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Cosmic wiggly string in black hole spacetime

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Abstract

We obtain two kinds of special solutions of test strings in the black hole spacetime because it is too difficult to obtain the general solutions of test string. One is the cosmic wiggly string of arc configuration which exhibits open string lying along the circular orbit in the equatorial plane outside the horizon. The other is the cosmic wiggly string of radial configuration which exhibits open string pointing toward the origin in the equatorial plane outside the horizon.

1 Introduction

A phase transition in the early Universe produced topological defects such as domain walls, strings and monopoles when the $U(1)$ symmetry breaks down in the unified theories. Topological defects are necessarily created due to the spontaneous symmetry breaking of vacuum state[see 1-4]. If the existence of the cosmic strings which are one of topological defects is confirmed, it is a strong evidence of vacuum phase transition in the universe. Besides the interests from the unified theories, it was proposed that cosmic strings can generate cosmological density fluctuations as seeds of the structure formation which can subsequently evolve into galaxies and large-scale structure[5,6]. However, this scenario was rejected due to the confliction with the precise observational data of cosmic microwave backgrounds[7,8].

Recently cosmic strings have been studied in the context of the superstring theories because fundamental strings and other string-like solitons such as D-strings could exist in the Universe as cosmic strings[9]. If such cosmic strings existin the Universe, it is an interesting subject how cosmic strings behave in the curved spacetime in general relativety. They have been investigatedby H. J. de Vega, N. G. Sánchez, V. P. Frolov and B. Carter[10-20]. The configuration of cosmic string in black hole spacetime is an interesting problem. It is well known how a test particle behaves in black hole spacetime. However, the motion of a test string in black hole spacetime is not known because the test string problem is significantly non-trivial due to the constraints. So we study the test string in detail. For the simplicity, we neglect the gravitational effects of string and assume that its thickness is zero.

We obtain two kinds of special solutions of test strings in the black hole spacetime because it is too difficult to obtain the general solutions of test string. One is the cosmic wiggly string of arc configuration which exhibits open string lying along the circular orbit in the equatorial plane outside the horizon. The other is the cosmic wiggly string of radial configuration which exhibits open string pointing toward the origin in the equatorial plane outside the horizon. We obtain interesting solutions though they are not the general solutions of test string.

Traditionally cosmic strings have been treated as fundamental strings and described by Nambu-Goto action$S_{N-G} = \int d\tau d\sigma \frac{1}{2\pi\alpha'}\sqrt{g^{ab}G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu}$. However, some literatures claim that the so-called cosmic wiggly string is more favorable than the standard Nambu-Goto string to explain the possible role of extended cosmic string in cosmology or astronomy[21-27]. For the Nambu-Goto string the energy density and the tension are equal, while their relation in the wiggly string model is more relaxed to the extent that the energy density and the tension are not equal. In particular, it is known that the wiggly string of which the product of the energy density and the tension is constant behaves well. If the energy density is much larger than the string tension, its shape tends to wiggle. We mainly treat cosmic wiggly strings as cosmic strings.

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2 Basic Equation of Cosmic Wiggly String

The induced metric on cosmic string is

$$h_{ab}(\tau,\sigma) = \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}, \qquad (a,b=\tau,\sigma), \tag{1}$$

where τ and σ are the string coordinates, X^ν is the background coordinate and $G_{\mu\nu}$ is the background metric.

The two-dimensional energy-momentum tensor is given by

$$t^{ab} = (\mu - \tilde{T})u^a u^b + \tilde{T} h^{ab}, \tag{2}$$

where $\mu(\tau,\sigma)$ is the energy per unit length of string, $\tilde{T}(\tau,\sigma)$ is its tension, and $u^a(\tau,\sigma)$ is the fluid velocity parallel to the string with the normalization:$u^a u_a = -1$. The string has the energy density and the tension. This can be covariantly described by the energy-momentum tensor localized at the string as Eq. (2) introducing the velocity field. The physical reason for the energy-momentum tensor t^{ab} is not the issue of the present paper. It may come from the configurations of Higgs and gauge fields or just particles depending on the model.

The four-dimensional energy-momentum tensor by a cosmic string configuration is given by

$$T^{\mu\nu}(x) = \int d\tau d\sigma \sqrt{-h}\, t^{ab}(\tau,\sigma)\partial_a X^\mu(\tau,\sigma)\partial_b X^\nu(\tau,\sigma)\frac{\delta^4(x - X(\tau,\sigma))}{\sqrt{-G(X)}}, \tag{3}$$

where $h = det(h_{ab})$ and $G = det(G_{\mu\nu})$.

We derive the basic equation of motion of the cosmic wiggly string. The motion of the string is dictated by $D_\mu T^{\mu\nu}(x) = 0$ i.e., the conservation of energy and momentum.

