

Yuichiro Sekiguchi, JGRG 22(2012)111601

“Gravitational-wave and neutrino emissions from black
hole-neutron star binary merger”

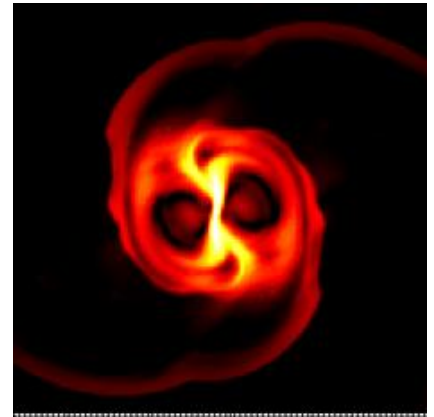
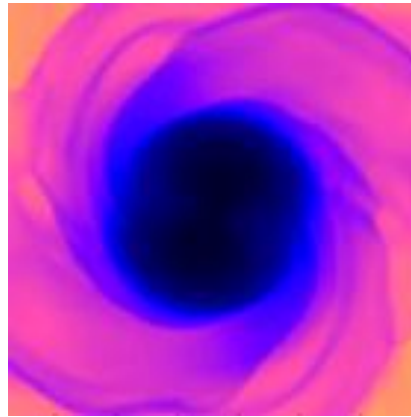
**RESCEU SYMPOSIUM ON
GENERAL RELATIVITY AND GRAVITATION**

JGRG 22

November 12-16 2012

Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan





Current Status of Numerical Relativity Simulations

Yuichiro Sekiguchi (YITP)

K. Kiuchi, K. Kyutoku, M. Shibata, K. Hotokezaka



What is Numerical Relativity ?

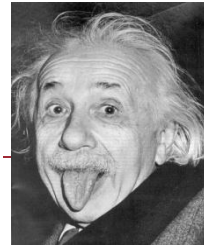
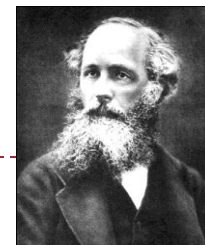
- ▶ Solving Einstein eq. and source field eqs. to clarify dynamical phenomena in the universe where strong gravity plays a role

$$G_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

$$\nabla_a T^{ab} = 0 \quad (T^{ab} = (T_{\text{Fluid}} + T_{\text{EM}} + T_{\nu} + \dots)^{ab})$$

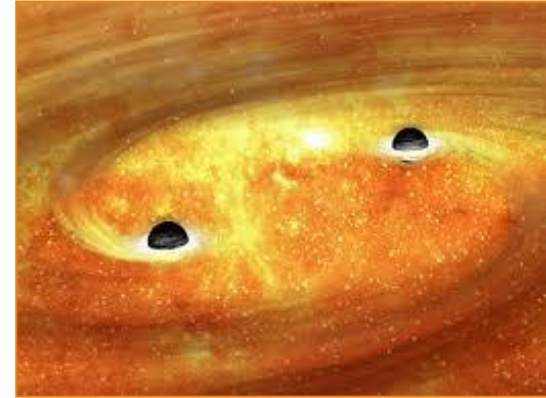
$$\nabla_a J^a = 0 \quad (J^a \sim (n_{\text{baryon}}, n_{\text{lepton}}(n_e, n_{\nu}, \dots), \dots)u^a)$$

- ▶ All four known interactions play important roles
 - ▶ **Gravity** : GR, BH formation, ISCO, etc
 - ▶ **Strong** : EOS (equation of state) of dense nuclear/hadronic matter
 - ▶ **EM** : MHD phenomena, EOS of dense matter
 - ▶ **Weak** : Electron capture, Neutrino production
 - ▶ 99% gravitational binding energy released is carried away by neutrinos in SNe



Targets of Numerical Relativity

- ▶ Dynamical phenomena with strong gravity
 - ▶ Black hole formation
 - ▶ Stellar core collapse
 - ▶ Merger of compact object binary



General relativistic gravity is important
Highly nonlinear and dynamical

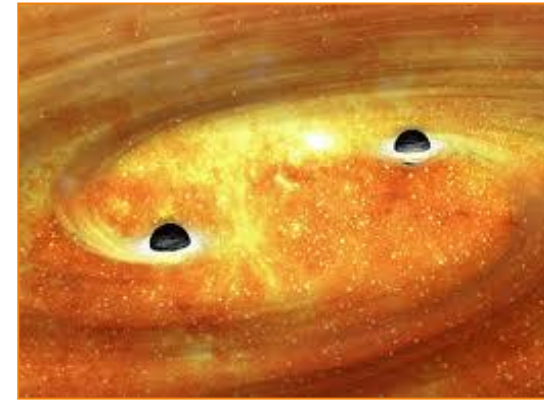


**Numerical
Relativity**



Targets of Numerical Relativity

- ▶ Dynamical phenomena with strong gravity
 - ▶ Black hole formation
 - ▶ Stellar core collapse
 - ▶ Merger of compact object binary
- ▶ **Gravitational waves from them**
 - ▶ **NR should provide GW templates**



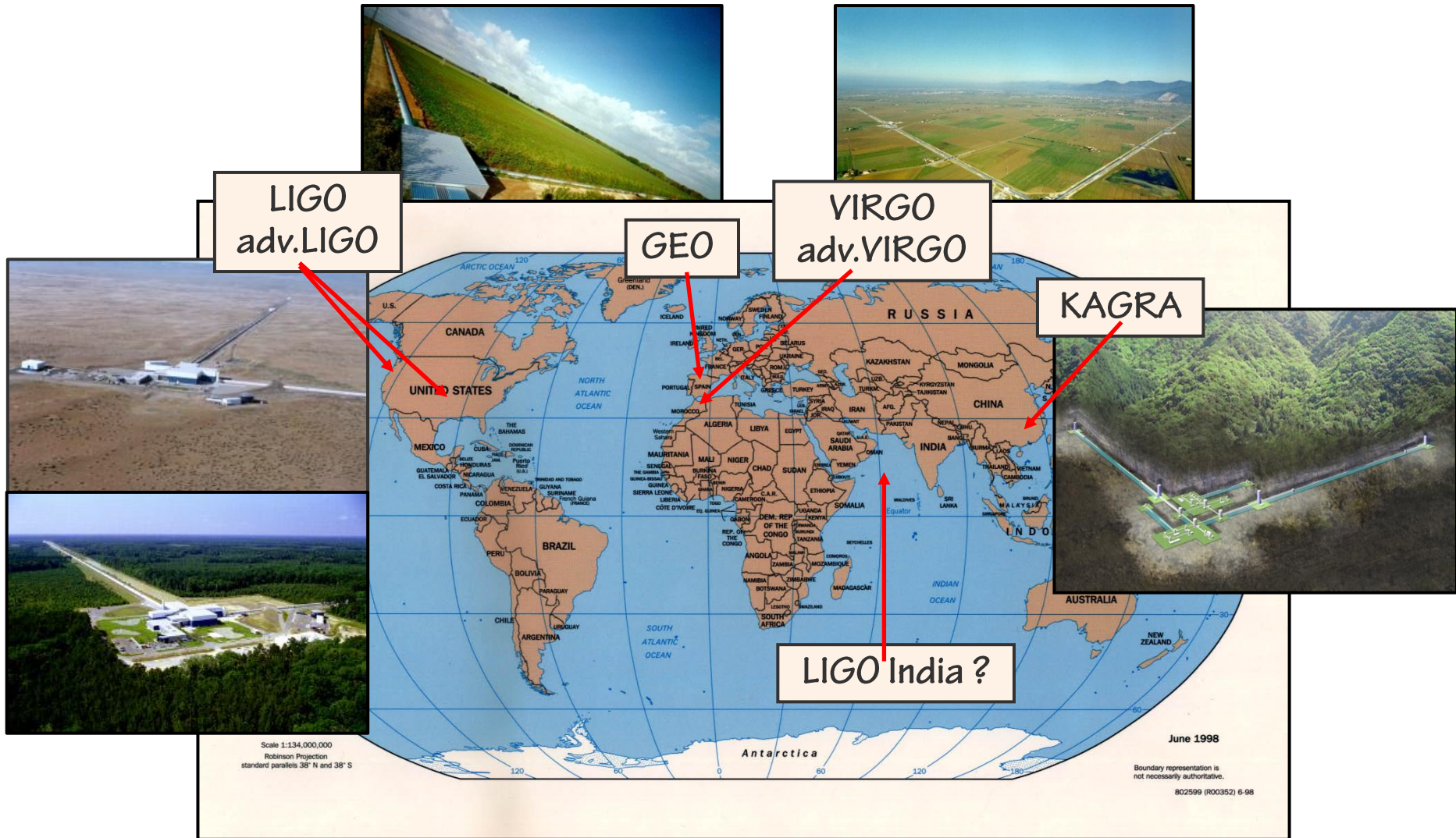
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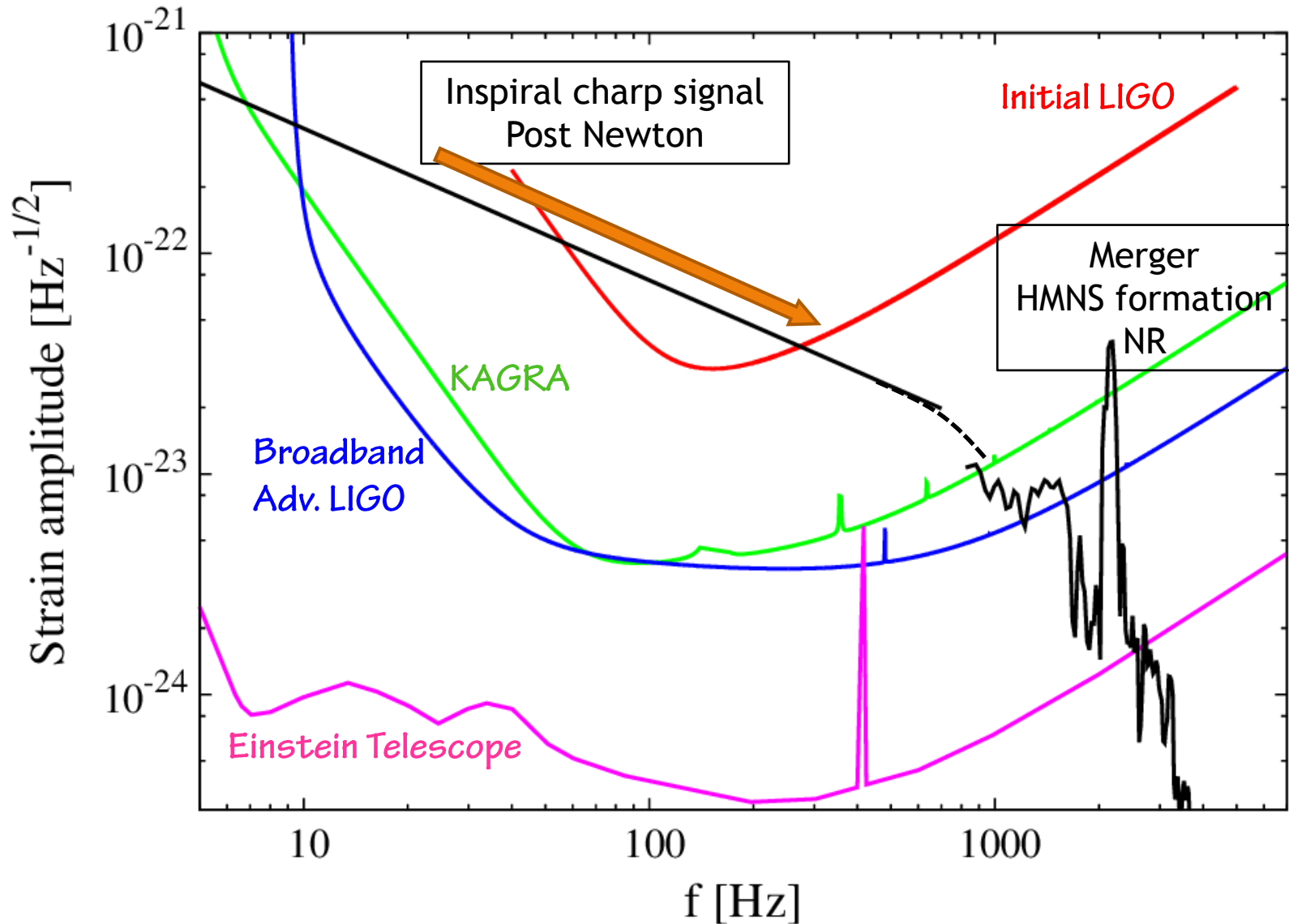
**Numerical
Relativity**



Current & up-coming GW detectors

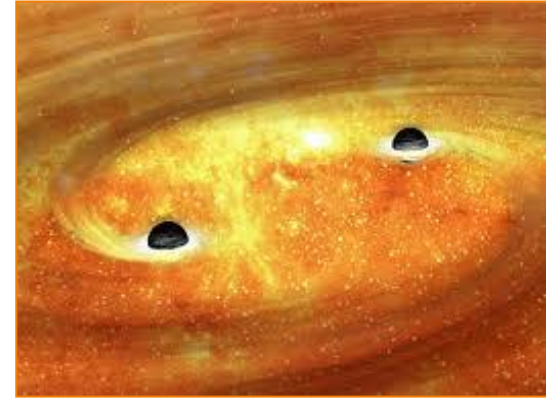


BNS 1.35-1.35Msolar optimal @ 100Mpc



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- ▶ Dynamical phenomena with strong gravity
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- ▶ Gravitational waves from them
 - ▶ NR should provide GW template
 - ▶ **Towards GW astronomy**
 - **Exploring physics and astrophysics by GW**



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Highly nonlinear and dynamical



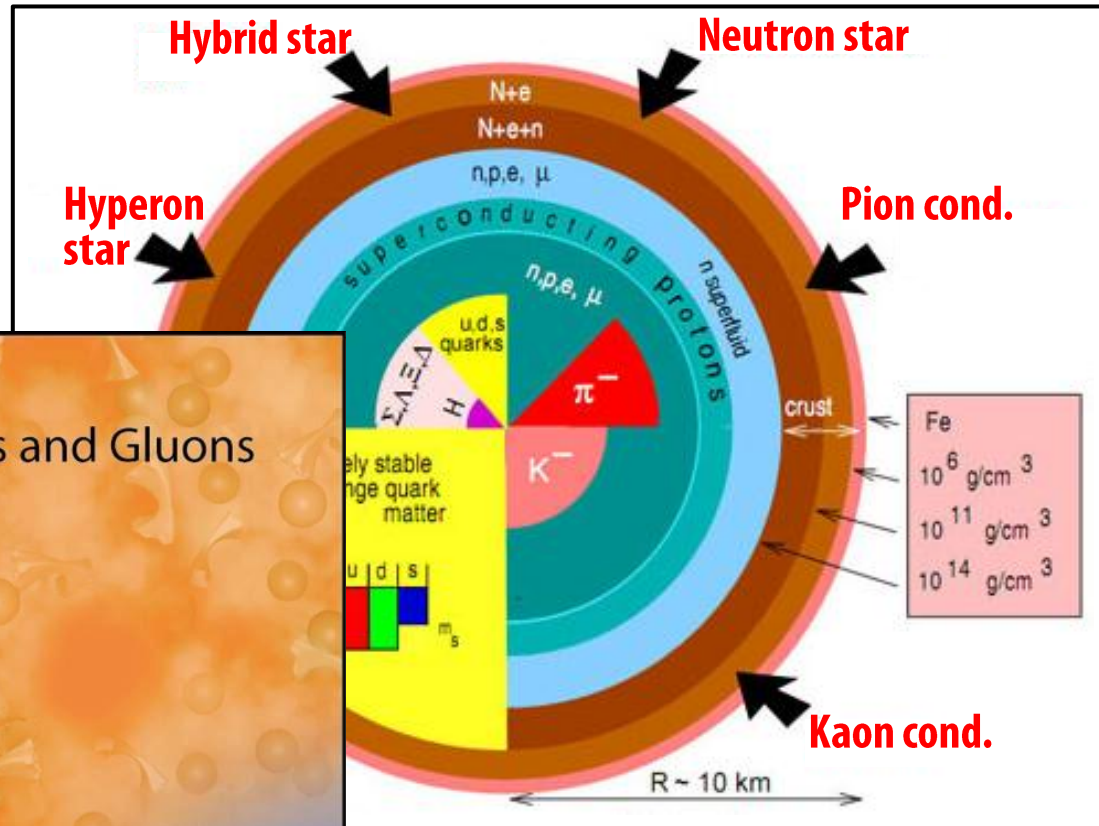
**Numerical
Relativity**



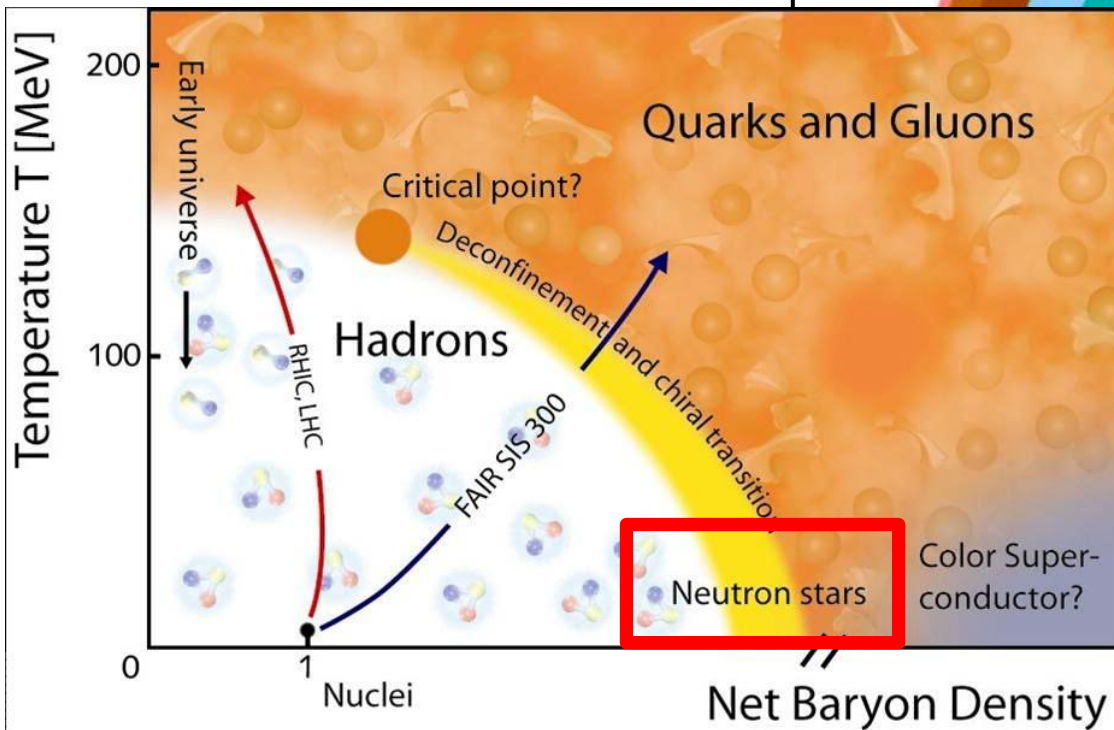
Neutron star & physics of dense matter

- ▶ Neutron star (NS) as a laboratory of dense matter physics
- ▶ There are a large number theoretical models
 - ▶ Equation of State (EOS)

$$P = P(\rho, T, X_i)$$

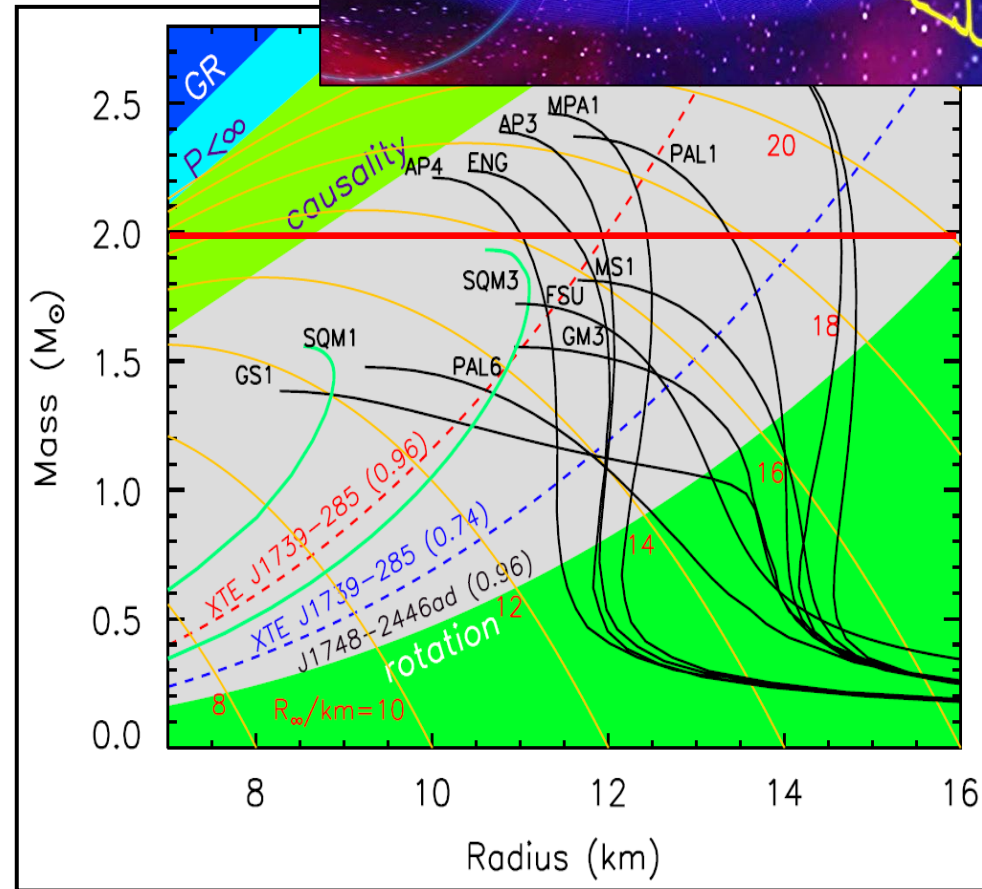


F. Weber (2005)



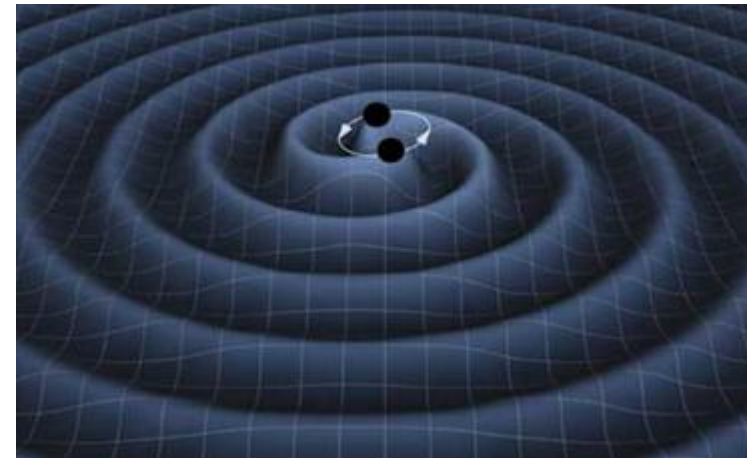
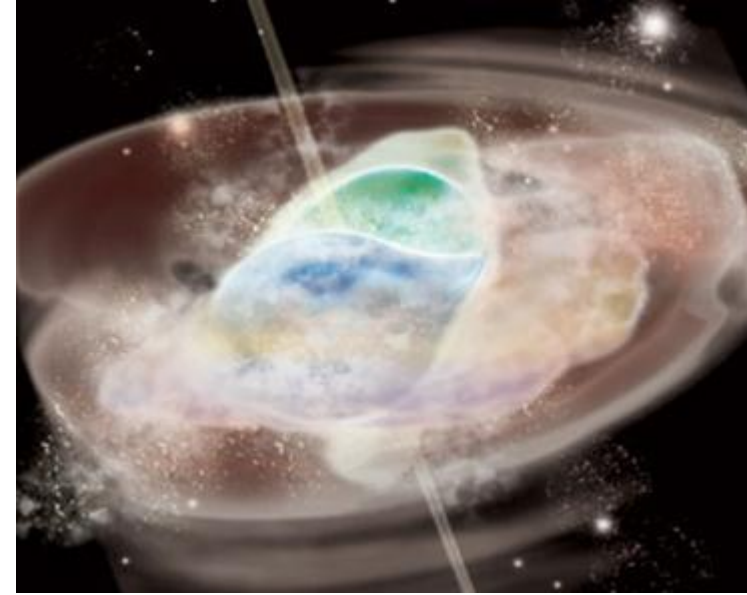
Open Question

