Yingli Zhang, JGRG 22(2012)111422

"Tunneling fields in non-linear massive gravity"

RESCEU SYMPOSIUM ON

GENERAL RELATIVITY AND GRAVITATION

JGRG 22

November 12-16 2012

Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

RESCEU SYMPOSIUM ON GENERAL RELATIVITY AND GRAVITATION

JGRG22

November 12 - 16 2012

Hawking-Moss instantons in non-linear Massive Gravity

Ying-li Zhang

YITP , Kyoto University

Cooperator: Ryo Saito, Misao Sasaki

arxiv: 1210.6224

In Massive Gravity (MG), the mass of graviton is non-vanishing, which breaks the gauge invariance

Generally speaking, the dof is

$$10 - 4 = 6$$

Constraint equations

Helicity $\pm 2, \ \pm 1, \ 0,$

Recently, a non-linear construction of massive gravity theory (dRGT) is proposed, where the BD ghost is removed by specially designed non-linear terms, so that the lapse function N becomes a Lagrangian Multiplier, which removes the ghost degree of freedom.

Non-linear Massive Gravity (dRGT)

C. de Rham, G. Gabadadze, Phys. Rev. D 82, 044020 (2010);
C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett 106, 231101 (2011);
S. F. Hassan and R. A. Rosen, JHEP 1107, 009 (2011)

$$S_{MG} = \int d^4x\, \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right] ,$$

where $[\mathcal{K}] = tr\,(K^\nu_\mu)$

$$\mathcal{L}_2 = \frac{1}{2} \left([\mathcal{K}]^2 - [\mathcal{K}^2] \right) ,$$

$$\mathcal{L}_3 = \frac{1}{6} \left([\mathcal{K}]^3 - 3\,[\mathcal{K}]\,[\mathcal{K}^2] + 2\,[\mathcal{K}^3] \right) ,$$

$$\mathcal{L}_4 = \frac{1}{24} \left([\mathcal{K}]^4 - 6\,[\mathcal{K}]^2\,[\mathcal{K}^2] + 3\,[\mathcal{K}^2]^2 + 8\,[\mathcal{K}]\,[\mathcal{K}^3] - 6\,[\mathcal{K}^4] \right) ,$$

$$\mathcal{K}^\mu_\nu \equiv \delta^\mu_\nu - \sqrt{g^{\mu\sigma} G_{ab}(\phi) \partial_\nu \phi^a \partial_\sigma \phi^b}.$$

fiducial metric

Stuckelberg field

Self-accelerating solution is found in context of non-linear massive gravity, where two branches exist with effective cosmological constant consists of a contribution from mass of graviton. A. E. Gumrukcuoglu et. al. JCAP 106, 231101(2011);

$$\Lambda_{\pm} = -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right)\pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right],$$

There seems to be some hope to explain the current acceleration, but…

still there exists the Cosmological Constant Problem

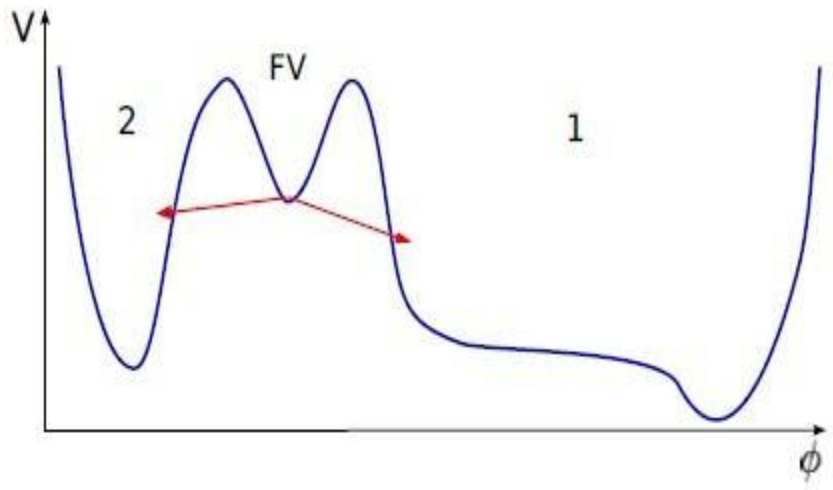
A possible resolution: Anthropic Landscape of Vacua

S. Weinberg, Rev. Mod. Phys. 61, 1 (1989)

- The Landscape has several local minima;

- the fields can (and will) tunnel from a metastable minimum to a lower one;

- this process is driven by instanton.