We differentiate Eq. (3) by x^μ and sum over μ :

$$\begin{aligned}
\frac{\partial T^{\mu\nu}}{\partial x^\mu} &= \int d\tau d\sigma \sqrt{-h}\, t^{ab}(\tau,\sigma)\partial_a X^\mu(\tau,\sigma)\partial_b X^\nu(\tau,\sigma)\frac{\partial \delta^4(x - X(\tau,\sigma))}{\partial x^\mu}\frac{1}{\sqrt{-G(X)}}, \\
&= \int d\tau d\sigma \frac{\partial}{\partial \xi^a}\left(\sqrt{-h}\, t^{ab}(\tau,\sigma)\partial_b X^\nu(\tau,\sigma)\frac{1}{\sqrt{-G(X)}}\right)\delta^4(x - X(\tau,\sigma)), \\
&= \int d\tau d\sigma \left(\frac{\partial(\sqrt{-h}\, t^{ab}\partial_b X^\nu)}{\partial \xi^a}\frac{1}{\sqrt{-G(X)}} + \sqrt{-h}\, t^{ab}\partial_b X^\nu \frac{\partial}{\partial \xi^a}\left(\frac{1}{\sqrt{-G(x)}}\right)\right)\delta^4(x - X), \\
&= \int d\tau d\sigma \left(\frac{\partial(\sqrt{-h}\, t^{ab}\partial_b X^\nu)}{\partial \xi^a} + \sqrt{-h}\, t^{ab}\partial_b X^\nu \sqrt{-G(x)}\frac{\partial}{\partial \xi^a}\left(\frac{1}{\sqrt{-G(x)}}\right)\right)\frac{\delta^4(x - X)}{\sqrt{-G(x)}}, \\
&= \int d\tau d\sigma \left(\partial_a(\sqrt{-h}\, t^{ab}\partial_b X^\nu) - \Gamma^\rho_{\mu\rho}\frac{\partial X^\mu}{\partial \xi^a}\sqrt{-h}\, t^{ab}\partial_b X^\nu\right)\frac{\delta^4(x - X)}{\sqrt{-G(x)}}, \\
&= \int d\tau d\sigma \partial_a(\sqrt{-h}\, t^{ab}\partial_b X^\nu)\frac{\delta^4(x - X)}{\sqrt{-G(x)}} - \Gamma^\rho_{\mu\rho}\int d\tau d\sigma \sqrt{-h}\, t^{ab}\partial_a X^\mu \partial_b X^\nu \frac{\delta^4(x - X)}{\sqrt{-G(x)}}, \\
&= \int d\tau d\sigma \partial_a(\sqrt{-h}\, t^{ab}\partial_b X^\nu)\frac{\delta^4(x - X)}{\sqrt{-G(x)}} - \Gamma^\rho_{\mu\rho}T^{\mu\nu}.
\end{aligned} \tag{4}$$

The covariant derivative of tensor $T^{\mu\nu}$ is defined by

$$D_\mu T^{\mu\nu} = \frac{\partial T^{\mu\nu}}{\partial x^\mu} + \Gamma^\mu_{m\mu}T^{m\nu} + \Gamma^\nu_{m\mu}T^{m\mu}. \tag{5}$$

From Eqs.(3),(4), Eq.(5) becomes

$$\begin{aligned}
D_\mu T^{\mu\nu} &= \int d\tau d\sigma \partial_a(\sqrt{-h}\, t^{ab}\partial_b X^\nu)\frac{\delta^4(x - X)}{\sqrt{-G(x)}} + \Gamma^\mu_{\nu\lambda}T^{\nu\lambda} \\
&= \int d\tau d\sigma \left(\partial_a(\sqrt{-h}\, t^{ab}\partial_b X^\nu) + \sqrt{-h}\,\Gamma^\mu_{\nu\lambda}t^{ab}\partial_a X^\nu \partial_b X^\lambda\right)\frac{\delta^4(x - X)}{\sqrt{-G(x)}}.
\end{aligned} \tag{6}$$

The conservation of energy and momentum is equivalent to

$$\frac{1}{\sqrt{-h}}\partial_a\left(\sqrt{-h}t^{ab}(\tau,\sigma)\partial_b X^\mu\right)+\Gamma^\mu_{\nu\lambda}\partial_a X^\nu\partial_b X^\lambda t^{ab}=0,\qquad (\mu=0,1,2,3). \tag{7}$$

Here $\Gamma^\mu_{\nu\lambda}$ is the affine connection computed from the background metric. This has to be solved for X^μ and u^a.

The covariant derivative of Einstein Eq. is given by

$$D^\mu(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R)=8\pi G D^\mu T_{\mu\nu}, \tag{8}$$

where the conservation of energy and momentum $D^\mu T_{\mu\nu}=0$ is derived because the left hand of Eq. (8) is zero by the Bianchi identity. The conservation law gives the equation of motion for topological defects which are either a particle or a string. If we consider a particle and a string as a small defect of spacetime, its motion is given by the motion of background spacetime. This concept is the characteristic of general relativity and was discussed in detail by Einstein, Infeld and Hoffmann[28].