- ▶ Given the theoretical uncertainty, which one
- ▶ Traditional method to constrain the models
 - ▶ Mass-Radius relation :
 - ▶ Estimation of mass and radius observation of X-ray binary
 - ▶ Large systematic error
 - ▶ Maximum mass :
 - ▶ Just find a massive NS
 - ▶ PSR J1614-2230 (NS-WD)
 - ▶ NS of 1.97Msolar
 - ▶ Mass measurement by Shapiro time delay
 - ▶ Too soft EOSs are excluded
- ▶ Still we have a number of theoretical models



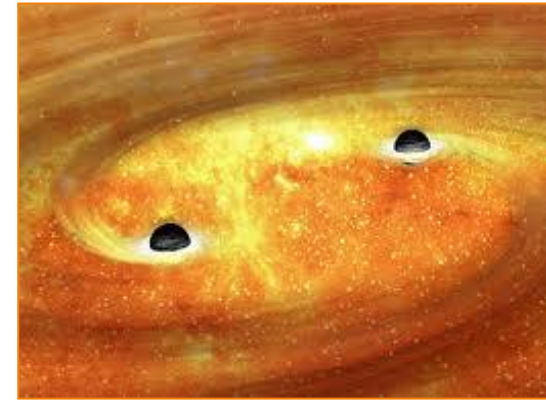
Compact binary merger as cosmological collider

- ▶ NS-NS merger
 - ▶ Collision of giant nuclear matter objects
 - ▶ Tell BH or NS by GW \Rightarrow maximum mass
 - ▶ Both M and R are contained in GW
 - ▶ We may explore the physics of dense matter by 'seeing' NS interior by GW
- ▶ BH-BH merger
 - ▶ Collision of strongest gravity sources
 - ▶ Testing gravity in extremely strong regime
 - ▶ Beyond Einstein gravity ?
 - ▶ Higher dimension ?
- ▶ Comparison of observations with NR modeling



Targets of Numerical Relativity

- ▶ Dynamical phenomena with strong gravity
 - ▶ Black hole formation
 - ▶ Stellar core collapse
 - ▶ Merger of compact object binary
- ▶ Gravitational waves from them
 - ▶ NR should provide GW template
 - ▶ Towards GW astronomy
 - E.g. NS as a laboratory for dense matter physics
- ▶ **High energy phenomena in astrophysics**
 - ▶ **Gamma-ray bursts**
 - ▶ Supernova explosions



General relativistic gravity is important
Highly nonlinear and dynamical



**Numerical
Relativity**

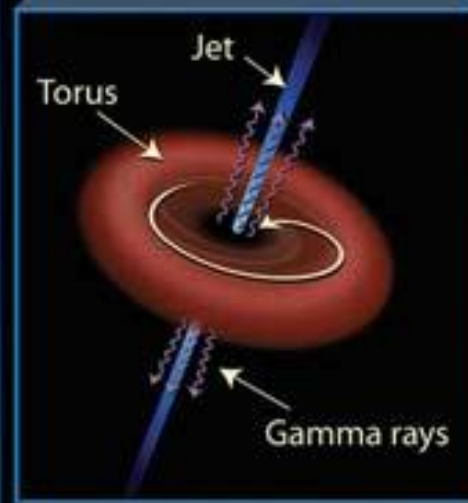
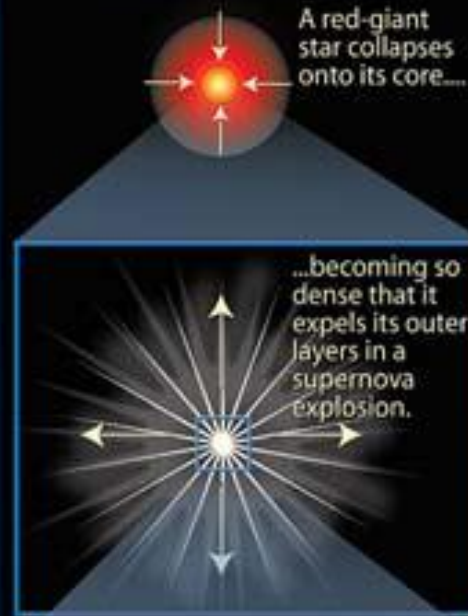


Central engine of

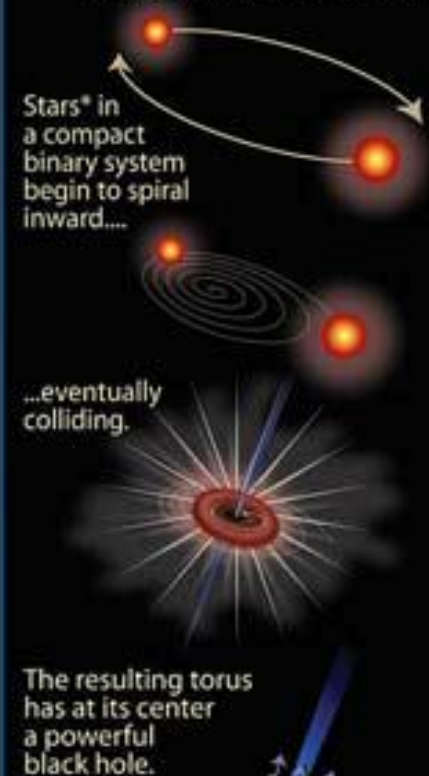
- ▶ Gamma-ray burst (GRB) : balanced energy release
 - ▶ Short and intense burst of gamma rays
 - ▶ Discovered accidentally in the late 1990s
 - ▶ With rapid time variability : $\Delta t \sim 10^{-3}$ s
 - ▶ Duration : $T \sim 0.01$ -1000 s.
 - ▶ Bimodal distribution :
 - ▶ $T < 2$ s : Short GRB (SGRB)
 - ▶ $T > 2$ s : Long GRB (LGRB)
 - ▶ Energy :
 - ▶ LGRB $\sim 10^{51}$ erg (with beaming)
 - ▶ SGRB $\sim 10^{49}$ erg
 - ▶ Central engine model
 - ▶ BH + accretion disk formed by
 - ▶ SGRB : NS-NS, BH-NS merger
 - ▶ LGRB : Stellar core collapse

Gamma-Ray Bursts (GRBs): The Long and Short of It

Long gamma-ray burst (>2 seconds' duration)

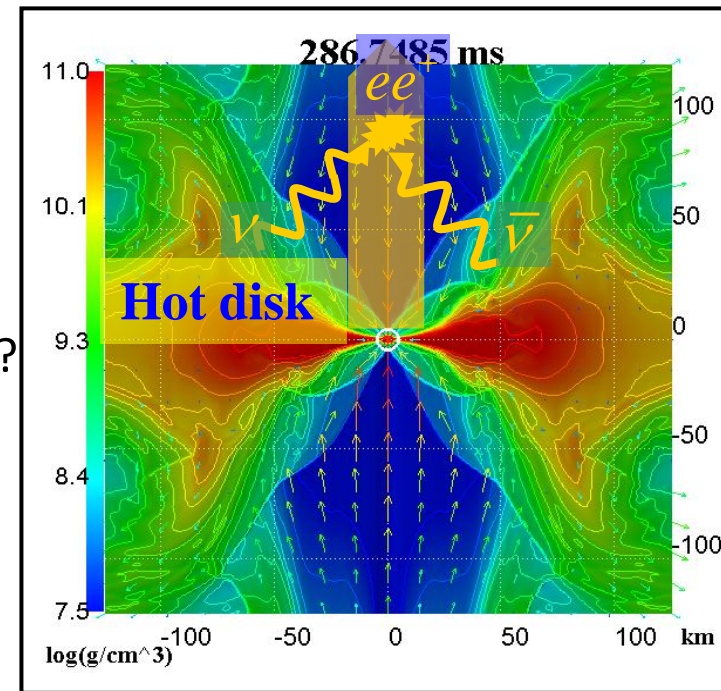


Short gamma-ray burst (<2 seconds' duration)



How does the GRB jet launch ?

- ▶ One possible scenario : **neutrino pair annihilation**
 - ▶ Emission of neutrinos in the hot accretion disk
 - ▶ Deposition of energy through neutrino annihilation in the baryon-poor funnel around the rotation axis driving a baryonic jet
 - ▶ Emission of gamma-ray photons in internal shocks
 - ▶ Energetics
 - ▶ Disk mass : 0.05Msolar
 - ▶ Gravitational energy at ISCO $\sim 10^{53}$ erg
 - ▶ Neutrino Luminosity \sim Gravitational energy ?
 - ▶ Neutrino pair annihilation efficiency of 0.1--1% ?
 - \Rightarrow jet energy of 10^{50-51} erg ?
 - \Rightarrow GRB energy of 10^{49} erg ?



Current status of NR (1)

▶ Solving Einstein equation ○

- ▶ ADM formulation (unstable) \Rightarrow BSSN formulation (stable)
 - ▶ Shibata & Nakamura (1995), Baumgarte & Shapiro (1999)
- ▶ Stable and less time-consuming coordinate conditions (1990~)

▶ Numerical scheme for GR hydrodynamics ○

- ▶ High resolution shock capturing scheme (Valencia, Munich 1990~)
- ▶ GR Magnetohydrodynamics (GRMHD; 2000~) Kiuchi-kun's talk

▶ Treatment of BH ○

- ▶ First successful binary BH simulation by Pretorius in 2005
- ▶ BSSN-puncture : adopt nice coordinates and variables (Campanelli+ 2006)

▶ Other issues ○

- ▶ Locating Apparent Horizon
- ▶ GW extraction techniques from the metric
- ▶ Mesh refinement techniques (E.g. Yamamoto+ 2008)
- ▶ Powerful Supercomputers

Current status of NR (2)

► Towards more 'realistic' or physical modeling

► Trend in 2010~

► Equation of state (EOS) ○

► Nuclear-theory-based finite temperature EOS tables

► Sekiguchi 2007,2010; Otsu et al. 2009

Takahashi, Ohsuga,
Sekiguchi, Inoue, & Tomida

► Neutrino treatment ○

► Weak interactions (Sekiguchi et al. 2007)

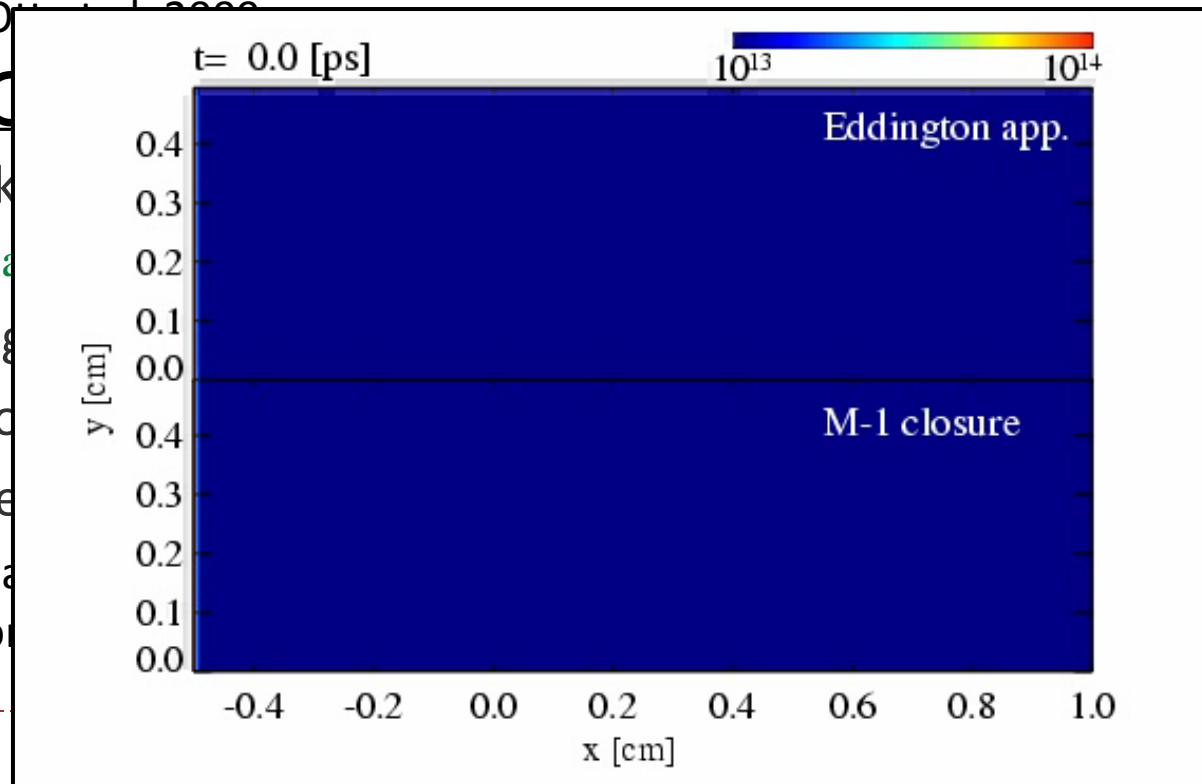
► e^\pm captures, e^\pm annihilation

► Neutrino cooling (Sekiguchi et al. 2007)

► Neutrino heating (Kuroda et al. 2007)

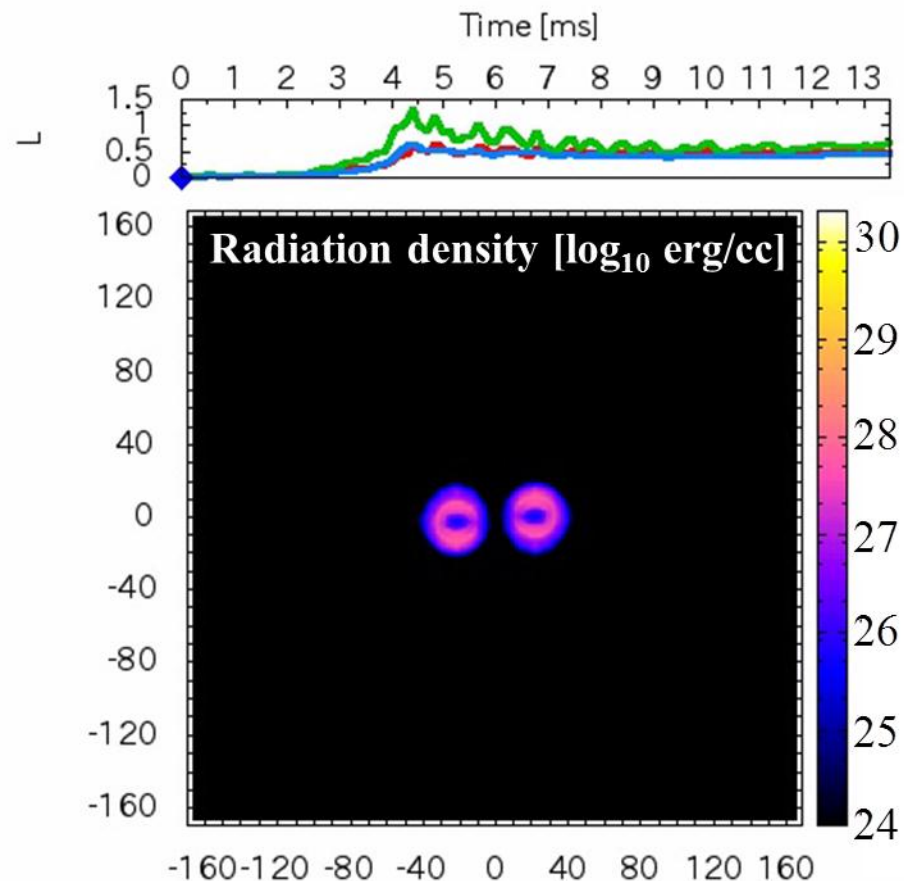
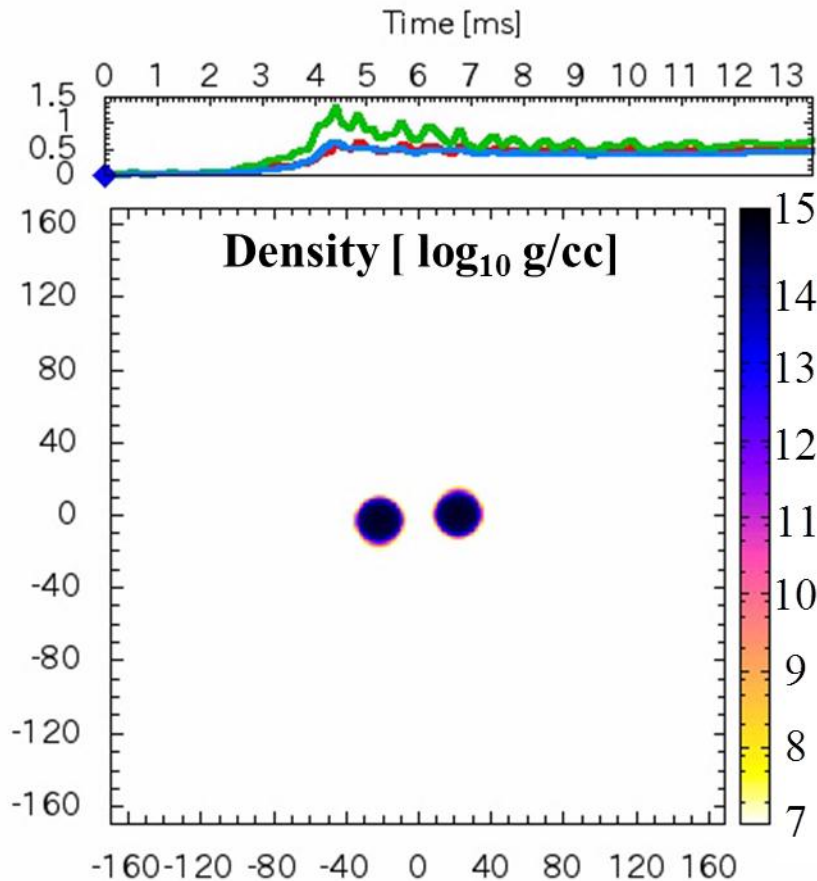
► **Neutrino transfer** based

► Solving Boltzmann equation
⇒ approximate solution



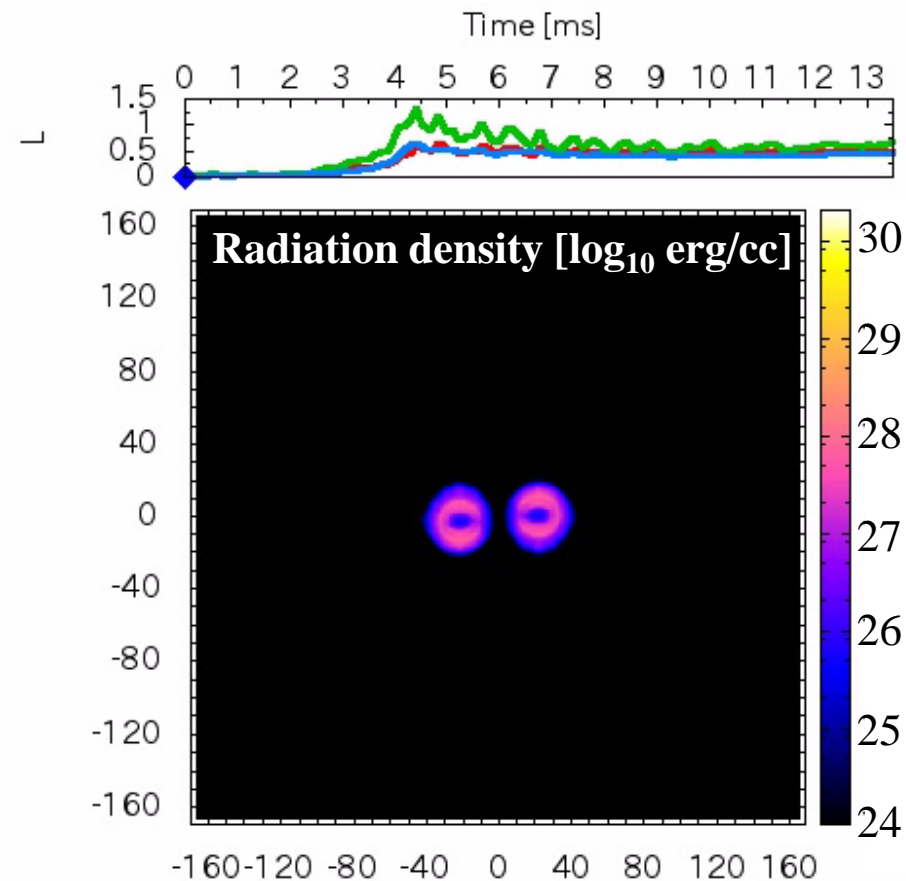
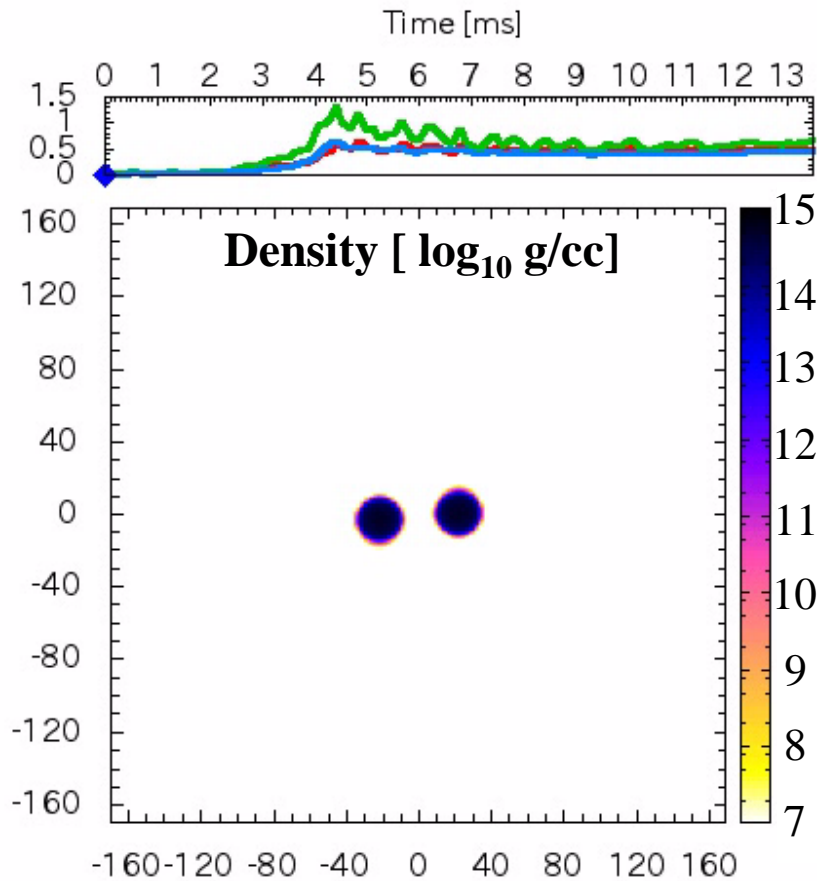
Neutrino transfer : last frontier in NR

- ▶ Solving Boltzmann equation (6+1 dims. !) is not feasible at current status
- ▶ Approximate solution by Moment scheme with a closure relation
 - ▶ Neutrino heating (absorption on proton/neutron) can be treated
 - ▶ Some approximate treatment is required for ν annihilation



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NR simulations with a physical modeling is now possible !

