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“Probing for massive gravitational-wave background with a ground-based detector network”

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Probing for massive GW background with a ground-based detector network

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Introduction

So far, various modified gravity theories have been suggested.
(Scalar-tensor theory, f(R) gravity, higher derivative gravity, bimetric gravity, nonlinear massive gravity etc.)

Those theories could alter tensor perturbations and predict the properties of GWs different from GR:

- massive gravitons
- different phase evolution of GWs
- additional GW polarizations (scalar & vector pols.)

⇩

GW observation can be utilized for

- direct test of general relativity
- probing the extended theories beyond GR

Here we focus on massive graviton and its detectability with GW detectors.

Massive graviton & GW

Dispersion relation of graviton $\omega^2 = m_g^2 + k^2$

- minimum frequency of GW $\omega_{\min} = m_g$
- propagating speed of GW (group velocity)

$$v_g(\omega; m_g) \equiv \frac{d\omega}{dk} = \sqrt{1 - \frac{m_g^2}{\omega^2}}$$

Modification of GW waveform from a compact binary

[Will 1998, Berti et al. 2005, Yagi & Tanaka 2010]

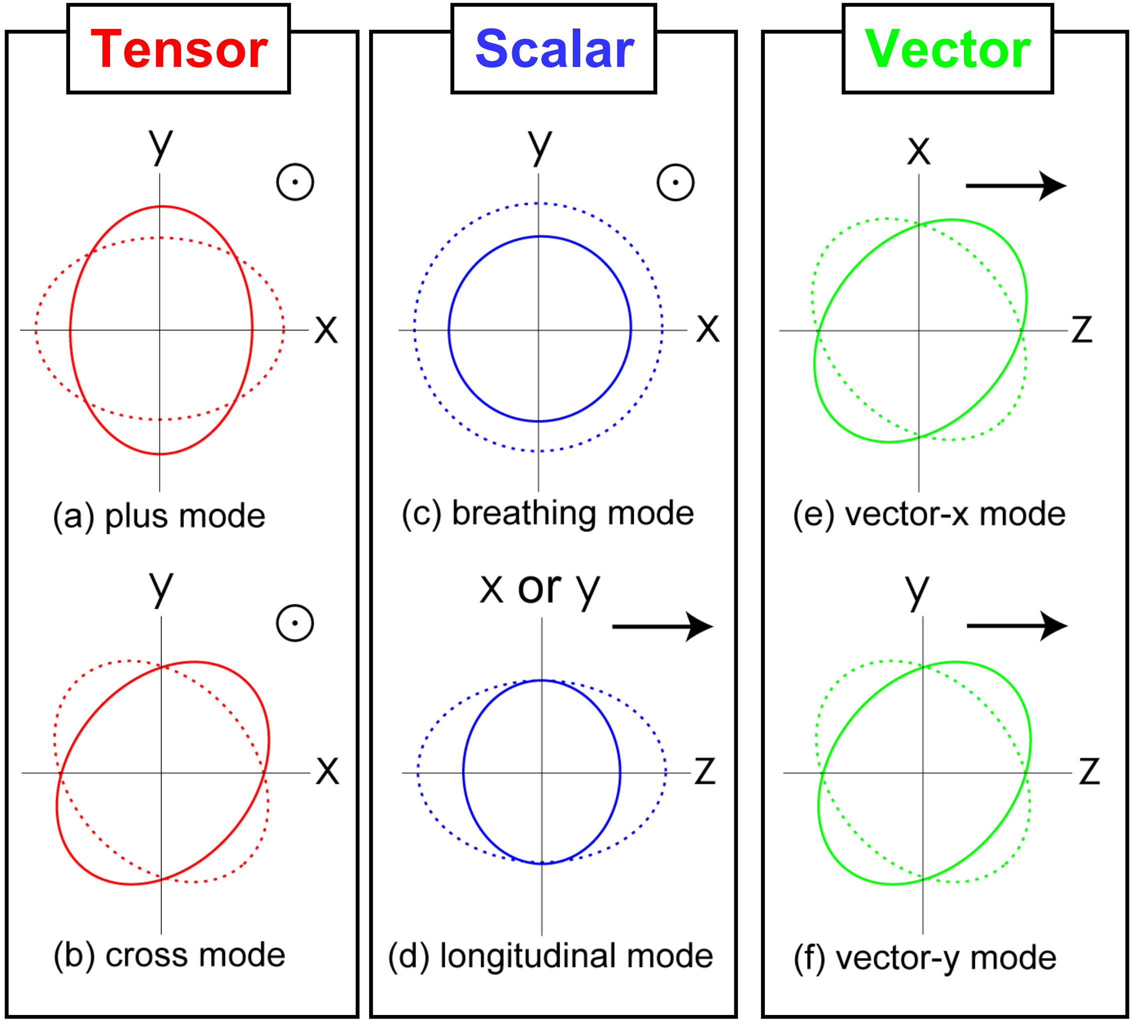
aLIGO: $m_g \lesssim 10^{-22}\,\mathrm{eV}$ LISA: $m_g \lesssim 10^{-25}\,\mathrm{eV}$

- phase velocity of GW

$$v_p(\omega; m_g) \equiv \frac{\omega}{k} = \left(\sqrt{1 - \frac{m_g^2}{\omega^2}}\right)^{-1}$$

GW polarizations

In general metric theory of gravity, six polarizations are allowed. [Eardley et al. 1973, Will 1993].



Current mass constraints

Solar system	$m_g \lesssim 10^{-21}\,\mathrm{eV}$	[Talmadge et al. 1988, Will 1998]
Galaxy cluster	$m_g \lesssim 10^{-29}\,\mathrm{eV}$	[Goldhaber & Nieto 1974]
Weak lensing	$m_g \lesssim 10^{-31}\,\mathrm{eV}$	[Choudhury et al. 2004]
CMB	$m_g \lesssim 10^{-30}\,\mathrm{eV}$	[Dubovsky et al. 2010]

The above is static bounds based on the modification of Newtonian potential (background level).

Binary pulsar $m_g \lesssim 10^{-20}\,\mathrm{eV}$ [Finn & Sutton 2002]

This bound is applied to only tensor polarization mode.

Constraints on scalar and vector mode of GW is NOT so strong and they can be quite massive.

GW background

Here we consider massive GW background.

Detector output of GW background

$$h(t,\vec{X}) = \sum_A \int_{S^2} d\hat{\Omega} \int_{-\infty}^{\infty} df\, \tilde{h}_A(f,\hat{\Omega})\, e^{2\pi i f(t-\hat{\Omega}\cdot\vec{X}/\underline{v_p})} \underline{F_A(\hat{\Omega})}$$

detector
response func.

$$v_p(\omega; m_g) \equiv \frac{\omega}{k} = \left(\sqrt{1-\frac{m_g^2}{\omega^2}}\right)^{-1}$$

Energy density of GW background

$$\boxed{\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm gw}}{d\ln f}}$$

tensor $\Omega_{\rm gw}^T \equiv \Omega_{\rm gw}^+ + \Omega_{\rm gw}^\times \qquad (\Omega_{\rm gw}^+ = \Omega_{\rm gw}^\times)\,,$

vector $\Omega_{\rm gw}^V \equiv \Omega_{\rm gw}^x + \Omega_{\rm gw}^y \qquad (\Omega_{\rm gw}^x = \Omega_{\rm gw}^y)\,,$

scalar $\Omega_{\rm gw}^S \equiv \frac{1}{3}\left(\frac{1+2\kappa}{1+\kappa}\right)(\Omega_{\rm gw}^b + \Omega_{\rm gw}^\ell)\,, \qquad \kappa \equiv \Omega_{\rm gw}^\ell/\Omega_{\rm gw}^b$

Correlation analysis of GW background

Single detector cannot distinguish GWB and random detector noise.
Also in most cases GW signal is small compared to noise.

$$h(t) \ll n(t)$$

Signal of detector 1: $s_1(t) = h(t) + n_1(t)$

Signal of detector 2: $s_2(t) = h(t) + n_2(t)$

↓ correlation

$$\int dt\, s_1(t)\, s_2(t) = \int dt\, \left[h^2(t) + \cancel{h(t)n_1(t)} + \cancel{h(t)n_2(t)} + n_1(t)n_2(t)\right]$$

$\propto\ T$

$\propto\ \sqrt{T}$

Signal to noise ratio

$$\mathrm{SNR} \approx \frac{\int dt\, h^2(t)}{\int dt\, n_1(t)n_2(t)} \propto \sqrt{T}$$

Correlation signal

Correlation signal in a frequency bin:

$$\mu_i(f) = \frac{3H_0^2}{10\pi^2} T f^{-3} \sum_A \gamma_i^A \Omega_{\rm gw}^A(f)\Delta f$$

Overlap reduction function

tensor $$\gamma_{IJ}^T(f;m_g^T) \equiv \frac{5}{2}\int_{S^2}\frac{d\hat{\Omega}}{4\pi}(F_I^+F_J^+ + F_I^\times F_J^\times)\exp\left[i\frac{2\pi f\,\hat{\Omega}\cdot\Delta\vec{X}}{v_p(f;m_g^T)}\right] ,$$

vector $$\gamma_{IJ}^V(f;m_g^V) \equiv \frac{5}{2}\int_{S^2}\frac{d\hat{\Omega}}{4\pi}(F_I^x F_J^x + F_I^y F_J^y)\exp\left[i\frac{2\pi f\,\hat{\Omega}\cdot\Delta\vec{X}}{v_p(f;m_g^V)}\right] ,$$

scalar $$\gamma_{IJ}^S(f;m_g^S) \equiv \frac{15}{1+2\kappa}\int_{S^2}\frac{d\hat{\Omega}}{4\pi}(F_I^b F_J^b + \kappa F_I^\ell F_J^\ell)\exp\left[i\frac{2\pi f\,\hat{\Omega}\cdot\Delta\vec{X}}{v_p(f;m_g^S)}\right]$$

