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“Testing gravity with galaxy cluster”

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Testing gravity with galaxy cluster

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Based on

TN, Yamamoto, JCAP (2012)

TN, Kobayashi, Yamauchi, Saito, work in progress

Outline

- Modified gravity
- Screening mechanism
- Cluster lensing
- Summary

Modified Gravity

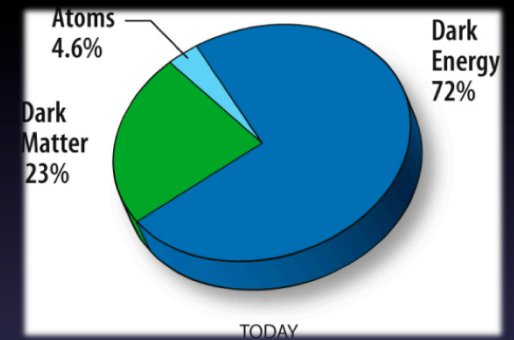
- Motivation: Discovery of current cosmic acceleration
[Riess, Schmidt et al. '98, Perlmutter et al. '99]

$$\ddot{a} > 0$$

→ Breakdown of GR at large-scales ?

- As an alternative to dark energy

- Modified gravity with extra fields ϕ



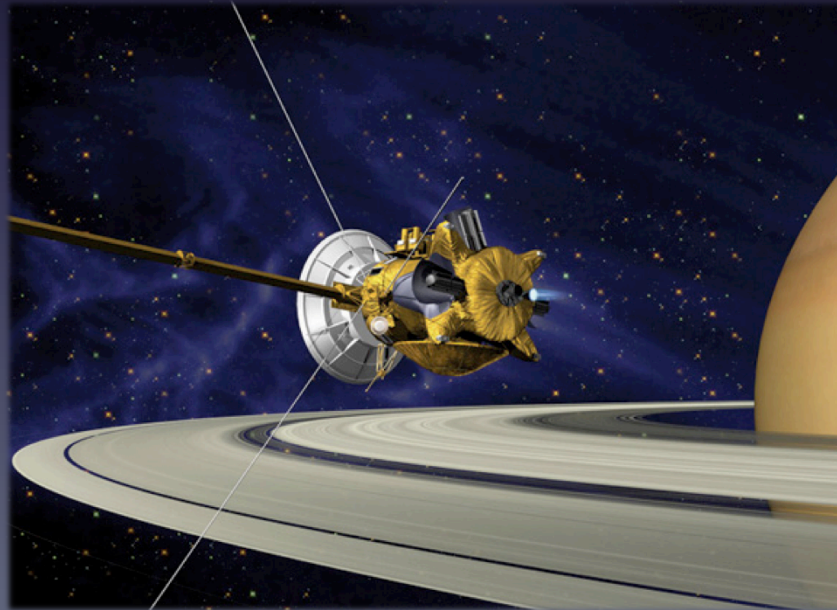
[WMAP]

Scalar-tensor gravity, $f(R)$ gravity, DGP model, Galileon model, Decoupling-limit of Ghost-free massive gravity, etc.

[Mukohyama-san's talk]

Local Gravity Constraints

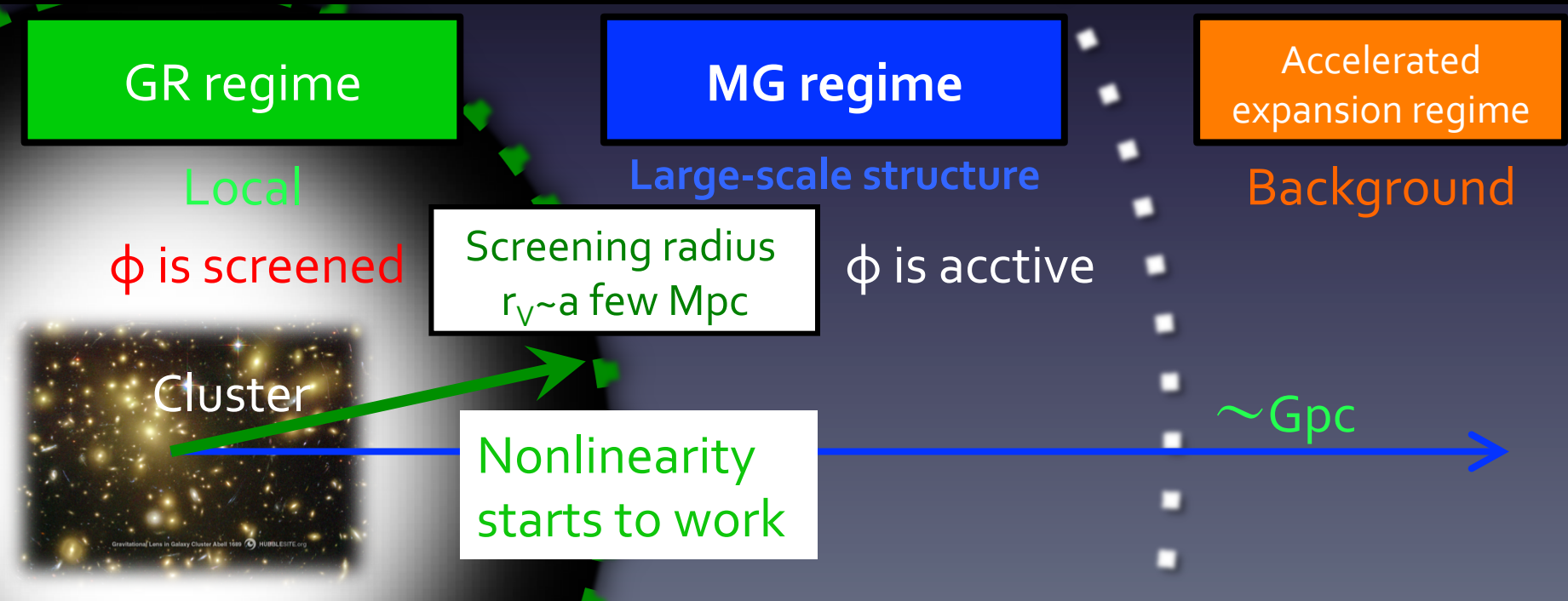
- In MG, new d.o.f. ϕ mediates fifth-force.
→ deviation from GR on large-scales.
- However, local gravity constraints are strong:
 - e.g. $\gamma_{\text{PPN}} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ by time-delay [Bertotti et al. '03]
- **MG need to evade local gravity tests with screening mechanisms!**