- ▶ Einstein's equations: Shibata-Nakamura (BSSN) formalism

- ▶ 4th order finite difference in space, 4th order Runge-Kutta time evolution
- ▶ Gauge conditions : 1+log slicing, dynamical shift

- ▶ GR Hydrodynamics *with neutrinos* (Sekiguchi 2010)

- ▶ **Nuclear-theory-based finite temperature EOS**

- ▶ **EOM of Neutrinos**

- ▶ **Lepton Conservations**

- ▶ **Weak Interactions**

- ▶ e^\pm captures, pair annihilation, plasmon decay, Bremsstrahlung

- ▶ **A detailed neutrino opacities**

- ▶ High-resolution-shock-capturing scheme

- ▶ **BH excision technique**

- ▶ **(Fixed) Mesh refinement technique**

$$\begin{aligned}\nabla_a T_b^a &= -Q_b^{(\text{leak})} \\ \nabla_a T_b^a (\nu, \text{stream}) &= Q_b^{(\text{leak})}\end{aligned}$$

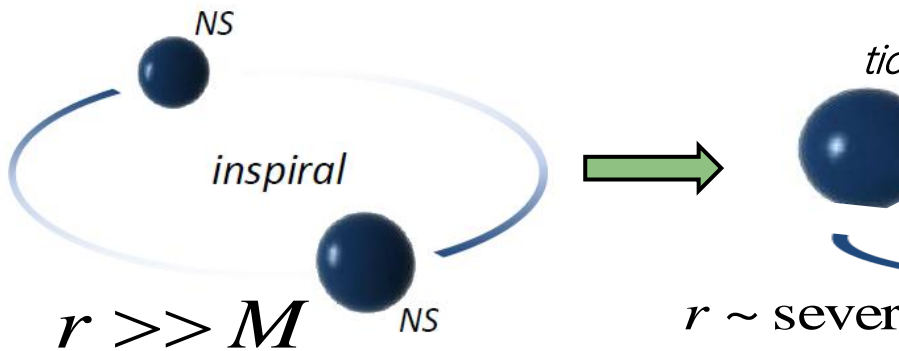
$$\begin{aligned}\nabla_a (\rho Y_e u^a) &= -\gamma_{e-\text{cap}} + \gamma_{e+\text{cap}} \\ \nabla_a (\rho Y_{\nu e} u^a) &= \gamma_{e-\text{cap}} + \gamma_{\text{pair}} + \gamma_{\text{plasmon}} + \gamma_{\text{Brems}} - \gamma_{\nu_e \text{leak}} \\ \nabla_a (\rho Y_{\bar{\nu} e} u^a) &= \gamma_{e+\text{cap}} + \gamma_{\text{pair}} + \gamma_{\text{plasmon}} + \gamma_{\text{Brems}} - \gamma_{\bar{\nu}_e \text{leak}} \\ \nabla_a (\rho Y_{\nu\mu, \tau} u^a) &= \gamma_{\text{pair}} + \gamma_{\text{plasmon}} + \gamma_{\text{Brems}} - \gamma_{\nu\mu, \tau \text{leak}}\end{aligned}$$

Compact object binary mergers

- ▶ NS–NS and BH–NS merger



Evolution of NS-NS



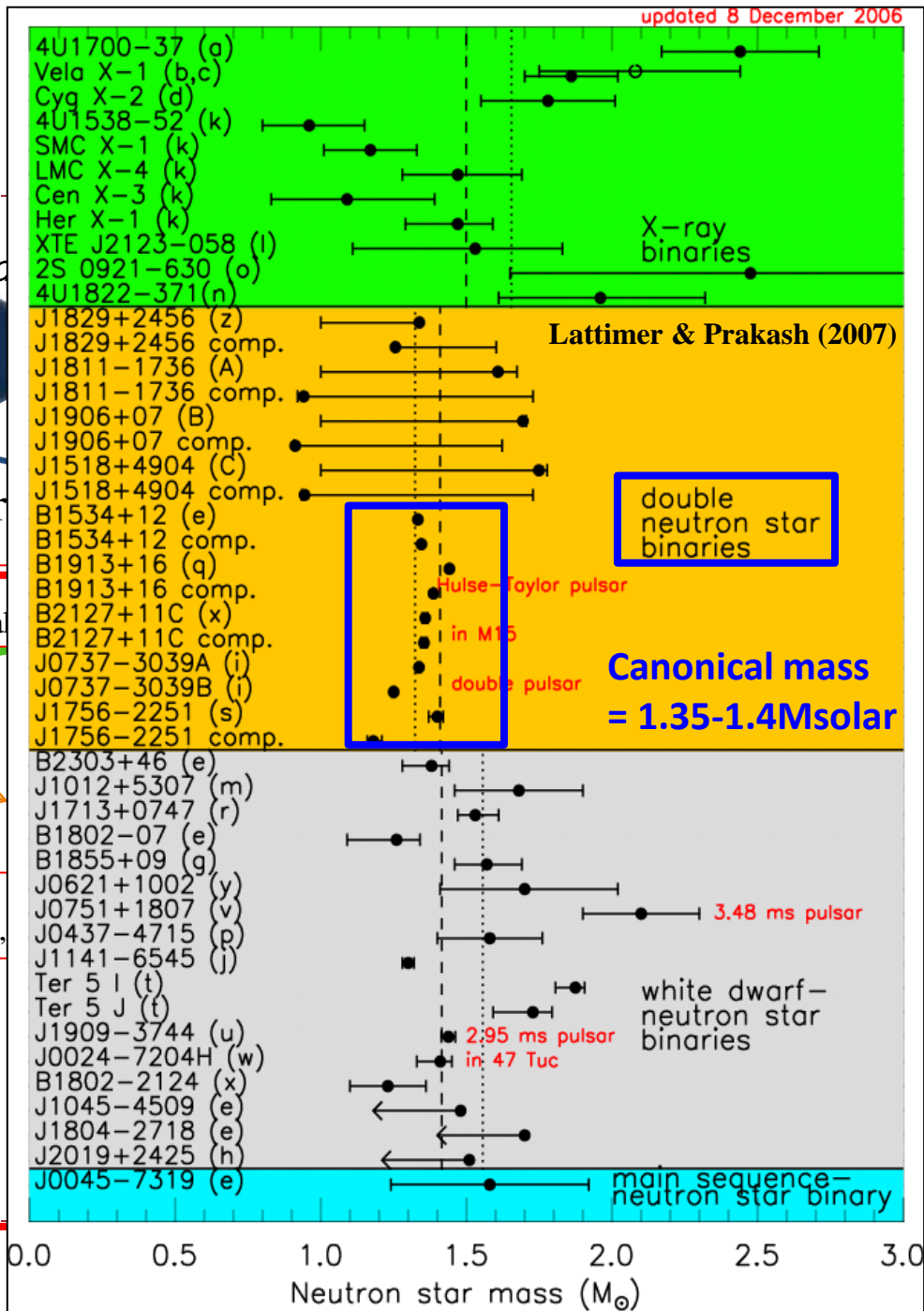
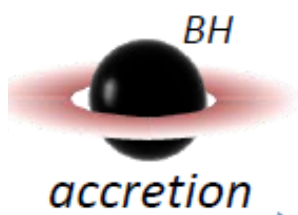
More Likely



$$M_{\text{total}} < M_{\text{NS,max}}$$

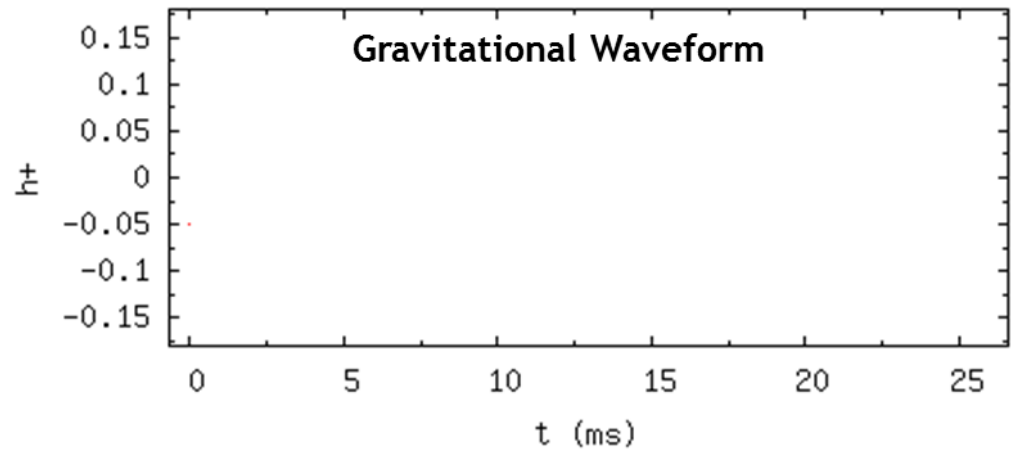
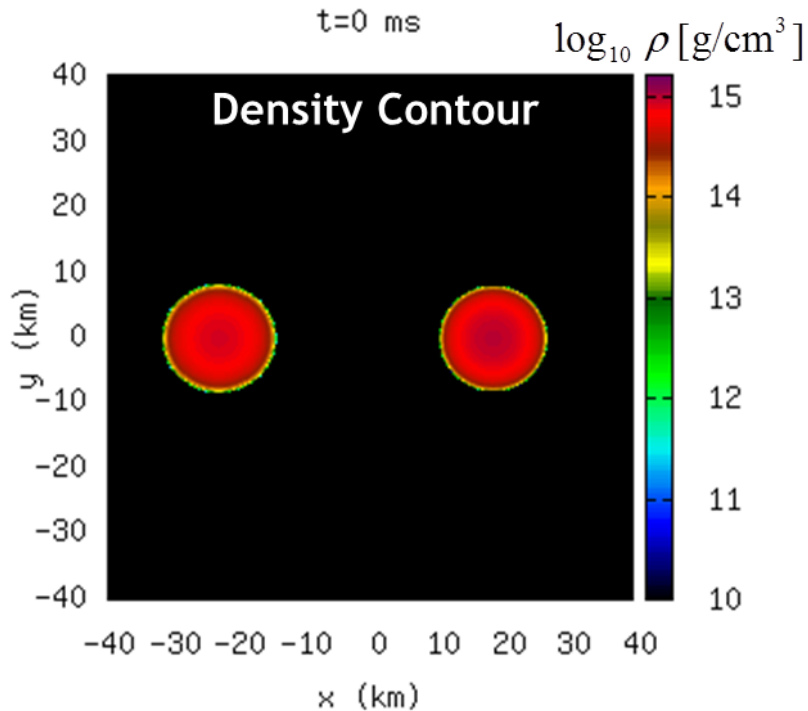


$$M_{\text{total}} > M_{\text{NS,max}}$$



GW from NS-NS (**long lived HMNS**)

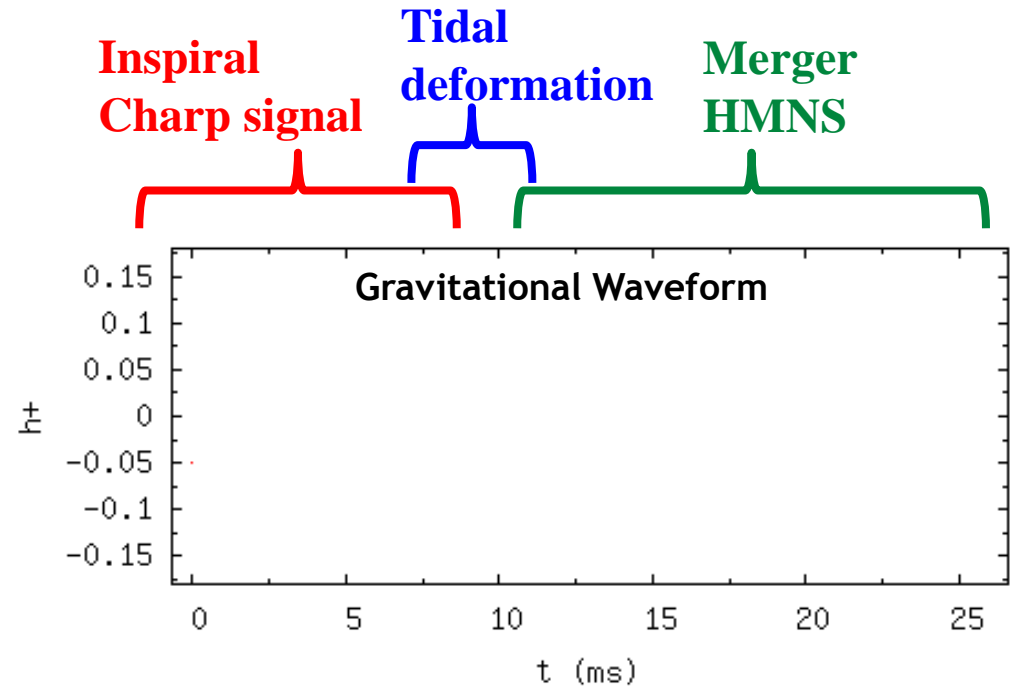
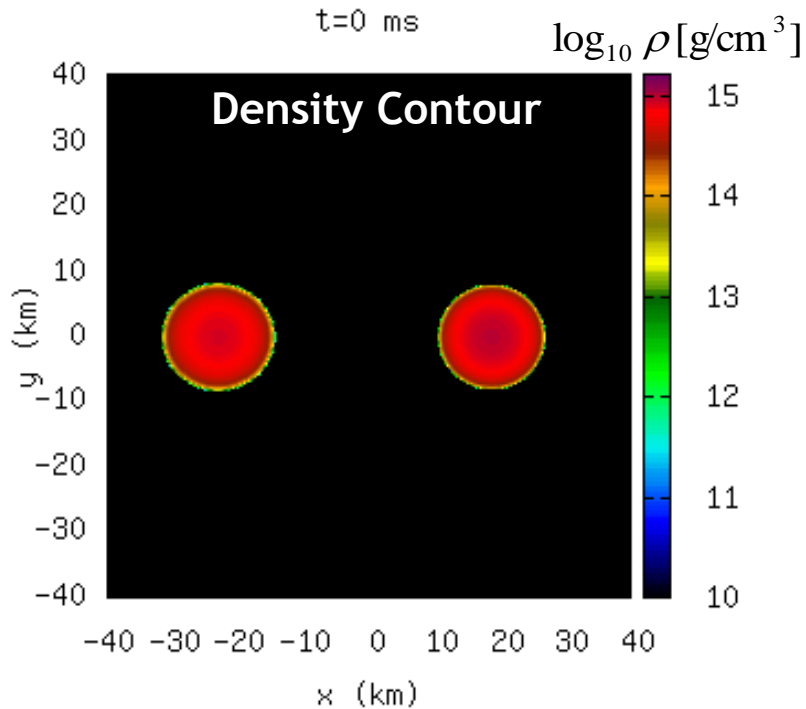
NS(1.2Msolar)-NS(1.5Msolar) binary (APR EOS)



Animation by Hotokezaka

GW from NS-NS (**long lived HMNS**)

NS(1.2Msolar)-NS(1.5Msolar) binary (APR EOS)

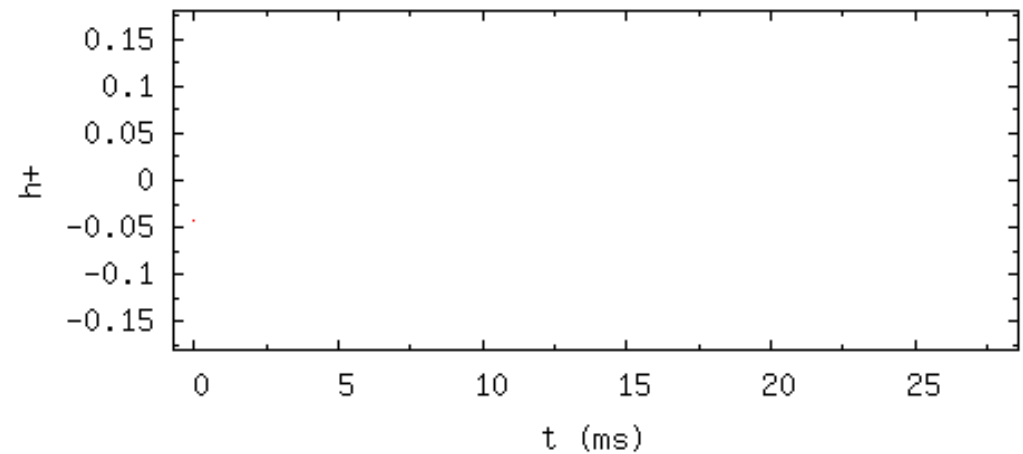
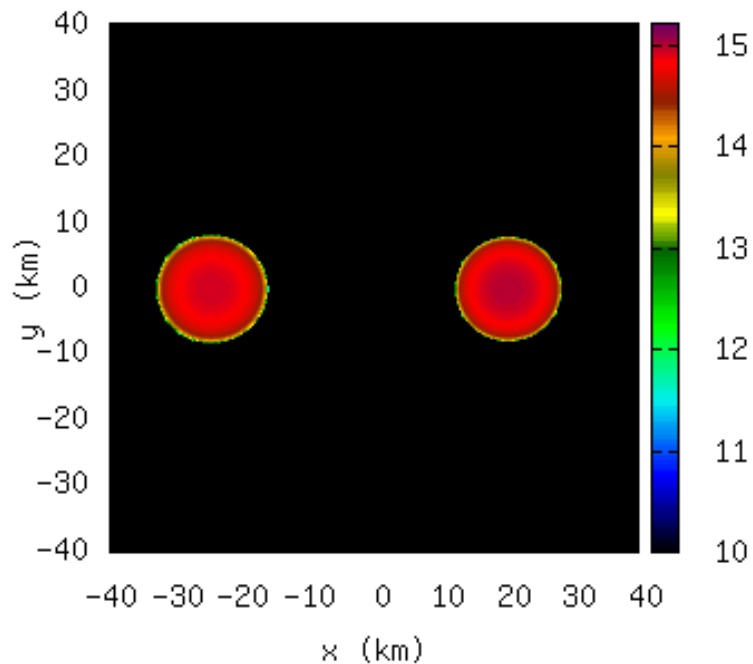


Animation by Hotokezaka

GW from NS-NS (**Prompt BH formation**)

NS(1.3Msolar)-NS(1.6Msolar) binary (APR EOS)

t=0 ms



Animation by Hotokezaka

Exploring Dense matter physics by GW

Inspiral phase

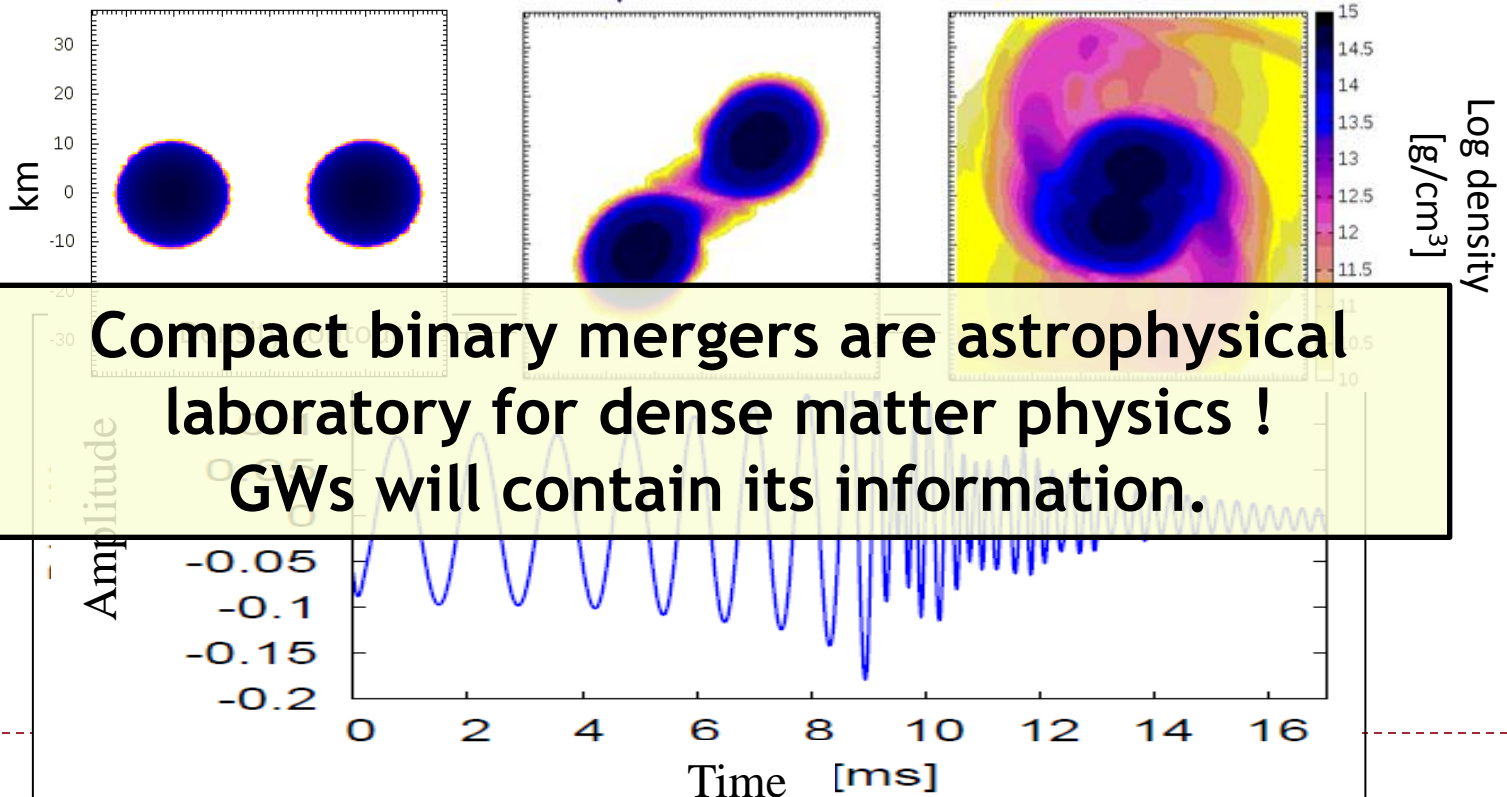
- Point particle approximation
- Information of orbits, neutron star mass etc.

