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“Features in the CMB spectrum as a probe of heavy physics  
during inflation”

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**RESCEU SYMPOSIUM ON  
GENERAL RELATIVITY AND GRAVITATION**

**JGRG 22**

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# RESCEU SYMPOSIUM ON GENERAL RELATIVITY AND GRAVITATION

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## Features in the CMB spectrum as a probe of heavy physics during inflation

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Ref: 1206.2164 [astro-ph.CO] (To appear in JCAP)

RS, J. Yokoyama, Y. Takamizu, in preparation

The simplest model of inflation:

## Single-field slow-roll inflation

### Predictions (with more technical assumptions)

The primordial fluctuations

- are adiabatic
- are nearly Gaussian-distributed
- have a nearly scale-invariant power spectrum

$$\Delta_{\mathcal{R}}^2(k) = \Delta_{\mathcal{R}}^2(k_0) (k/k_0)^{n_s - 1}$$

No strong evidence to show that the Universe is described in a more complicated way.

“Single-field” does not mean that there is no field other than inflaton.

From a model-building point of view, in general, there are

**many (scalar) degrees of freedom other than inflaton,**

especially in models embedded in supergravity/string theory:

Moduli fields, Kaluza-Klein modes, Scalar SUSY-partner of inflaton,..

Usually, they are so **heavy**,  $m \gg H$ , that the models are treated *effectively* as single-field models.

The power spectrum is calculated by a single-field analysis with a reduced potential *assuming heavy fields are stuck to their potential minima*. [Yokoyama & Yamaguchi]

**Is there any chance to probe heavy fields?**

Heavy fields are *not necessarily* stuck to their potential minima and *can* affect the correlation functions.

They are displaced from their minima if they

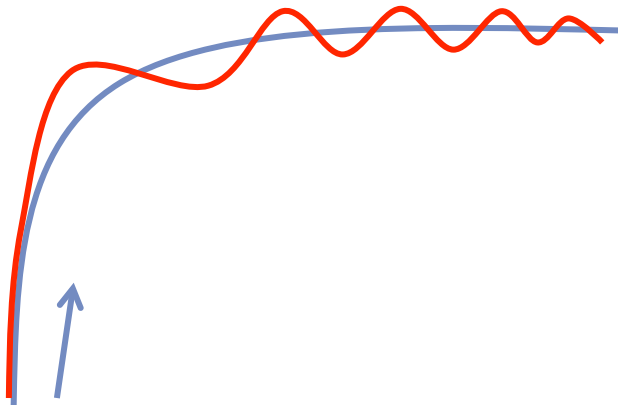
- happen to be excited at the beginning of inflation.

[Burgess, Cline, Lemieux, & Holman]

- are dynamically excited by e.g. turns of the inflaton trajectory.

[Achucarro et al.], [Shiu & Xu], [Chen & Wang], [Sespedes et al.], [Pi & Sasaki],

[Gao, Langlois, & Mizuno], [Burgess et al.]..



Heavy fields are displaced from their minima due to a centrifugal force.

When the turn is very sharp, the oscillation along the heavy direction can be excited after the turn.

What kind of effects are expected when heavy fields are excited from the potential minima?

– Large mixing between the light and heavy modes.

[Achucarro et al.], [Shiu & Xu], [Chen & Wang], [Sespedes et al.], [Pi & Sasaki],  
[Gao, Langlois, & Mizuno],...

– **Resonance between the excited oscillation and the fluctuations.**

[Chen] [RS, Nakashima, Takamizu, & Yokoyama]

