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# Quantum Interferometry in Chern-Simons modified gravity



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## Abstract

Possible effects of Chern-Simons (CS) gravity on a quantum interferometer turn out to be dependent on the latitude and direction of the interferometer on the Earth in orbital motion around the Sun.

## 1 Chern-Simons (CS) gravity

CS gravity modifies GR via the addition of a correction

$$S_{CS} = \frac{1}{16\pi G} \int d^4x \frac{1}{4} f R^* R, \quad (1)$$

Following [1], let us consider a system of nearly spherical bodies in the standard PPN point-particle approximation. The CS correction to the metric becomes

$$\delta_{CS} g_{0i} = \frac{2G}{c^3} \sum_A \frac{\dot{f}}{r_A} \left[ \frac{m_A}{r_A} (\vec{v}_A \times \vec{n}_A)^i - \frac{J_A^i}{2r_A^2} + \frac{3}{2} \frac{(\vec{J}_A \cdot \vec{n}_A)}{r_A^2} n_A^i \right]. \quad (2)$$

## 2 Phase shifts in a quantum interferometer

We consider a quantum interferometer that consists of a closed path  $C$  (its area  $S$ ) on the Earth, as shown by Fig 1.  $\Delta$  is a phase difference induced by  $g_{0i}$ . By using Stokes theorem,  $\Delta$  is rewritten in the surface integral form over  $S$

$$\Delta = \frac{mc}{\hbar} \oint_C \vec{g} \cdot d\vec{r} = \frac{mc}{\hbar} \int_S (\vec{\nabla} \times \vec{g}). \quad (3)$$

