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“Graphical approach to the Lovelock black holes”

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Graphical Approach to the Lovelock black hokles

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ABSTRACT

Recently a large number of studies have focused on the question of higher dimensions. One of the reasons for this is the fascinating new picture of our Universe called the braneworld universe. Because the fundamental scale could be about TeV in this senario, these models suggest that creation of tiny black holes in the upcoming large hadron collider will be possible. The braneworld model is an underlying fundamental theory, superstring/M-theory. Although superstring/M-theory has been highly elaborated, it is not enough to understand black hole physics in the string context. Hence at present to take string effects perturbatively into classical gravity is one approach to the study of quantum gravity effects.

With the restriction that the tension of a string be large as compared to the energy scale of other variables, i.e., in the α-expansion, the Gauss-Bonnet terms appear as the first curvature correction term to general relativity. Though many complicated combinations may appear for the higher order corrections, here we will adopt Lovelock gravitational theory expecting some aspects of higher curvature corrections are obtained. Lovelock gravitational theory is the most general theory which gives the e.o.m. with up to the second order derivative. In this theory we investigate the black hole solution and the spacetimes with product metric which are Nariai and Bertotti-Robinson types. The study of the black hole solution is the direct extension of the works by Whitt (1988) and Myers & Simon (1988). The study of the product metric will be applied to the spontaneous compactification mechanism without fluxes.

black hole solutions

In the first part of the poster we investigate black hole solutions in Lovelock gravity. Under the static ansatz the gravitational equation of motion becomes algebraic equation of the single metric function. Although it can not be solved analytically except for some simple cases, we show a technique to find the spacetime structure of the solution. We also consider the topological black hole solution whose submanifold has curvature k=0, -1. As some examples we study the solution in Gauss-Bonnet gravity and "M-theory" model.

Nariai and Bertotti-Robinson solutions

In the second part of the poster we investigate the solution with product metric in Lovelock gravity. Under the decomposition into 2-dim. Riemaninan manifold and (n-2)-dim. Euclidean submanifold with maximal symmetry, the gravitational equation of motion becomes algebraic equation of the single metric function. We will give the way to analyze such equation and to classify the solution into Nariai, anti-Nariai, Bertotti-Robindon, and Plebanski-Hachyan solutions. As some examples we study the solution in general relativity and Gauss-Bonnet gravity.

MODEL

Lagrangian

- We consider the Lovelock lagrangian up to Nth order, which gives a quasi-linear 2nd order gravitational equation

$$N \le \frac{n-1}{2}$$

$$\mathcal{L} = \sum_{p=0}^{N} \frac{\alpha_p}{(n-2p)(n-2)!}\mathcal{L}_p$$

$$\mathcal{L}_p = 2^{-p}\delta^{\mu_1\nu_1\cdots\mu_p\nu_p}_{\rho_1\sigma_1\cdots\rho_p\sigma_p} R^{\rho_1\sigma_1}{}_{\mu_1\nu_1}\cdots R^{\rho_p\sigma_p}{}_{\mu_p\nu_p}.$$

α : coefficients

$\mathcal{L}_0$: cosmological constant

$\mathcal{L}_1$: Einstein-Hilbelt action

$\mathcal{L}_2$: Gauss-Bonnet term

Metric

- We assume the n-dimensional static spacetime whose metric is descrived by

$$ds^2 = -f(r)e^{-2\delta(r)}dt^2 + f^{-1}(r)dr^2 + R(r)^2 \underbrace{d\Omega^2_{n-2}}_{\text{(n-2)-dim maximally sym. space}},$$