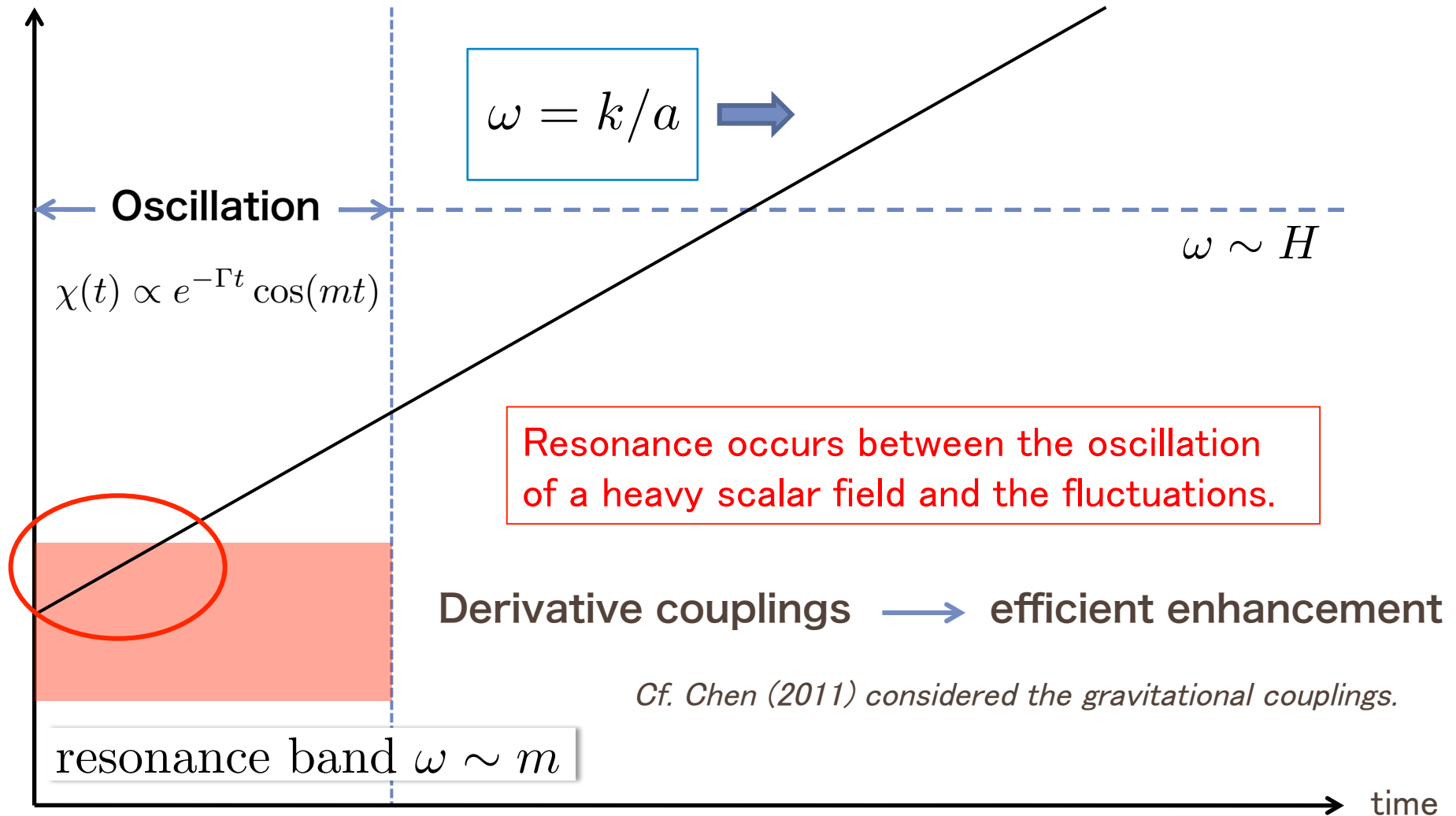
**The correction becomes larger for larger mass.**

→ **It opens a possibility to probe heavy physics during inflation.**

# Basic idea

[Chen] [RS, Nakashima, Takamizu, & Yokoyama]