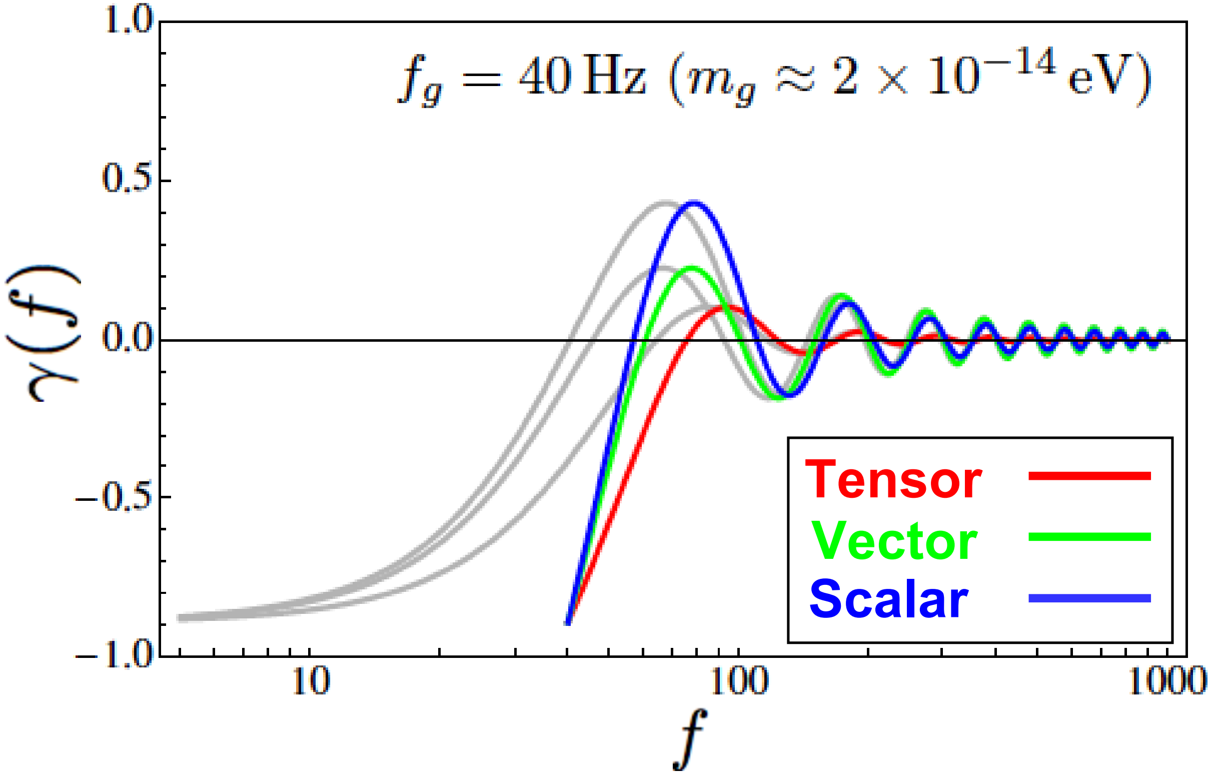
$$\kappa \equiv \Omega_{\rm gw}^\ell/\Omega_{\rm gw}^b$$

Overlap reduction function

$$\gamma^T_{IJ}(f;m^T_g) \equiv \frac{5}{2}\int_{S^2}\frac{d\hat{\Omega}}{4\pi}\left(F^+_I F^+_J + F^\times_I F^\times_J\right)\exp\left[i\,\frac{2\pi f\,\hat{\Omega}\cdot\Delta\vec{X}}{v_p(f;m^T_g)}\right]$$

@ low freq. ⟶ Const. @ high freq. ⟶ Damping oscillation

LIGO H1-L1 pair



For massive graviton, effective distance between detectors is smaller than massless case.

$$\frac{\Delta\vec{X}}{c} \longrightarrow \frac{\Delta\vec{X}}{v_p}$$

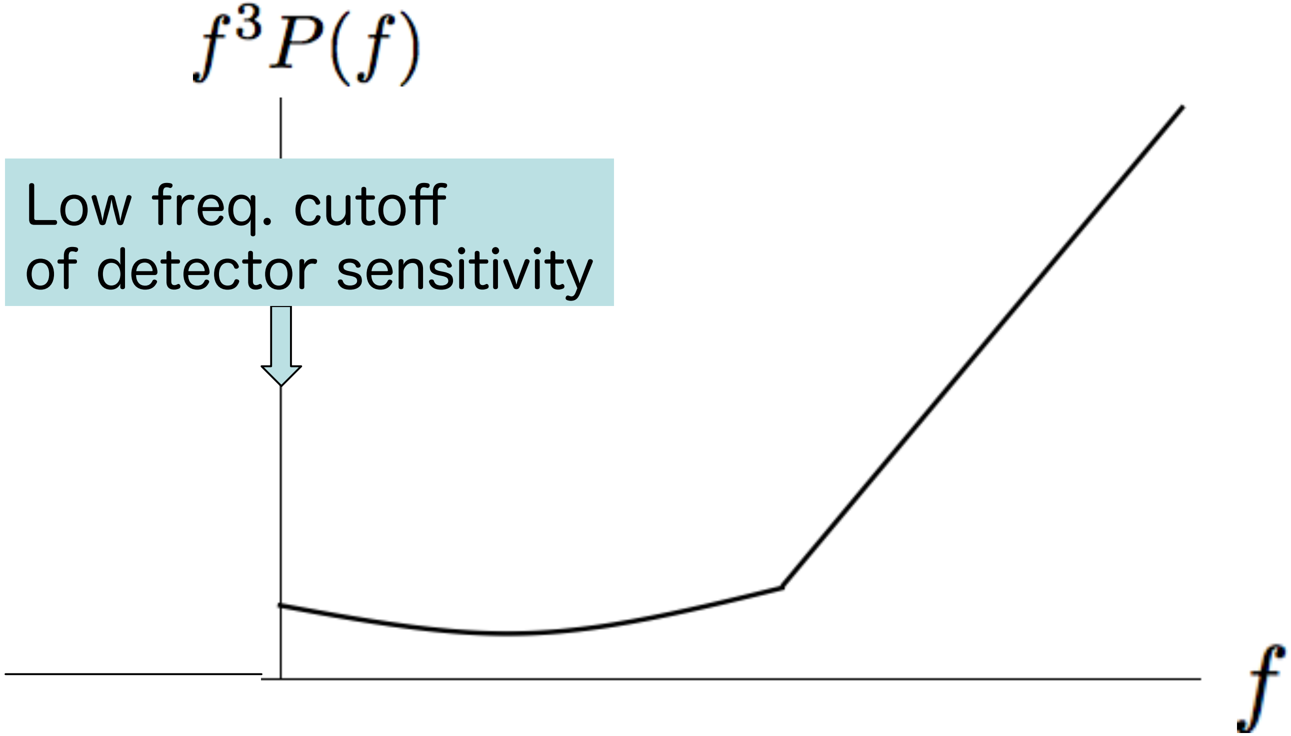
- stronger correlation
- low freq. cutoff

