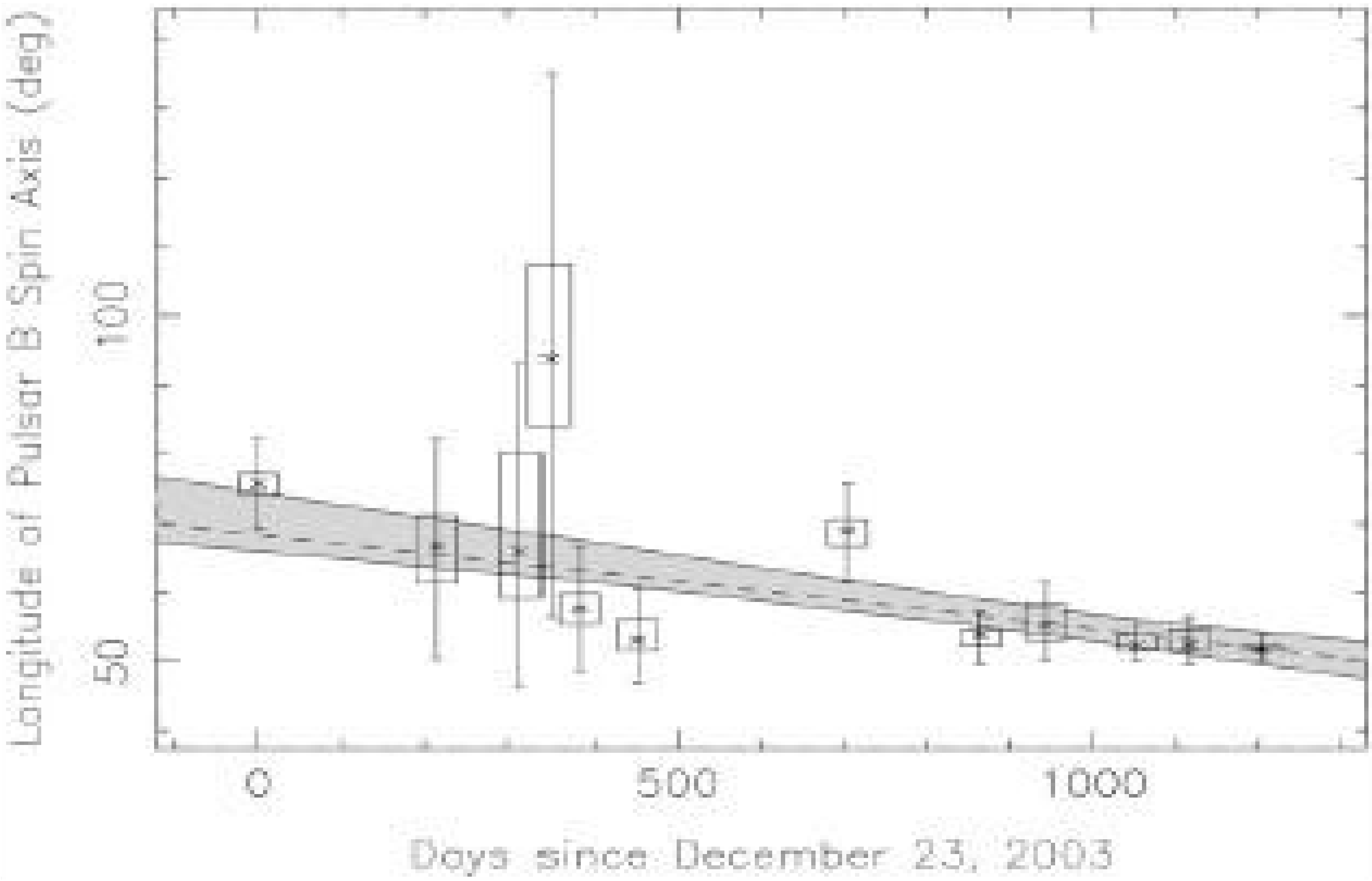
Kramer et al. (2006)

w: precession of periastron
g: time dilation gravitational redshift
r: Shapiro time delay (range)
S: Shapiro time delay (shape)
Pb: sec. change of the orbital period
R: mass ratio

Double pulsar magnetosphere

Breton et al. (2006)

Geodetic precession



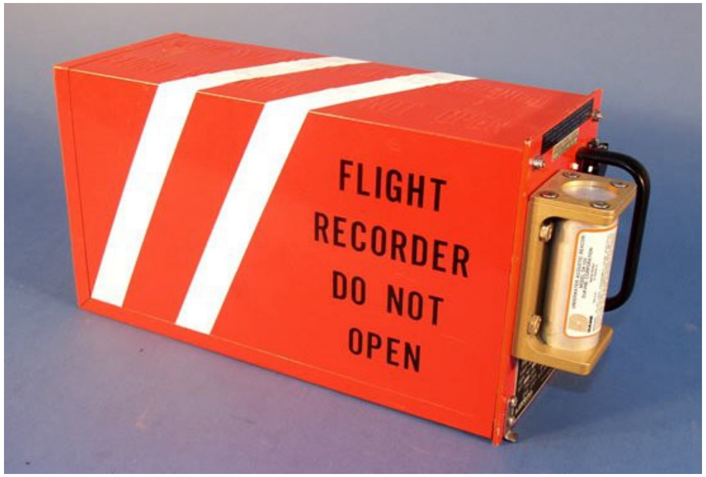
Breton et al. (science) Only available for the double pulsar.

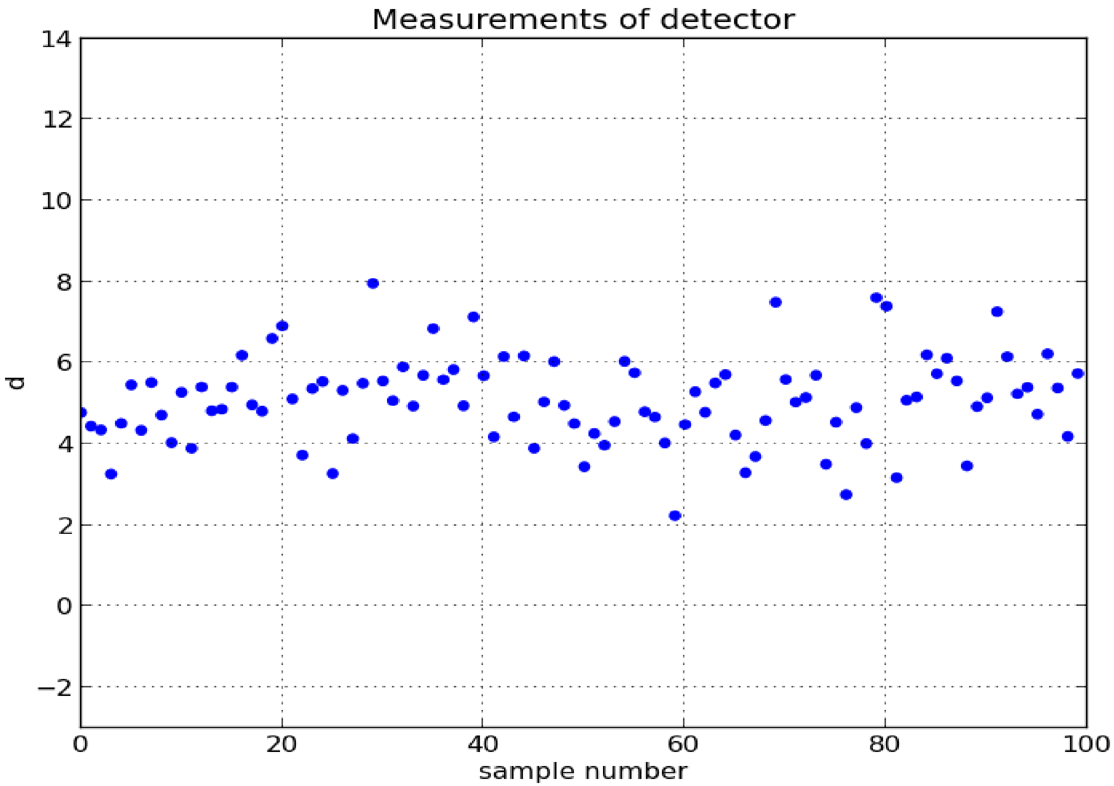
Data analysis outline

The goal is to give you an idea how to do gravitational-wave searches in practice. Main problems are different from ground-based detectors.

- Data analysis – toy problem
- Gravitational waves: detection & upper limits
- The IPTA Mock Data Challenge
- Python packages to do all this at home

Data analysis – toy problem



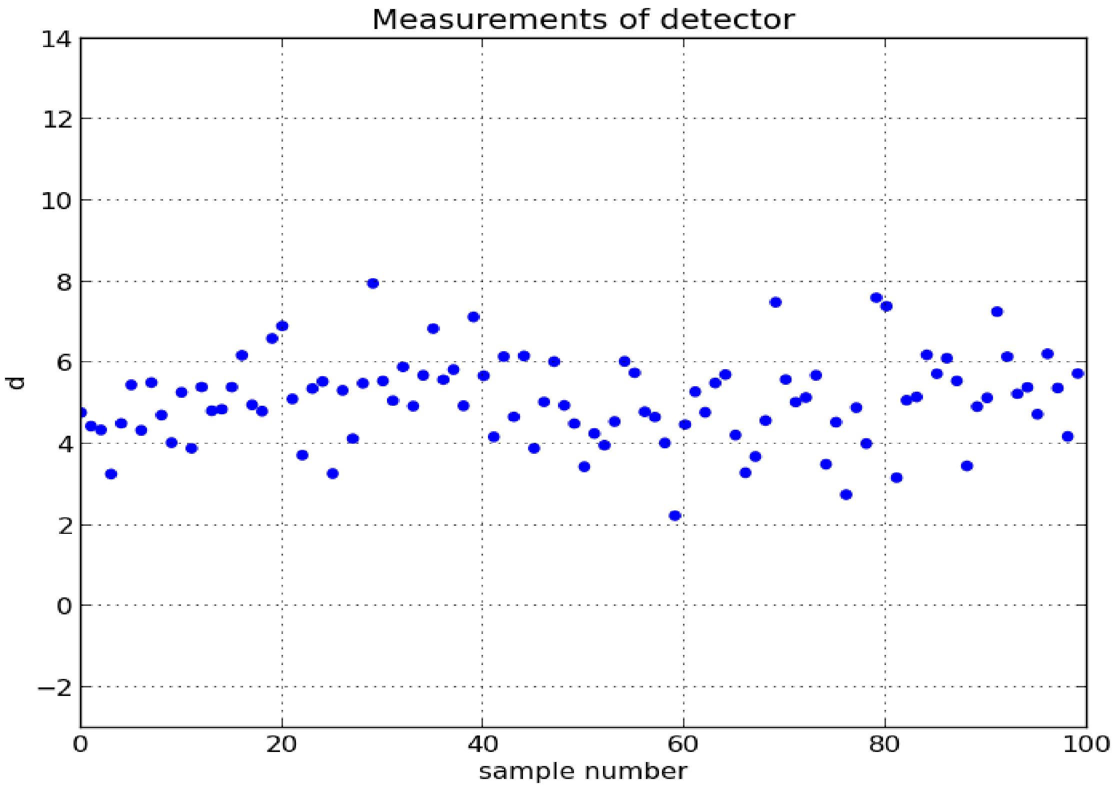


Black box detector. Know nothing about the data, except for data points.

Question:
What can we say about the data and the detector?

Calculate mean, variance. Take a Fourier transform

What we need is a model



With a model, we can test the model against the data: discover model parameters, compare models, good fit/bad fit.

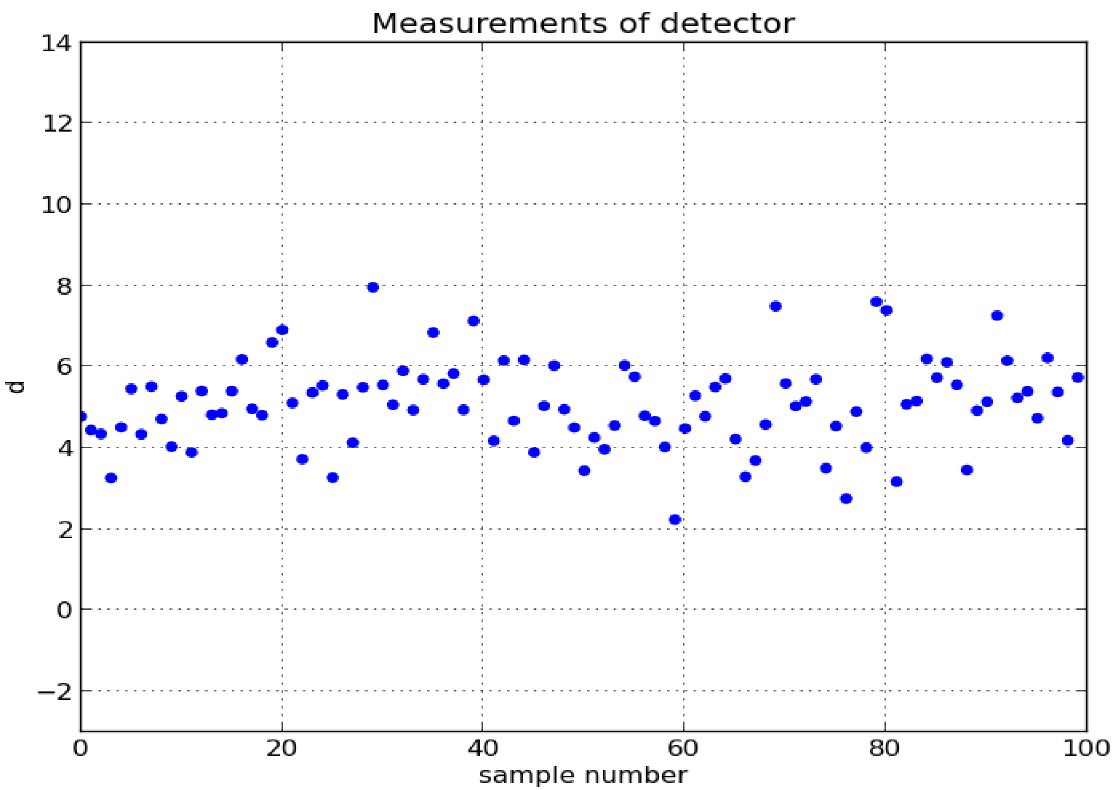
Our 'toy' model is…. see next slide

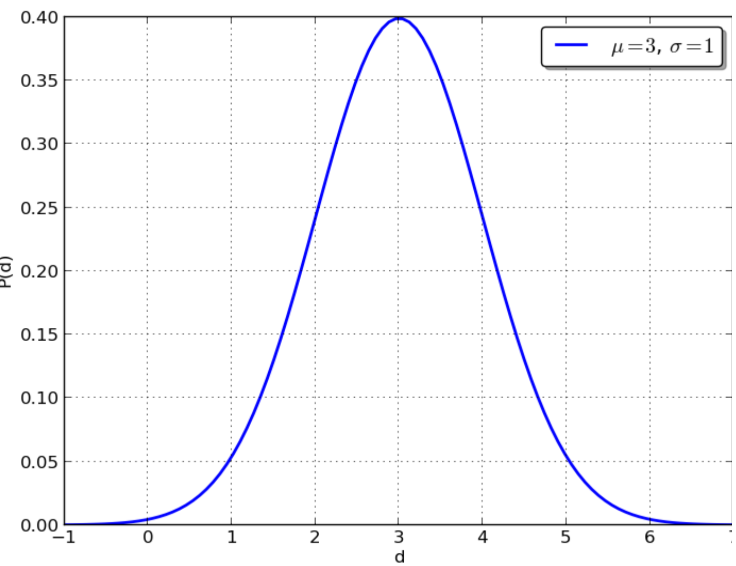
What we need is a model

$$d_i = \mu + \epsilon_i$$

$$N(0, \sigma) \rightarrow \epsilon_i$$

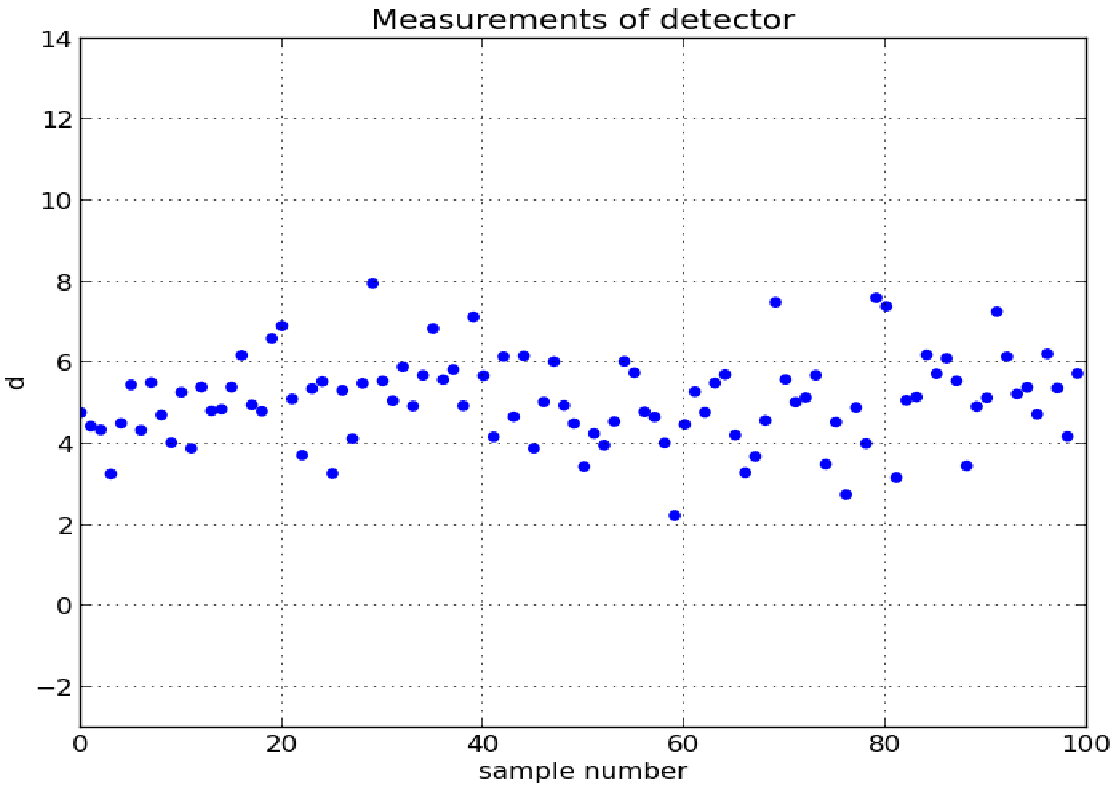
Our toy model is a Gaussian. It is described by two parameters: mean and standard deviation



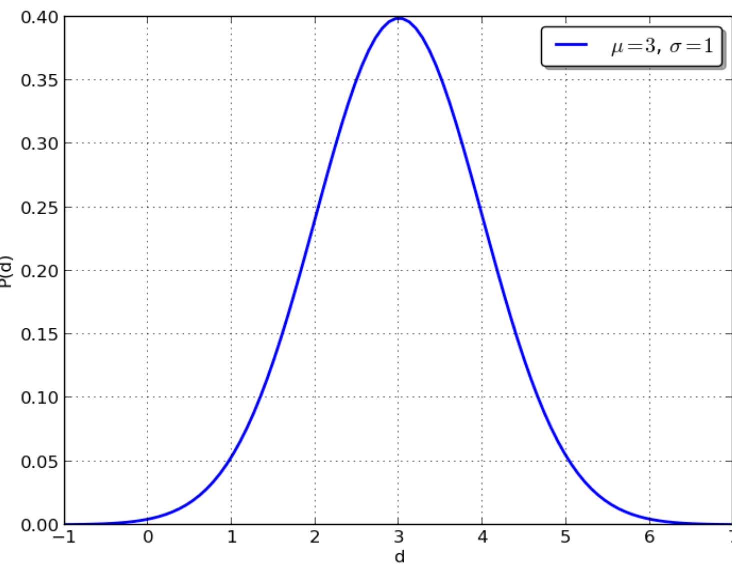


$$P(\vec{d}|\mu,\sigma) = \frac{1}{\sigma^n \sqrt{(2\pi)^n}} \exp\left[\underbrace{\frac{-1}{2}\sum_i \left(\frac{d_i - \mu}{\sigma}\right)^2}_{\chi^2}\right]$$

Question: what are the parameters?