Physical scale



# Model

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**Action:** inflaton + a heavy scalar field with **derivative couplings**

$$S_m \equiv - \int dx^4 \sqrt{-g} \left[ \frac{1}{2} (\partial\phi)^2 + V(\phi) + \frac{1}{2} (\partial\chi)^2 + \frac{m^2}{2} \chi^2 + K_n + K_d \right],$$

where

$$K_n \equiv \frac{\lambda_n}{2\Lambda_n} \chi (\partial\phi)^2$$

$$K_d \equiv \frac{\lambda_{d1}}{4\Lambda_d^4} (\partial\chi)^2 (\partial\phi)^2 + \frac{\lambda_{d2}}{4\Lambda_d^4} (\partial\chi \cdot \partial\phi)^2$$

Decay rate of  $\chi$ :  $\Gamma$



## Kn & Kd from the effective-field-theory point of view

[Weinberg]

$$\mathcal{L}_0 \equiv - \left[ \frac{1}{2} (\partial\phi)^2 + V(\phi) + \frac{1}{2} (\partial\chi)^2 + \frac{m^2}{2} \chi^2 \right],$$

is just the first term of the derivative expansion in a generic effective field theory.

Kn & Kd are next corrections to  $\mathcal{L}_0$ : most general ones if we assume parity & shift symmetry for the inflaton field.

They are usually unimportant, but could play an important role in the resonance region, which is deep inside the horizon and the heavy scalar field rapidly oscillates and then the derivatives are large.

## Kn in specific models

- A heavy scalar field with a non-minimal coupling (incl. a model with higher curvature terms),
- A pseudo-NG boson + a symmetry breaking field,
- Supergravity + higher order terms in the Kahler potential,..

## Kd in specific models

- DBI action (Dp-brane):

$$S_{\text{DBI}} = -T \int d^4x \int d^{p-3} \lambda \sqrt{-\det (G_{MN} \partial_m \phi^M \partial_n \phi^N)} \quad (M, N = 0, \dots, D-1)$$

Kd couplings appear expanding the square root in the action.

heavy scalar field = brane coordinates other than inflaton, KK modes

# Setup

- Hierarchy of the energy scales

$$H < \Gamma < m < \Lambda_n, \Lambda_d$$

- Higher order terms are negligible

$$\chi < \Lambda_n, \dot{\phi}, \dot{\chi} < \Lambda_d^2$$

- The heavy scalar field is subdominant

$$f_\chi \equiv \frac{\rho_\chi}{\rho} \simeq \frac{\dot{\chi}^2 + m^2 \chi^2}{6M_p^2 H^2} \ll 1$$

- Flat potential

$$\epsilon_V \equiv \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1, \quad \eta_V \equiv M_p^2 \frac{V''}{V} \ll 1$$

# Power spectrum

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We can make some simplifications (in the flat gauge):

The resonance takes place deep inside the horizon.



The gravitational effects are not important.

(The metric perturbations can be neglected.)

Small couplings & Large mass.



The fluctuations for the heavy field are smaller than those in the inflaton field and also do not satisfy the resonance condition (  $\omega = \sqrt{m^2 + (k/a)^2} > m$  ).

(The perturbations in the heavy field can be neglected.)

Only the inflaton perturbations are relevant:  $\phi = \phi_0 + \varphi$

Second-order action (flat gauge):

$$S_2 \simeq \int dt d^3x \frac{z_\phi^2}{2} [\dot{\varphi}^2 - c_s^2 (\nabla \varphi)^2 / a^2]$$

where

$$z_\phi^2 = a^3 \left[ 1 + \lambda_n \frac{\chi}{\Lambda_n} + (\lambda_{d1} + \lambda_{d2}) \frac{\dot{\chi}^2}{2\Lambda_d^4} \right]$$

$$c_s^2 \simeq 1 - \frac{\lambda_{d2} \dot{\chi}^2}{2\Lambda_d^4} + O\left(\frac{\dot{\chi}^4}{\Lambda_d^8}\right) \quad (\text{the speed of sound})$$

