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"Gravitational lensing by modified lens gravity"

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Gravitational lensing by modified lens gravity

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Abstract: IS DEMAGNIFICATION AN EVIDENCE FOR WORMHOLES ??

1 Deflection angle of light in the modified space-time metric (inverce power form)

In weak field limit, We consider a modified space-time metric as

$$ds^2 = -(1 - \frac{\varepsilon_1}{r^n})dt^2 + (1 + \frac{\varepsilon_2}{r^n})dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi). \tag{1}$$

This metric is rewritten up to O(ε) by comformal transformation as

$$ds^2 = (\frac{1}{1 - \frac{\varepsilon_1}{r^n}})\tilde{ds}^2, \tag{2}$$

where

$$\tilde{ds}^2 \approx -dt^2 + (1 + \frac{\varepsilon}{R^n})dR^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{3}$$

with $R \equiv r^2/(1 - \frac{\varepsilon_1}{r^n})$ and $\varepsilon \equiv n\varepsilon_1 + \varepsilon_2$. We derivated deflection angle of light in this space-time as with method of schwartzchild case.

Derivated deflection angle is

$$\alpha = \frac{\varepsilon}{b^n} \cdot \int_0^{\frac{\pi}{2}} \cos^n\theta d\theta. \tag{4}$$

Especially, Equation (4) is rewritten as

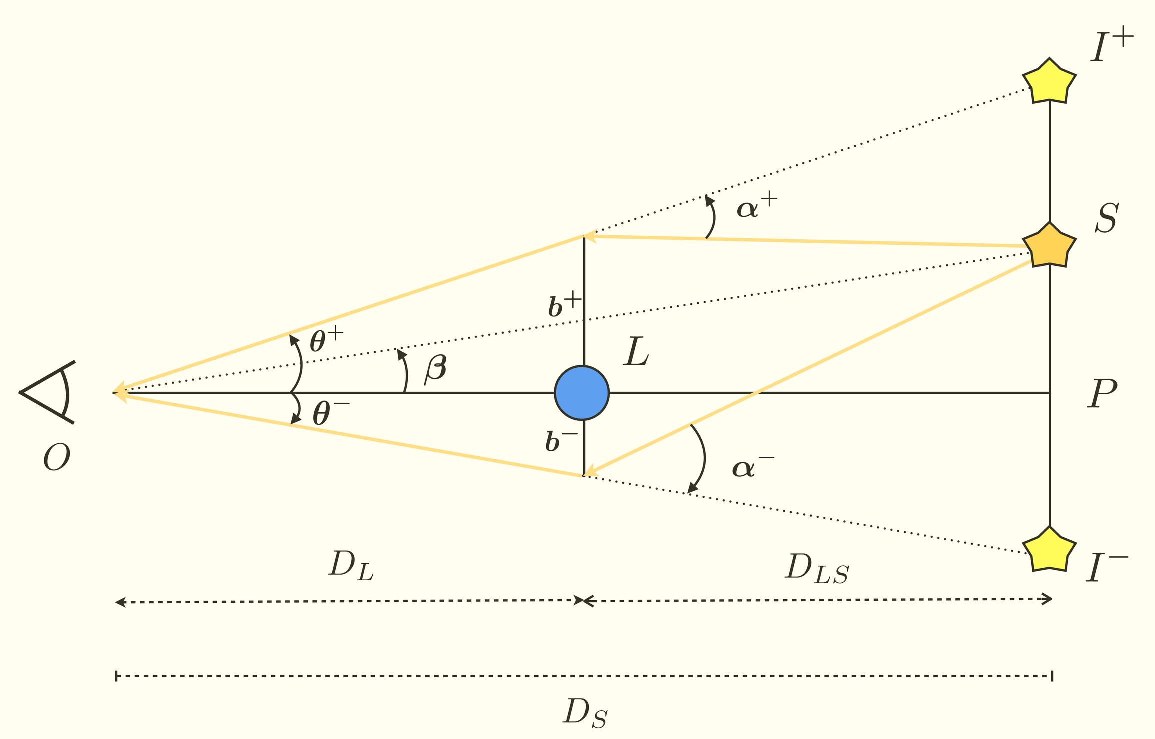
$$\alpha = \frac{\varepsilon}{b^n} \cdot \frac{(n-1)!!}{n!!}\frac{\pi}{2} \quad \text{(n is even)}, \tag{5}$$

$$\alpha = \frac{\varepsilon}{b^n} \cdot \frac{(n-1)!!}{n!!} \quad \text{(n is odd)}. \tag{6}$$

b is impact parameter.

