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"Stable null bound orbits around a black ring"

RESCEU SYMPOSIUM ON

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Stable Bound Null Orbit around a Black Ring

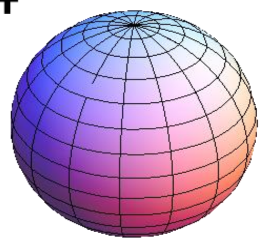
Hideki Ishihara

T.Igata, H.Ishihara, and Y.Takamori
(Osaka City University)

Introduction

Higher-dim. Black Hole is a key
for a verification of extra dimensions?

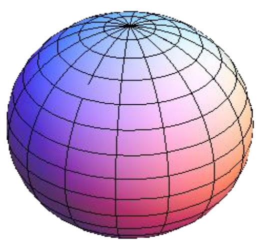
$D = 4$



S^2

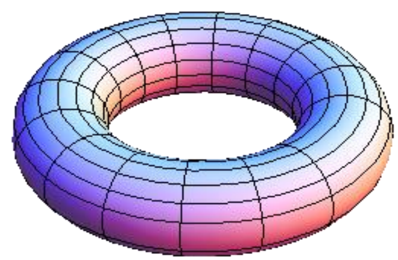
Kerr Black Hole

$D = 5$



S^3

Black Hole
Myers & Perry (1986)



$S^2 \times S^1$

Black Ring
Emparan & Reall (2002)

Geodesics around a Black Hole

Geodesic particles are important probes of gravitational field around a black hole.

$D = 4$

- ISCO appears for timelike particles
- Unstable circular orbits exist for null particles
- Geodesic equations in Kerr geometry are separable

$D = 5$

For spherical black holes (the Myers-Perry metric)

- No stable circular orbit exists for timelike particles
- Unstable circular orbits exist for null and timelike particles
- Geodesic equations are separable

V.P.Frolov and D.Stojkovic(2003)

For black rings (the Emparan-Real metric)

- Stable stationary orbits exist for timelike particles
- Geodesic equations would not be separable

J.Hoskisson(2008), M.Durkee(2009),
T.Igata, H.Ishihara, and Y.Takamori(2010),(2011)
S.Grunau, V.Kagramanova, J.Kunz, C.Lammerzahl(2012)

Stable Bound Orbits

Orbits stable against small perturbations, and bounded in a finite domain outside the black hole horizon.

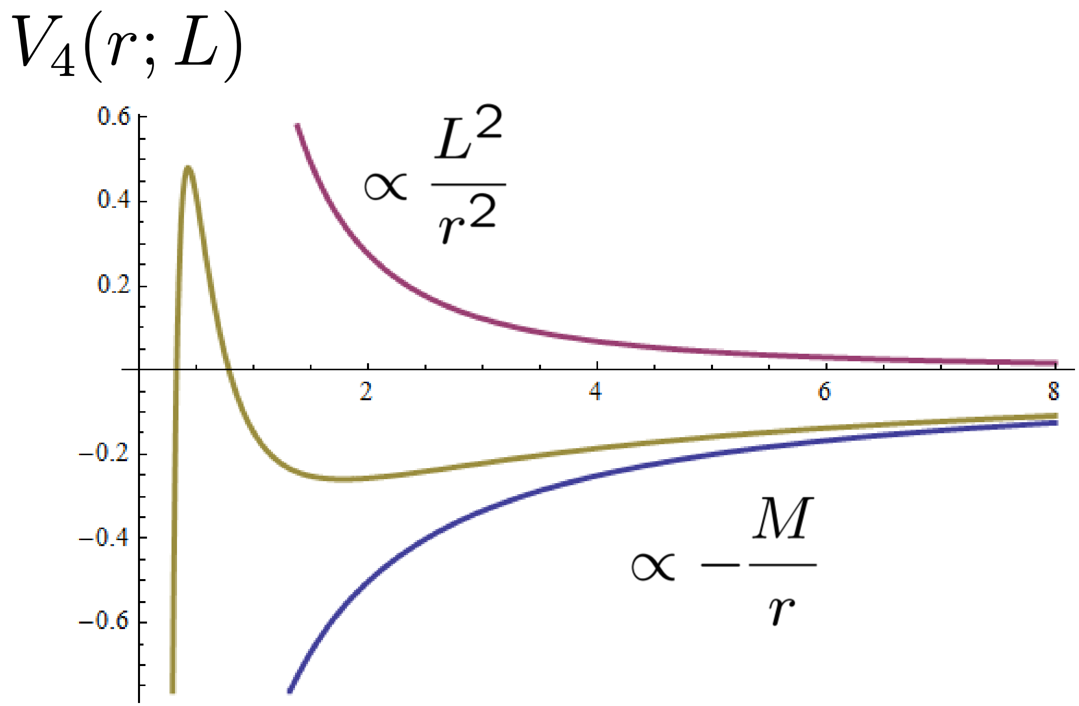
	Stable Bound Orbit	
	Timelike	Null
4-D Black Holes	Yes	No
5-D Black Holes	No	No
5-D Black Rings	Yes	?

T.Igata, H.Ishihara, and Y.Takamori, Phys. Rev. D82, 101501 (2010)

Particle Motion around a Black Hole

4–dim. Schwarzschild BH case

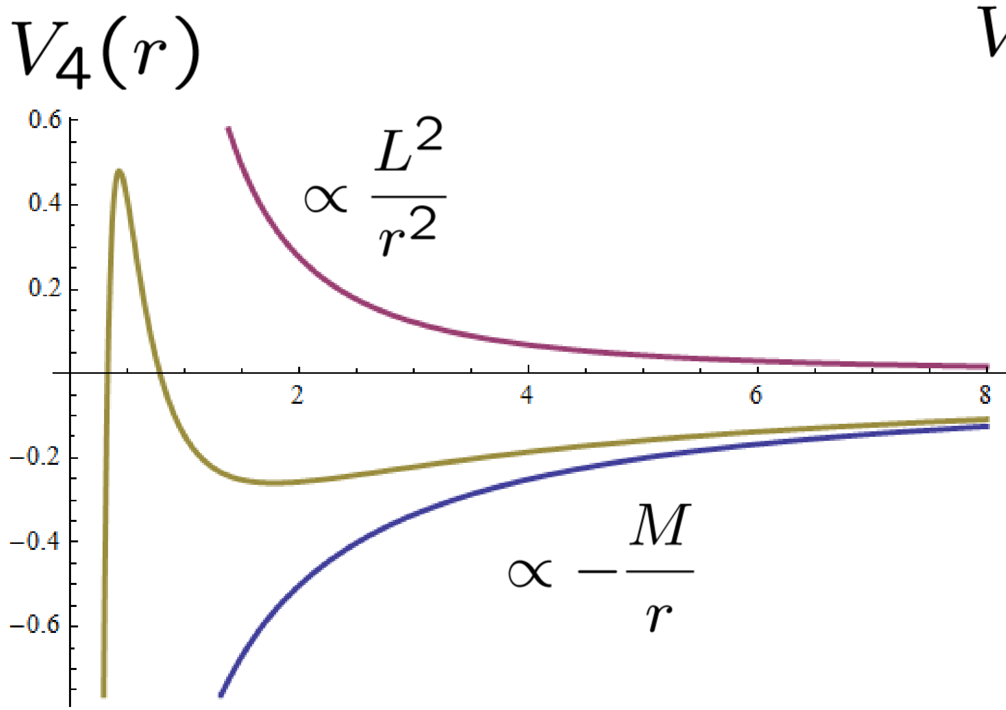
Effective potential : $$V_4(r;L) = \frac{L^2}{r^2} - \frac{M}{r} - \frac{ML^2}{r^3}$$

