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Excess power method with the Hilbert-Huang transform in search for gravitational wave signals

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Abstract

We investigate the application of the Hilbert-Huang transform (HHT) to search for gravitational waves. The HHT is a novel and adaptive approach to time-series analysis. The HHT consists of an empirical mode decomposition and Hilbert spectral analysis. It can be applied to the analysis of gravitational-wave bursts, because this analysis does not assume the waveform. Thus, in this paper, we propose the excess power method with the HHT and estimate the detection efficiency of the proposed method.

1 Introduction

The Hilbert-Huang transform (HHT) [1, 2, 3] is a novel analysis of time series data which contain physical oscillatory modes. It is used to detect the signal in the noise and characterize physical oscillatory modes. The HHT consists of an empirical mode decomposition (EMD), followed by a Hilbert spectral analysis (HSA). It has a higher resolution of time-frequency than traditional analysis methods, because the EMD is an adaptive time-frequency decomposition. Thus, the HHT can be applied to non-linear and non-stationary time series data. On the other hand, traditional analysis methods such as the Fourier transform and wavelet transform also assume that the data is linear and stationary.

The HHT has been applied to various fields; biomedical engineering, financial engineering, image processing, seismic studies, ocean engineering, etc.

In this paper, we investigate the excess power method with the HHT to detect gravitational-wave bursts. This analysis does not require any knowledge of gravitational wave, for example, waveform etc. We also estimate the detection efficiency of the proposed method.

2 Hilbert spectral analysis and empirical mode decomposition

The Hilbert transform of function $u(t)$ is defined by

$$v(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{u(t')}{t-t'} dt', \tag{1}$$

where P indicates the Cauchy principal value. If the function $u(t) \in L^p(\mathbf{R})$ for $1 < p < \infty$, $v(t)$ is imaginary part of analytic function $F(t) = u(t) + iv(t) = a(t)e^{i\theta(t)}$. Then, an instantaneous amplitude (IA) $a(t)$ and instantaneous frequency (IF) $f(t)$ is defined by

$$a(t) = \sqrt{u^2(t) + v^2(t)}, \tag{2}$$

$$\theta(t) = \tan^{-1}\Big(\frac{v(t)}{u(t)}\Big), \tag{3}$$

$$f(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}. \tag{4}$$

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Generally, it is not guaranteed that we obtain the physically meaningful IF. In order to obtain the meaningful IF, Huang et al. [2] showed that the data $u(t)$ should be satisfy the following two conditions: (I) A number of extrema and number of zero crossing equal or differ at most by one, (II) The mean value of the envelope defined using the local maxima and the envelope defined using the local minima is zero.

To satisfy the above conditions, we perform the EMD which is a series of high-pass filters in a sense. The EMD decomposes the data $u(t)$ to intrinsic mode functions (IMFs) and residual $r(t)$,

$$u(t) = \sum_i \mathrm{IMF}i + r(t). \tag{5}$$

Then, each IMF has a locally monochromatic frequency scale that is obtained empirically. The EMD algorithm is as follows:

- $h_1(t) = u(t)$
- for $i = 1$ to $i_{\max}$
  - ▷ $h_{i,1}(t) = h_i(t)$
  - ▷ for $k = 1$ to $k_{\max}$
    - ◦ Identify the local maxima and minima of $h_{i,k}(t)$
    - ◦ $U_{i,k}(t)$ = the upper envelope joining the local maxima using a cubic spline
    - ◦ $L_{i,k}(t)$ = the lower envelope joining the local minima using a cubic spline
    - ◦ $m_{i,k}(t) = (U_{i,k}(t) + L_{i,k}(t))/2$
    - ◦ $h_{i,k+1}(t) = h_{i,k}(t) - m_{i,k}(t)$

    Exit from the loop k if a stoppage criterion $\dfrac{\sum_j |m_{i,k}(t_j)|}{\sum_j |h_{i,k}(t_j)|} < \epsilon$
  - ▷ $\mathrm{IMF}i = c_i(t) = h_{i,k}(t)$
  - ▷ $h_{i+1}(t) = h_i(t) - c_i(t)$
- residual: $r(t) = h_{i_{\max}+1}(t)$

The EMD frequently occurs mode mixing which is defined as a single IMF either consisting of signals of widely desperate scales or a signal of a similar scale residing in different IMF components. To suppress mode mixing, we perform the ensemble EMD (EEMD) [4],
(1) Add a white noise series to data,