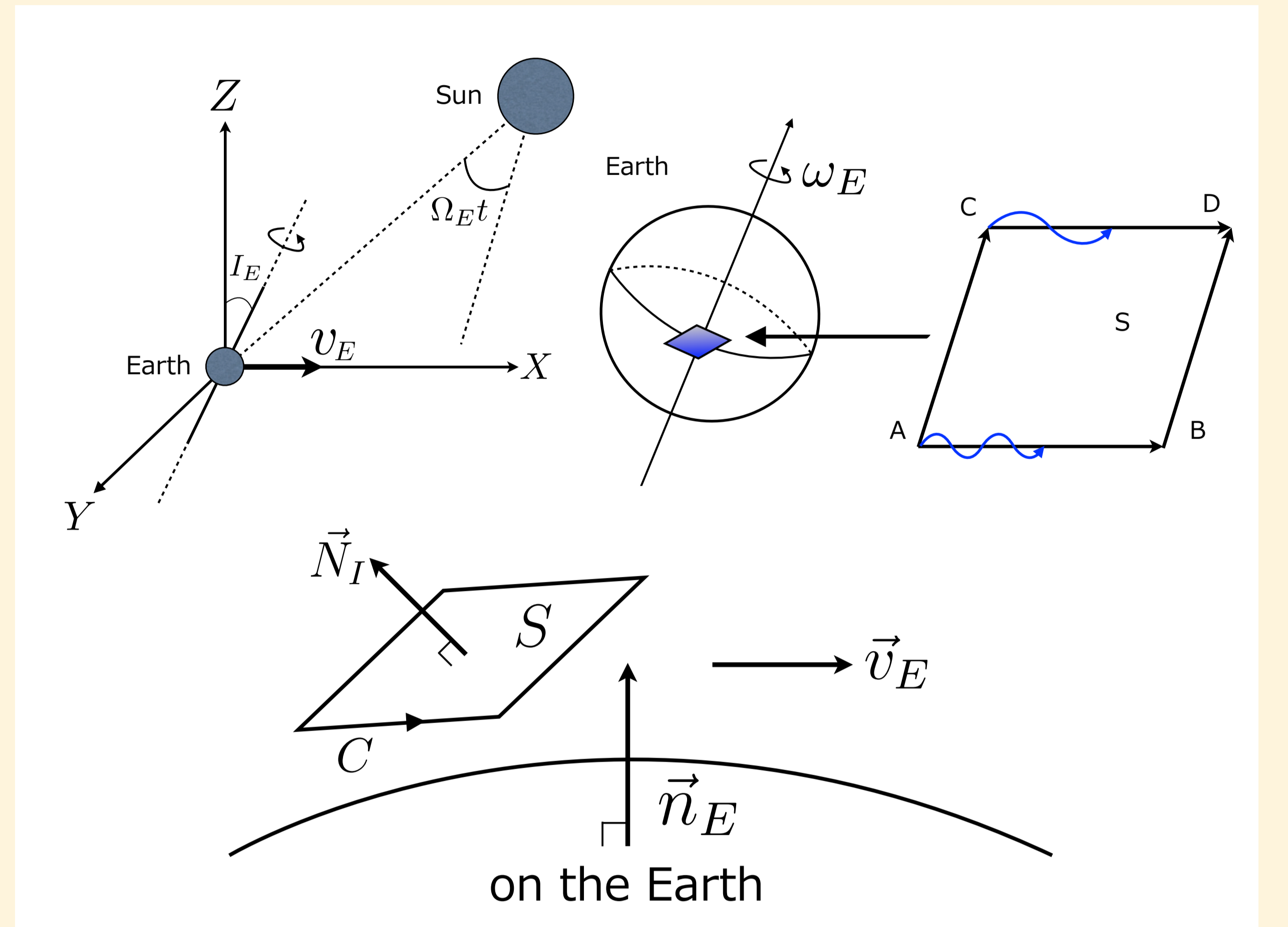


Figure 1: Quantum interferometer on the Earth orbiting around the Sun.

## 3 Phase shifts for Chern-Simons (CS) gravity

Let us substitute the CS term of Eq. (2) into Eq. (3) to obtain  $\Delta$  for CS gravity. We focus on the Earth mass in CS gravity and use  $r_E \gg \sqrt{S}$ . Hence,

$$\Delta_{CS} = 2\dot{f} \frac{mGM_E S}{\hbar c^2 r_E^3} \tilde{\Delta}_{CS}, \quad (4)$$

$$\tilde{\Delta}_{CS} = [3(\vec{v}_E \cdot \vec{n}_E) \vec{n}_E - \vec{v}_E] \cdot \vec{N}_I. \quad (5)$$

Eq. (5) depends on the latitude and direction, and changes with the Earth's spin and orbital motion.