3 Cosmic wiggly strings of arc configuration in black hole spacetime

It is well known that the minimal radius of stable circular orbit for a test particle around the Schwarzschild black hole is three times the Schwarzschild radius. We concerns with the minimal radius of stable circular orbit for a test string around the black holes. We study the classical motion of cosmic wiggly strings of arc configuration in the various black holes and obtain the new special solutions of them. The cosmic wiggly strings exhibit open strings lying along the circular orbit in the equatorial plane outside horizon in the various black holes. The solutions in the various black holes are compared with them in the Schwarzschild black hole.

At first, because of the simplicity of the equations, we study the cosmic wiggly string in the Schwarzschild black hole. In the presence of small scale wiggles the macroscopically average value of the energy density μ would exceed its Nambu-Goto value, while the tension $\tilde{T}$ decreases. The particular case of equation of state for a wiggly cosmic string

$$\mu\tilde{T}={\mu_0}^2,\qquad (\mu_0 : const.) \tag{9}$$

is simple in the analysis.

We make the ansatz

$$\begin{aligned}\theta &= \frac{\pi}{2}, &(10)\\ r &= const., &(11)\\ ct &= d\tau+e\sigma, &(12)\\ \varphi &= m\tau+n\sigma, &(13)\end{aligned}$$

where d,e,m and n are constants to be determined in what follows.

The conservation of energy and momentum (7) becomes

$$\frac{1}{\sqrt{-h}}\partial_\tau\left(\sqrt{\frac{-\left(1-\frac{2MG}{r}\right)e^2+n^2r^2}{-\left(1-\frac{2MG}{r}\right)d^2-m^2r^2}}\left(-\mu(\tau)+2\frac{{\mu_0}^2}{\mu(\tau)}\right)\dot{X}^\mu\right)$$
$$+\Gamma^r_{tt}\partial_a t\partial_b t t^{ab}+\Gamma^r_{\varphi\varphi}\partial_a\varphi\partial_b\varphi t^{ab}=0. \tag{14}$$

As we can choose the off-diagonal element of the induced metric $h_{\tau,\sigma}$ in Eq. (1) to be zero, the solution of Eq. (14) for an open string must satisfy the following condition:

$$-\left(1-\frac{2MG}{r}\right)de+mnr^2=0. \tag{15}$$

The condition for the solution of Eq. (15) to be outside the horizon $r > 2MG$ reads

$$\frac{de}{mn} \geq 27(MG)^2. \tag{16}$$

From Eqs.(12)-(15), we obtain a particular solution[29]

$$r = \frac{\mu(dn + 2em) - 3\frac{\mu_0{}^2}{\mu}em}{\mu em + \frac{\mu_0{}^2}{\mu}(dn - 2em)}MG, \tag{17}$$

where the solution is outside the horizon if the tension $\frac{\mu_0{}^2}{\mu}$ of string is zero or the average value of energy density μ of string is zero and $2 > \frac{em}{dn} > 1/2$ (Fig.1). The angular velocity ω, the angular momentum M^{xy} and the energy P^0 are generally defined by

$$\omega := \frac{d\varphi}{dt} = \frac{m}{d}, \tag{18}$$

$$M^{xy} := \int (xT^{y0} - yT^{x0})dxdydz = \pi\sqrt{\omega en}(\mu - 3\frac{\mu_0{}^2}{\mu})r^2, \tag{19}$$

$$P^0 := \int T^{00}dxdydz = \pi(\sqrt{\frac{en}{\omega}}(\mu - 2\frac{\mu_0{}^2}{\mu}) - e\sqrt{\frac{e\omega}{n}}\frac{\mu_0{}^2}{\mu}). \tag{20}$$

In the strongly wiggly case($\mu \gg \mu_0$), we have a simple expression for the radius r in terms of the angular velocity ω from Eqs. (15), (17) and (18),

$$r = \frac{1}{\omega^{2/3}}(MG)^{1/3}. \tag{21}$$

We can determine the parameters d, e, m, n if the angular velocity ω, the energy P^0 and the angular momentum M^{xy} are given, except for the overall magnitude of d and m, which comes from the arbitrariness of the scale of τ.

