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"An improved method for CMB lensing reconstruction and its cosmological applications"

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An Improved Method for CMB Lensing Reconstruction and Its Cosmological Applications

Toshiya Namikawa (The University of Tokyo)

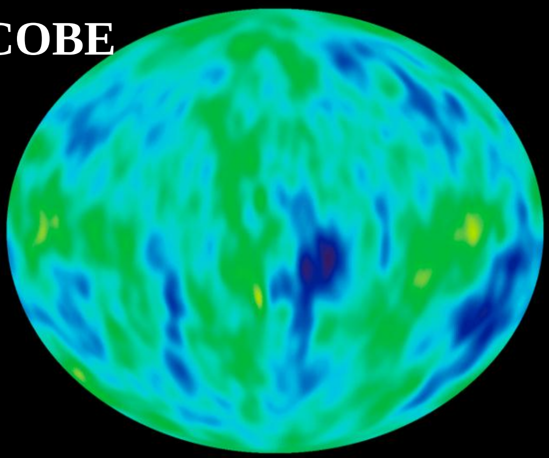
Nov.12-16, 2012, JGRG @ Tokyo

Introduction

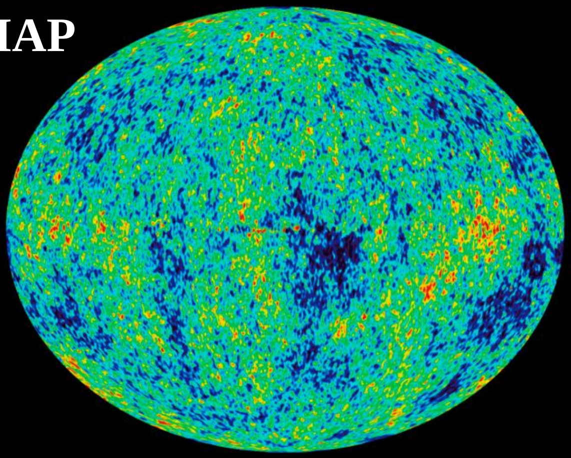
Cosmic Microwave Background (CMB)

- **Precise measurements of CMB fluctuations**

COBE



WMAP



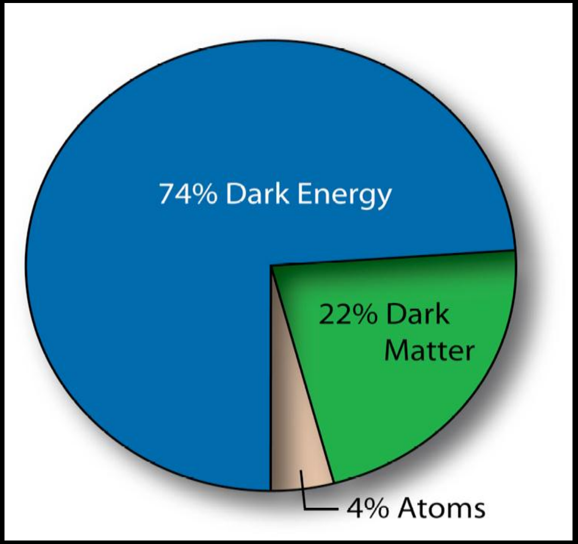
from LAMBDA web site (http://lambda.gsfc.nasa.gov/)

- **The energy components of Universe is well described by ΛCDM model**

  ✓ **dark energy, dark matter**

  ✓ **mass of neutrinos**

  ⋮



Cosmological observations

➢ Observations

- CMB temperature/ polarizations
- Type-Ia Super Novae
- Baryon Acoustic Oscillations
- Cluster abundance
- Weak Lensing
- 21cm brightness temperature

➢ CMB lensing

✓ Sensitive to both geometry and density fluctuations at z=1~5
✓ The most high-z source for weak lensing
✓ The physical properties of source (CMB) is well known

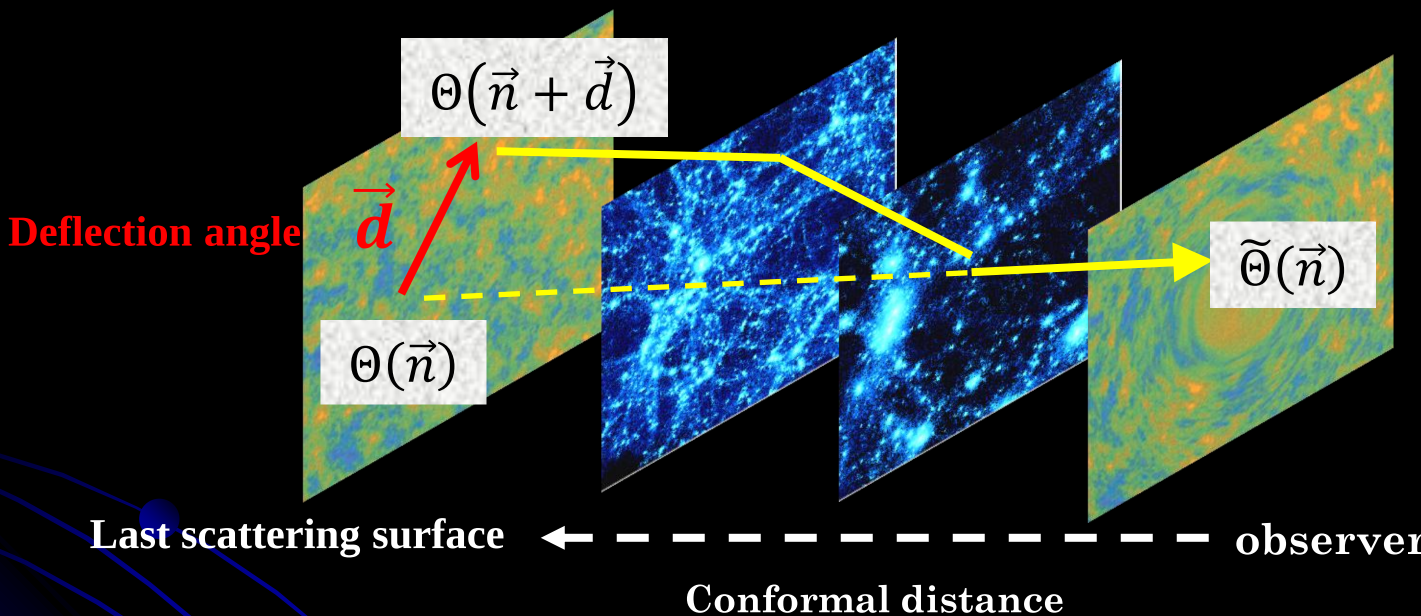
dark energy, neutrino mass, …

primordial gravitational waves, primordial non-Gaussianity

Lensing effect on CMB

- **CMB Lensing**
  = Distortion of the pattern of the temperature/polarizations anisotropies

(Reviews : Lewis&Challinor'06, Hanson+'10)



- **Lensed anisotropies**

$$\tilde{\Theta}(\vec{n}) = \Theta(\vec{n} + \vec{d}(\vec{n}))$$

Lensing potential

Gravitational potential

$$\vec{d}(\vec{n}) = \nabla\left(-2\int_0^{\chi_s} d\chi \frac{\chi_s - \chi}{\chi\chi_s} \psi(\eta_0 - \chi, \chi\vec{n})\right)$$

Lensed temperature anisotropies

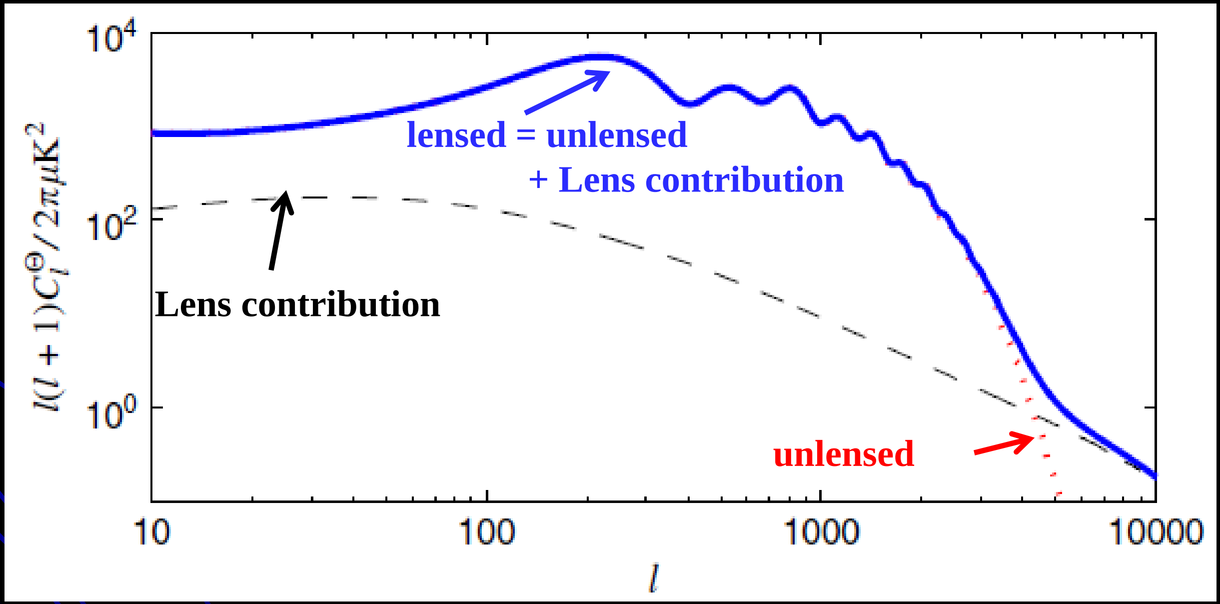
- **Angular power spectrum**

$$\tilde{\Theta}(\vec{n}) \Rightarrow \tilde{\Theta}_{\ell m} = \int d\vec{n}\, Y^{*}_{\ell m}(\vec{n})\tilde{\Theta}(\vec{n}) \Rightarrow \langle \tilde{\Theta}_{\ell m}\tilde{\Theta}^{*}_{\ell' m'}\rangle = \delta_{\ell\ell'}\delta_{mm'}\tilde{C}^{\Theta\Theta}_{\ell}$$