(n-2)-dim maximally sym. space

$$k = 1, 0, -1$$

BLACK HOLE SOLUTION

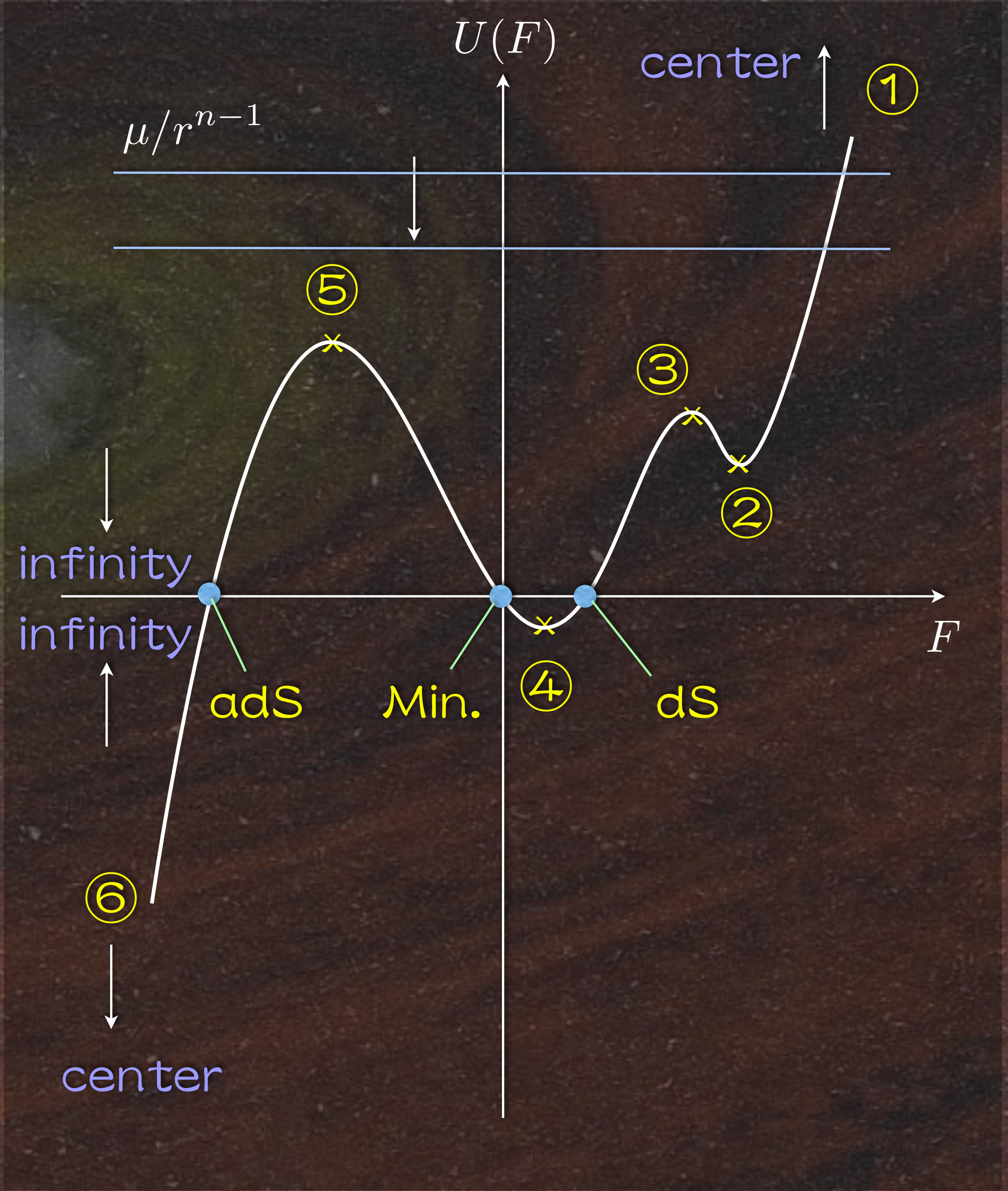
- gravitational equation

$R \neq \text{const.} \xrightarrow{\text{gauge}} R = r$

$$f(r) = k - r^2 F(r)$$

mass

$$U(F) := \sum_{p=0}^{N} \alpha_p F^p = \frac{\mu}{r^{n-1}} \qquad \delta \equiv 0$$



- vacuum: $U(F) = 0$
- infnity: $U(F_\infty) = 0$

$F_\infty = 0$: asymptotically flat

$F_\infty > 0$: asymptotically de Sitter

$F_\infty < 0$: asymptotically anti-de Sitter

❖ singularity:

◆ central singularity: $r \to 0$

- n-1 > 2N
  - ① $F_0 > 0$: spacelike
  - ⑥ $F_0 < 0$: timelike
- n-1 = 2N
  - $\bar{k} > 0$: timelike
  - $\bar{k} = 0$: null
  - $\bar{k} < 0$: spacelike

$\bar{k} := k \pm \left(\frac{\mu}{\alpha_p}\right)^{\frac{1}{p}}$ even N

$\bar{k} := k - \left(\frac{\mu}{\alpha_p}\right)^{\frac{1}{p}}$ odd N

◆ branch singularity: $r \to r_b > 0$

② $|r| \to |r_b|_{+0}$: outer branch sing.