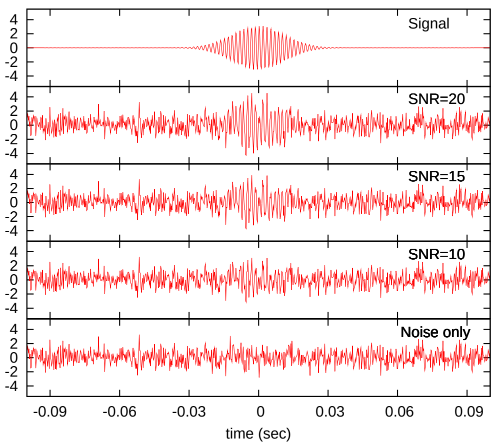

Figure 1: The sine-Gauss signal + the white-Gaussian noise

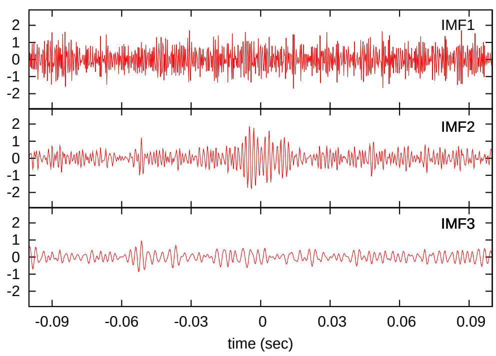

Figure 2: EEMD of the sine-Gauss signal in the white-Gaussian noise with SNR = 10

(2) Decompose the data with added white noise into IMFs,
(3) Repeat step (1) and (2) in many times with different white noise series each time,
(4) Obtain the ensemble means of corresponding IMFs of the decompositions as the final result.

To perform the EMD, we must be set the empirical EMD parameters which depend on the characteristic of the data [5]. End-of-loop-i condition is that residual becomes monotonic functions. We set the stoppage criterion $\epsilon = 10^{-4}$, consequently, k_{max} is almost equal to or lower than 100 in our simulations. The parameters for the EEMD are the size of ensemble N_e and standard deviation of white noise σ_e. We set $N_e = 200$ and $\sigma_e = 1.0$. We tried other values of σ_e but we found that this values is optimal. As for N_e, we verified that the results hardly change even with $N_e > 100$ but the value $N_e = 50$ is too small.

We apply the HHT to the sine-Gauss signal

$$h(t) = a\exp[-(t/\tau)^2]\sin(2\pi f t). \tag{6}$$

We set frequency of signal $f = 600$ Hz and $\tau = 0.1$ sec$/2\pi = 0.016$ sec. Moreover, we use a white-Gaussian noise with zero mean $\mu = 0$ and standard deviation $\sigma = 1.0$. Figure 1 plots the data used in our simulations. The signal-to-noise ratio (SNR) is defined by

$$\mathrm{SNR} = \sqrt{\sum_j h^2(t_j)}/\sigma. \tag{7}$$

The sampling frequency and data length is 4096 Hz and 1.0 sec, respectively. The results of EMD are showed in Fig. 2. The signal appears only in IMF2, because the sine-Gauss signal used in this case has a single frequency component.

3 Excess power methods

We propose an excess power method with the HHT to detect gravitational-wave bursts. If the IA > IA$_c$ for a duration δt_c in some IMF, then we define the candidate of the detection for gravitational-wave bursts. Figure 3 plots the IAs for each IMF1, 2 and 3. The blue line represent the maxinum IA of noise only data. We evaluate the detection efficiency by using the receiver operating characteristics (ROC) curve, which shows the detection rate (DR) as a function of the false alarm rate (FAR). We perform the EEMD procedure for 1000 samples of each data which generated noise with a different seed.

Figure 4 plots the ROC curves for IMF1, 2 and 3 ($\delta t_c = 0$ msec). In IMF1 and IMF3, it does not detect gravtational-wave bursts for SNR = 10 and 15. We found that we obtatined the high detection efficiency in IMF2 : DR > 0.95 and FAR < 0.05 for SNR = 10.

Figure 5 plots the ROC curves of IMF2 for each SNR = 10, 15 and 20 and different δt_c. For SNR = 15 and 20, we obtain better detection efficiency for any δt_c. When $\delta t_c = 4$ msec, we find DR > 0.97 and FAR < 0.05 for SNR = 10.