Tidal deformation

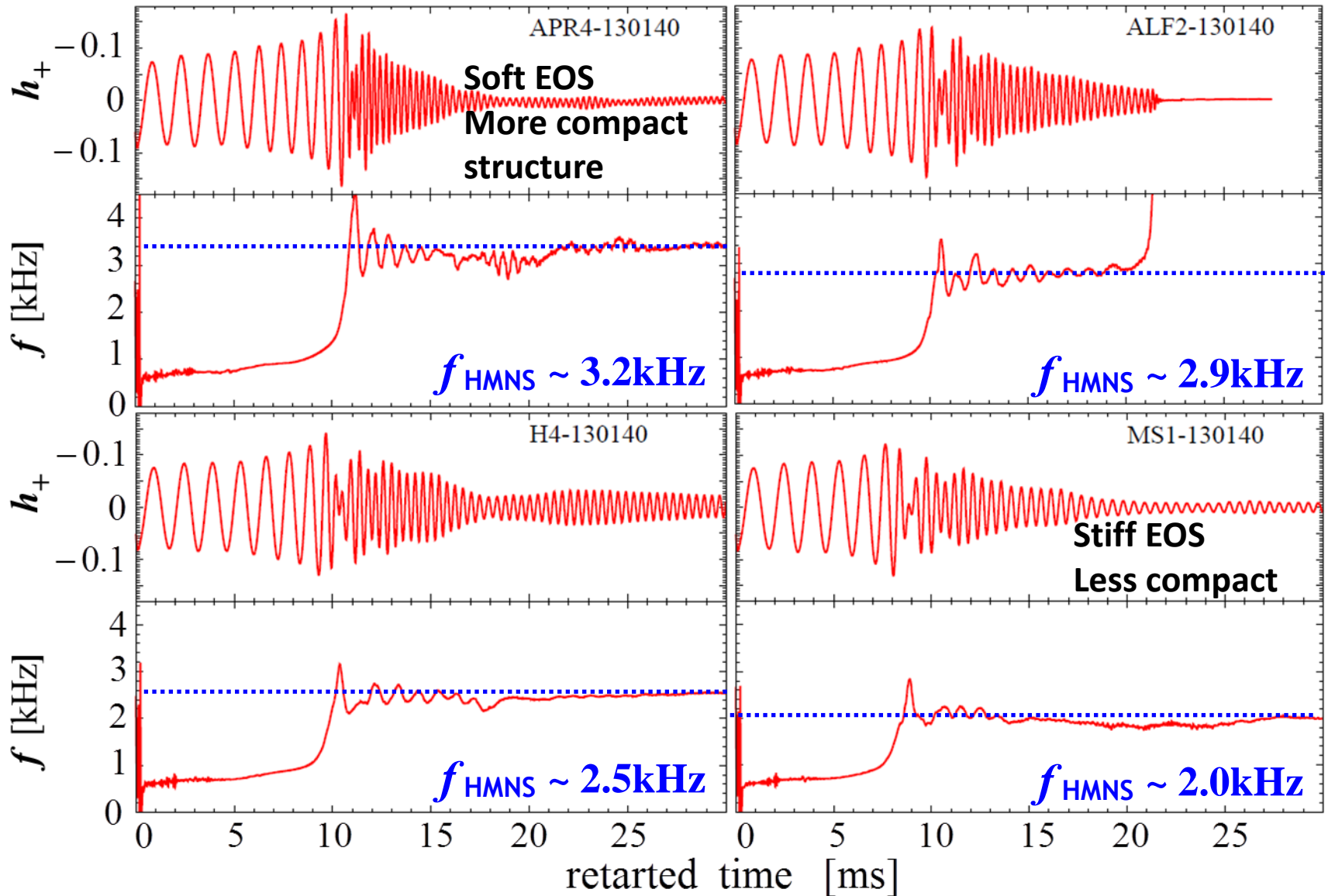
- Finite size effect
- Deviation from charp \Rightarrow NS radius

Merger and oscillation of HMNS

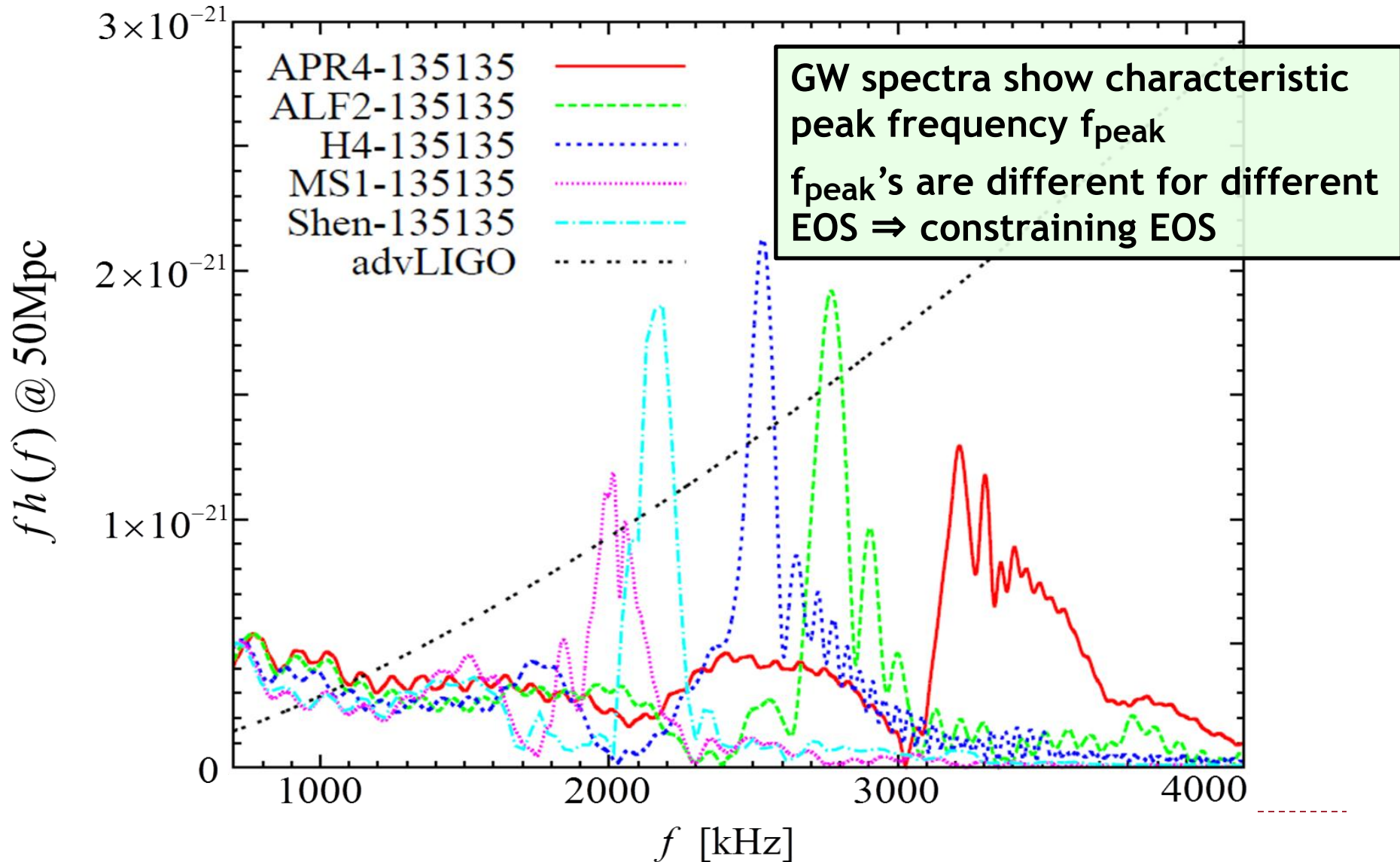
- BH or NS \Rightarrow maximum mass
- GW from rotating HMNS \Rightarrow NS radius (and EOS)



GWs from HMNS (1.3-1.4 Msolar Merger)

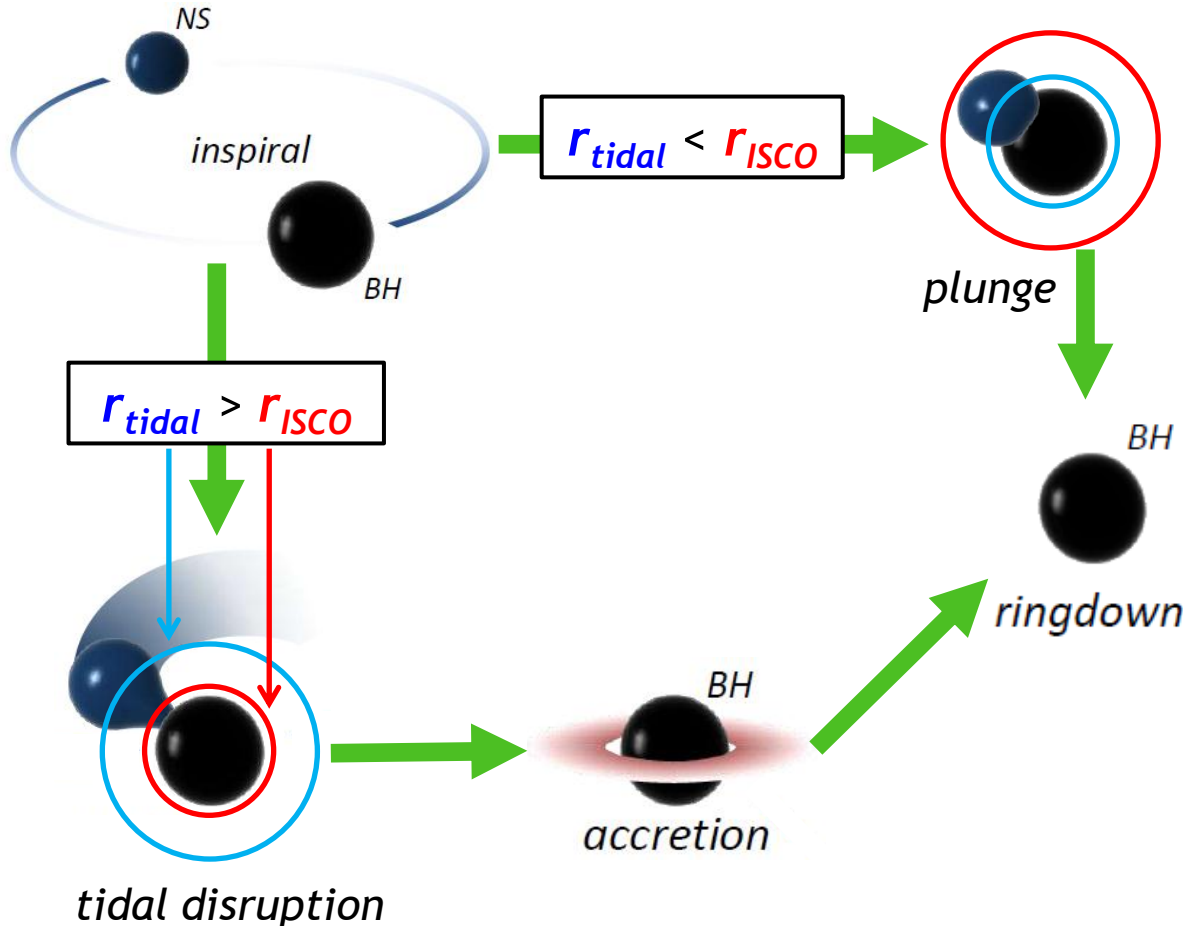


GW spectra (1.35-1.35 Msolar)



Evolution of BH-NS

Shibata & Taniguchi (2008)
Kyutoku et al. (2010), (2011)



$$r_{\text{ISCO}} = r_{\text{ISCO}}(M_{\text{BH}}, a_{\text{BH}})$$

BH spin dependence
larger $a_{\text{BH}} \Rightarrow$ smaller r_{ISCO}

tidal force = self gravity of NS

$$\frac{r_{\text{tidal}}}{M_{\text{BH}}} = \left(\frac{M_{\text{NS}}}{M_{\text{BH}}}\right)^{2/3} \left[\frac{M_{\text{NS}}}{R_{\text{NS}}}\right]^{-1}$$

Compactness of NS
 \Rightarrow **NS structure (EOS)**

GW from BH-NS merger

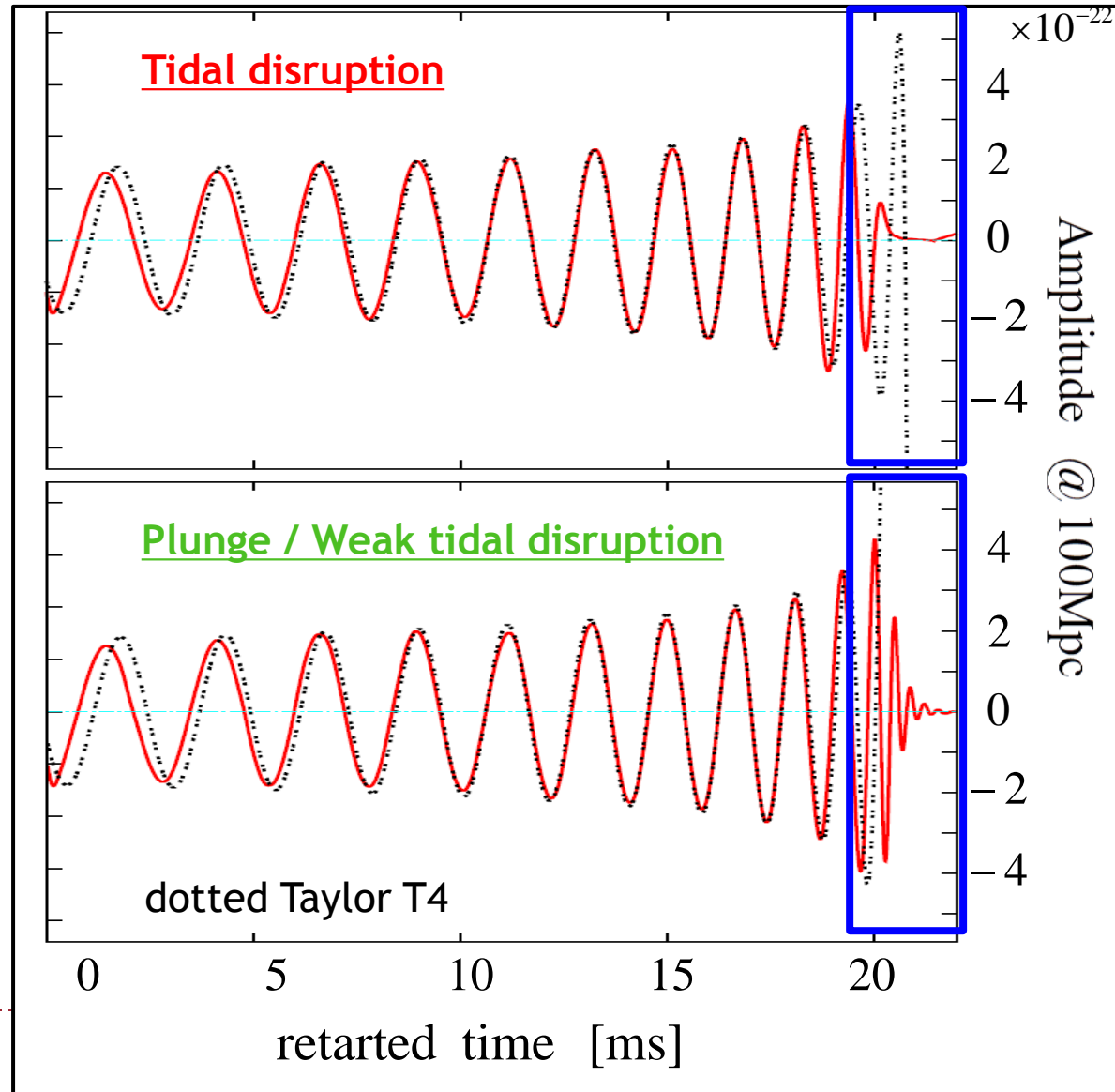
Kyutoku et al. (2010), (2011)

▶ Tidal disruption

- ▶ GW amplitude shutdown suddenly
- ▶ Widespread tidal arm and accretion disk form

▶ Plunge/Weak disruption

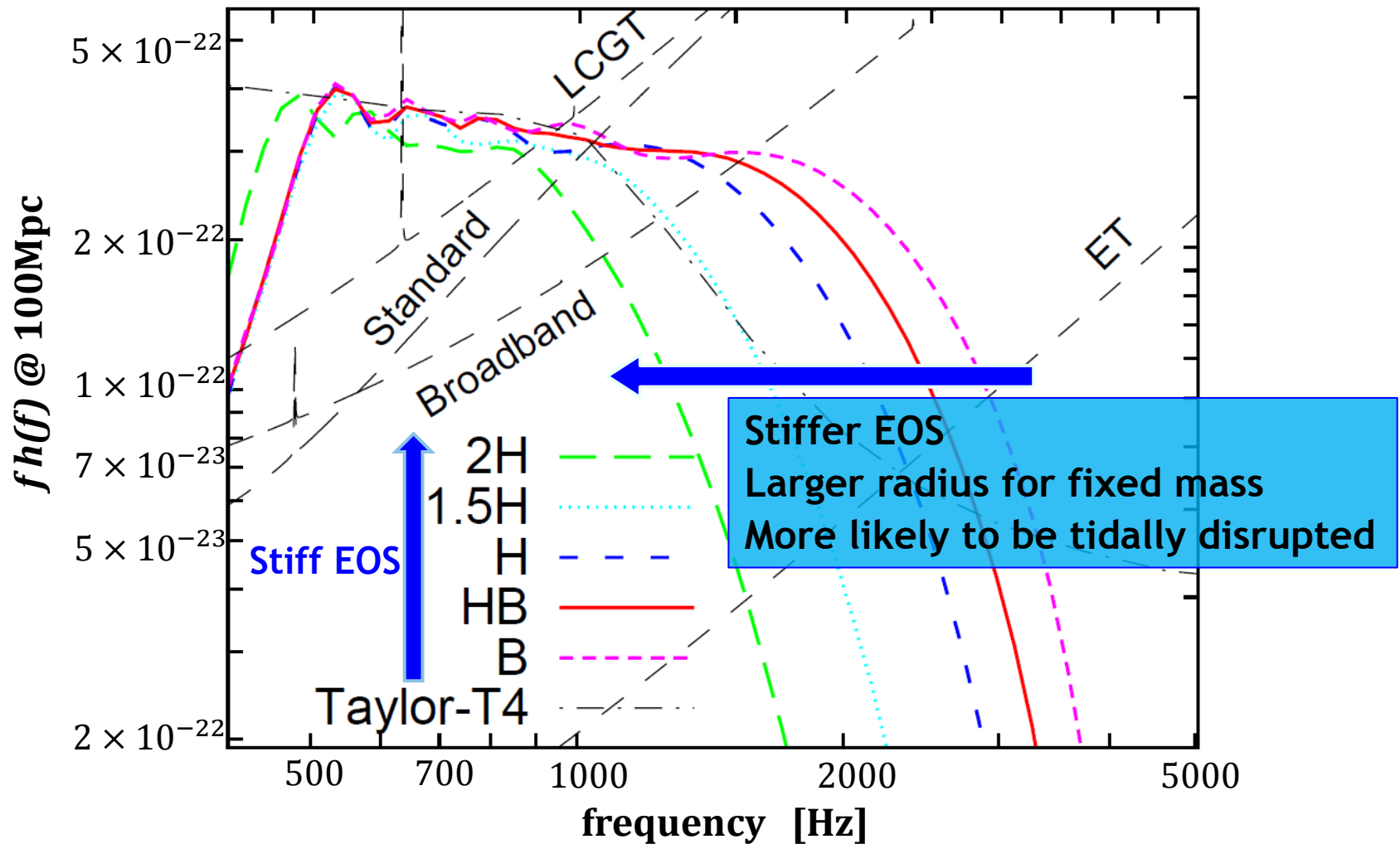
- ▶ inspiral orbit sustains in more inner regions
- ▶ NS hits BH and quasi-normal mode is excited