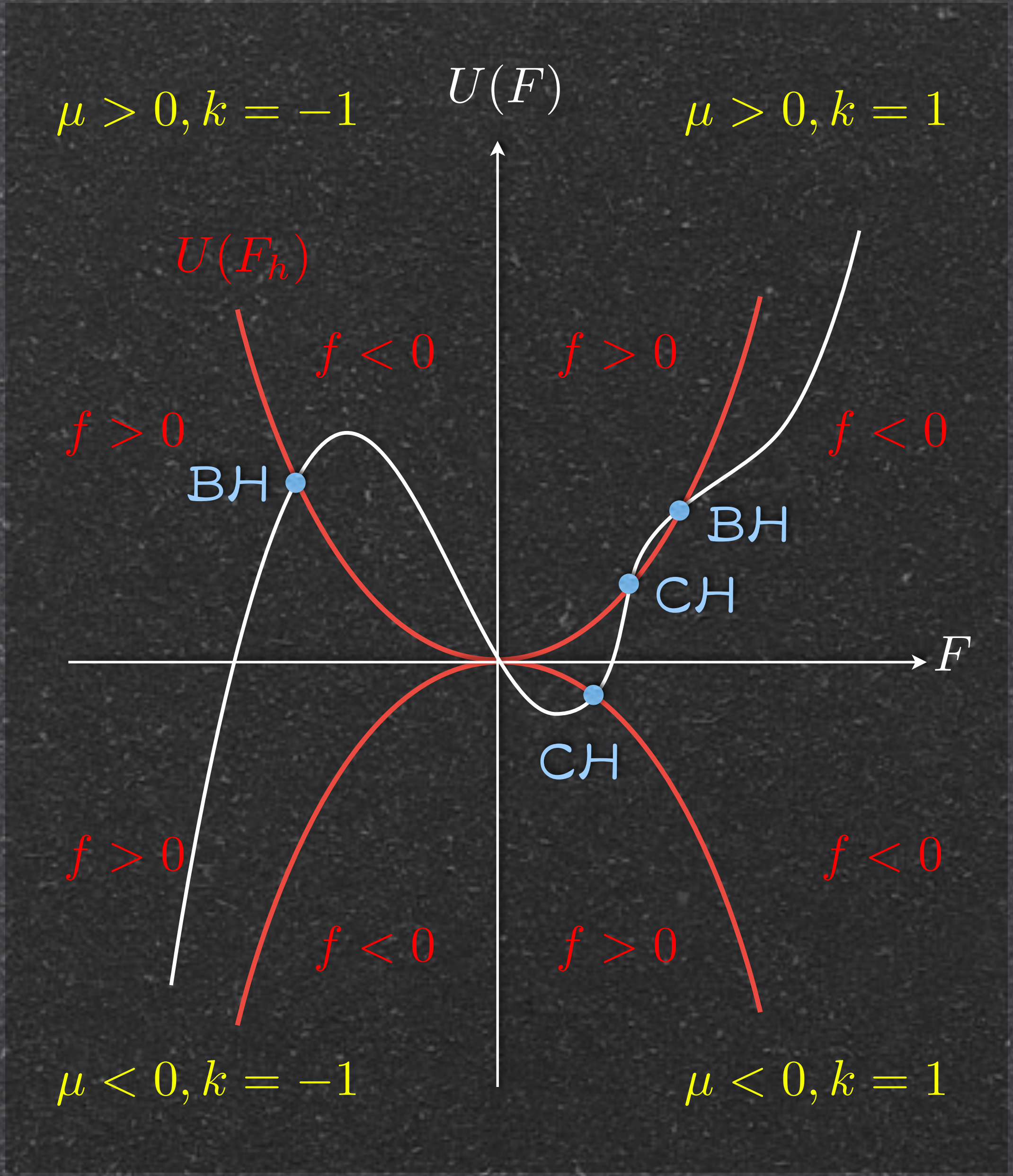
③-⑤ $|r| \to |r_b|_{-0}$: inner branch sing.