Question:
What are the parameters I used to generate the data in this plot?

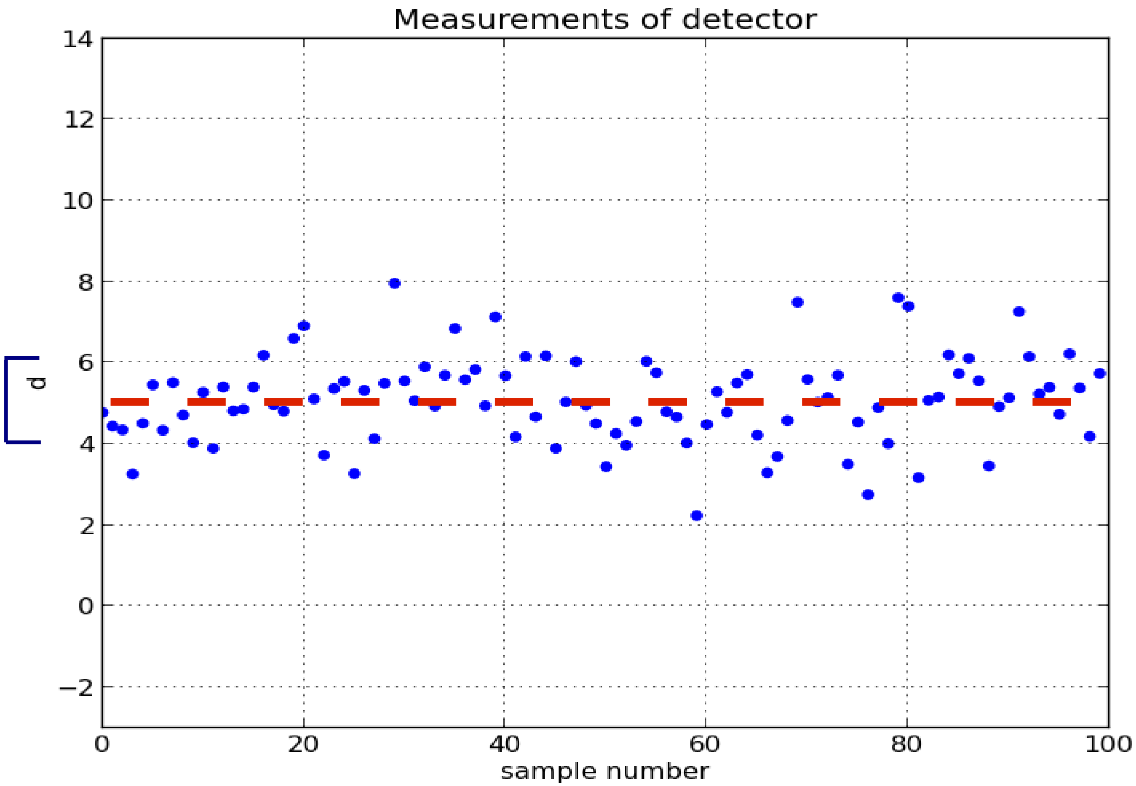


$$P(\vec{d}|\mu,\sigma)=\frac{1}{\sigma^n\sqrt{(2\pi)^n}}\exp\left[\frac{-1}{2}\underbrace{\sum_i\left(\frac{d_i-\mu}{\sigma}\right)^2}_{\chi^2}\right]$$

Use mean and rms

Mean is the average:

$$\bar{\mu}=\frac{1}{n}\sum_i d_i \quad (\text{Min}\,\chi^2)$$



Root-mean-square error:

$$\bar{\sigma}=\sqrt{\frac{1}{n}\sum_i (d_i-\mu)^2}$$

How estimate that if we do not know the mean?

$$\bar{\sigma}_{n-1}=\sqrt{\frac{1}{n-1}\sum_i (d_i-\bar{\mu})^2}$$

The 'n-1' comes from the number of parameters (regressors) we determine from the data. Degrees of freedom = n-1

Different approach

Use the probability distribution to do inference of the parameters:

$$P(\vec{d}|\mu,\sigma)=\frac{1}{\sigma^n\sqrt{(2\pi)^n}}\exp\left[\frac{-1}{2}\sum_i\left(\frac{d_i-\mu}{\sigma}\right)^2\right]$$

Bayes theorem:

$$P(\mu,\sigma|\vec{d})\,P(\vec{d})=P(\vec{d}|\mu,\sigma)\,P_0(\mu,\sigma)$$

Posterior | Marginal likelihood | Likelihood | Prior

Output | Input

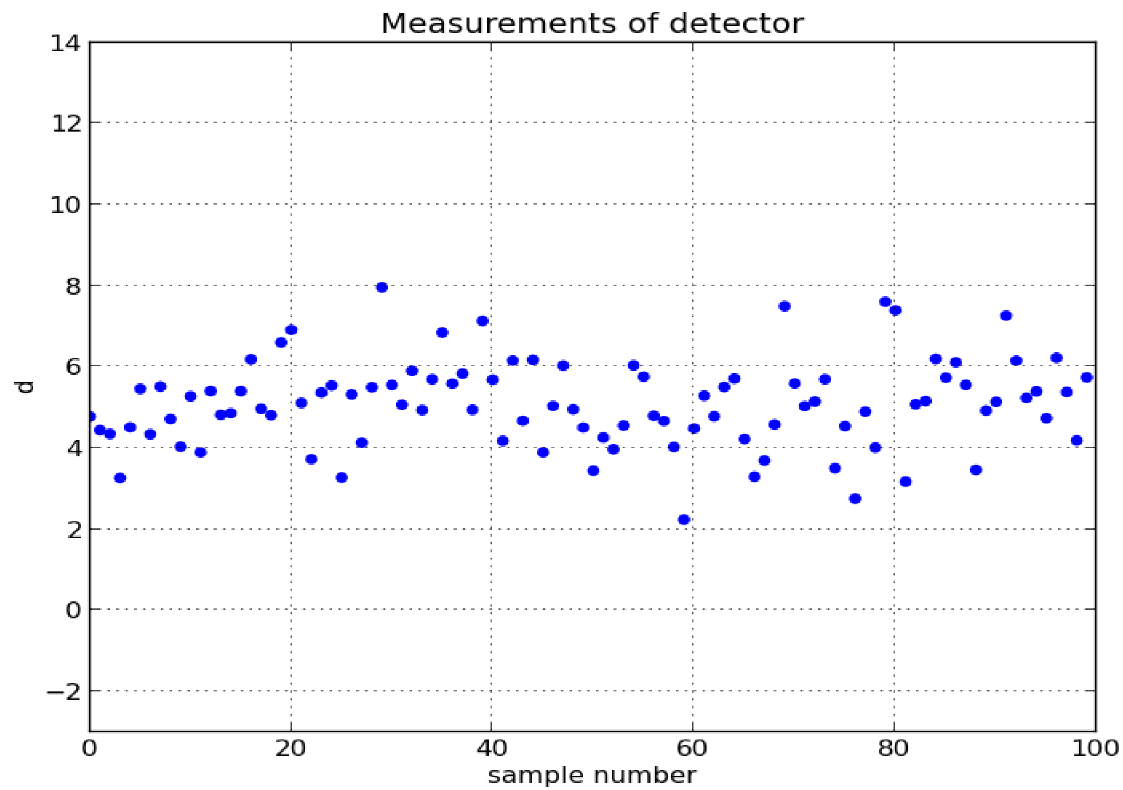
Comes from: $P(x,y)=P(x|y)P(y)=P(y|x)P(x)$

Bayesian analysis: do the same

Instead of calculating the mean and the spread with 'estimators', we are going to inspect the likelihood function of the problem.

The likelihood is just the probability distribution of the data, as a function of the model parameters.

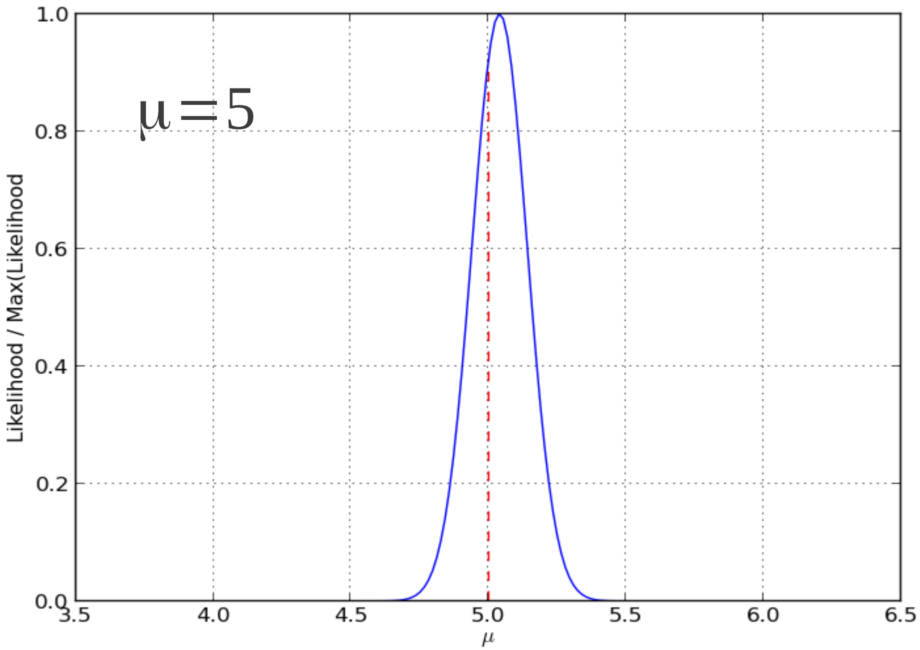
Homework exercise: what are the maximum likelihood estimators?

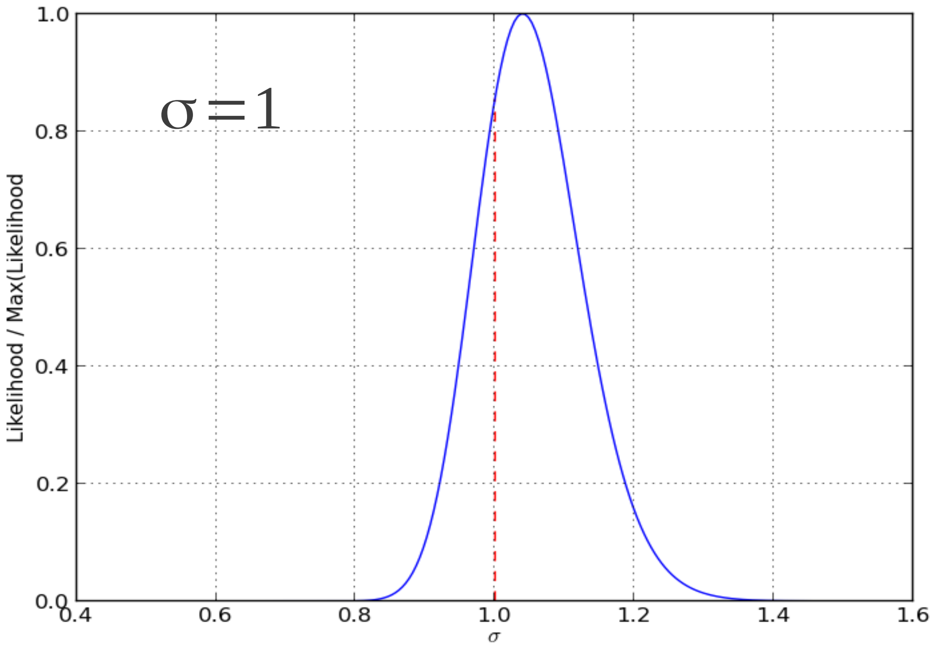


Normalisation

$$P(\mu,\sigma|\vec{d}) = \frac{M}{\sigma^n \sqrt{(2\pi)^n}} \exp\left[\underbrace{\frac{-1}{2}\sum_i \left(\frac{d_i-\mu}{\sigma}\right)^2}_{\chi^2}\right]$$

Inspect the posterior





Posterior, assuming that we know the standarddeviation

$$\bar{\mu} = \frac{1}{n} \sum_i d_i = 5.04$$

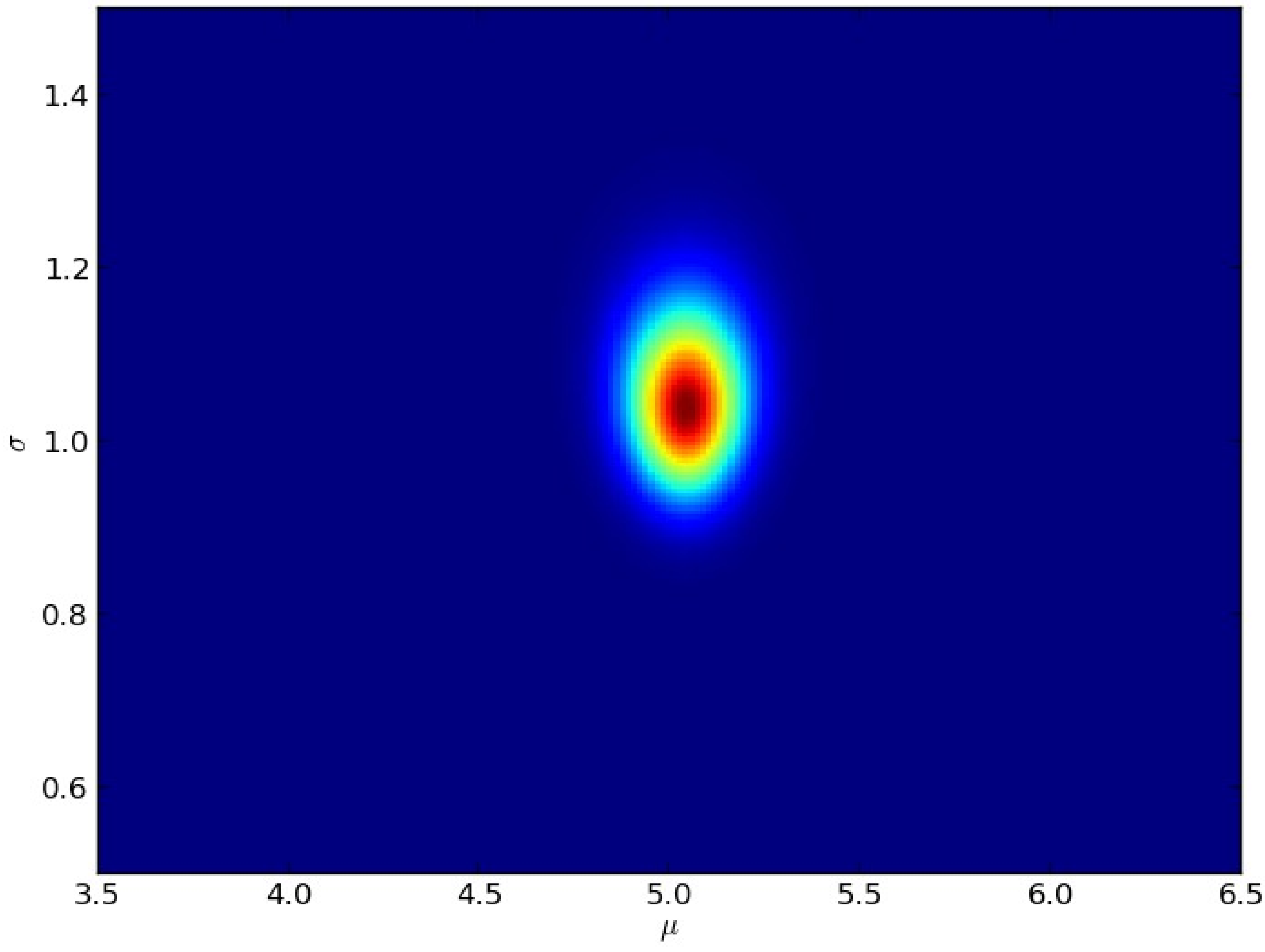
Posterior, assuming that we know the mean

$$\bar{\sigma} = \sqrt{\frac{1}{n} \sum_i (d_i - \mu)^2} = 1.039$$

$$\bar{\sigma}_{n-1} = \sqrt{\frac{1}{n-1} \sum_i (d_i - \bar{\mu})^2} = 1.044$$

Can do 2D as well

$\mu=5$
$\sigma=1$



Recap

Data generation: $$P(\vec{d}|\mu,\sigma)=\frac{1}{\sigma^n\sqrt{(2\pi)^n}}\exp\left[\underbrace{\frac{-1}{2}\sum_i\left(\frac{d_i-\mu}{\sigma}\right)^2}_{\chi^2}\right]$$

Break up in two parts: $d_i=\mu+\epsilon_i$ with $N(0,\sigma)\rightarrow\epsilon_i$

e.g. 'Fit'

e.g. Fourier transform

Method 1

Deterministic (μ): min χ^2

Stochastic (σ): other 'statistic'

Method 2

Inspect the posterior distribution

The actual likelihood function

Examples::

$$P(\vec{d}|\mu,\sigma)=\frac{1}{\sigma^n\sqrt{(2\pi)^n}}\exp\left[\frac{-1}{2}\sum_i\left(\frac{d_i-\mu}{\sigma}\right)^2\right]$$