Harmonics space **Angular power spectrum**

- **Lensed angular power spectrum**

(Lewis&Challinor'06)



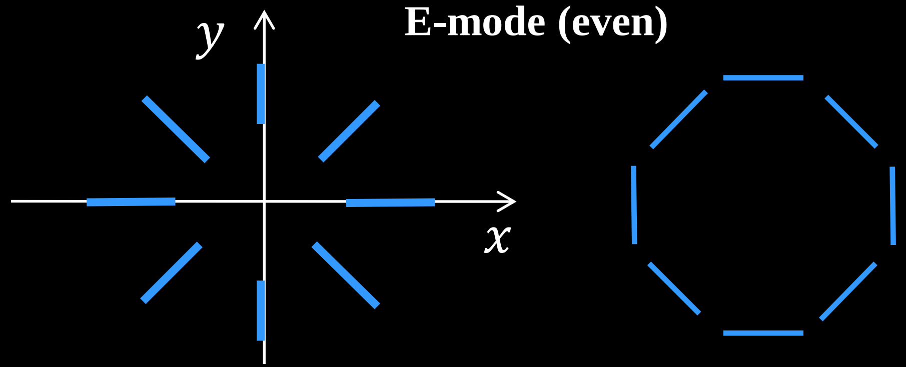
Lensing effect becomes dominate at Silk damping scale : $\ell > 2000$ (~ few arcmin)

→ **A few arcmin angular resolution is required**

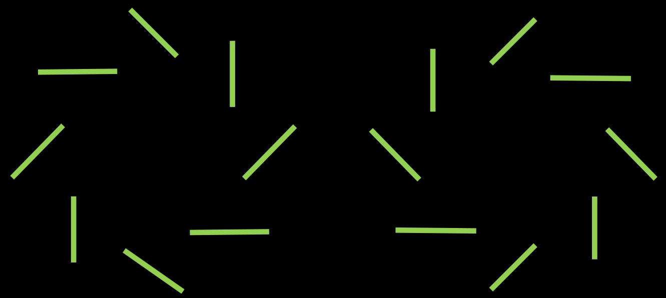
Lensed polarizations anisotropies

- **CMB polarizations**
  - **Decomposition with parity** (patterns of E/B-modes)

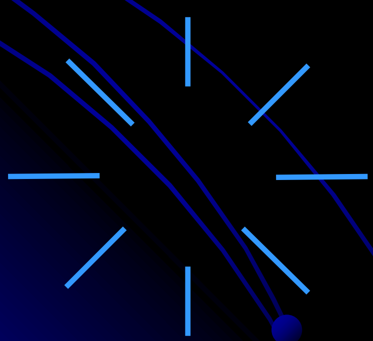
E-mode (even)



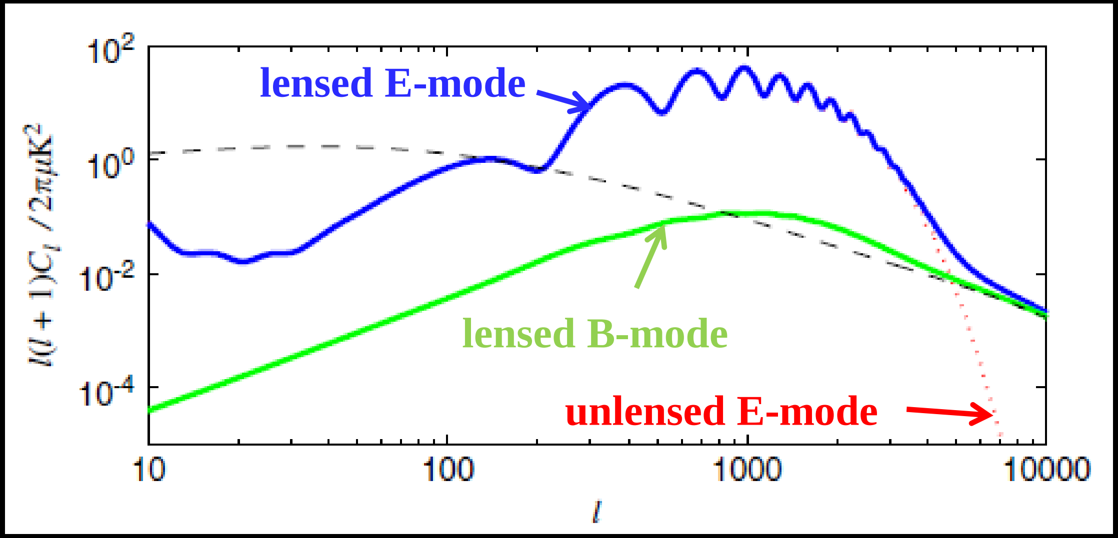
B-mode (odd)



- **Lensed E/B-modes angular power spectra**
  - **B-mode from lensing**



(Lewis&Challinor'06)

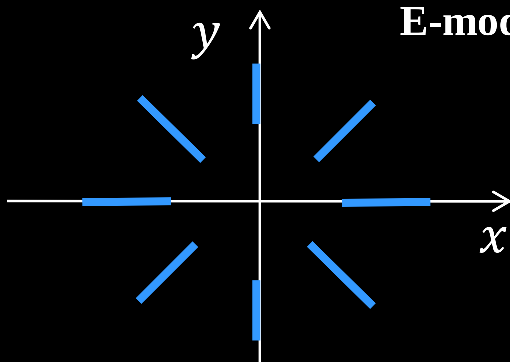


Lensed polarizations anisotropies

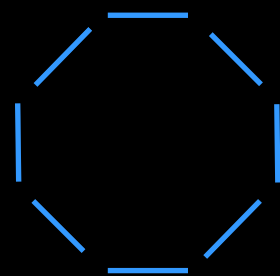
➢ CMB polarizations

- Decomposition with parity

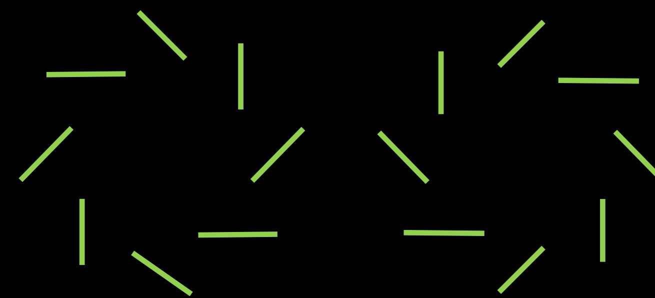
(patterns of E/B-modes)



E-mode (even)

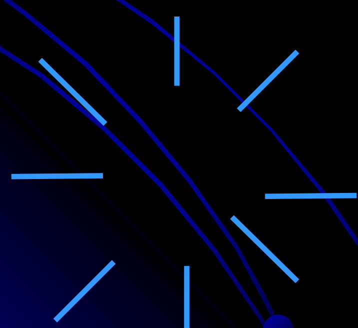


B-mode (odd)

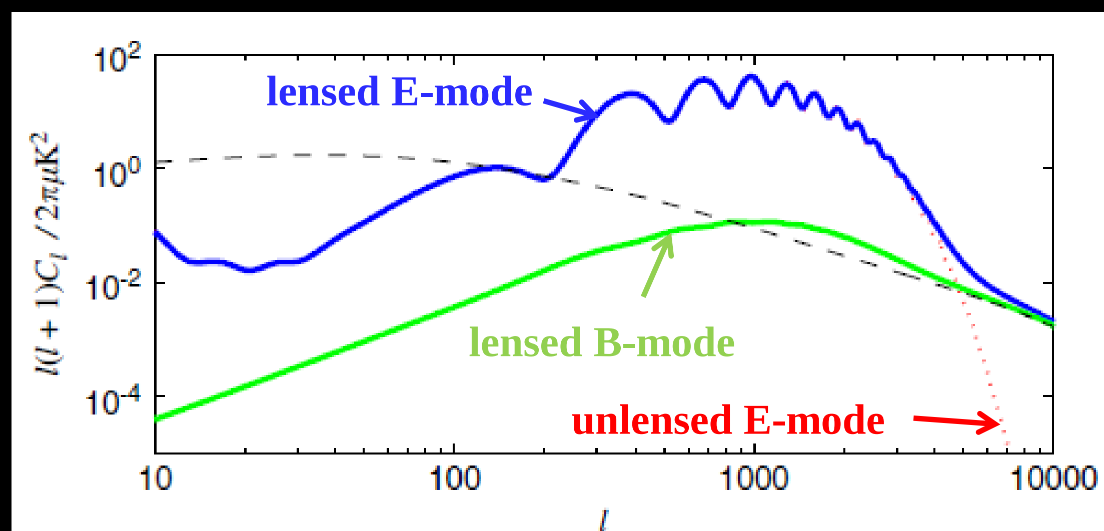


➢ Lensed E/B-modes angular power spectra

- B-mode from lensing



(Lewis&Challinor'06)