We have obtained the solutions for open cosmic wiggly strings of circular configuration in the Schwarzschild black hole. For a closed cosmic string we can obtain a circular solution by restricting the parameter $n = 2$. The extended solutions for open cosmic wiggly strings of circular configuration outside the horizon have been found. The Nambu-Goto limit of the cosmic string has only a point-like solution, while the fundamental generic string has only a null particle solution. This is because the freedom of Nambu-Goto cosmic string is more than the one of the fundamental string. In contrast to our present work, Frolov and the co-authors got open string solutions lying in the radial direction and Ishihara and co-authors classified closed string solutions in Minkowski spacetime. We have obtained the solutions around the Schwarzschild black holes under the gauge condition $ct \propto (\tau \pm \alpha\sigma)$ while usually people choose the gauge condition $ct = \tau$. It remains to be clarified why only this particular class of gauge fixing condition works.

In the case of the Reisner-Nordström black hole, the open cosmic wiggly string lies in the equatorial plane and rotates along a circular orbit of a shorter radius outside the horizon comparing with the circular orbit in the Schwarzschild spacetime because of the existence of charge[30]. One might be interested in the string configuration in the extreme case (Q=M). However, no qualitatively distinctive feature arises in the extreme limit. On the other hand, there are only point-like solutions for the Nambu-Goto cosmic string because of its high symmetry, i.e. more constraints than the cosmic wiggly string.

In the case of the Kerr black hole, the open cosmic wiggly string lies in the equatorial plane and rotates in the same direction as the Kerr spacetime along a circular orbit of a smaller radius outside the horizon comparing with the circular orbit in the Schwarzschild spacetime because of the existence of the angular momentum[31]. We have compared the strongly wiggly string solution in the case of small Kerr parameter with the one in the Schwarzschild spacetime. In the case of positive Kerr parameter, the obtained radius is smaller than that of the solution for the Schwarzschild spacetime, while it is larger in the case of the negative Kerr parameter. This feature of stable circular orbit of a test string conforms with that of a test particle in the Kerr spacetime. The parameter regions for the wiggly string solution to be outside of the horizon have been indicated(Fig.2).

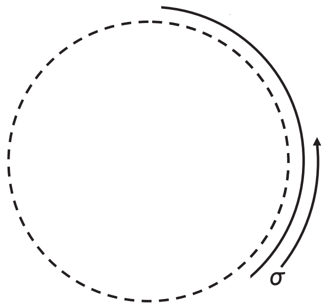

Figure 1: The string solution of arc configuration is shown. The dotted circle shows the horizon of spherically symmetric black hole, while the solid curve shows the string of arc configuration parametrized by the world sheet coordinate σ.

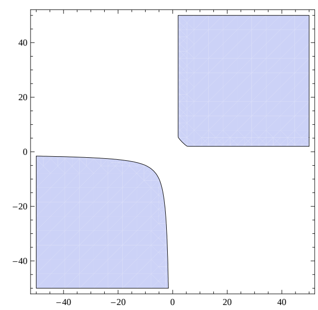

Figure 2: The shaded are parameter regions allowed in the case of $a = 1$, where the horizontal and vertical coordinate axes indicate d/m and e/n, respectively.

We have studied the relation of the string radius and the angular momentum in the case of a constant energy. The radius is larger with the decrease of the angular momentum for the positive Kerr parameter, while for the negative Kerr parameter the radius is smaller with the decrease of the angular momentum. The mechanism behind this phenomenon seems complicated and has not been fully understood. One might be interested in the string configuration in the expreme case($a = M$). However, no qualitatively distinctive feature arises in the extreme limit. On the other hand there are only point-like solutions for the Nambu-Goto cosmic string because of its high symmetry, i.e., more constraints. But we do not have any intuitive explanation for the existence of the extended solution in the case of the wiggly string. For example, we have found a curious but not fully understood phenomenon that the radius of the strongly wiggly string approaches the horizon as the angular velocity becomes larger.

We have obtained the solutions for open cosmic wiggly string in the Kerr-Newman black hole[32]. It is theoretically important to study the wiggly string solutions in the Kerr-Newman spacetime because the Kerr-Newman black hole is the most general stationary black hole. However, most of the black holes in the universe are generically rotating but there are no observational evidence that a black hole has a charge. We have showed that the solutions for the cosmic wiggly string reduced to a string-like extended

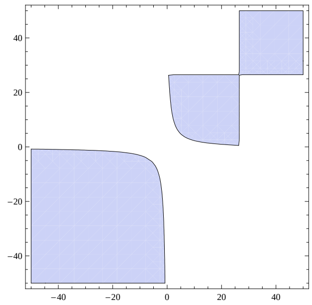

Figure 3: In the case of $a = 0.15$.

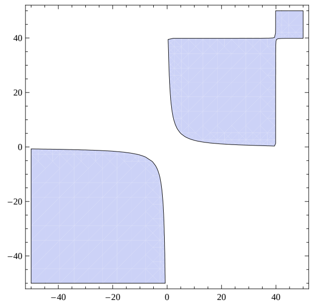

Figure 4: In the case of $a = 0.1$.

configuration in the Kerr-Newman spacetime and compared the solution with the cases of the Reisner-Nordström spacetime and the Kerr spacetime. We have found a new special solution of open wiggly strings in the (3+1)-dimensional stationally axially symmetric charged rotating black hole, though we cannot find any extended solution for the Nambu-Goto open string.

