

Jaume Garriga, JGRG 22(2012)111603

“Vacuum transitions and the arrow of time”

---

**RESCEU SYMPOSIUM ON  
GENERAL RELATIVITY AND GRAVITATION**

**JGRG 22**

November 12-16 2012

Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan



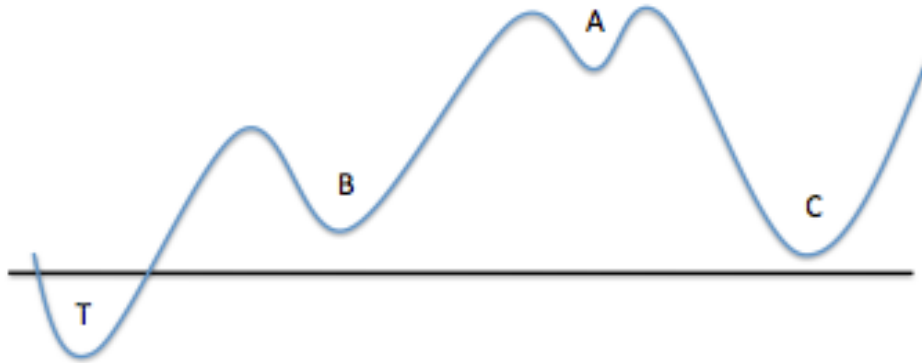
# Vacuum transitions and the arrow of time

Jaume Garriga (U. Barcelona)

ジャウマ ガリガ (Kyoto YITP)

# Inflationary multiverse:

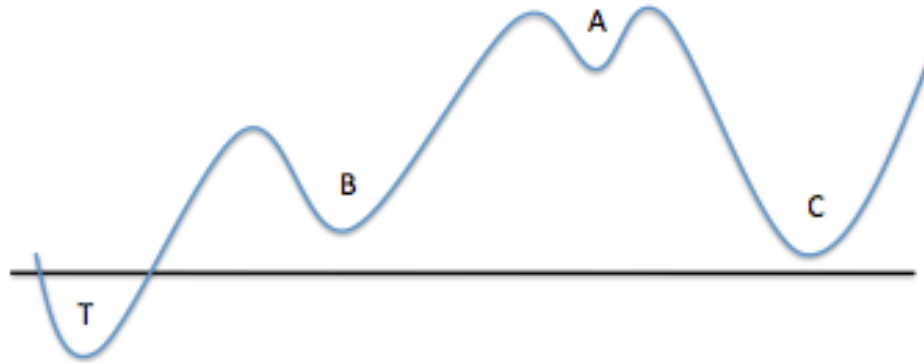
Gravity coupled to a field theory with several metastable vacua



Very interesting dynamics, driven by inflation and vacuum transitions.

## Inflationary multiverse:

Gravity coupled to a field theory with several metastable vacua



### Our universe could well be that system:

- There is some evidence for primordial inflation.
- And some more for cosmic acceleration.
- This scenario may offer a solution of the cc problem (and all other observed coincidences), given a vast landscape of vacua.

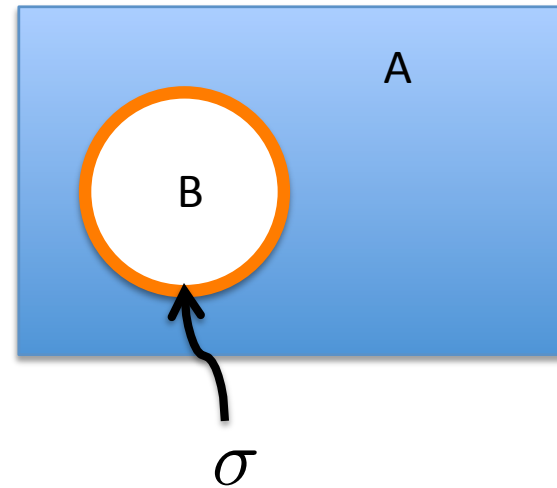
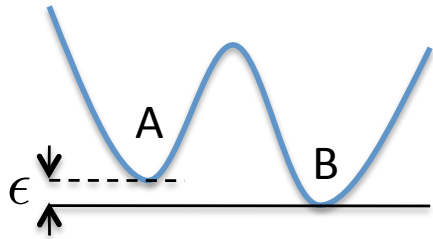
- Vacuum transitions:**
- Slow roll
  - Quantum diffusion
  - Bubble nucleation

These are semiclassical mechanisms (there may be other transition mechanisms in a full quantum theory).

## **Robust features of theories with long lived inflating vacua:**

- Inflation is generically **eternal to the future**.
- **All vacua** which are accessible are **eventually populated**. There will be some fraction of the volume of the universe
- The **distribution of vacua** [e.g.  $\text{Vol}(\phi, t)$ ] approaches an **attractor** at late times.

# Bubble formation

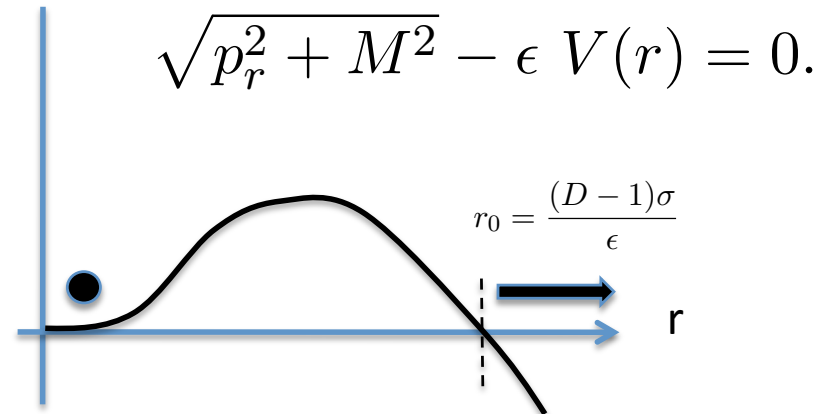


Voloshin, Kobzarev, Okun (75)

