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Effect of dark matter halos around IMBHs on the gravitational waves

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Introduction

IMBHs formation and their property

The purpose of this work is to estimate the effect of a dark matter (DM) halo around an intermediate mass black hole (IMBH) on GW. According to Ref.[3], there are two different IMBH formation scenarios. The first scenario is that IMBH is a remnant of collapse of a Pop III star. The second scenario is that IMBH is formed by cold gas within the early forming halos. Both scenarios predict that IMBH in the present is surrounded by dark matter mini-halo. (See Fig.1).

Dark matter mini-halo distribution

The DM profile of a mini-halo can be well approximated by Navarro, Frenk, and White (NFW) profile. But for simplicity, we assume that the DM density is distributed in a spherically symmetric manner around the IMBH and that the radial distribution is described by a power law between r_{min} and r_{max},

$$\rho(\boldsymbol{x}) = \rho_0 \left(\frac{r_0}{r}\right)^{\alpha} \quad (r_{\min} \le r \le r_{\max})$$

where ρ_0 500 $M_\odot$ /pc^3 and r_0 is 1.6 pc according to Ref[2]. r_{min} is a minimum value of the radial distance beyond which stable circular orbits are no longer allowed, that is, an Innermost Stable Circular Orbit (ISCO). So $r_{min} = r_{ISCO} =$ $6GM_{BH}/c^2$. r_{max} is the size of the DM mini-halo.

Formulation

Dark matter mini-halo distribution

We consider the binary systems which consist of a IMBH which has M_{BH}~10^3 $M_\odot$ and a star which has μ~1 $M_\odot$. The IMBH is surrounded by a DM halo which has a spherical density distribution and exists between r_{max} and r_{min}. We suppose that the DM density is still unperturbed even when the star orbits in the DM halo, which is justified by the fact that the DM total mass~10^6 $M_\odot$ is much larger than the star mass. In other words, we treat the star as a test-particle and we call it a "particle" in the following. The system that we examine here is idealized in comparison with the real astrophysical sources. However this idealization allows us to estimate the order of magnitude of DM effects on GW. In this situation, the equation of motion for the particle can be written as

$$\frac{d^2 r}{dt^2} = -\frac{GM_{\rm eff}}{r^2} - \frac{F}{r^{\alpha-1}} + \frac{l^2}{r^3}$$

where l is the angular momentum of the particle per its mass, and M_{eff} and F is

$$M_{\rm eff} = \begin{cases} M_{\rm BH} + \dfrac{4\pi r_0^\alpha \rho_0}{3-\alpha}\left(r_{\max}^{3-\alpha} - r_{\min}^{3-\alpha}\right) \\ M_{\rm BH} - \dfrac{4\pi r_0^\alpha \rho_0}{3-\alpha} r_{\min}^{3-\alpha} \\ M_{\rm BH} \end{cases}, \quad F = \begin{cases} 0 & (r \ge r_{\max}) \\ \dfrac{4\pi G r_0^\alpha \rho_0}{3-\alpha} & (r_{\min} \le r \le r_{\max}) \\ 0 & (r \le r_{\min}) \end{cases}.$$

In the first term of the right hand side of this equation of motion, the dark matter mini-halo modifies the effective mass of the central BH. The second term contains interesting information of the DM halo radial distribution. The third term is regarded as a centrifugal force. We assume that the second term is much smaller than the first term,

$$\varepsilon r^{3-\alpha} \ll 1 \qquad \left(\varepsilon \equiv \frac{F}{GM_{\rm eff}}\right),$$

that is, we can treat the term which involves DM halo as a perturbation. When the particle orbits circularly around the IMBH, the left hand side of the equation of motion vanishes, and we get the circular orbit radius F. Computing the GW waveforms, we obtain the amplitudes for h_+ and $h_\times$,

$$h_+ = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\frac{1+\cos^2\iota}{2}\cos(2\omega_s t)$$

$$h_\times = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\cos\iota \sin(2\omega_s t)$$

where ι is the inclination which is the angle between the normal to the orbit and the line-of-sight, and $2\omega_s$ is the GW frequency.

Waveforms including GW back-reaction

Then we include the effect of the GW back-reaction within linearized theory. The orbital radius and frequency is not constant any longer, because GW radiation energy E_{GW} is taken from the rotational energy E_{orbit} of the particle.

$$\frac{dE_{\rm orbit}}{dt} = -\frac{dE_{\rm GW}}{dt}.$$

This differential equation of time gives the relation between the orbital radius R and the time. Using this relation, we can compute the orbital frequency ω_s depending on time. So we replace ω_s and R in time-independent waveforms with time-dependent ω_s and R to get the waveform including the GW back-reaction.