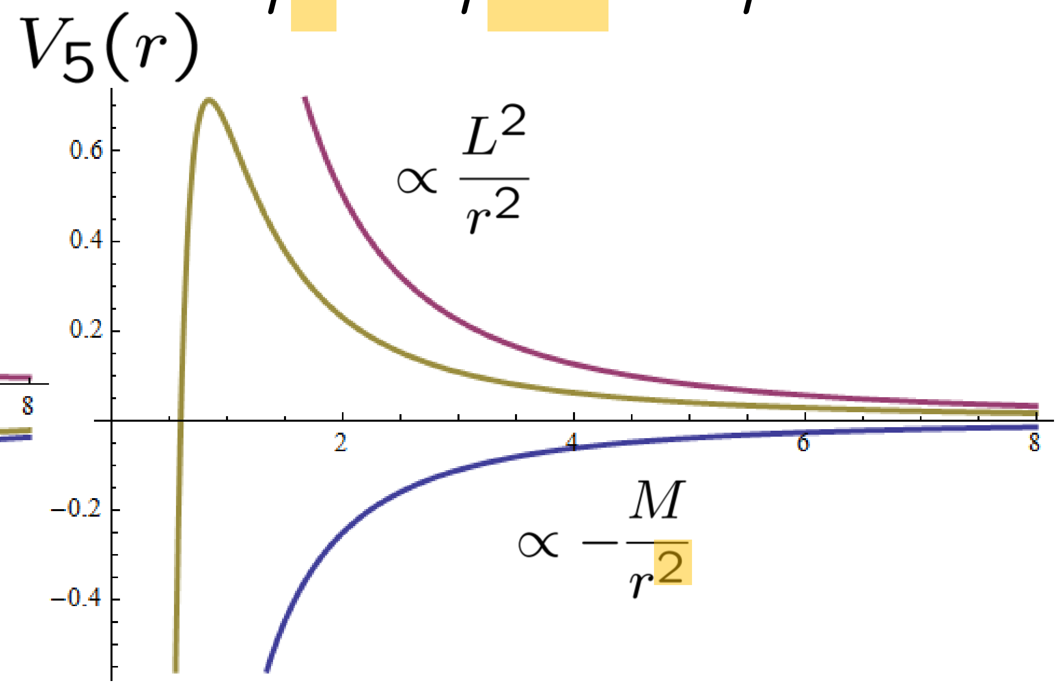


Particle Motion around a Black Hole

D−dim. Schwarzschild BH case

Effective potential : $V_D(r;L) = \dfrac{L^2}{r^2} - \dfrac{M}{r^{D-3}} - \dfrac{ML^2}{r^{D-1}}$





No stable bound orbits for $D \geq 5$

Stationary Points Set for Effective Potential

Effective potential for D−dim. Schwarzschild BH

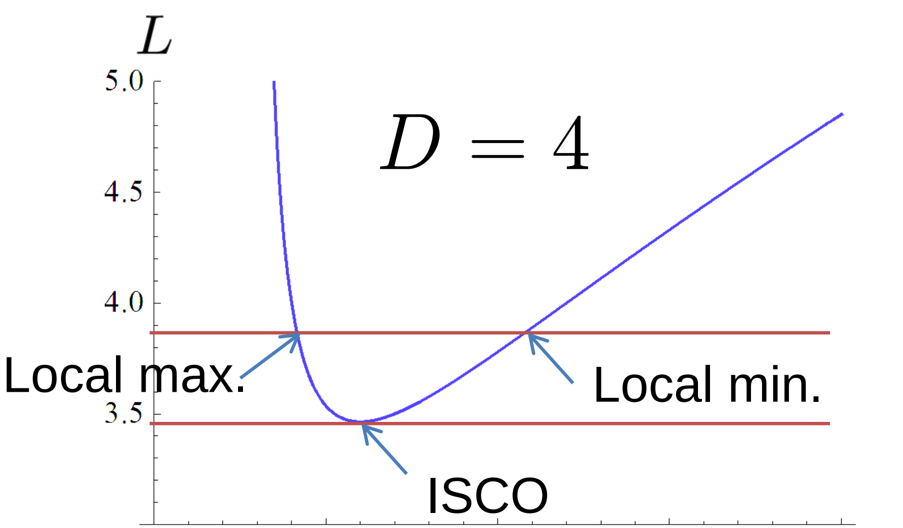
$$V_D(r;L) = \frac{L^2}{2r^2} - \frac{M}{r^{D-3}} - \frac{ML^2}{r^{D-1}}$$

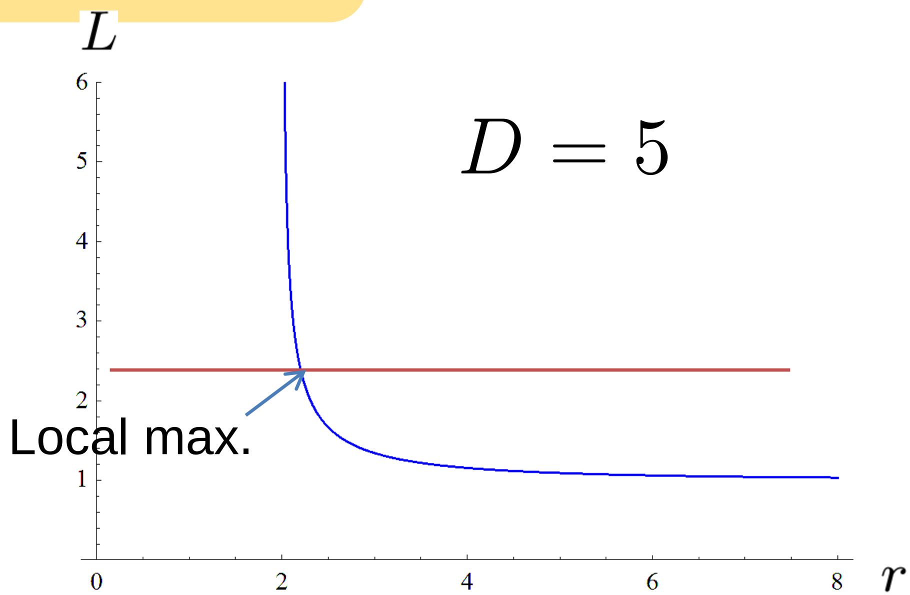
Stationary condition

$$\partial_r V_D(r;L) = -\frac{L^2}{r^3} + (D-3)\frac{M}{r^{D-2}} + (D-1)\frac{ML^2}{r^D} = 0$$

Stationary Points Set

$$L(r) = \sqrt{\frac{(D-3)Mr^2}{r^{D-3}-(D-1)M}}$$





no stable bound orbits

How about

Black Ring ?