4 Cosmic wiggly strings of radial configuration in black hole spacetime

We have obtained the classical new special solutions of cosmic wiggly string of arc configuration in Sec. 3. Here we study classical motion of cosmic wiggly strings of radial configuration in the black holes as the extension of cosmic wiggly strings of arc configuration. The cosmic wiggly strings exhibit open strings pointing toward the origin in the equatorial plane outside the horizon in the various black holes. We show that there are two kinds of solutions : static and stationary solutions.

At first, because of the simplicity of the equations, we study classical motion of a string in (3+1)-dimensional spherically symmetric neutral black holes. We make the ansatz

$$\theta = \frac{\pi}{2}, \tag{22}$$

$$\phi = const., \tag{23}$$

$$ct = \tau, \tag{24}$$

where we first study the static cosmic wiggly string by Eq. (23) for simplicity. Our string can be considered as an open string because our string solutions satisfy the free boundary condition at the end points $\sigma = 0$ and π,

$$\frac{\partial X_{\perp}{}^{\mu}}{\partial \sigma}|_{0,\pi} = 0, \tag{25}$$

where $X_{\perp}{}^{\mu}$ is the radial coordinate of the string, because $\phi = const.$ in our case.

As we can choose the off-diagonal element of the induced metric $h_{\tau\sigma}$ in Eq. (2) to be zero while the two-dimensional energy-momentum tensor $t^{\sigma\sigma}$ has a finite value, the solution for a static string is given

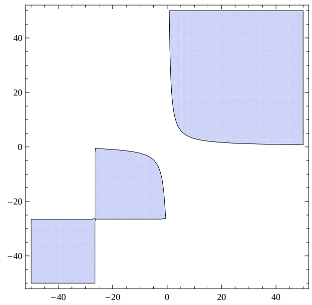

Figure 5: In the case of $a = -0.15$.

by the following equation:

$$\dot{r} = 0. \tag{26}$$

The essence of the dynamics of cosmic string can be described as the conservation of energy and stress. The conservation of energy and momentum (7) becomes

$$\mu M r'^2 + \frac{{\mu_0}^2}{\mu}\left(-5Mr'^2 + r(r-2M)r''\right) = 0, \tag{27}$$

where $'$ is the derivative with respect to σ.

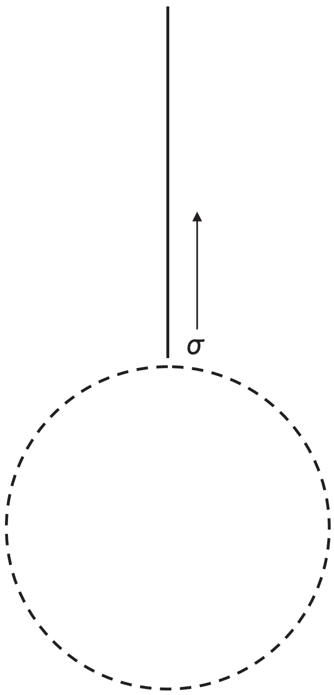

Figure 6: The string solution of radial configuration is shown. As the same as Fig. 1, the dotted circle shows the horizon of spherically symmetric black hole, while the solid line shows the string of radial configuration parametrized by the world sheet coordinate σ.

From Eq. (27), we obtain the solution of equation[33] (Fig. 6) as

$$\sigma = \frac{1}{b}\int_{r_0}^{r} dr \left(\frac{r}{r-2M}\right)^{\frac{1}{2}(5-\frac{\mu^2}{{\mu_0}^2})}, (0 < \sigma < \pi) \tag{28}$$

where b is a constant that is defined by the following equation

$$b = r'\Big(\frac{r}{r-2M}\Big)^{\frac{1}{2}(5-\frac{{\mu_0}^2}{\mu^2})}. \tag{29}$$

In the special case of the ratio $\frac{\mu^2}{{\mu_0}^2} = 4$ of the energy density to the tension we obtain the solution of equation as

$$\sigma(r=2) = 0, \tag{30}$$

$$\sigma = 2\Big(\sqrt{\Big(\frac{r}{2}-1\Big)\frac{r}{2}} + \log\Big(\sqrt{\frac{r}{2}} + \sqrt{\frac{r}{2}-1}\Big)\Big). \tag{31}$$

The obtained cosmic wiggly string lies in the radial direction in the equatorial plane outside horizon and the string gets longer as the ratio $\frac{\mu^2}{{\mu_0}^2}$ of the energy density to the tension increases.