What GW spectra tell us

Kyutoku et al. (2010), (2011)

$$(M_{NS}/M_{BH}=0.5, M_{NS}=1.35, a_{BH}=0.75)$$



Kyutoku et al. (2011)

NR simulations with microphysics

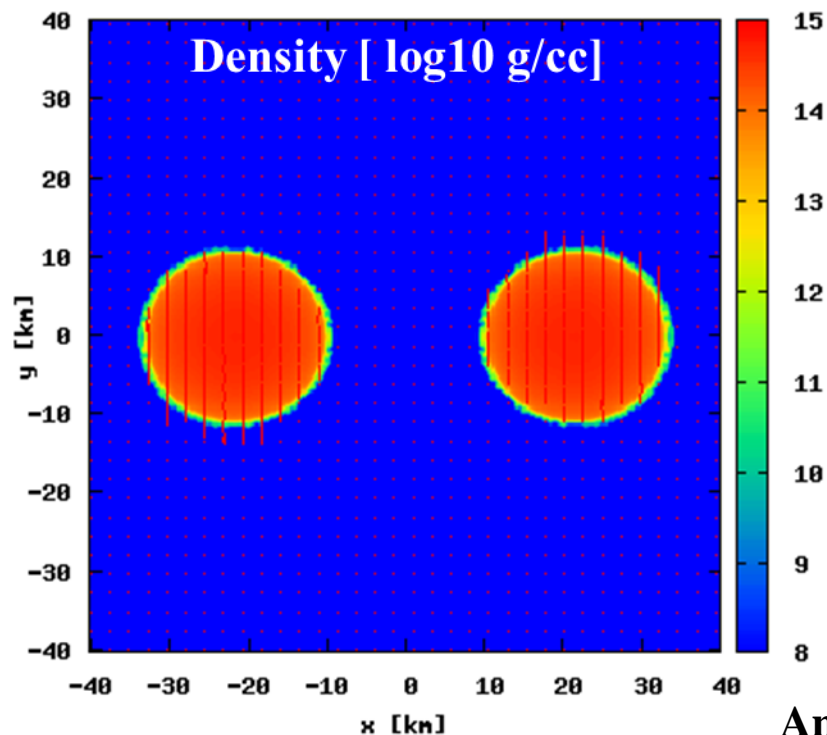
- ▶ Towards central engine of GRB



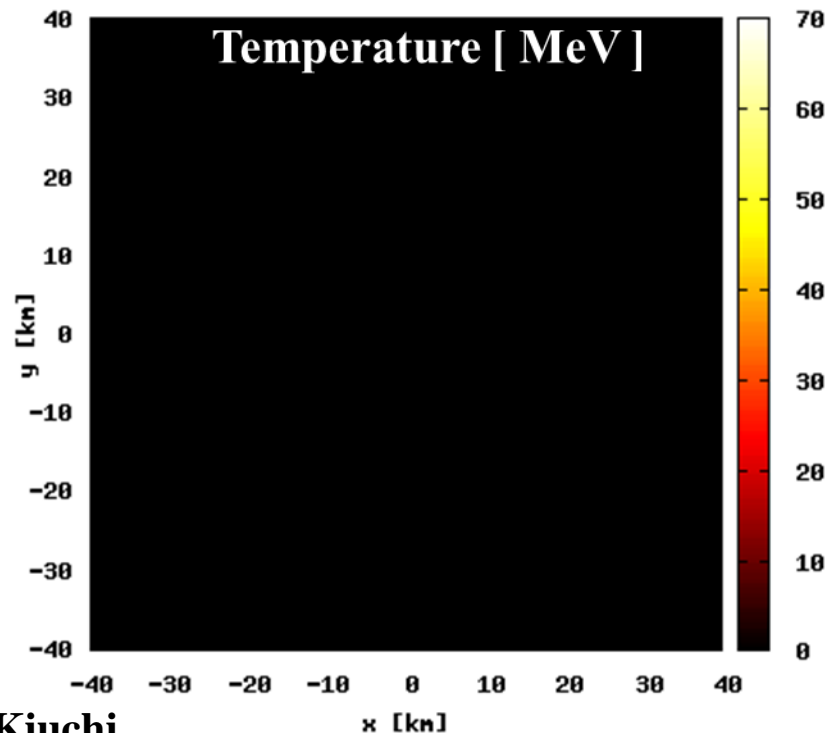
NS-NS merger (1.6-1.6Msolar)

- ▶ Hyper massive NS (HMNS) is first formed
 - ▶ Temperature increases significantly by compression and shock heating
 - ▶ Shocks occur in spiral arms
- ▶ HMNS eventually collapse to a BH due to emission of GW and neutrinos
 - ▶ accretion disk (with $M_{\text{disk}} < 0.1 M_{\text{solar}}$) forms around the BH

$t = 0 \text{ ns}$



$t = 0.02101 \text{ ns}$

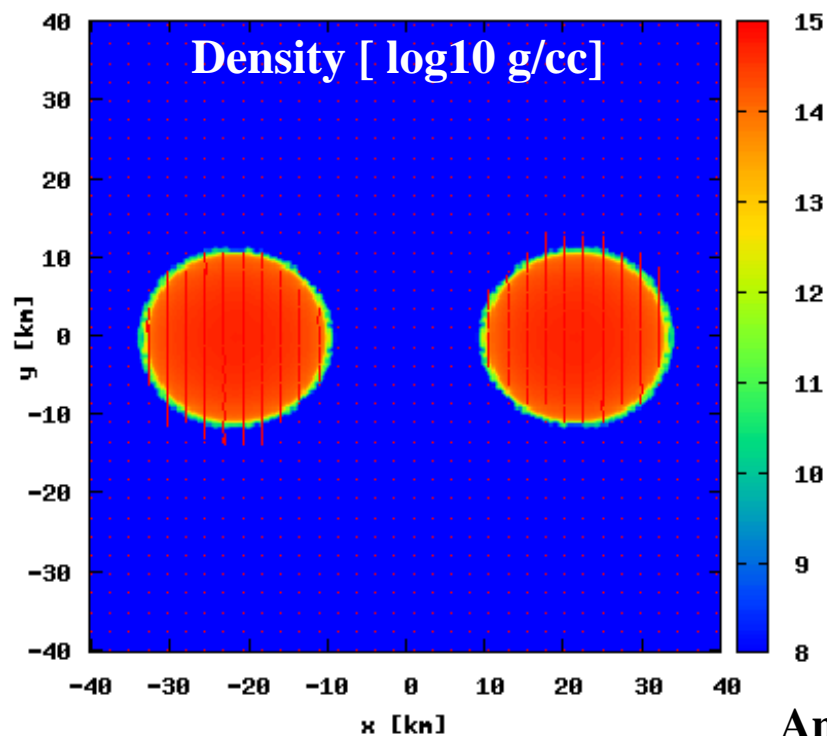


Animation by Kiuchi

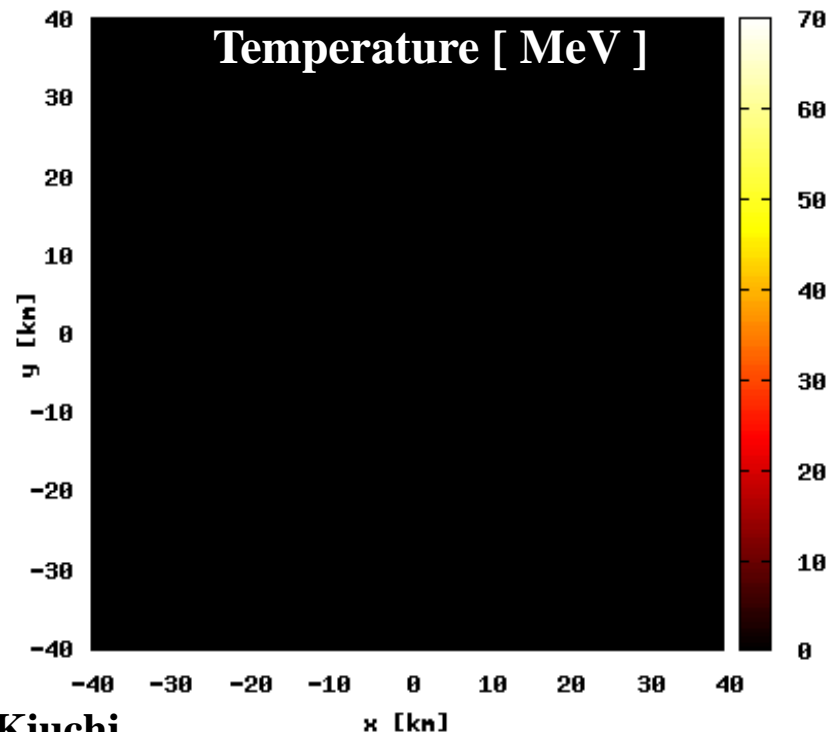
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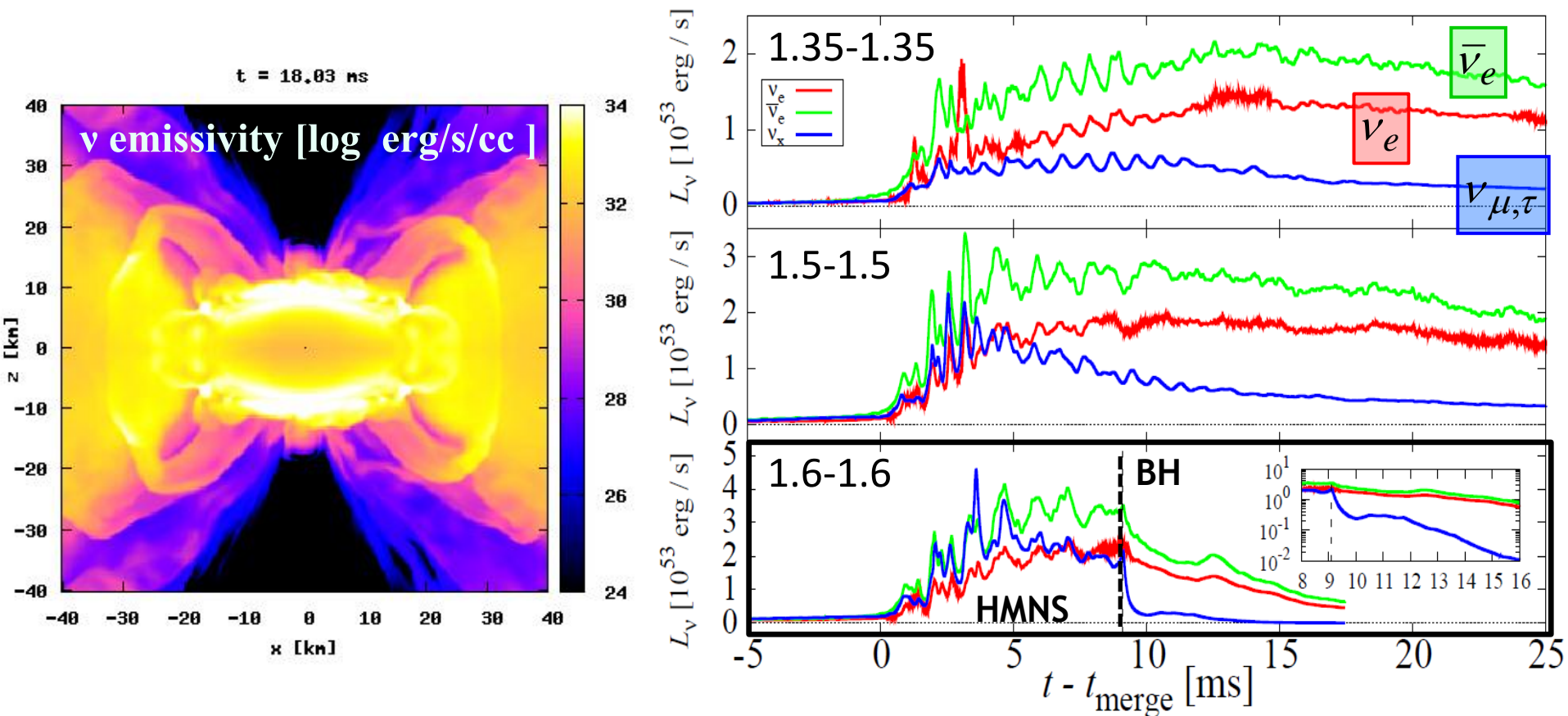
t = 0.02101 ns



Animation by Kiuchi

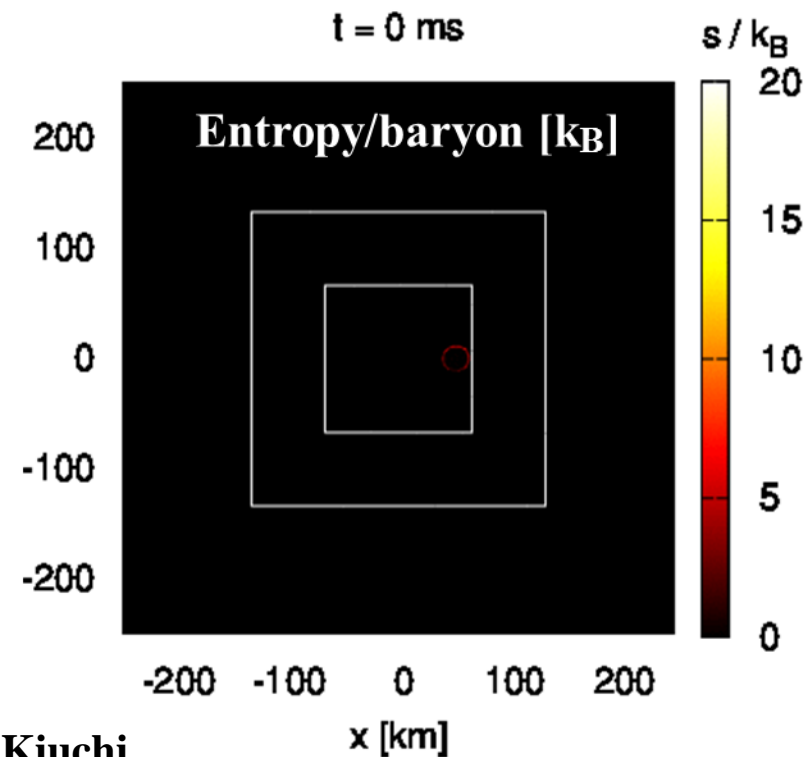
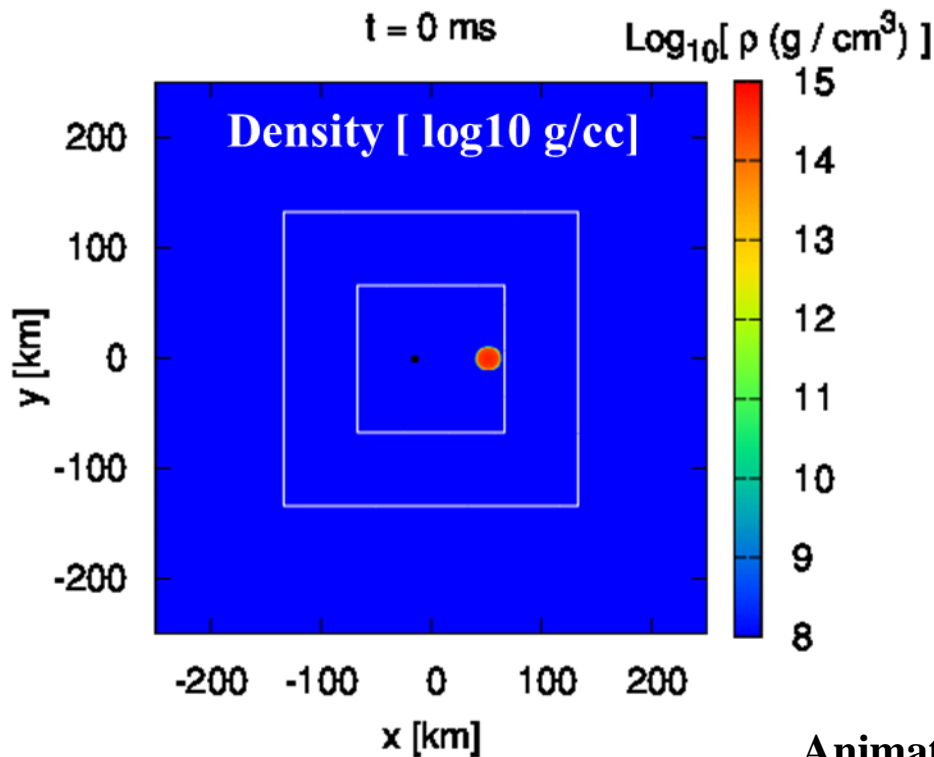
Neutrino emission (NS-NS)

- ▶ HMNS emits copious neutrinos : $L_\nu \sim 3 \times 10^{53}$ erg/s ($E_\nu = 20\text{-}30$ MeV)
 - ▶ Events within 5 (1) Mpc can be detected by Hyper Kamiokande (SK)
- ▶ Large neutrino luminosity of $\sim 10^{53}$ erg/s even after the BH formation



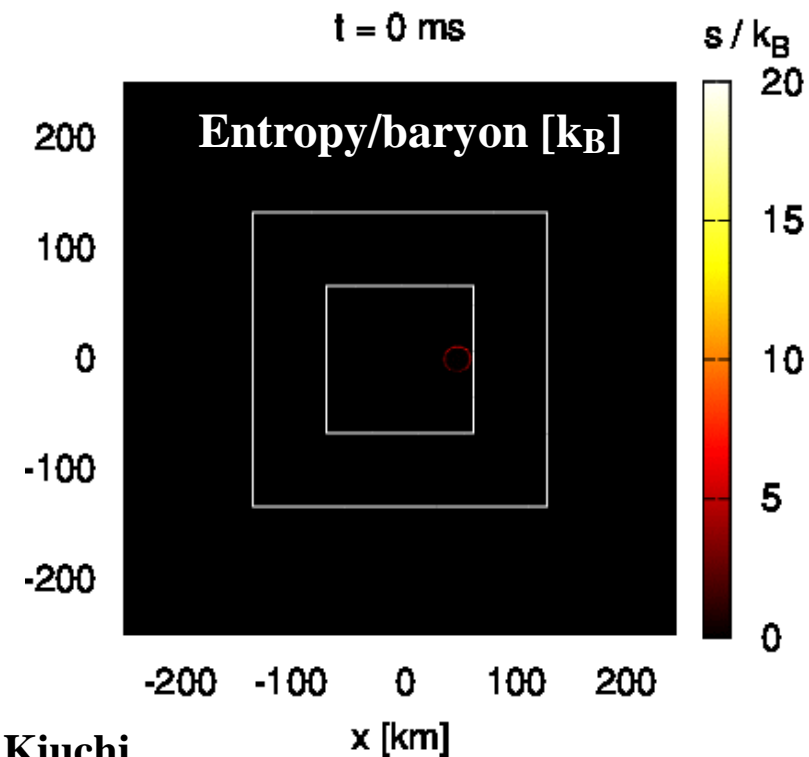
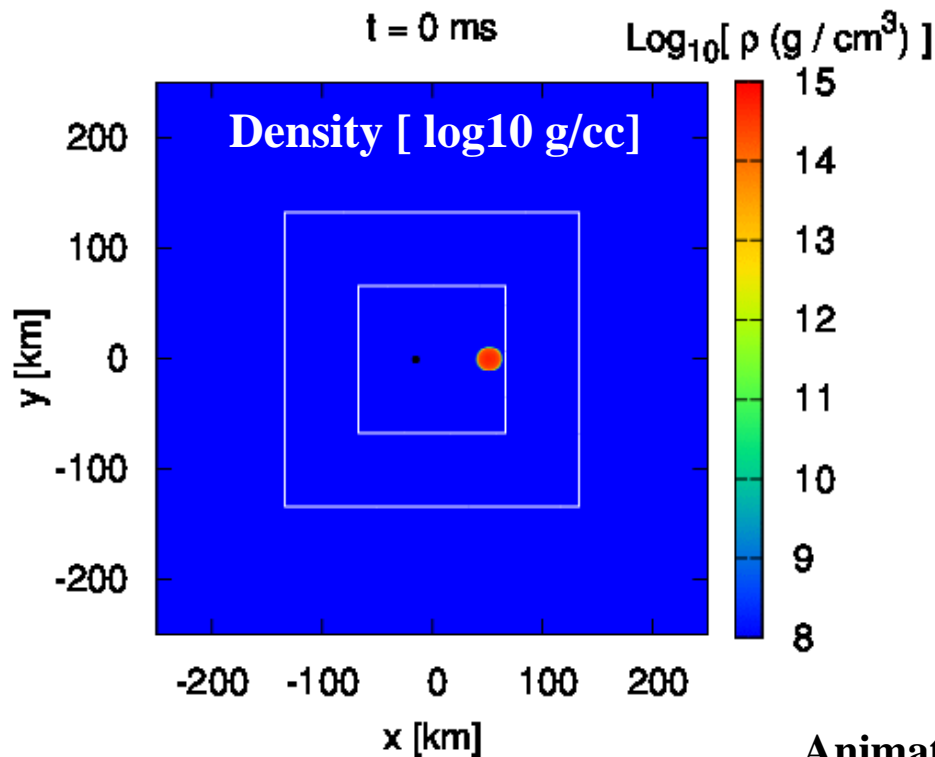
BH-NS merger (4 - 1.35 Msolar, $a_{\text{BH}} = 0.5$)

- ▶ NS is tidally disrupted and single spiral arm is formed
- ▶ The spiral arm interacts with itself and shock wave occur there
- ▶ A massive ($O(0.1\text{Msolar})$) and hot accretion disk eventually forms around the BH



BH-NS merger (4 - 1.35 Msolar, $a_{\text{BH}} = 0.5$)

