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"Gravitational collapse of the Einstein cluster in the Lovelock gravity"

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Gravitational Collapse of the Einstein Cluster in Lovelock gravity

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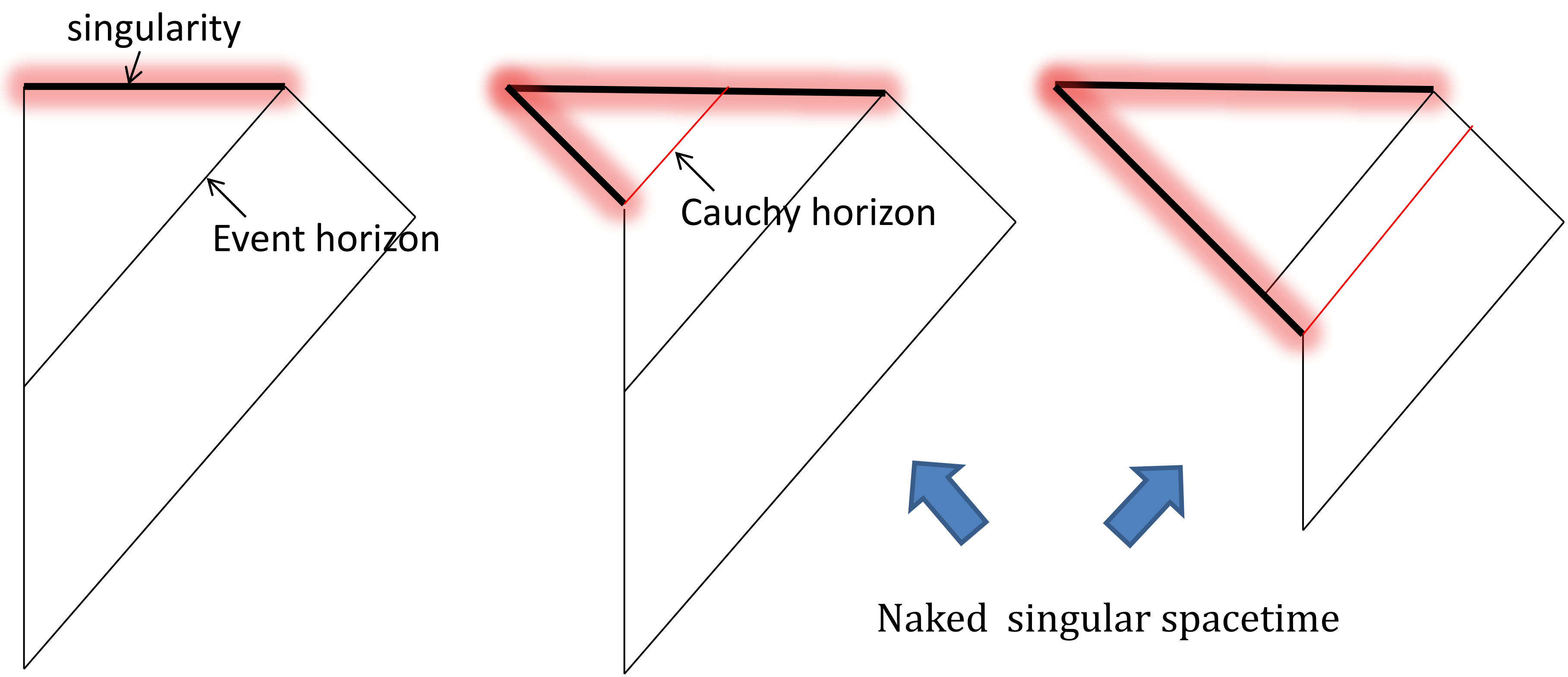
1. 1 Introduction

Goal: Verify the Cosmic Censorship Conjecture

Cosmic Censorship Conjecture

No naked singularities form under physically reasonable condition.

- Which is the final state of gravitational collapse ?



Naked singular spacetime

1. 2 Introduction

- Gravitational collapse and singularity formation are high energetic phenomena

- String theory is formulated in higher dimensional spacetime and predicts higher curvature correction to Einstein theory.

- It is important to study the gravitational collapse in higher dimensional spacetime in generalized Einstein gravity and verify whether the Cosmic Censorship Conjecture holds or not.