S. Coleman and F. de Luccia, Phys.Rev. D21, 3305, (1980)

→ it is interesting to investigate how the stability of a vacuum is determined in the context of non-linear Massive Gravity

Setup of model

$$S = S_{MG} + S_m,$$

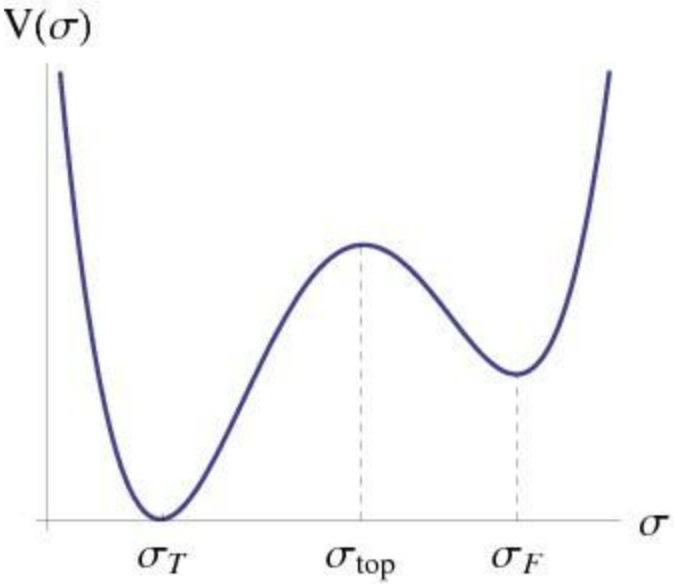
$$S_m \equiv -\int d^4x\, \sqrt{-g}\left[\frac{1}{2}(\partial\sigma)^2 + V(\sigma)\right],$$

- potential $V(\sigma)$

  local minima: σ_F

  global minima: σ_T

  local max: $\sigma_{\rm top}$



- tunneling probability per unit time per unit volume

$$\Gamma/V = Ae^{-B},$$

$$B = S_E[g_{\mu\nu,B}, \phi_B] - S_E[g_{\mu\nu,F}, \phi_F],$$

bounce solution ↑ ‘false vacuum’ ↑

Lowest action ↓

usually, bounce solutions are explored by assuming an O(4) symmetry

➢ spacetime metric: Euclidean

$$g_{\mu\nu}dx^\mu dx^\nu = N(\xi)^2 d\xi^2 + a(\xi)^2 \Omega_{ij} dx^i dx^j,$$

$$\Omega_{ij} \equiv \delta_{ij} + \frac{K\delta_{il}\delta_{jm}x^l x^m}{1 - K\delta_{lm}x^l x^m}, \quad K > 0$$

Note: the fiducial metric may not respect the symmetry

- fiducial metric: deSitter

$$G_{ab}(\phi)d\phi^a d\phi^b \equiv -(d\phi^0)^2 + b(\phi^0)^2 \Omega_{ij} d\phi^i d\phi^j,$$

$$b(\phi^0) \equiv F^{-1}\sqrt{K}\cosh(F\phi^0).$$

↑ fiducial Hubble parameter

⟶ the O(4)-symmetric solutions are obtained by setting

$$\phi^0 = f(\xi), \quad \phi^i = x^i.$$

Inserting these ansatz into the action, we obtain the constraint equation by varying with respect with f

$$(i\dot{a} + N b_{,f}) \left[\left(3 - \frac{2b}{a}\right) + \alpha_3 \left(1 - \frac{b}{a}\right)\left(3 - \frac{b}{a}\right) + \alpha_4 \left(1 - \frac{b}{a}\right)^2 \right] = 0,$$

$$\dot{a} \equiv \frac{da}{d\xi} \qquad b_{,f} \equiv \frac{db}{df} = \sqrt{K}\sinh(Ff)$$

→ Branch I $\quad N b_{,f} = -i\dot{a},$ (equivalent to branch II)

Branch II $\quad \left(3 - \frac{2b}{a}\right) + \alpha_3 \left(1 - \frac{b}{a}\right)\left(3 - \frac{b}{a}\right) + \alpha_4 \left(1 - \frac{b}{a}\right)^2 = 0.$

→ $$b = X_{\pm} a, \qquad X_{\pm} \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}.$$