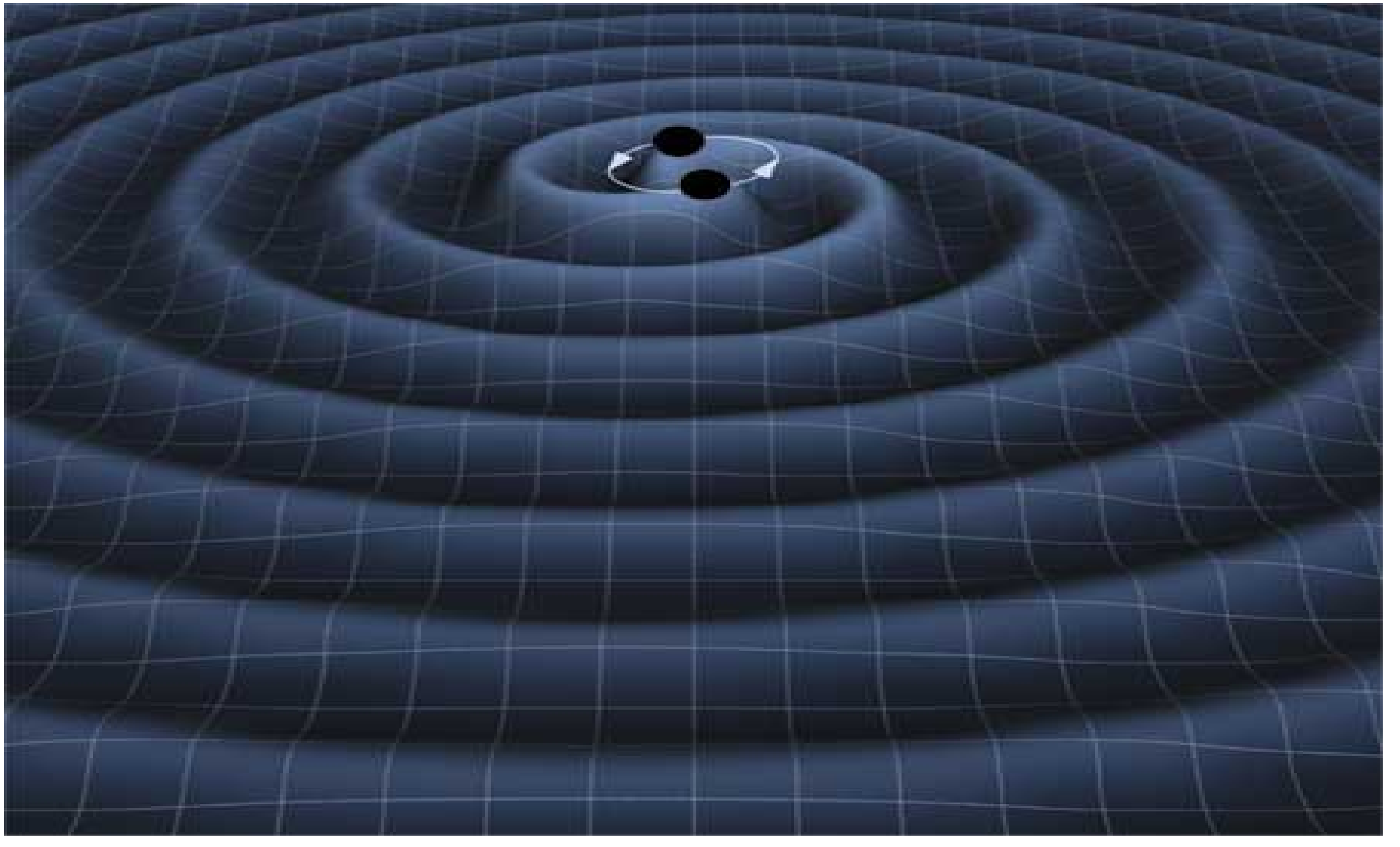
Actual:

$$P(\vec{d}|\vec{a},\vec{\theta})=\frac{\exp[-(\vec{d}-f(\vec{a}))C^{-1}(\vec{d}-f(\vec{a}))/2]}{\sqrt{(2\pi)^n det\,C}}$$

$$C=C(\tau,\vec{\theta})=\int df\cos(f\tau)P(f)$$

The likelihood function used in pulsar timing is 'just' a **multivariate Gaussian**. The matrix C is calculated from the power spectrum of the signal.

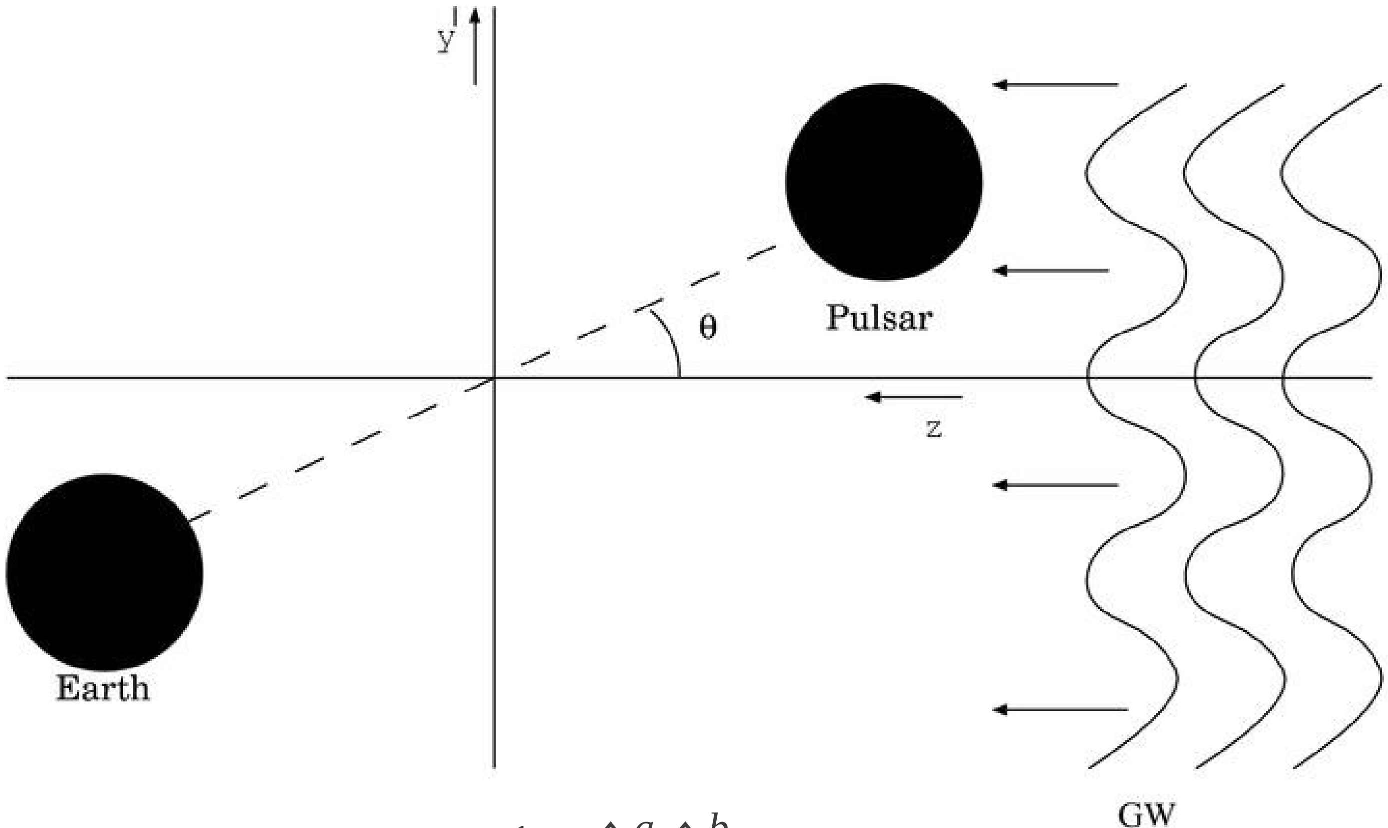
Gravitational waves: how to detect



We are going to focus on 2 types of sources only:

- Stochastic background, caused by ensemble of supermassive BH binaries
- Single inspirals of supermassive BH binaries

Earth-term / pulsar term

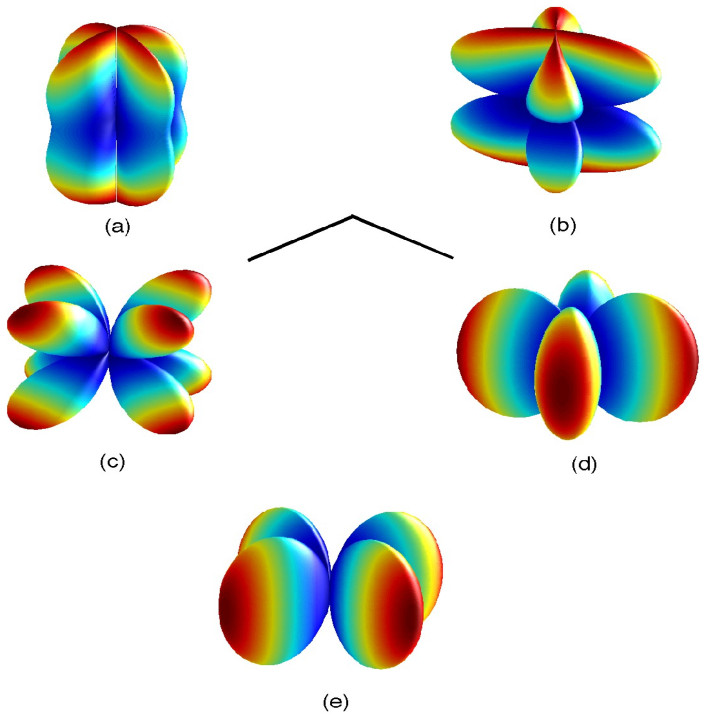


$$\frac{\delta\nu}{\nu} = e^{A}_{ab}(\hat{\Omega})\,\frac{1}{2}\,\frac{\hat{p}^{a}\,\hat{p}^{b}}{1+\hat{\Omega}\cdot\hat{p}}\left(h_e(t_e)-h_p(t_p)\right)$$

Earth-term

Pulsar-term

Antenna pattern response



a,b: +,x polarisation
c,d: vector x,y modes
e: scalar mode

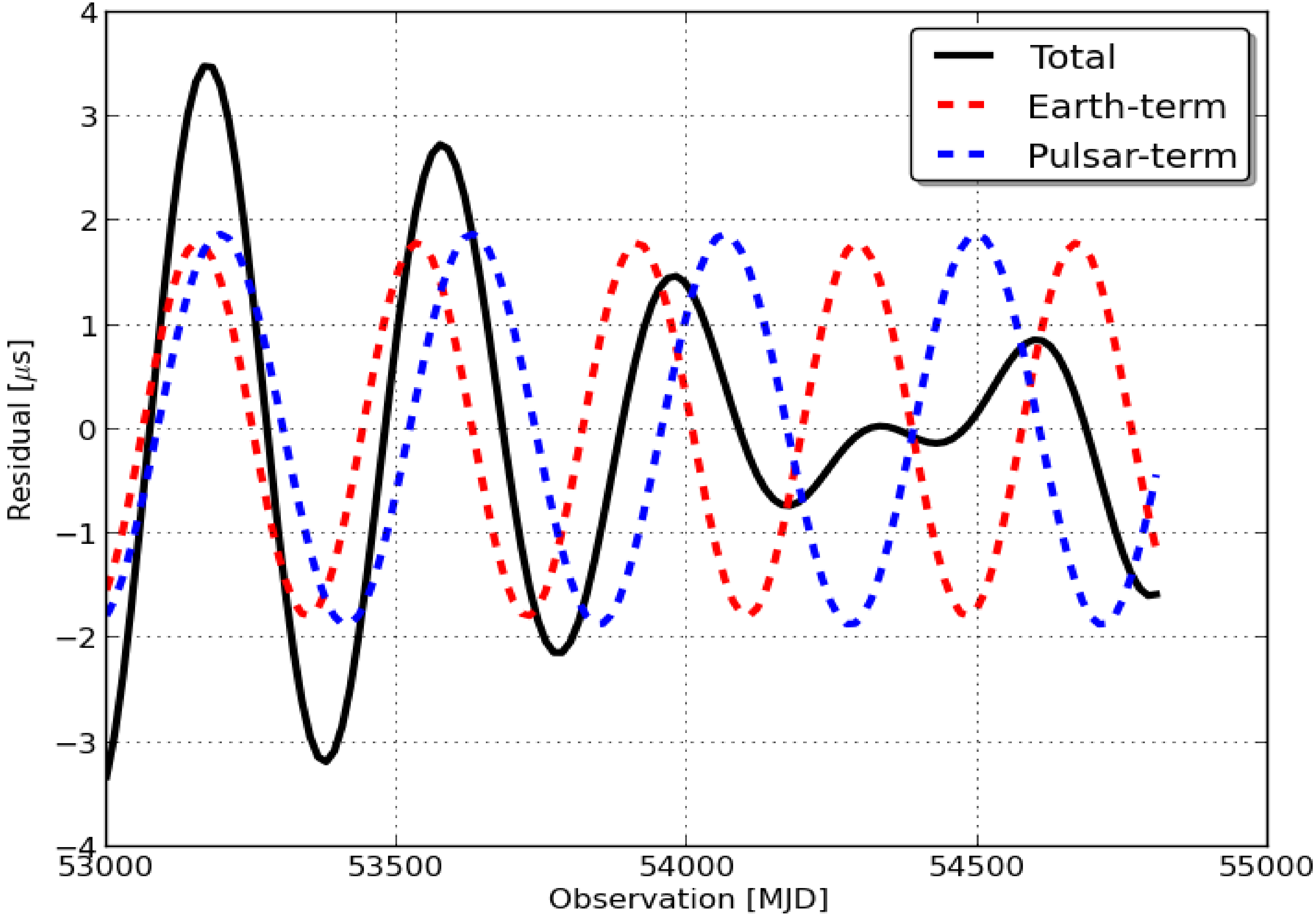
Most efforts focus on the usual +,x polarisations.

$$\frac{\delta\nu}{\nu} = e^{A}_{ab}(\hat{\Omega})\frac{1}{2}\frac{\hat{p}^{a}\hat{p}^{b}}{1+\hat{\Omega}\cdot\hat{p}}\left(h_e(t_e) - h_p(t_p)\right)$$

Earth-term

Pulsar-term

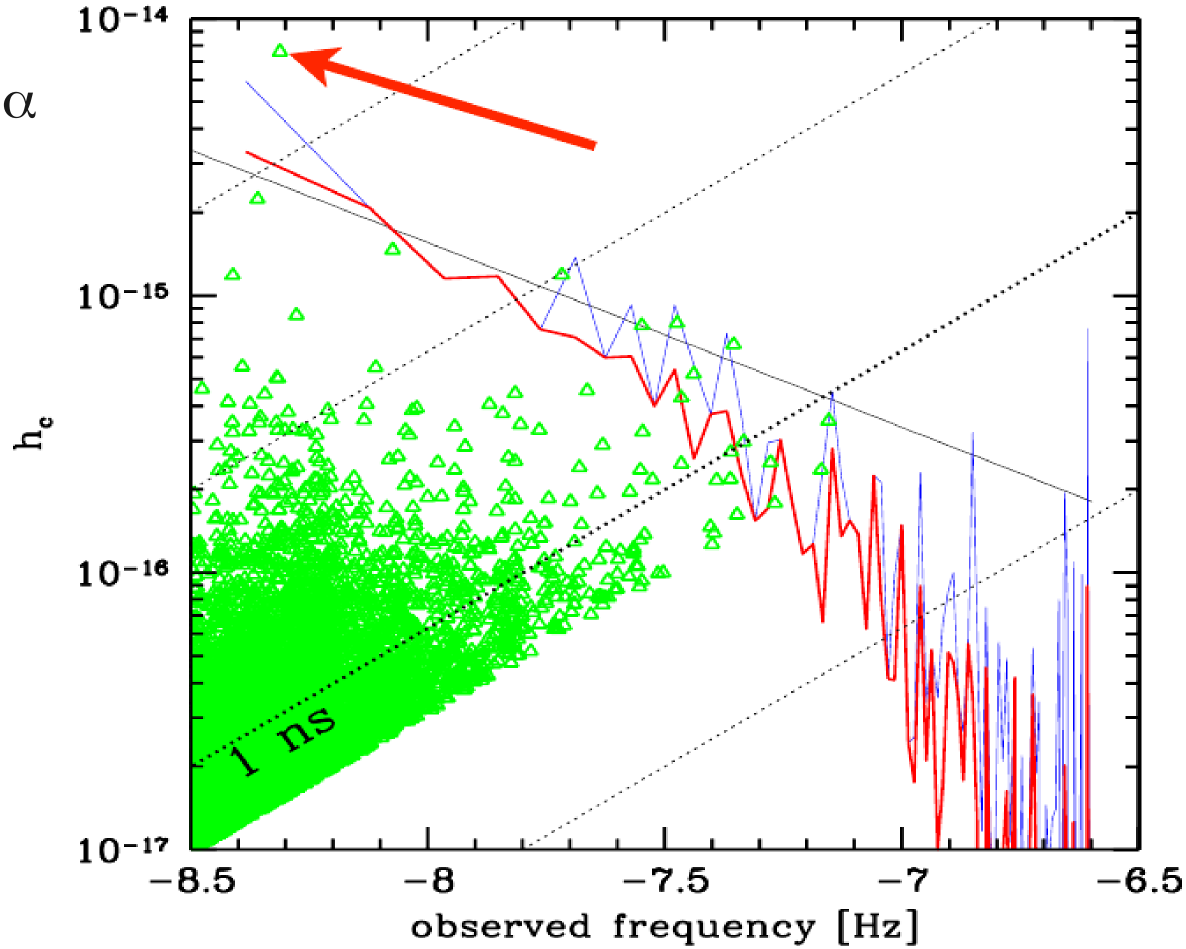
Earth-term / pulsar term



Remember: background at low freq.

$$h_c(f) = h_c \times (f/f_0)^{\alpha}$$
$$\alpha = -2/3$$

Phinney 01
Jaffe & Backer 03
Wyithe & Loeb 03
Sesana et al. 07, 09

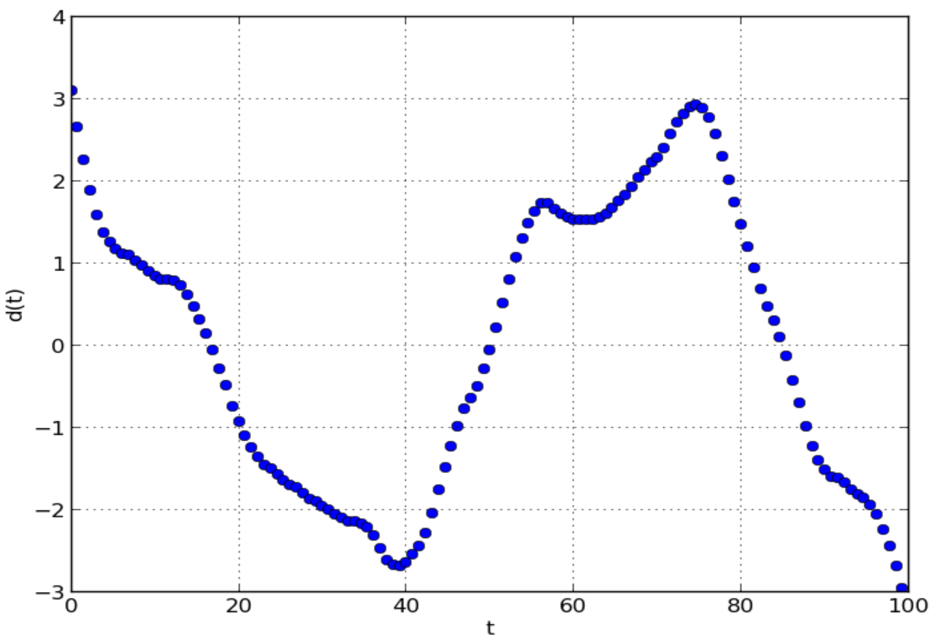


Sesana et al. (2008), Ravi et al. (2012): Theory and simulations suggest there is a non-zero probability that individual sources have SNR above the background.