[Cassini spacecraft]

Screening Mechanisms of ϕ in Local Region

- Kinetic type: Vainshtein mechanism [Vainshtein '72]
 - Large kinetic term \leftrightarrow weak coupling: Galileon-type
- Potential type: Chameleon mechanism, Symmetron [Khouri & Weltman '04, Hinterbichler & Khouri '10]
 - Large curvature of potential \leftrightarrow massive scalar field m_ϕ



Observational Test of Modified Gravity

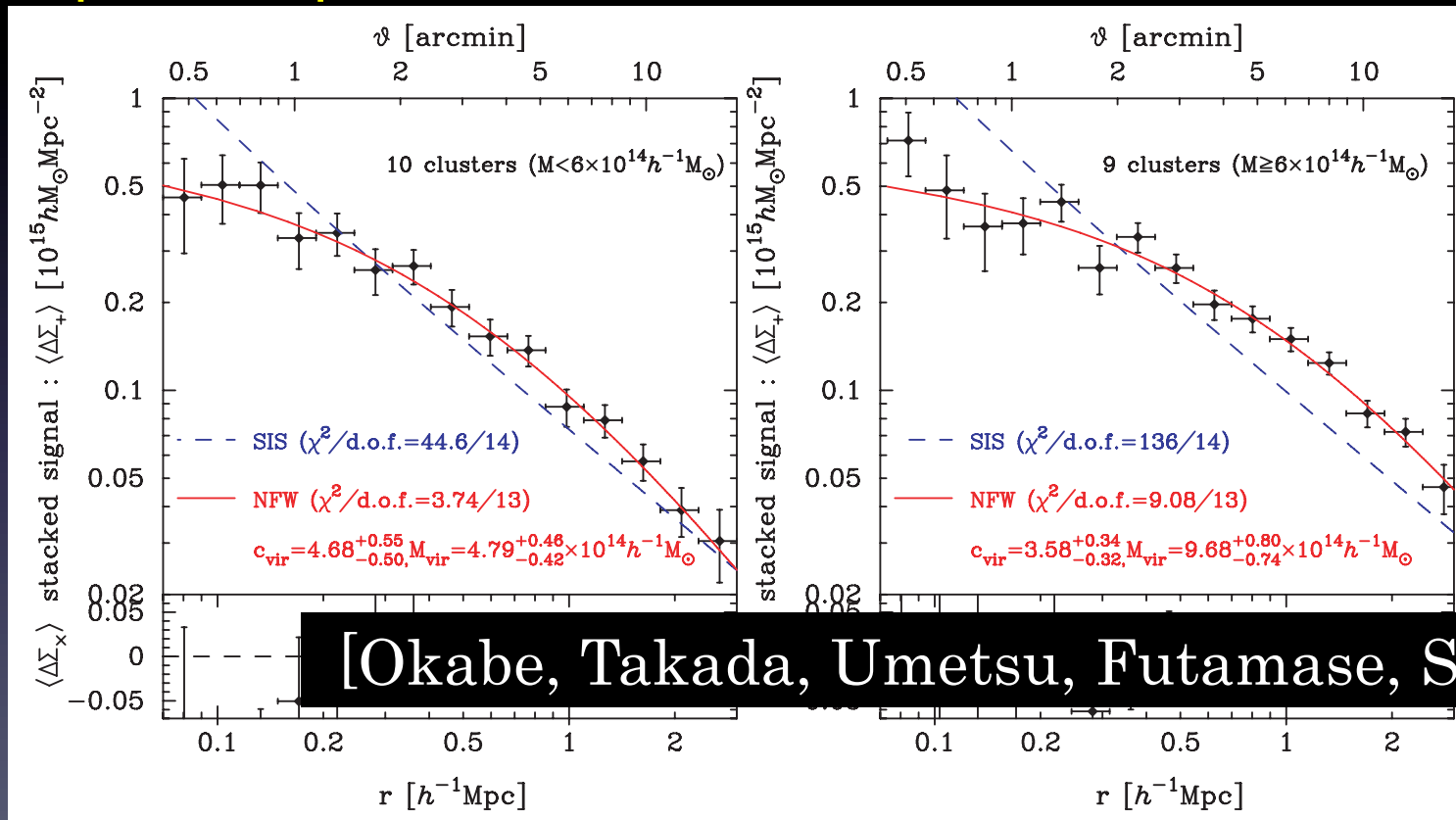
- Whether the screening mechanism works completely or not around high-density regions ?
- Gravitational lensing of galaxy clusters may be useful to test modified gravity
- Key: lensing potential $\Delta\Phi_+ \equiv \Delta(\Phi + \Psi)/2$
[TN, Yamamoto, '12]
- Large amount of data of cluster lensing will be provided by Hyper Suprime-Camera (HSC)
[Takada-san's talk]