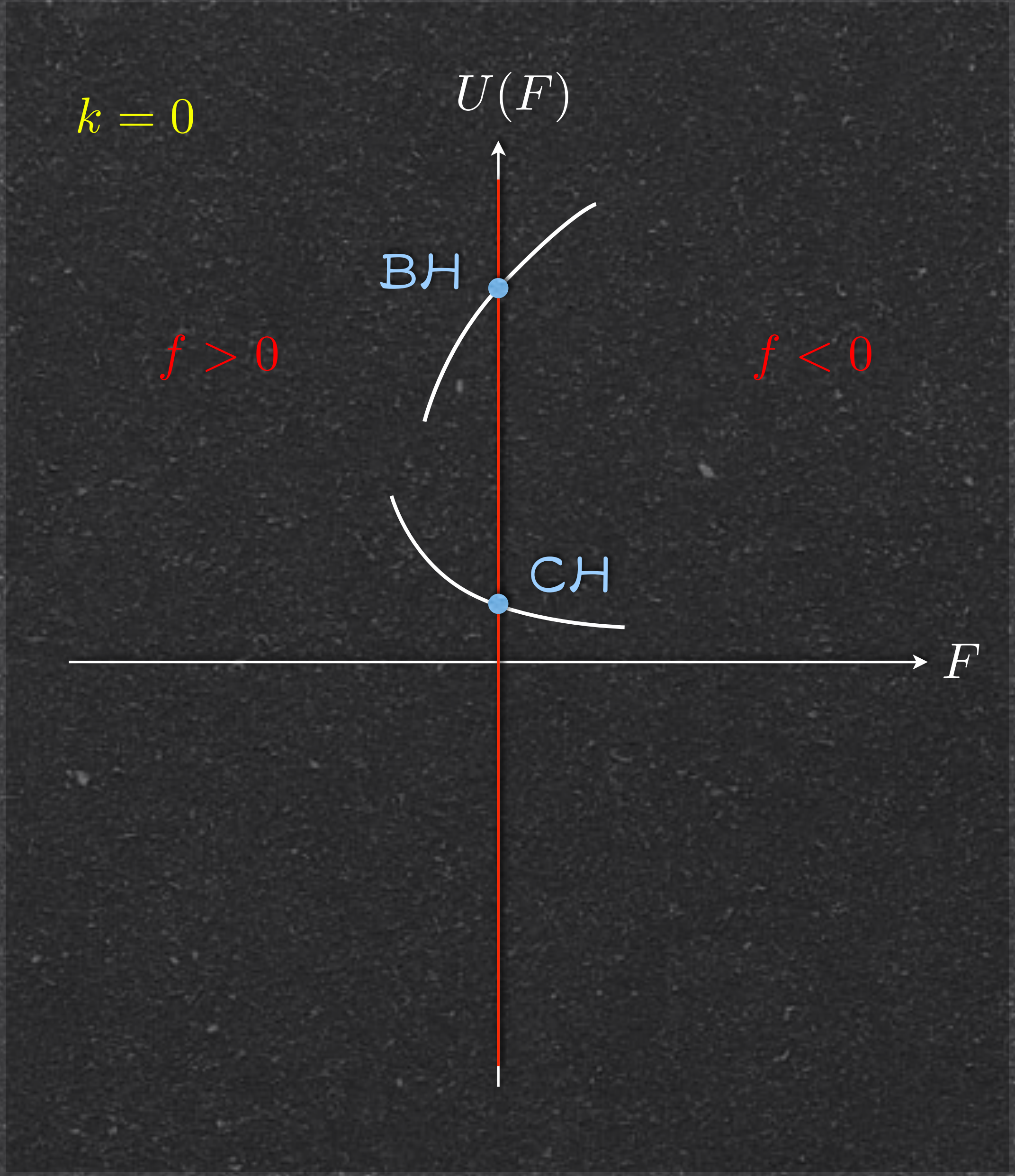
$$k + r_b^2 F_b \begin{cases} > 0 & : \text{timelike} \\ < 0 & : \text{spacelike} \\ = 0 & \begin{cases} : \text{timelike} \\ : \text{spacelike} \end{cases} \end{cases}$$

F-U diagram

❖ event horizon

$$f(r_h) = k - r_h^2 F_h = 0 \quad \Longrightarrow \quad \begin{cases} k = 0: & F_h = 0 \\ k \neq 0: & \mu\left(\dfrac{F_h}{k}\right)^{\frac{n-1}{2}} = 0 \end{cases}$$