Isotropic background with 1 pulsar

Example: isotropic stochastic background of gravitational-waves.

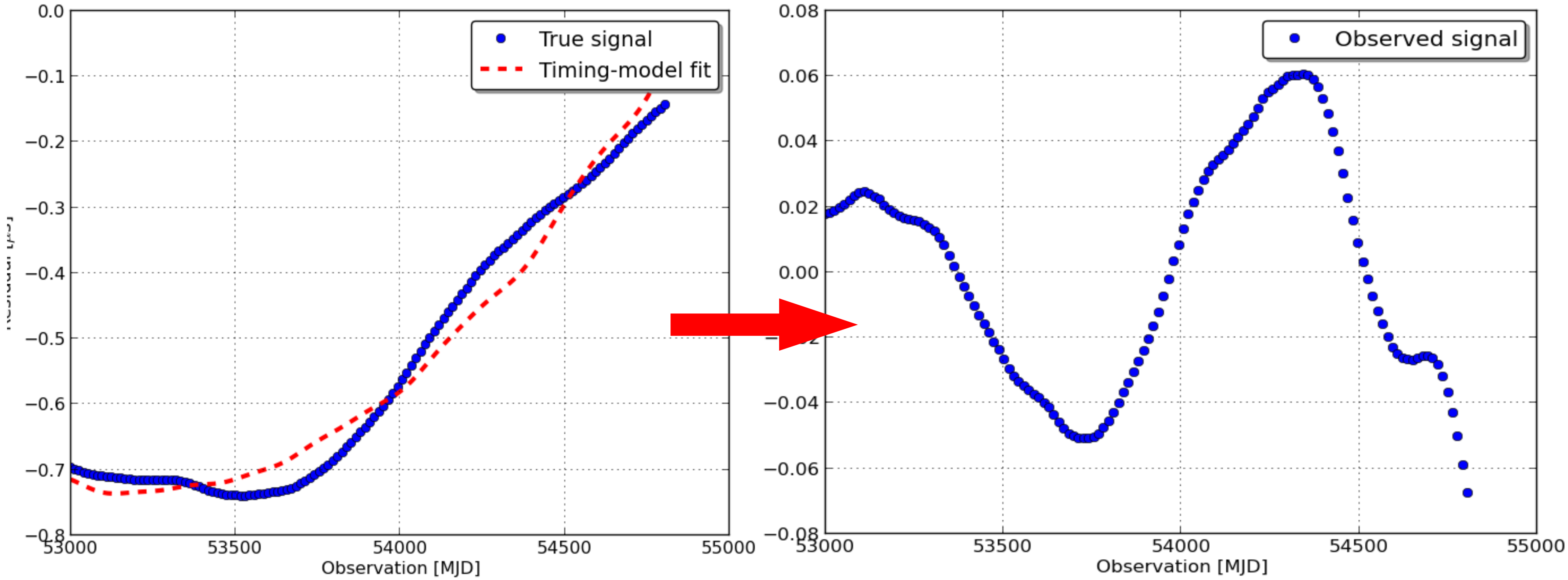
Stochastic signal, with spectrum $P(f) = h_c^2 \times \left(\frac{f}{f_0}\right)^{3-2\alpha}$



RMS: $\bar{\sigma}_{n-1} = \sqrt{\frac{1}{n-1}\sum_i (d_i - \bar{\mu})^2}$

The fitting for the timing model, like discussed earlier, kind of complicates this....

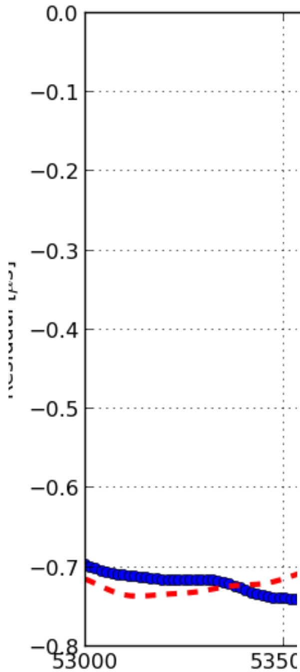
The effect of fitting

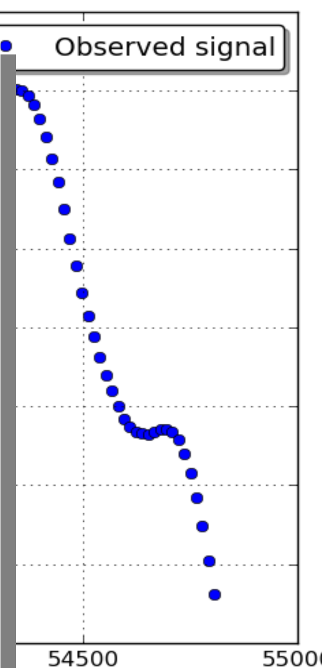


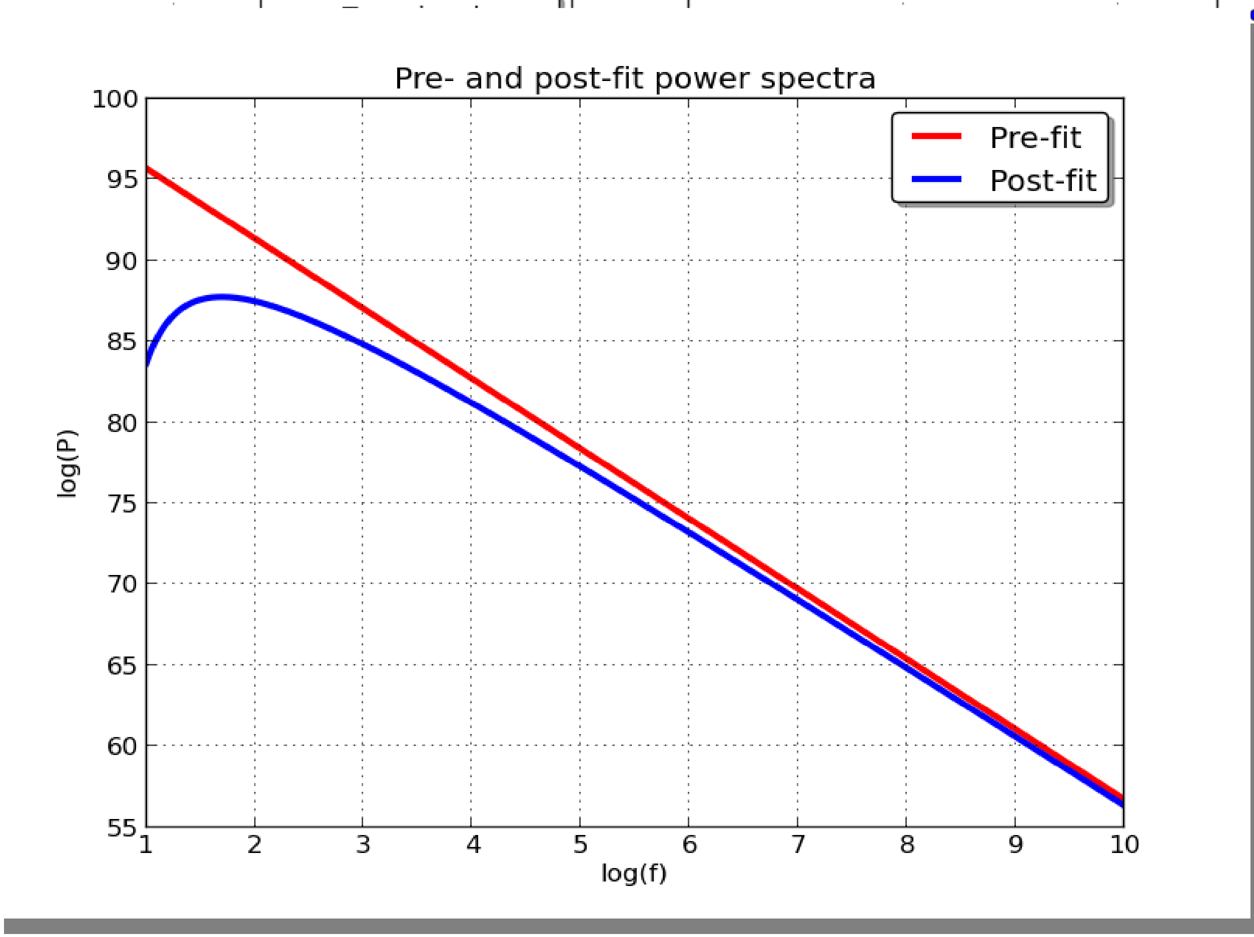
A lot of power is absorbed by the fitting process.
Most comes from fitting a quadratic shape

But: **the RMS still proportional to signal amplitude**

The effect of fitting



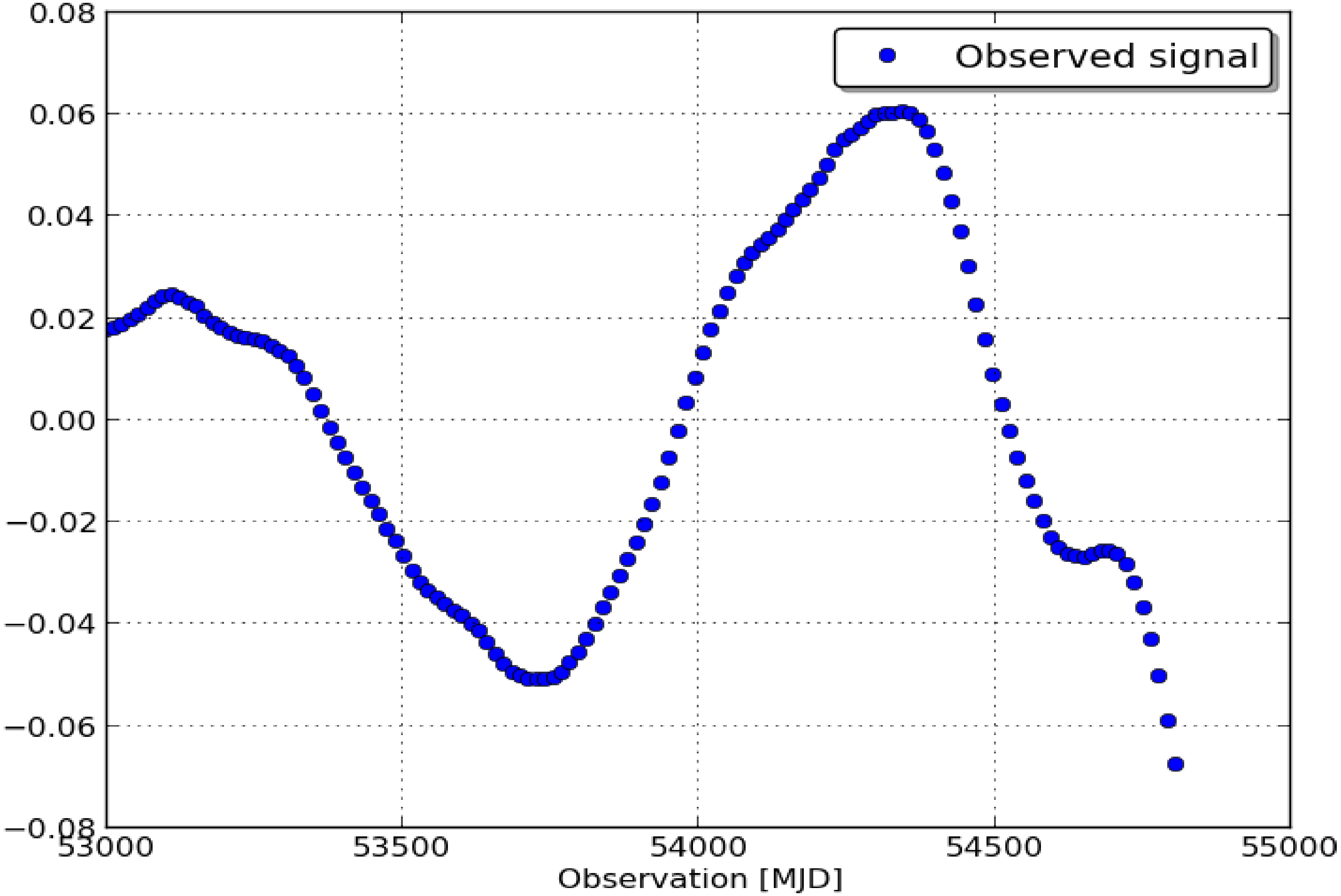




A lot of p
Most co

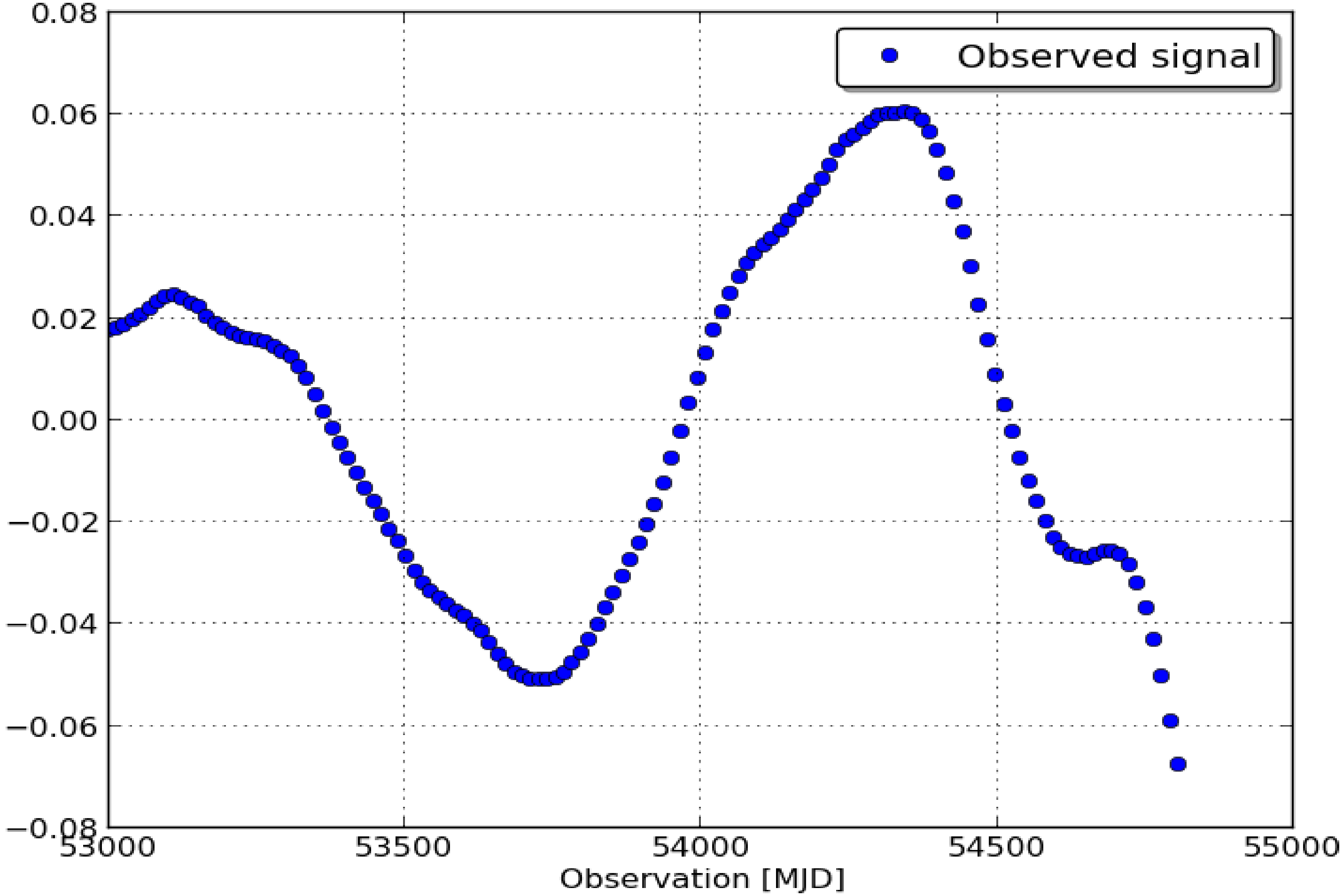
But: **the RMS still proportional to signal amplitude**

Ok.. so what if we find this



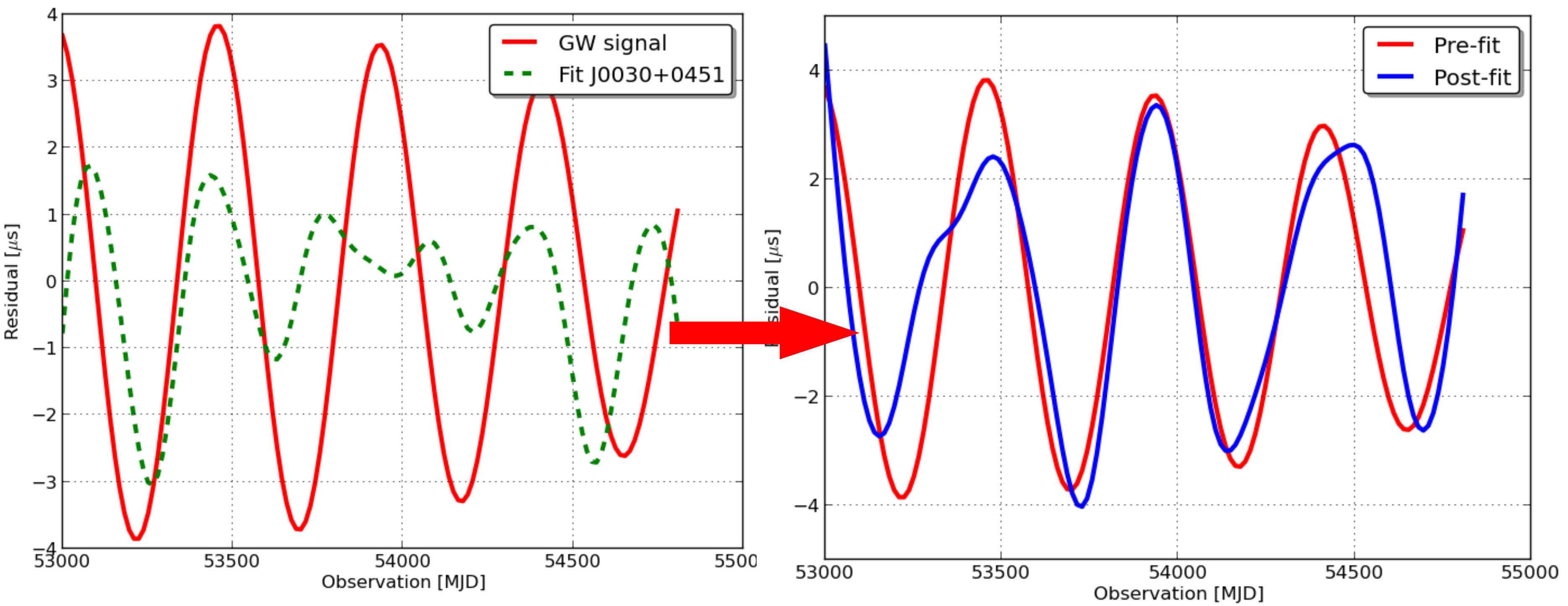
Question: is this indeed a GW?

