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"Ghost in multimetric gravity"

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Ghost in Multimetric Gravity

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1. Introduction

Recently, the ghost problem in non-linear massive gravity has been solved.

Massive gravity : a challenge to give mass to graviton in general relativity

linear level : Pauli-Fiertz mass term ・・・ no problem

non linear level : a serious difficulty emerges

graviton has spin 2 ・・・ mssless : 2 degrees of freedom

massive : 5 degrees of freedom

In linear level, there are certainly 5 degrees of freedom.

However, in non-linear level, extra 6^{th} degree of freedom emerges and turns out to be a ghost particle.

→ **BD ghost**

ghost particle : a particle having negative kinetic energy

For the consistency of the theory, ghost particles must be excluded.

Extension to bimetric gravity

In non-linear massive gravity, metric is always decomposed into backgroundand and fluctuation.

↓ The background metric can be promoted to a dynamical variable.

a theory of two dynamical metrics

It has been shown that no BD ghost is contained.

Question

Can we construct ghost-free theories with more than two metrics？

①：Trimetric case as a naïve extension of bimetric gravity is proposed. (2012, N. Khosravi et al)

⇒ The problem of BD ghost is unsolved.

②：Ghost-free multimetric gravity with vielbeins is proposed. (2012, K. Hinterbichler and R. A. Rosen)

⇒ ② dose not contain ①.

⇒ We solve the ghost problem in ①, and further develop our analysis to more general multimetric gravity.

2. Bimetric gravity g, f : metric

$$S_{bi} = M_g^2 \int d^4x \sqrt{-\det g}\, R[g] + M_f^2 \int d^4x \sqrt{-\det f}\, R[f]$$

Einstein Hilbert term

$$+ a \int d^4x \sqrt{-\det g}\, \Phi\left(\sqrt{g^{-1} f}\right)$$

Interaction term

M_g, M_f : Planck mass , $R[g], R[f]$: curvature , a : coupling constant

$\Phi(A)$: linear combination of $Tr(A)$, $(TrA)^2 - Tr(A^2)$,....

(up to fourth order in A)

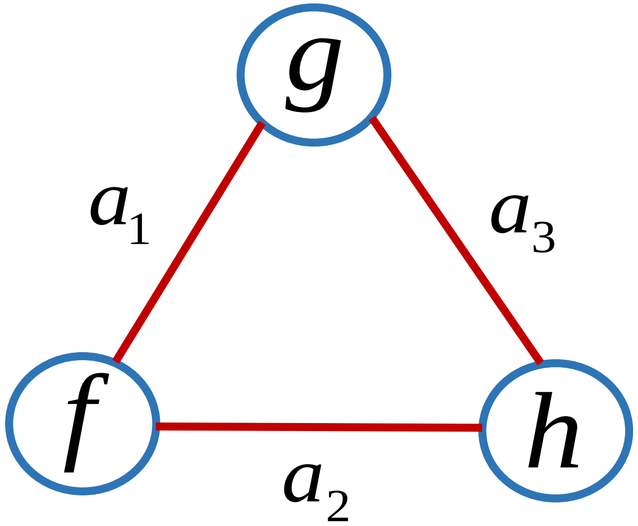
No BD-ghost is contained,
and there exist one massless and one massive graviton.
→ degrees of freedom : 2 + 5 = 7

3. Trimetric gravity g, f, h: metric

$$S_{tri} = M_g^2 \int d^4x \sqrt{-\det g}\, R[g] + M_f^2 \int d^4x \sqrt{-\det f}\, R[f] + M_h^2 \int d^4x \sqrt{-\det h}\, R[h]$$
$$+ a_1 \int \sqrt{-\det g}\, \Phi_1\left(\sqrt{g^{-1} f}\right) + a_2 \int \sqrt{-\det f}\, \Phi_2\left(\sqrt{f^{-1} h}\right) + a_3 \int \sqrt{-\det h}\, \Phi_3\left(\sqrt{h^{-1} g}\right)$$