In order to compare the theoretical waveforms with the experimental sensitivities, we must perform the Fourier transform of the waveforms. Using the stationary phase method with which only the slowly varying term in the Fourier integral can survive , we express the amplitude in Fourier space as expansions in ε,

$$\tilde{h}_+(f) = \left(\frac{5}{24}\right)^{\frac{1}{2}} \frac{e^{i\Psi_+(f)}}{\pi^{\frac{2}{3}} f^{\frac{7}{6}}} \frac{c}{r} \left(\frac{GM_c}{c^3}\right)^{\frac{5}{6}} \frac{1+\cos^2\iota}{2} \left[1 + \frac{7-2\alpha}{3}\left(\frac{GM_{\rm eff}}{\pi^2 f^2}\right)^{\frac{3-\alpha}{3}} \varepsilon + \cdots\right]$$

$$\tilde{h}_\times(f) = \left(\frac{5}{24}\right)^{\frac{1}{2}} \frac{e^{i\Psi_\times(f)}}{\pi^{\frac{2}{3}} f^{\frac{7}{6}}} \frac{c}{r} \left(\frac{GM_c}{c^3}\right)^{\frac{5}{6}} \cos\iota \left[1 + \frac{7-2\alpha}{3}\left(\frac{GM_{\rm eff}}{\pi^2 f^2}\right)^{\frac{3-\alpha}{3}} \varepsilon + \cdots\right]$$

$$\Psi_+ = 2\pi f\left(t_c + \frac{r}{c}\right) - \Phi_0 - \frac{\pi}{4} + 2\left(\frac{GM_c}{c^3} 8\pi f\right)^{-\frac{5}{3}}$$
$$\times \left[1 + \frac{10}{3}\frac{2\alpha-5}{2\alpha-11}\left(\frac{GM_{\rm eff}}{\pi^2 f^2}\right)^{\frac{3-\alpha}{3}} \varepsilon - \frac{5}{9}\frac{(2\alpha-1)(4\alpha-11)}{4\alpha-17}\left(\frac{GM_{\rm eff}}{\pi^2 f^2}\right)^{\frac{2(3-\alpha)}{3}} \varepsilon^2 + \cdots\right]$$

$$\Psi_\times = \Psi_+ + \frac{\pi}{2}$$

where t_c is the retarded time at coalescence, Φ_0 is the value of the phase at coalescence, M_c is the chirp mass, and $\Psi_\pm$ is the phase of the waveform $h_\pm$. These expansions are valid for the frequency f in which the higher order terms are negligible.

Definition of signal-to-noise ratio

To estimate the detectability, we must compute the signal-to-noise ratio, which is defined as S/N,

$$\left(\frac{S}{N}\right)^2 = 4\frac{\left[\int_{f_{\rm ini}}^{\infty} df \, \frac{\tilde{h}(f)\tilde{h}_t^*(f)}{S(f)}\right]^2}{\int_{f_{\rm ini}}^{\infty} df \, \frac{\left|\tilde{h}_t(f)\right|^2}{S(f)}}$$

where h(f) is the GW signal that is coming in the detector, $h_t(f)$ is the template which is the GW signal we infer in advance, and S(f) is the spectral density that depends on the detector. We consider that the GW is detected by the evolved Laser Interferometer Space Antenna (eLISA). According to Ref [4], the sensitivity of eLISA is shown in Fig.2, which is the square root of the spectral density S(f).