Ok.. so what if we find this



Answer: No idea. We do not know our noise well enough.
But we can set an upper limit!

Fits and continuous waves

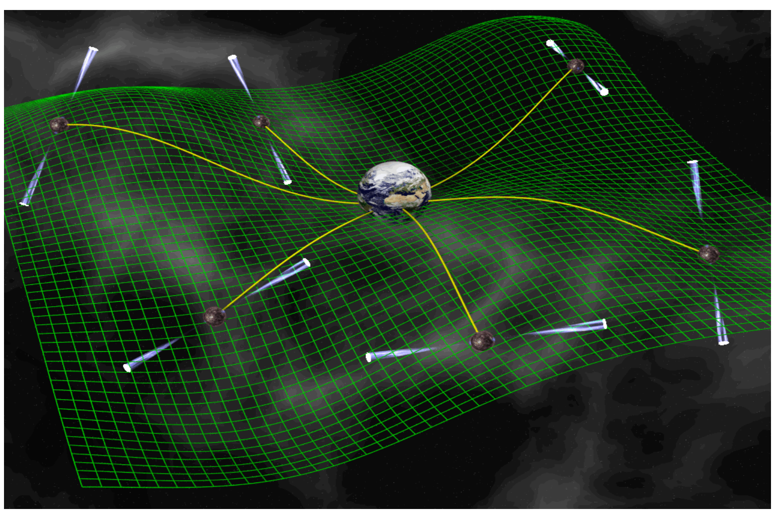


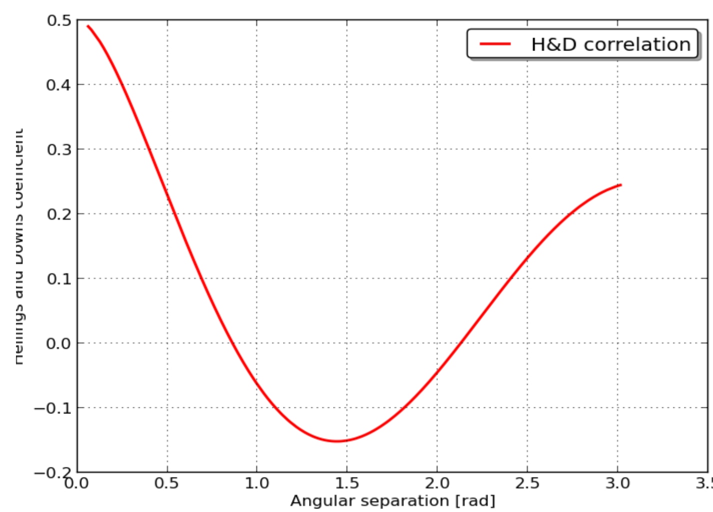
This is the signal of a single BHB inspiral. Also a lot of power was absorbed.

Question: what is the power absorbed by? (freq ~= 1 yr^-1)

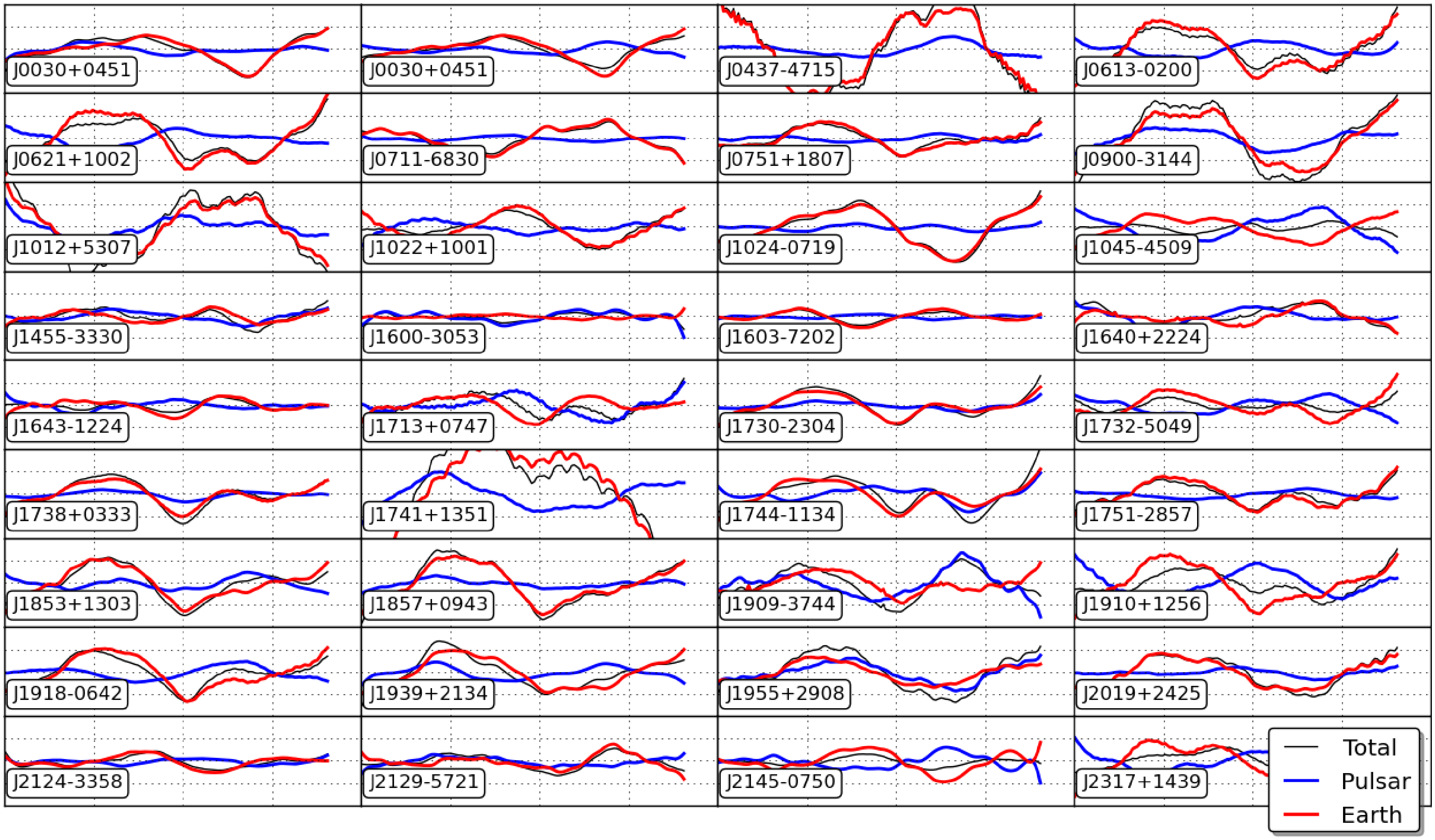
Correlations

- Correlations between pulsar pairs are the 'smoking gun'. Unique signature for general relativity
- Unlikely a detection will be generally accepted by the broad scientific community, unless confirmed with a whole 'array' of pulsars

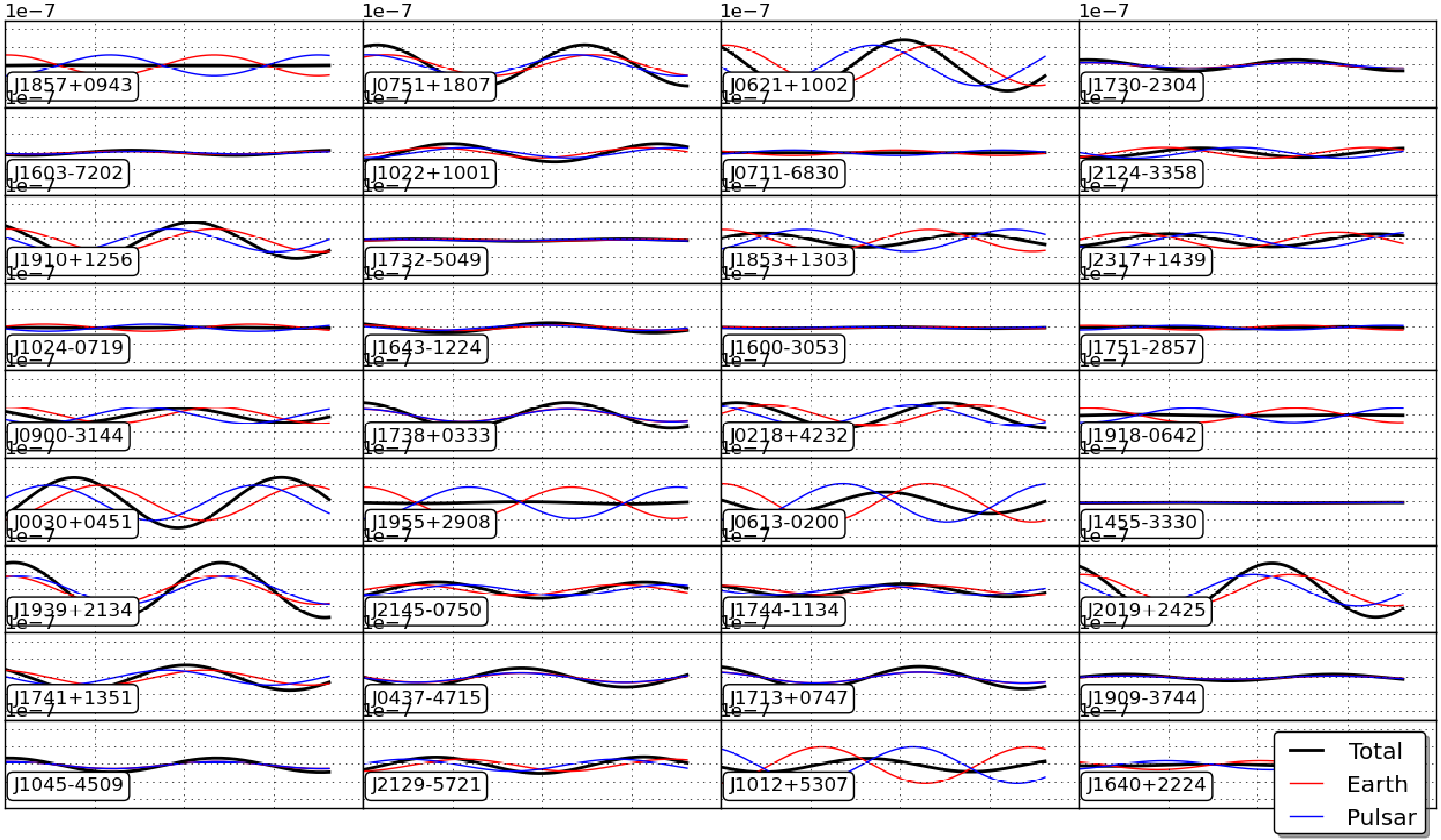




One isotropic GWB realisation

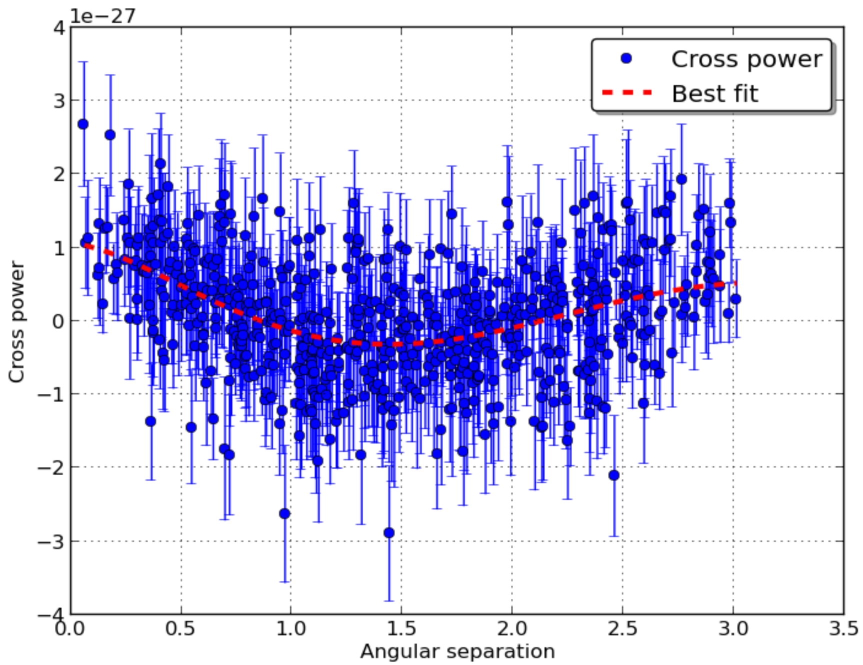


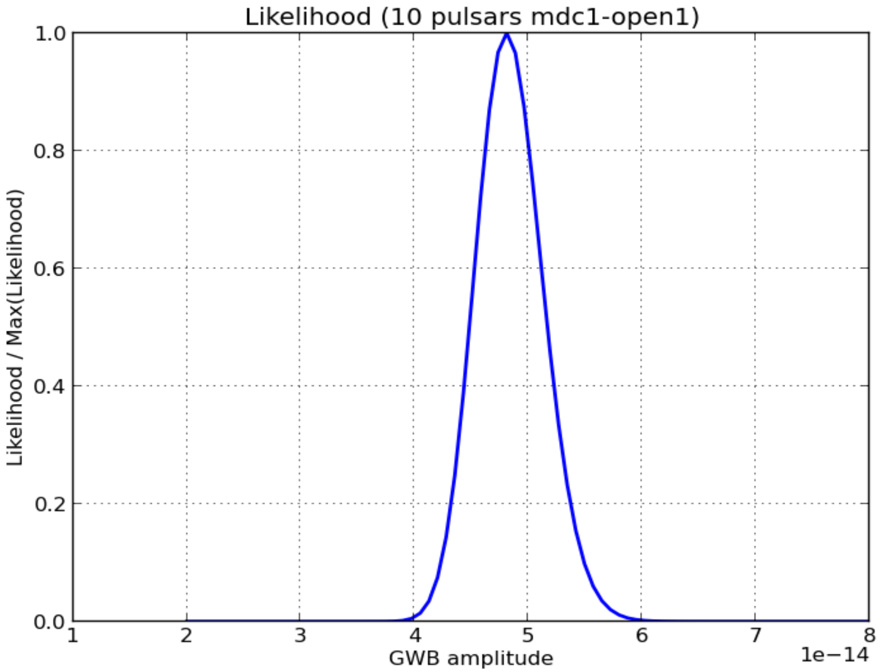
One CW realisation



Detection of GWs: isotropic background

“We have found evidence of a signal with GR correlations in an array of pulsars”