$$M(r) = \sigma \Omega r^{D-2}$$

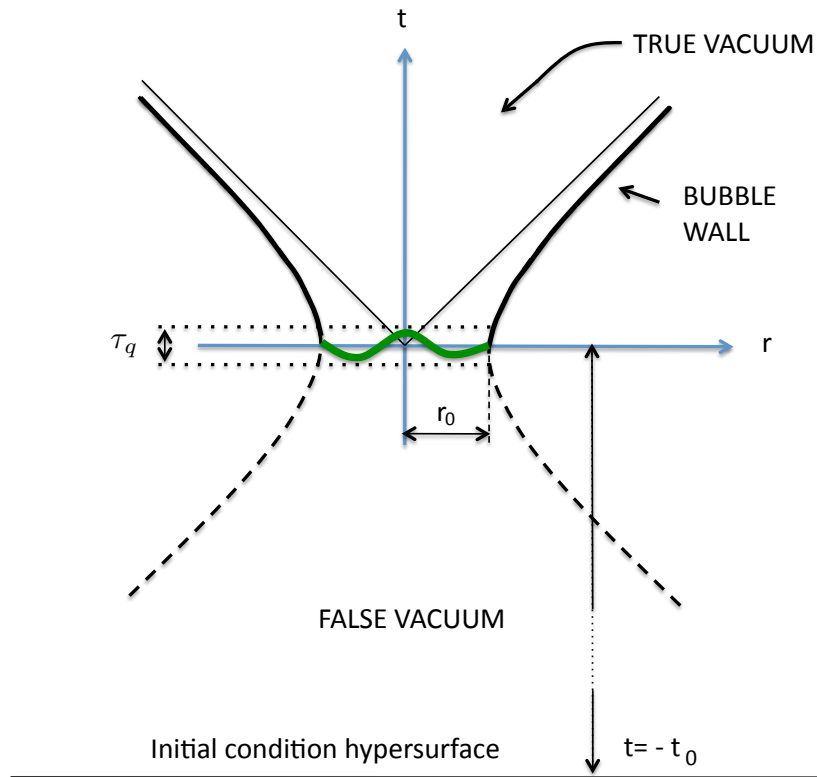
$$V(r) = \frac{\Omega}{D-1} r^{D-1}$$

$$p_r = \gamma M \dot{r}$$



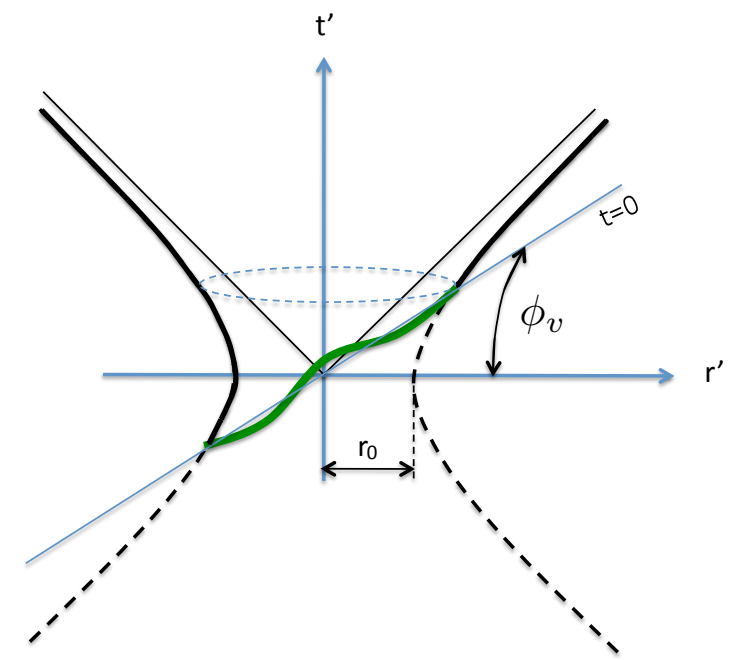
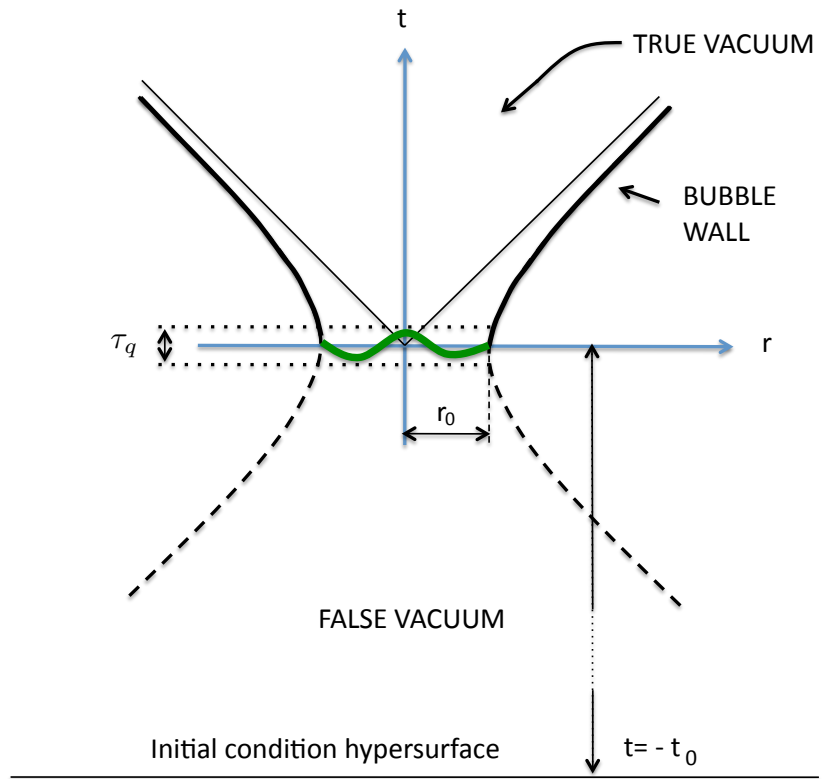
$$\Gamma \sim e^{-\int_{r=0}^{r_0} |p_r| dr}$$

**Critical bubble nucleates at rest, and then expands**



False vacuum approximately Lorentz invariant.

