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“Testing gravity with galaxy cluster”
Testing gravity with galaxy cluster

Tatsuya Narikawa
(RESCEU, U-Tokyo)

Outline

- Modified gravity
- Screening mechanism
- Cluster lensing
- Summary

Based on
TN, Yamamoto, JCAP (2012)
TN, Kobayashi, Yamauchi, Saito, work in progress
Modified Gravity

- Motivation: Discovery of current cosmic cosmic acceleration
  [Riess, Schmidt et al. ‘98, Perlmutter et al. ‘99]
  →Breakdown of GR at large-scales?

- As an alternative to dark energy

- Modified gravity with extra fields $\phi$
  Scalar-tensor gravity, $f(R)$ gravity, DGP model, Galileon model, Decoupling-limit of Ghost-free massive gravity, etc.

[Mukohyama-san’s talk]
Local Gravity Constraints

- In MG, new d.o.f. $\phi$ mediates fifth-force deviation from GR on large-scales.

- However, local gravity constraints are strong:
  - e.g. $\gamma_{\text{PPN}}^{-1}=(2.1\pm2.3)\times10^{-5}$ by time-delay [Bertotti et al. ‘03]

- MG need to evade local gravity tests with screening mechanisms!

[Cassini spacecraft]
Screening Mechanisms of $\phi$ in Local Region

- **Kinetic type**: Vainshtein mechanism [Vainshtein ‘72]
  - Large kinetic term $\Leftrightarrow$ weak coupling: **Galileon-type**
- **Potential type**: Chameleon mechanism, Symmetron
  [Khoury & Weltman ‘04, Hinterbichler & Khoury ‘10]
  - Large curvature of potential $\Leftrightarrow$ massive scalar field $m_\phi$
Observational Test of Modified Gravity

- Whether the screening mechanism works completely or not around high-density regions?
- Gravitational lensing of galaxy clusters may be useful to test modified gravity

Key: lensing potential \[ \triangle \Phi_+ \equiv \triangle (\Phi + \Psi)/2 \]

[TN, Yamamoto, ‘12]

- Large amount of data of cluster lensing will be provided by Hyper Suprime-Camera (HSC)
  [Takada-san’s talk]
Stacked Cluster Lensing Analysis

The average distortion profile

- is less sensitive to individual substructures/asphericity
- helps to improve the S/N

[Okabe, Takada, Umetsu, Futamase, Smith ’10]

- may be also useful to test modified gravity
Galileon-type Modified Gravity

• Motivation: decoupling limit of DGP model
  \[ \mathcal{L}_{\text{int}} \sim X \Box \phi : \text{higher-derivative term} \]
  where \( \phi \) : scalar field, \( X \equiv -\frac{1}{2} (\partial \phi)^2 \)

enjoy Galileon shift symmetry:
  \[ \partial_\mu \phi \rightarrow \partial_\mu \phi + c_\mu \]
→ 「 • Cosmic acceleration
  • 2nd order EOM
  • Vainshtein mechanism 」

[DLofDGP] Luty, Porrati & Rattazzi ‘03, Nicolis & Rattazzi ’04
Vainshtein Mechanism in Simple Model

\[ \mathcal{L}_\phi = 3 \phi \Box \phi - \frac{2}{\Lambda^3} (\partial \phi)^2 \Box \phi + \frac{2}{M_{\text{Pl}}} \phi T \]

\( \phi \text{EOM} \)

\[ 3 \Box \phi + \frac{1}{\Lambda^3} \left( (\Box \phi)^2 - (\partial_\mu \partial_\nu \phi)^2 \right) = -\frac{1}{M_{\text{Pl}}} T \]

At \( r << r_V \): Large kinetic term \( \Leftrightarrow \) Weak coupling to matter

\[ \phi'(r) \sim \left( \frac{M}{2M_{\text{Pl}}} \frac{\Lambda^3}{r} \right)^{1/2} \ll \partial_r \Phi_N \leftarrow \text{screened!} \]

where Vainshtein radius: \( r_V \equiv (M/M_{\text{Pl}}) \Lambda^{-1} \)