Stacked Cluster Lensing Analysis

The average distortion profile

[Itoh-san's talk]

- is less sensitive to individual substructures/asphericity
- helps to improve the S/N



[Okabe, Takada, Umetsu, Futamase, Smith '10]

- may be also useful to test modified gravity

Galileon-type Modified Gravity

- Motivation: decoupling limit of DGP model

$$\mathcal{L}_{\text{int}} \sim X \square \phi : \text{higher-derivative term}$$

where ϕ : scalar field, $X \equiv -\frac{1}{2}(\partial\phi)^2$

enjoy Galileon shift symmetry: $\partial_\mu \phi \rightarrow \partial_\mu \phi + c_\mu$

→ [• Cosmic acceleration

• 2nd order EOM


• Vainshtein mechanism]

[DLofDGP] Luty, Porrati & Rattazzi '03, Nicolis & Rattazzi '04

Vainshtein Mechanism in Simple Model

$$\mathcal{L}_\phi = 3\phi\Box\phi - \frac{2}{\Lambda^3} (\partial\phi)^2\Box\phi + \frac{2}{M_{\text{Pl}}} \phi T$$

ϕ EOM


$$3\Box\phi + \frac{1}{\Lambda^3} \left((\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2 \right) = -\frac{1}{M_{\text{Pl}}} T$$

At $r \ll r_V$: Large kinetic term \Leftrightarrow Weak coupling to matter

$$\phi'(r) \sim \left(\frac{M}{2M_{\text{Pl}}} \frac{\Lambda^3}{r} \right)^{1/2} \ll \partial_r \Phi_N \quad \leftarrow \text{screened!}$$

where Vainshtein radius: $r_V \equiv (M/M_{\text{Pl}})\Lambda^{-1}$

[Luty, Porrati & Rattazzi '03, Nicolis & Rattazzi '04]

The Most General Second-Order Scalar-Tensor Theory [Horndesky '74]

$$S = \int d^4x \sqrt{-g} [\mathcal{L} + \mathcal{L}_m] \quad \text{assume matter do not directly couple to } \phi$$

$$\begin{aligned} \mathcal{L} = & K(\phi, X) - G_3(\phi, X) \square \phi \\ & + G_4(\phi, X) R + G_{4X} \times (\text{field derivatives}) \\ & + G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi \\ & - \frac{1}{6} G_{5X} \times (\text{field derivatives}) \end{aligned}$$

where $G_{iX} \equiv \partial G_i / \partial X$ 4 arbitrary functions of ϕ, X

is equivalent to the generalized galileon

[Deffayet et al. '11, Kobayashi, Yamaguchi, Yokoyama '11]

Background solution

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\phi = \phi_0 = \text{const}, \quad X = 0$$

In order to admit this solution, we require that

$$K(\phi_0, 0) = 0, \quad K_\phi(\phi_0, 0) = 0$$

Spherical symmetric perturbations produced by a nonrelativistic matter

$$ds^2 = -[1 + 2\Phi(r)]dt^2 + [1 - 2\Psi(r)]\delta_{ij}dx^i dx^j$$

$$\phi = \phi_0 + \varphi(r)$$

All the coefficients are evaluated at the background.
We will ignore the mass term $K_{\phi\phi}$.