No preferred frame of nucleation.

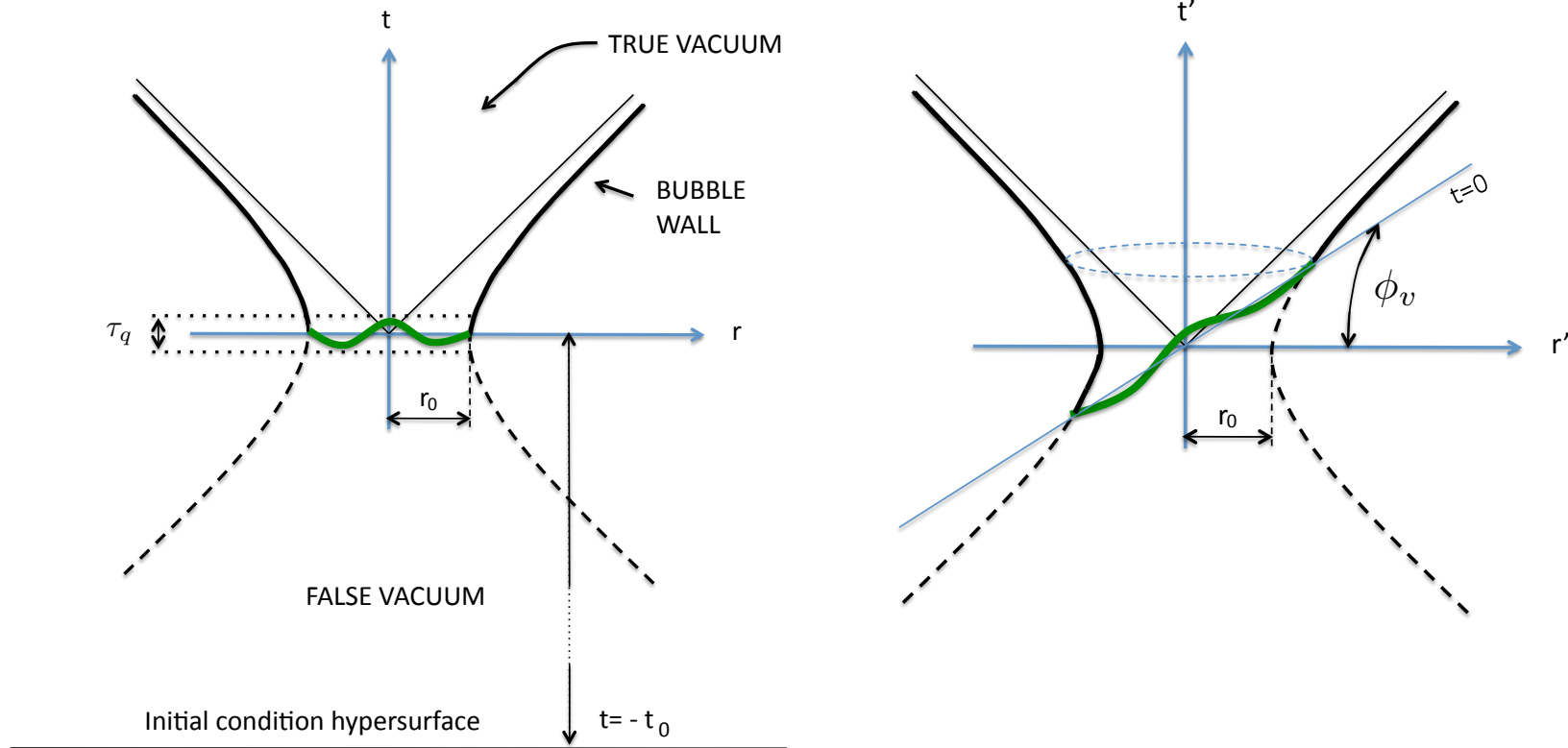


False vacuum approximately Lorentz invariant.

No preferred frame of nucleation.

Integrate over all possible frames?  $\Gamma_{total} = \Gamma \Omega \int d\phi_v (\sinh \phi_v)^{D-2} = \infty$





False vacuum approximately Lorentz invariant.