Equation of motion (on subhorizon scales):

$$\frac{d^2 v_k}{dz^2} + [A_k - 2q \cos(2z)] v_k = 0$$

where

$$z \equiv mt,$$

$$v_k \equiv z_\phi \varphi,$$

$$A_k \equiv \left( \frac{k}{am} \right)^2,$$

$q_d$

$$q \equiv e^{-2\Gamma t} \left[ \lambda_{d2} \frac{m^2 \chi_0^2}{8\Lambda_d^4} \left( \frac{k}{am} \right)^2 + (\lambda_{d1} + \lambda_{d2}) \frac{m^2 \chi_0^2}{\Lambda_d^4} \right]$$

Form of the Mathieu equation  $\longrightarrow$  **Parametric resonance**

A similar equation for the Kn-coupling case:

$$m \rightarrow m/2, \Gamma \rightarrow \Gamma/2, q \rightarrow \lambda_n q_n e^{-\Gamma t} \equiv \lambda_n \frac{\chi_0}{\Lambda_n} e^{-\Gamma t}$$

## Condition for the parametric resonance

$$q \equiv e^{-2\Gamma t} \left[ \lambda_{d2} \frac{m^2 \chi_0^2}{8\Lambda_d^4} \left( \frac{k}{am} \right)^2 + (\lambda_{d1} + \lambda_{d2}) \frac{m^2 \chi_0^2}{\Lambda_d^4} \right] < 1$$

➔ **Narrow resonance**

$$|A_k - 1| < q \quad \text{➔} \quad (1 - \tilde{q}/2)m < k/a < (1 + \tilde{q}/2)m$$

where

$$\tilde{q} \equiv (2\lambda_{d1} + 3\lambda_{d2}) \frac{q_d}{8} e^{-2\Gamma t} \sim \left( \frac{V^{1/4}}{\Lambda_d} \right)^4 f_\chi$$

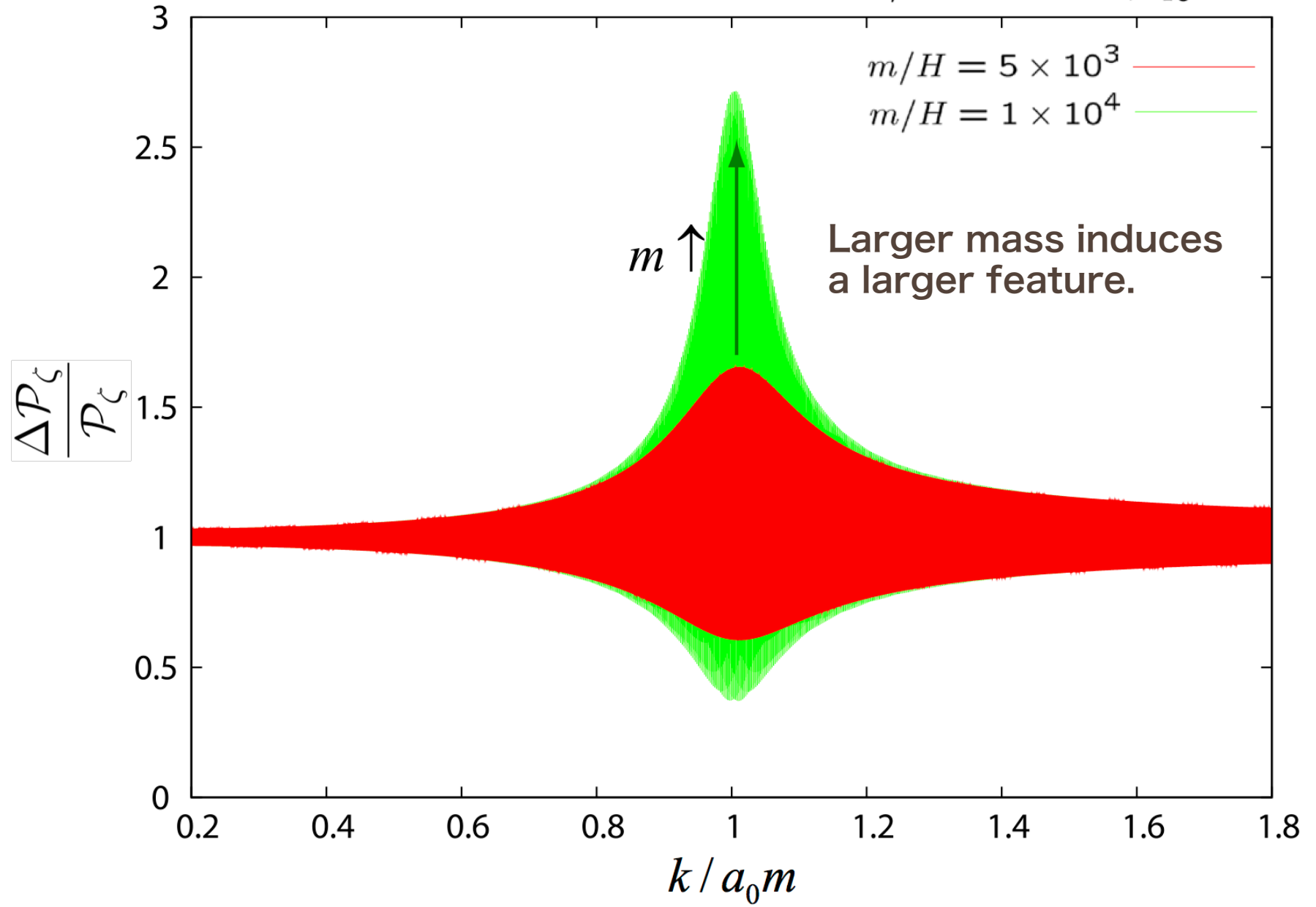
Number of resonance periods:

$$\Delta z \sim \frac{m}{H} \min(\tilde{q}, H/\Gamma)$$

**The resonance is more efficient for larger mass.**

# Spectra for different values of the mass

$$\Gamma/H = 5 \times 10^2, \quad q_0 = 0.1$$





# Bispectrum

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The oscillation can also affect the higher-order correlation functions (non-Gaussianity, NG).

e.g., Bispectrum

$$\langle \zeta_{\mathbf{k}_1}(t) \zeta_{\mathbf{k}_2}(t) \zeta_{\mathbf{k}_3}(t) \rangle = (2\pi)^7 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) (\mathcal{P}_\zeta)^2$$

## NG from the resonance

### Resonant non-Gaussianity

[Chen, Easter, & Lim], [Flauger & Pajer], [Chen],  
[Behbahani et al.], [Gwyn Rummel, & Westphal],...

## Relevant interactions

- contains an oscillatory component
- contains more derivatives

(The mixing with the fluctuations in the heavy field can be neglected.)

### Kn couplings

$$\mathcal{L}_3 \supset -\frac{\lambda_n \chi}{\Lambda_n} \frac{\dot{\phi}}{2M_p^2 H} \left[ \frac{1}{2} \varphi \left( \dot{\varphi}^2 + \frac{1}{a^2} (\partial\varphi)^2 \right) - \partial_i (\partial^{-2} \dot{\varphi}) \dot{\varphi} \partial_i \varphi \right],$$

### Kd couplings

$$\mathcal{L}_3 \supset -\frac{\lambda_{d1} \dot{\chi}^2}{2\Lambda_d^4} \frac{\dot{\phi}}{2M_p^2 H} \left[ \frac{1}{2} \varphi \left( \dot{\varphi}^2 + \frac{1}{a^2} (\partial\varphi)^2 \right) - \partial_i (\partial^{-2} \dot{\varphi}) \dot{\varphi} \partial_i \varphi \right] \\ - \frac{\lambda_{d2} \dot{\chi}^2}{2\Lambda_d^4} \frac{\dot{\phi}}{2M_p^2 H} \varphi \dot{\varphi}^2.$$