Mass detection

Case (i): small mass

Case (ii): intermediate mass

Case (iii): large mass



Low freq. cutoff
of detector sensitivity

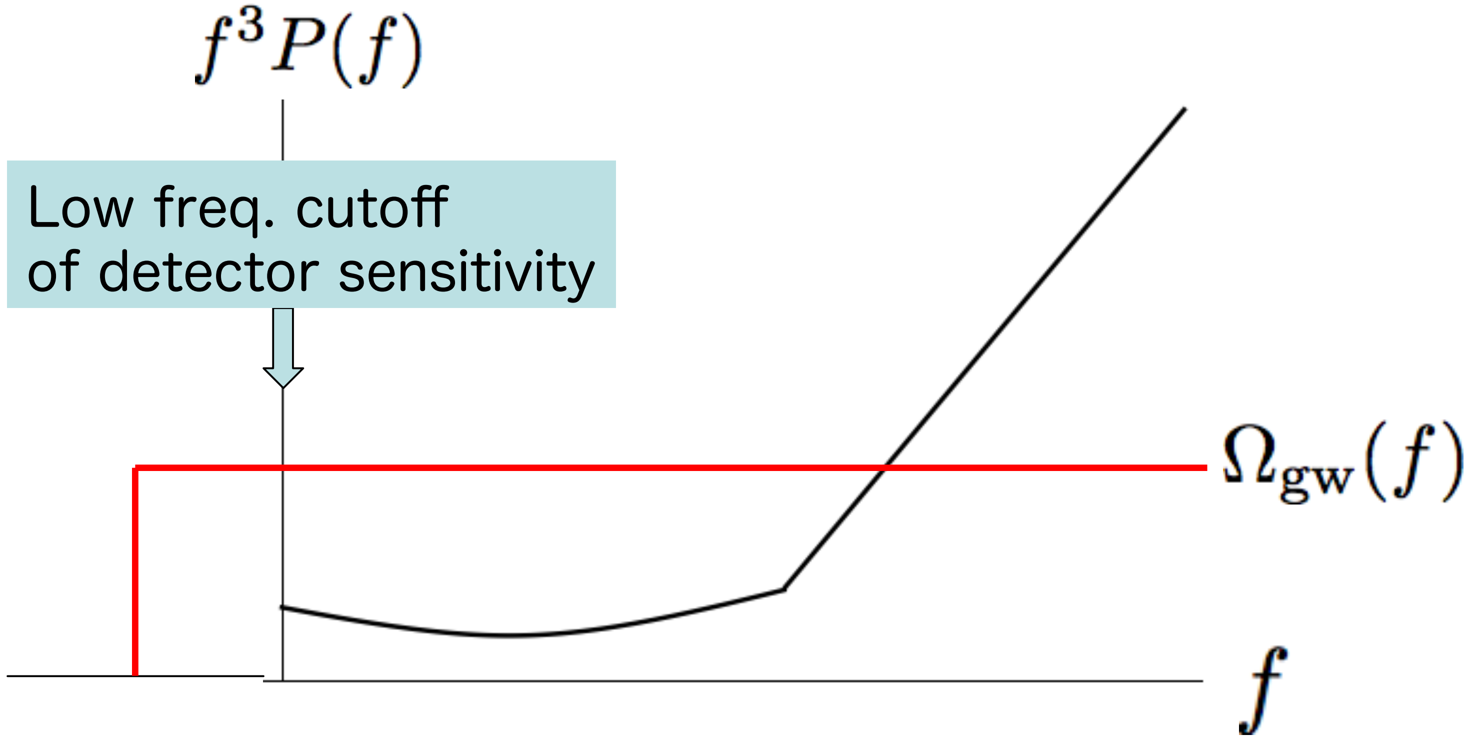
Mass detection

Case (i): small mass

Case (ii): intermediate mass

Case (iii): large mass

Indistinguishable
from massless case



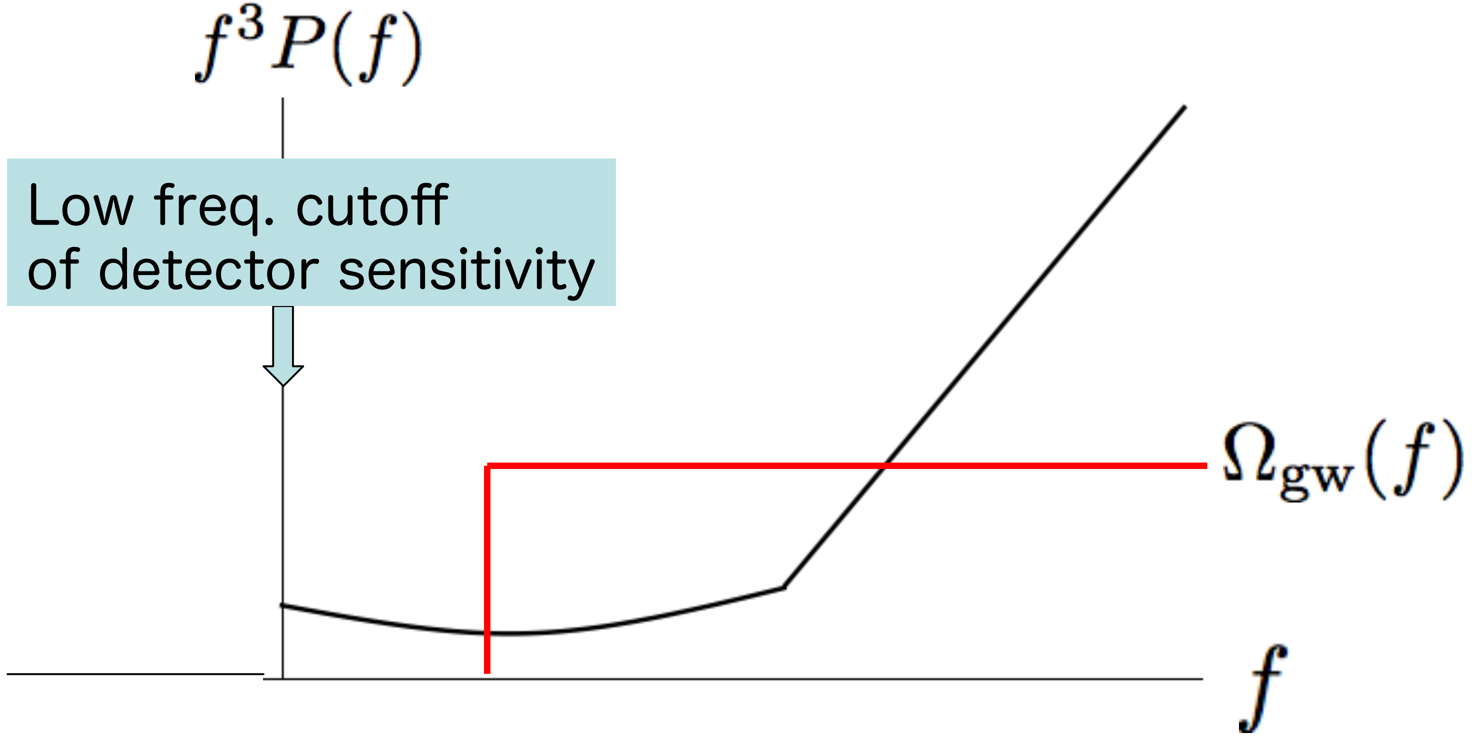
Mass detection

Case (i): small mass

Case (ii): intermediate mass

Case (iii): large mass

Characteristic jump of GWB spectrum is seen.



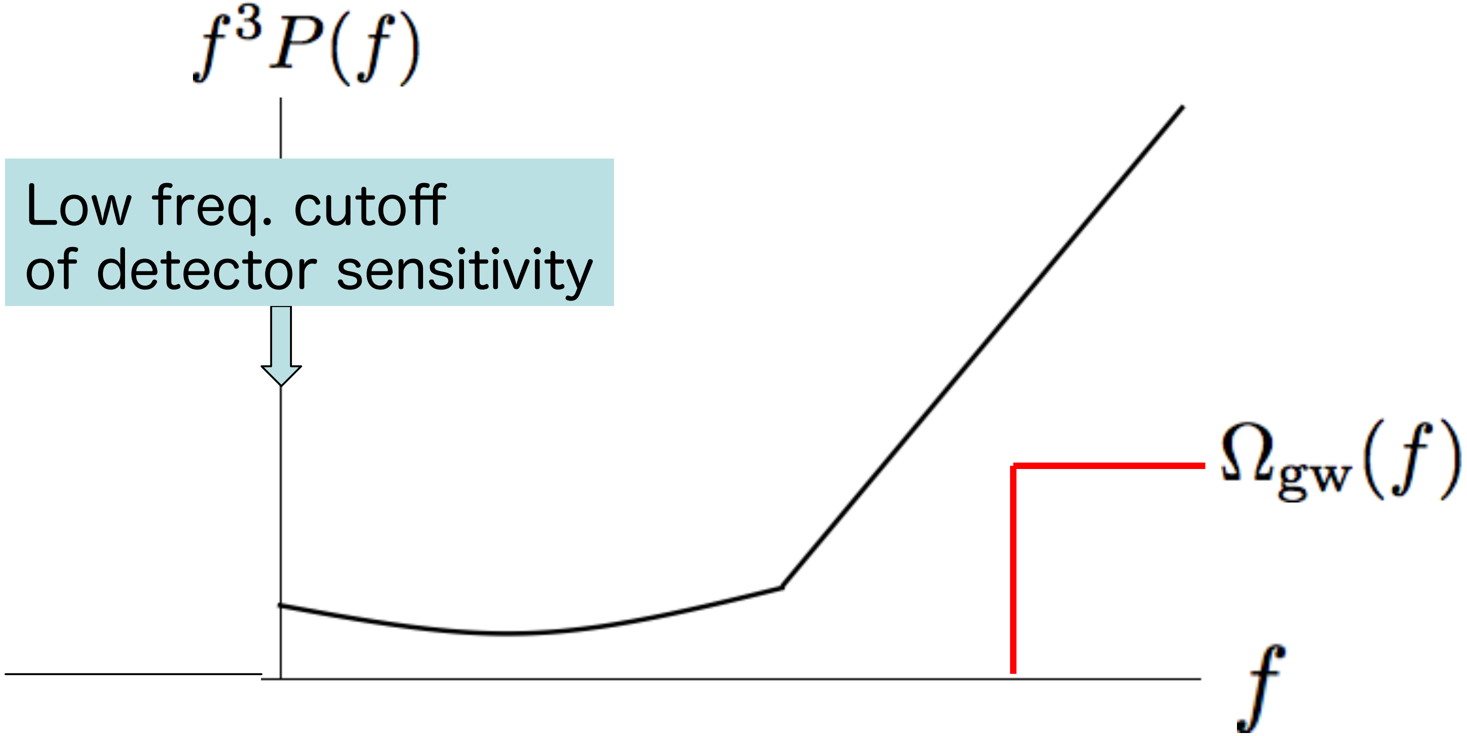
Mass detection

Case (i): small mass

Case (ii): intermediate mass

Case (iii): large mass

Even if large GWB exists,
we see nothing.



Low freq. cutoff
of detector sensitivity

Fisher matrix & graviton mass determination

Typical mass scale detectable with a GW detector:

$$m_g \approx 6.58 \times 10^{-14} \left(\frac{f_g}{100\,\mathrm{Hz}} \right) \mathrm{eV}$$

Use Fisher matrix to estimate measurement accuracy of m_g

$$F_{ab} = \sum_{i=1}^{N_{\mathrm{pair}}} \sum_{A} \int_0^\infty df \left[\frac{C(f)\{\gamma_i^A(f)\}^2 \partial_a \Omega_{\mathrm{gw}}(f) \partial_b \Omega_{\mathrm{gw}}(f)}{\mathcal{N}_i(f)} + \frac{\partial_a \gamma_i^A(f) \partial_b \gamma_i^A(f) + \gamma_i^A(f) \partial_a \partial_b \gamma_i^A(f)}{3\{\gamma_i^A(f)\}^2} \right]$$

$$C(f) \equiv \frac{9H_0^4 T}{50\pi^4 f^6}, \qquad \mathcal{N}_i(f) \equiv P_I(f) P_J(f)$$

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$$C(f) \equiv \frac{9H_0^4 T}{50\pi^4 f^6}\,, \qquad \mathcal{N}_i(f) \equiv P_I(f) P_J(f)$$

We ignore the contribution from the 2nd term for safety.
Then our estimate is conservative one.

Computation setup

Model of GW background:

$$\Omega_{\rm gw}(f) = \Omega_{\rm gw,0}\left(\frac{f}{f_0}\right)^{n_t}\Theta[f-f_g]$$

We assume only a single pol. mode exists.
(not mixture of 3 pols.)