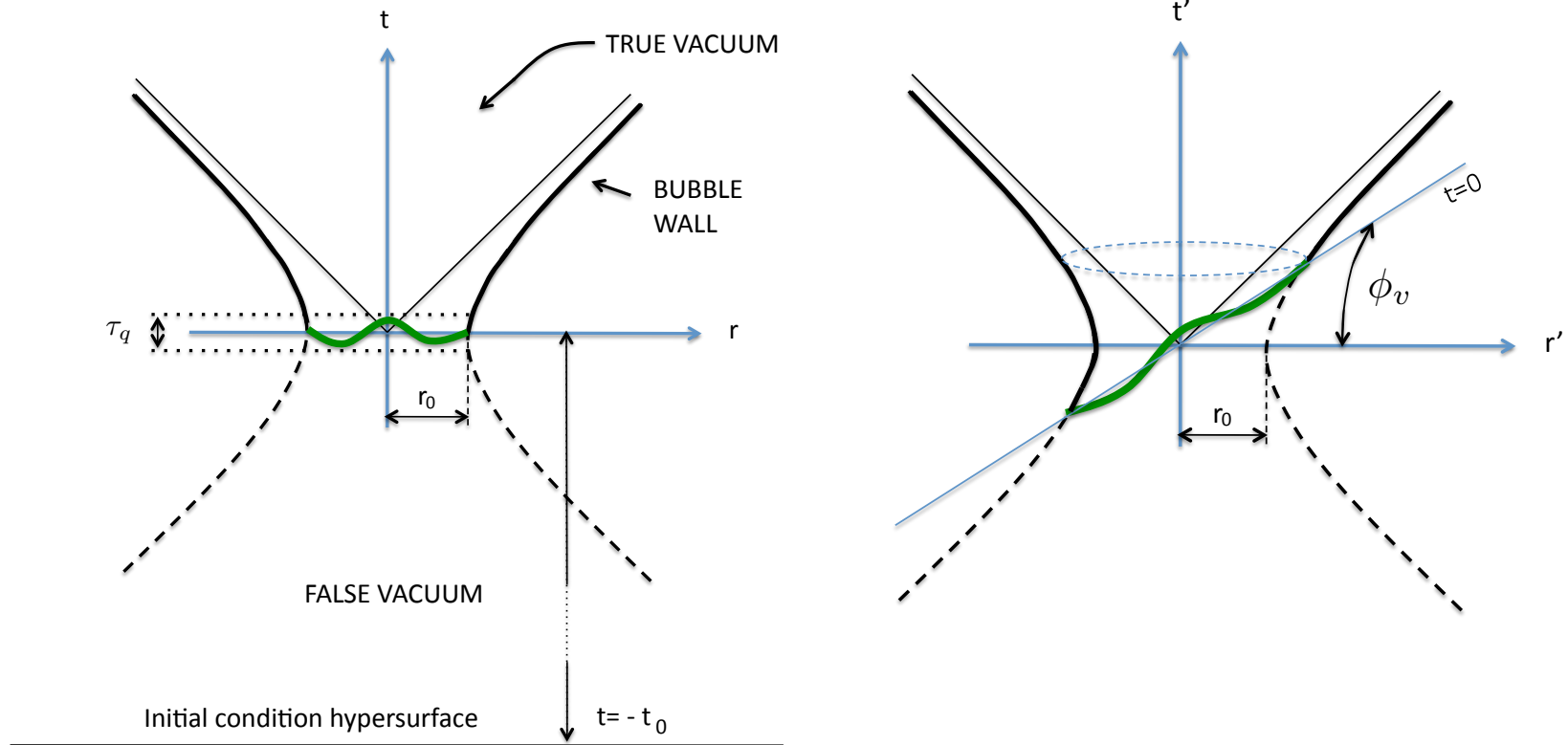
No preferred frame of nucleation.

Integrate over all possible frames?

~~$$\Gamma_{total} = \Gamma \Omega \int d\phi_v (\sinh \phi_v)^{D-2} = \infty$$~~

$\Gamma = A e^{-S_E}$

Coleman (77) showed that the **rate is finite** (integration over rest frame does not play a role)



**Remaining question:** if it is true that the false vacuum approximately Lorentz invariant.

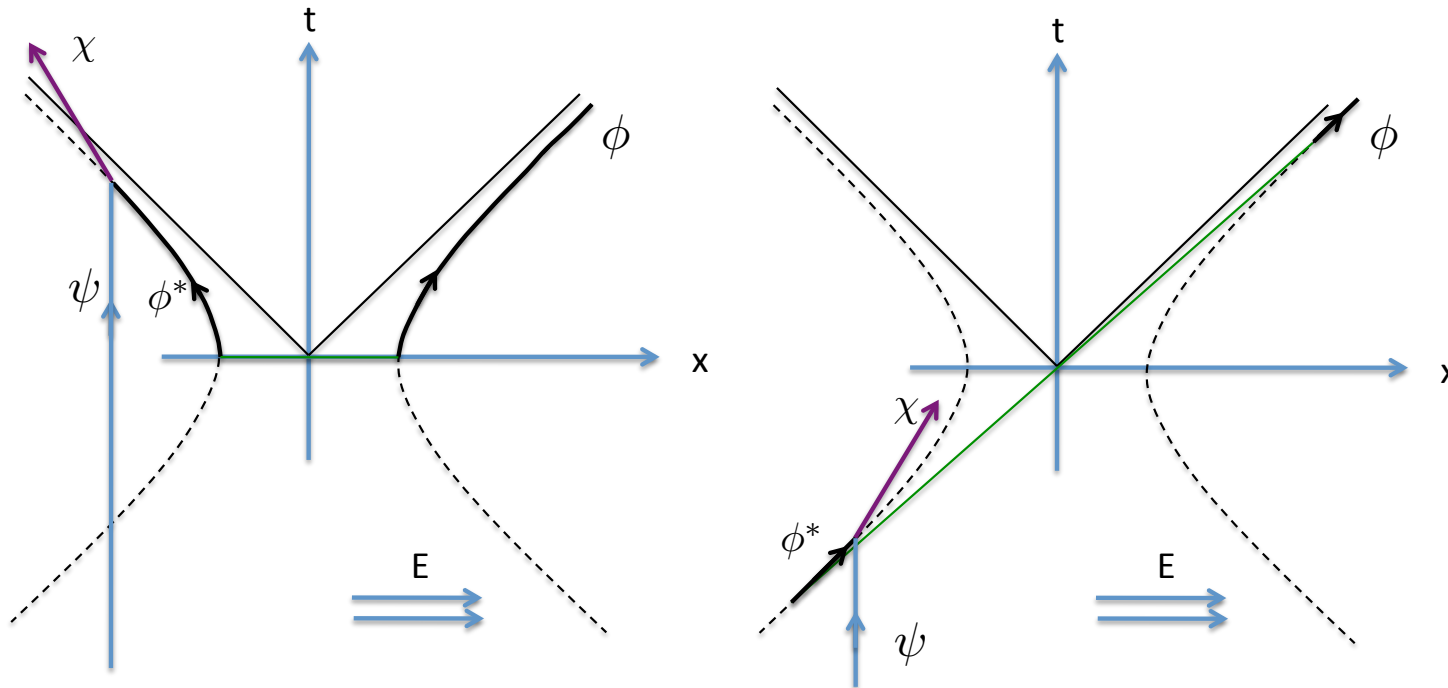
**What is it that determines the frame of nucleation?**

