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"Gravitational field of a rotating ring around a Schwarzschild black hole"

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P19

GRAVITATIONAL FIELD OF A ROTATING RING AROUND A SCHWARZSCHILD BLACK HOLE

～ Using Hertz Potential ～

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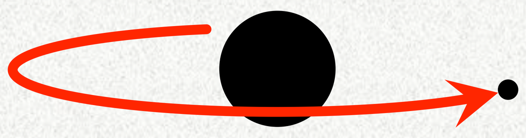
We calculated the perturbed space time metric of a Schwarzschild black hole (BH) and a rotating mass ring using Teukolsky equation and CCK (Chrzanowski, Cohen & Kegeles) formalism. We also visualized the result with tendex line and vortex line.

Introduction



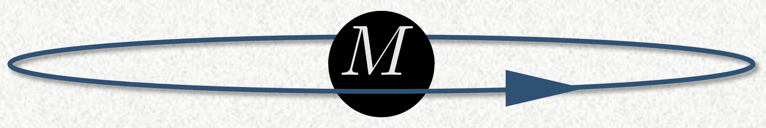
- Black hole perturbation (1st order)
  - mass ratio (source/BH) as the small parameter

$$g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h_{\alpha\beta}$$

BH metric ↑

- "Black hole + moving point mass" perturbed metric → Self-force, gravitational wave
  - For Kerr BH, it is very difficult to calculate $h_{\alpha\beta}$
- Illuminating the method available with Kerr spacetime