Free parameters: $\Omega_{\rm gw,0},\ n_t,\ f_g$

Fiducial values: $\Omega_{\rm gw,0}=10^{-7},\ n_t=0.$ & all f_g

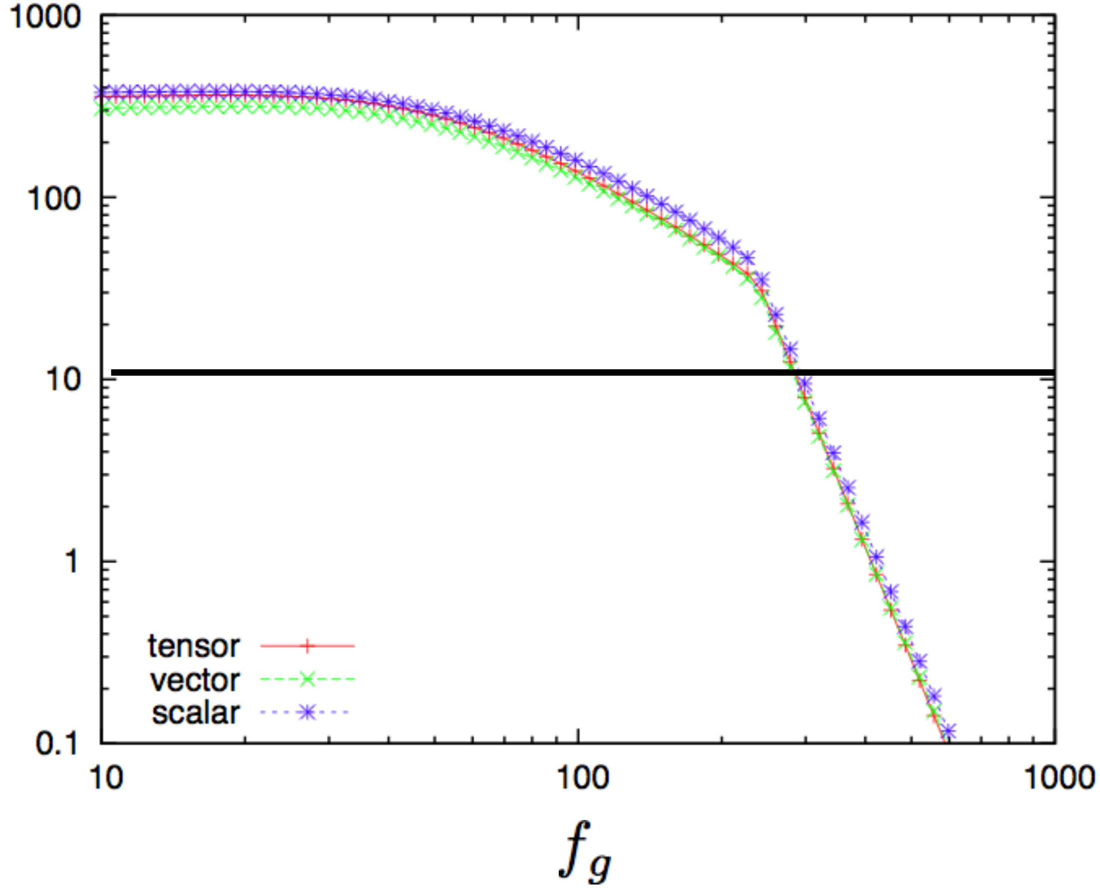
Detector network:

Consider 4 GW detectors: aLIGO (H1&L1), aVIRGO, KAGRA

Correlation pairs are HL, HV, LV, HK, KL, KV.

(all noise spectra are assumed to be that of aLIGO.)

SNR of a detector network

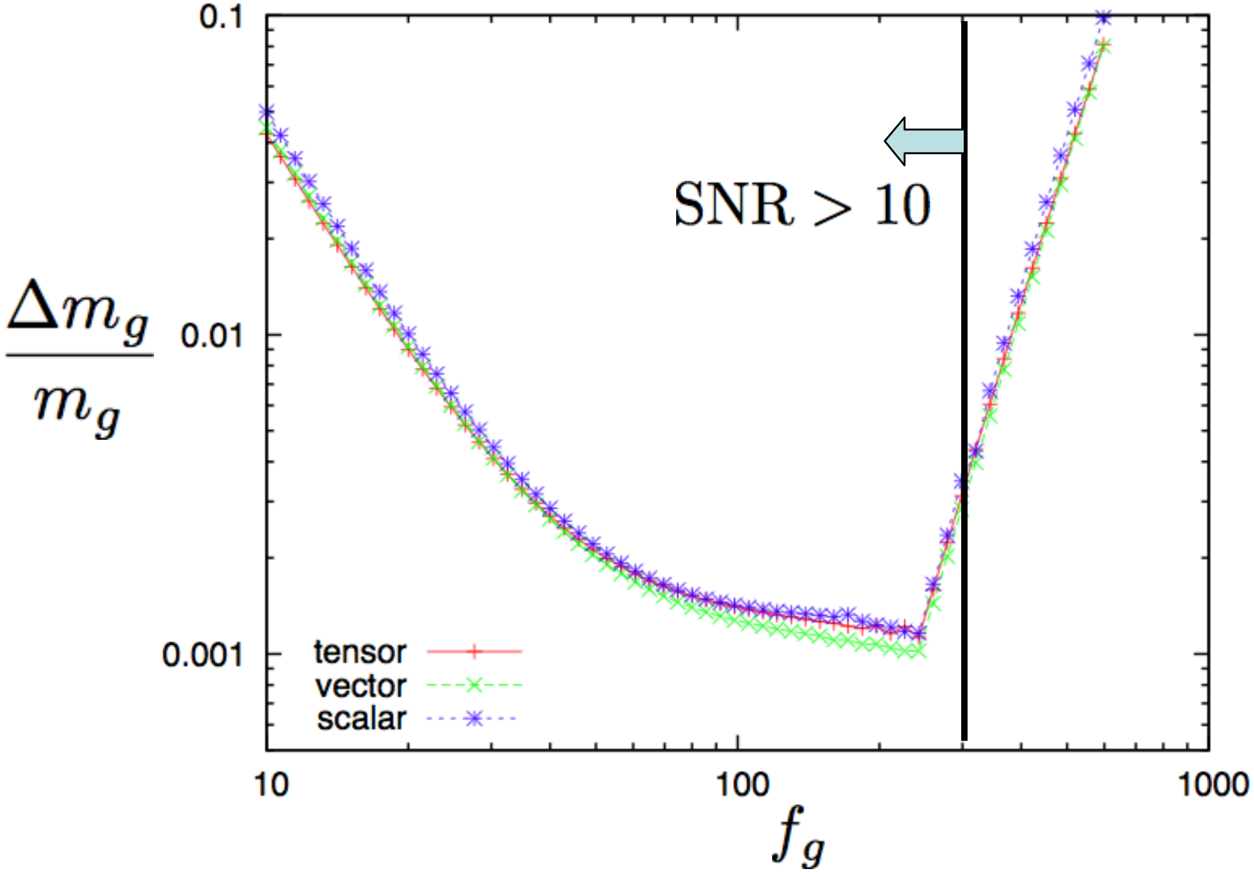


No significant difference between polarization modes. A detector network has almost the same sensivity to GWB.

Detector low freq. cutoff = 10 Hz.

SNR threshold = 10 $\longrightarrow$ High freq. cutoff = 300 Hz

Mass measurement accuracy



In the available frequency range, graviton mass is well determined.

$$10\,\mathrm{Hz} \le f \le 300\,\mathrm{Hz}$$

⇒ $$\boxed{6.7 \times 10^{-15}\,\mathrm{eV} \le m_g \le 2.0 \times 10^{-13}\,\mathrm{eV}}$$

for $\Omega_{\mathrm{gw},0} = 10^{-7}$
$n_t = 0$

Summary

- Search for graviton mass and polarization enable us to perform model-independent test of gravity and to constrain alternative theory of gravity.
- We considered massive GWB and showed that if GWB is detected, advanced-detector network can search for graviton mass in the range.

$$6.7 \times 10^{-15}\,\mathrm{eV} \leq m_g \leq 2.0 \times 10^{-13}\,\mathrm{eV}$$

Note1: If the correlation signal is a mixture of 3 pol. modes, we can robustly separate these mode with a detector network as shown in

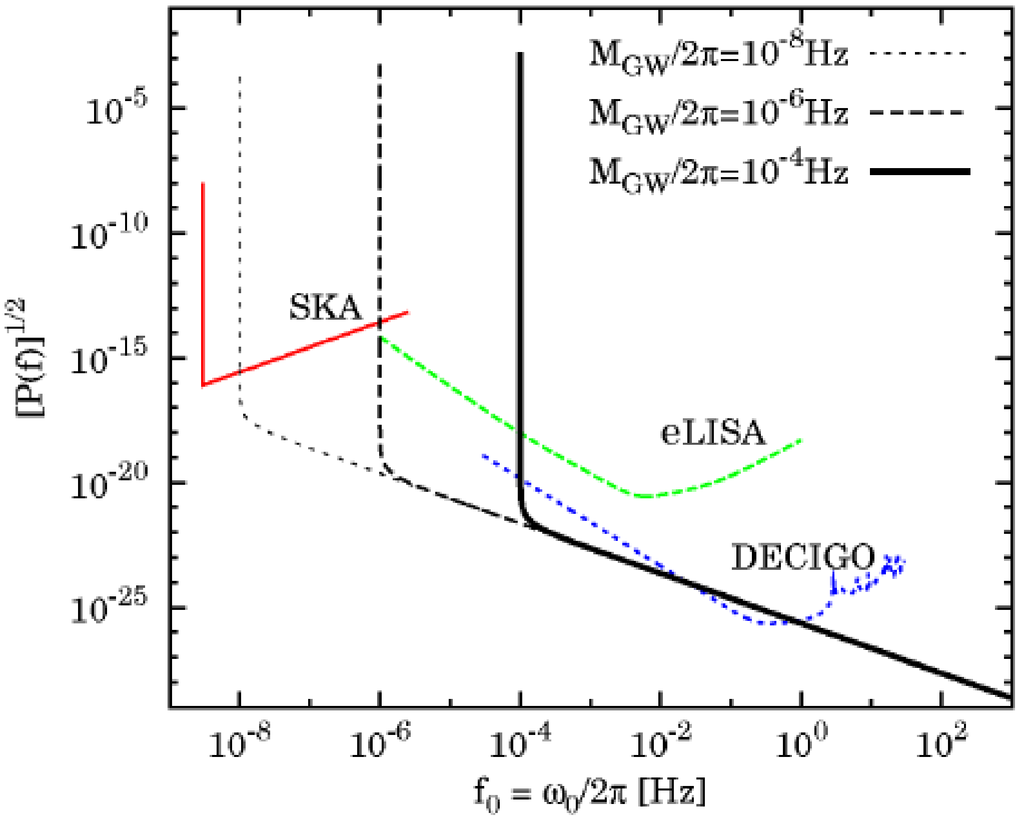
[AN et al., PRD 79, 082002 (2009); PRD 81, 104043 (2010)]

Note2: If we take the Fisher matrix for $\gamma(f)$ into account, detectable mass range would broaden.