Static-Spherically Symmetric Configurations

Metric EOM:

$$\frac{M_{\text{Pl}}^2}{2} \frac{(r^2 \Psi')'}{r^2} - M_{\text{Pl}} \xi \frac{(r^2 \varphi')'}{2r^2} - \frac{M_{\text{Pl}}}{\Lambda^3} \alpha \frac{[r(\varphi')^2]'}{2r^2} - \frac{3M_{\text{Pl}}}{\Lambda^6} \beta \frac{[(\varphi')^3]'}{6r^2} = -\frac{1}{4} T_t^t$$

$$M_{\text{Pl}}^2 (\Psi' - \Phi') - 2M_{\text{Pl}} \xi \varphi' - \frac{M_{\text{Pl}}}{\Lambda^3} \alpha \frac{(\varphi')^2}{r} = 0$$

ϕ EOM:

$$\eta \frac{(r^2 \varphi')'}{r^2} - 2 \frac{\mu}{\Lambda^3} \frac{[r(\varphi')^2]'}{r^2} + 2M_{\text{Pl}} \xi \frac{[r^2(2\Psi - \Phi)']'}{r^2} + 4 \frac{M_{\text{Pl}}}{\Lambda^3} \alpha \frac{[r\varphi'(\Psi' - \Phi)']'}{r^2} + 2 \frac{\nu}{\Lambda^6} \frac{[(\varphi')^3]'}{r^2} - \frac{6M_{\text{Pl}}}{\Lambda^6} \beta \frac{[(\varphi')^2 \Phi']'}{r^2} = 0$$

where six dimensionless parameters:

$\xi, \eta, \mu, \alpha, \nu, \beta$ are functions of $K_X, G_{3\phi}, G_{3X}, G_{4X}, G_{5\phi}, \dots$

(cf. Vainshtein mechanism under considering background evolution [Kimura, Kobayashi, Yamamoto '12])

Quintic Scalar-Field Equation

We arrive at

$$P(x, A) := \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2\right) x + [\mu + 6\alpha\xi - 3\beta A(r)] x^2 \\ + (\nu + 2\alpha^2 + 4\beta\xi) x^3 - 3\beta^2 x^5 = 0$$

where we define

$$x = \frac{1}{\Lambda^3} \frac{\varphi'}{r}, \quad A(r) = \frac{1}{M_{\text{Pl}} \Lambda^3} \frac{M(r)}{8\pi r^3}$$

which are dimensionless.

$M(r)$ is the enclosed mass.

Scalar-Field Equation [cf. Sbisa, Niz, Koyama, Tasinato '12]

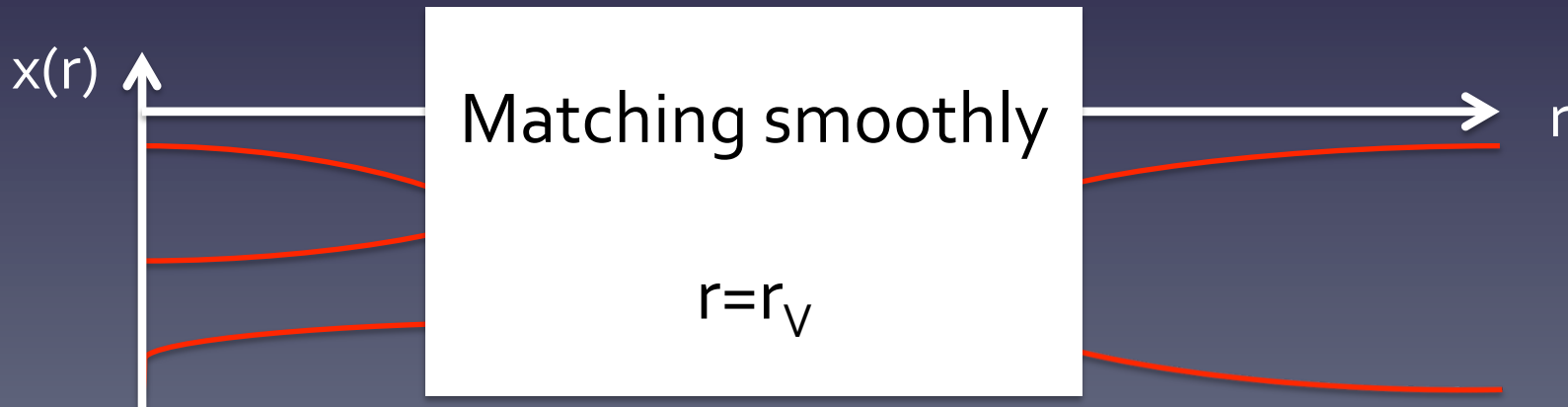
We solve

$$x = \frac{1}{\Lambda^3} \frac{\varphi'}{r}, \quad A(r) = \frac{1}{M_{\text{Pl}} \Lambda^3} \frac{M(r)}{8\pi r^3}$$

$$P(x, A) := \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2 \right) x + [\mu + 6\alpha\xi - 3\beta A(r)] x^2 \\ + (\nu + 2\alpha^2 + 4\beta\xi) x^3 - 3\beta^2 x^5 = 0$$

for the inner region ($A \gg 1$) and the outer region ($A \ll 1$).

We then derive the conditions under which the two solutions are matched smoothly in an intermediate region.