The energy P^0 is explicitly evaluated as

$$\begin{aligned} P^0 : &= \int T^{00} dxdydz \\ &= \frac{1}{c}\Big(\mu - 2\frac{{\mu_0}^2}{\mu}\Big)\Big(r + 2M\log(r-2M) + A\Big), \end{aligned} \tag{32}$$

where $T^{\mu\nu}$ is the four-dimensional energy-momentum tensor given by Eq. (3) and A is an integral constant.

we study a slowly stationary rotating cosmic wiggly string solution in the radial direction in the equatorial plane outside horizon in the Schwarzschild black hole.

We make the ansatz

$$\theta = \frac{\pi}{2}, \tag{33}$$

$$\phi = \beta\tau, \tag{34}$$

$$ct = \tau, \tag{35}$$

where β is a constant. Note that Eq. (34) implies a stationary rotation in contrast to Eq. (23).

The conservation of energy and momentum (7) becomes

$$\begin{aligned} &\mu(-\beta^2 r^4 + 2M\beta^2 r^3 + Mr - 2M^2){r'}^2 \\ &+\frac{{\mu_0}^2}{\mu}\Big((3\beta^2 r^4 - 4M\beta^2 r^3 - 5Mr + 10M^2){r'}^2 + r(-\beta^2 r^3 + r - 2M)(r-2M)r''\Big) \\ &= 0, \end{aligned} \tag{36}$$

where $'$ is the derivative with respect to σ and which corresponds to Eq. (27) in the case of $\beta = 0$.

The obtained cosmic wiggly string is rotating very slowly outside horizon[33]. The maximum angular velocity of the cosmic wiggly string which starts from $r(\sigma = 0) = 2.01$ is about $\beta = 0.03509$ where the cosmic wiggly string shrinks to a point. In the case of the ratio $\frac{\mu^2}{{\mu_0}^2} \leq 11$ of the energy density to the tension the maximum angular velocity of the cosmic wiggly string which starts from $r(\sigma = 0) = 2.1$ is about $\beta = 0.10391$ where the cosmic wiggly string shrinks to a point. In the case of the ratio $\frac{\mu^2}{{\mu_0}^2} \geq 11.9$ of the energy density to the tension the cosmic wiggly string which starts from $r(\sigma = 0) = 2.1$ gets longer as the angular velocity β increases and the wiggly string solution suddenly disappears. In the case of the intermediate ratio $\frac{\mu^2}{{\mu_0}^2} = 11.5$ of the energy density to the tension for example the cosmic wiggly string which starts from $r(\sigma = 0) = 2.1$ gets longer until the angular velocity β reaches 0.045. and gets shorter and shrinks to a point at about $\beta = 0.10391$.

The energy P^0 is generally defined by Eq. (32). The angular momentum M^{xy} is given by

$$\begin{aligned} M^{xy} &:= \int dxdydz(xT^{y0} - yT^{x0}) \\ &= (\mu - 2\frac{{\mu_0}^2}{\mu})\beta\int dr\frac{r^3}{\sqrt{(r-2M)(-\beta^2 r^3 + r - 2M)}}. \end{aligned} \tag{37}$$

The solutions for open cosmic wiggly strings of radial configuration in the Reisner-Nordström black hole have been obtained in [33]. We have obtained the special solution of open cosmic wiggly string in the radial direction in a black hole spacetime which gets shorter with the increase of the charge. With the increase of the angular velocity the cosmic string gets longer or shorter depending on the strong or weak wiggliness. We can observe a curious behavior for the marginally wiggly case that is a little smaller than a certain value. The cosmic wiggly string gets shorter with the increase of the angular velocity and becomes a point when the ratio of the energy density to the tension is very smaller than the certain value (Fig. 7). In Figure 7, the flat regions indicate the regions where the cosmic wiggly string becomes a point. With the increase of charge the flat region shrinks and the maximum value of angular velocity β_{max} becomes larger.

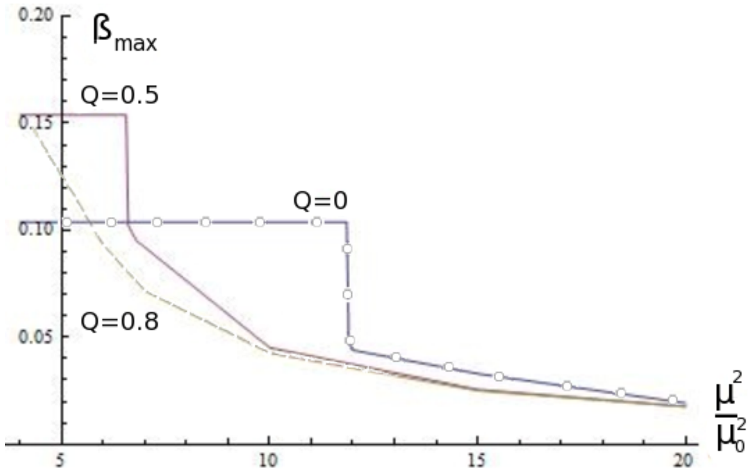

Figure 7: The relations between the ratio of the enaergy density to the tension $\frac{\mu^2}{{\mu_0}^2}$ and the maximum value of angular velocity β_{max} are shown in the cases of the charge $Q = 0, 0.5, 0.8$, where the horizontal and vertical coordinate axes indicate $\frac{\mu^2}{{\mu_0}^2}$, β_{max}, respectively.