J.G., Kanno, Sasaki, Soda, Vilenkin (2012)  
 J.G., Kanno, Tanaka (in preparation).

# What is it that determines the frame of nucleation?

The question can be addressed by using a model detector

We need a fully quantum description of bubble nucleation: Schwinger pair production in (1+1)



$$g(\phi\psi^*\chi + hc)$$

$$w_\phi + w_\psi = w_\chi$$

## Possible outcomes of this “experiment”:

- (A) - Frame of nucleation is determined by the rest frame of the detector.
- (B) - Perhaps the frame of nucleation is not very well determined and the detector can probe part of the contracting branch.
- (C) - The frame of nucleation is determined by initial conditions.

---

$$S_E \equiv \pi \lambda \gg 1$$

For tunneling to be semiclassical

$$r_0$$

Size of the critical bubble

### Time-scales:

$$\tau_{nuc} \sim r_0$$

Timescale of nucleation

$$\tau_q \sim \lambda^{-1/3} r_0 \ll r_0$$

Quantum fuzziness scale around the turning point

### Let us consider

$$\tau_q \ll r_0 \ll t_0 \rightarrow \infty \quad (\text{time elapsed since initial conditions})$$

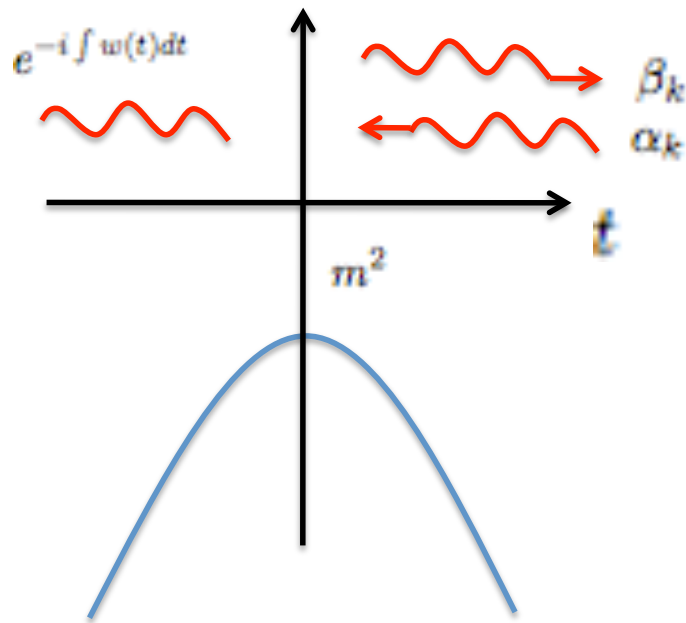
This will take care of option (C)

## In a constant external electric field

$$\left[ \frac{d^2}{dt^2} + m^2 + (k + eEt)^2 \right] \phi_k = 0.$$

$$k_{\text{phys}} = k + eEt.$$

$$\phi_k = \alpha_k \phi_k^{\text{out}} + \beta_k \phi_k^{\text{out}*}$$



$$|\beta_k|^2 = e^{-\pi\lambda} = e^{-\frac{\pi m^2}{eE}}.$$

(Independent of  $k$ )

## Lorentz invariance of the “in” vacuum ?

Gauge invariant Wightman function

$$G^+(x^\mu, y^\mu) \equiv {}_{\text{in}}\langle 0 | \phi^\dagger(x^\mu) e^{-ie \int_x^y A_\nu dx^\nu} \phi(y^\mu) | 0 \rangle_{\text{in}},$$

$$G^+(\Delta x^\mu) = \frac{|\alpha|^2}{4\pi} \Gamma(-\nu^*) \frac{W_{i\lambda/2, 0}(ieE \Delta s^2/2)}{\sqrt{ieE \Delta s^2/2}}, \quad \text{for } \Delta t - \Delta x > 0$$

$$G^+(\Delta x^\mu) = \frac{|\alpha|^2}{4\pi} \Gamma(-\nu) \frac{W_{-i\lambda/2, 0}(-ieE \Delta s^2/2)}{\sqrt{-ieE \Delta s^2/2}}, \quad \text{for } \Delta t - \Delta x < 0.$$

$$\Delta s^2 = -(\Delta t)^2 + (\Delta x)^2$$

  $|0\rangle_{\text{in}}$  translation and boost invariant.

