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“Non-inertial effects on Landau levels”

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Non-Inertial Effects on Landau Levels

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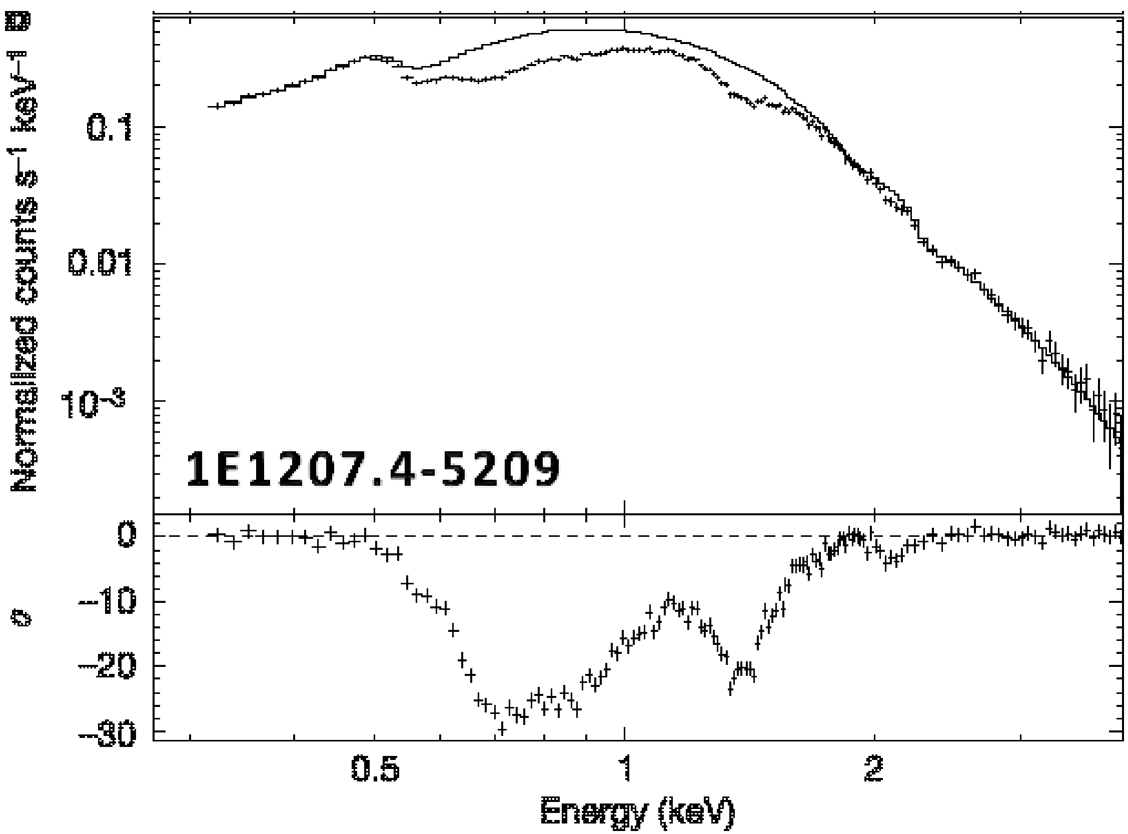
K. Konno and R. Takahashi
"Space-time Rotation-Induced Landau Quantization"
Phys. Rev. D 85, 061502(R) (2012)
(arXiv:1201.5188)

Landau levels on a neutron star surface

e.g. G.F. Bignami et al., Nature 423, 725 (2003);
F. Haberl, Ap&SS 308, 181 (2007).



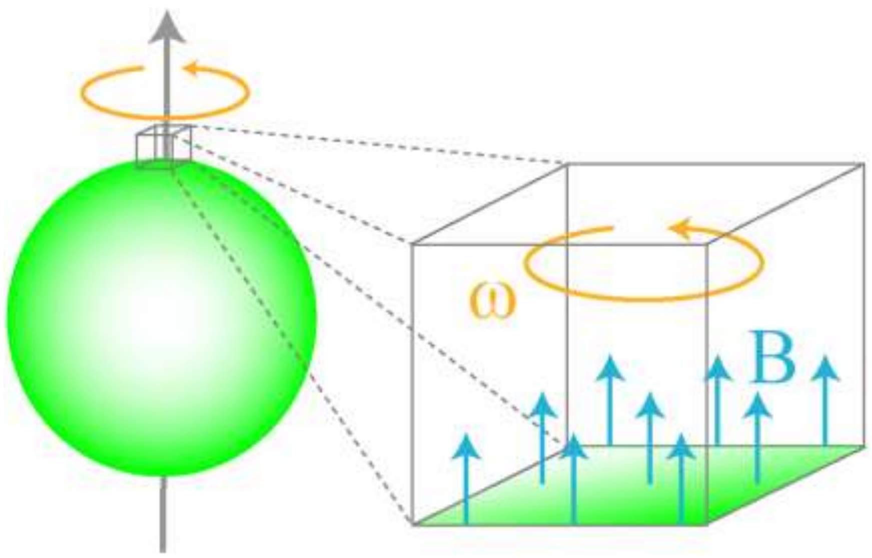
Other objects：

- RX J1605.3+3249
- RX J0720.4-3125
- RBS 1223
- RBS 1774
- etc

Quantum phenomena under strong gravity

Objective of our work

We consider quantum systems on the surface of a relativistic star to investigate quantum phenomena in the presence of both gravitational field and electromagnetic field.