[Luty, Porrati & Rattazzi ‘03, Nicolis & Rattazzi ‘04]
The Most General Second-Order Scalar-Tensor Theory [Horndesky '74]

\[ S = \int d^4 x \sqrt{-g}[\mathcal{L} + \mathcal{L}_m] \]

assume matter do not directly couple to \( \phi \)

\[ \mathcal{L} = K(\phi, X) - G_3(\phi, X) \Box \phi \]

\[ + G_4(\phi, X) R + G_{4X} \times (\text{field derivatives}) \]

\[ + G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi \]

\[ - \frac{1}{6} G_{5X} \times (\text{field derivatives}) \]

where \( G_{iX} \equiv \partial G_i / \partial X \)

4 arbitrary functions of \( \phi, X \)

is equivalent to the generalized galileon

[Deffayet et al. '11, Kobayashi, Yamaguchi, Yokoyama '11]
Background solution

\[ ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \]

\[ \phi = \phi_0 = \text{const}, \quad X = 0 \]

In order to admit this solution, we require that

\[ K(\phi_0, 0) = 0, \quad K_\phi(\phi_0, 0) = 0 \]

Spherical symmetric perturbations produced by a nonrelativistic matter

\[ ds^2 = -[1 + 2\Phi(r)] dt^2 + [1 - 2\Psi(r)] \delta_{ij} dx^i dx^j \]

\[ \phi = \phi_0 + \phi(r) \]

All the coefficients are evaluated at the background. We will ignore the mass term \( K_{\phi\phi} \).
Static-Spherically Symmetric Configurations

Metric EOM:

\[
\frac{M_{\text{Pl}}^2}{2} \frac{(r^2 \Psi')'}{r^2} - M_{\text{Pl}} \xi \frac{(r^2 \phi')'}{2r^2} - \frac{M_{\text{Pl}}}{\Lambda^3} \alpha \frac{[r(\phi')^2]'}{2r^2} - \frac{3M_{\text{Pl}}}{\Lambda^6} \beta \frac{[(\phi')^3]'}{6r^2} = -\frac{1}{4} T_t^t
\]

\[
M_{\text{Pl}}^2 \left( \Psi' - \Phi' \right) - 2M_{\text{Pl}} \xi \phi' - \frac{M_{\text{Pl}}}{\Lambda^3} \alpha \left( \phi' \right)^2 = 0
\]

ΦEOM:

\[
\eta \frac{(r^2 \phi')'}{r^2} - 2 \frac{\mu}{\Lambda^3} \frac{[r(\phi')^2]'}{r^2} + 2M_{\text{Pl}} \xi \frac{[r^2(2\Psi - \Phi)']}{r^2} + 4 \frac{M_{\text{Pl}}}{\Lambda^3} \alpha \frac{[r\phi'(\Psi' - \Phi')]}{r^2} +
\]

\[
2 \frac{\nu}{\Lambda^6} \frac{[(\phi')^3]'}{r^2} - 6M_{\text{Pl}} \frac{[(\phi')^2 \Phi']}{\Lambda^6} = 0
\]

where six dimensionless parameters: \( \xi, \eta, \mu, \alpha, \nu, \beta \) are functions of \( K_X, G_{3\phi}, G_{3X}, G_{4X}, G_{5\phi}, \ldots \)

(cf. Vainshtein mechanism under considering background evolution [Kimura, Kobayashi, Yamamoto ‘12])
Quintic Scalar-Field Equation

We arrive at

\[ P(x, A) := \xi A(r) + \left( \frac{\eta}{2} + 3\xi^2 \right) x + \left[ \mu + 6\alpha\xi - 3\beta A(r) \right] x^2 \]
\[ + \left( \nu + 2\alpha^2 + 4\beta\xi \right) x^3 - 3\beta^2 x^5 = 0 \]

where we define

\[ x = \frac{1}{\Lambda^3} \frac{\varphi'}{r}, \quad A(r) = \frac{1}{M_{\text{Pl}} \Lambda^3} \frac{M(r)}{8\pi r^3} \]

which are dimensionless.

\( M(r) \) is the enclosed mass.
Scalar-Field Equation [cf. Sbisa, Niz, Koyama, Tasinato ‘12]

We solve

\[
P(x, A) := \xi A(r) + \left( \frac{\eta}{2} + 3\xi^2 \right) x + \left[ \mu + 6\alpha\xi - 3\beta A(r) \right] x^2
\]

\[
+ \left( \nu + 2\alpha^2 + 4\beta\xi \right) x^3 - 3\beta^2 x^5 = 0
\]

for the inner region (A>>1) and the outer region (A<<1).