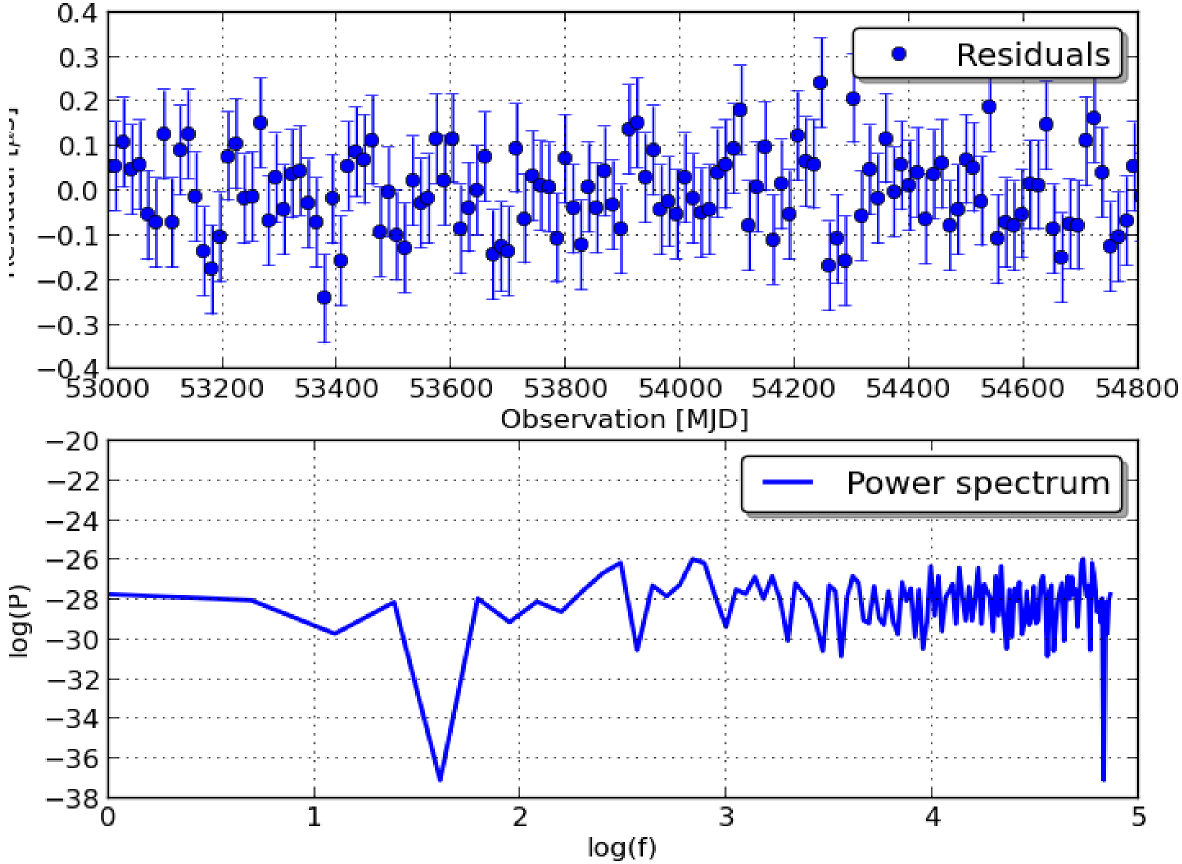




Placing upper-limits

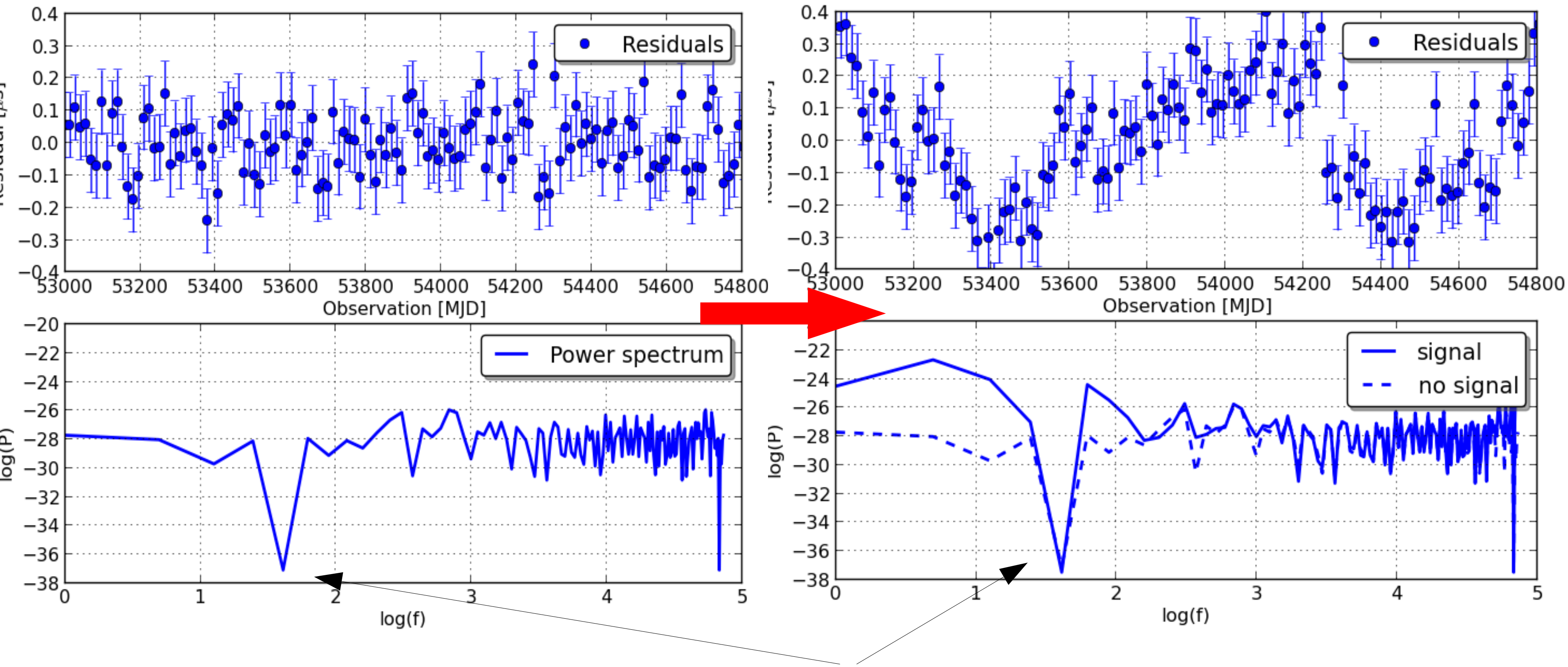
“If there is a signal, it is not larger than this amplitude”

Example: look at power spectrum



Placing upper-limits: spectrum & rms

Signal shows up in FFT. Can also use RMS

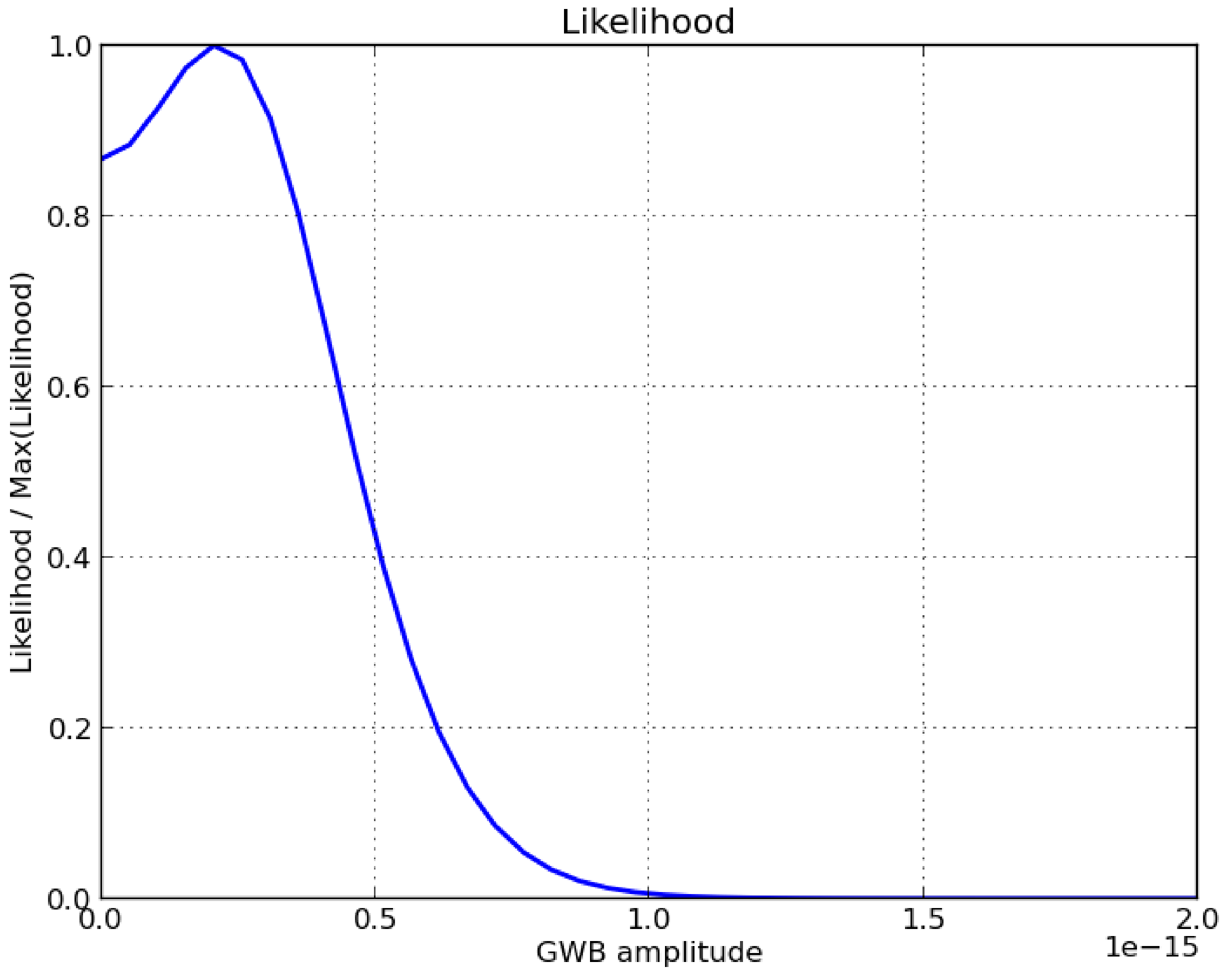


Question: is this a GW detection?

Question: what is this dip here?

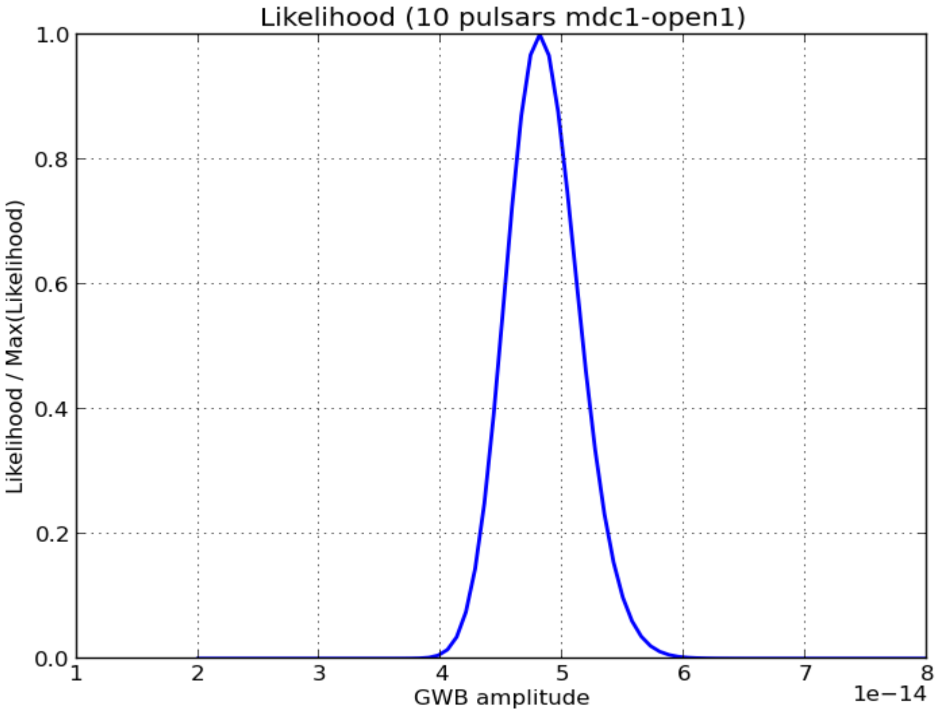
Placing upper-limits: posterior distribution

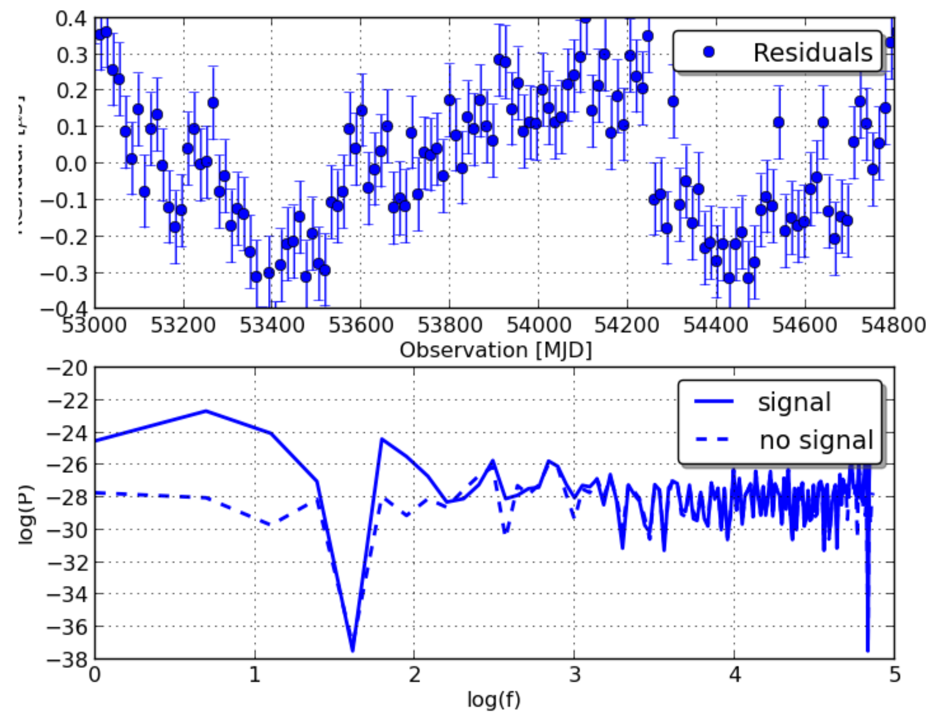
Find the 95% confidence limit



Upper-limit vs detection

Posterior: make sure to ask the right question

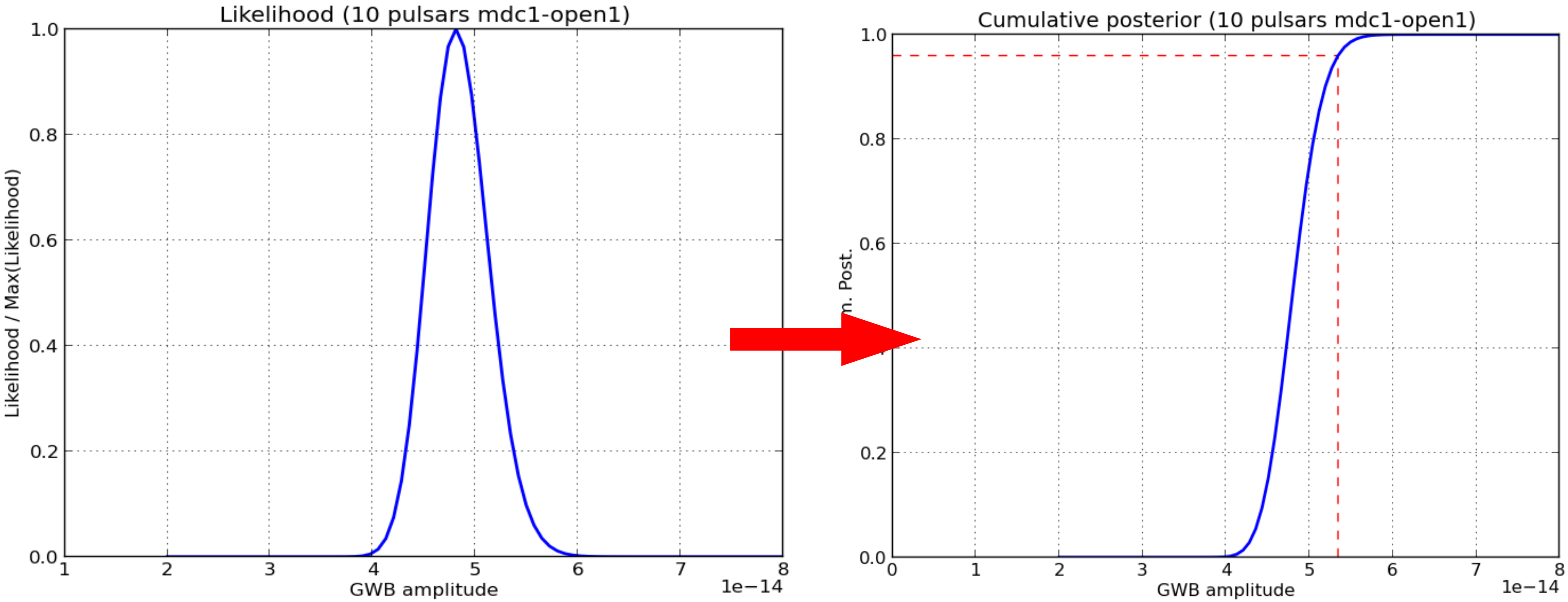




Use a reasonable model = include all noise contributions

Upper-limit likelihood: conservative noise

Integrate the posterior distribution



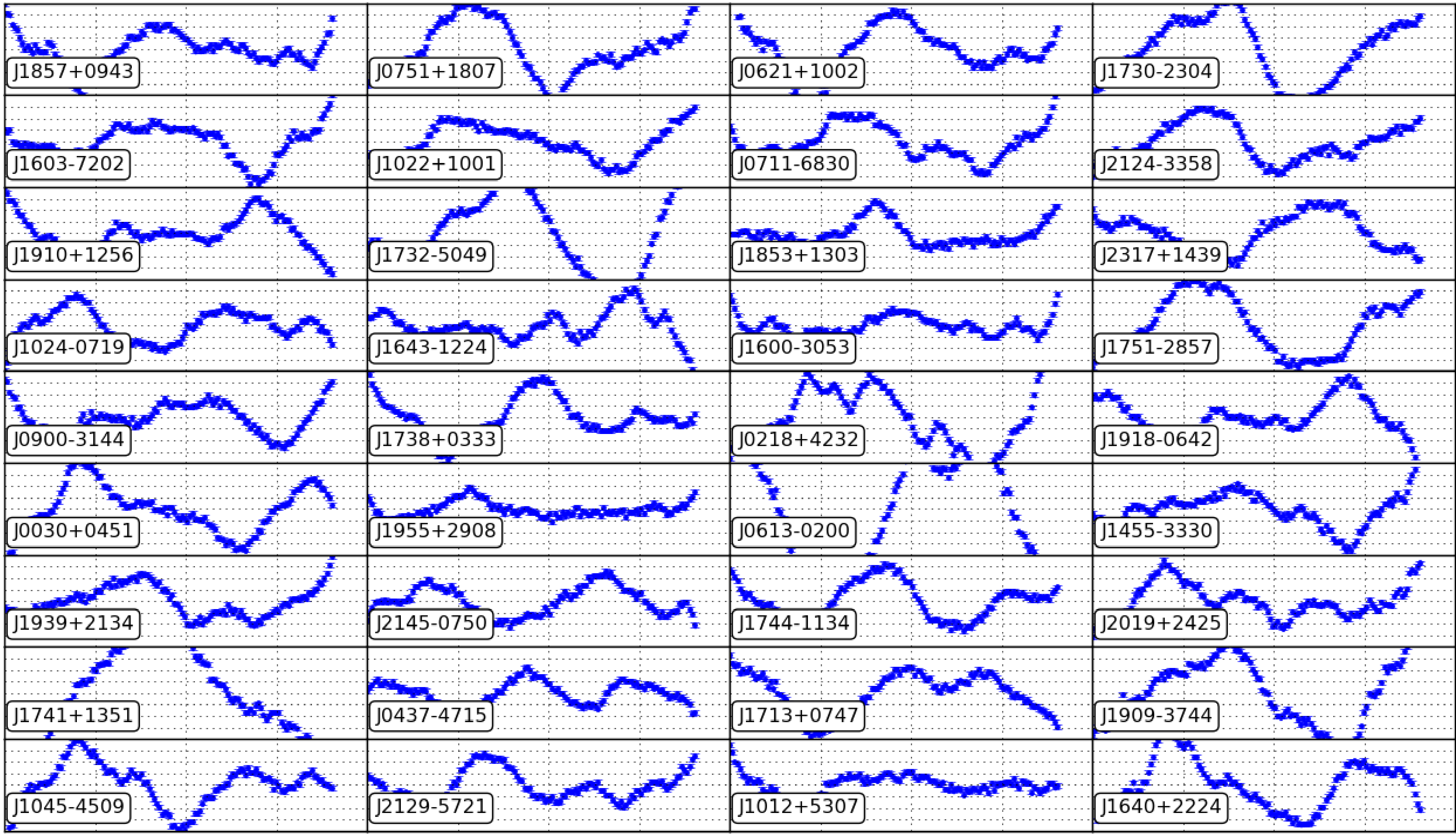
With conservative noise, detection is upper limit. Robust & easy