Note3: It'd be interesting to consider space-based detectors and pulsar timing, which can constrain different mass range.

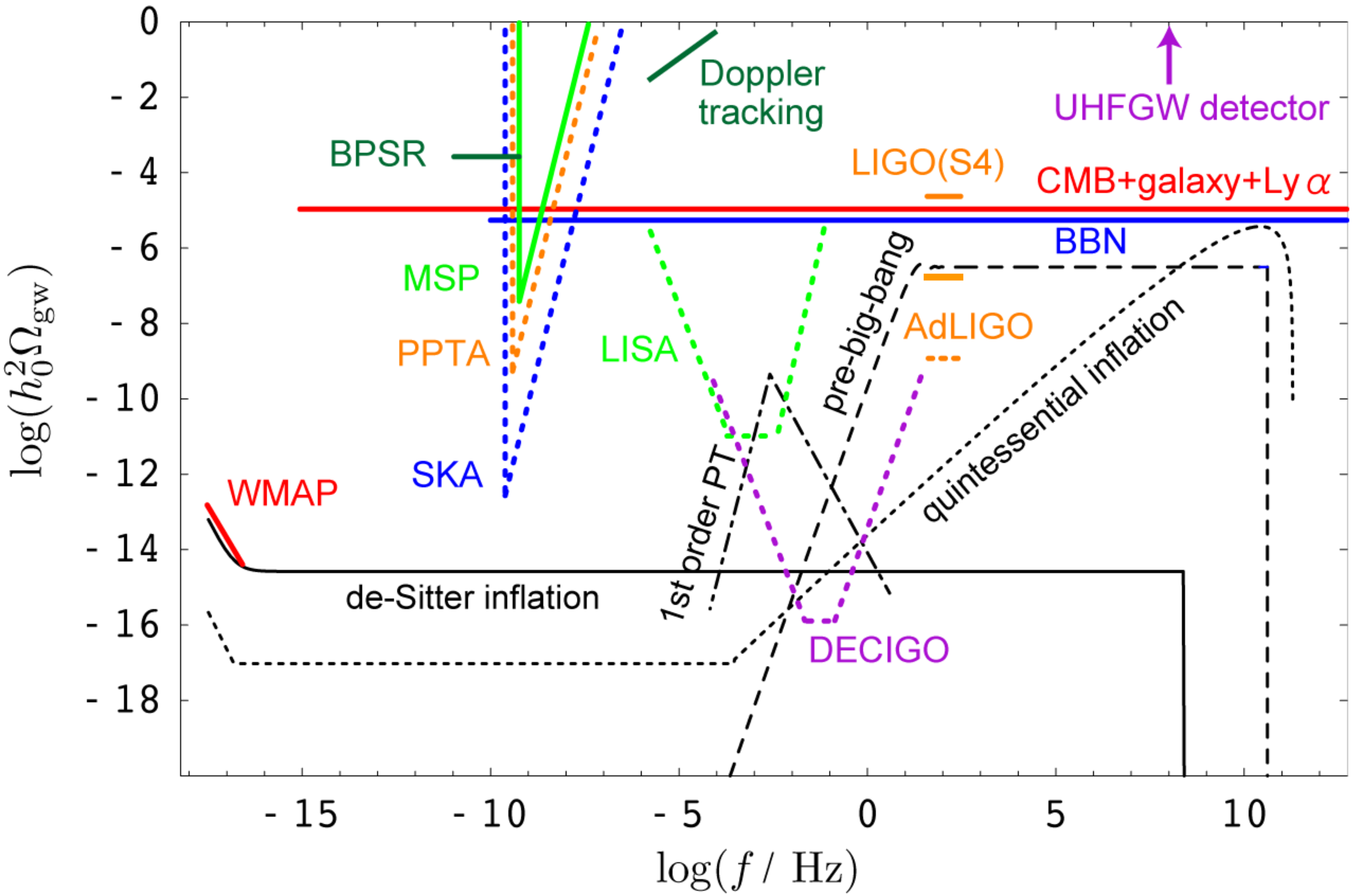
Large peak on GWB spectrum?

[Gumrukcuoglu et al., arXiv:1208.5975]



$$\mathcal{P}(\omega_0) \equiv \frac{d}{d\ln\omega_0}\langle\gamma_{ij}\gamma^{ij}\rangle\Big|_{t=t_0} = \frac{\omega_0^2}{\omega_0^2 - M_{GW,0}^2}\frac{2k^3}{\pi^2}|\gamma_k(t_0)|^2$$

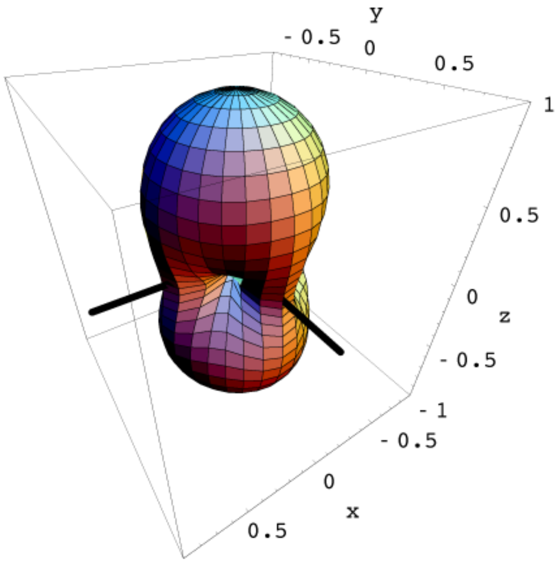
Observational constraints on GWB



Angular response functions

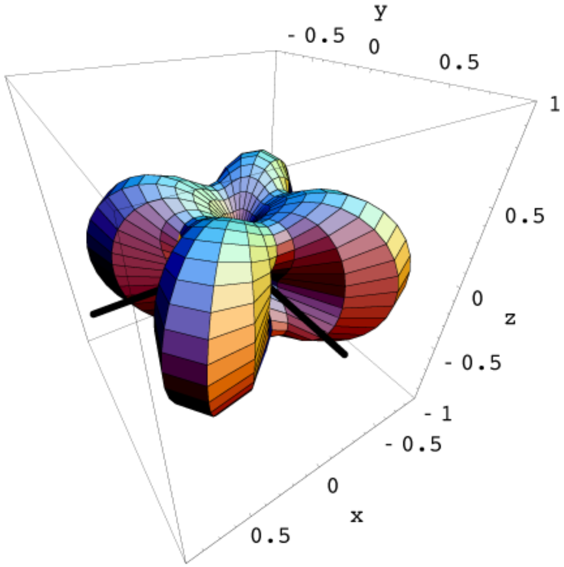
[Tobar, Suzuki & Kuroda 1999]