5D Singly Rotating Black Ring

Metric

Emparan & Reall (2002)

$$ds^2 = -\frac{F(y)}{F(x)}\left(dt - CR\frac{1+y}{F(y)}d\psi\right)^2 + \frac{R^2}{(x-y)^2}F(x)\left(-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right),$$

$$F(\xi) = 1 + \lambda\xi, \quad G(\xi) = (1-\xi^2)(1+\nu\xi), \quad C = \sqrt{\lambda(\lambda-\nu)\frac{1+\lambda}{1-\lambda}}, \quad \lambda = \frac{2\nu}{1+\nu^2}$$

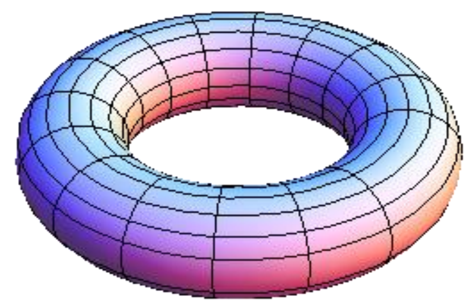
Free parameters

R : ring radius

ν : thickness $\quad$ (0 $<$ ν $<$ 1) (thin $\nu \to 0$, fat $\nu \to 1$)

Killing vectors

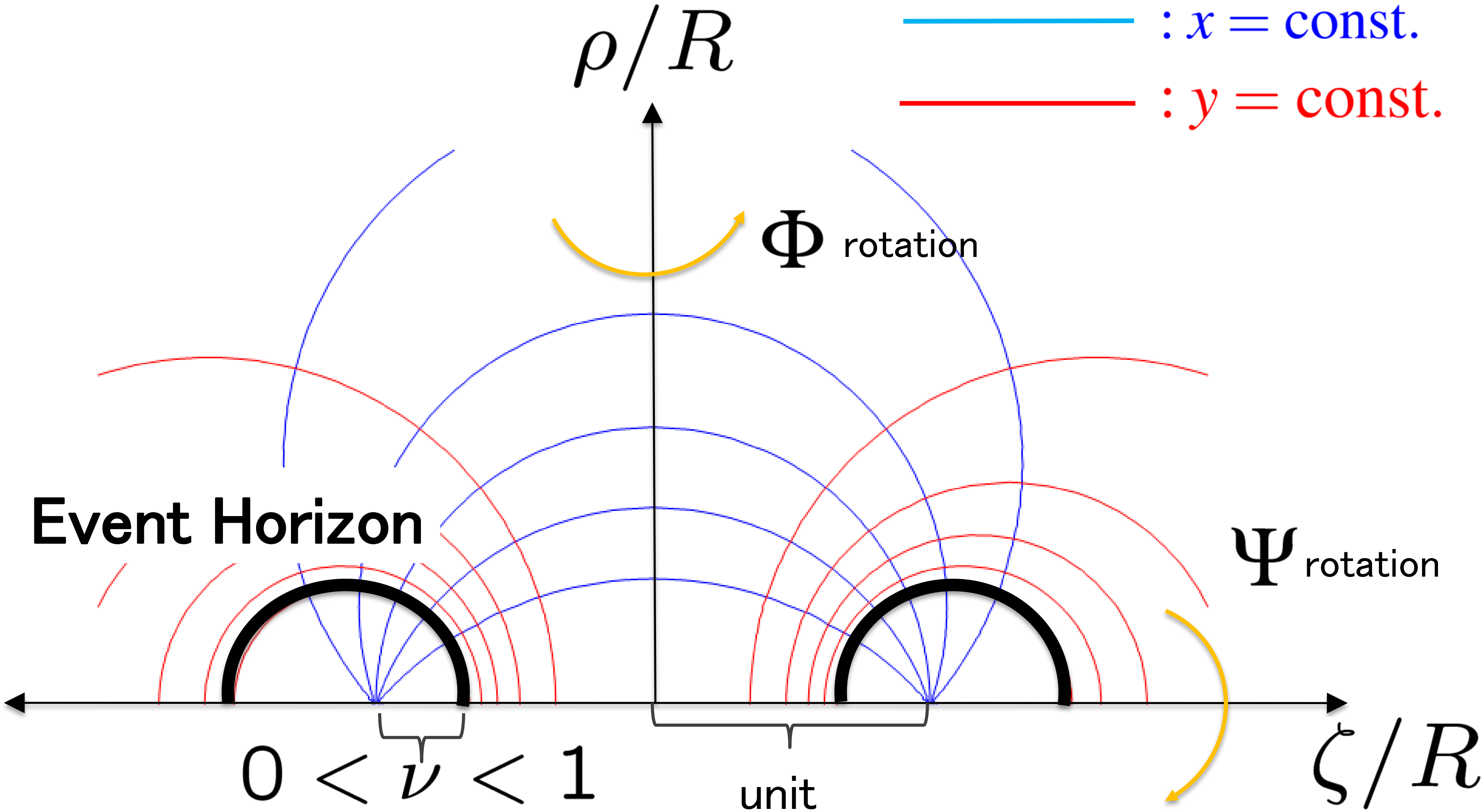
$$\partial_t, \quad \partial_\phi, \quad \partial_\psi$$



Horizon topology

$$\mathrm{S}^2 \times \mathrm{S}^1$$

Coordinates for the Black Ring



(t, Φ, Ψ) are suppressed.

(ρ, Φ), (ζ, Ψ): polar coordinates on the independent two planes, respectively.