- Here we consider the Lovelock gravity as the generalized Einstein gravity theory

2.1 Lovelock gravity

Lovelock gravity is the natural extension of Einstein gravity to higher dimension and the most general theory which satisfies 3 conditions

- Field equation is second rank tensor

$$\mathcal{G}_{\mu\nu} = \mathcal{G}_{\nu\mu}$$

- Field equation is divergence free

$$\nabla_\mu \mathcal{G}^{\mu\nu} = 0$$

- Field equation contains the derivatives of metric tensor up to second order

$$\mathcal{G}(g, \partial g, \partial^2 g)$$

Einstein gravity is uniquely characterized by above conditions

2.2 Action for Lovelock gravity

- The most general action which satisfies abone 3 condition is

$$\mathcal{L} = \sum_{m=1}^{k} \frac{a_m}{m} \mathcal{L}_m$$

where

$$\mathcal{L}_m = \frac{1}{2^m} \delta^{\nu_1 \nu_2 \ldots \nu_{2m-1} \nu_{2m}}_{\mu_1 \mu_2 \ldots \mu_{2m-1} \mu_{2m}} R_{\nu_1 \nu_2}{}^{\mu_1 \mu_2} \ldots R_{\nu_{2m-1} \nu_{2m}}{}^{\mu_{2m-1} \mu_{2m}}$$

$$k = [(D-1)/2]$$

a_m : arbitrary constants

$\delta^{\nu_1 \nu_2 \ldots \nu_{2m-1} \nu_{2m}}_{\mu_1 \mu_2 \ldots \mu_{2m-1} \mu_{2m}}$: totally anti-symmetric tensor

3. 1 Collapse of Einstein Cluster

- We consider the gravitational collapse with **spherical symmetry**.
  In the previous paper , we study the spherical dust collapse in the Lovelock gravity .

Summary of dust case

Even dimensional case	Odd dimensional case
Only central singularity can be naked	Singularity at **finite radius** as well as central one can be naked

- Dust cloud, however, is not the realistic matter model. It is important to include the pressure.
  Here we consider the gravitational collapse of Einstein cluster.
  Einstein cluster is counter rotating dust cloud, which is equivalent to fluid which has **tangential pressure.**

3.2 Metrics and Field equation

- Metric for Einstein cluster

$$ds^2 = -A^2 dt^2 + \left(\left(\frac{R'}{R} \right)^2 \frac{J^2 + R^2}{W^2} \right) dr^2 + R^2 d\Omega^2$$

- Field equation for Einstein cluster

$$\sum_{m=1}^{k} c_m \left(\left(\frac{\dot{R}}{RA} \right)^2 + \frac{1}{R^2} - \frac{W^2}{J^2 + R^2} \right)^m = \frac{F(r)}{R^{n+1}}$$

$$c_l := \begin{cases} 1 & \text{if } \ l = 1 \\ \frac{a_l}{l} \prod_{p=1}^{2l-2} (n-p) & \text{if } \ 2 \leq l \leq k \end{cases}$$

$F(r)$:mass function

$J(r)$:specific angular momentum （J=0 corresponds to dust）

3.3 Singularity, apparent horizon and bounce point

Condition for Singularity formation $R = 0$

Odd dim

Apparent horizon $$H_{odd}(R) \equiv \sum_{m=1}^{k=(n+1)/2} c_m R^{n-2m+1} = F(r)$$

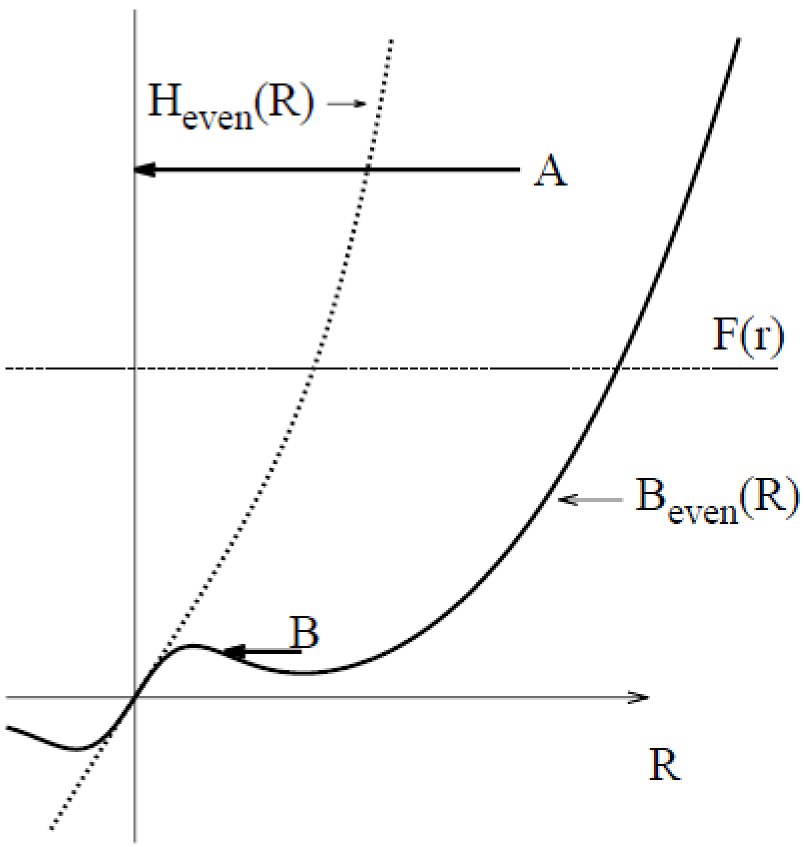
Bounce $$B_{odd}(R,J) \equiv \sum_{m=1}^{k=(n+1)/2} c_m R^{n+1-2m} \left(\frac{J^2}{J^2+R^2}\right)^m = F(r)$$