How to measure lensing effect from CMB

- **Lensed CMB power spectrum, e.g., C_ℓ^{BB}**
  - ✓ **easy to mitigate biases from, e.g., masking compared to reconstruction**
  - ✓ **estimation of power spectrum of lensing potentials, $C_\ell^{\phi\phi}$, is difficult**

- **CMB lensing reconstruction = estimate lensing potentials itself**
  - ✓ **useful for cross-correlation studies with, e.g., cosmic shear, galaxy clustering, etc**
  - ✓ **also useful to study, e.g., three-point correlation of lensing potentials, etc**

CMB lensing reconstruction

- **Essence of lensing reconstruction** (Review: Hanson+'10)

Lensing induces mode coupling in the temperature anisotropies

$$\tilde{\Theta}_{\vec{\ell}} = \Theta_{\vec{\ell}} - \int d^2L\,(\vec{\ell}\cdot\vec{L})\underline{\phi_{\vec{L}-\vec{\ell}}}\Theta_{\vec{L}}$$

Lensing potentials would be estimated from mode coupling $\tilde{\Theta}_{\ell}\tilde{\Theta}_{\vec{L}}\ (\ell \neq L)$

- **Estimator** (e.g., Hu&Okamoto'02; Hanson+'09)

$$\hat{\phi}_L^{(C)} = \hat{\phi}_L^{(S)} - \langle\hat{\phi}_L^{(S)}\rangle$$

Mean-field bias: induced by non-lensing effect (mask, inhomogeneous noise, beam asymmetry, …)

$$\hat{\phi}_L^{(S)} = \int d^2\ell\ F^{\phi}_{\ell,L}\,\boxed{\bar{\Theta}_{\ell}\bar{\Theta}_{\vec{L}-\vec{\ell}}} \qquad \left(\bar{\Theta}_{\ell} = \frac{\tilde{\Theta}_L}{C_L^{\Theta\Theta}}\right)$$

$(L \neq 0)$

includes "observed lensed" Cl's

Lensing reconstruction from current data

- **Smith+ '07** **(3.4σ)**

  WMAP
  + NVSS (NARO VLA Sky Survey)

- **Hirata+ '08** **(2.5σ)**

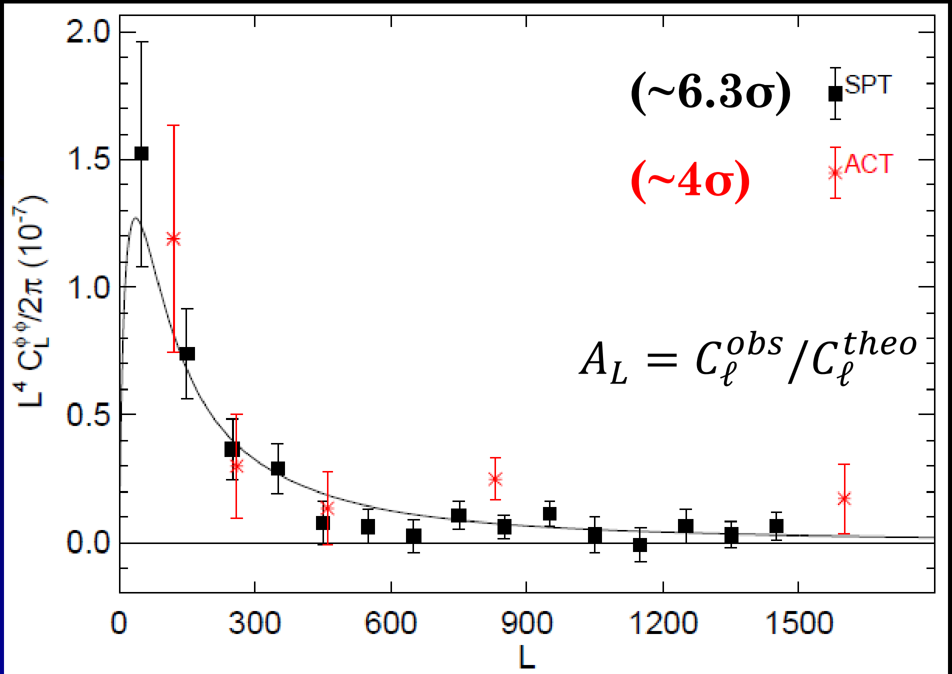
  WMAP
  + NVSS + SDSS (Sloan Digital Sky Survey)

- **Das+ '11**

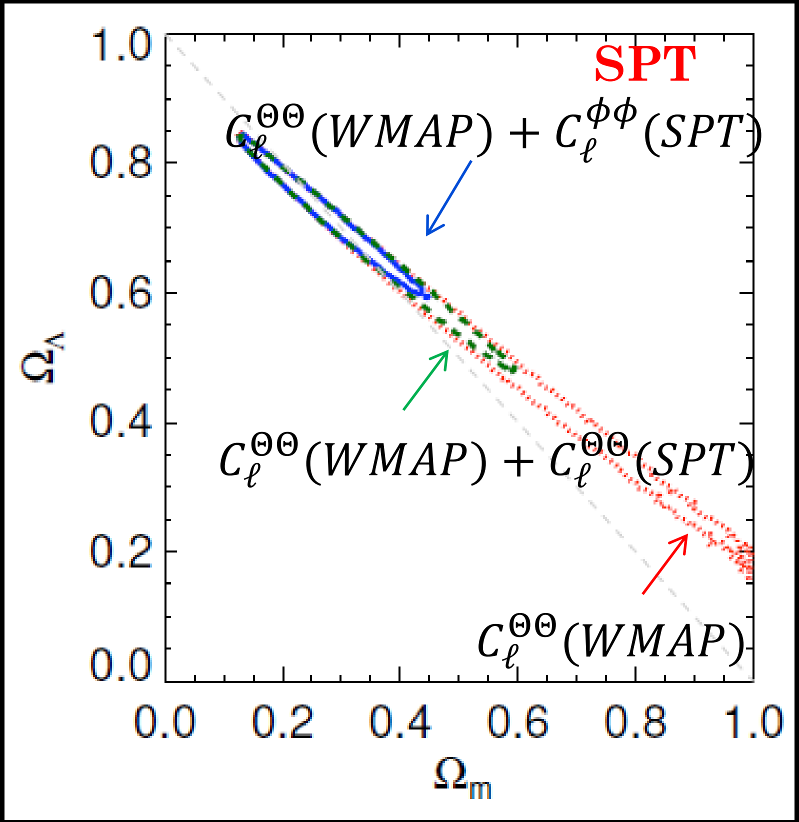
  WMAP
  + Atacama Cosmology Telescope (ACT)

- **van Engelen+ '12**

  WMAP + South Pole Telescope (SPT)