(Cannot determine the frame of nucleation)

# Detector $g(\phi\psi^*\chi + hc)$

$$g(t) = g e^{-t^2/T^2}$$

We connect the interaction for a finite time T

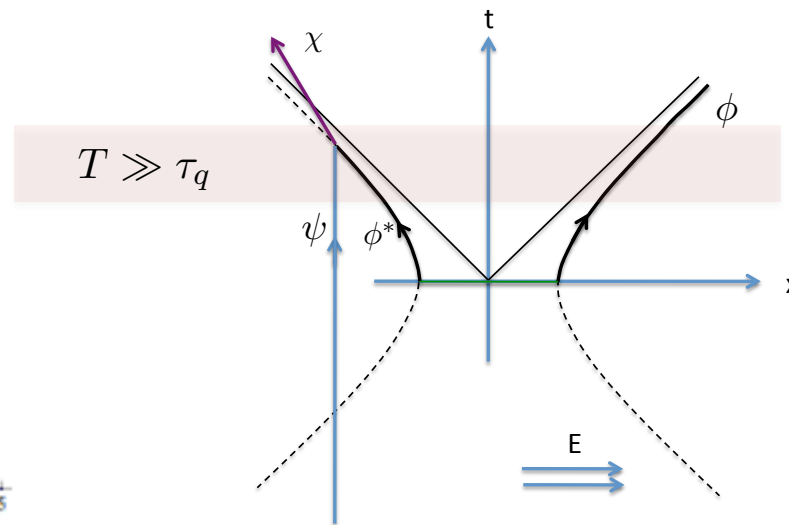
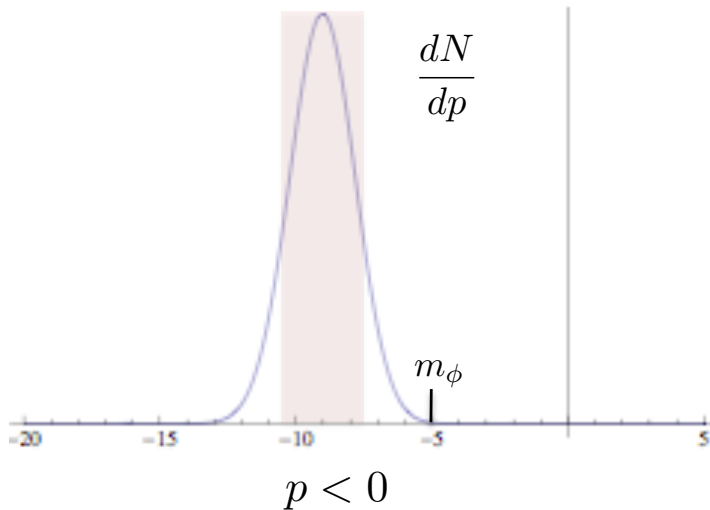
$$\frac{dN_\chi}{dp} \propto |\mathcal{A}_\chi(p; q=0)|^2 \propto \exp\left[-\frac{(p + \omega_p - m_\psi)^2}{(eET)^2}\right]$$

$$m_\chi - m_\psi \gg m_\phi$$

$$T \gg \tau_q$$

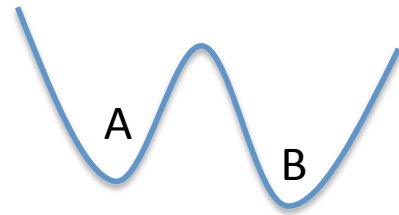
$$\bar{p} = -\frac{1}{2m_\psi}(m_\chi^2 - m_\psi^2)$$

Negative and in accordance with kinematics



In good agreement with option (A)

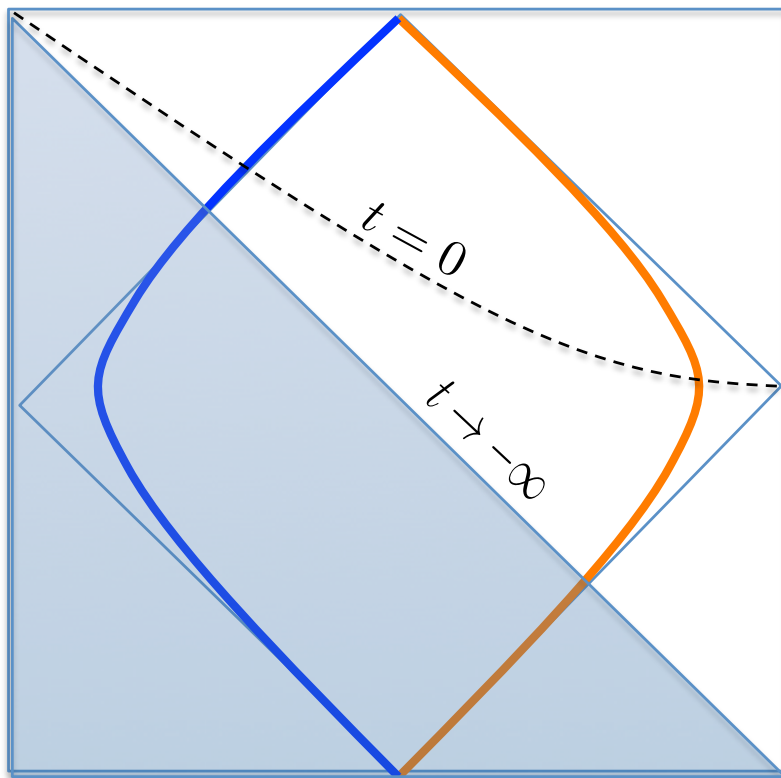
# De Sitter



A  $\longrightarrow$  B (+)

B  $\longrightarrow$  A (-)

## Schwinger process in 1+1 de Sitter (J.G. 94)



Electric field does not dilute in 1+1

$\longrightarrow$   $|\beta_k|^2 \approx \exp\left(-\frac{2\pi}{H^2} [(\mathcal{M}^2 H^2 + e^2 E_0^2)^{1/2} + eE_0]\right)$ ,

$\longrightarrow$   $|\beta_k|^2 \approx \exp\left(-\frac{2\pi}{H^2} [(\mathcal{M}^2 H^2 + e^2 E_0^2)^{1/2} - eE_0]\right)$

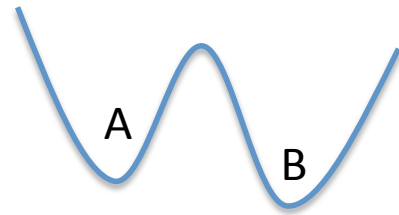
$|\beta_k|^2 \approx e^{-S_E}$  In agreement with instanton methods.