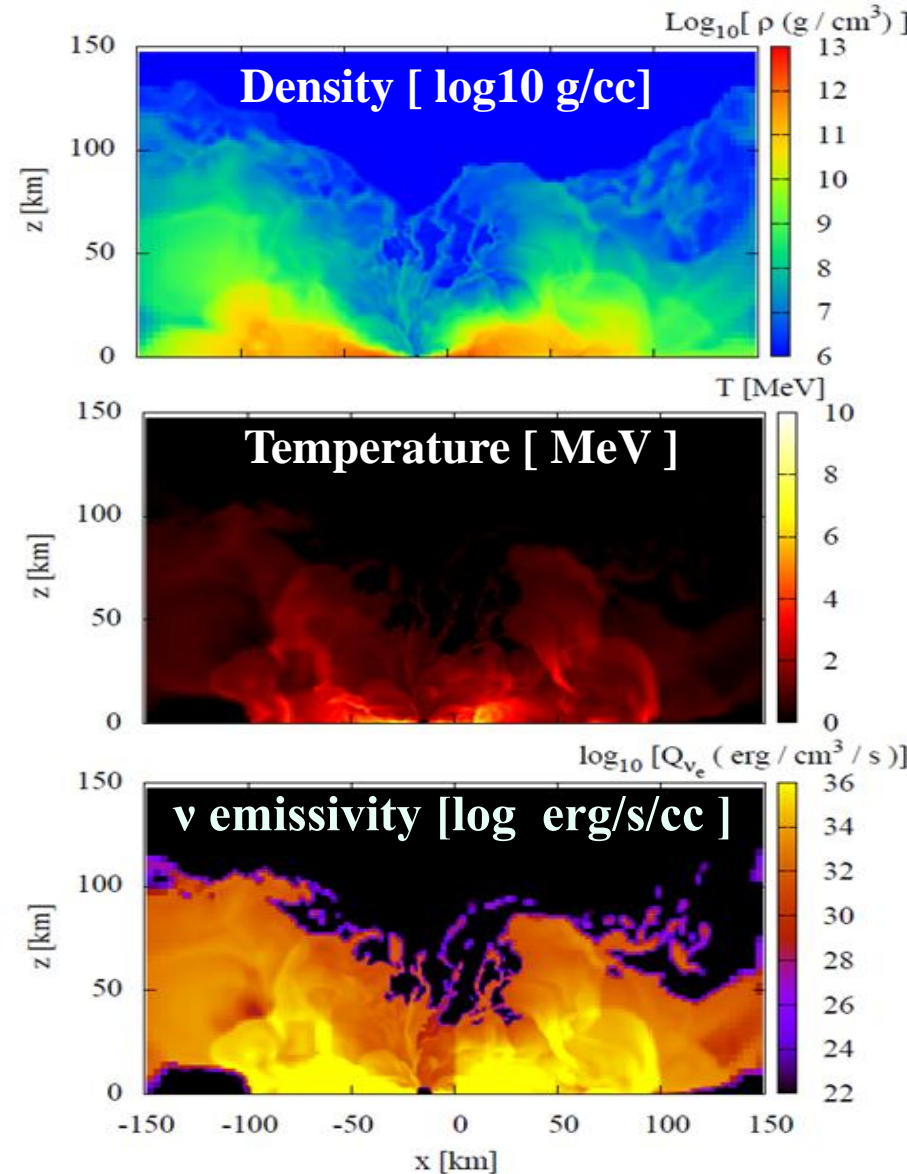
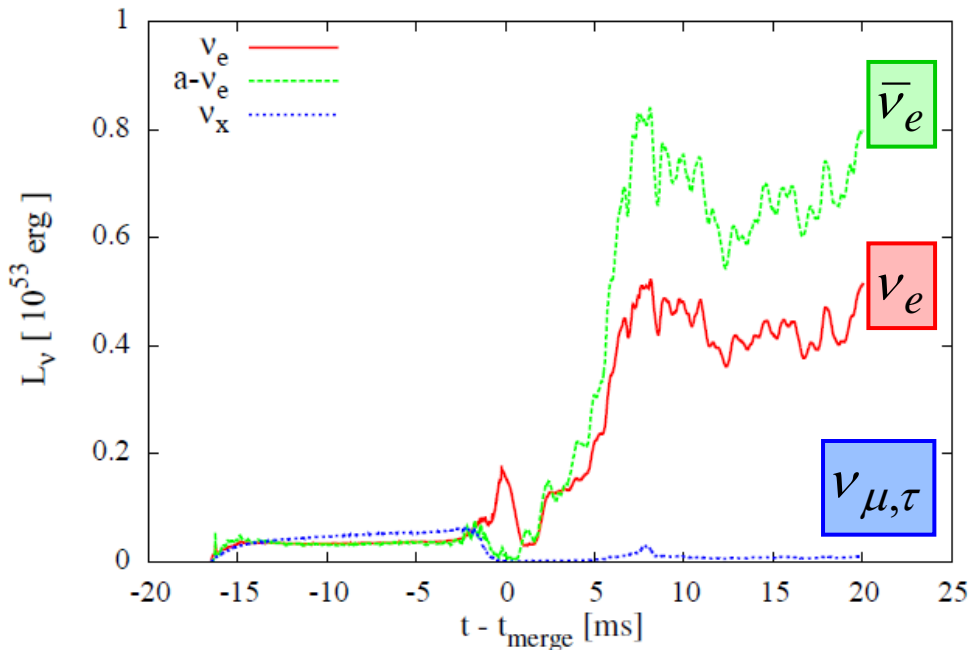
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- ▶ A massive ($O(0.1\text{Msolar})$) and hot accretion disk eventually forms around the BH



Animation by Kiuchi

Neutrino emission (BH-NS)

- ▶ Copious neutrinos ($5-8 \times 10^{52}$ erg/s) are emitted from the hot disk
- ▶ L_ν is smaller than NS-NS merger case
 - ▶ Shock waves are weaker
 - ▶ More dense disk : longer diffusion time
- ▶ Low density region above BH
 - ▶ A potential site for ν -pair annihilation

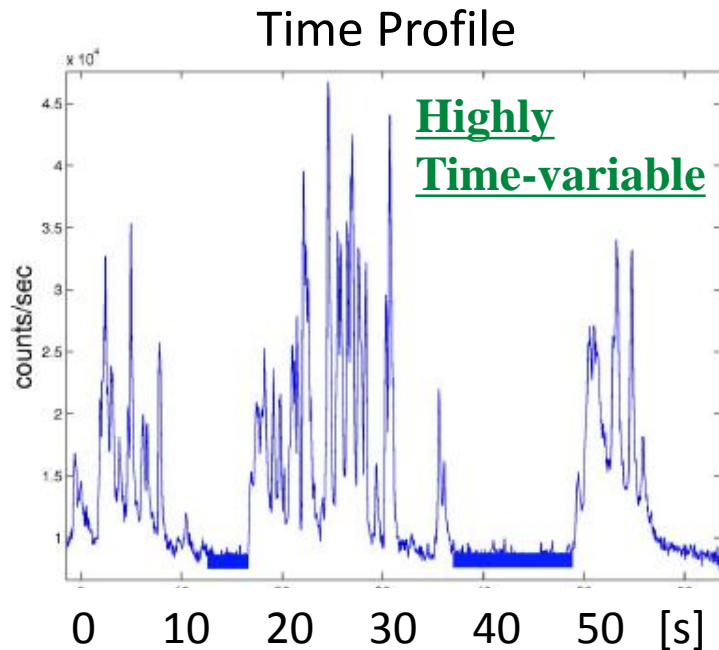


BH formation in stellar core collapse



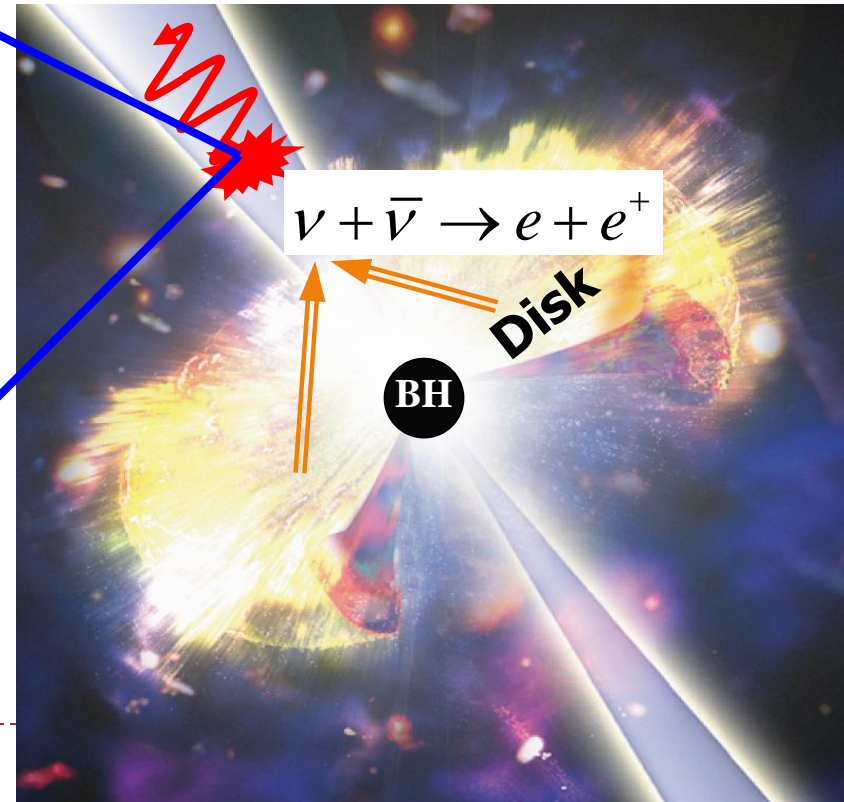
BH formation and Long GRBs

- ▶ Collapse of massive stellar core to BH + Disk
 - ▶ Promising theoretical candidate of central engine of **Long Gamma-ray Bursts (LGRBs)**

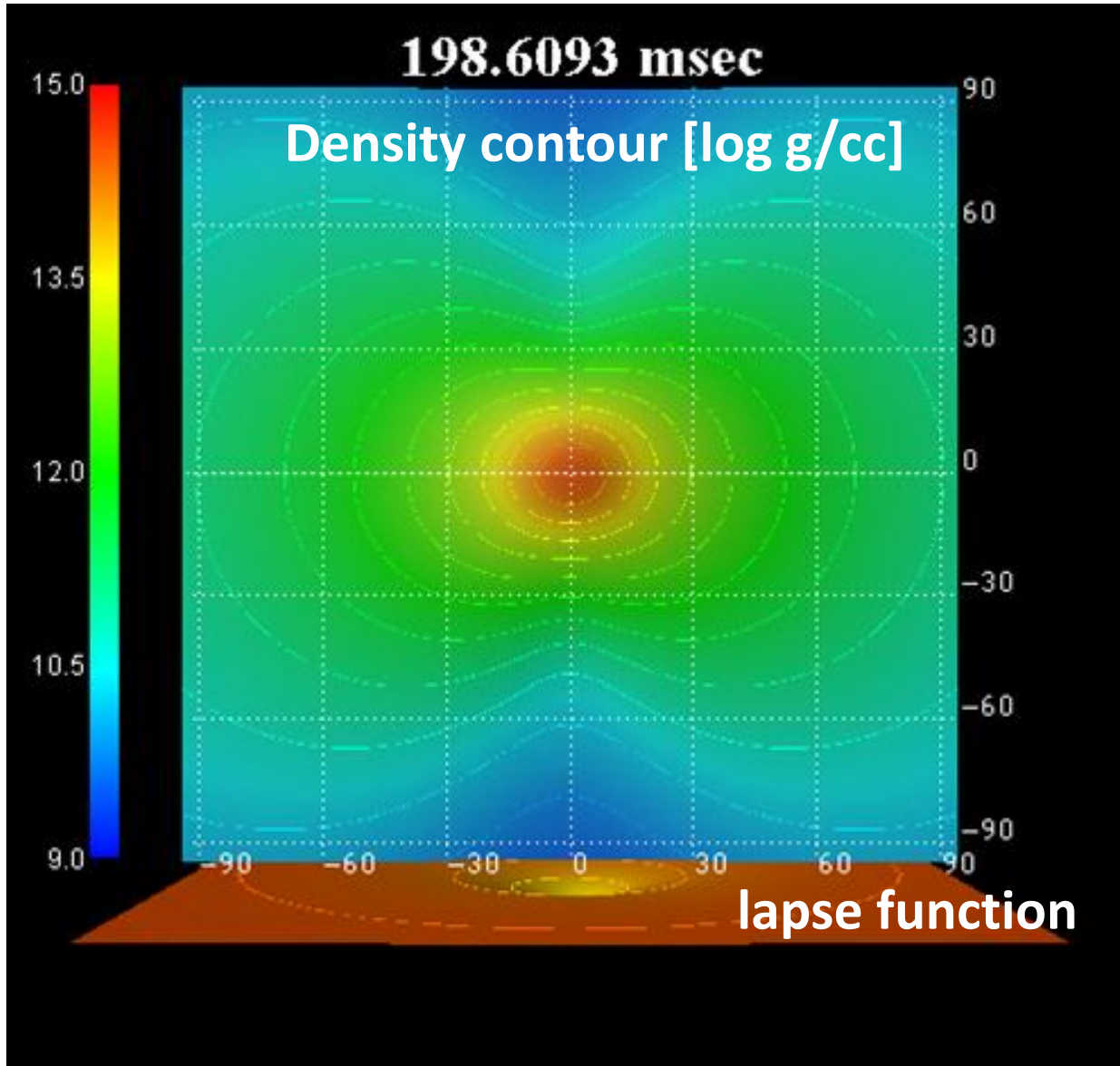


$$10^{50} \text{erg/s} < L_{\gamma} < 10^{52} \text{erg/s}$$

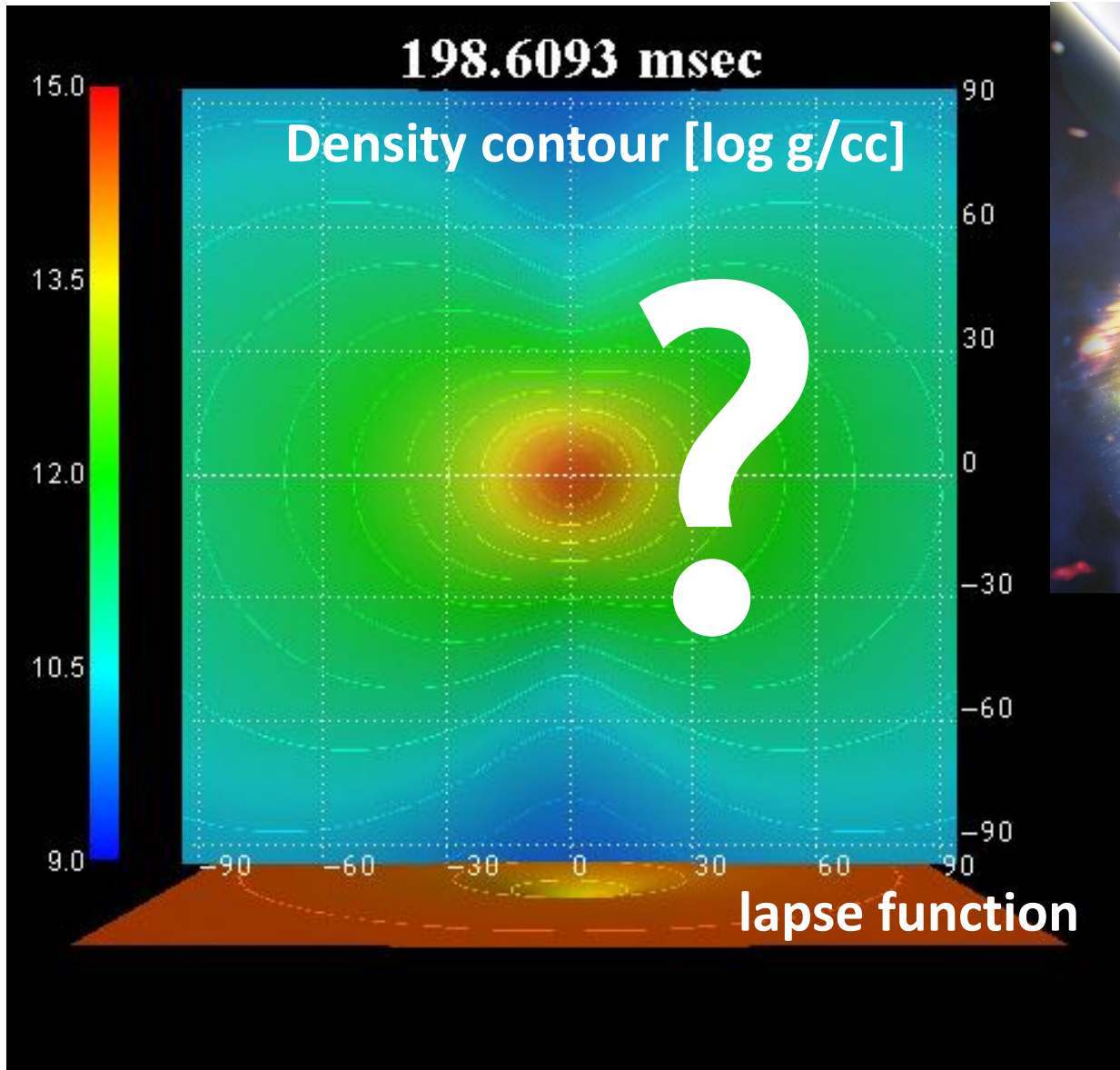
Most violent explosion in the universe



BH formation in 2005

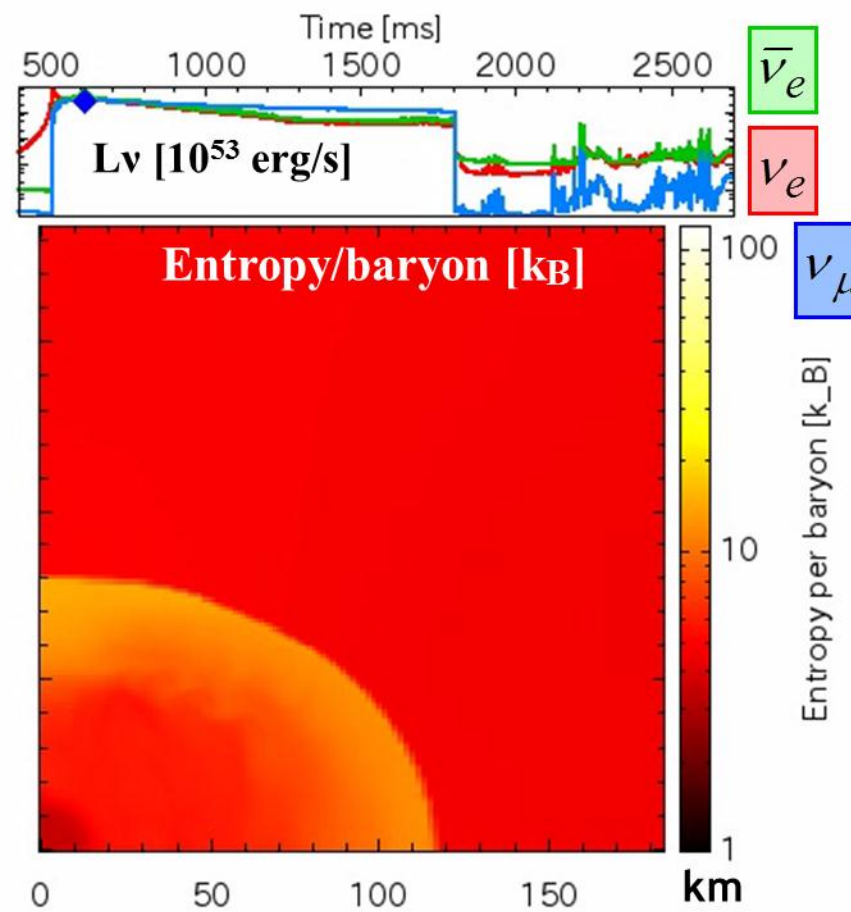
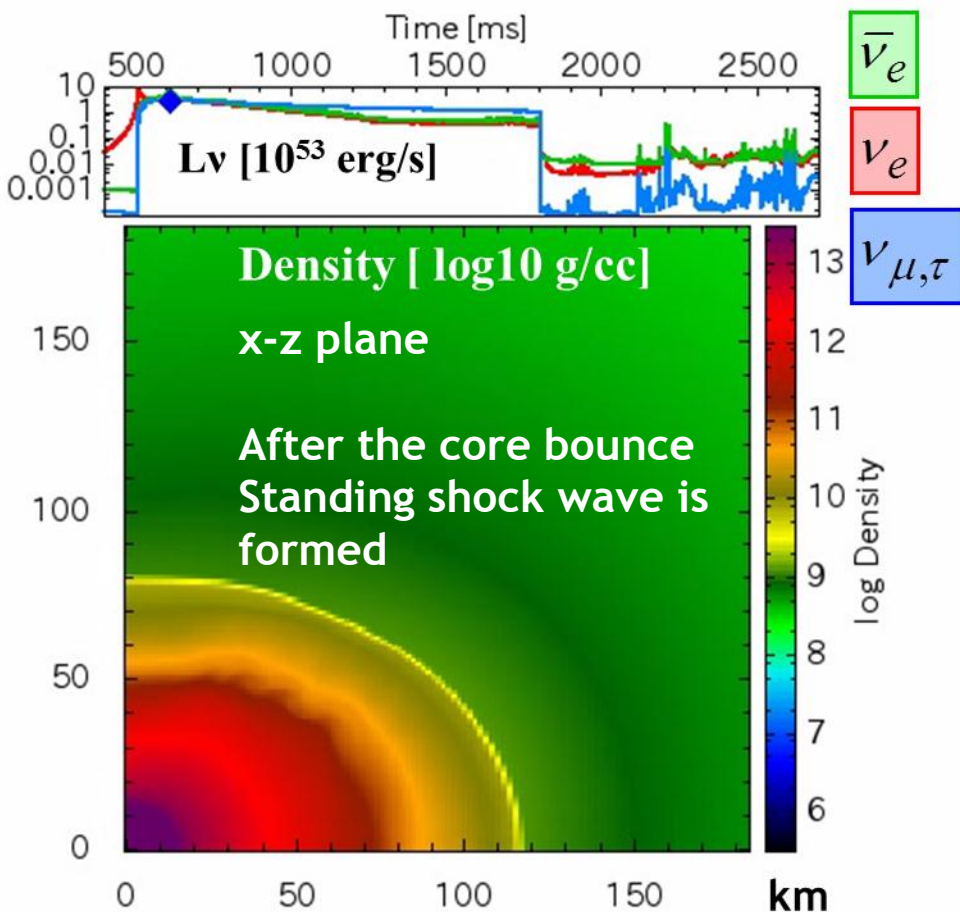