Hamiltonian formalism for a Null particle

Constants of motion

$$E = -k_t, L_\phi = k_\phi, L_\psi = k_\psi$$

Hamiltonian

$$\begin{aligned} H &= \frac{1}{2} g^{\alpha\beta} k_\alpha k_\beta \\ &= \frac{1}{2}\left(g^{\zeta\zeta} k_\zeta^2 + g^{\rho\rho} k_\rho^2 + E^2\, U\right) = 0, \end{aligned}$$

Effective Potential

$$U(\zeta, \rho; l_\psi, l_\phi) = g^{tt} + g^{\phi\phi} l_\phi^2 + g^{\psi\psi} l_\psi^2 - 2 g^{t\psi} l_\psi$$

$$(l_\phi := L_\phi / E, \quad l_\psi := L_\psi / E)$$

Stationary Orbits

Stationary Solutions

$$\dot{\rho} = \dot{\zeta} = 0 \longrightarrow k_\rho = k_\zeta = 0$$

$$\longrightarrow \partial_\zeta U(\zeta, \rho; l_\psi, l_\phi) = \partial_\rho U(\zeta, \rho; l_\psi, l_\phi) = 0$$

$$\longrightarrow l_\psi = l_\psi(\zeta, \rho), \qquad l_\phi = l_\phi(\zeta, \rho)$$

Stationary Points Set

$$\Sigma = \{(\zeta, \rho, l_\psi(\zeta, \rho), l_\phi(\zeta, \rho))\}$$

defines 2-dimensional surface embedded
in the 4-dimensional space

$$\mathcal{N} = \{(\zeta, \rho, l_\psi, l_\phi)\}$$

Null condition for the stationary orbits is $U\big|_\Sigma = 0$

Stability conditions

Local minimum of $U(\zeta,\rho;l_\psi,l_\phi)$ $\Leftrightarrow$ Stable Stationary orbit

$\Updownarrow$

Two eigenvalues of Hessian matrix

$$\mathcal{H}(U)=\begin{pmatrix} \partial_\zeta^2 U & \partial_\zeta\partial_\rho U \\ \partial_\rho\partial_\zeta U & \partial_\rho^2 U \end{pmatrix}$$

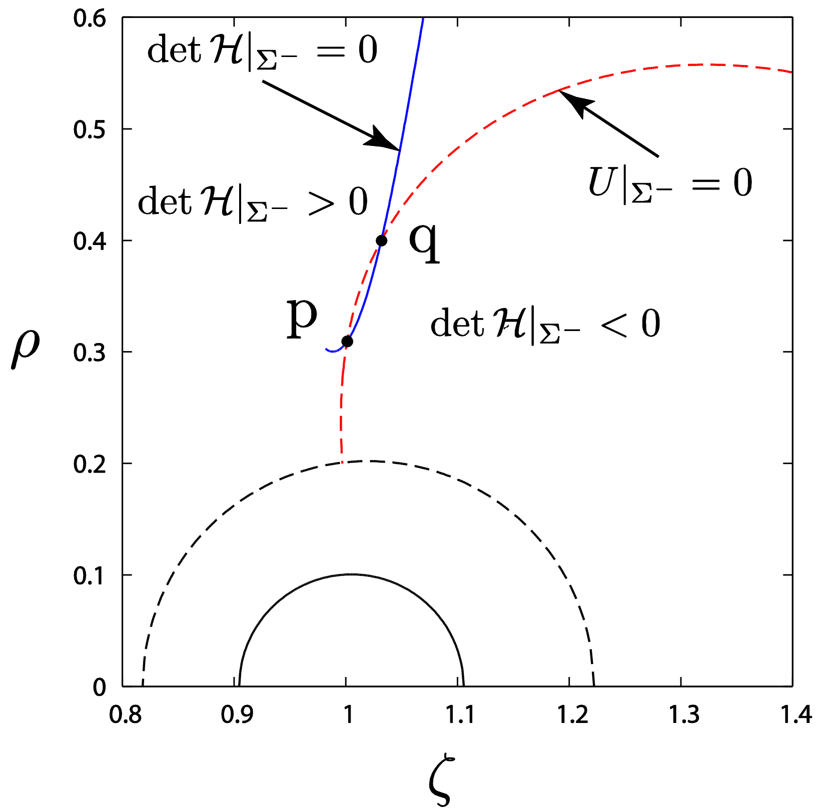
are positive at the stationary point.