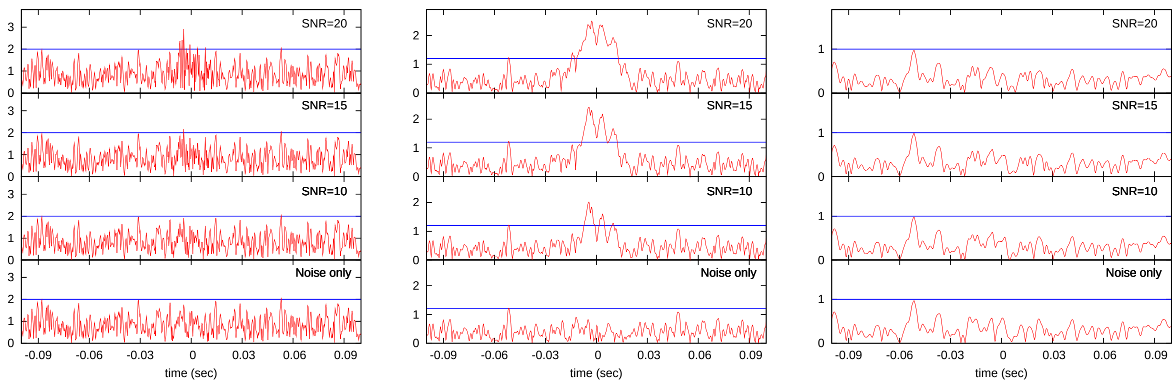

Figure 3: The IAs for each IMF1 (left), 2 (center), 3 (right)

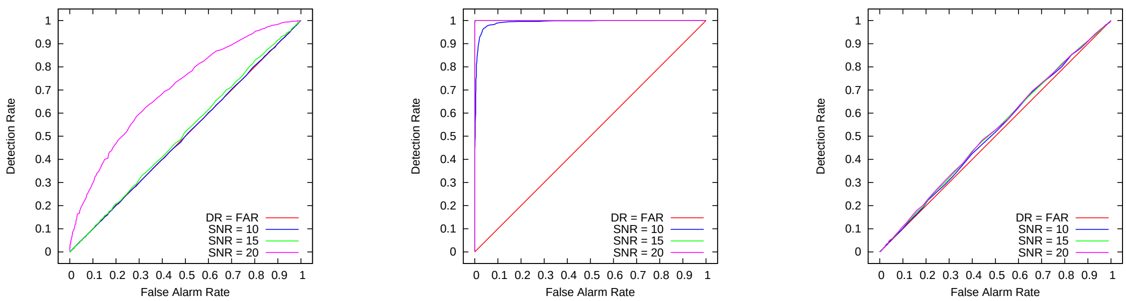

Figure 4: The ROC curves for each IMF1 (left), 2 (center), 3 (right)

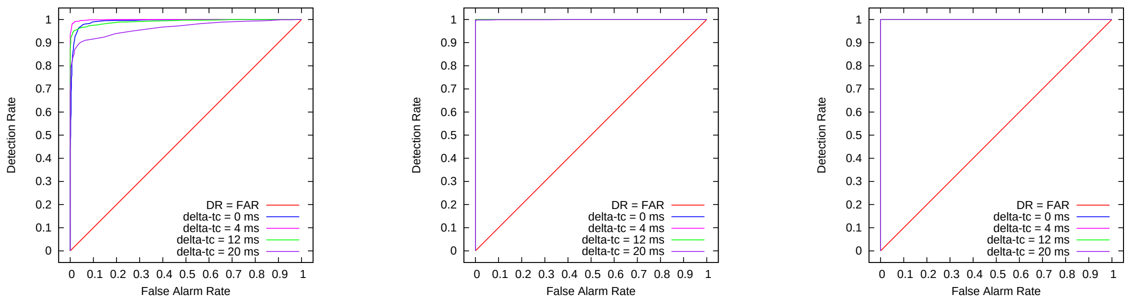

Figure 5: The ROC curves of IMF2 for each SNR = 10 (left), 15 (center), 20 (right)

4 Conclusion

We investigated the property of excess power method with the HHT for gravitational-wave bursts. We found the high detection efficiency only in IMF2. The signal almost appeared in IMF2, since the sine-Gauss signal used this paper had a single frequency component ($f = 600$ Hz).

Even if SNR $= 10$, we obtained DR > 0.95 and the FAR < 0.05. Moreover, when the duration time was $\delta t_c = 4$ msec, DR and FAR were better than that of $\delta t_c = 0$ msec.

We used the signal which had constant frequency. However, the real gravitational waves have complicate and large-scale frequency modulation. Since we expect to appear the signal in multiple IMFs, it is possible for us to obtain the better detection efficiency. We will report elsewhere.

Acknowledgments

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