Outer Solution: Asymptotically Flat

- In the outer region ($A \ll 1$), there is always a decaying solution,

$$x \approx x_f := -\frac{2\xi A(r)}{\eta + 6\xi^2}$$

which is obtained by neglecting the nonlinear terms in $P(x, A) = 0$.

The other solutions do not correspond to an asymptotically flat spacetime.

Inner Solution: Vainshtein

- In the inner region ($A \gg 1$), $P=0$ reduces to

$$P(x, A) \approx \xi A - 3\beta A x^2 - 3\beta^2 x^5 \approx 0$$

We have a solution ($\xi\beta > 0$):

$$x \approx x_{\pm} := \pm \sqrt{\frac{\xi}{3\beta}} = \text{const}$$

($P(x_{\pm}, A)$ does not depend on A .)

For this solution, we have Newtonian behavior:

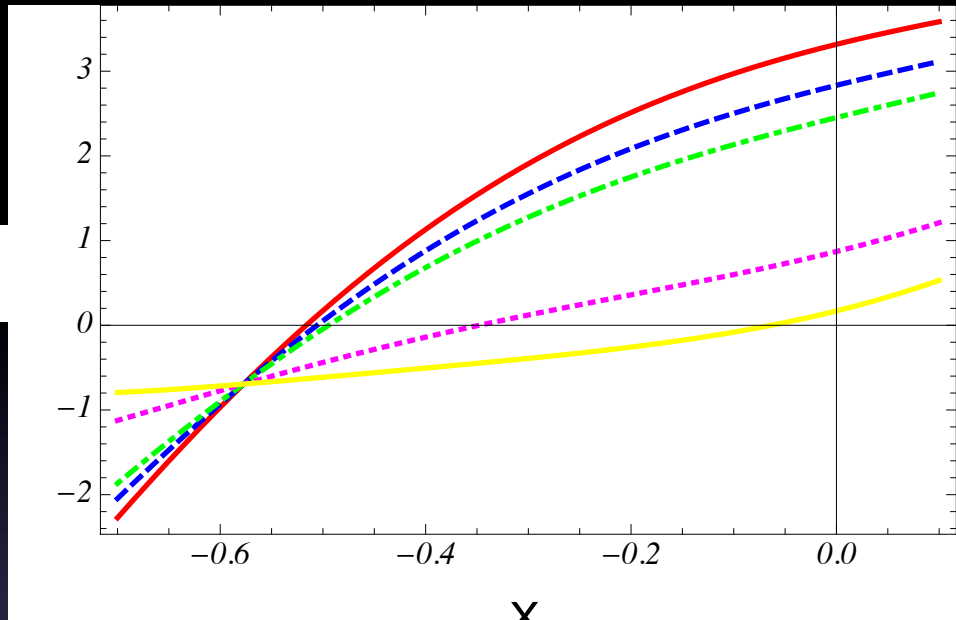
$$\Psi' / r \simeq \Phi' / r \propto A$$

This is therefore the Vainshtein solution.

For simplicity we focus on the case of $\xi > 0$.

Matching of Two Solutions

$P(x,A)$



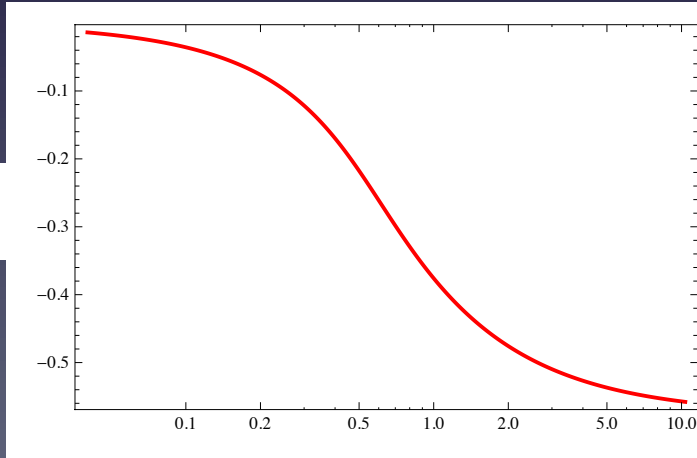
↑
A
↓

In this case, $P(x)=0$ has a single real root in $(x_-,0)$ for any $A>0$.

x_-

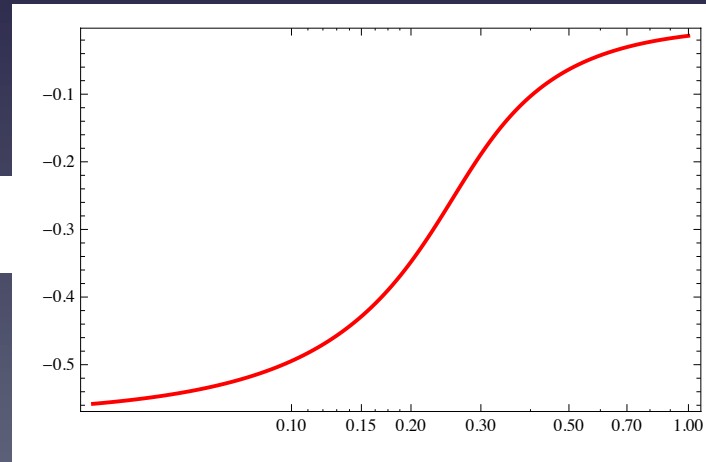
x

x



A

x



r

Decoupling Limit of Massive Gravity

[de Rham, Heisenberg '11]

Decoupling limit: $M_{\text{pl}} \rightarrow \infty$, $m_g \rightarrow 0$, $\Lambda^3 = M_{\text{pl}} m_g^2 = \text{fixed}$.