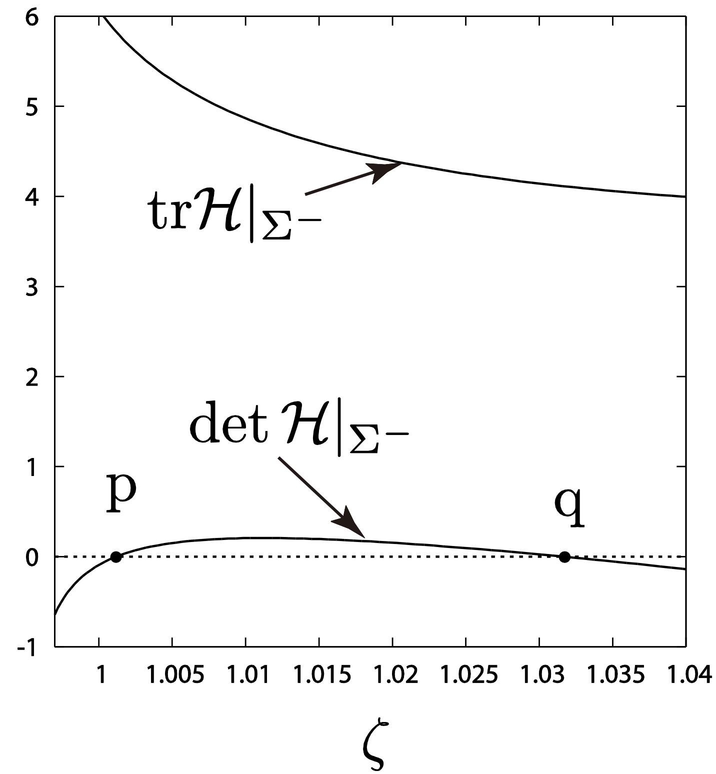
$\Updownarrow$

$\det\mathcal{H}(U)|_\Sigma > 0$ and $\mathrm{tr}\,\mathcal{H}(U)|_\Sigma > 0$

Existence of Stable Stationary Orbits

Projection of Σ into the ζ - ρ plane for $\nu = 0.1$



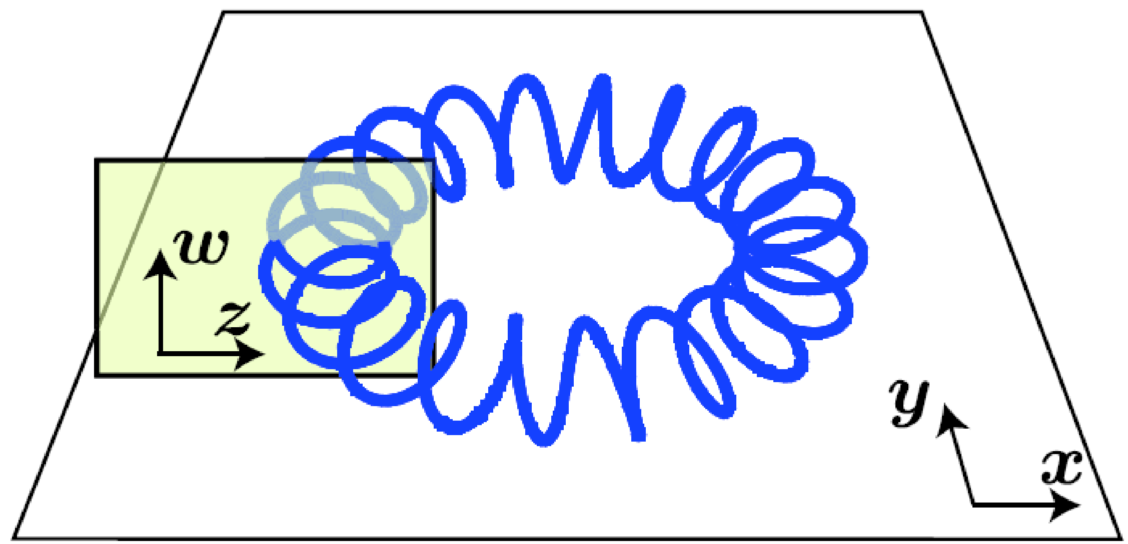


Stable Toroidal Spiral Orbits

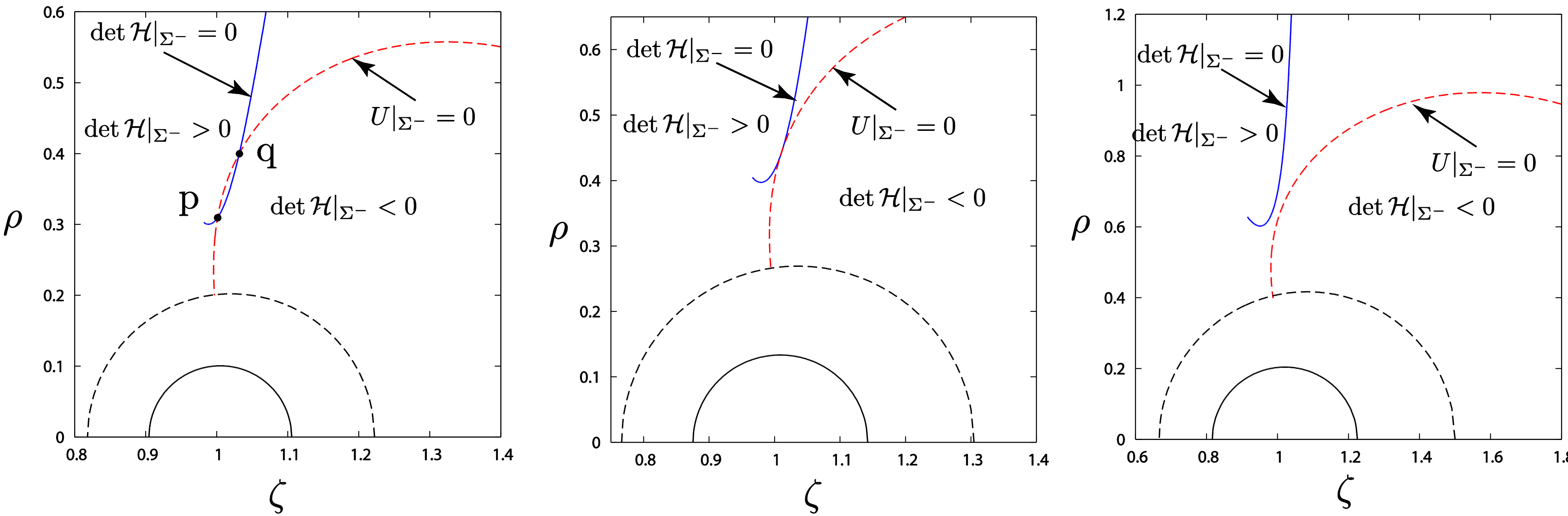
The stable stationary orbit is tangent to a null Killing vector

$$\partial_t + \alpha(l_\psi, l_\phi)\partial_\psi + \beta(l_\psi, l_\phi)\partial_\phi \,.$$

The projection of a orbit on a $t = const$ surface is a toroidal spiral on $\mathrm{S}^1 \times \mathrm{S}^1$.



Critical Thickness



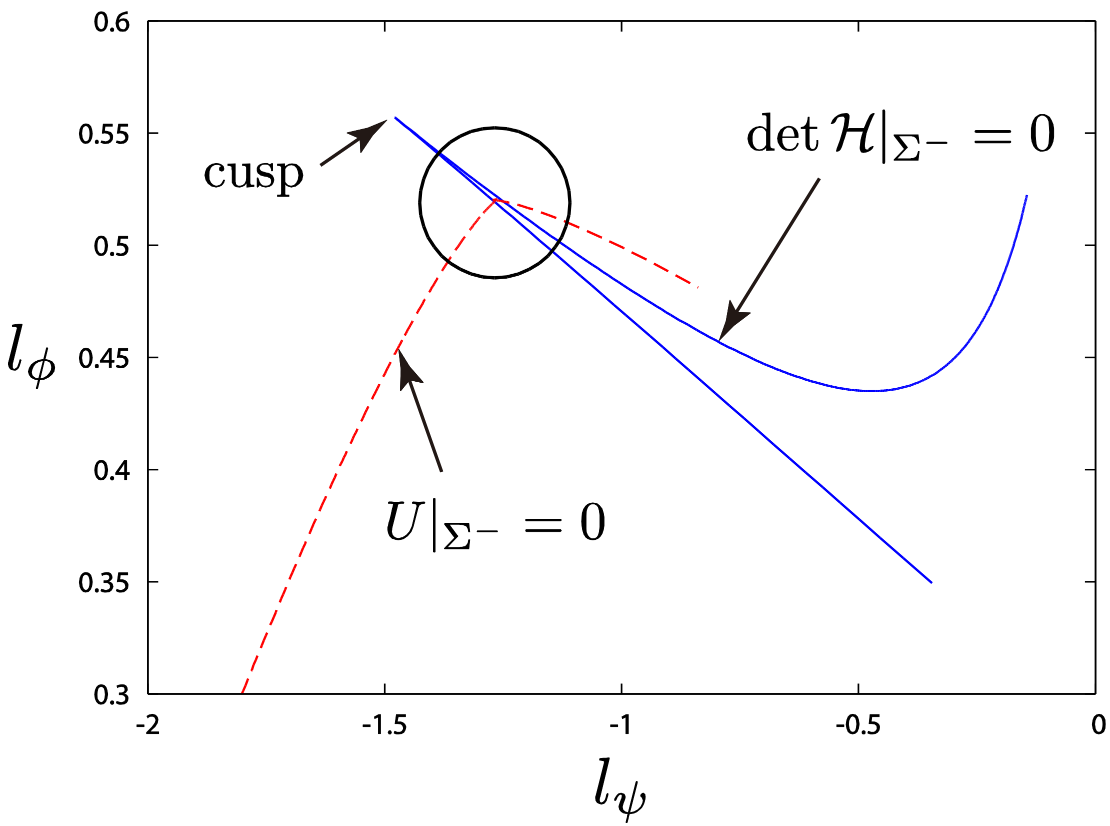
$\nu = 0.1$ $\qquad$ $\nu = 0.13224 \simeq \nu_c$ $\qquad$ $\nu = 0.2$