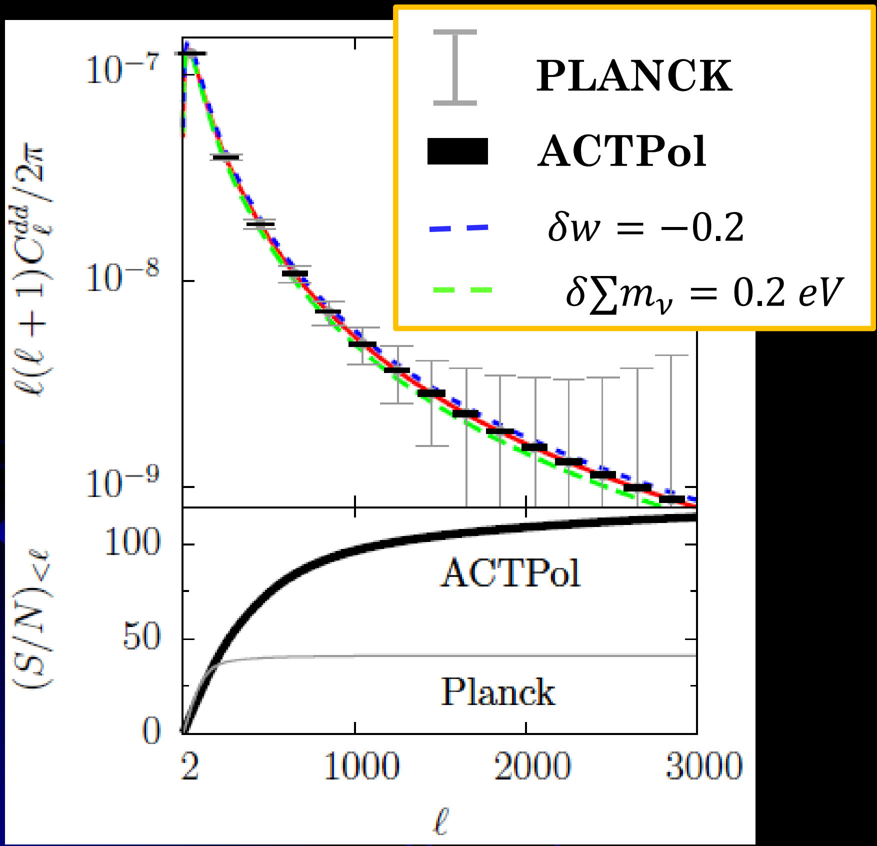


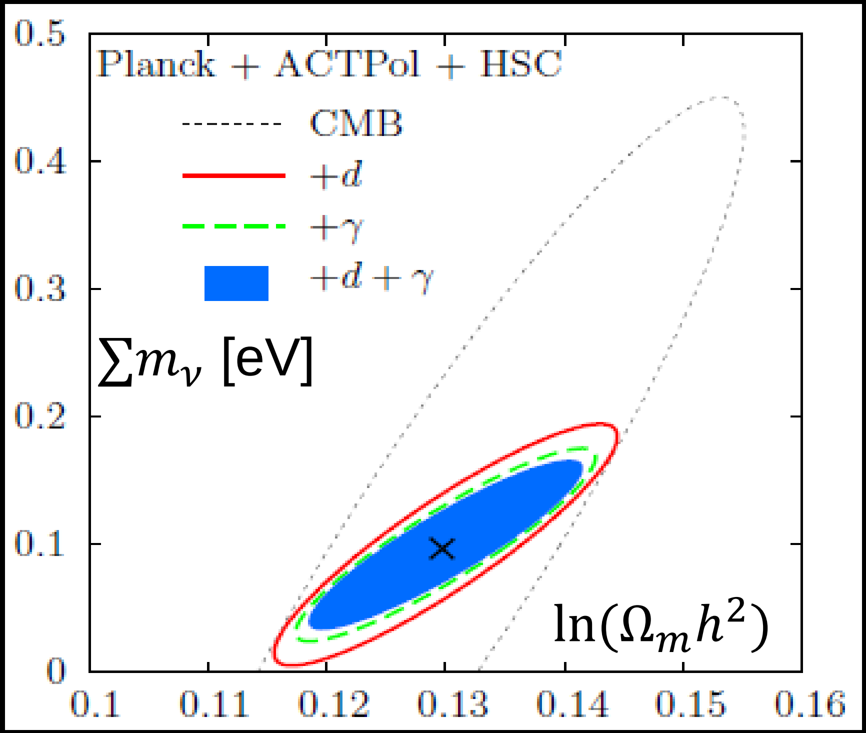
Angular power spectrum of lensing potentials



Future prospects

- **Ongoing/upcoming and next generation CMB experiments**
  - ✓ **Space** PLANCK, CMBPol, LiteBIRD, …
  - ✓ **Ground** PolarBear, ACTPol, SPTpol, Polar…





TN, Saito & Taruya '10

Lensing signals would be detected with enough precision to probe, e.g., dark energy and massive neutrinos

Lensing reconstruction from CMB map and Its Cosmological Applications

Gradient/Curl modes

- **Deflection angle**

2 components
- Gradient --- linear density fluctuations…
- Curl --- any vector and tensor sources (gravitational waves (GWs), cosmic strings, magnetic field…)

2D Levi-Chivita tensor

$$d_i(\vec{n}) = \underbrace{\partial_i \phi(\vec{n})}_{\text{gradient}} + \underbrace{\epsilon_{ij}\partial_j \omega(\vec{n})}_{\text{curl}}$$

- **Application of curl-mode reconstruction**
  - Probing, e.g., cosmic strings, GWs, magnetic field …
  - Check systematics

- **Outline**

1. derive estimator for gradient and curl modes
2. reconstruct from current CMB data
3. discuss future prospects

Estimator

- **Estimator** 　　　　**TN, Yamauchi & Taruya '12**

$$\hat{\phi}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} F^{XY}_{\ell L_1 L_2} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

$$\hat{\omega}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} G^{XY}_{\ell L_1 L_2} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

mode coupling

$\tilde{X}_{\ell m}$, $\tilde{Y}_{\ell m}$: **Lensed quantities** ($\tilde{\Theta}_{\ell m}, \tilde{E}_{\ell m}$, **or** $\tilde{B}_{\ell m}$)

- **Weight functions are determined so that**

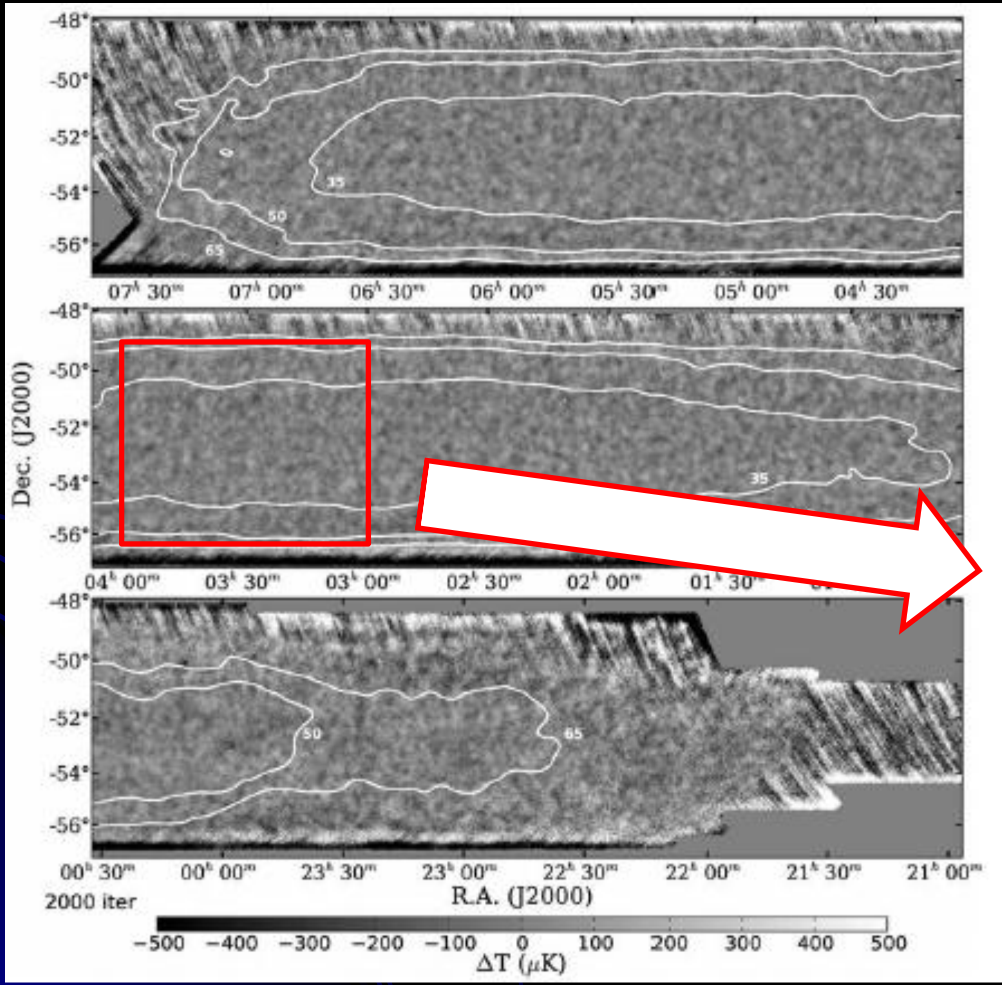
1. **Gradient / curl mode estimators do not include curl / gradient mode, respectively**
2. **The variance of estimator is minimized**

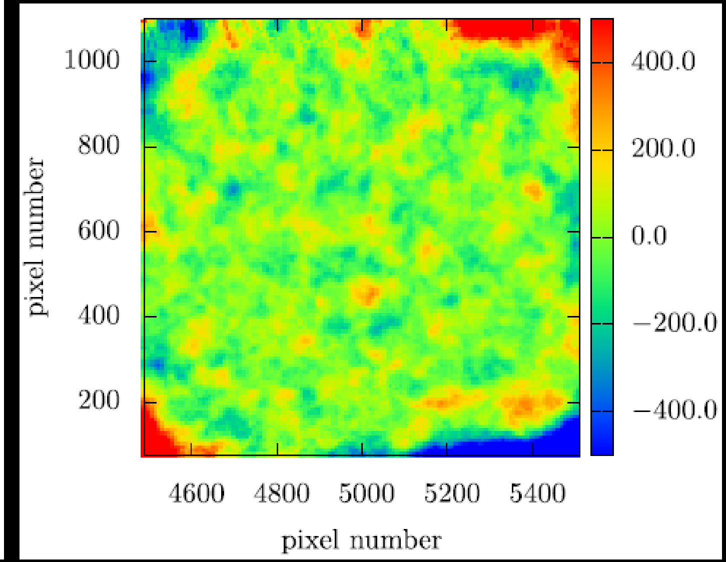
Thanks to the distinctive property of parity (∇ and $\star\,\nabla$), we can estimate gradient and curl mode separately

Lensing reconstruction from current data

- **Data** **ACT**

(Dunner+'12)





Data is taken from http://lambda.gsfc.nasa.gov/product/act/act_prod_table.cfm (LAMBDA)