The solutions for open cosmic wiggly strings of radial configuration in the Kerr spacetime have been obtained[34]. We have obtained the special solution of open cosmic wiggly string in the radial direction in the Kerr spacetime which gets longer with the increase of the ratio of the energy density to the tension and shorter with the increase of the absolute of the Kerr parameter. With the increase of the angular velocity the cosmic string gets longer or shorter depending on the strong or weak wiggliness. We can observe a curious behavior for the marginally wiggly case that is slightly smaller than a certain value. The cosmic wiggly string gets shorter with the increase of the angular velocity and becomes a point when the ratio of the energy density to the tension is very smaller than the certain value (Fig. 8). Especially in the case of $\beta a < 0$ the behavior is more complicated comparing the case of the Schwarzschild spacetime. In Figure 8, the flat regions indicate the regions where the cosmic wiggly string becomes a point. With the increase of Kerr parameter a the flat region is enlarged and the maximum value of angular velocity β_{max} becomes smaller. In the case of negative Kerr parameter, the flat region is larger than that of the case of positive Kerr parameter and the maximum value of angular velocity β_{max} takes the same value. One might be interested in the wiggly string configuration in the extreme case($a = 1$). However, nothing qualitatively distinctive feature arises in the extreme limit.

The solutions for open cosmic wiggly strings of radial configuration in the Kerr-Newman spacetime have been obtained[35]. The difference is the critical values of $\frac{\mu^2}{{\mu_0}^2}$ where the behavior of the string configuration drastically changes;shrinking to a point or the total disappearance after reaching the maximum radius. We have obtained the special solution of open cosmic wiggly string in the radial direction in the Kerr-Newman spacetime which gets longer with the increase of the ratio of the energy density to the tension, slightly shorter with the increase of the charge and shorter with the increase of the absolute of the Kerr parameter. This conclusion corresponds to the one in the Reisner-Nordström black hole and the Kerr black hole. With the increase of the angular velocity the cosmic wiggly string gets longer or shorter

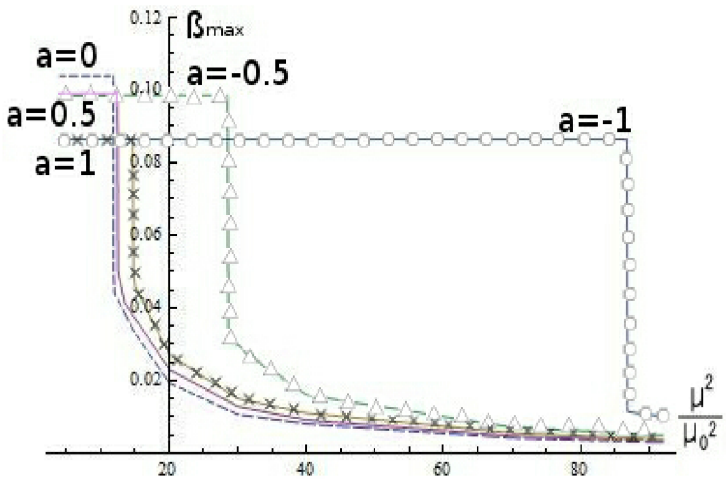

Figure 8: The relations between the ratio of the energy density to the tension $\frac{\mu^2}{\mu_0{}^2}$ and the maximum value of angular velocity β_{max} in the cases of the Kerr parameter $a = 0, 0.5, 1, -0.5, -1$

depending on the strong or weak wiggliness. We can observe a curious behavior for the marginally wiggly case that is a little smaller than a certain value. The cosmic wiggly string gets shorter with the increase of the angular velocity and becomes a point when the ratio of the energy density to the tension is very smaller than the certain value. Especially in the case of $\beta a < 0$ the behavior is more complicated comparing the case of the Schwarzschild spacetime. One might be interested in the wiggly string configuration in the extreme case($a = 1$). However, nothing qualitatively distinctive feature arises in the extreme limit.

In contrast to our present work, V. Frolov *et al.* got rigidly rotating open string solutions lying in the radial direction which are the curved strings and not straight[19]. H. J. de Vega *et al.* got a straight nonoscillating string, radially disposed which rotates uniformly around the symmetry axis of the spacetime and which becomes an infinite static fundamental string from the horizon to infinity as the angular velocity approaches to zero[15]. H. Ishihara and H. Kozaki classified string solutions with geometrical symmetries in Minkowski spacetime[21].