Friedmann equation & EOM for tunneling field

$$\begin{cases} \dfrac{3}{a^2}\left(\dfrac{da}{d\tau}\right)^2 - \dfrac{3K}{a^2} = \dfrac{1}{2}\left(\dfrac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm, \\[2ex] \dfrac{d^2\sigma}{d\tau^2} + 3\left(\dfrac{da}{d\tau}\right)\dfrac{d\sigma}{d\tau} - V_{,\sigma}(\sigma) = 0 \end{cases}$$

where $\mathrm{d}\tau \equiv N\mathrm{d}t,$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right],$$

Hawking-Moss(HM) solution

- HM solution can be found at the local maximum of the potential $\sigma = \sigma_{\rm top}$ under boundary condition $a_{\rm HM}(H_{\rm HM}\tau = \pm\pi/2) = 0\,,$

$$a_{\rm HM}(\tau) = H_{\rm HM}^{-1}\sqrt{K}\cos\left(H_{\rm HM}\tau\right)\ ,$$

$$d\tau \equiv N d\xi$$

$$H_{\rm HM} \equiv \sqrt{\frac{\Lambda_\pm + V(\sigma_{top})}{3}}$$

- inserting this result into the Euclidean action and evaluate by integrating in the range $H_{\rm HM}\tau = -\pi/2 \longrightarrow \pi/2$, we finally express the HM action

$$Y_\pm \equiv 3(1-X_\pm) + 3\alpha_3(1-X_\pm)^2 + \alpha_4(1-X_\pm)^3,$$

$$S_E[a_{\rm HM}, \sigma_{\rm top}] = -\frac{8\pi^2}{H_{\rm HM}^2}\left[1 - \frac{Y_\pm X_\pm}{6\alpha^4}\left(\frac{m_g}{H_{\rm HM}}\right)^2\left(2 - \sqrt{1-\alpha^2}(2+\alpha^2)\right)\right]$$