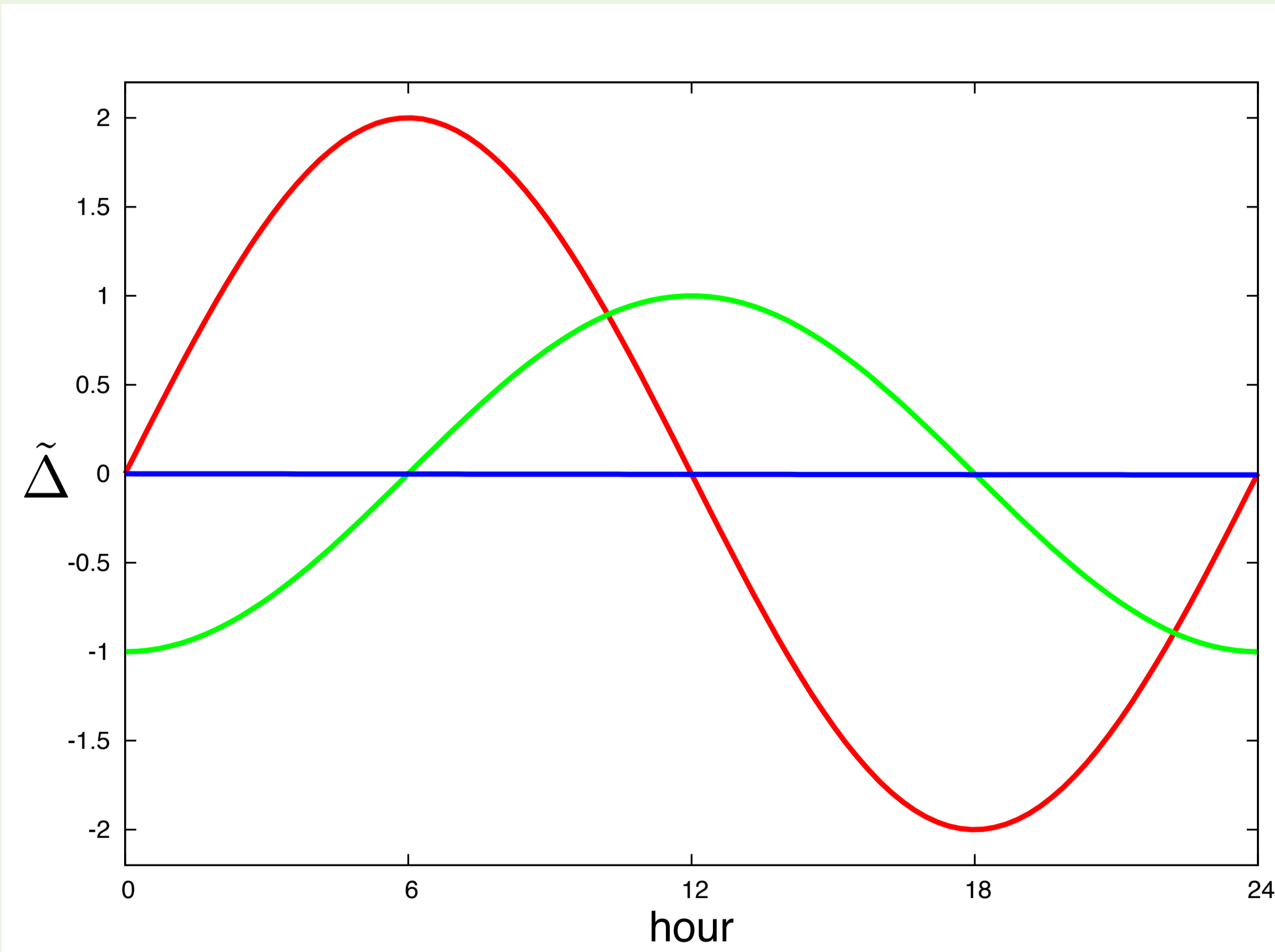


Figure 2: Daily variation in phase differences by CS effects. The red, green, and blue curves correspond to  $\vec{N}_I$  for a horizontal plane and two vertical ones (one facing the East and the other facing the North), respectively.

## 4 Time variation and the latitude

By using the coordinate rotation  $\vec{N}_I(t) = R(\omega_E t) \vec{N}_{I0}$ ,  $\vec{n}_E(t) = R(\omega_E t) \vec{n}_{E0}$ ,  $\tilde{\Delta}_{CS}$  is rewritten in the rotating matrix  $R(t)$

$$\tilde{\Delta}_{CS} = (R(t)^{-1} \vec{v}_E)^T [3(\vec{n}_{E0} \cdot \vec{N}_{I0}) \vec{n}_{E0} - \vec{N}_{I0}], \quad (6)$$

$$R(t)^{-1} \vec{v}_E = \{R(\phi)\}^{-1} \{R(\omega_E t)\}^{-1} \{R(\Omega_E t)\}^{-1} R(I_E) R(\Omega_E t) \begin{pmatrix} v_E \\ 0 \\ 0 \end{pmatrix}. \quad (7)$$