Even dim

Apparent horizon $$H_{even}(R) \equiv \sum_{m=1}^{k=n/2} c_m R^{n-2m+1} = F(r)$$

Bounce $$B_{even}(R,J) \equiv \sum_{m=1}^{k=n/2} c_m R^{n+1-2m} \left(\frac{J^2}{J^2+R^2}\right)^m = F(r)$$

3.4 Result : EVEN dimension



$r \neq 0$ case

- If J is sufficiently small, B_{even} has local maximum and shell can Bounce
- All the singularities are covered by apparent horizon and cannot be naked

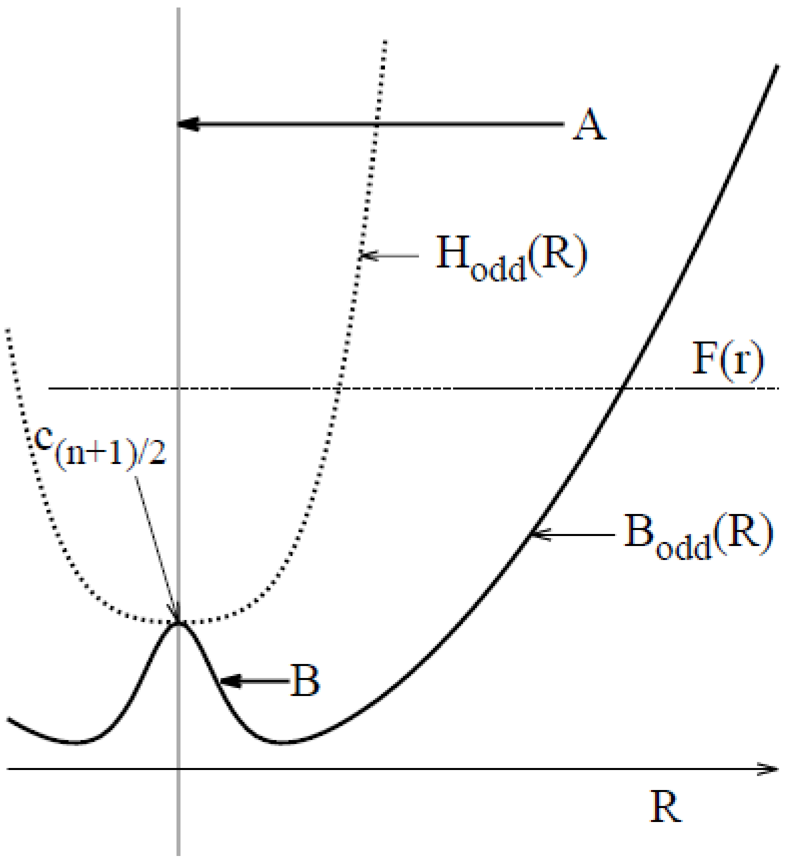
$r = 0$ case

$J^2(r) = j_4 r^4 + O(r^6)$

$j_4 \neq 0$ ⇒ shell bounces

$j_4 = 0$ ⇒ shell does not bounce and singularity can be naked

3.5 Result : ODD dimension



$r \neq 0$ case

- The shells which satisfy $F(r) < c_{(n+1)/2}$ inevitably bounce
- The other shells hit the singularity which cannot be naked

$r = 0$ case

$J^2(r) = j_4 r^4 + O(r^6)$

$j_4 \neq 0$ ⇒ shell bounces

$j_4 = 0$ ⇒ shell does not bounce and singularity can be naked

4. Summary

- We consider the gravitational collapse of Einstein cluster in Lovelock gravity, and investigate the effect of angular momentum (or pressure) on the Cosmic Censorship conjecture.

	Even dimension	Odd dimension
$r \neq 0$	J is sufficiently small → bounce No singularity can be naked.	$F(r) < c_{(n+1)/2}$ → bounce Other shell hit the singularity , but cannot be naked.
$r = 0$	$j_4 \neq 0$ → shell bounces $j_4 = 0$ → shell does not bounce and singularity can be naked	