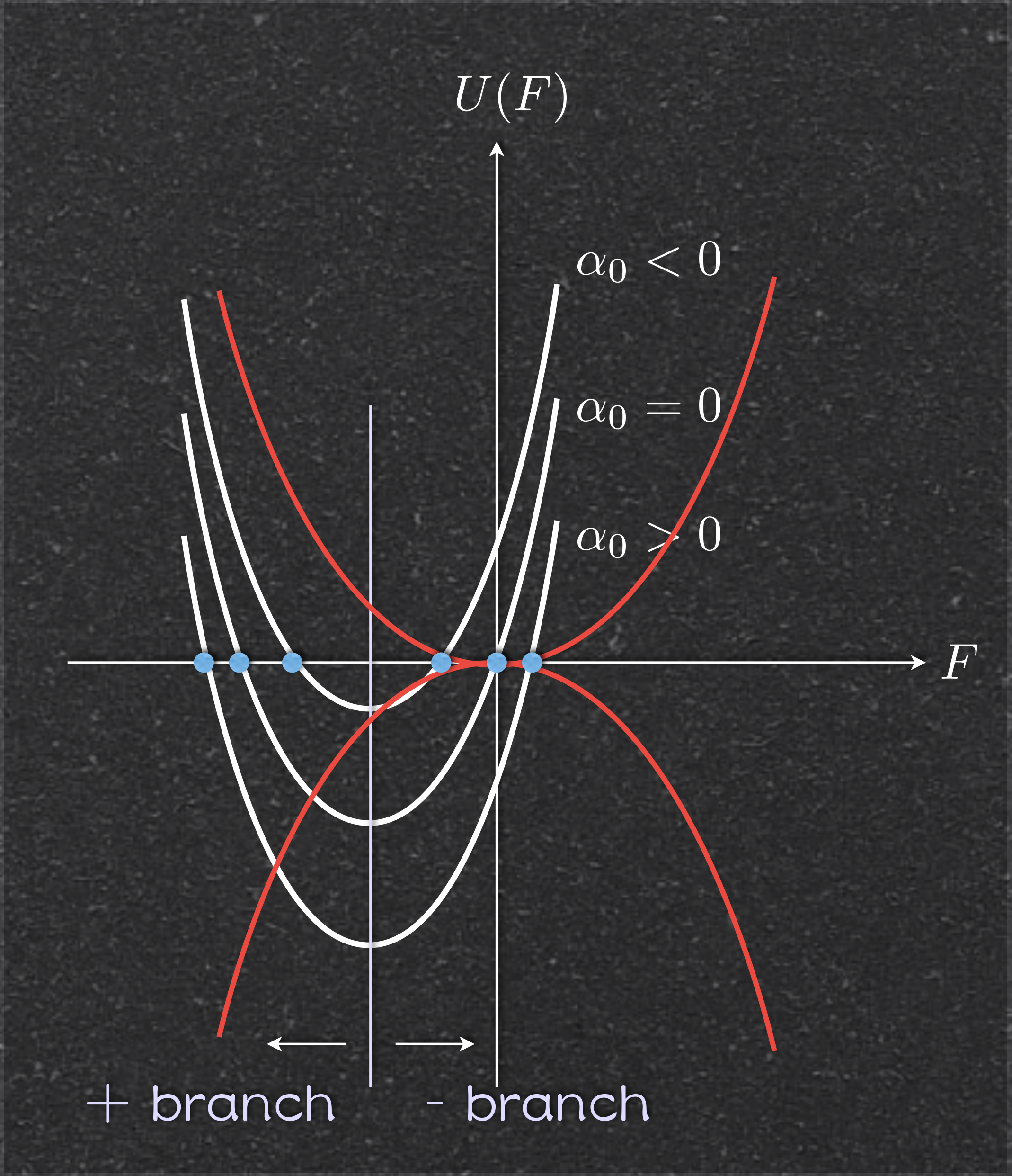




◆ Only the adS-branch solution has BH for k=0, -1.