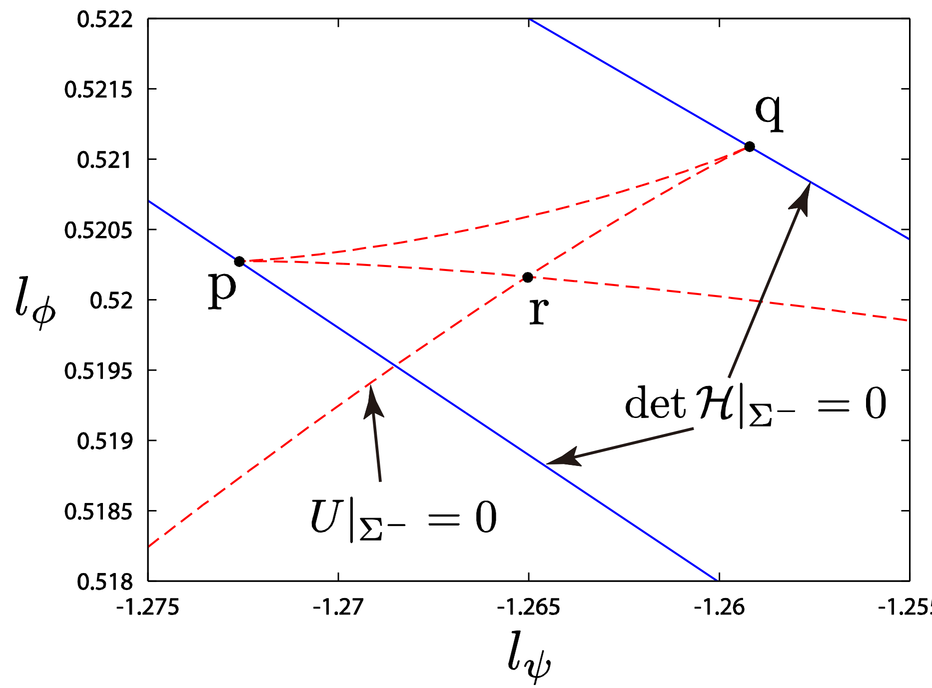
p : Innermost Stable Toroidal Spiral Orbit (ISTSO)

q : Outermost Stable Toroidal Spiral Orbit (OSTSO)

Cusps

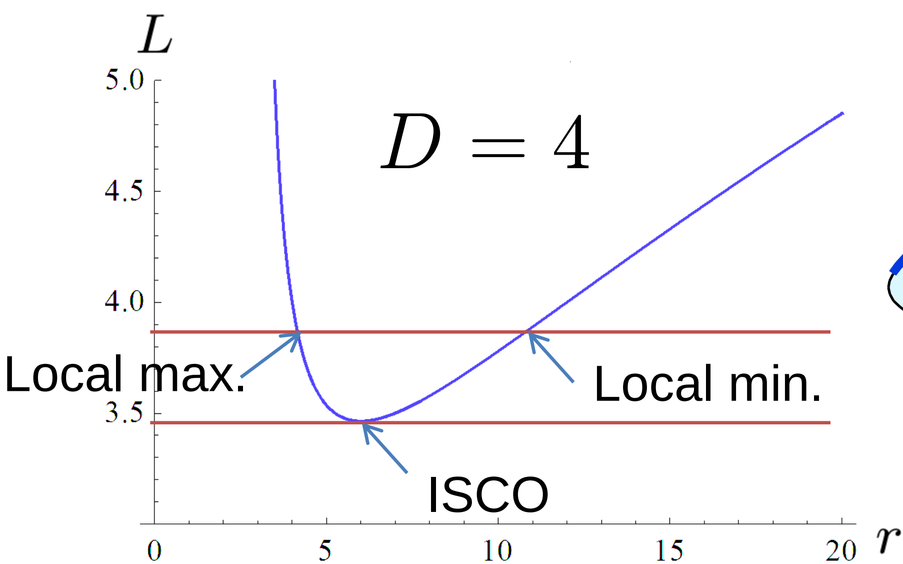
Projection of Σ into the l_ψ - l_ϕ plane for $\nu = 0.1$

