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Effect of Metric Perturbations in Vacuum 5D Universe on Material 4D Universe

Takao Fukui
Dokkyo University
1-1 Gakuencho, Soka, Saitama, 340-0042, Japan
tfukui@dokkyo.ac.jp

1.5D STM Universe [1)]

★ Field equations of the 5D Space (123) -Time (0) -Matter (4) Universe

(A bar over any quantity denotes its unperturbed value)

$$ {}^{5D}\overline{G}_{AB} = 0 \qquad (A,B = 0,123,4) $$

$$ {}^{5D}\overline{G}_{\alpha\beta} = {}^{4D}\overline{G}_{\alpha\beta} + {}^{5thD}Extra \quad terms = 0 \quad (\alpha,\beta = 0,123) $$

$$ {}^{5D}\overline{G}_{\alpha 5} = 0, \qquad {}^{5D}\overline{G}_{55} = 0 $$

★ Geometrical properties of the 5th coordinate

= Physical properties of the 4D Universe [2)]

$$ {}^{4D}\overline{G}_{\alpha\beta} = -{}^{5thD}Extra \;\; terms = {}^{4D}Physical \;\; properties $$
$$ = 8\pi\overline{T}_{\alpha\beta} = 8\pi\left[(\overline{\rho}+\overline{p})\overline{u}_\alpha\overline{u}_\beta - \overline{p}\overline{g}_{\alpha\beta}\right] $$

$c = G = 1$ unit system is employed here.

★ Metric

$$ ds^2 = \sigma\left\{dt^2 - a^2\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]\right\} + \varepsilon b^2 dl^2 $$

★ Solutions [3)]

$$ a^2 = -\frac{(2Ft-g)^2+\kappa}{4F} $$

$$ b = -\frac{4\varepsilon\sigma f\hat{f}F^2t^2 - 2\hat{g}F^2t + g\hat{g}F - \varepsilon\sigma f\hat{f}\left(g^2+\kappa\right)}{4afF^2} $$

★ The solutions determine the properties of matter of the 4D Universe through the unperturbed energy-momentum tensor for the perfect fluid

$$ {}^{4D}\overline{G}_{\alpha\beta} = 8\pi\overline{T}_{\alpha\varphi} = 8\pi\left[(\overline{\rho}+\overline{p})\overline{u}_\alpha\overline{u}_\beta - \overline{p}\overline{g}_{\alpha\beta}\right] $$

$$\frac{8\pi\bar{\rho}}{3} = -\frac{\varepsilon f^2}{a^2} - \frac{\kappa}{4\sigma a^4}$$

$$8\pi\bar{p} = \frac{\varepsilon f^2}{a^2} - \frac{\kappa}{4\sigma a^4}$$

$$F(l) \equiv \varepsilon\sigma f^2 + k, \quad f = f(l), \quad g = g(l), \quad \hat{f} \equiv \frac{\partial f}{\partial l}, \quad \hat{g} \equiv \frac{\partial g}{\partial l}$$

2.First-order perturbations [4)]

$$g_{AB} = \bar{g}_{AB} + h_{AB}(t,r,l)$$

$$R_{AB} = \bar{R}_{AB} + H_{AB} + O(h^2)$$

★ First order field equation of 5D STM theory

$$^{5D}G_{AB} = \bar{R}_{AB} - \frac{1}{2}\bar{g}_{AB}\bar{R} + H_{AB} - \frac{1}{2}\bar{g}_{AB}H - \frac{1}{2}h_{AB}\bar{R} + \frac{1}{2}\bar{g}_{AB}\bar{g}^{CD}\bar{g}^{EF}h_{DF}\bar{R}_{CE}$$

★ Nonzero solutions of the off-diagonal components of $^{5D}G_{AB} = 0$ for h_{AB}

$$h_{00} = \frac{\zeta_0^{1/2} r^2}{4\sqrt{\varepsilon\sigma}\, f^3 q^{3/2}}$$

$$h_{55} = \frac{\zeta_0^{1/2} r^2 \hat{f}^2 q^{1/2}}{8(\varepsilon\sigma)^{5/2} f^5}$$