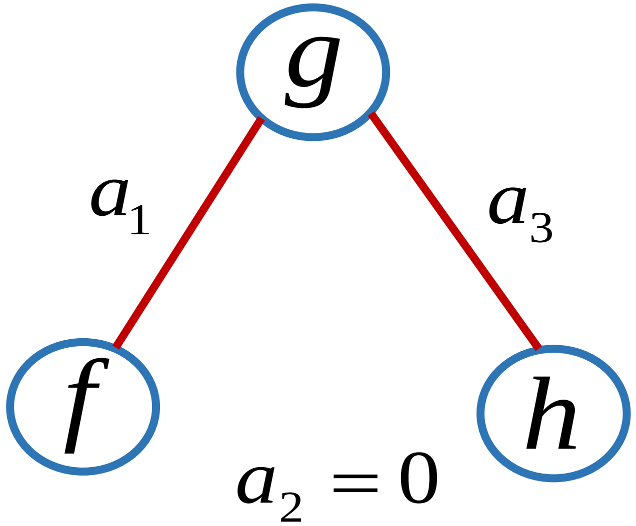
a nive extension of bimetric case

In this case, we verified the exsistence of a BD-ghost.



cutting one of interactions

Ghost-freedom is already shown in the vielbein formalism.



We perform the Hamiltonian analysis in homogeneous model.

$$g_{\mu\nu}dx^\mu dx^\nu = -N(t)^2 dt^2 + \gamma(t)_{ij} dx^i dx^j$$

$$f_{\mu\nu}dx^\mu dx^\nu = -L(t)^2 dt^2 + \omega(t)_{ij} dx^i dx^j$$

$$h_{\mu\nu}dx^\mu dx^\nu = -Q(t)^2 dt^2 + \rho(t)_{ij} dx^i dx^j$$

One of the metrics can be diagonalized through spacial rotations.

Hamiltonian : $H = NC_N + LC_L + QC_Q$

$N, L, Q:$ Laglange multipliers

Constraints : $C_N = 0, C_L = 0, C_Q = 0$

Degrees of freedom : $3+6+6=15$

Constraints must be preserved in the time evolution.

$$\dot{C}_N = \{C_N, H\} = \{C_N, C_L\}L + \{C_N, C_Q\}Q \approx 0$$

$$\dot{C}_L = \{C_L, H\} = \{C_L, C_N\}N + \{C_L, C_Q\}Q \approx 0$$

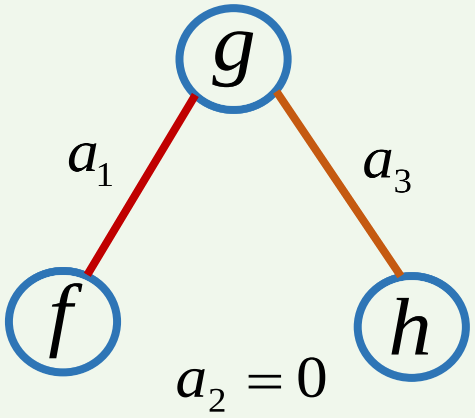
$$\dot{C}_Q = \{C_Q, H\} = \{C_Q, C_N\}N + \{C_Q, C_L\}L \approx 0$$

$$\{C_N, C_L\} \propto a_1, \quad \{C_L, C_Q\} \propto a_2, \quad \{C_N, C_Q\} \propto a_3$$

$$\begin{pmatrix} 0 & a_1 & a_3 \\ -a_1 & 0 & a_2 \\ -a_3 & -a_2 & 0 \end{pmatrix} \begin{pmatrix} N \\ L \\ Q \end{pmatrix} = 0$$

We must determine the Lagrange multipliers to satisfy this condition.

Case 1:



$$\begin{pmatrix} 0 & a_1 & a_3 \\ -a_1 & 0 & 0 \\ -a_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} N \\ L \\ Q \end{pmatrix} = 0$$

Secondary Constraints $a_1 \propto \{C_N, C_L\} \approx 0, \quad a_3 \propto \{C_N, C_Q\} \approx 0$

consistency in the time evolution

$$d\{C_N, C_L\}/dt = NC_{NL,N} + LC_{NL,L} + QC_{NL,Q} \approx 0$$

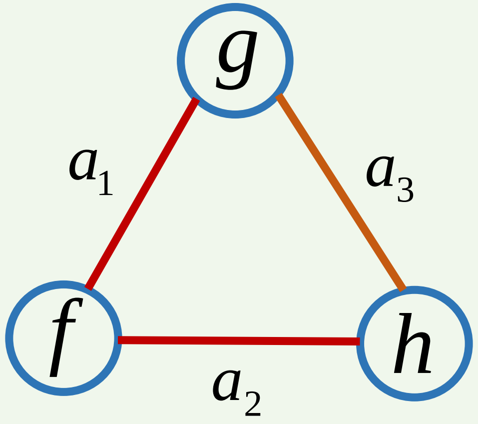
$$d\{C_N, C_Q\}/dt = NC_{NQ,N} + LC_{NQ,L} + QC_{NQ,Q} \approx 0$$

- One of the Lagrange multipliers is left undetermined. (one gauge freedom)
- The total number of constraints is five.