The IPTA Mock Data Challenge

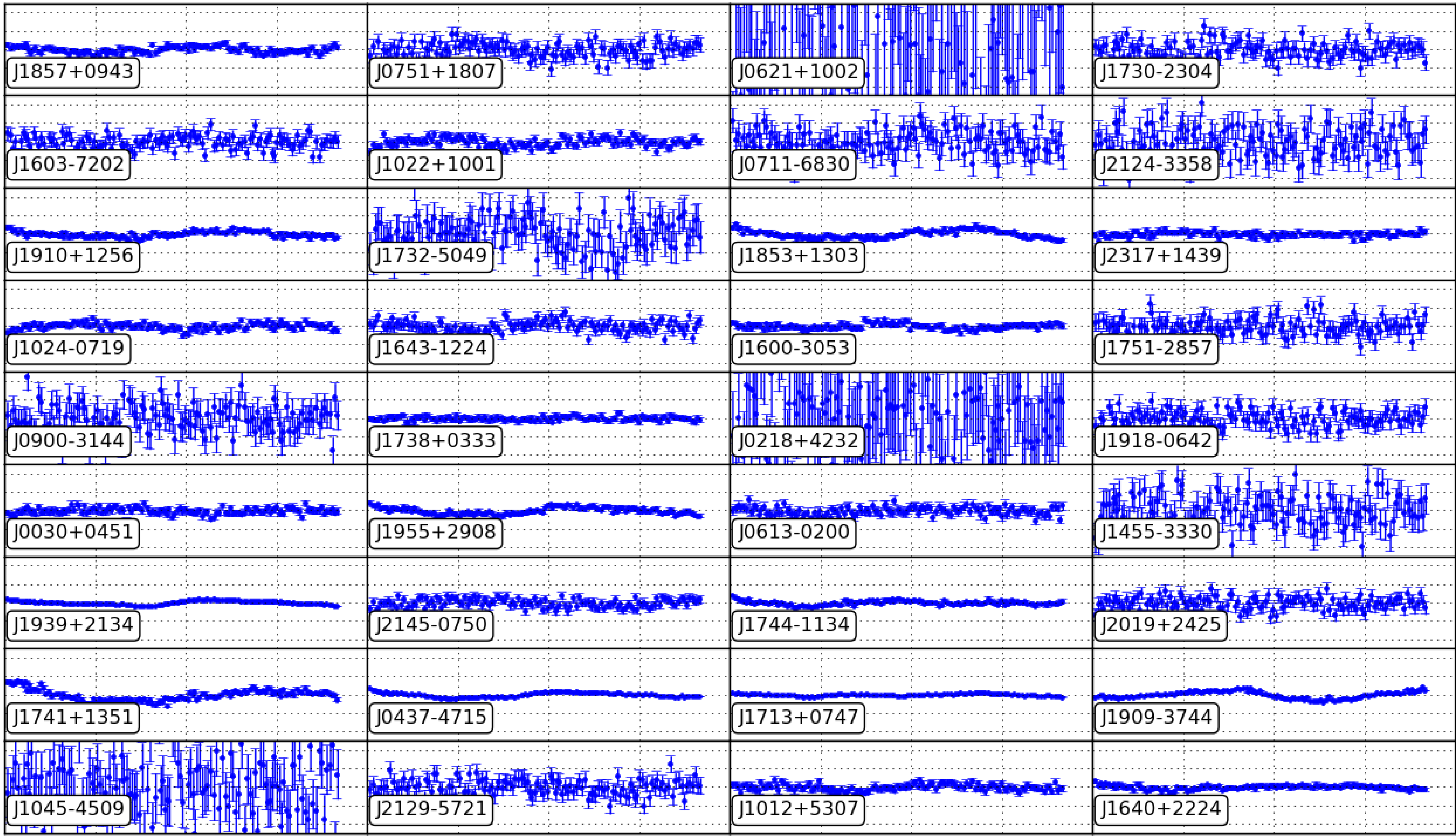
- The challenge: find the injected signals
- Overly simplistic datasets
- 3 difficulties – three sub-challenges
- Open sets & closed sets

- MDC1 (done): isotropic stochastic background
- MDC2 (upcoming): continuous waves (?)

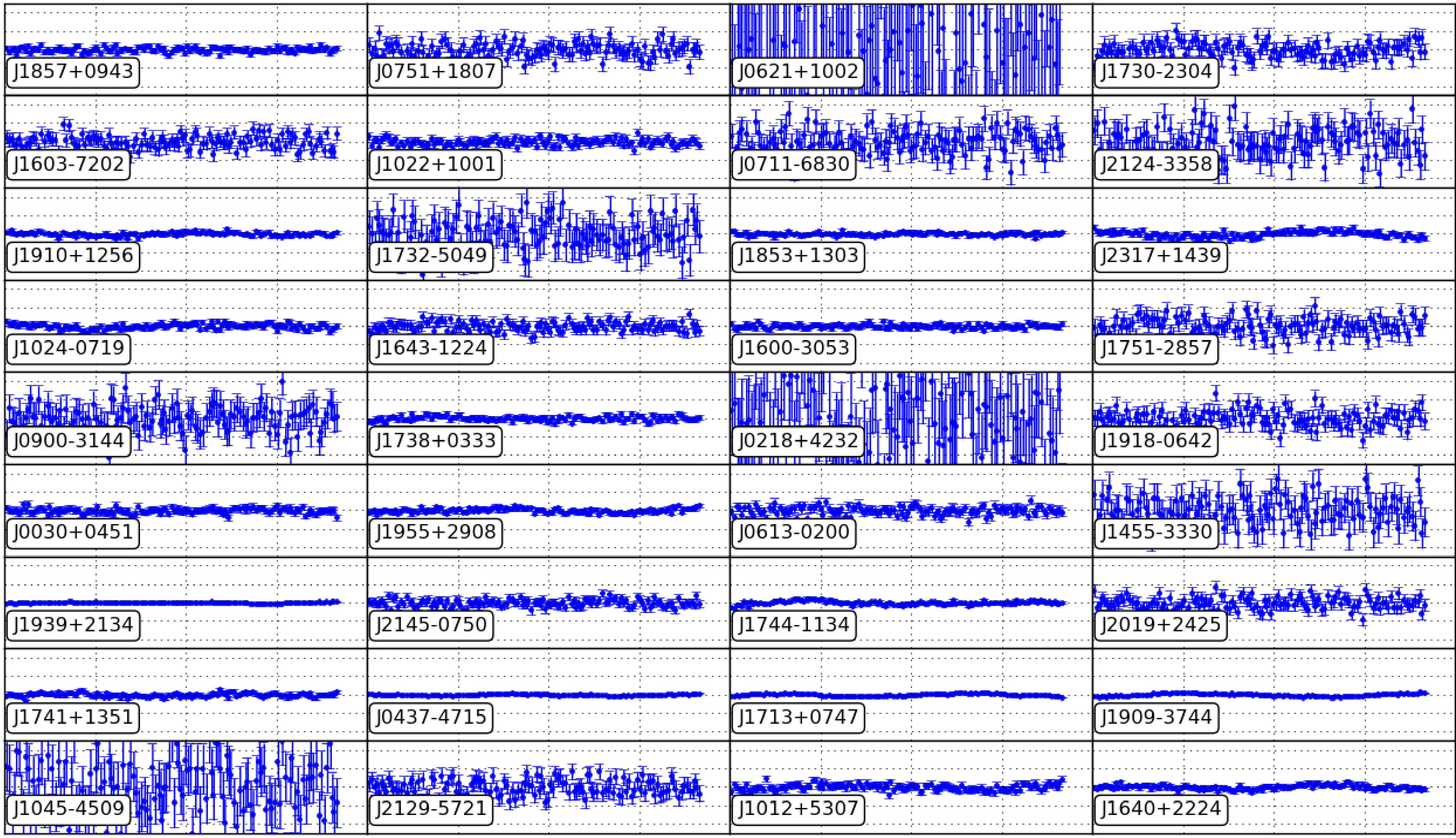
MDC1 open challenge 1



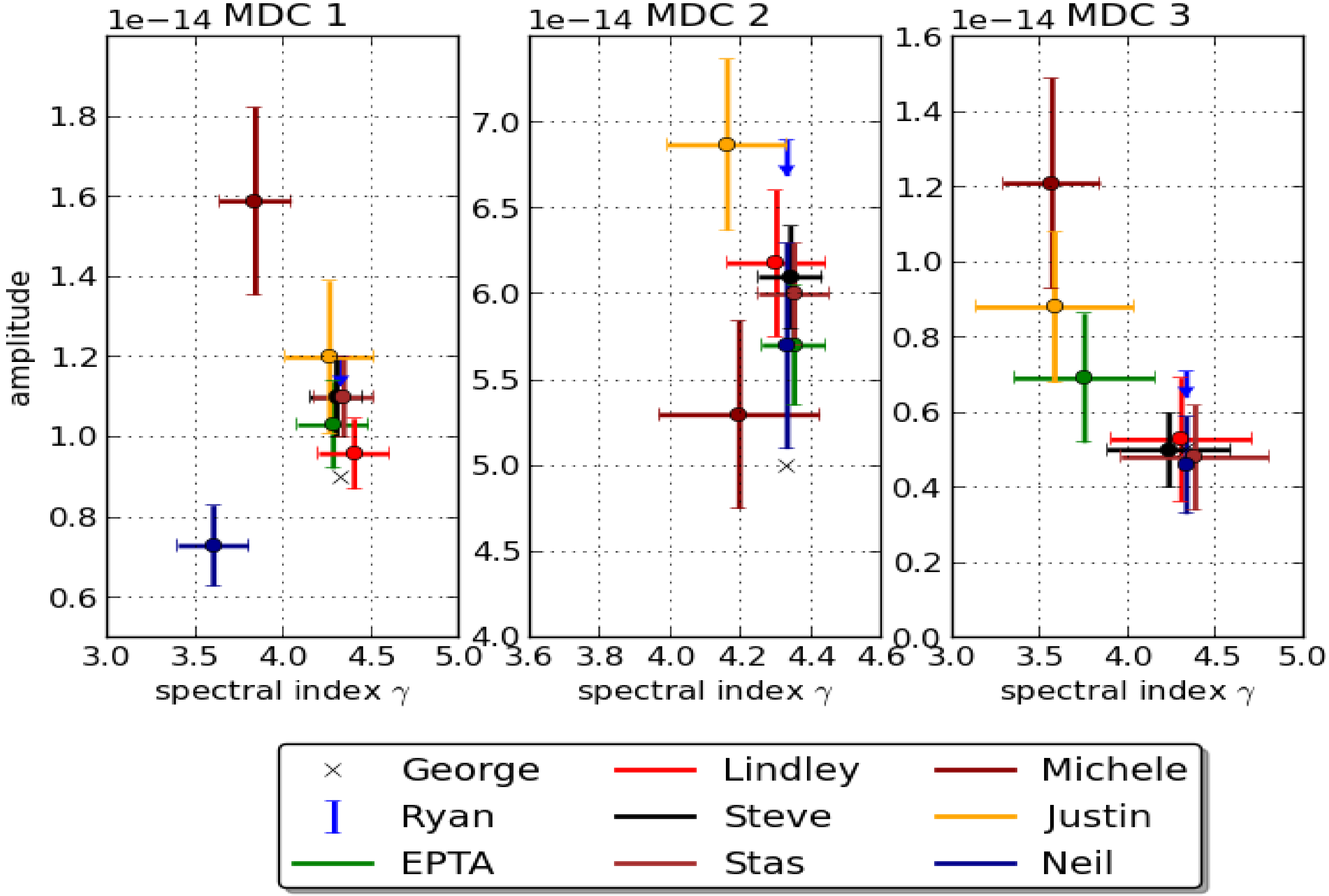
MDC1 open challenge 3



MDC1 closed challenge 3



Data challenge results

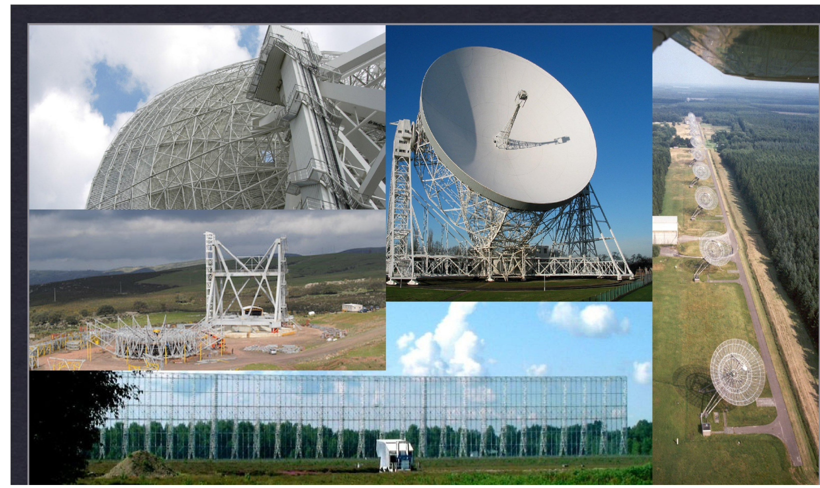


Software to try this at home

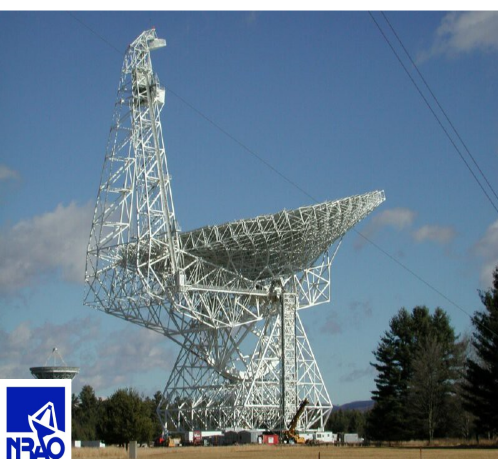
- Tempo2: http://tempo2.sourceforge.net/
- Libstempo: http://github.com/vallis/
- Working python/numpy installation
- More info: http://www.ipta4gw.org – site down :(

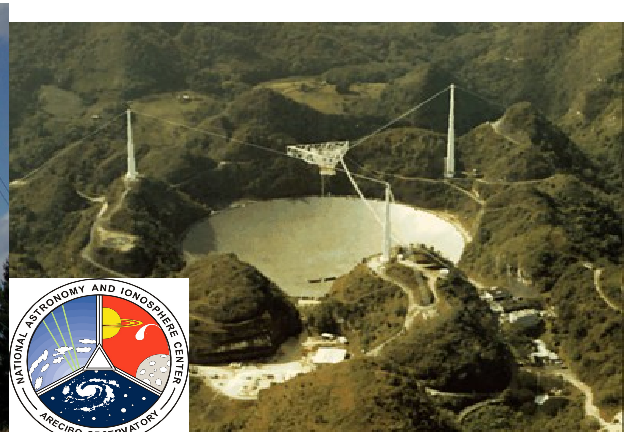
The Pulsar Timing Arrays

Parkes Pulsar Timing Array: Parkes radio telescope (64m). Oldest fully organised PTA effort. Best timing residuals to date. Southern Hemisphere



European Pulsar Timing Array: Effelsberg (100m), Westerbork synthesis (14x25m), Nancay (94m), Lovell (76m), Sardinia (64m). Most dishes.



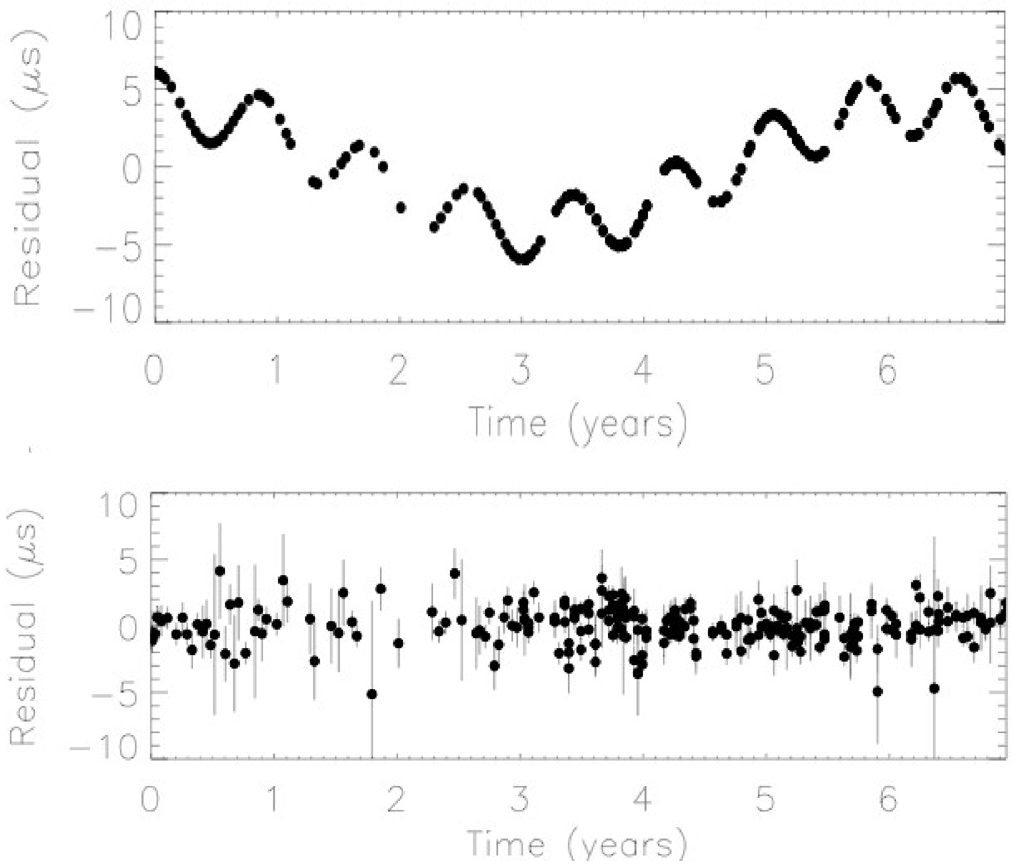


NANOGrav: GreenBank (100m), Arecibo (300m). Biggest dishes.