In view of other solutions found by Frolov, de Vega and Ishihara et al. we suspect there will be rich variety of wiggly string solutions in a black hole background. The evolution of wiggly cosmic strings will be more interesting if they are trapped by black holes. However, its cosmological significance remains to be studied.

5 Conclusion

The cosmic wiggly string solutions have been obtained outside the horizon in all stationary black hole spacetimes. One is the cosmic wiggly string solutions of arc configuration. The other is the cosmic wiggly string solution of radial configuration in black hole spacetime, which exhibits finite open strings pointing toward the origin in the equatorial plane outside the horizon. The test string can exist near the horizon of black hole though the test particle cannot exist near the horizon of black hole. We hope that this investigation of test string as a probe further enchances the understanding of black holes.

References

[1] T. W. B. Kibble, *J. Phys.* **A9**, 1387 (1976).

[2] M. B. Hindmarsh and T. W. B. Kibble, *Rept. Prog. Phys.* **58**, 477 (1995). [arXiv:hep-ph/9411342].

[3] A. Vilenkin and E. P. S. Shellard,*Cosmic Strings and Other Topological Defects*(Cambridge University Press, 1994).

[4] M. R. Anderson, *The Mathematical Theory of Cosmic Strings* (Institute of Physics, Bristol 2003).

[5] Y. B. Zeldovich, *Mon. Not. Roy. Astron. Soc.* **192**, 663 (1980).

[6] A. Vilenkin, *Phys. Rev. Lett.* **46**, 1169 (1981).

[7] A. Albrecht, R. A. Battye and J. Robinson, *Phys. Rev. Lett.* **79**, 4736 (1997).

[8] A. Albrecht, R. A. Battye and J. Robinson, *Phys. Rev.* **D59**, 023508 (1998).

[9] E. J. Copeland, R. C. Myers and J. Polchinski, *High Energy Phys.* JHEP06, 013 (2004).

[10] H. J. de Vega and N. G. Sánchez, *Phys. Lett.* **B197**, 320 (1987).

[11] H. J. de Vega and N. G. Sánchez, *Phys. Rev.* **D47**, 3394 (1993) .

[12] H. J. de Vega, A. L. Larsen and N. G. Sánchez, *Nucl. Phys.* **B427**, 643 (1994).

[13] A. L. Larsen and N. G. Sánchez, *Phys. Rev.* **D50**, 7493 (1994).

[14] A. L. Larsen and N. G. Sánchez, *Phys. Rev.* **D51**, 6929 (1995).

[15] H. J. de Vega and I. L. Egusquiza, *Phys. Rev.* **D54**, 7513 (1996).

[16] *Phys. Rev.* **D60**, 125014 (1999).

[17] V. P. Frolov, V. D. Skarzhinsky, A. I. Zelnikov and O. Heinrich, *Phys. Lett.* **B224**, 255 (1989).

[18] B. Carter and V.P. Frolov, *Class. Quantum Grav.* **6**, 569 (1989).

[19] V. P. Frolov, S. Hendy and J. P. de Villiers, *Class. Quantum Grav.* **18**, 1535(2001)

[20] V. P. Frolov and D. Fursaev, *Class. Quantum Grav.* **14**, 1099(1997).

[21] H. Ishihara and H. Kozaki, *Phys. Rev.* **D72**, 061701 (2005).

[22] J. Hong, J. Kim and P. Sikivie, *Phys. Rev. Lett.* **69**, 2611 (1992).

[23] J. Kim and P. Sikivie, *Phys. Rev.* **D50**, 7410 (1994).

[24] B. Carter, *Phys. Rev.* **D41**, 3869 (1990).

[25] B. Carter, *Phys. Rev. Lett.* **74**, 3098 (1995).

[26] B. Carter, *Phys. Rev.* **D60**, 083502(1999).

[27] A. Arazi and C. Simeone, *Mod. Phys. Lett.* **A15**, 1369 (2000).

[28] A. Einstein, L. Infeld, and B. Hoffmann, *Ann. Math.* **39**, 65(1938).

[29] H. Suzuki, *Prog. Theor. Phys.* **120**, 865(2008).

[30] H. Suzuki, *Mod. Phys. Lett.* **A25**, 1233(2010).

[31] H. Suzuki, *Prog. Theor. Phys.* **124**, 181(2010).

[32] H. Suzuki, *Mod. Phys. Lett.* **A26**, 1221(2010).

[33] H. Suzuki, *Int. J. Mod. Phys.* **A26**, 3275(2011).

[34] H. Suzuki, *Mod. Phys. Lett.* **A26**, 1933(2011).

[35] H. Suzuki, *Gen. Relativ. Gravit.* **44**, 1937(2012).