The corresponds to

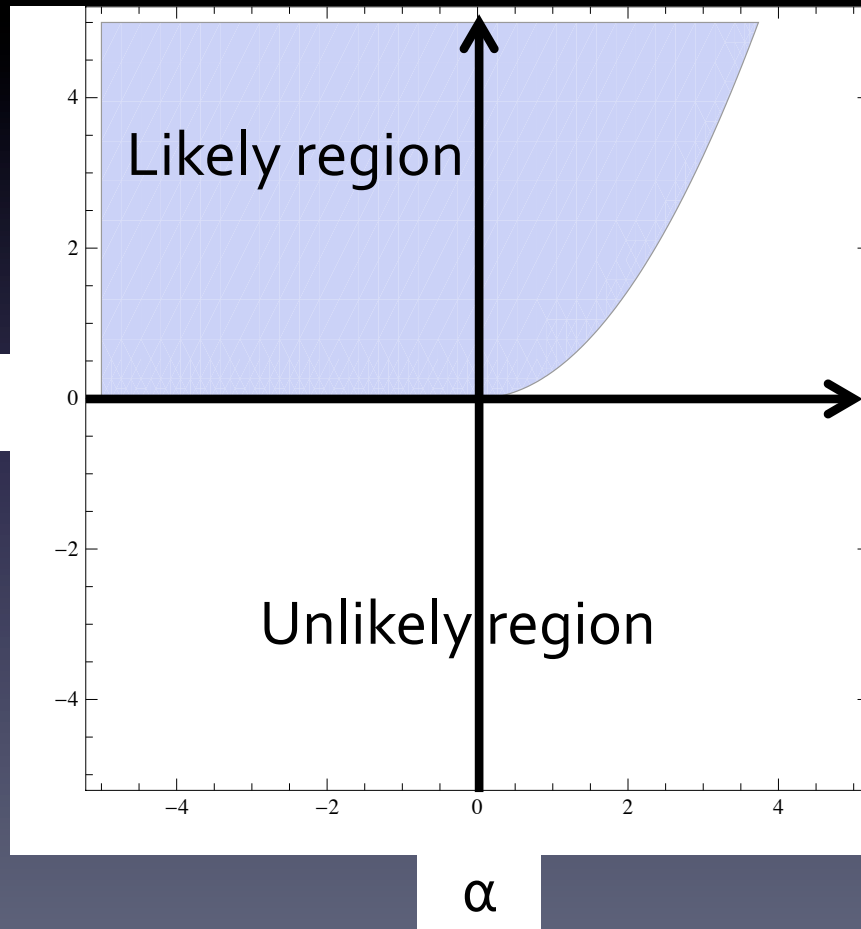
$$K = 0 = G_3, \quad G_4 = \frac{M_{\text{pl}}^2}{2} + M_{\text{Pl}} \phi + \frac{M_{\text{Pl}}}{\Lambda^3} \alpha X, \quad G_5 = -3 \frac{M_{\text{Pl}}}{\Lambda^6} \beta X$$

One finds

$$\eta = \mu = \nu = 0, \quad \xi = 1, \quad \alpha \neq 0, \quad \beta \neq 0$$

The condition of smooth matching of the two solutions:

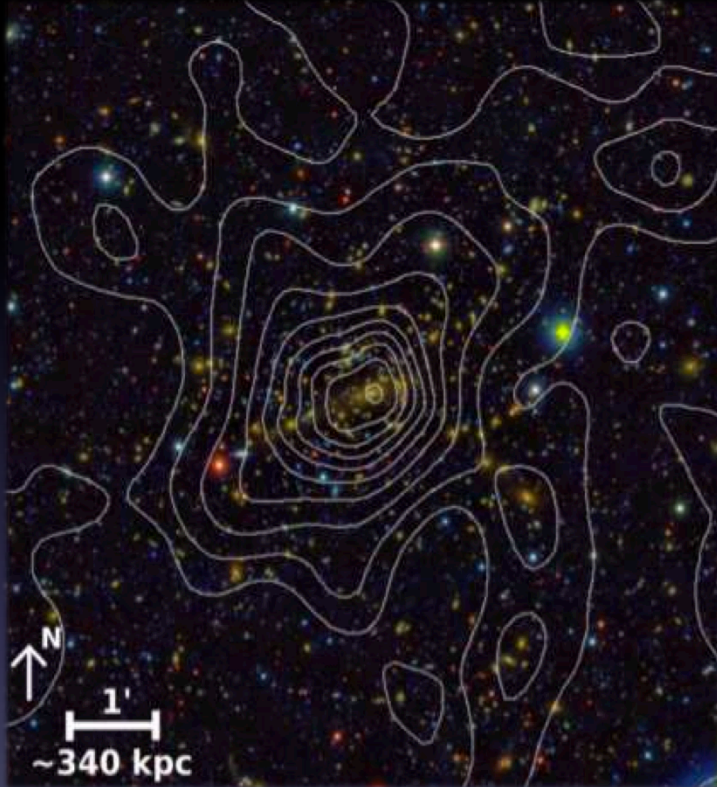
$$\alpha < 0 \text{ or } \frac{\sqrt{\beta}}{\alpha} \geq \sqrt{\frac{5 + \sqrt{13}}{24}} \sim 0.6$$