The Equation (4) coinside with schwarzschild case with $n = 1$ and Ellis wormhole case with $n = 2$.

2 Modified lens equation



Lens equation for modified space-time is

$$\beta = \theta - \frac{1}{\theta^n} \quad (\theta > 0), \tag{7}$$

$$\beta = \theta + \frac{1}{(-\theta)^n} \quad (\theta < 0), \tag{8}$$

in the units of $\theta_E = \varepsilon^{\frac{1}{n+1}}$ that is Einstein ring radius with $\beta = 0$. We call this equation as **Modified lens equation**.

We want to obtain analytical solution for modified lens equations. But no fomula of solution for fifth-order(or higher) equation. Therefore, we solve it using **asymptotic expansion**.

Then we consider source object pass through nearzone in proportion with Einstein ring radius ($\beta \ll |\theta_E|$).For $\beta < 1$, Equation [7,8] are iteratively solved as

$$\theta_+ = 1 + \frac{1}{n+1}\beta + \frac{1}{2}\frac{n}{(n+1)^2}\beta^2 \quad (\theta > 0), \tag{9}$$

$$\theta_- = -1 + \frac{1}{n+1}\beta - \frac{1}{2}\frac{n}{(n+1)^2}\beta^2 \quad (\theta < 0). \tag{10}$$

3 Approximate prediction for amplification

Total amplification is

$$A \equiv \left|\frac{\theta_+}{\beta}\frac{d\theta_+}{d\beta}\right| + \left|\frac{\theta_-}{\beta}\frac{d\theta_-}{d\beta}\right|. \tag{11}$$

substitute Equations [9,10] in Equation (11):

$$A = \frac{1}{n+1}\frac{2}{\beta}. \tag{12}$$

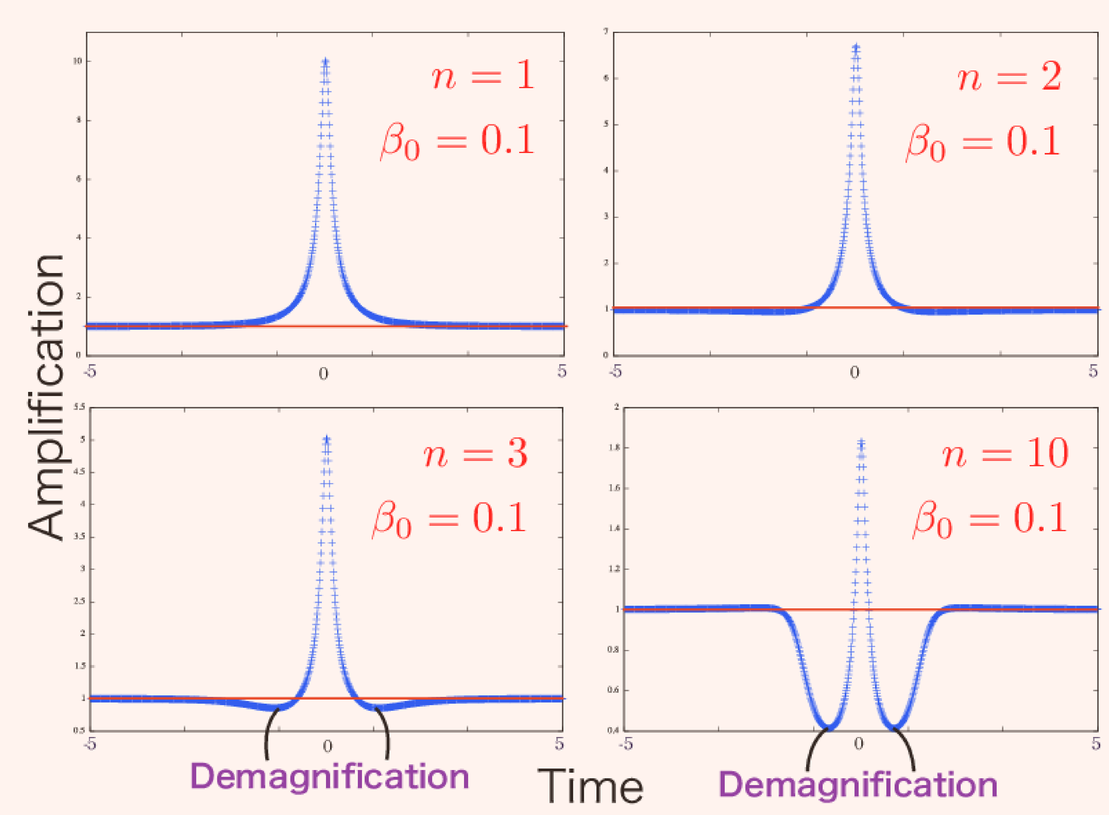
Thus, magnifying condition ($A > 1$) is rewritten as