Setting

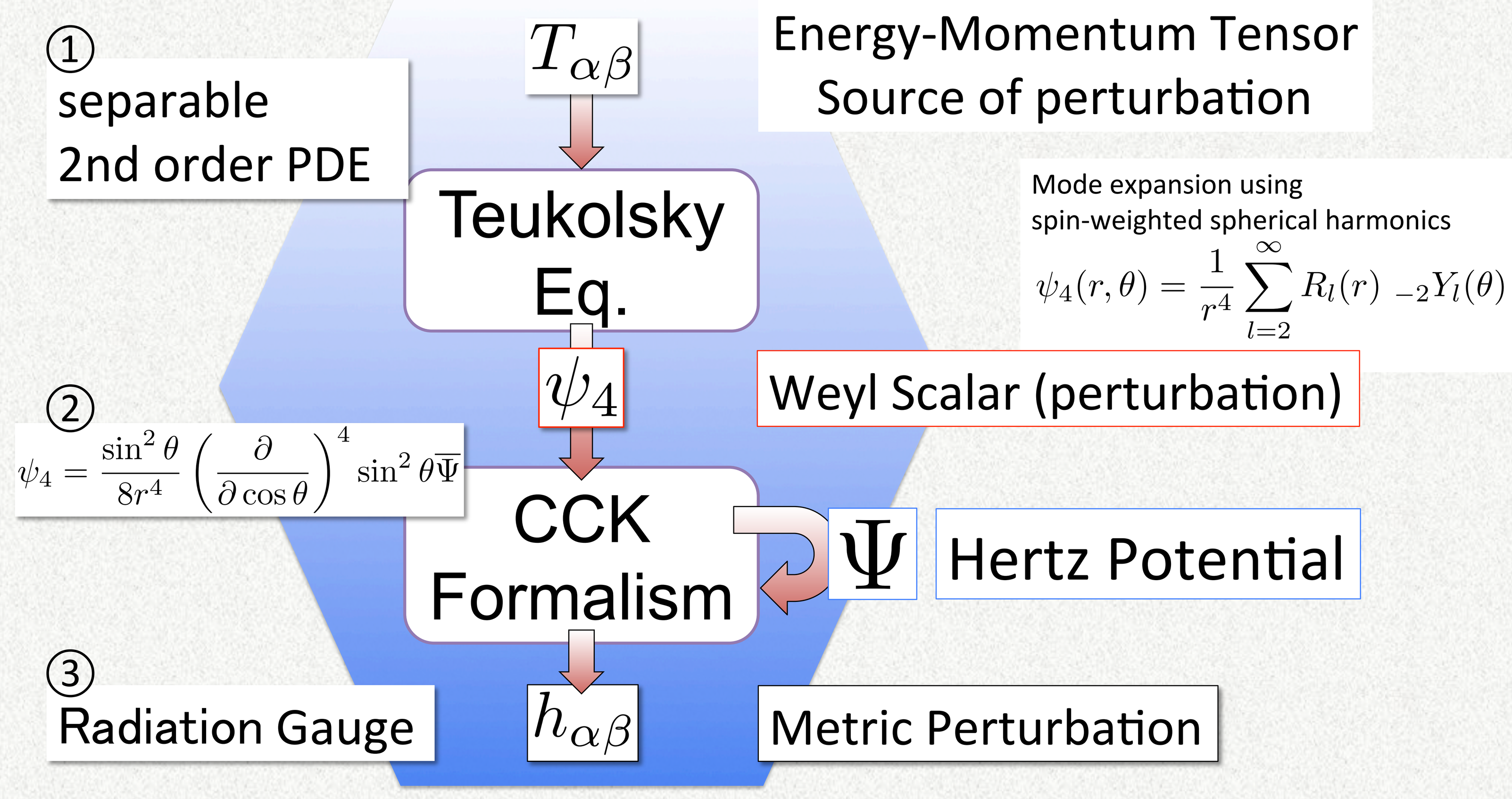


- Schwarzschild black hole + Rotating circular ring
  - Ring: a set of point particles in circular geodetic motion
  - Axisymmetric & steady problem
- Energy-momentum tensor of the ring ↓
  - $2\pi m$: rest mass
  - u^α: four velocity
  - r_0: radius

$$T^{\alpha\beta} = \frac{m u^\alpha u^\beta}{u^t {r_0}^2}\delta(r - r_0)\delta(\cos\theta)$$

Method

- Finding $h_{\alpha\beta}$ via Hertz potential Ψ



- Weyl scalar: Components of the Weyl tensor
  - 5 complex components (ψ_0, ψ_1, ψ_2, ψ_3, ψ_4)
- CCK: Chrzanowski ('75), Cohen & Kegeles ('79)
  In "Radiation gauge", the perturbation $h_{\alpha\beta}$ is given by 2nd order partial derivative of Hertz potential
  - Hertz potential satisfies source free Teukolsky Eq.

Hertz potential $\Psi = \Psi_P + \Psi_H$

- Assuming Hertz potential in the same mode expansion as Weyl scalar $(l \geq 2)$,
  Eq.② reduces to algebraic Eq. ➡ The particular solution Ψ_P obtained easily
  - HOWEVER, Ψ_P gives singular Weyl tensor field, that is not continuous at the ring radius
- Determination of the Homogeneous solution Ψ_H
  - Some of degrees of freedom in Ψ_H are physical parameters (mass and angular momentum) $(l = 0,\ 1$ mode)
    ➡ By computing the mass and angular momentum, one can determine those parameters

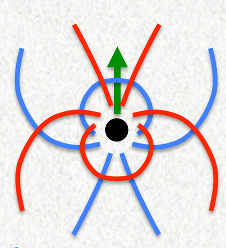
$$M_{\rm ring} = -2\pi m u_\alpha \xi^\alpha \qquad J_{\rm ring} = 2\pi m u_\alpha \psi^\alpha$$

$\xi^\alpha = \left(\frac{\partial}{\partial t}\right)^\alpha$ $\psi^\alpha = \left(\frac{\partial}{\partial \phi}\right)^\alpha$: Killing vectors

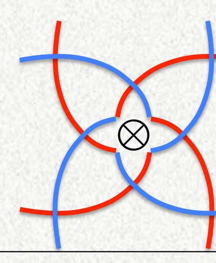
  - $J_{\rm ring}$ completed the imaginary parts of all Weyl scalars, cancelling the discontinuities
  - $M_{\rm ring}$, on the other hand, unexpectedly not cancelled the discontinuities of real parts of Weyl scalars

Visualizing (vortex line)