$$\sqrt{F_+^2 + F_\times^2}$$



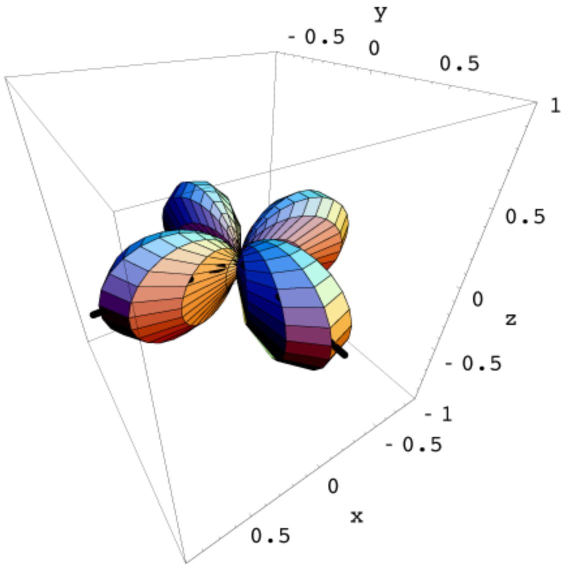
Tensor

$$\sqrt{F_x^2 + F_y^2}$$



Vector

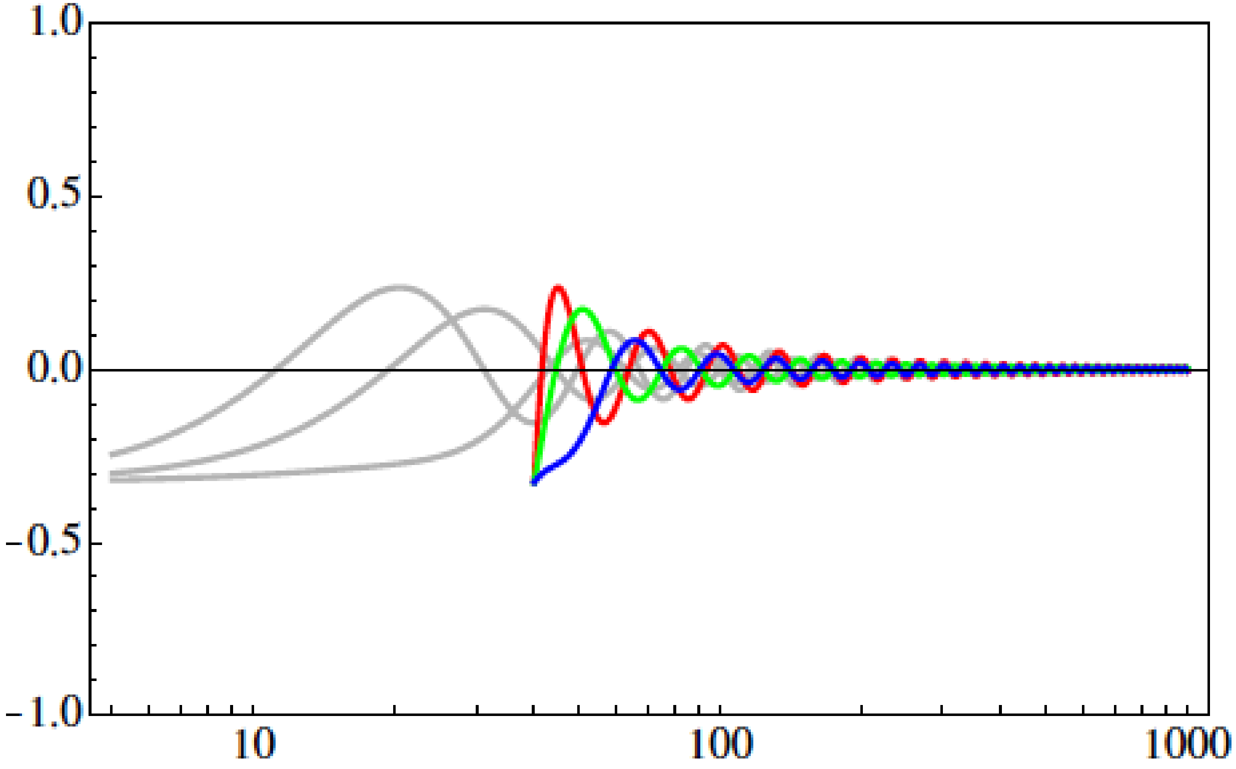
$$\sqrt{F_b^2 + F_\ell^2}$$



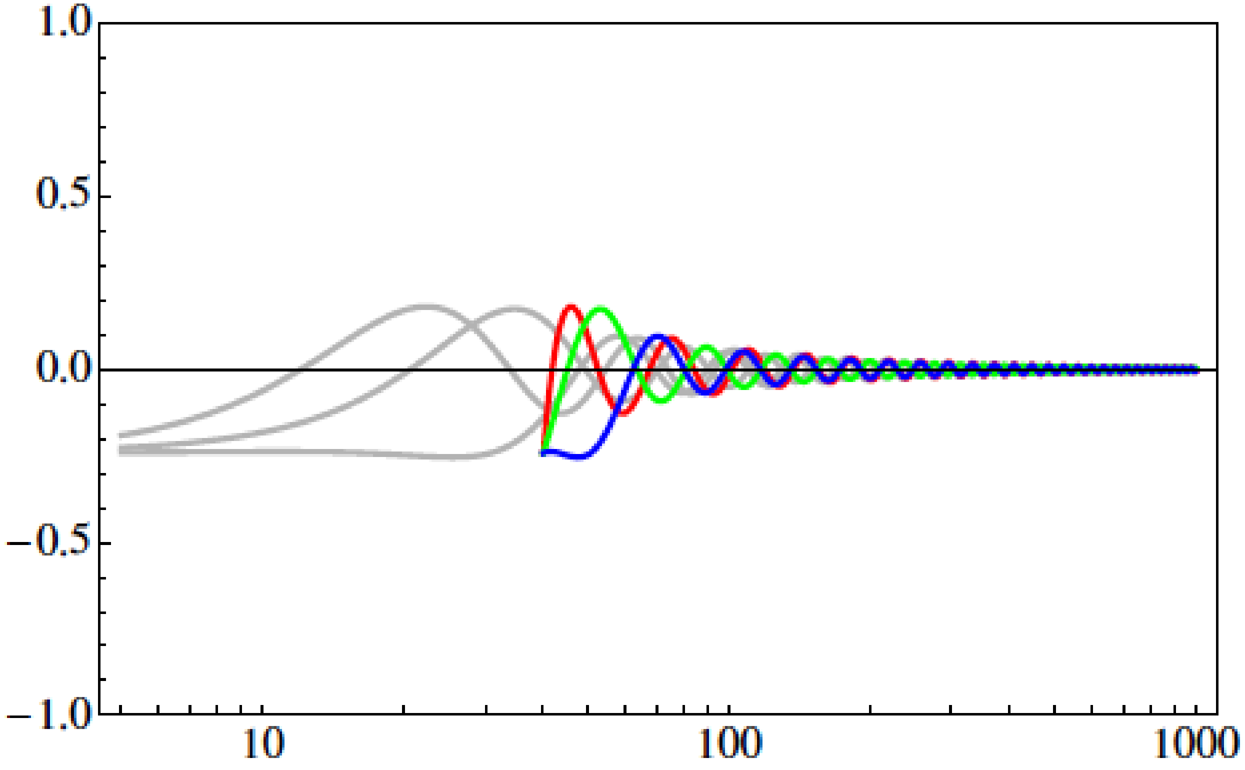
Scalar

Vector and scalar modes are also detectable with an interferometer.

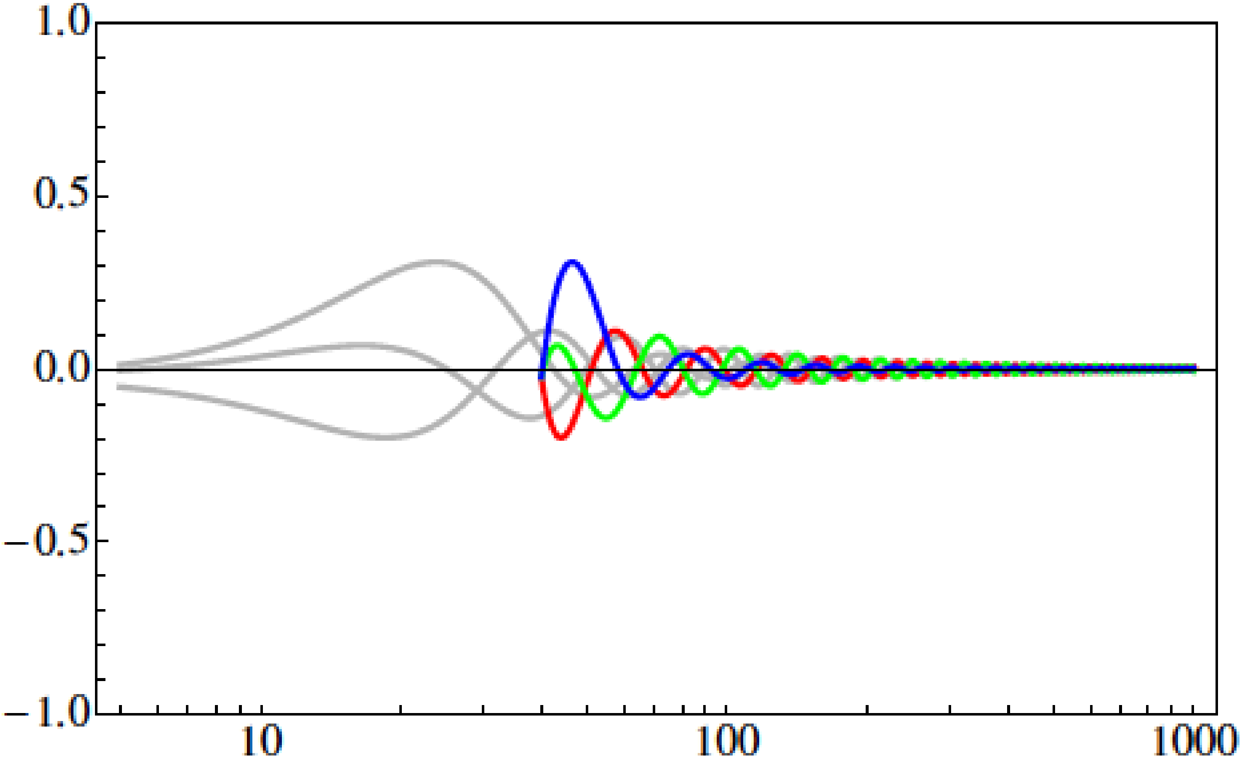
Overlap reduction function (KV)



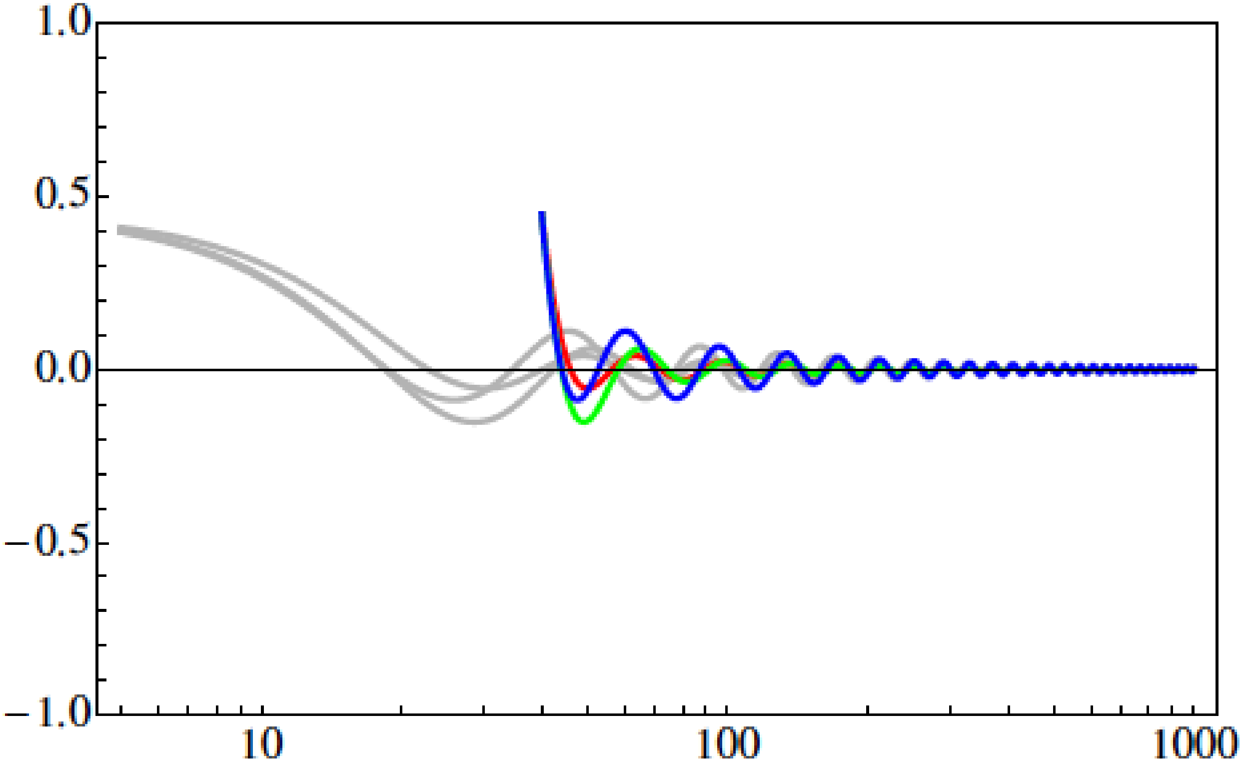
Overlap reduction function (LV)