“in” vacuum state is Hadamard.

Pairs are created at a constant rate, but they are diluted by the expansion. Healthy UV limit.



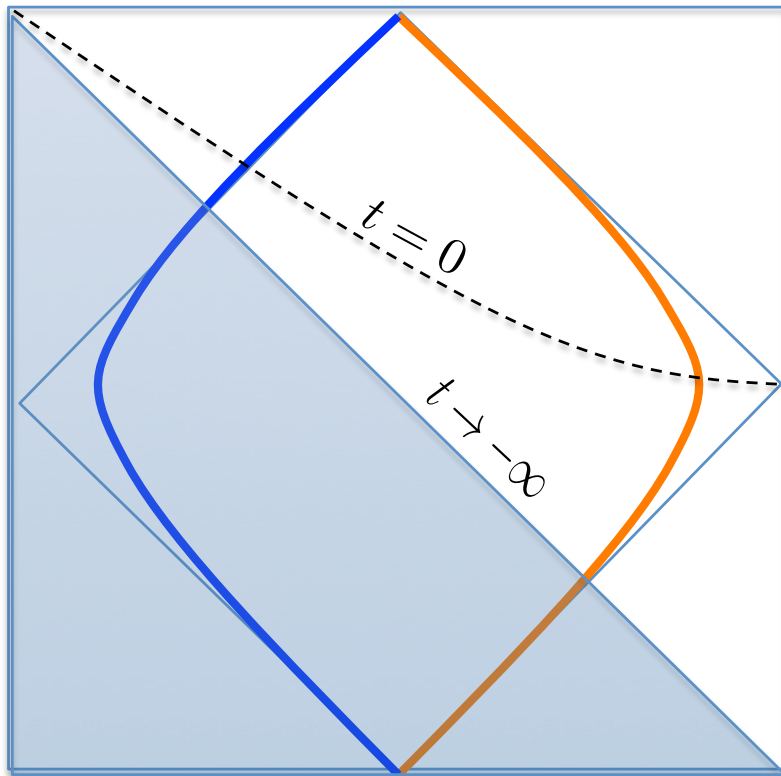
# De Sitter



$$A \xrightarrow{\text{orange}} B \quad (+)$$

$$B \xrightarrow{\text{blue}} A \quad (-)$$

## Schwinger process in 1+1 de Sitter (J.G. 94)



Electric field does not dilute in 1+1

$$\xrightarrow{\text{blue}} |\beta_k|^2 \approx \exp\left(-\frac{2\pi}{H^2} [(\mathcal{M}^2 H^2 + e^2 E_0^2)^{1/2} + eE_0]\right),$$

$$\xrightarrow{\text{orange}} |\beta_k|^2 \approx \exp\left(-\frac{2\pi}{H^2} [(\mathcal{M}^2 H^2 + e^2 E_0^2)^{1/2} - eE_0]\right)$$

$|\beta_k|^2 \approx e^{-S_E}$  In agreement with instanton methods.

“in” vacuum state is Hadamard.

Pairs are created at a constant rate, but they are diluted by the expansion. Healthy UV limit.

(GKSSTV 2012) “in” vacuum breaks de Sitter invariance  $\langle J_\mu \rangle_{in} = \delta_\mu^1 (J^+ - J^-) = \text{finite} \neq 0$  !!

## Spontaneous breakdown of dS invariance

Persistent stationary current  $\langle J_\mu \rangle \langle J^\mu \rangle = \text{const} \neq 0$ .

There is no dS invariant Hadamard state in the presence of electric field E

**Arrow of time**  $t^\mu \propto \epsilon^{\mu\nu} \langle J_\nu \rangle$

determined by the choice of initial conditions. It lasts forever.

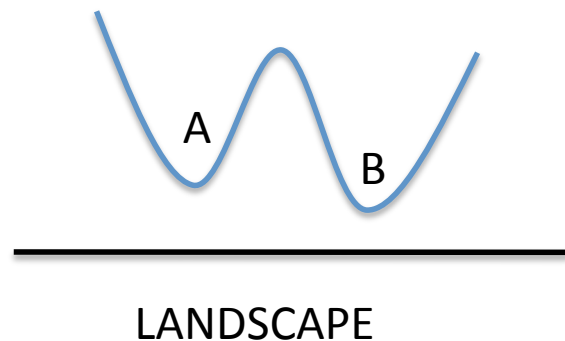
## Discussion:

### Is this be related to an arrow of time in the multiverse?

- If we include backreaction, the electric field disappears, and the current disappears.
- However vacuum transitions will continue to occur,
- Bubble nucleation is still **time asymmetric**:

**we see expanding bubbles but not contracting ones**

- This seems to imply a small deviation from ergodicity even in the case of a dS landscape



$$\frac{\Gamma_{BA}}{\Gamma_{AB}} = e^{S_B - S_A}$$

Ratio of transition rates per unit volume (from instanton formalism).

$$\frac{\kappa_{BA}}{\kappa_{AB}} = \left(\frac{H_B}{H_A}\right)^{D-1} e^{S_B - S_A}$$