$$q \equiv 2\varepsilon\sigma t - f_0$$

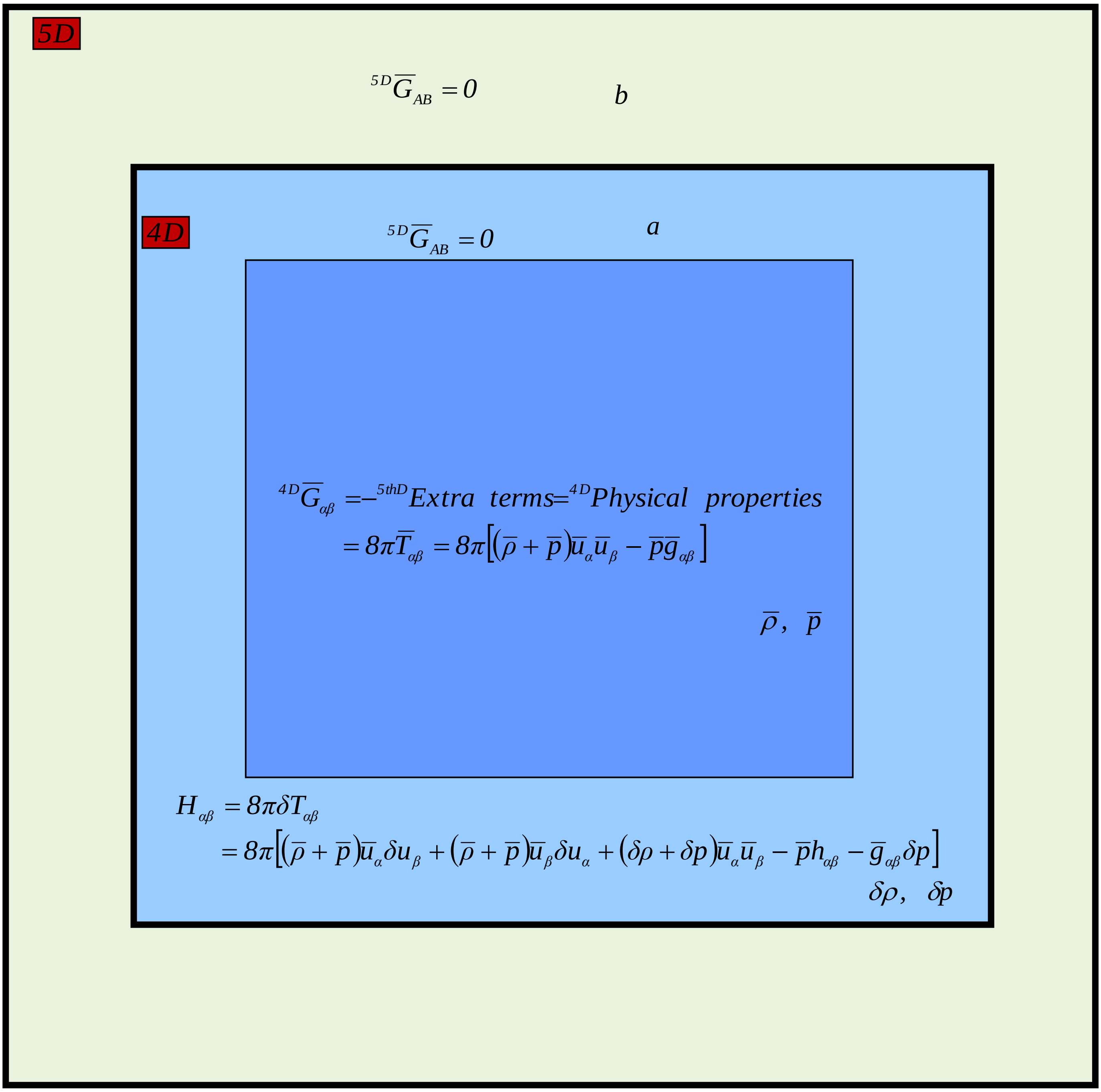
★ The solutions determine the effects of the perturbations on ρ and p through

$$\begin{aligned} H_{\alpha\beta} &= 8\pi\delta T_{\alpha\beta} \\ &= 8\pi\left[(\bar{\rho}+\bar{p})\bar{u}_\alpha\delta u_\beta + (\bar{\rho}+\bar{p})\bar{u}_\beta\delta u_\alpha + (\delta\rho+\delta p)\bar{u}_\alpha\bar{u}_\beta - \bar{p}h_{\alpha\beta} - \bar{g}_{\alpha\beta}\delta p\right] \end{aligned}$$

★ The dark side of the Universe [5)] through $\delta\rho$ and δp

$$8\pi\delta\rho = \frac{3\zeta_0^{1/2} r^2}{2\sqrt{\varepsilon\sigma}\, f^3 q^{7/2}} + \frac{6\zeta_0^{1/2}}{(\varepsilon\sigma)^{3/2} f^5 q^{7/2}}$$

$$8\pi\delta p = -\frac{2\zeta_0^{1/2}}{(\varepsilon\sigma)^{3/2} f^5 q^{7/2}}$$



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