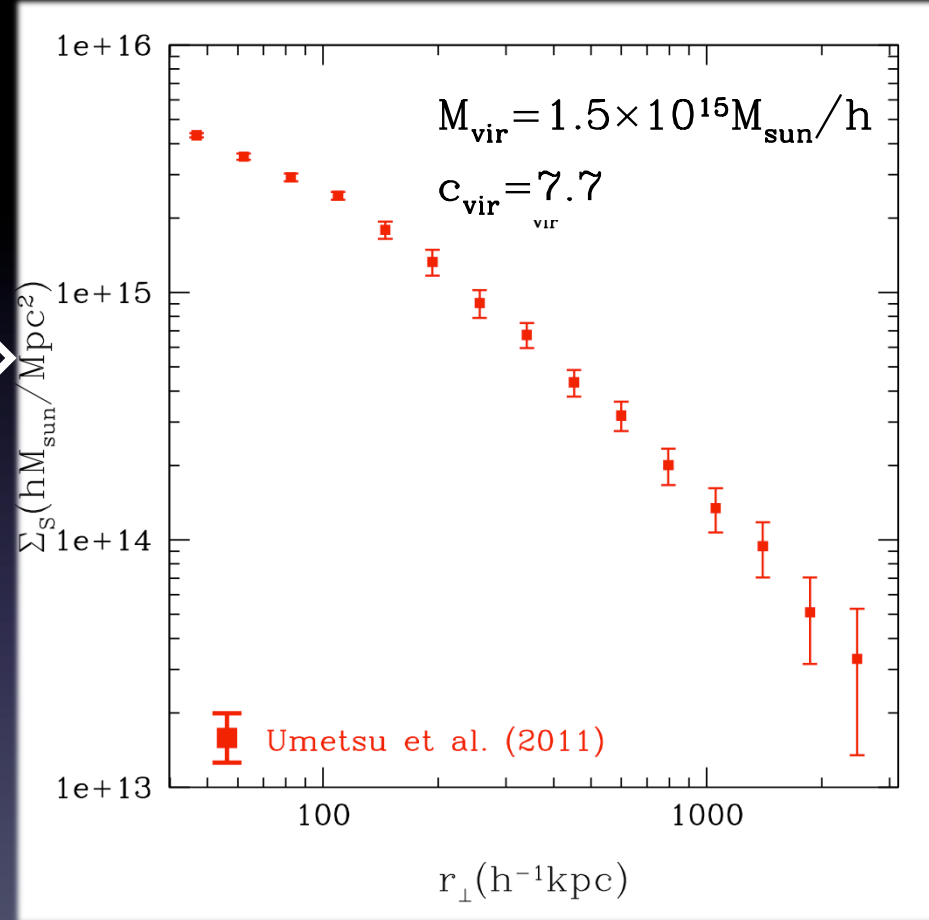
Explore the observationally allowed region with cluster lensing.

Cluster Lensing

Mass map (shear)



Surface mass density $\Sigma_S(r_{\text{perp}})$



[Bartelmann's talk]

[Umetsu et al. '11,
cf. Oguri et al. '12]

Gravitational Lensing in Massive Gravity

- Geodesic equation

$$\frac{d^2}{d\chi^2} (\chi\theta^i) = 2\Phi_{+,i}, \quad i = 1, 2 \quad \Phi_+ \equiv (\Phi + \Psi)/2$$

- Surface mass density $\Sigma_S(\mathbf{r}_{\text{perp}}) \propto \kappa(\mathbf{r}_{\text{perp}})$