Example 1: Gauss-Bonnet gravity

$\alpha_0 \neq 0, \ \alpha_1, \alpha_2 > 0$



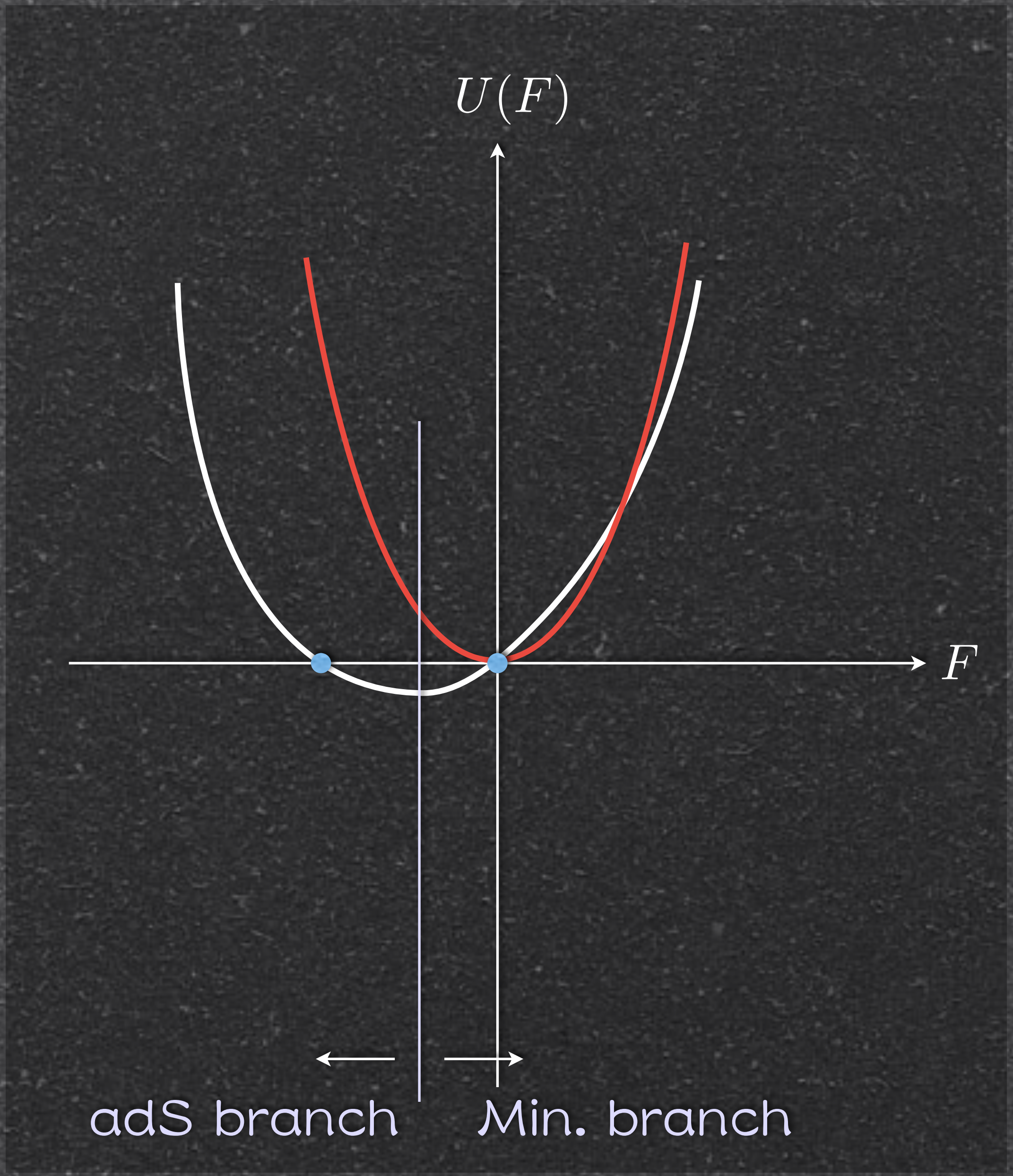
- $\alpha_1^2 - 4\alpha_0\alpha_2 > 0$ for the existence of relevant vacuum.
- Solutions in minus branch are asymptotically Mincowski/dS/adS for zero/negative/positive α_0, respectively.
- Solutions in plus branch are asymptotically adS.
- Negative mass solutions have branch singularity.
- Solutions in plus branch have BH horizons for k=-1.

- For details see
TT & Maeda, PRD71, 124002 (2005)

Example 2: "M-theory" model

Curvature corrections to M-theory consist of Lovelock part and other terms which gives higher derivative equation of motion. Here we consider only the Lovelock part.

$$\alpha_1 > 0, \alpha_4 \neq 0, \ n = 10$$



- Minkowski and adS branch exist.
- For positive mass, Minkowski-branch solution has spacetime central singularity, while adS-branch solution has timelike one.
- For positive mass, properties of the branch singularity depends on $\tilde{k}$.
- Horizon
  - Minkowski branch, positive mass: There is one BEH for k=1.
  - Minkowski branch, negative mass: There is one CEH for k=-1.
  - adS branch, positive mass: There are at least 2 horizons for k=-1.
  - adS branch, positive mass: There is at least 1 horizon for k=-1.