Estimating lensing power spectrum

- **Power spectrum of lensing estimator** (e.g., Hanson+'11)

$$\hat{x}_L = \int d\vec{\ell}\ F^x_{\ell,L}\ \widetilde{\Theta}_\ell \widetilde{\Theta}_{\vec{L}-\vec{\ell}}$$

$$\longrightarrow \langle|\hat{x}_L|^2\rangle = \int d\vec{\ell}_1 \int d\vec{\ell}_2\, F^x_{L,\ell_1} F^x_{L,\ell_2} \langle \widetilde{\Theta}^*_{\ell_1} \widetilde{\Theta}^*_{\vec{L}-\vec{\ell}_1} \widetilde{\Theta}_{\ell_2} \widetilde{\Theta}_{\vec{L}-\vec{\ell}_2} \rangle$$

decomposed into disconnected/connected part

$$\longrightarrow \langle|\hat{x}_L|^2\rangle \simeq N^{x,(0)}_\ell + C^{xx}_\ell + \sum_x N^{x,(1)}_\ell + O[(C^{xy}_\ell)^2]$$

disconnected part (Gaussian bias)

connected part

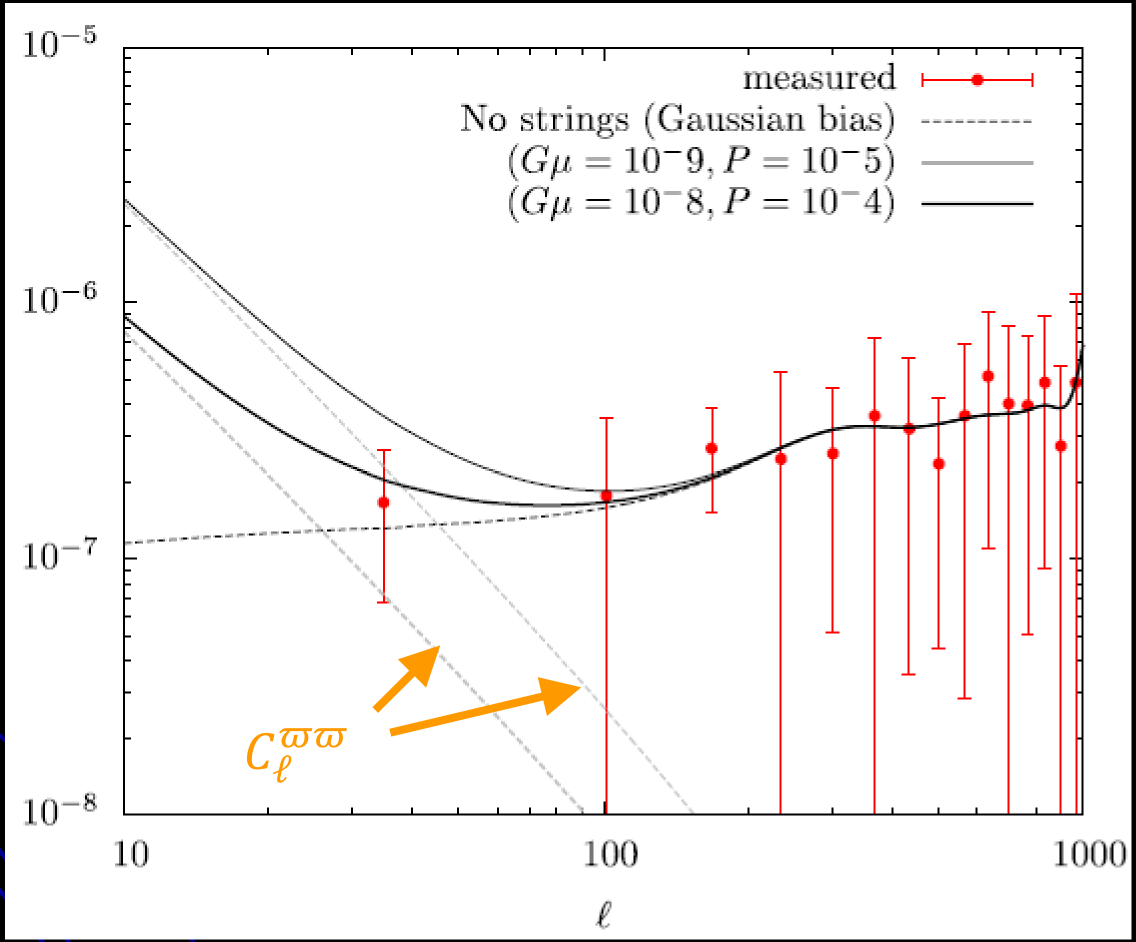
[Note] several techniques for power spectrum estimation can reduce uncertainties in Gaussian bias

(**Hu'02, Sherwin&Das'11, *TN*,Hanson&Takahashi'12**)

Cosmological implications

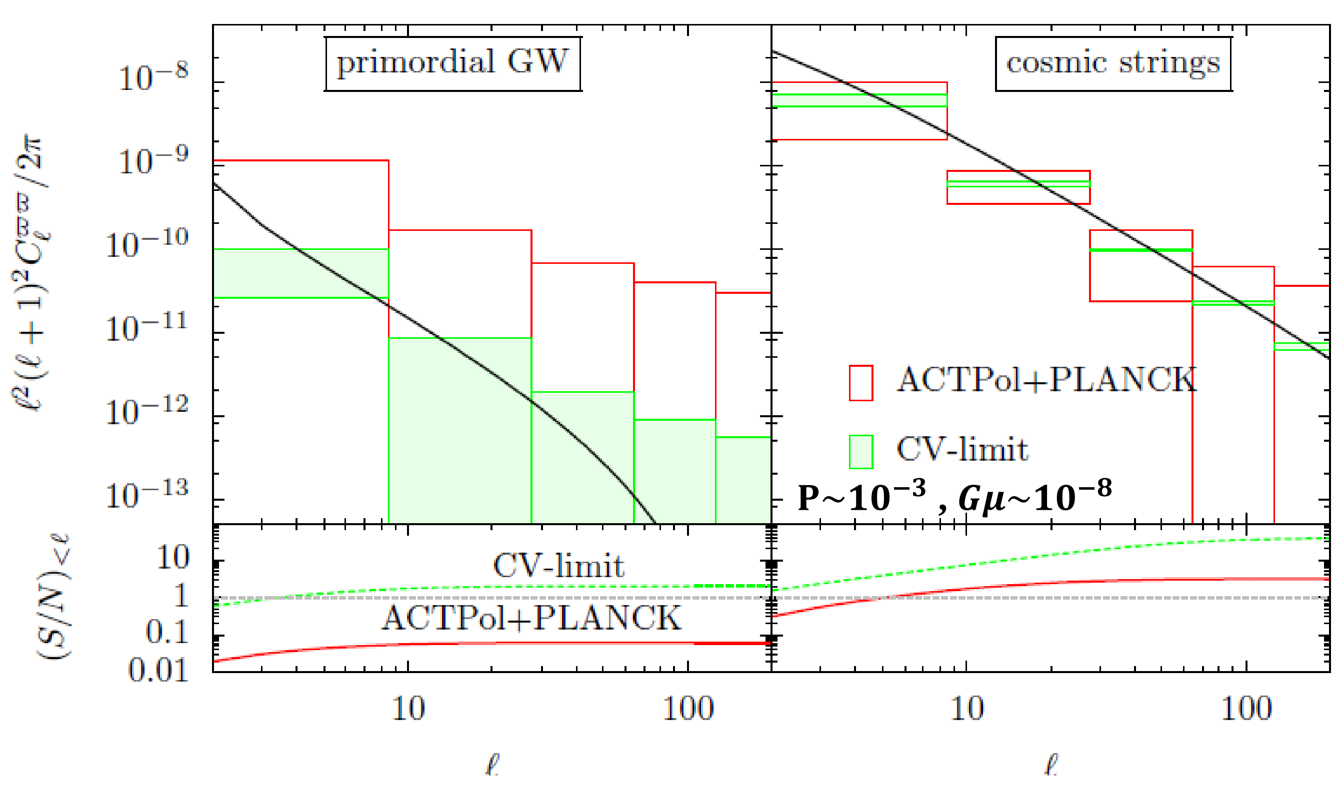
- **An example of cosmological implications from curl mode**

$\ell^4 |\hat{\varpi}_L|^2 / 2\pi$



From ACT data, parameter region which is not ruled out from CMB temperature power spectrum, e.g., $G\mu \sim 10^{-9}$ with P $\sim 10^{-5}$ seems to be ruled out

Cosmological implications (Future Prospects)



TN, Yamauchi & Taruya '12

- ✓ **Primordial GWs --- even r=0.1 would be difficult to detect.**
- ✓ **Cosmic strings --- can be explored with upcoming experiments**

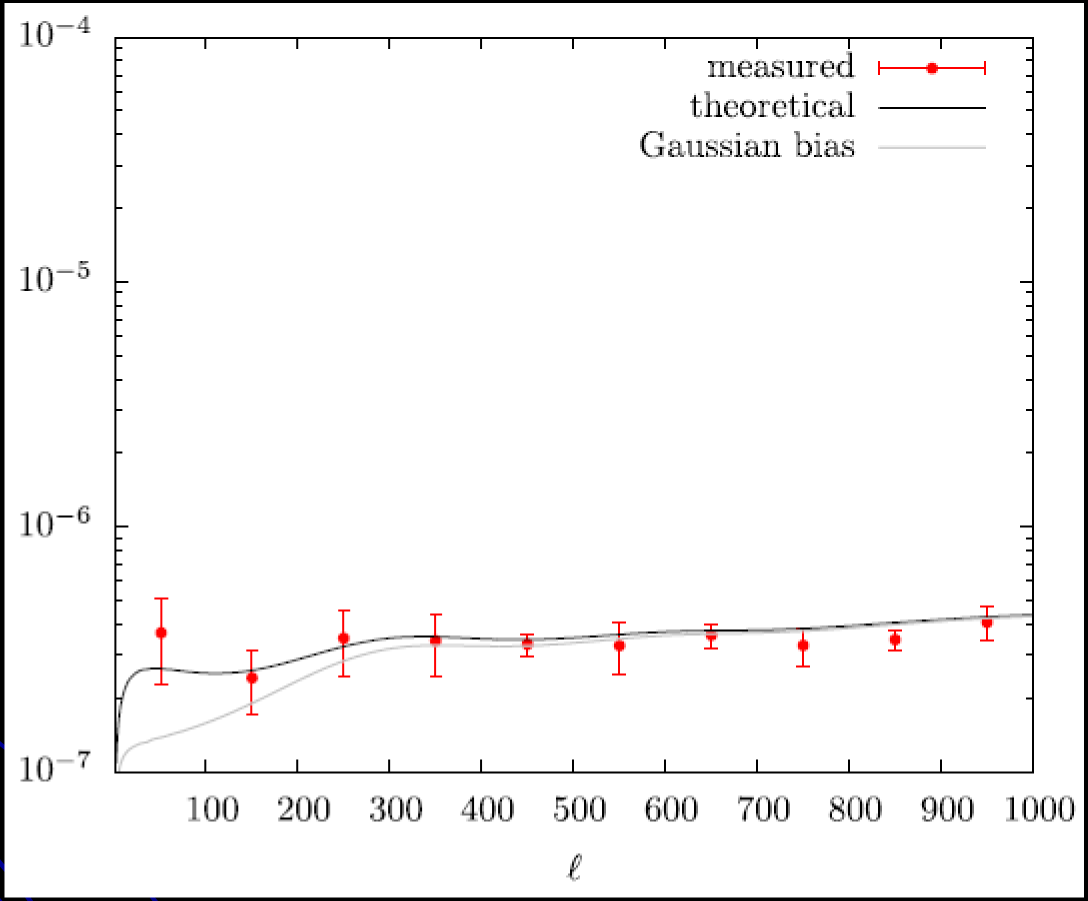
(This would be a new probe of cosmic strings from CMB)