standard HM solution

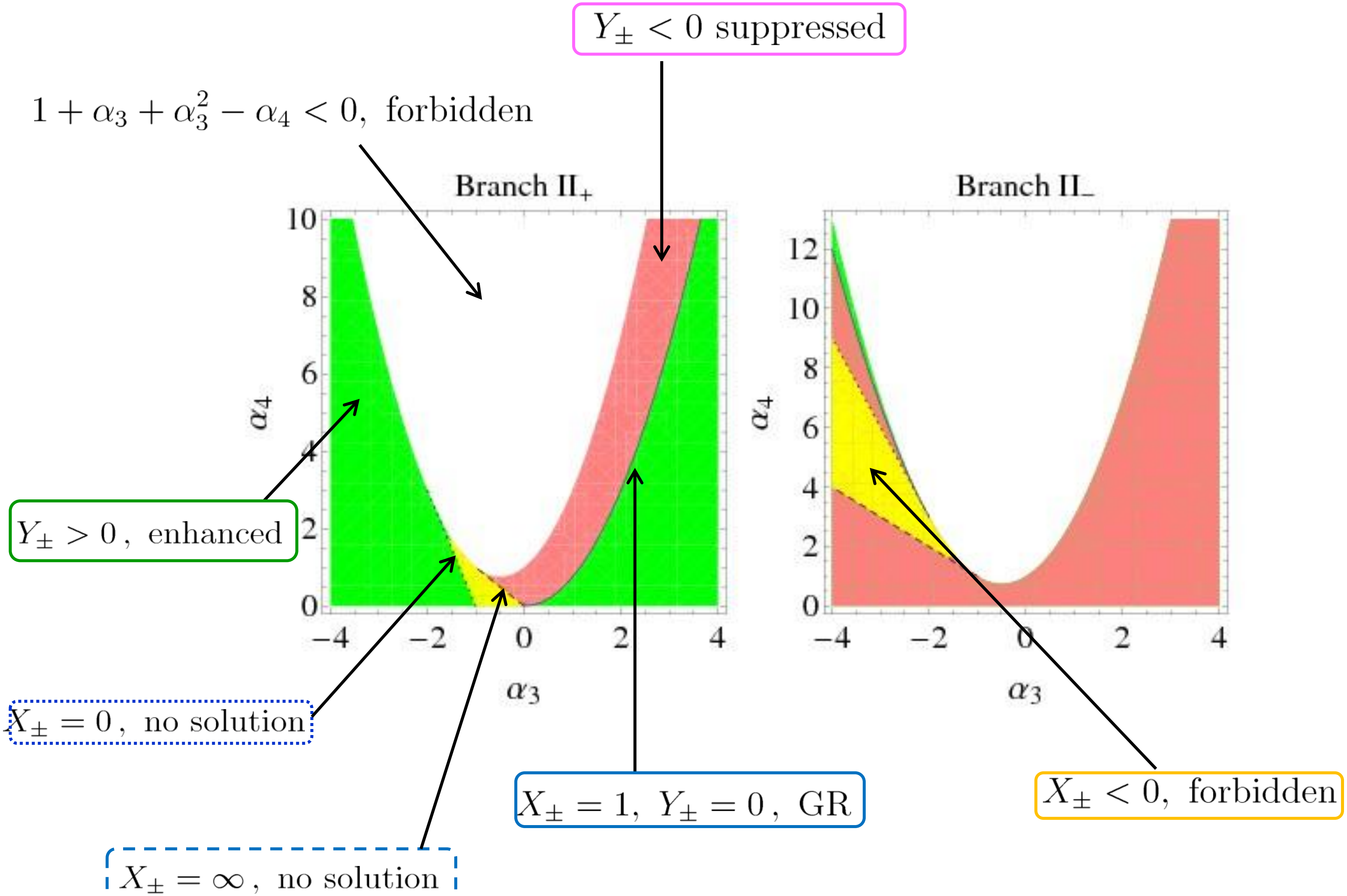
Correction due to the mass of graviton

$$\alpha \equiv X_\pm \frac{F}{H_{\rm HM}} \in [0,1]$$

Comparing with GR case, recalling the tunneling probability $\Gamma/V = Ae^{-B}$, we obtains:

$$\Delta B \equiv B^{\rm (MG)} - B^{\rm (GR)} = CY_\pm\,, \quad C < 0$$

Tunneling rate is enhanced for $Y_\pm > 0$,
suppressed for $Y_\pm < 0$.



$Y_\pm < 0$ suppressed
$1+\alpha_3+\alpha_3^2-\alpha_4<0$, forbidden
Branch II$_+$
Branch II$_-$
α_4
α_3
$Y_\pm > 0$, enhanced
$X_\pm = 0$, no solution
$X_\pm = 1$, $Y_\pm = 0$, GR
$X_\pm < 0$, forbidden
$X_\pm = \infty$, no solution

Summary and future work

- We constructed a model in which the tunneling field minimally couples to the non-linear massive gravity;
- corrections of HM solution from mass term is found, which implies suppression or enhancement of tunneling rate, depending on the choices of parameters;
- analysis of Colemann-DeLuccia solutions is under work;
- it would be interesting to investigate the case where the tunneling field couples to the non-linear massive gravity non-minimally.

• Appendix

$$Y_\pm \equiv 3(1-X_\pm) + 3\alpha_3(1-X_\pm)^2 + \alpha_4(1-X_\pm)^3,$$

$$S_E[a_{\rm HM},\sigma_{\rm top}] = \int d^3x\sqrt{\Omega}\int_{-\pi/2H_{\rm HM}}^{\pi/2H_{\rm HM}} d\tau\; a_{HM}^3\left(2\Lambda_{\pm,\rm eff} - \frac{6K}{a_{\rm HM}^2} + m_g^2 Y_\pm\sqrt{-\left(\frac{df_{\rm HM}}{d\tau}\right)^2}\right)$$

$$\Lambda_{\pm,\rm eff} \equiv \Lambda_\pm + V(\sigma_{\rm top})$$

$$b_{\rm HM} = F^{-1}\sqrt{K}\cosh(F f_{\rm HM}) = X_\pm a_{\rm HM} \Longrightarrow \left(\frac{df_{\rm HM}}{d\tau}\right)^2 = \frac{X_\pm^2\sin^2(H_{\rm HM}\tau)}{\alpha_{\rm HM}^2\cos^2(H_{\rm HM}\tau)-1}$$

$$\alpha_{\rm HM} \equiv X_\pm\frac{F}{H_{\rm HM}} \in [0,1]$$

- Note: for the Minkowski fiducial metric, $b_{\rm HM} = \sqrt{K} f_E$, by setting $f_E = -if$

$$\frac{df_{E,\rm HM}}{d\tau} = -X_\pm \sin(H_{\rm HM}\tau)$$

so we recover the Minkowski one by setting $\alpha_{\rm HM} = 0$.