NARIAI & BERTOTTI-ROBINSON SOLUTION

- gravitational equation

$$R(r) \equiv B^2 = const.$$

$\delta \equiv 0$ (gauge)

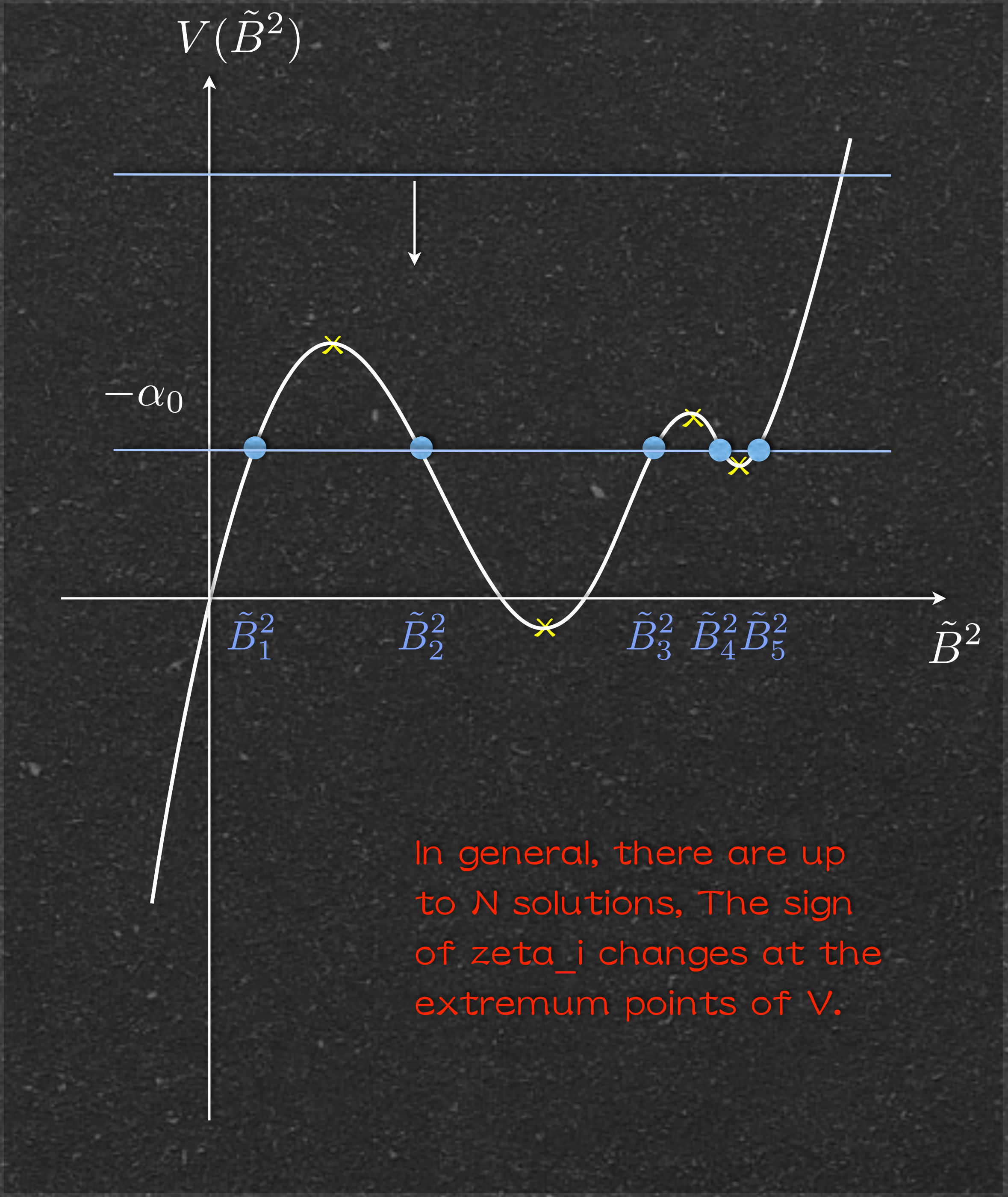
$$V(\tilde{B}^2) := \sum_{p=1}^{N} \alpha_p (k\tilde{B}^2)^p = -\alpha_0$$

$$f = 1 - \zeta_i r^2$$

$$\zeta_i := -\frac{\sum_{p=0}^{N} \alpha_p (n-2)_{2p+2} (k\tilde{B}_i^{-2})^p}{\sum_{p=0}^{N} \alpha_p (n-2)_{2p} (k\tilde{B}_i^{-2})^{p-1}}$$