BH formation in 2005



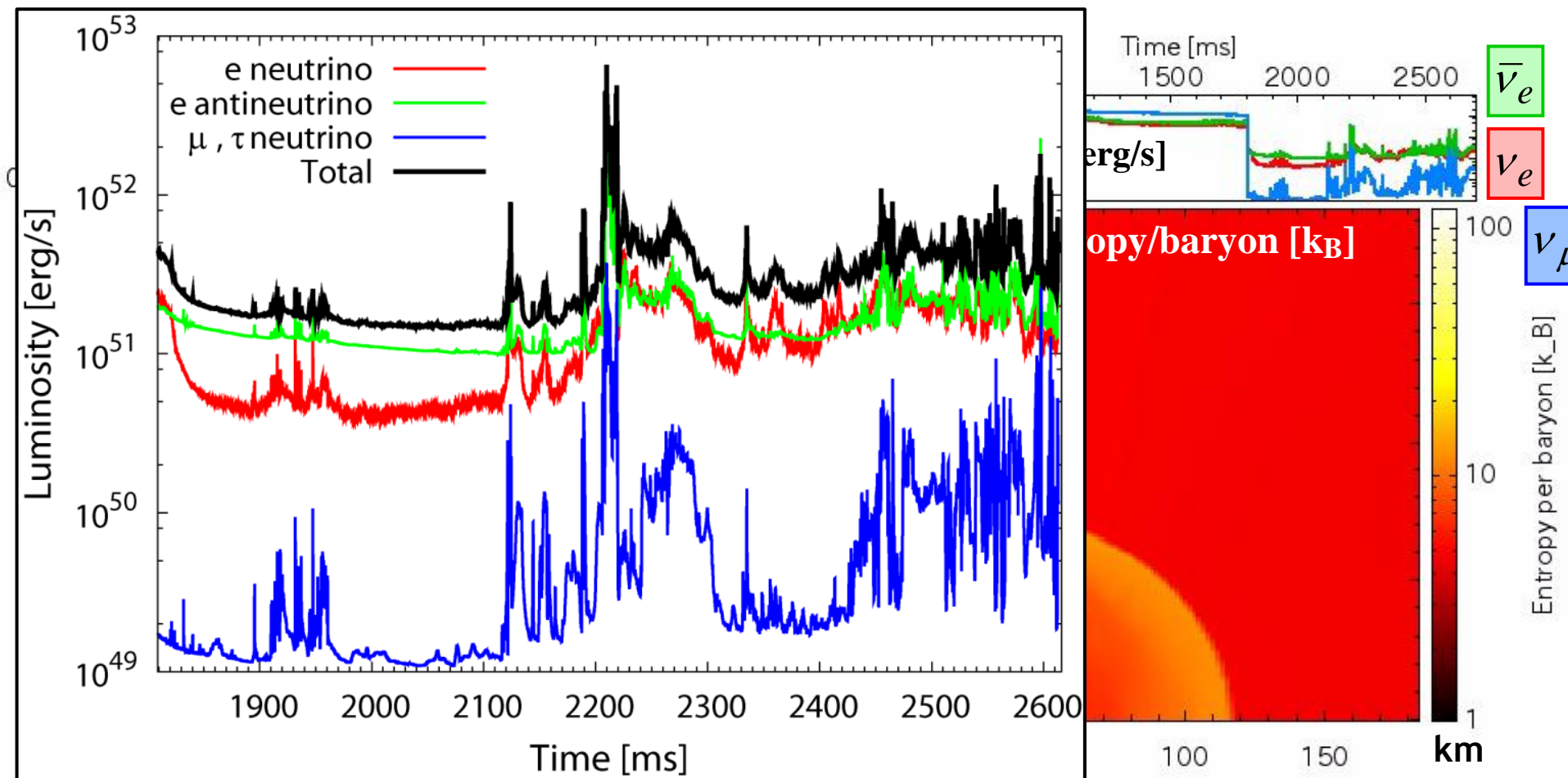
BH+Disk formation in stellar core collapse

- ▶ 100Msolar model by Umeda & Nomoto (2008) + rotation
- ▶ Torus-structured shock : accumulation of matter to the proto-NS
- ▶ Time varying, large ($\sim 10^{52}$ erg/s) neutrino luminosity after BH formation

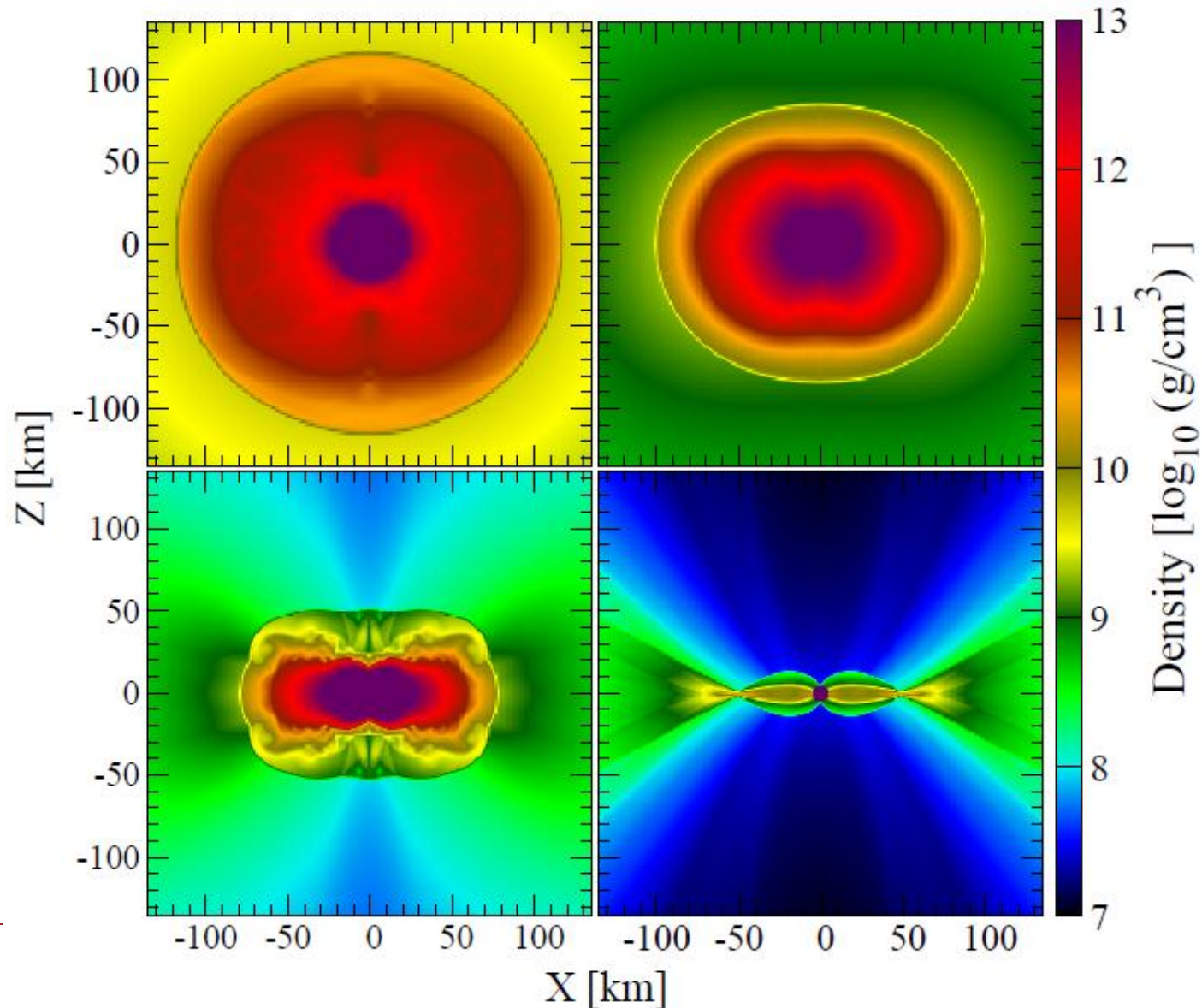


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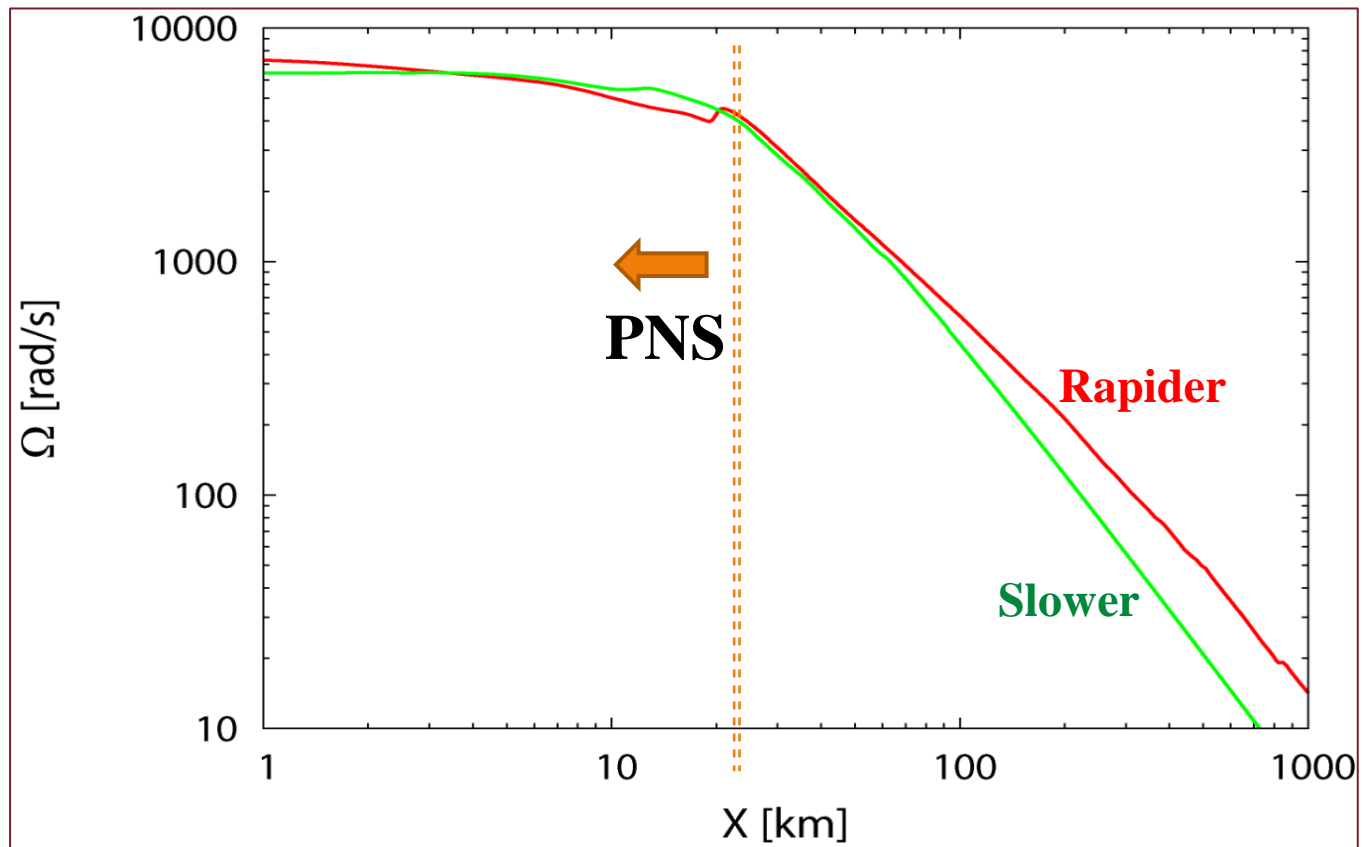


Same model but with Slower Rotation: Spheroidal configuration, No time variability

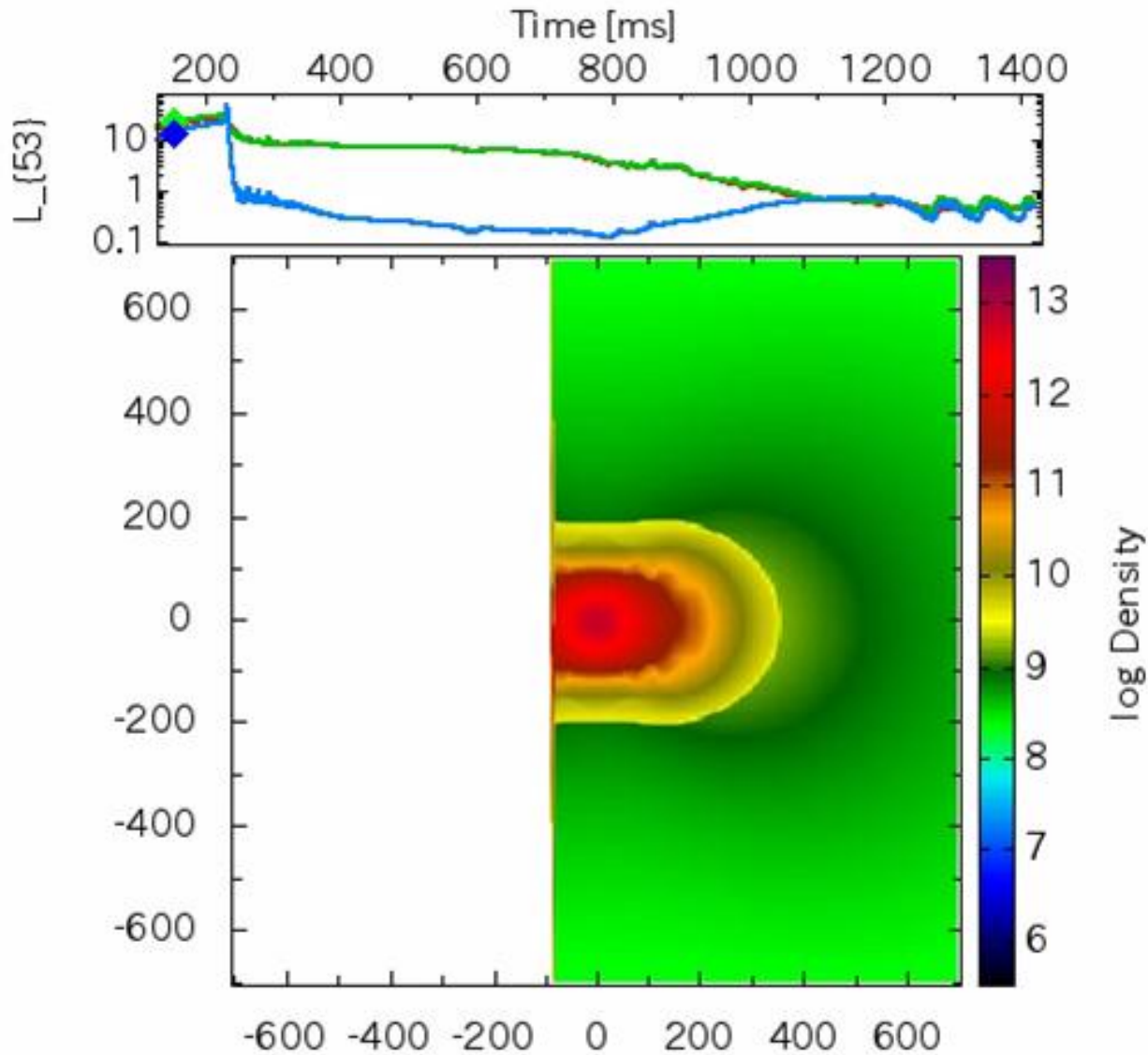


Comparison of Rotational Profile

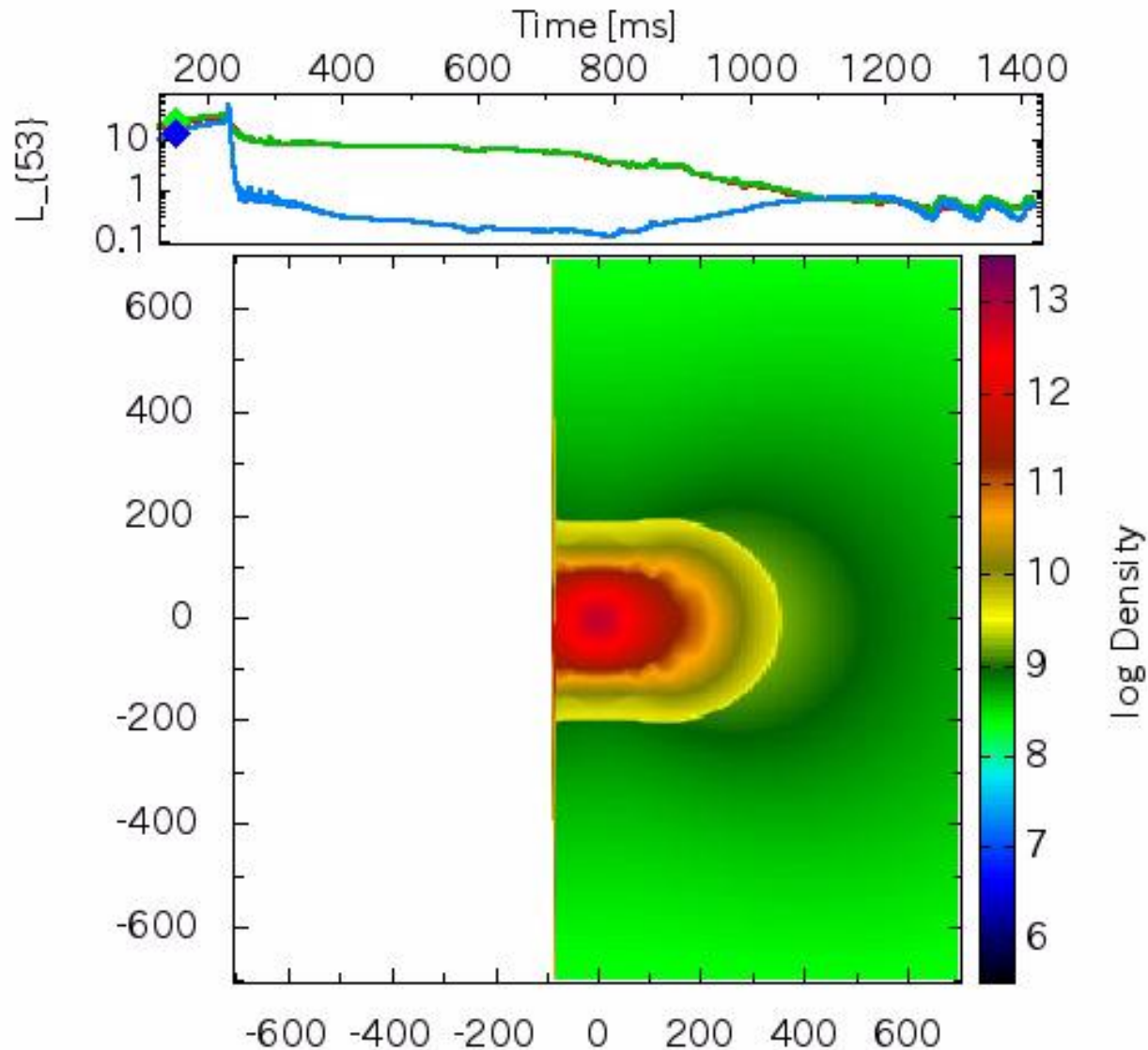
- ▶ Rotational profiles of Proto-Neutron Star are similar
- ▶ Small difference in rotational profile of outer region results in large difference in dynamics



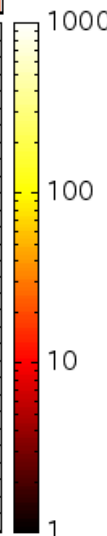
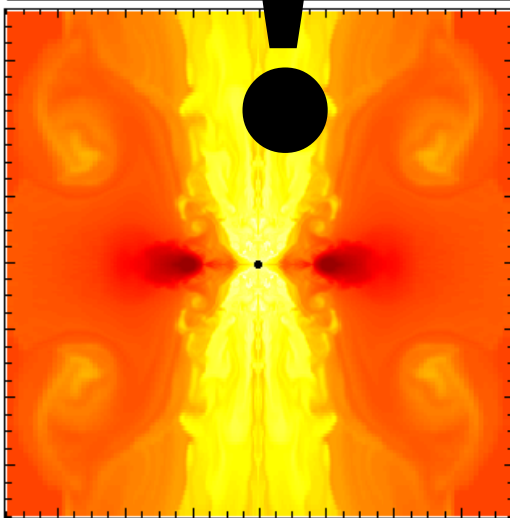
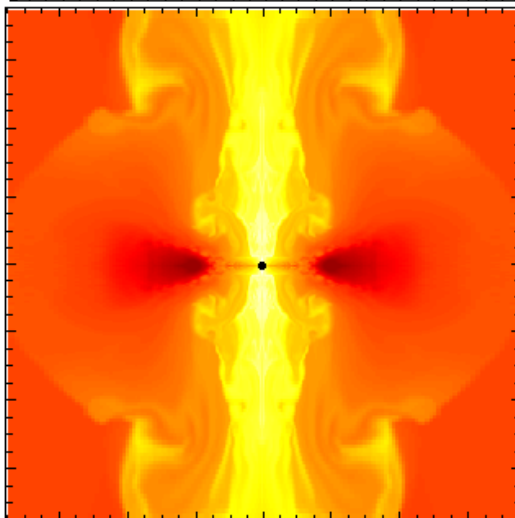
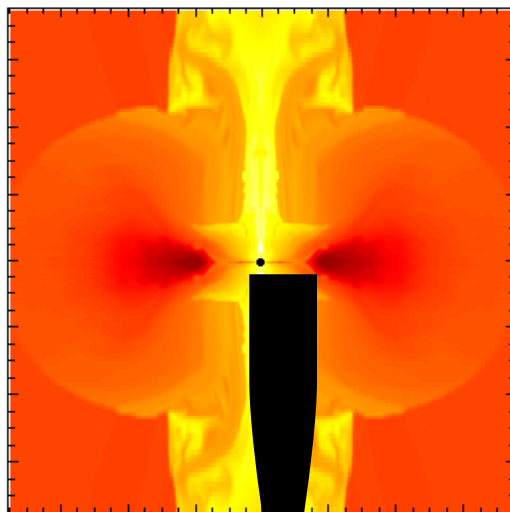
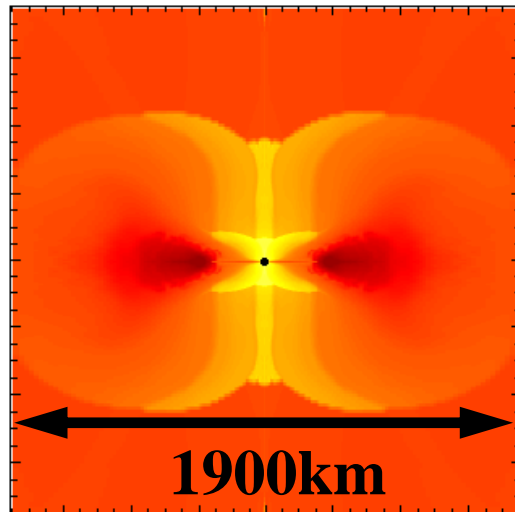
500Msolar-PopIII core collapse: Outflow appears even when BH is formed directly