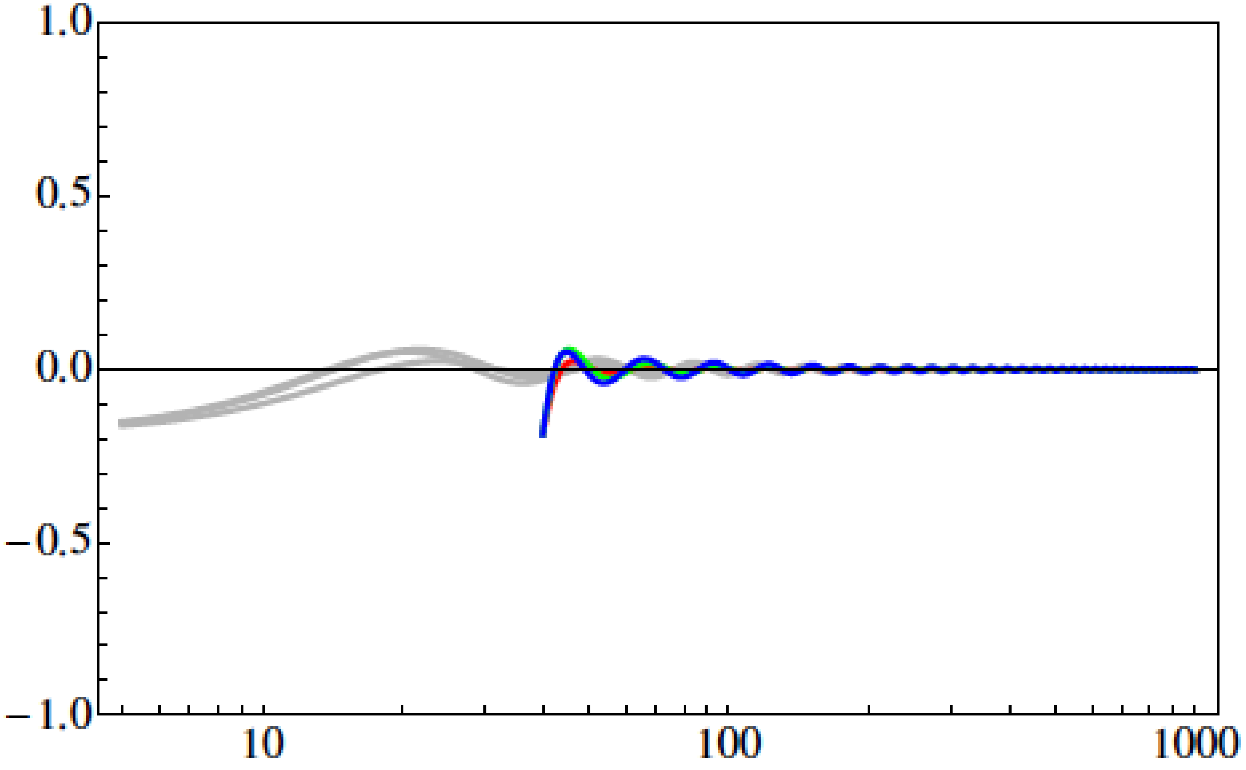
Overlap reduction function (HV)



Overlap reduction function (KH)



Overlap reduction function (KL)



Mode separation

Correlation signal of GW at a frequency bin

$$Z_{IJ}(f) \equiv \Omega^T_{\rm gw}(f)\gamma^T_{IJ}(f) + \Omega^V_{\rm gw}(f)\gamma^V_{IJ}(f) + \xi\Omega^S_{\rm gw}(f)\gamma^S_{IJ}(f)$$

In principle, three detectors allow us to separate the modes.

$$\begin{pmatrix} Z_{12} \\ Z_{23} \\ Z_{31} \end{pmatrix} = \mathbf{\Pi} \begin{pmatrix} \Omega^T_{\rm gw} \\ \Omega^V_{\rm gw} \\ \xi\Omega^S_{\rm gw} \end{pmatrix} \qquad \mathbf{\Pi} \equiv \begin{pmatrix} \gamma^T_{12} & \gamma^V_{12} & \gamma^S_{12} \\ \gamma^T_{23} & \gamma^V_{23} & \gamma^S_{23} \\ \gamma^T_{31} & \gamma^V_{31} & \gamma^S_{31} \end{pmatrix}$$

Mode separation

$$\begin{pmatrix} \Omega^T_{\rm gw} \\ \Omega^V_{\rm gw} \\ \xi\Omega^S_{\rm gw} \end{pmatrix} = \mathbf{\Pi}^{-1} \begin{pmatrix} Z_{12} \\ Z_{23} \\ Z_{31} \end{pmatrix}$$

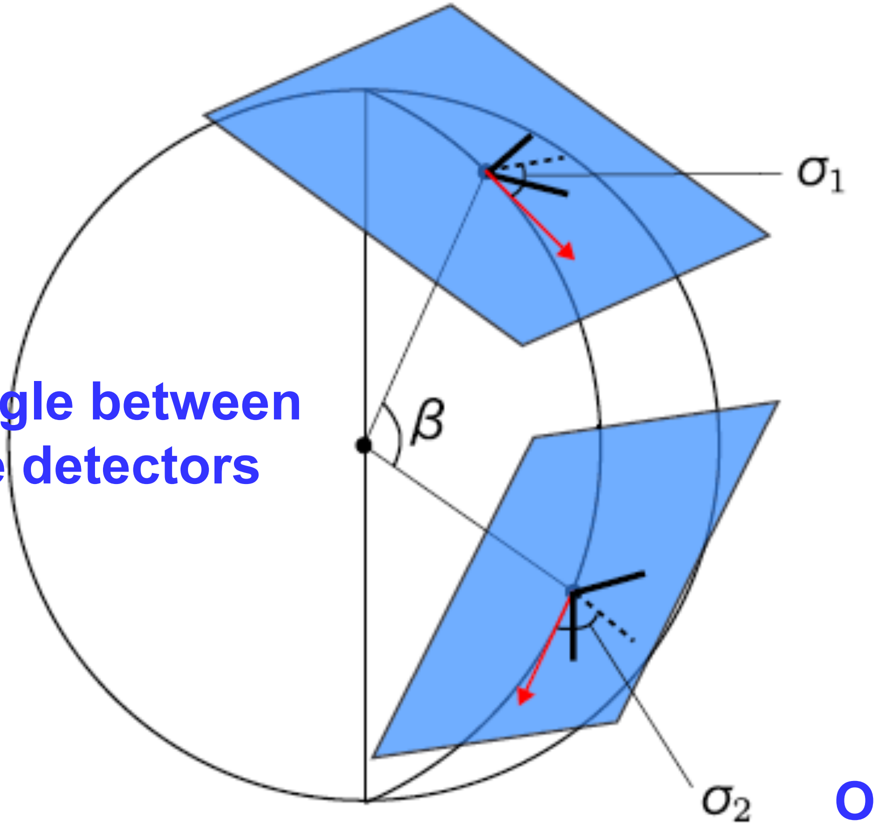
Separability strongly depends on $\det \mathbf{\Pi}$. $\mathrm{SNR}(f) \propto \det \mathbf{\Pi}(f)$

If the modes are not separable ($\det \mathbf{\Pi} = 0$),
GWB signal does not contribute to the SNR at the frequencies.

Detectors & Earth coordinate

5 advanced detectors on the ground.
[A=AIGO, C=LCGT, H=AdvLIGO(H1), L=AdvLIGO(L1), V=AdvVIRGO.]

Detector pair is completely characterized by three parameters.



Orientation of det. 1

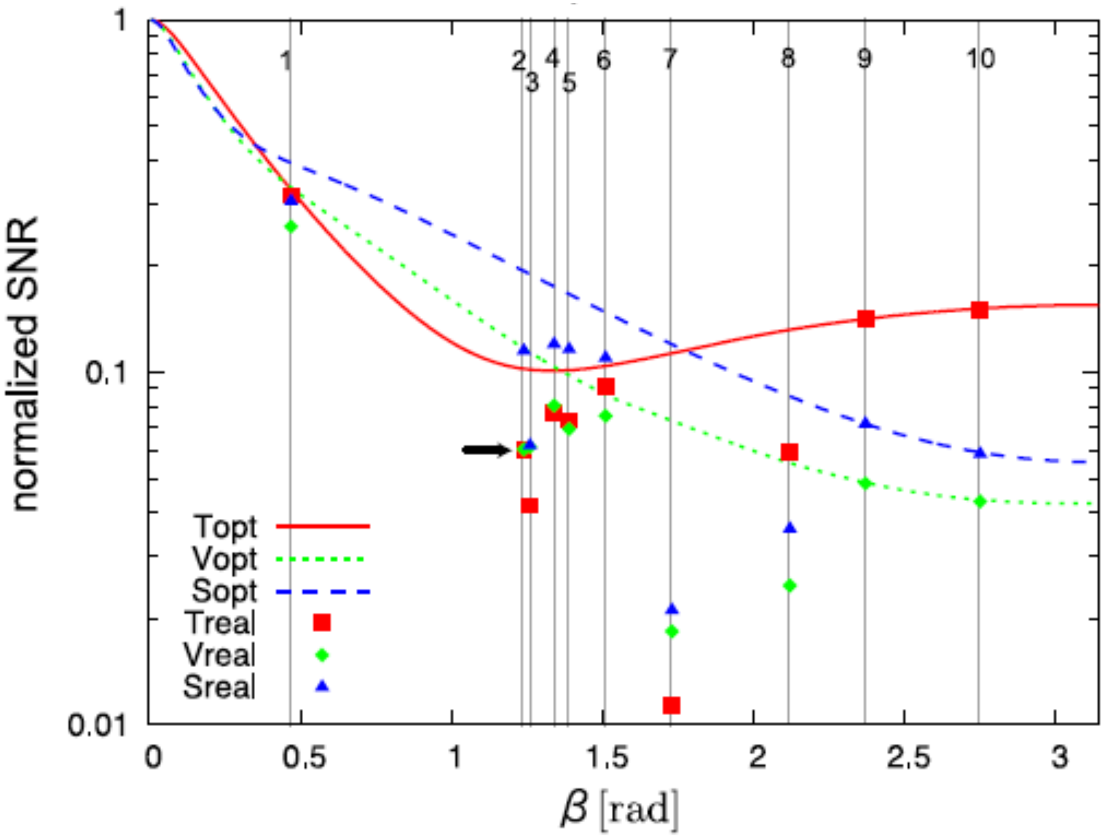
Angle between the detectors

Orientation of det. 2

SNR (single pol.)