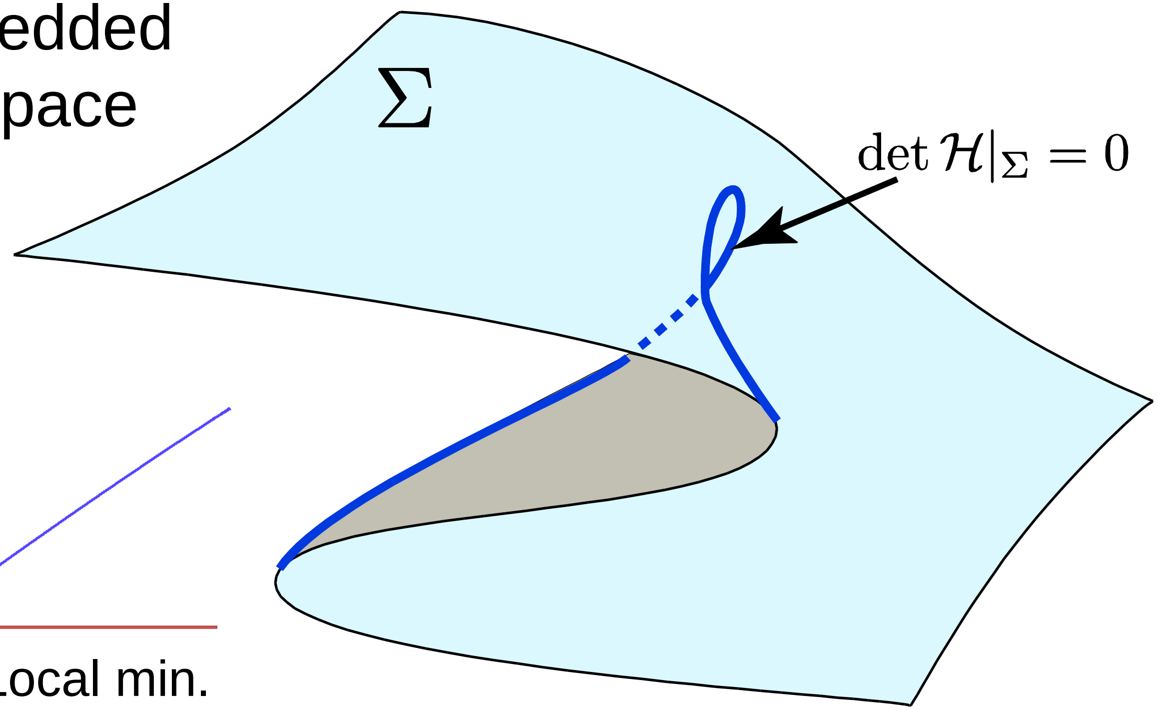




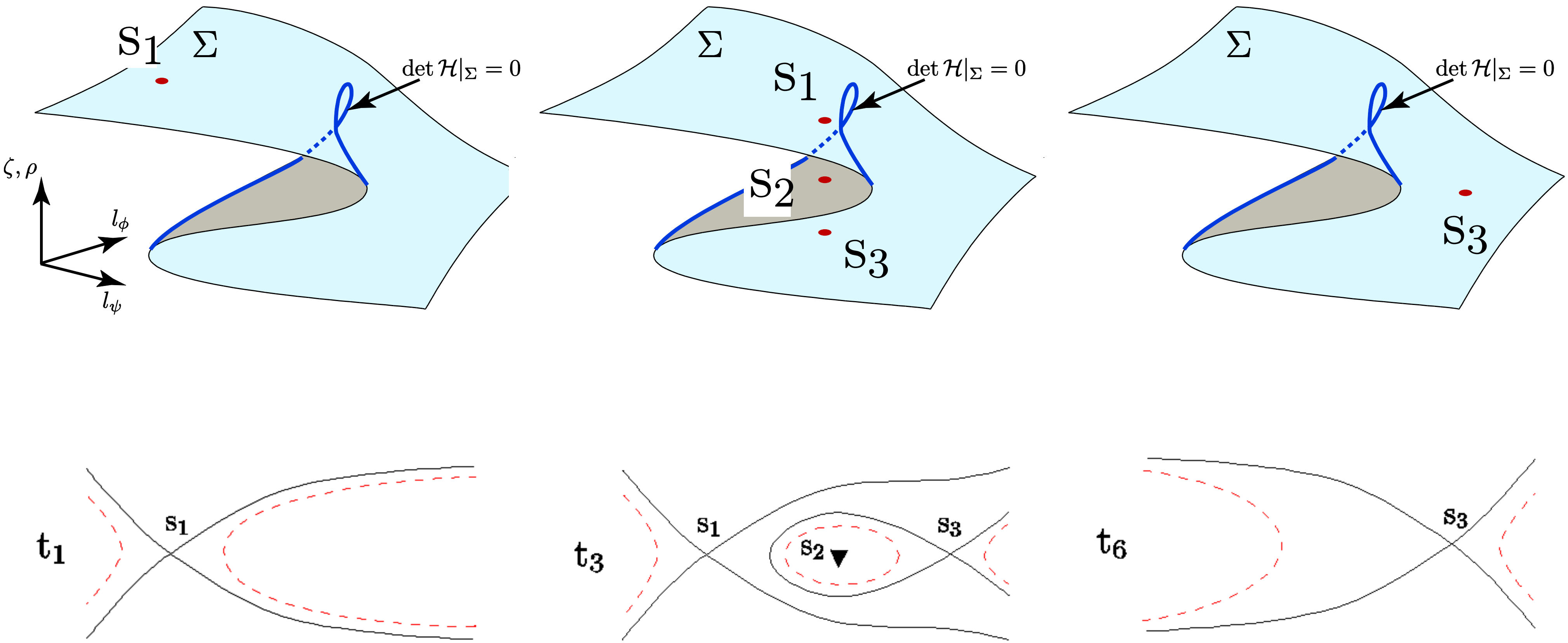
Stationary Points Set

2-dim. surface embedded
in 4-dimensional space

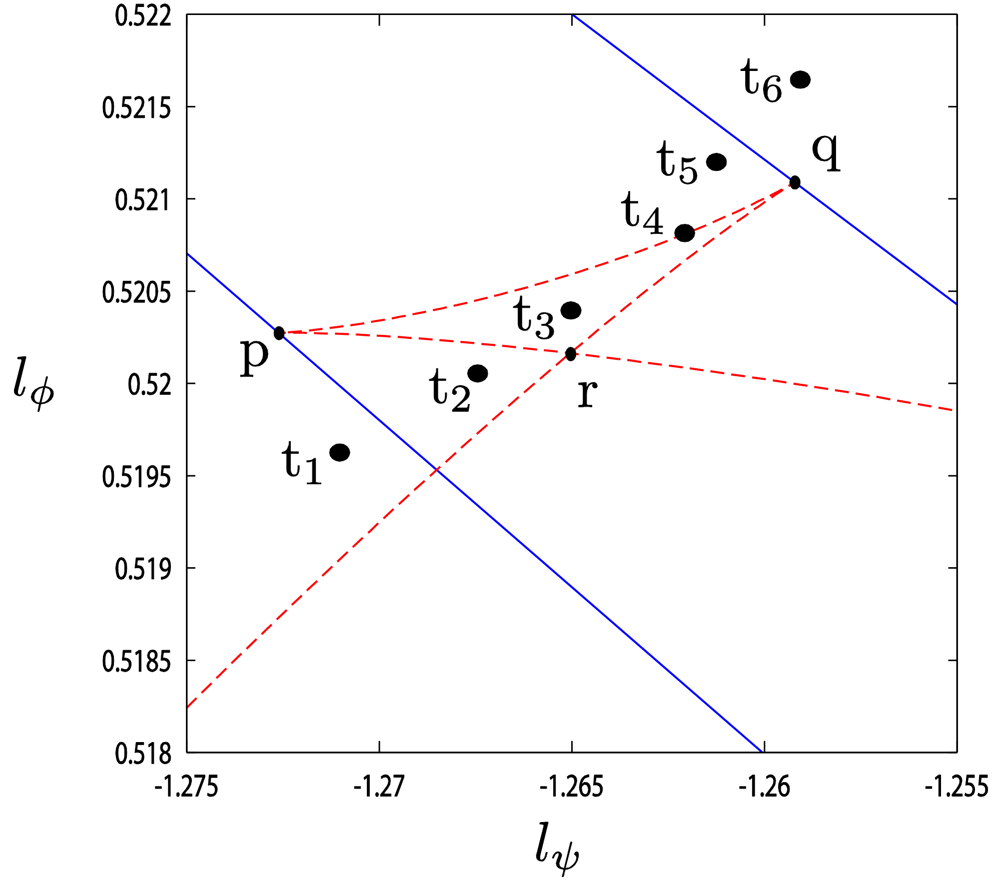


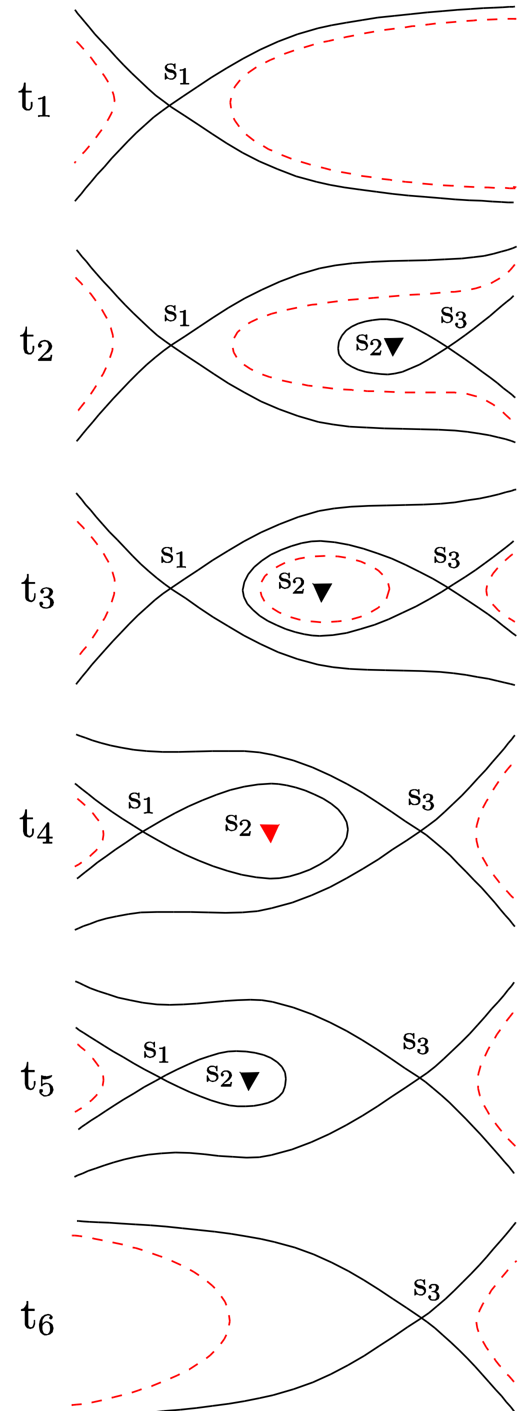


Stationary Points



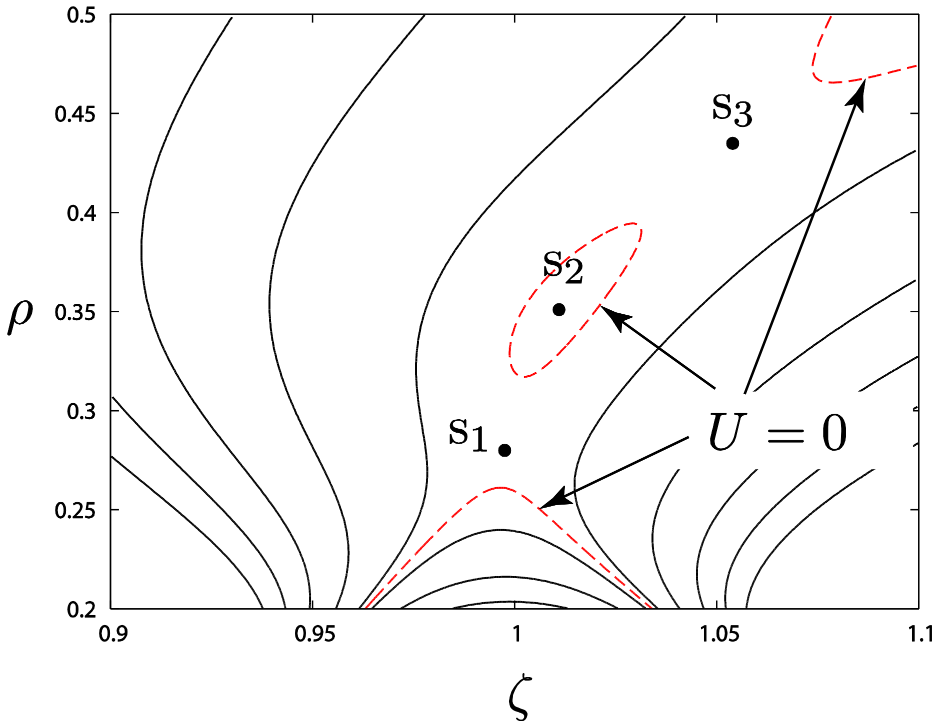
Stationary Points