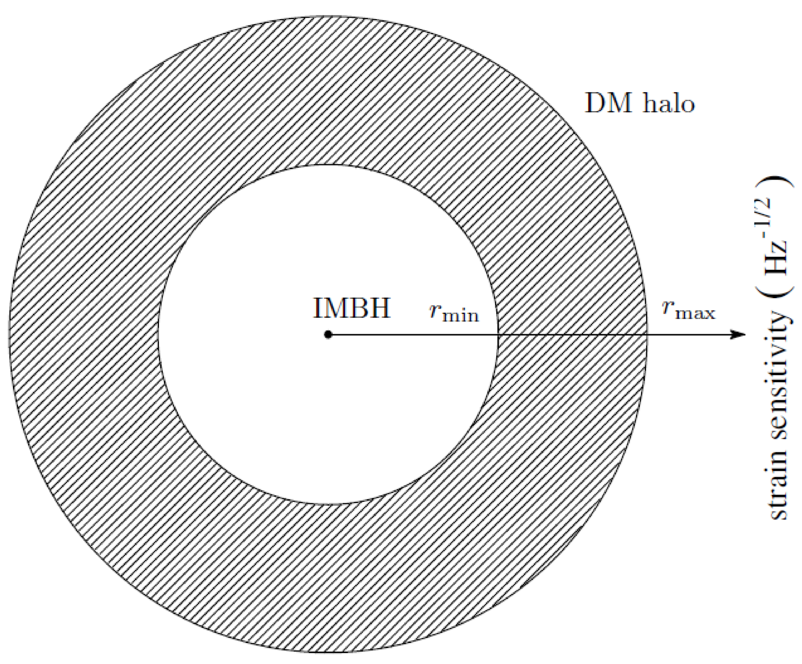

Fig.1: IMBH surrounded by DM mini-halo

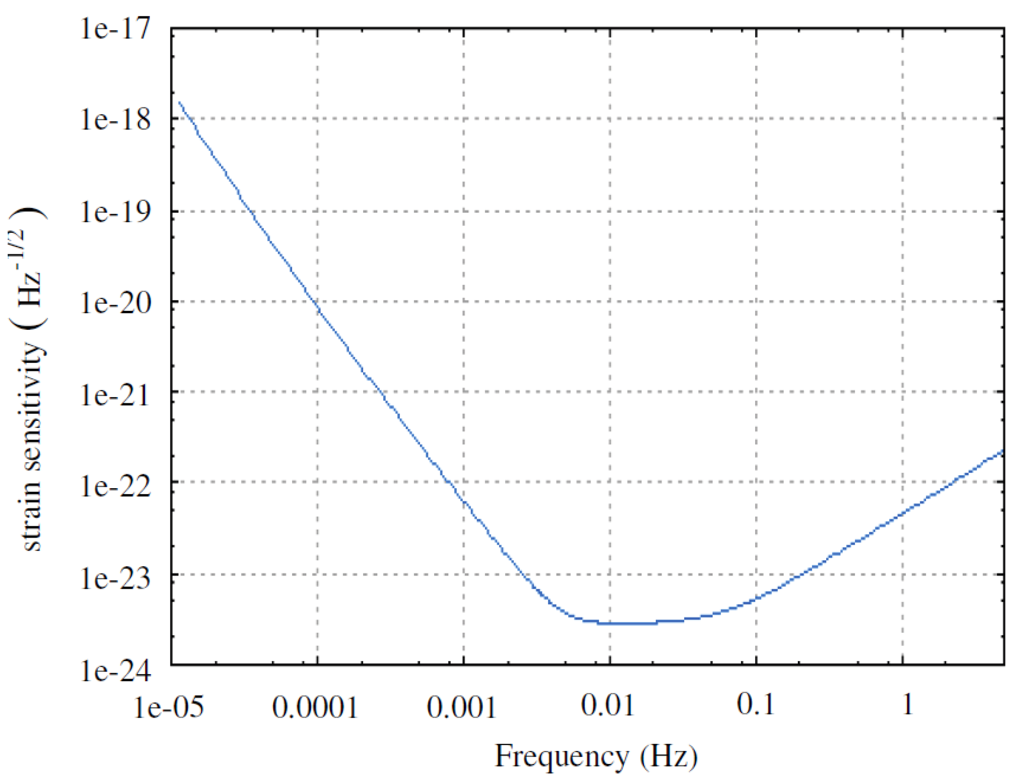

Fig.2: Sensitivity of eLISA

Conclusions

We assume that eLISA observes the GW for 5.0 years until the coalescence, then the interval of the frequency integration is from 22.7 mHz to infinity. P is defined as S/N in which a template is calculated including both the IMBH and the DM mini-halo gravitational potential and Q is defined as S/N in which a template is calculated including only IMBH. According to Ref [3], the power law index is α=7/3. So,

P/Q = 1.6×10^-3

This result means that in order to detect the GW from a binary where a star orbits around an intermediate mass black hole surrounded by a mini-halo in practice we must calculate the template including the effect of the DM halo. P/Q strongly depends on the power law index α. So if the GW is detected, the power law index would be determined very accurately.

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