We then derive the conditions under which the two solutions are matched smoothly in an intermediate region.
Outer Solution: Asymptotically Flat

- In the outer region \((A<<1)\), there is always a decaying solution,

\[
x \approx x_f := -\frac{2\xi A(r)}{\eta + 6\xi^2}
\]

which is obtained by neglecting the nonlinear terms in \(P(x,A)=0\).

The other solutions do not correspond to an asymptotically flat spacetime.
Inner Solution: Vainshtein

- In the inner region (A>>1), P=0 reduces to
  \[ P(x, A) \approx \xi A - 3\beta A x^2 - 3\beta^2 x^5 \approx 0 \]

We have a solution (\(\xi \beta > 0\)):

\[
x \approx x_\pm := \pm \sqrt{\frac{\xi}{3\beta}} = \text{const}
\]

(P(x-, A) does not depend on A.)

For this solution, we have Newtonian behavior:

\[
\Psi'/r \approx \Phi'/r \propto A
\]

This is therefore the Vainshtein solution.

For simplicity we focus on the case of \(\xi > 0\).
Matching of Two Solutions

In this case, \( P(x) = 0 \) has a single real root in \((x-, 0)\) for any \( A > 0 \).
Decoupling Limit of Massive Gravity
[de Rham, Heisenberg ‘11]

Decoupling limit: $M_{\text{pl}} \rightarrow \infty$, $m_g \rightarrow 0$, $\Lambda^3 = M_{\text{pl}} m_g^2 = \text{fixed}$.

The corresponds to

$$K = 0 = G_3, \quad G_4 = \frac{M_{\text{pl}}^2}{2} + M_{\text{Pl}} \phi + \frac{M_{\text{Pl}}}{\Lambda^3} \alpha X, \quad G_5 = -3 \frac{M_{\text{Pl}}}{\Lambda^6} \beta X$$

One finds

$$\eta = \mu = \nu = 0, \quad \xi = 1, \quad \alpha \neq 0, \quad \beta \neq 0$$
The condition of smooth matching of the two solutions:

\[ \alpha < 0 \text{ or } \frac{\sqrt{\beta}}{\alpha} \geq \sqrt{\frac{5 + \sqrt{13}}{24}} \sim 0.6 \]

Explore the observationally allowed region with cluster lensing.
Cluster Lensing

Mass map (shear)

Surface mass density $\Sigma_S(r_{\text{perp}})$

$M_{\text{vir}} = 1.5 \times 10^{15} M_{\odot}/h$
$c_{\text{vir}} = 7.7$

[Bartelmann’s talk]

[Umetsu et al. (2011)]

Gravitational Lensing in Massive Gravity

• Geodesic equation

\[
\frac{d^2}{d\chi^2} (\chi \theta^i) = 2 \Phi_+^i, i = 1, 2 \quad \Phi_+ \equiv (\Phi + \Psi)/2
\]

• Surface mass density \( \Sigma_S(r_{\text{perp}}) \propto k(r_{\text{perp}}) \)

\[
\Sigma_S \propto \int_0^\infty dZ \Delta^{(2D)} \Phi_+ \quad r = \sqrt{r_{\perp}^2 + Z^2}
\]

where lensing potential in massive gravity:

\[
\Delta \Phi_+ = \frac{\Lambda^3}{M_{\text{Pl}}} \left[ (\alpha x^2 + 2\beta x^3 + 2A) \frac{r^3}{2r^2} \right]'
\]

Assume \( \delta \rho(r) \) as NFW profile: \( \rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2} \)
Surface Mass Density in Massive Gravity

Parameters: \{\alpha, \beta, \Lambda; \rho_s, r_s\}
(\Lambda = 0.01/H_0, \rho_s, r_s \text{ fixed})

Dip appears at \( r \sim r_V := (r_s M_{Pl}/\Lambda^3)^{1/3} \)
→ allows us to put constraint
Summary

• Static-spherically symmetric solutions of $\phi$EOM in the context of Horndeski’s theory.
• We explore the possibility of testing modified gravity exhibiting the Vainshtein mechanism against observation of cluster lensing.

• Key effects on cluster lensing $\Sigma_s$ is that $x'(<=\phi'')$ can substantially be large at the transition from screened to unscreened regions.

→ This allows us to put observational constraints on modified gravity.
Thank you for your attention