Summary

- **Formulation of CMB lensing reconstruction in the presence of both gradient and curl modes**
  - ✓ **Gradient and curl modes are estimated separately thanks to the distinctive property of parity**
- **Lensing reconstruction from current CMB map**
  - ✓ **Show an example of cosmological implications from curl mode**
- **Future prospects of CMB lensing reconstruction**
  - ✓ **Lensing signals from upcoming and future CMB experiments and galaxy imaging surveys would constrain not only mass of neutrinos, property of dark energy, but also e.g., cosmic strings and other non-scalar components**
  - ✓ **If $r<0.01$, lensing reconstruction is important to probe primordial gravitational waves from B-polarization**

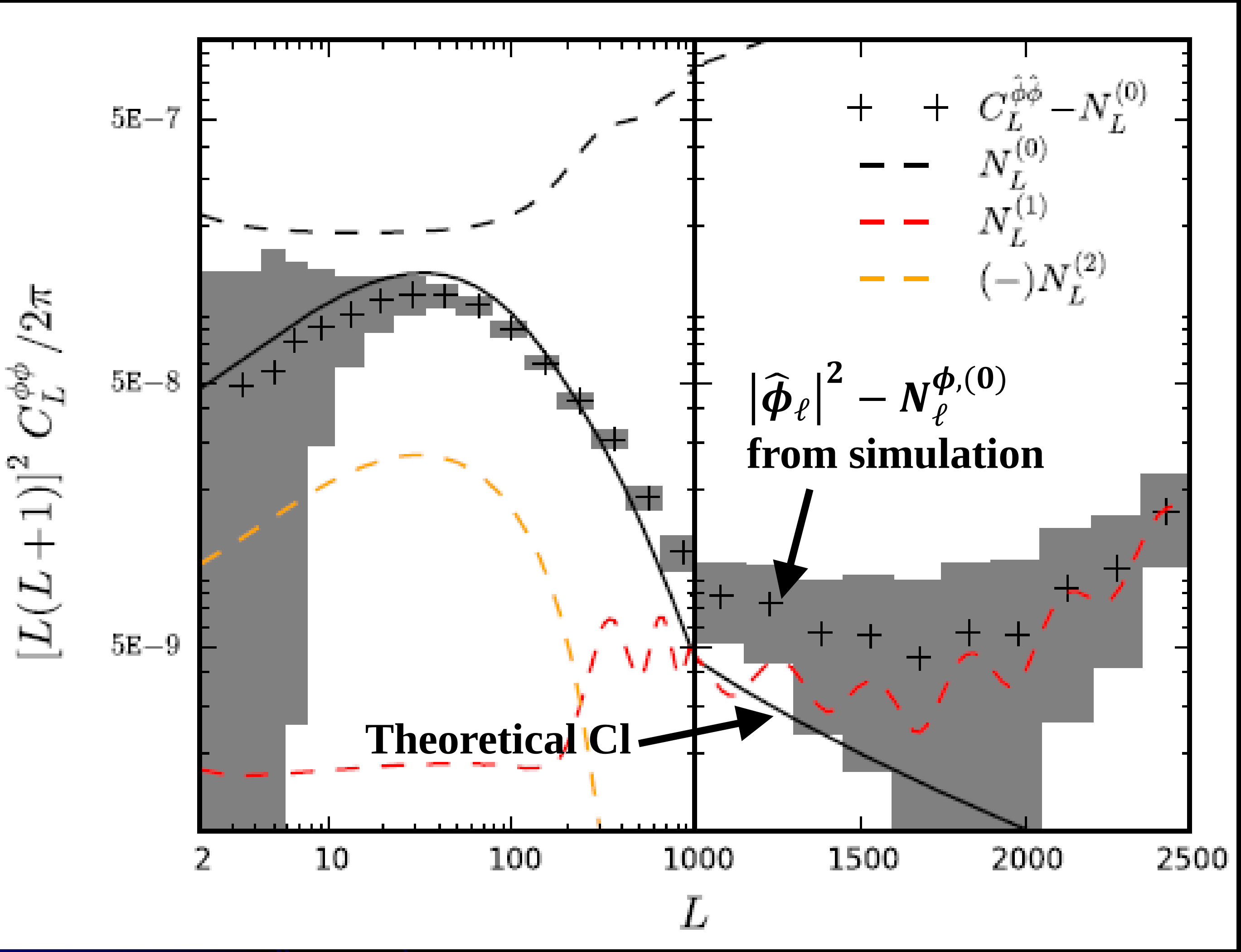
- **Gradient mode**

$\ell^4 \left|\hat{\phi}_L\right|^2 / 2\pi$



Estimating lensing power spectrum

- **Bias contributions** (e.g., Hanson+'11)



Reduced-bias estimator

(2) improve "standard" estimator to remove "mean field"

$$\langle \hat{\phi}_{\vec{\ell}} \rangle = R_\ell^{\phi M} M_\ell \quad \longrightarrow \quad \langle \hat{\phi}_{\vec{\ell}}^{BR} \rangle = 0$$

We consider "lensing" and "mask" estimators:

$$\widehat{M}_\ell = N_\ell^{MM} \int d\vec{L} \frac{f_{\ell,L}^{\phi} f_{\ell,L}^{M}}{2\tilde{C}_\ell^{\Theta\Theta} \tilde{C}_L^{\Theta\Theta}}$$

$$f_{\ell,L}^{\phi} = \vec{\ell} \cdot \vec{L} \tilde{C}_L^{\Theta\Theta} + \vec{\ell} \cdot (\vec{\ell} - \vec{L}) \tilde{C}_{|\vec{\ell}-\vec{L}|}^{\Theta\Theta}$$

$$f_{\ell,L}^{M} = [\tilde{C}_L^{\Theta\Theta} + \tilde{C}_{|\vec{\ell}-\vec{L}|}^{\Theta\Theta}]$$

$$R_\ell^{ab} = N_\ell^{aa} \int d\vec{L} \frac{f_{\ell,L}^{a} f_{\ell,L}^{b}}{2\tilde{C}_\ell^{\Theta\Theta} \tilde{C}_L^{\Theta\Theta}}$$

$$[N_\ell^{aa}]^{-1} = \int d\vec{L} \frac{f_{\ell,L}^{a} f_{\ell,L}^{a}}{2\tilde{C}_\ell^{\Theta\Theta} \tilde{C}_L^{\Theta\Theta}}$$

Since

$$\langle \hat{\phi}_\ell \rangle_{CMB} = \phi_\ell + R_\ell^{\phi M} M_\ell$$

$$\langle \widehat{M}_\ell \rangle_{CMB} = R_\ell^{M\phi} \phi_\ell + M_\ell$$

we define a new estimator

$$\hat{\phi}_\ell^{BR} = \frac{\hat{\phi}_\ell - R_\ell^{\phi M} \widehat{M}_\ell}{1 - R_\ell^{\phi M} R_\ell^{M\phi}}$$

Appendix: Filter functions

- **"standard" quadratic estimator**

$$F^a_{\vec{\ell},\vec{L}} = -N^{aa}_\ell f^a_{\ell,L}$$

$$f^\phi_{\ell,L} = \vec{\ell}\cdot\vec{L}C^{\Theta\Theta}_L + \vec{\ell}\cdot(\vec{\ell}-\vec{L})C^{\Theta\Theta}_{|\vec{\ell}-\vec{L}|}$$

$$f^\varpi_{\ell,L} = (\star\,\vec{\ell})\cdot\vec{L}C^{\Theta\Theta}_L + (\star\,\vec{\ell})\cdot(\vec{\ell}-\vec{L})C^{\Theta\Theta}_{|\vec{\ell}-\vec{L}|}$$

$$[N^{aa}_\ell]^{-1} = \int d\vec{L}\,\frac{f^a_{\ell,L}f^a_{\ell,L}}{2\tilde{C}^{\Theta\Theta}_\ell\tilde{C}^{\Theta\Theta}_L}$$