500Msolar-PopIII core collapse: Outflow appears even when BH is formed directly



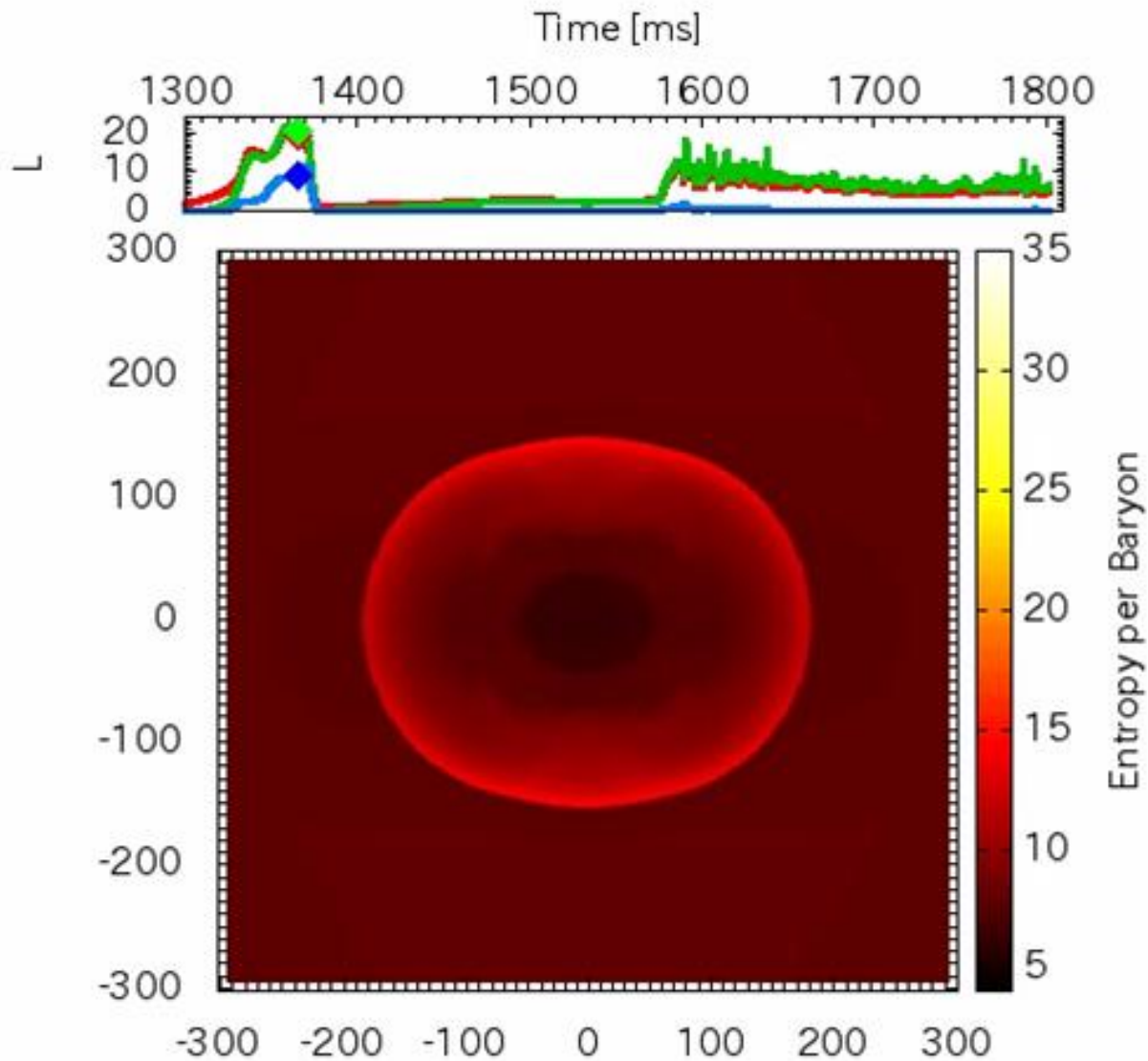
500Msolar-PopIII core collapse: Outflow appears even when BH is formed directly



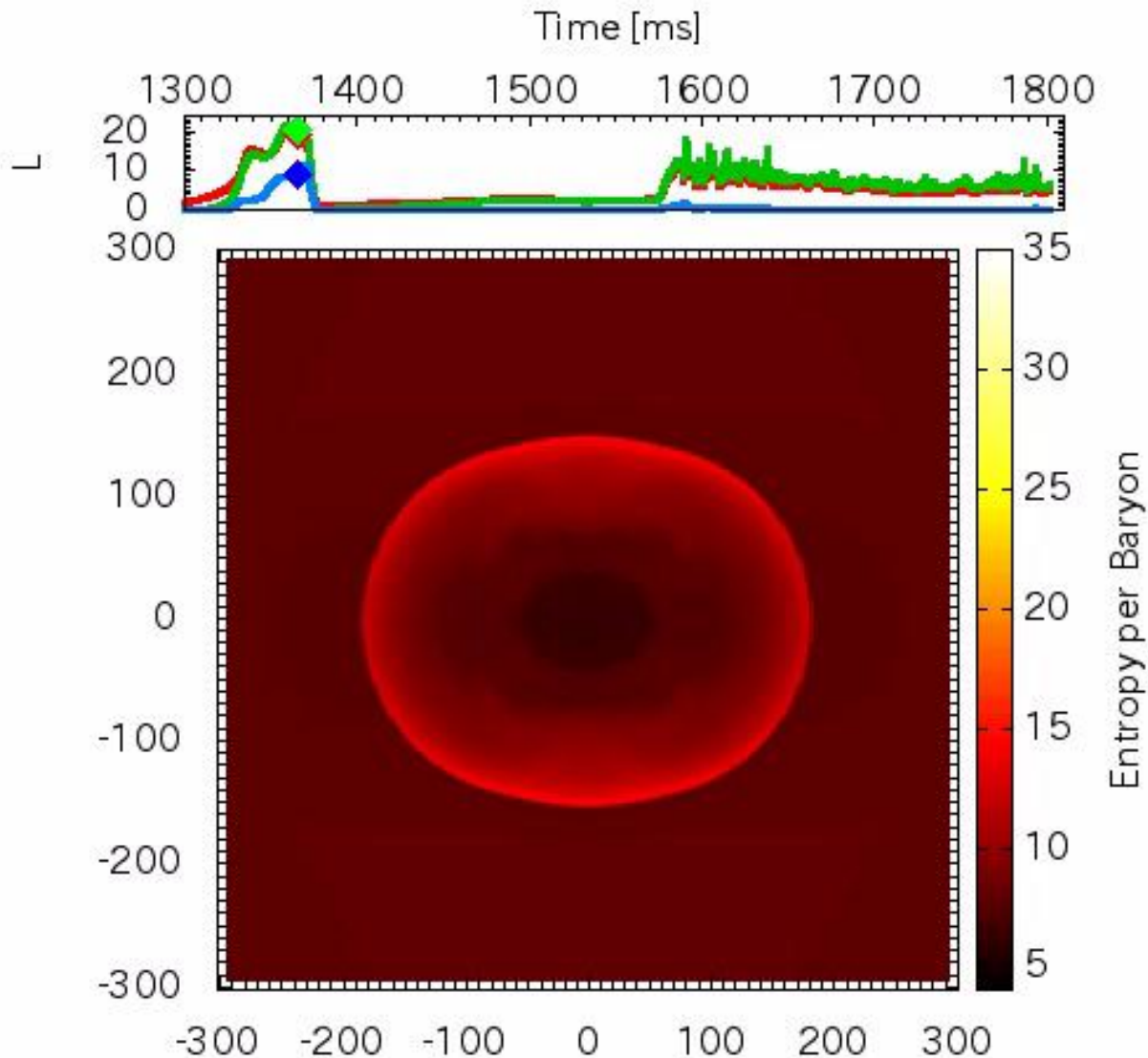
- ▶ Inefficient advection cooling
- ▶ Thermal energy is stored
- ▶ Outflow



Moderate rotation : BH formation zoo



Moderate rotation : BH formation zoo



Summary

- ▶ Numerical Relativity is the unique tool to study dynamical phenomena in the universe where strong gravity plays a role
 - ▶ Recent developments enable us to perform simulations in physical modeling
- ▶ NS-NS and BH-NS are very interesting phenomena both in physics and astrophysics
 - ▶ Promising sources of ground-based GW detectors
 - ▶ As laboratory for exploring physics of dense matter
 - ▶ Central engine of SGRB
- ▶ BH formation process in stellar core collapse is quite dynamical, accompanying oblique shock, convection, and outflows
 - ▶ The dynamics is sensitive to the initial rotational profile
 - ▶ The resulting system has preferable features for LGRBs
- ▶ More systematic studies with physical modeling will be done in the near future



Appendix

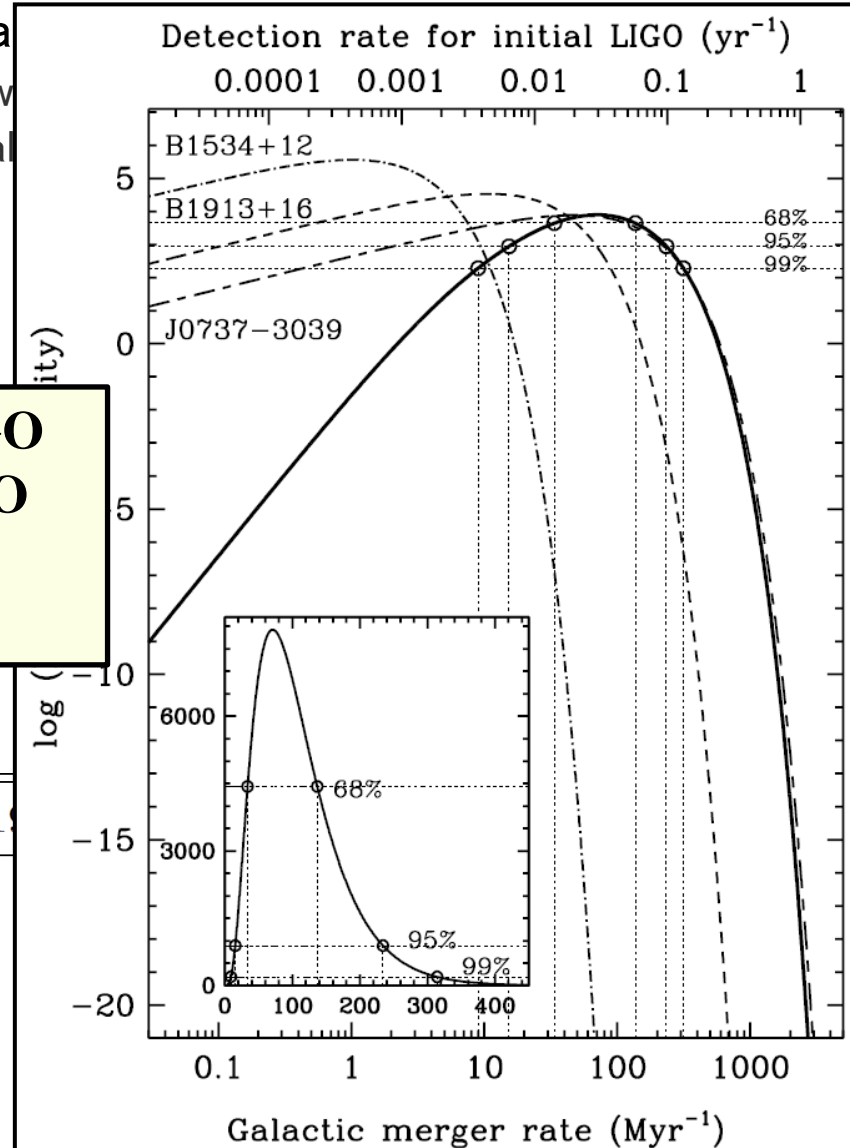


Expected Merger Rate

- ▶ Binary Neutron Star (BNS, NS-NS) and ca
 - ▶ 6 Binaries with pulsar are expected to merge w
 - ▶ Empirical NS-NS merger rate: 3–190 Myr⁻¹ /gal
- ▶ Merger rate from population synthesis
 - ▶ NS-NS : 10–200 Myr⁻¹/gal. (Kalogera et al. 2004)
 - ▶ BH-NS : 0.1–5 Myr⁻¹/gal. (Belczynski 2007)

NS-NS : ~10 - 100 events/yr for advLIGO
BH-NS : ~ 1 - 30 events/yr for advLIGO
Not so rare events !
We can do GW astronomy

$\log_{10}(\tau_g/[yr])$	7.9	12.4
Masses measured?	Yes	No
	B1820–11	J1829+2456
P [ms]	279.8	41.0
P_b [d]	357.8	1.18
e	0.79	0.14
$\log_{10}(\tau_c/[yr])$	6.5	10.1
$\log_{10}(\tau_g/[yr])$	15.8	10.8
Masses measured?	No	No

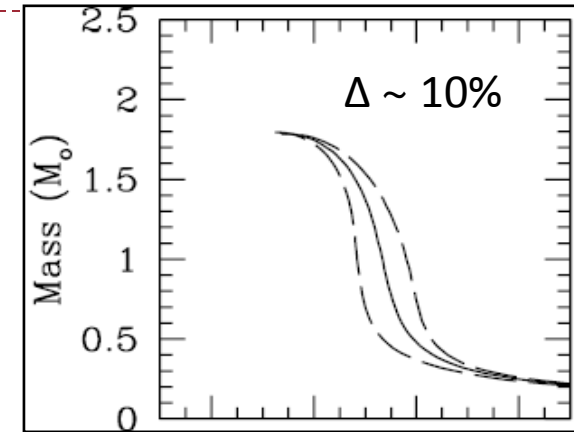


Author	NS-NS		BH-NS		Method
	LIGO	AdLIGO	LIGO	AdLIGO	
Kim et al. [143]	5e-3	27			Empirical
Nakar et al. [198]		~ 2		~ 20.0	SGRBs
Guetta & Stella [128]	7.0e-3	22	7.0e-2	220	SGRBs
Voss & Tauris [323]	6.0e-4	2.0	1.2e-3	4.0	Pop. Synth. – SFR
de Freitas Pacheco et al. [79]	8.0e-4	6.0			Pop. Synth. – SFR
Kalogera et al. [140]	1.0e-2	35	4.0e-3	20	Pop. Synth. – NS-NS
O’Shaughnessy et al. [218]	1.0e-2	10	1.0e-2	10	Pop. Synth. – NS-NS

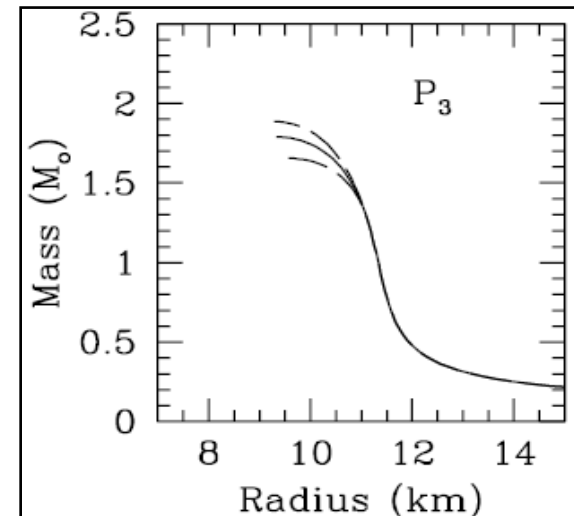


Why GWs from NS-NS are interesting ?

- ▶ **One of most promising source of GWs**
 - ▶ Next generation interferometer can see $\sim 350\text{Mpc}$
 - ▶ Expected event rate : more than 10/yr
- ▶ **Unique window to ‘see’ inside dense matters**
 - ▶ Very small cross section with matter
- ▶ **Dynamical response of dense matter**
 - ▶ By contrast with static, isolated neutron star
- ▶ **Multiple information of equation of state**
 - ▶ **Tidal deformation (radius) : relatively low density**
 - ▶ Maximum mass : most high density
 - ▶ Oscillation :
- ▶ **Less uncertain parameters**
 - ▶ Inspiral waveform provides information of mass
 - ▶ Mass should be determined in isolated neutron star
- ▶ **Simple in a complementary sense**
 - ▶ Essentially quadrupole formula
 - ▶ By contrast with optical observation



Radius is sensitive to relatively low density parts



Maximum mass depends on most dense parts

HMNS

▶ $M_{\text{crit}} \approx M_{\text{max,sph.cold.NS}} + \Delta M_{\text{rot}}^{\text{rigid}} + \Delta M_{\text{rot}}^{\text{diff}} + \Delta M_{\text{thermal}}$

- ▶ $M_{\text{max,sph.cold.NS}}$: maximum mass of spherical NS at $T = 0$, depends on EOS
 - ▶ Most massive NS accurately observed : 1.97 Msolar (Demorest et al. 2010)
- ▶ $\Delta M_{\text{rot}}^{\text{rigid}}$: effects of rigid rotation $\sim O(10\%)$
- ▶ $\Delta M_{\text{rot}}^{\text{diff}}$: effects of differential rotation typically $\sim O(10\%)$
- ▶ $\Delta M_{\text{thermal}}$: effects of finite temperature $\sim O(10\%)$
- ▶ HMNS formed after the merger is very hot as $T \sim O(10\text{MeV})$
 - ▶ Thermal contribution is not negligible \Rightarrow Finite temperature EOS
 - ▶ Neutrino cooling plays an important role \Rightarrow Microphysics



Development in Numerical Relativity (1)

- ▶ **ADM /3+1decomposition** (Arnowitt, Deser, & Misner 1962; York 1978)
 - ▶ General Relativity : Theory on spacetime manifold
 - ▶ Time and spatial derivatives appear in equations in a mixed manner
 - ▶ It is not clear the type of equation (elliptic, hyperbolic ?)
 - ▶ Formulation as an initial (and boundary) problem
- ▶ **Basic ideas on coordinate conditions** (Smarr, York, ... in 1970's)
 - ▶ There is no absolute spacetime and no preferred frame of reference
 - ▶ Those who perform simulations must specify coordinate
 - ▶ Use this degree of freedom to avoid singularities and to resolve the frame dragging
 - ▶ Development of faster conditions (Shibata, Alccubierre, Brugmann, Bona, ... ~2000)
- ▶ **Some pioneering studies** (Nakamura, Ohara, Teukolsky, ...1980's)
 - ▶ First full GR simulation of gravitational collapse (Nakamura 1980's)



Development in Numerical Relativity (2)

- ▶ **BSSN formalism** (Shibata & Nakamura 1995; Baumgarte & Shapiro 1999)
 - ▶ Einstein equation : constrained system
 - ▶ Maxwell eq. : Gauss's law, No-monopole condition
 - ▶ Einstein eq. : **Hamiltonian(~energy), Momentum constraint equations**
 - ▶ **ADM formalism : violation of constraints grows monotonically in time and simulation clash in a short time**
 - ▶ Stable, long-term simulations become possible
- ▶ **Quasi-equilibrium configurations of NS-NS** (Uryu, Gourgoulhon, Taniguchi, Cook, Shibata, ... 90's ~)
 - ▶ **The first full GR simulation of NS-NS merger (Shibata & Uryu 2000)**
 - ▶ NS-BH initial data (Taniguchi, Shibata, Uryu, Grandclement, Kyutoku, ... 2006~)
 - ▶ **The first full GR simulation of BH-NS merger (Shibata & Uryu 2006)**



Development in Numerical Relativity (3)

- ▶ **Evolving BH spacetime** (Pretorius 2005; Campanelli et al. 2006)
 - ▶ **BH excision** : no information from BH interior \Rightarrow excise and set boundary cond.
 - ▶ The first BH-BH merger simulation (Pretorius 2005)
 - ▶ Need experienced craftsmanship
 - ▶ **BSSN-Puncture** : adopt nice coordinate conditions and variables
 - ▶ Easy to implement : most difficult simulation (BH-BH) becomes relatively easier one
 - ▶ A large number of BH-BH merger simulations (2006~)
- ▶ **(General) Relativistic hydrodynamics** (Font, Marti, Muller, Del Zanna, ... 90's~)
- ▶ **(General) Relativistic MHD** (Hawley, Komissarov, Anton (Valencia), Duez (Illinoi), Shibata-Sekiguchi, ... 2000~)



Recent development and Future direction

▶ Toward more physical modeling

▶ Numerical Relativity simulations with Microphysics (Sekiguchi 2010)

- ▶ Nuclear-theory-based finite temperature EOS (Table EOS !)
- ▶ Weak interactions : e^\pm capture, neutrino scatterings, neutrino capture
 - $\tau_{\text{weak}} \ll \tau_{\text{dyn}}$: Two very different timescales
 - ⇒ **Numerically, very ‘stiff’ source terms**
- ▶ Neutrino cooling : simplified treatment

▶ Towards GR (neutrino)radiation-hydrodynamics

- ▶ An early attempt : Farris et al. (2008)
- ▶ Covariant formulation based on Thorne’s moment formalism : Shibata et al. (2011)
- ▶ The first Full GR radiation-MHD simulation : Shibata & Sekiguchi (2011)
 - Simplified modeling of BH-Disk system
- ▶ 1D core collapse simulation : O’Connor & Ott (2012)
- ▶ An semi-implicit scheme : Roedig et al. (2012)



Why Microphysics ?

- ▶ High density ($>10^{12}$ g/cc) and T ($> 1-10$ MeV) regions
 - ▶ $\lambda_\nu \gg \lambda_\gamma, \lambda_e \Rightarrow$ neutrinos drive the thermal / chemical evolution
 - ▶ 99% of energy released *in stellar core collapse* is carried away by neutrinos
 - ▶ **Neutrino : Weak interactions** should be taken into account
 - ▶ Strong dependences of weak rates on T \Rightarrow **Finite temperature EOS**
- ▶ NS-NS, BH-NS mergers
 - ▶ Inspiral : NS is cold ($k_B T / E_F \ll 1$) \Rightarrow **zero T EOS**
 - ▶ Merger : Compression, shock heating ($k_B T / E_F \sim O(0.1)$) \Rightarrow **finite T EOS**
 - ▶ **Prompt BH formation \Rightarrow hot region quickly swallowed by BH**
 - ▶ **Effects of finite temperature would be minor**
 - ▶ **HMNS, late time BH, and massive disk formation (more likely)**
 - ▶ **Shock heating, neutrino cooling, etc. are important**