Early success: 3C66B

3C66B was/is a proposed supermassive binary black hole system. No continuous waves in data B1855+09. System was ruled out with published parameters.

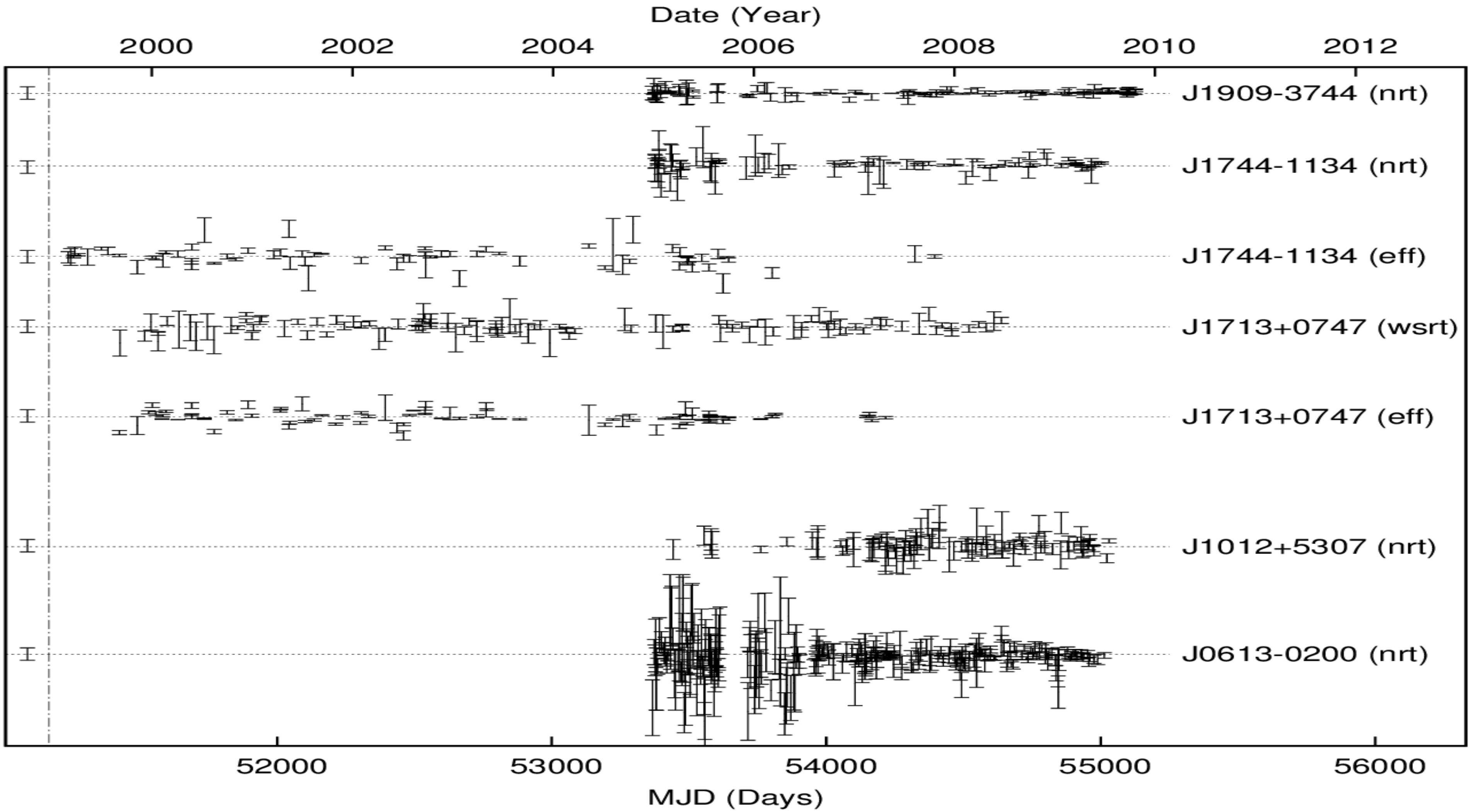
Newly proposed parameters make this system below the noise limit.



Jenet et al. (2004)

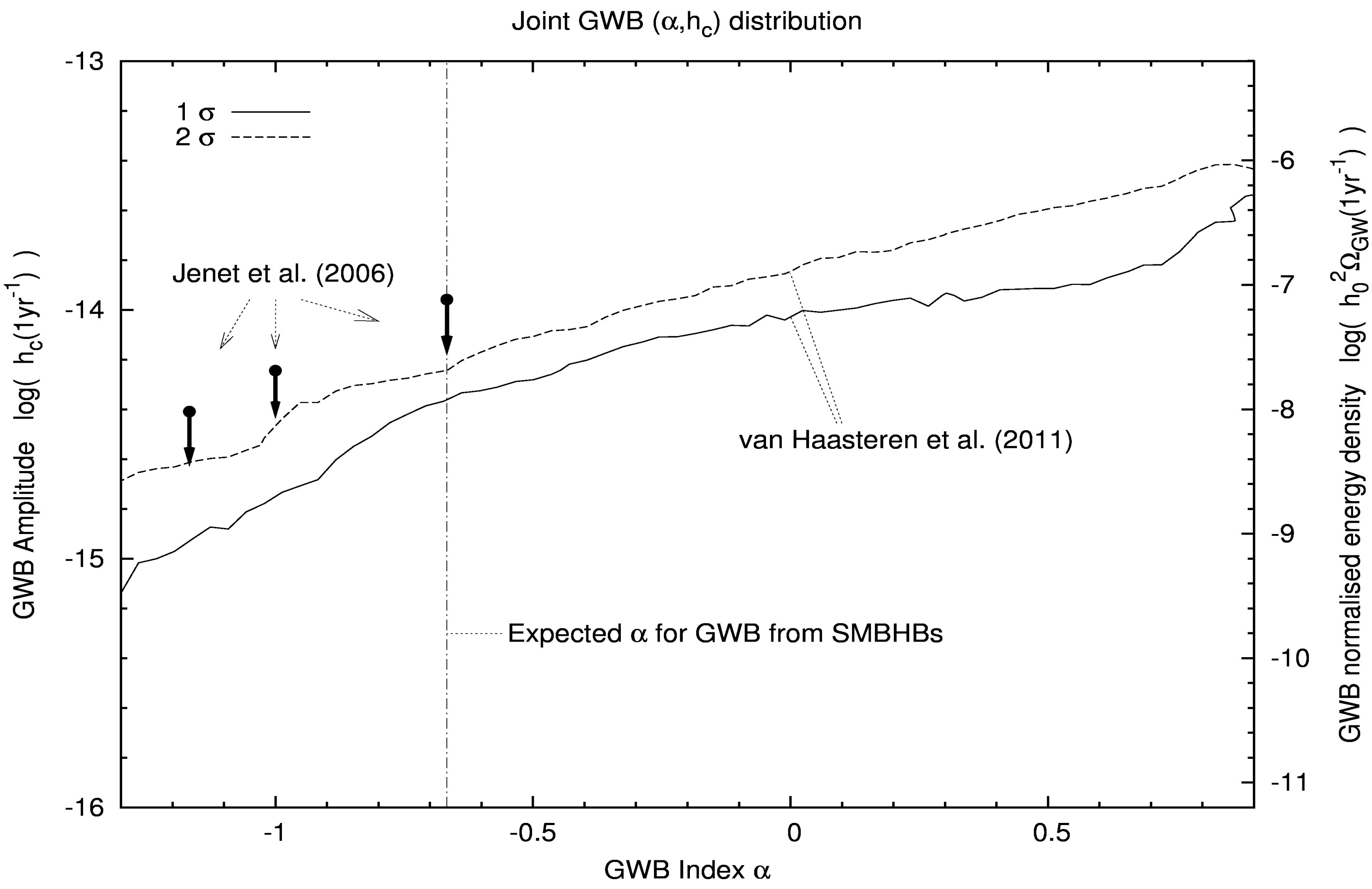
Data from Kaspi, Taylor, Ryba (1994) of pulsar PSR B1855+09

Published upper limits: EPTA



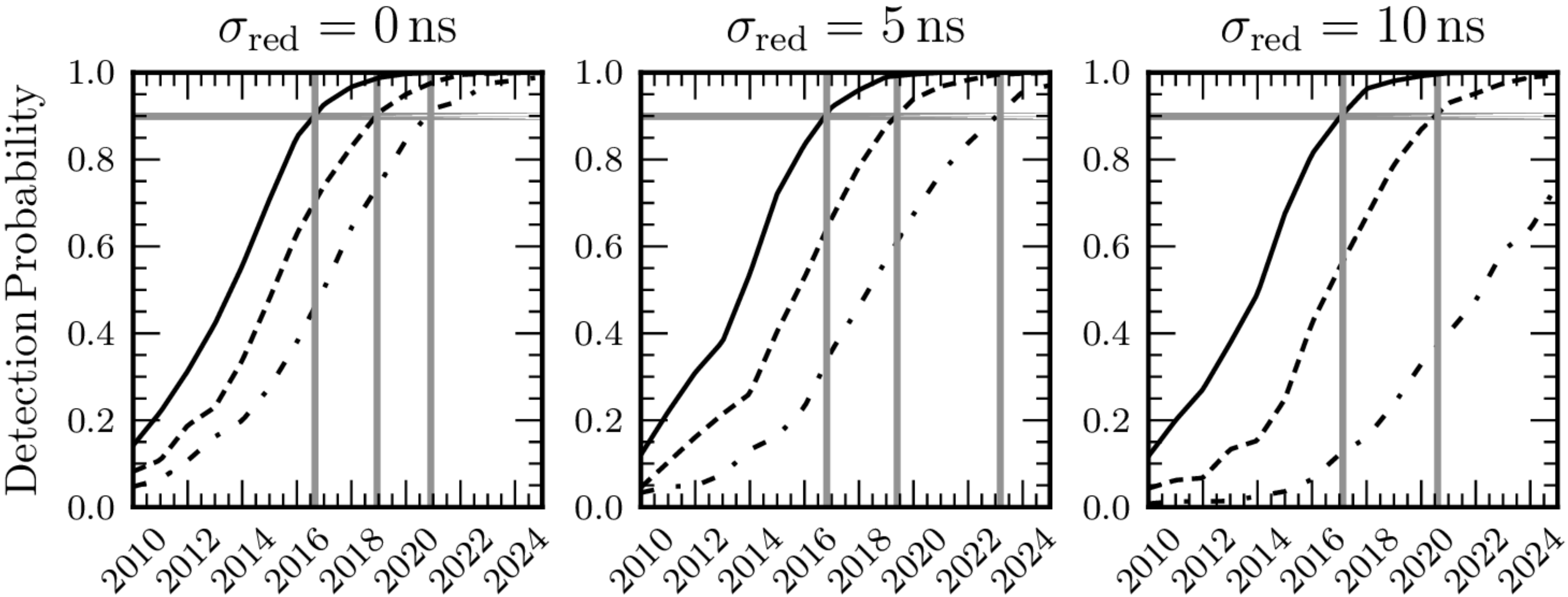
Spectrum: $h_c(f) = h_c(f/f_0)^{-\alpha}$

Published upper limits: EPTA



Spectrum: $h_c(f) = h_c f^{-\alpha}$ h_c < 6e-15

Prospects for detection

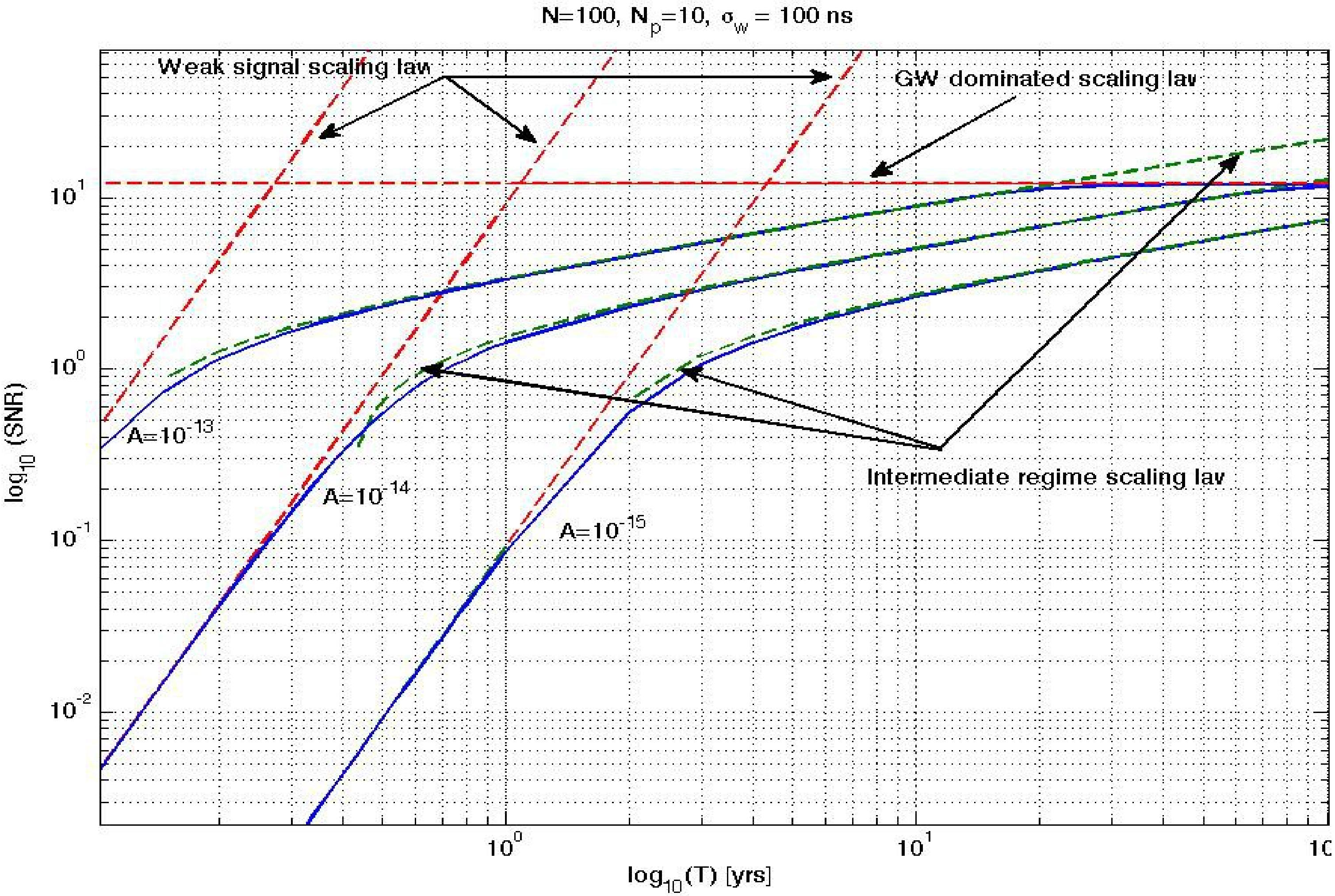


Siemens et al. (in prep.)

Assumptions: keep observing all pulsars, and add 3 per year

Note: red noise estimate is <u>very</u> uncertain

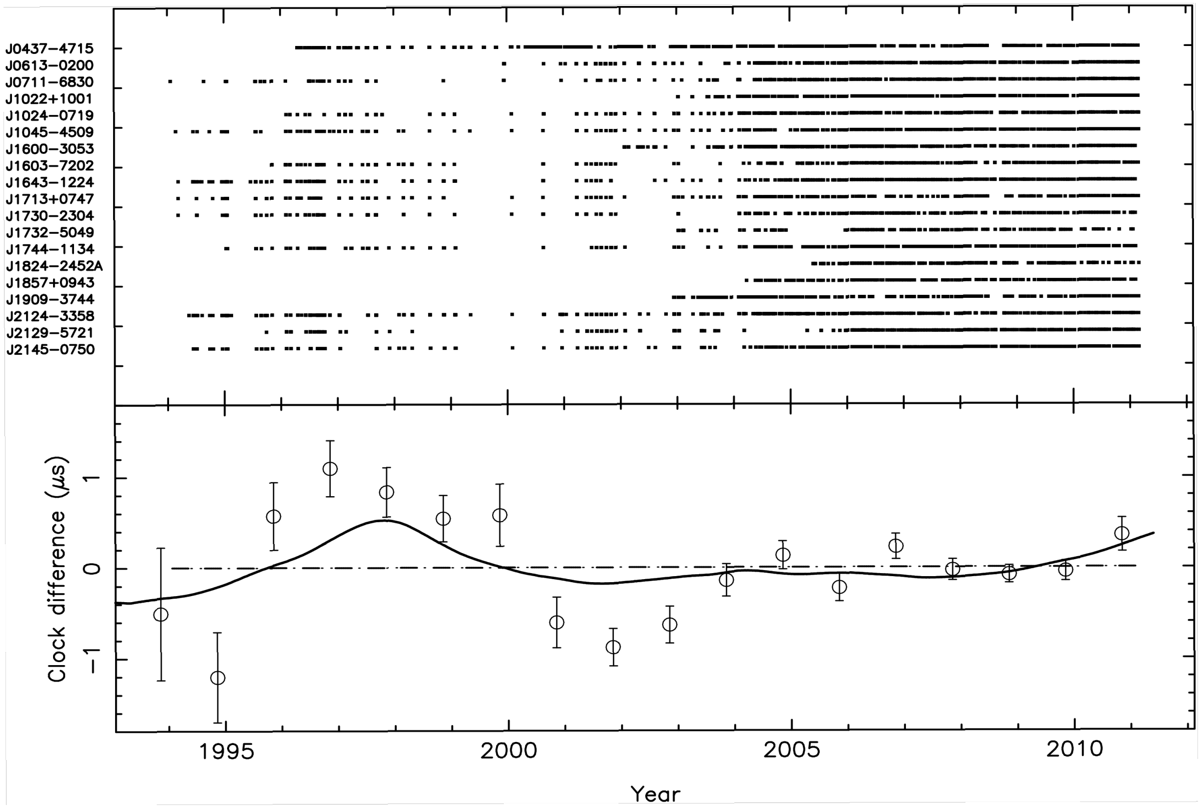
Scaling laws



Cannot 'just wait'. Need more pulsars

Siemens et al. (in prep.)

Other applications



Hobbs et al. (2012)

Pulsars can be used to construct a timescale, independent from atomic clocks

Other uses include: studying the solar system ephemeris (planet masses), cosmic strings, interstellar navigation, ...

Conclusions

- Pulsars can be used as sensitive instruments
- Lots of fundamental science done
- Ideal for testing gravitational theories
- Observing GWs in the near future with pulsar timing arrays: 5-15 years??

- Plenty of other uses for PTAs
- Join in on data challenge!