Total degrees of freedom : $(15 \times 2 - 5 - 1)/2 = 12$

- Masless graviton : 1
- Massive graviton : 2
- BD ghost : 0

Case 2:



$$\begin{pmatrix} 0 & a_1 & a_3 \\ -a_1 & 0 & a_2 \\ -a_3 & -a_2 & 0 \end{pmatrix} \begin{pmatrix} N \\ L \\ Q \end{pmatrix} = 0$$

No seconday constraint

$$N = \frac{a_2}{a_1} Q, \quad L = -\frac{a_3}{a_1} Q, \qquad Q : \text{ arbitrary}$$

Constraints : 3
Gauge freedom : 1

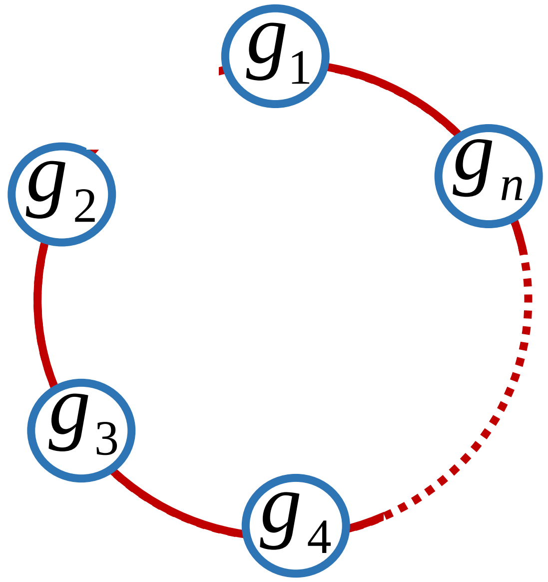
Total degrees of freedom : $(15 \times 2 - 3 - 1)/2 = 13$

- Masless graviton : 1
- Massive graviton : 2
- BD ghost : 1

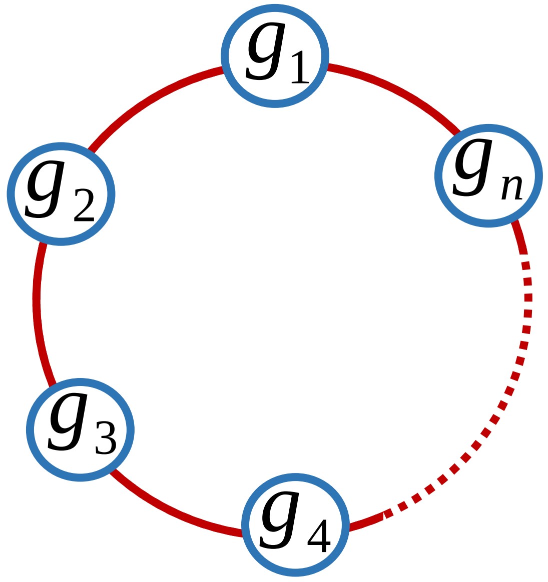
4. More general Multimetric gravity

$g_1, g_2, \ldots, g_n$: metrics

$$S_n = \sum_{k=1}^{n} M_k^2 \int d^4x \sqrt{-\det g_k}\, R[g_k] + \sum_{k=1}^{n} a_k \int d^4x \sqrt{-\det g_k}\, \Phi_k\left(\sqrt{g_k^{-1} g_{k+1}}\right)$$



BD ghost : 0



BD ghost : $(n-1)/2$ 個 (n:odd）

$(n-2)/2$ 個 (n:even）

5. summary

・ We showed that there exists a BD-ghost in the trimetric gravity as a naive extension of the recently proposed ghost-free bimetric gravity.

・ We also studied more general multi metric gravity, and showed there always exist BD-ghosts if loop type interactions are contained.