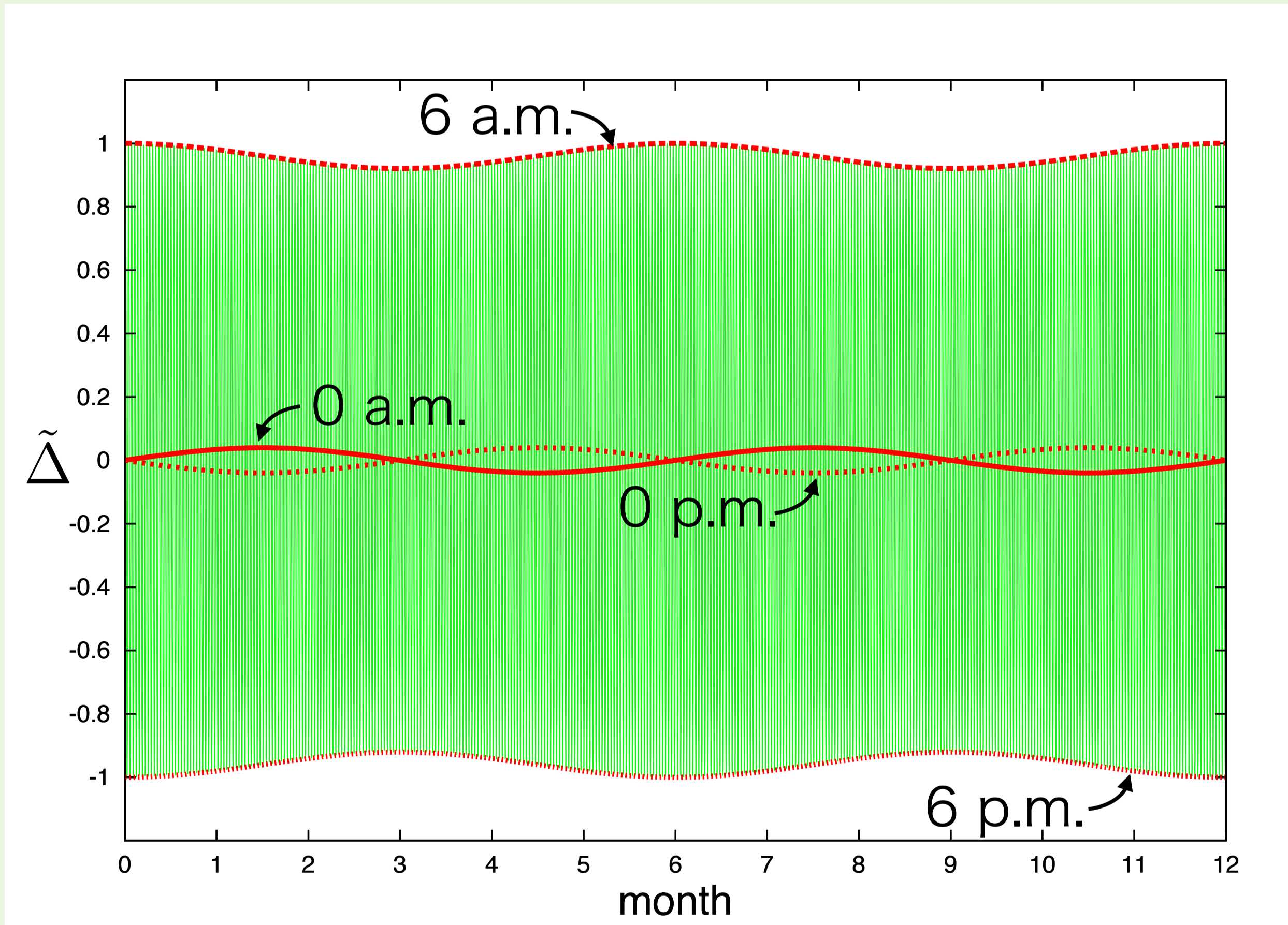


Figure 3: Seasonal variation in phase differences by CS effects. The green solid is full data points.

## 5 Possible constraint on $\dot{f}$

$\dot{f}$  induces the phase shift

$$|\Delta_{CS}| \sim 10^{-3} \text{s}^{-1} \times \left( \frac{mc^2}{1\text{GeV}} \right) \left( \frac{\dot{f}}{c} \right) \left( \frac{S}{0.4\text{m}^2} \right). \quad (8)$$

Current phase measurement accuracy at  $O(10^{-3}) \rightarrow \dot{f} c^{-1} < 10^0 \text{s}$  bound.  
GPB (Gravity Probe B), LAGEOS space mission  $\rightarrow \dot{f} c^{-1} < 10^{-3} \text{s}$ .

## 6 Conclusion

We considered effects of CS gravity on a quantum interferometer.

- Daily and seasonal variations in phase shifts are predicted with an estimate of the size of the effects.
- Neutron interferometry with  $\sim 5$  meters arm length and  $\sim 10^{-4}$  phase measurement accuracy would place a bound on a CS parameter comparable to Gravity Probe B satellite [2].

## References

- [1] S. Alexander and N. Yunes, Phys. Rev. Lett. **99**, 241101 (2007). [2] H. Okawara, K. Yamada, H. Asada, accepted Phys. Rev. Lett. (2012) (arXiv: 1210.4628).