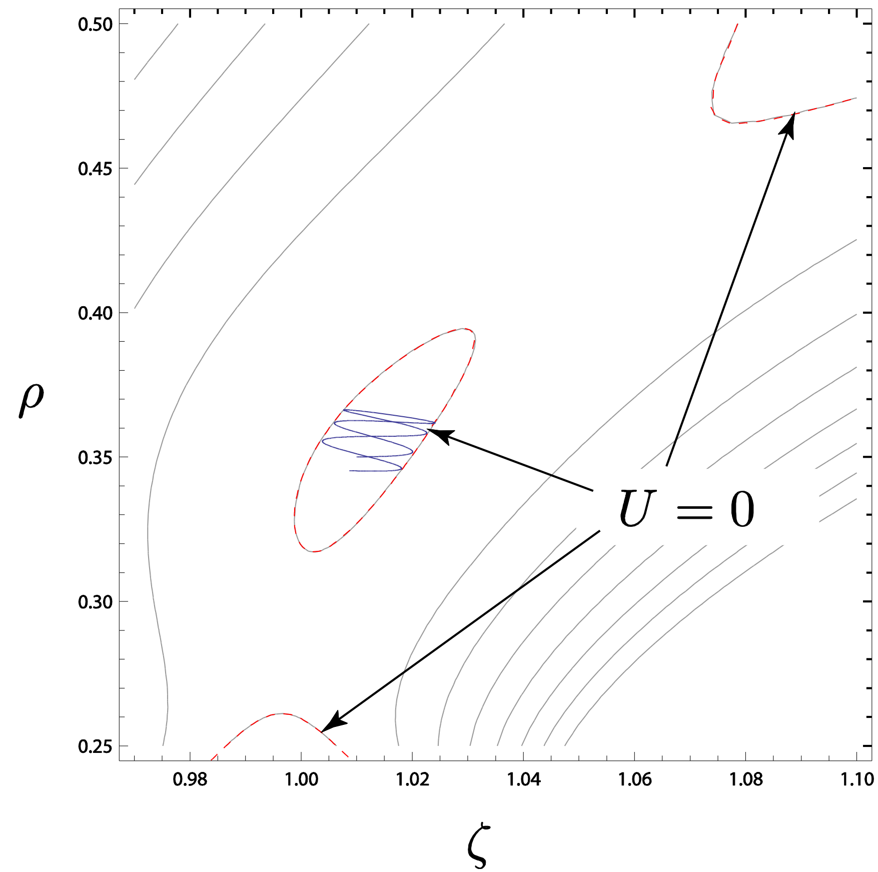




Bounded Null Orbits

Non stationary bounded orbits appear





Summary

Stable bound null orbits exist around a Black Ring.

for $\nu \leq \nu_{\rm c} = 0.13224\cdots$

	Stable Bound Orbit	
	Timelike	Null
4-D Black Holes	Yes	No
5-D Black Holes	No	No
5-D Black Rings	Yes	Yes