- **Our "bias-reduced" estimator**

$$F'^{\,\phi}_{\vec{\ell},\vec{L}} = \frac{F^\phi_{\ell,L} - R^{\phi M}F^M_{\ell,L}}{1 - R^{\phi M}R^{M\phi}}$$

$$f^\phi_{\ell,L} = \vec{\ell}\cdot\vec{L}\tilde{C}^{\Theta\Theta}_L + \vec{\ell}\cdot(\vec{\ell}-\vec{L})\tilde{C}^{\Theta\Theta}_{|\vec{\ell}-\vec{L}|}$$

$$f^\varpi_{\ell,L} = (\star\,\vec{\ell})\cdot\vec{L}\tilde{C}^{\Theta\Theta}_L + (\star\,\vec{\ell})\cdot(\vec{\ell}-\vec{L})\tilde{C}^{\Theta\Theta}_{|\vec{\ell}-\vec{L}|}$$

$$f^M_{\ell,L} = [\tilde{C}^{\Theta\Theta}_L + \tilde{C}^{\Theta\Theta}_{|\vec{\ell}-\vec{L}|}]$$

$$R^{ab}_\ell = N^{aa}_\ell\int d\vec{L}\,\frac{f^a_{\ell,L}f^b_{\ell,L}}{2\tilde{C}^{\Theta\Theta}_\ell\tilde{C}^{\Theta\Theta}_L}$$

Lensing Reconstruction

Information on the deflection angle is included in the observed lensed anisotropies

$$\tilde{\Theta}(\vec{n}) = \Theta\left(\vec{n} + \vec{d}\right) \simeq \Theta(\vec{n}) + d(\vec{n}) \cdot \nabla\Theta(\vec{n})$$

⇨ **We need to reconstruct the deflection angle only from the statistical properties of lensed and unlensed CMB**

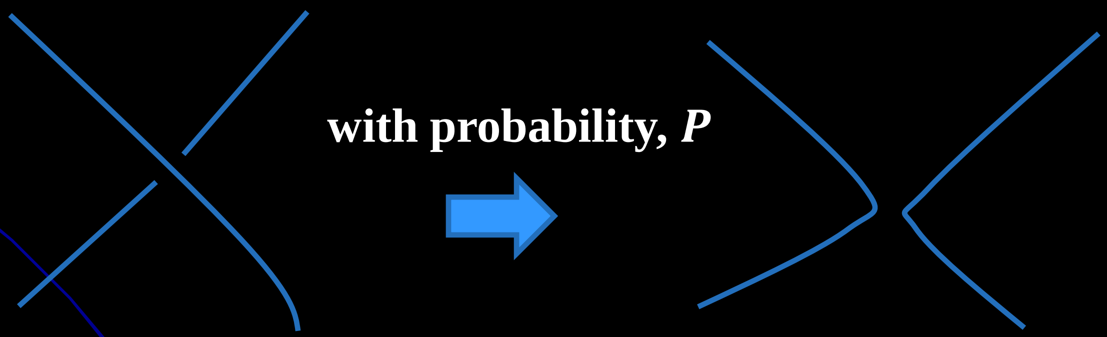
Result 2. Cosmological applications

We consider two sources of curl mode: primordial GWs and cosmic strings

Model of cosmic strings

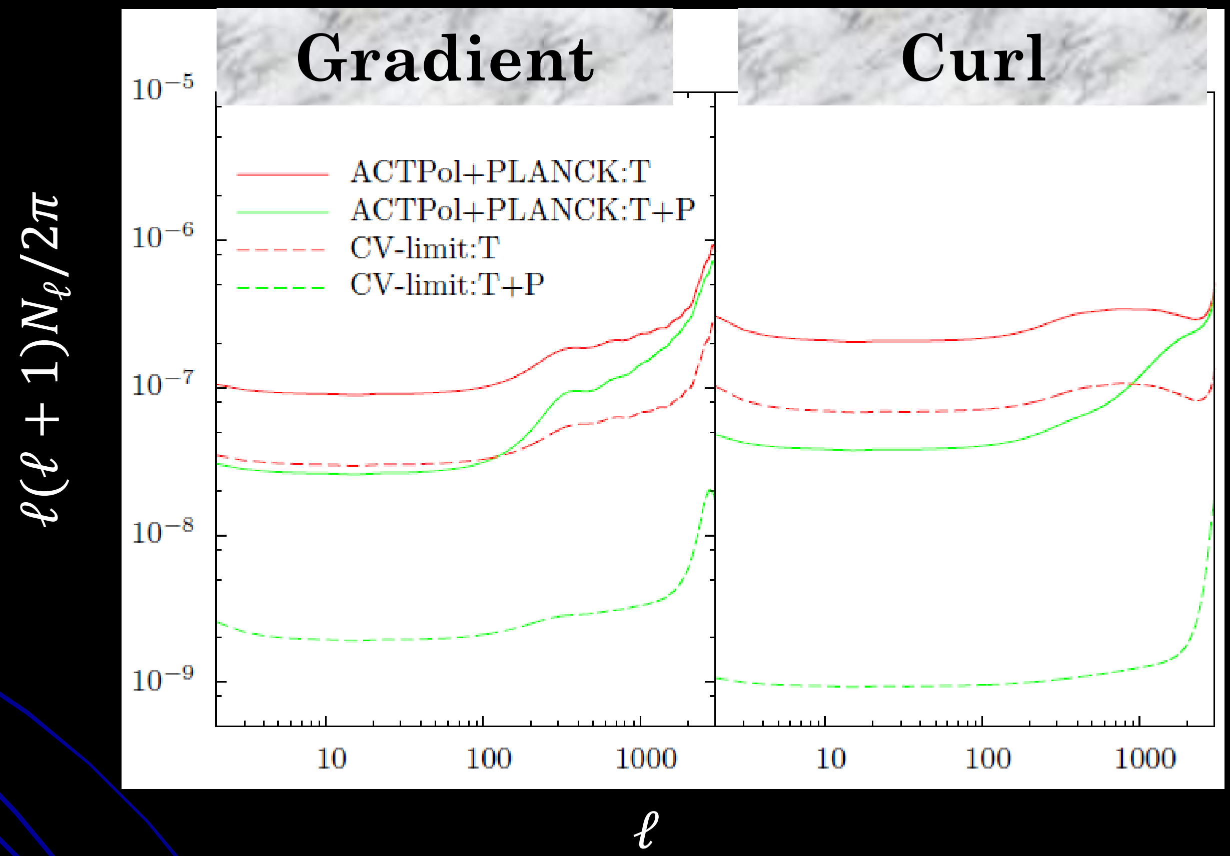
See Yamauchi, *TN* & Taruya '12 for details of cosmic strings

- **We consider straight string, randomly oriented**
- **Motion of strings is determined by velocity-dependent one scale model which depends on string tension, $G\mu$ and intercommuting probability, P**

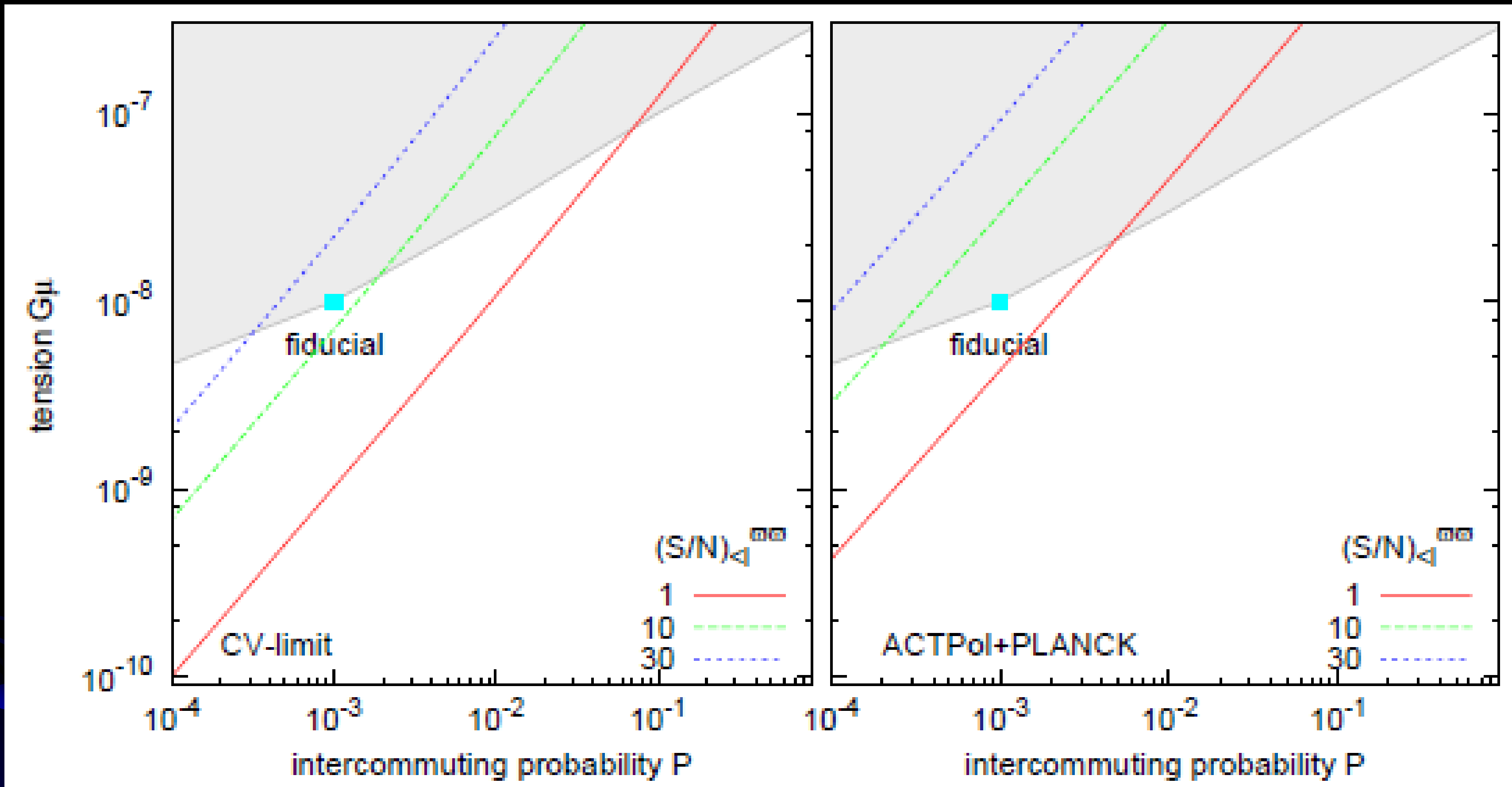


- **Lensing is induced by metric perturbations from strings in our line-of-site**