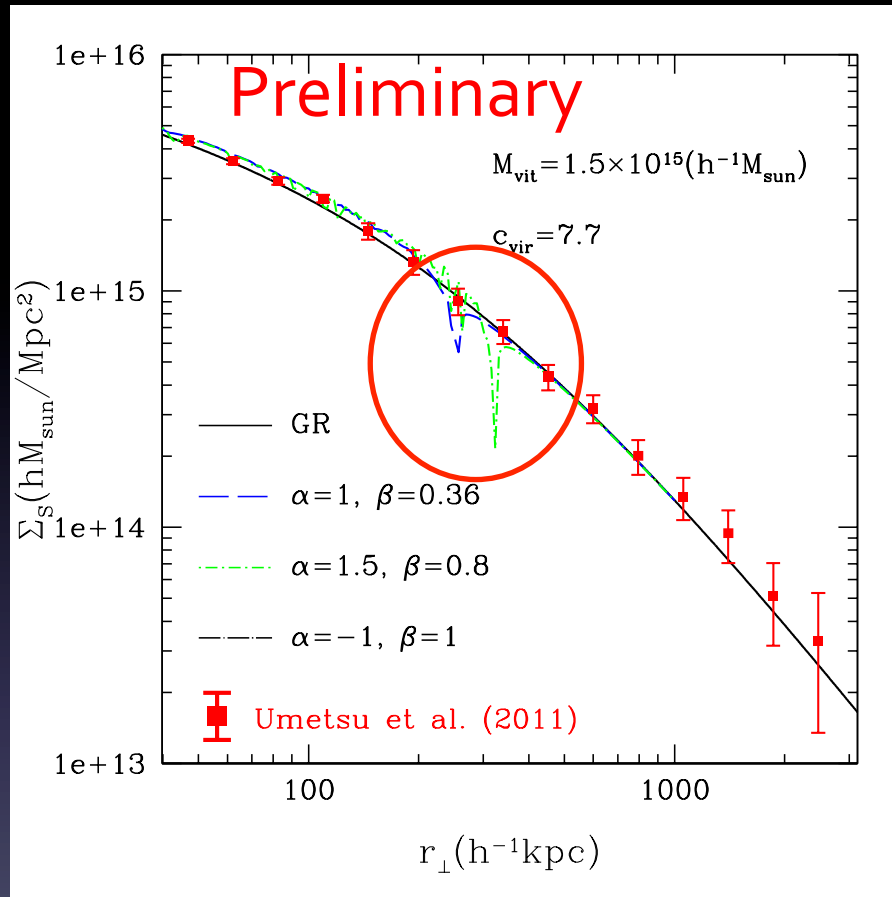
$$\Sigma_S \propto \int_0^\infty dZ \Delta^{(2D)} \Phi_+ \quad r = \sqrt{r_\perp^2 + Z^2}$$

where lensing potential in massive gravity:

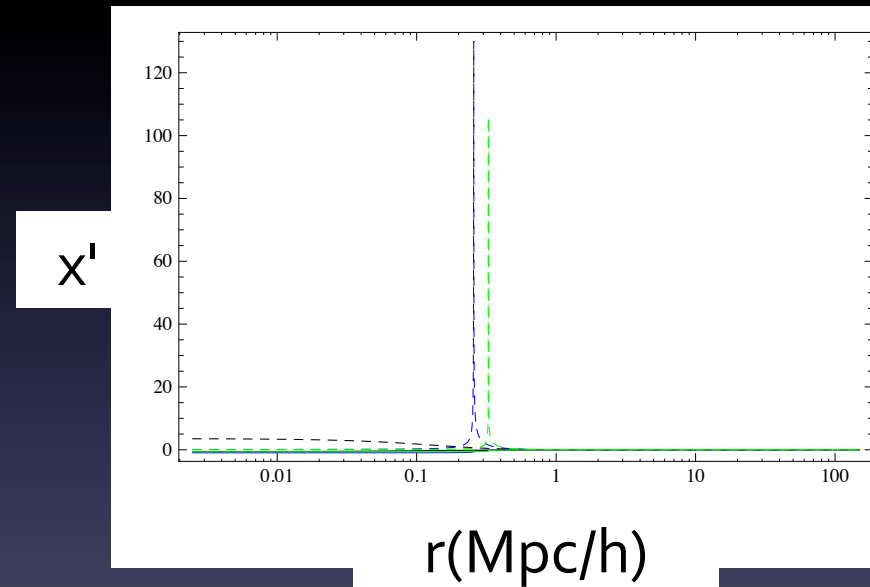
$$\Delta\Phi_+ = \frac{\Lambda^3}{M_{\text{Pl}}^2} \frac{[(\alpha x^2 + 2\beta x^3 + 2A) r^3]'}{2r^2}$$

Assume $\delta\rho(r)$ as NFW profile: $\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$

Surface Mass Density in Massive Gravity



Parameters: $\{\alpha, \beta, \Lambda; \rho_s, r_s\}$
 ($\Lambda = 0.01/H_0, \rho_s, r_s$ fixed)



Dip appears at $r \sim r_V := (r_s M_{\text{pl}} / \Lambda^3)^{1/3}$

→ allows us to put constraint

Summary

- Static-spherically symmetric solutions of ϕ EOM in the context of Horndeski's theory.
- We explore the possibility of testing modified gravity exhibiting the Vainshtein mechanism against observation of cluster lensing.
- Key effects on cluster lensing Σ_5 is that $x'(\Leftrightarrow\phi'')$ can substantially be large at the transition from screened to unscreened regions.
→ This allows us to put observational constraints on modified gravity.

Thank you for your attention