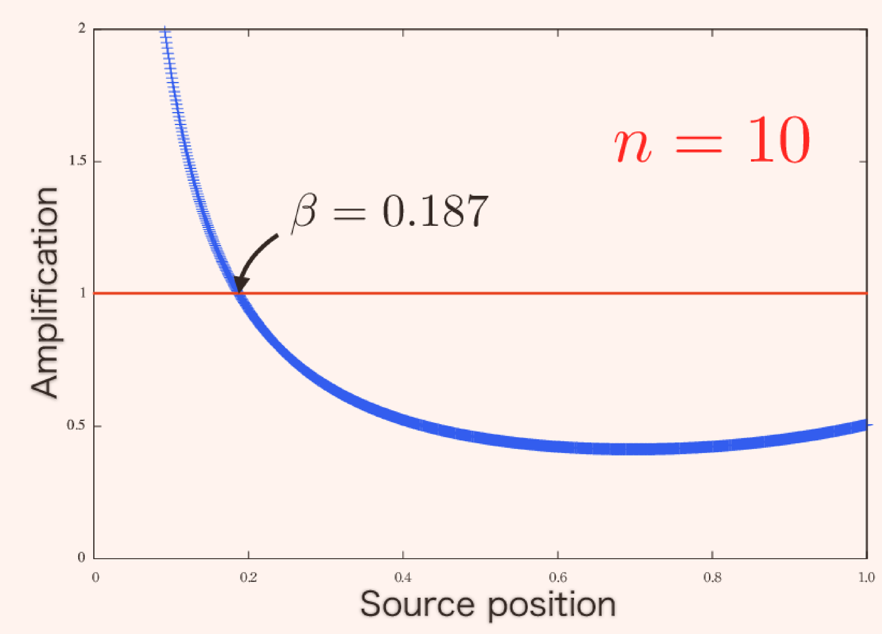
$$\frac{2}{n+1} > \beta. \tag{13}$$

The meaning of Equation (13) is changing from magnification to demagnification with $\beta = 2/(n+1)$.

e.g. $n = 10$, $\beta = 0.182$

4 Light curve by numerical calculations





Blue line is light curve, and **Red line** is the brightness of source. β_0 is the closest position from lens to source in units of θ_E.

Demagnified by **10 %, 60 %** from source brightness with $n = 3$, $n = 10$. The second figure shows β in the point which changes from magnification to demagnification is 0.187, this value is close to value of approximate prediction for foregoing section.

5 Conclusion

- **We obtained light curve for modified space-time metric**
- **Demagnification is an evidance EWH. But not always prove it !!**

Future work...

- Applying to modified gravity and exotic matter
- Mechanism of demagnification

References

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