$$\tilde{B} := B^{-1}$$

$$(n-p)_q = (n-p)(n-p-1)\cdots(n-q)$$



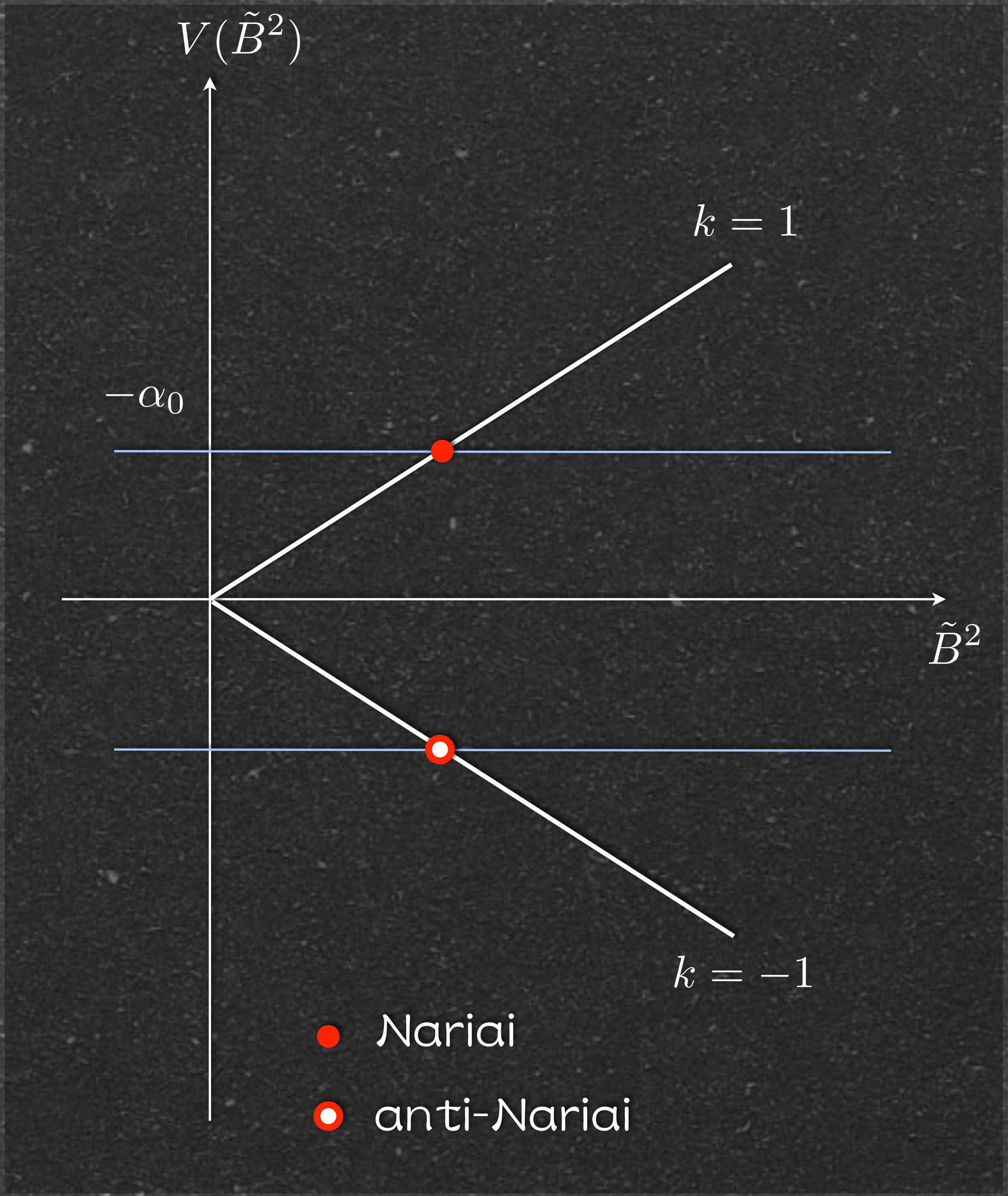
Example 1: general relativity

$\alpha_o \neq 0,\ \alpha_1 > 0$

$$f = 1 + \frac{2\alpha_o}{(n-2)\alpha_1} r^2$$

$$B^2 = -\frac{\alpha_1}{\alpha_0} k \quad \Longrightarrow \quad \alpha_0 k < 0$$

- k=1 and positive c.c. :
  - Nariai $(dS_2 \times S^{n-2})$
- k=-1 and negative c.c. :
  - anti-Nariai $(adS_2 \times H^{n-2})$



Example 2: Gauss-Bonnet gravity

$\alpha_o \neq 0,\ \alpha_1, \alpha_2 > 0$

$$V(\tilde{B}^2) = \alpha_1 k \tilde{B}^2 + \alpha_2 \tilde{B}^4$$

- Bertotti-Robinson solution exist for k=-1 case.
- These solution corresponds to extreme black hole solutions in the same system. There are certain coordinate transformation from them.