## Interaction Hamiltonian

$$H^{\text{int}} = \frac{\epsilon_\phi^{1/2}}{2\sqrt{2}a^2(2\pi)^6 M_p} \iiint d^3k_1 d^3k_2 d^3k_3 k_2 k_3 C_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3} \varphi_{\mathbf{k}_1} \varphi_{\mathbf{k}_2} \varphi_{\mathbf{k}_3} \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3),$$

where

$$C_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3} \equiv C_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3}^n e^{-\Gamma t} \underline{\cos(mt)} + C_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3}^d e^{-2\Gamma t} \underline{\sin^2(mt)},$$

$$C_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3}^n \equiv \lambda_n \left[ 1 + \hat{\mathbf{k}}_2 \cdot \hat{\mathbf{k}}_3 \left( 1 - \frac{k_1}{k_2} \right) \right] q_n,$$

$$C_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3}^d \equiv \left\{ \frac{\lambda_{d1}}{2} \left[ 1 + \hat{\mathbf{k}}_2 \cdot \hat{\mathbf{k}}_3 \left( 1 - \frac{k_1}{k_2} \right) \right] - \lambda_{d2} \right\} q_d,$$

The Bispectrum can be estimated by using the in-in formalism:

$$\begin{aligned} \langle \varphi_{\mathbf{k}_1}(t) \varphi_{\mathbf{k}_2}(t) \varphi_{\mathbf{k}_3}(t) \rangle &= i \int dt' \langle [H^{\text{int}}(t'), \varphi_{\mathbf{k}_1}(t) \varphi_{\mathbf{k}_2}(t) \varphi_{\mathbf{k}_3}(t)] \rangle \\ &= \text{Re} \left( i \int dt' \langle H^{\text{int}}(t') \varphi_{\mathbf{k}_1}(t) \varphi_{\mathbf{k}_2}(t) \varphi_{\mathbf{k}_3}(t) \rangle \right) \end{aligned}$$

**Resonance at  $K/a_* \simeq m$**

$$H^{\text{int}}(t') \sim \cos(mt') \varphi_{\mathbf{k}_1}(t') \varphi_{\mathbf{k}_2}(t') \varphi_{\mathbf{k}_3}(t') \sim \cos(mt') e^{-iK\eta'}$$

(  $K = k_1 + k_2 + k_3$  )

Resonance

➔ An additional contribution from the oscillatory region.

(There is no contribution from the subhorizon region in the usual case.)

# Analytic estimation

Using the method of the steepest descent,

Dimensionless

$$\begin{aligned} & F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) k_1^2 k_2^2 k_3^2 \\ &= -\frac{\epsilon_\phi}{16H} \int_0^\infty dt' \frac{k_2 k_3}{a^5} C_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3} \cos(K\eta') + \text{cyclic} \\ & \quad \text{The time of the excitation} \\ & \sim \frac{\sqrt{2\pi}\epsilon_\phi}{16} \left(\frac{2m}{H}\right)^{3/2} \left(\frac{k_2 k_3}{K^2}\right) \left(\frac{K}{2m}\right)^{-3-2\Gamma/H} \cos\left\{\frac{2m}{H} \left[\ln\left(\frac{K}{2m}\right) + 1\right]\right\} C_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3}^d + \text{cyclic} \end{aligned}$$

A similar equation for the Kn-coupling case:

$$m \rightarrow m/2, \Gamma \rightarrow \Gamma/2, C_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3}^d \rightarrow C_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3}^n$$

\* We set the lower limit of integration as  $-\infty$  instead of 0 to obtain the final expression. The function F is suppressed for  $K < m$  if we use 0 as the lower limit properly. (by a few factor for  $K=m$ )

**A very large feature also appears in the Bispectrum.**

$$\sim \epsilon_\phi q \left(\frac{m}{H}\right)^{3/2} \quad \text{at} \quad K \simeq m \text{ (for } Kn), 2m \text{ (for } Kd)$$

# Summary

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- We could obtain some hints on heavy physics during inflation by analyzing the local features of the CMB power spectrum.
- A heavy scalar field could leave non-negligible signatures in the CMB spectrum through parametric resonance between its background oscillations and the inflaton fluctuations.
- A large features could be induced in the higher-correlation functions, Non-Gaussianity, even when the feature in the power Spectrum is too small to be observed.
- If they are detected, they will improve our understanding of physics behind inflation.