Assume that GWB has only one polarization mode.

$$\text{normalized SNR} = \frac{\text{SNR}(\beta)}{\text{SNR}(\beta = 0)},$$



Detector pair

1=HL,
2=AC,
3=CH,
4=LV,
5=HV,
6=CV,
7=CL,
8=AV,
9=AH,
10=AL.

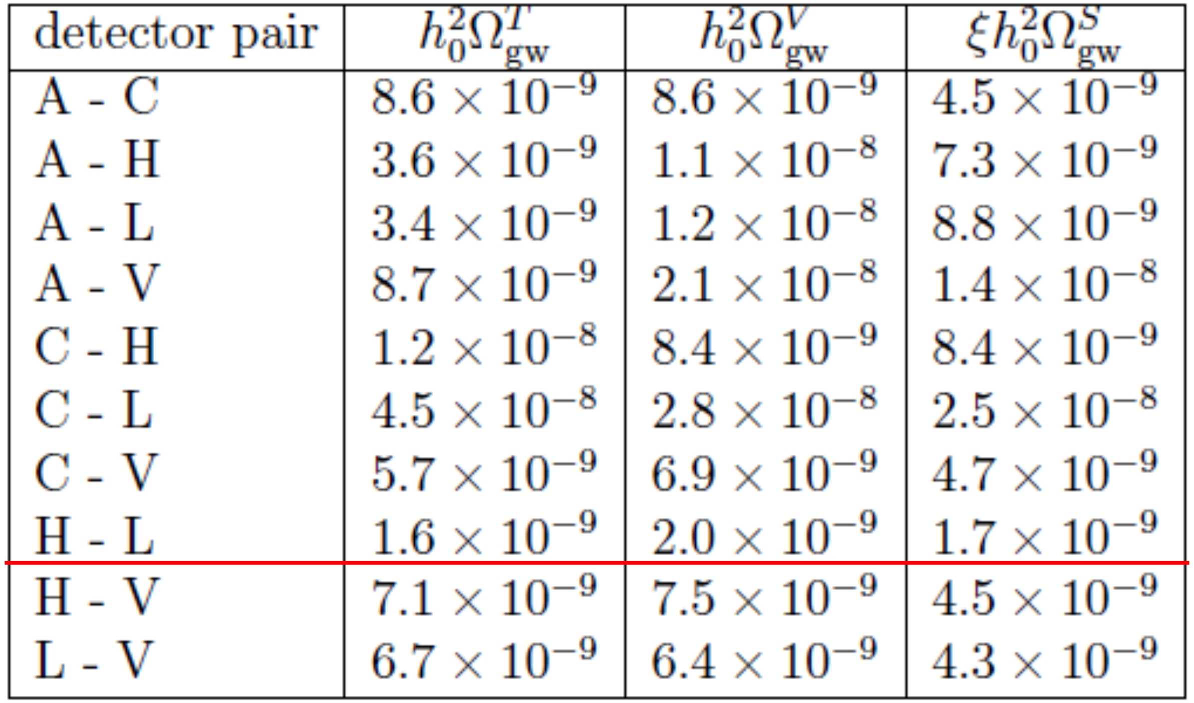
This is also true for current detectors.

Detectable GWB with single pol.

5 advanced detectors on the ground.
[A=AIGO, C=LCGT, H=AdvLIGO(H1), L=AdvLIGO(L1), V=AdvVIRGO.]

All detectors have the same noise spectrum as that of AdvLIGO.

Observation time $T = 3\text{yr.}$ $\text{SNR} = 5$



detector pair	$h_0^2\Omega_{\rm gw}^T$	$h_0^2\Omega_{\rm gw}^V$	$\xi h_0^2\Omega_{\rm gw}^S$
A - C	8.6×10^{-9}	8.6×10^{-9}	4.5×10^{-9}
A - H	3.6×10^{-9}	1.1×10^{-8}	7.3×10^{-9}
A - L	3.4×10^{-9}	1.2×10^{-8}	8.8×10^{-9}
A - V	8.7×10^{-9}	2.1×10^{-8}	1.4×10^{-8}
C - H	1.2×10^{-8}	8.4×10^{-9}	8.4×10^{-9}
C - L	4.5×10^{-8}	2.8×10^{-8}	2.5×10^{-8}
C - V	5.7×10^{-9}	6.9×10^{-9}	4.7×10^{-9}
H - L	1.6×10^{-9}	2.0×10^{-9}	1.7×10^{-9}
H - V	7.1×10^{-9}	7.5×10^{-9}	4.5×10^{-9}
L - V	6.7×10^{-9}	6.4×10^{-9}	4.3×10^{-9}

most sensitive

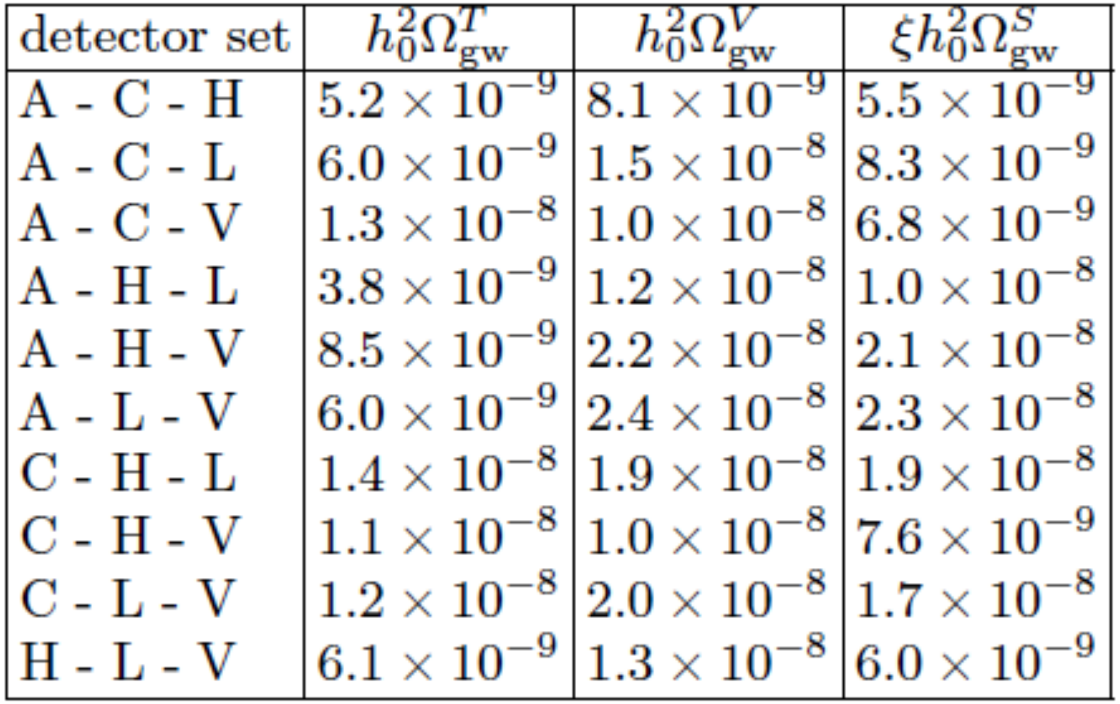
All modes are detectable with almost the same SNRs.

Detectable GWB after mode separation

- Advanced detectors on the ground

[A=AIGO, C=LCGT, H=AdvLIGO(H1), L=AdvLIGO(L1), V=AdvVIRGO.]

- Assume the same noise spectrum as that of AdvLIGO.



detector set	$h_0^2\Omega_{\rm gw}^T$	$h_0^2\Omega_{\rm gw}^V$	$\xi h_0^2\Omega_{\rm gw}^S$
A - C - H	5.2×10^{-9}	8.1×10^{-9}	5.5×10^{-9}
A - C - L	6.0×10^{-9}	1.5×10^{-8}	8.3×10^{-9}
A - C - V	1.3×10^{-8}	1.0×10^{-8}	6.8×10^{-9}
A - H - L	3.8×10^{-9}	1.2×10^{-8}	1.0×10^{-8}
A - H - V	8.5×10^{-9}	2.2×10^{-8}	2.1×10^{-8}
A - L - V	6.0×10^{-9}	2.4×10^{-8}	2.3×10^{-8}
C - H - L	1.4×10^{-8}	1.9×10^{-8}	1.9×10^{-8}
C - H - V	1.1×10^{-8}	1.0×10^{-8}	7.6×10^{-9}
C - L - V	1.2×10^{-8}	2.0×10^{-8}	1.7×10^{-8}
H - L - V	6.1×10^{-9}	1.3×10^{-8}	6.0×10^{-9}

$T = 3\mathrm{yr.}$

$\mathrm{SNR} = 5$

Mode separation hardly degrade the SNRs.
(Almost the same sensitivity to GWB in the presence of a single pol. mode)