Noise spectrum



- The variance of curl-mode estimator is similar to that of gradient-mode
- If we include polarization, the variance of curl-mode estimator is improved efficiently compared to that of gradient mode



$$\omega(\theta,\varphi) = \int_0^{\chi_s} d\chi \frac{\chi_s - \chi}{\chi_s \chi} \frac{1}{\sin\theta} \frac{d}{d\chi} \left(\frac{\partial \Omega_\theta}{\partial \varphi} - \frac{\partial \Omega_\varphi}{\partial \theta} \right)$$

Vector and tensors contributions

Purposes

➢ Previous work including curl-mode reconstruction

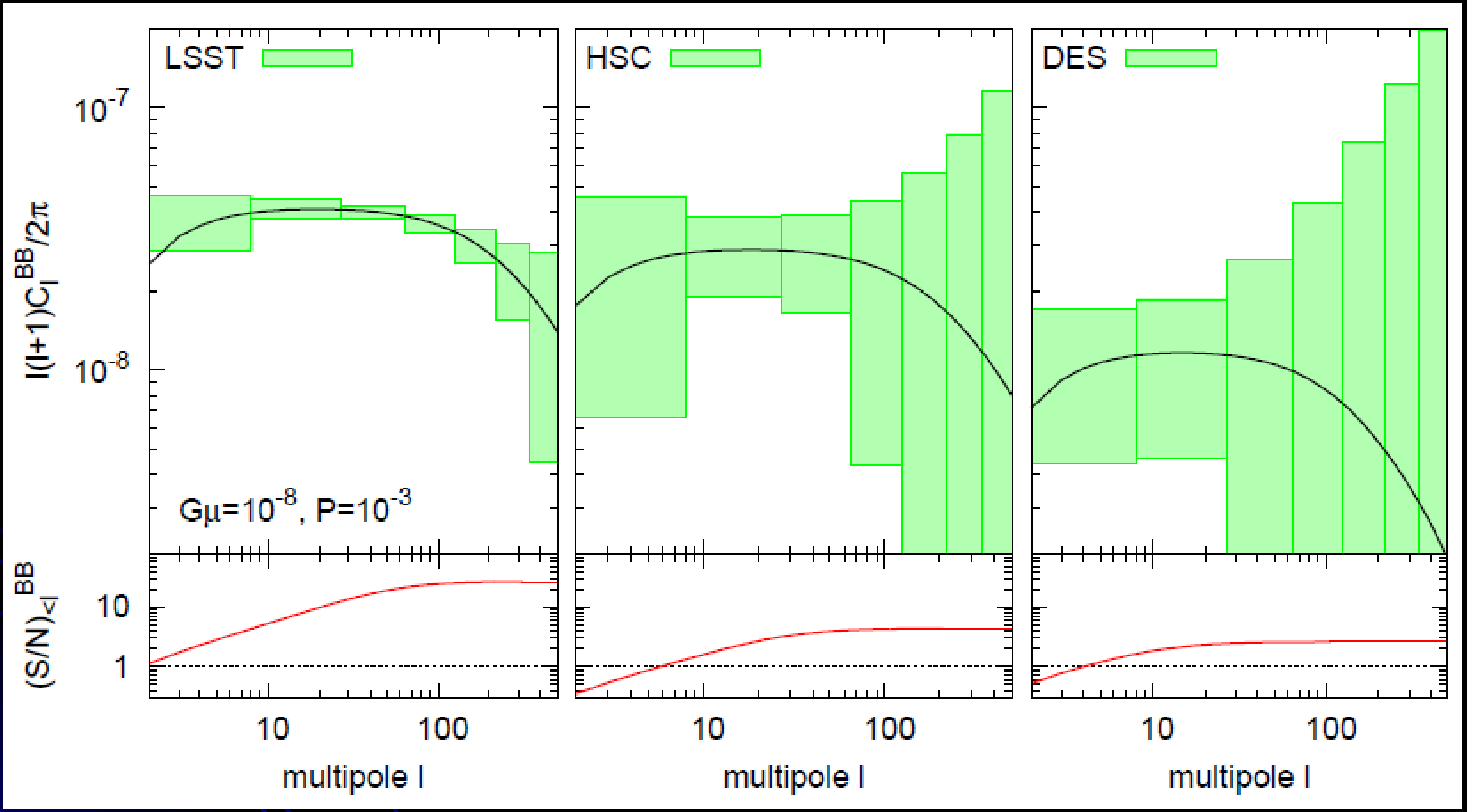
✓ Cooray+'05

- empirically defined a quadratic estimator in flat-sky
- claimed that the gradient-mode estimator given in previous studies are biases in the presence of curl mode

➢ Our purposes

1. Derive estimator including curl mode
2. Lensing reconstruction from current CMB data and show cosmological implications
3. Compute the detectability of curl mode from primordial GWs and cosmic strings with full-sky estimator., and also compare with that of B-mode shear generated from cosmic strings.

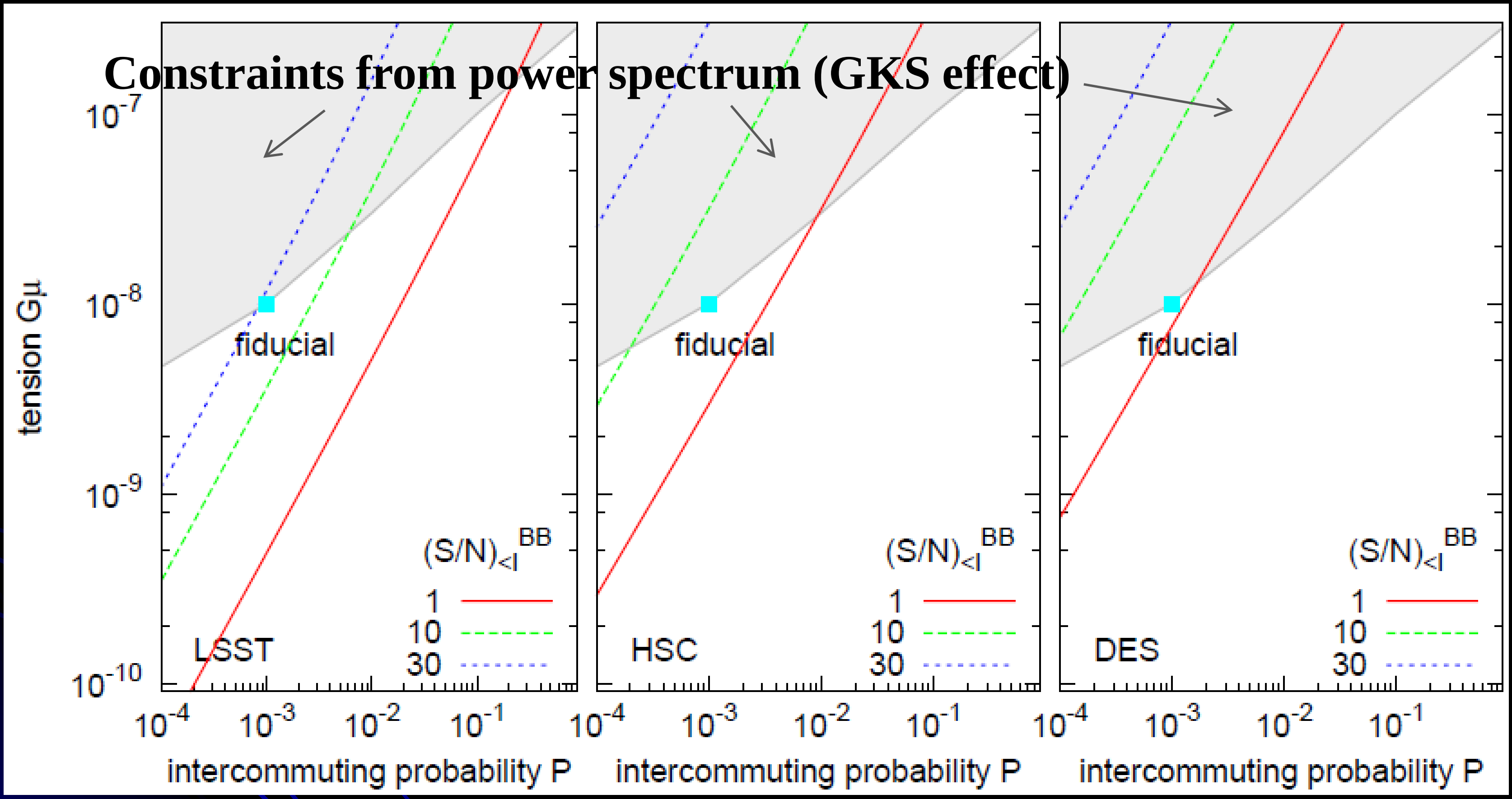
Comparison with B-mode shear



Yamauchi, TN & Taruya '12

Comparison with power spectrum

- S/N on $P - G\mu$ plane



Yamauchi, TN & Taruya ’12

LSST has sensitivity to cosmic strings with P~0.1 and $G\mu \sim 10^{-7}$