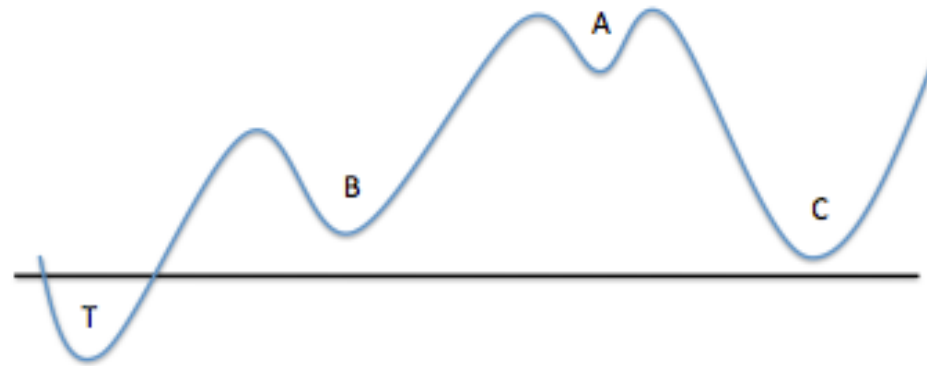
Ratio of transition rates



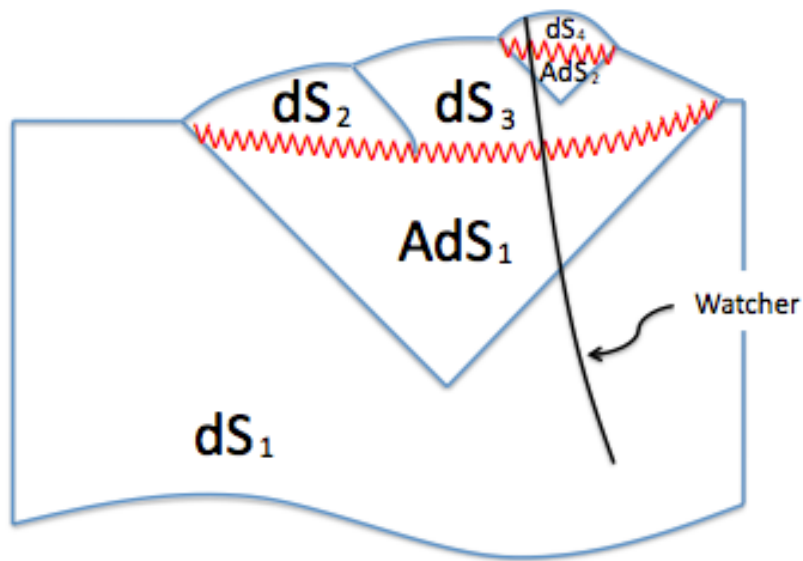
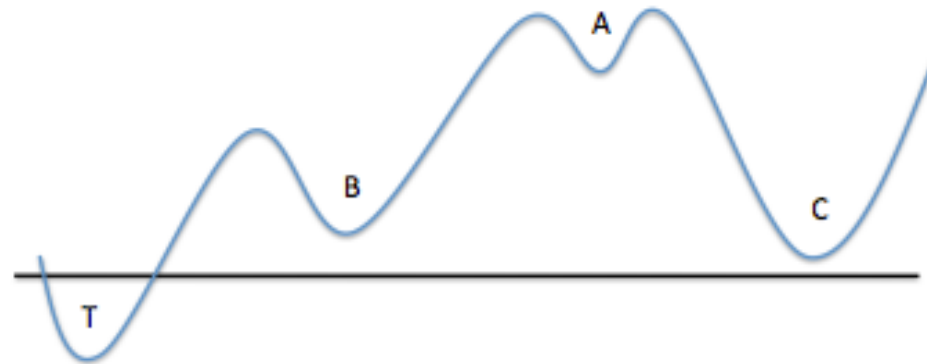
Slight departure from ergodicity! **Persistent arrow of time.**

(J.G.+ Vilenkin '12)

In real life we need something more drastic



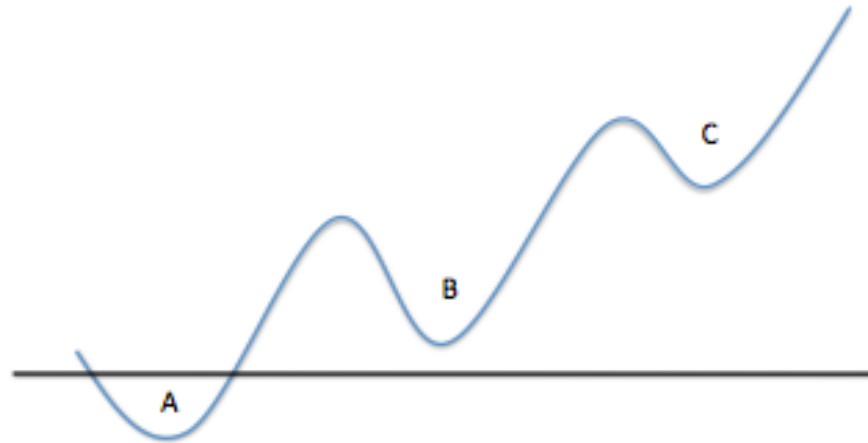
In real life we need something more drastic



Transitions through crunches, if allowed, are likely to be very far from ergodic.

We expect in this case a very well defined arrow of time.

(J.G.+ Vilenkin '12)



$$f_C/f_B \sim \exp(S_C - S_B) \quad \text{If crunches are terminal}$$

$$f_C/f_B \sim \kappa_{AB}/\kappa_{BC} \quad \text{If we can pass through crunches}$$