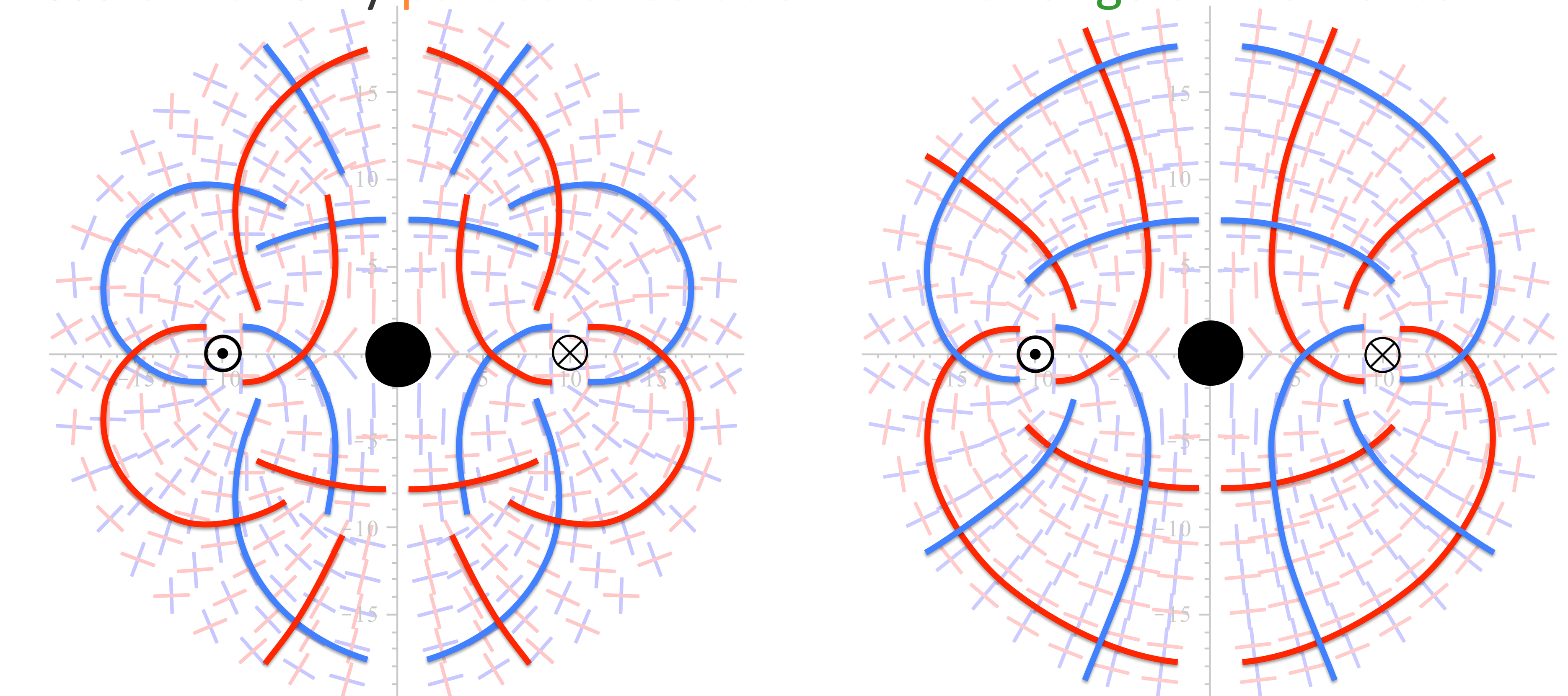
- A tensor representing frame-dragging effect
  - Weyl tensor $C_{\alpha\beta\gamma\delta}$ projected onto a 3 dimensional space $\mathcal{B}_{\hat{i}\hat{j}} = -\frac{1}{2}\epsilon_{\hat{i}\hat{p}\hat{q}}C_{\hat{0}\hat{j}}{}^{\hat{p}\hat{q}}$
- Draw integral curves of the eigen vector field
- Ex. 1: Kerr BH
  - Outward curves in polar region



- Ex. 2: Line mass flow
  - Spiral around the (See only blue or red)



Result with only particular solution | With angular momentum



Not continuous at the ring radius
Outward curves upside down in color

Spiral pattern around the ring
Continuous and interpretable