Assumptions:

- The star is rotating.
- The star has an axisymmetric magnetic field
- The rotational axis is aligned with the magnetic axis.
- Uniform magnetic field in the polar region

Our approach

Covariant Klein-Gordon equation

$$\left[g^{\mu\nu} \left(\nabla_\mu - \frac{ie}{\hbar} A_\mu \right) \left(\nabla_\nu - \frac{ie}{\hbar} A_\nu \right) + \frac{m^2 c^2}{\hbar^2} \right] \phi = 0$$

Neglecting intrinsic spin

Post-Newton expansion

$$\phi(x) = \Psi(x) \exp\left[-i \left(mc^2 / \hbar \right) t \right]$$

Schrodinger equation with relativistic corrections

Approximations

- **Slow rotation** ··· Kerr parameter: $\tilde{a} \ll 1$
- **Weak gravitational field** ··· $GM / \left(c^2 R\right) \ll 1$
- **Uniform magnetic field** ··· $\lambda_{\text{particle}} \ll L_B$

Deriving non-relativistic equation

$$\phi(x) = \Psi(x)\exp\left[-i\frac{mc^2}{\hbar}t\right]$$

Metric: $g_{\mu\nu}$,

4-Vector: $A_\mu = \left(0,\ -\frac{B}{2}y,\ \ \frac{B}{2}x,\ 0\right)$

$$\left[g^{\mu\nu}\left(\nabla_\mu - \frac{ie}{\hbar}A_\mu\right)\left(\nabla_\nu - \frac{ie}{\hbar}A_\nu\right) + \frac{m^2c^2}{\hbar^2}\right]\phi = 0$$

Spacetime metric

Slow rotation limit of the Kerr metric

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \frac{4GMa}{cr} \sin^2\theta \, dt d\varphi \\
&\quad - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)
\end{aligned}
$$

Transformation to the surface of a rotating star

$$
\begin{cases}
x = x' \cos\Omega t - y' \sin\Omega t \\
y = x' \sin\Omega t + y' \cos\Omega t \\
z = z' + R
\end{cases}
$$

Schrodinger equation with relativistic corrections in the polar region

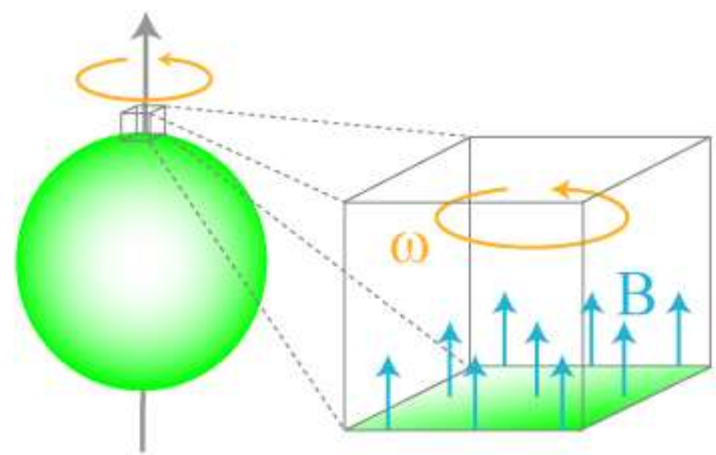
Neglecting the order of $O(1/c^2)$

$$i\hbar\frac{\partial}{\partial t}\Psi=\left[-\frac{\hbar^2}{2m}\nabla^2-\frac{GMm}{R}+mgz-\left(\frac{eB}{2m}+\varpi\left(R\right)\right)\hat{L}_z+\left(\frac{e^2B^2}{8m}+\frac{eB}{2}\varpi\left(R\right)\right)\left(x^2+y^2\right)\right]\Psi$$

where

$$\varpi\left(R\right)=\Omega-\frac{2GMa}{cR^3}$$

$$\hat{L}_z=-i\hbar\left(x\partial_y-y\partial_x\right)$$



Separation of variables

Cylindrical coordinates: $\Psi(r,\theta,z)=\psi(r,\theta)\varphi(z)$

Vertical direction:

$$K\varphi=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+V(z)\right]\varphi$$

Horizontal direction :

$$(E-mU-K)\psi$$
$$=\left[-\frac{\hbar^2}{2m}\left(\partial_r{}^2+\frac{1}{r}\partial_r\right)+\frac{\hat{p}_\theta{}^2}{2mr^2}-\left(\frac{eB}{2m}+\varpi(R)\right)\hat{p}_\theta+\left(\frac{e^2B^2}{8m}+\frac{eB}{2}\varpi(R)\right)r^2\right]\psi$$

where

$$U=-GM/R,\quad \varpi(R)=\Omega-2GMa/\left(cR^3\right),$$
$$\hat{p}_\theta=-i\hbar\partial_\theta,\quad K=\text{constant}$$

Wave equation

$$\psi(r,\theta) = r^{\ell} \exp\left[i \frac{p_\theta}{\hbar} \theta \right] \exp\left[-\frac{\beta}{2} r^2 \right] F(r),$$

$$\ell = \pm \frac{p_\theta}{\hbar} \quad \text{for} \quad q = \pm e, \quad \beta = \left(\frac{q^2 B^2}{4\hbar^2} + \frac{mqB}{\hbar^2} \varpi(R) \right)^{1/2},$$

$$x = \beta r, \quad \varepsilon = \frac{2m}{\hbar^2}\left(E - mU - K_n\right) + \left(\frac{qB}{\hbar^2} + \frac{2m}{\hbar^2} \varpi(R) \right) p_\theta,$$

Transformation of variables

Confluent hypergeometric equation

$$\left[x \frac{d^2}{dx^2} + \left\{ (\ell + 1) - x \right\} \frac{d}{dx} - \left(\frac{\varepsilon}{4\beta} - \frac{\ell + 1}{2} \right) \right] F(x) = 0$$

Integrability condition $\Rightarrow$ $\frac{\varepsilon}{4\beta} - \frac{\ell + 1}{2} = n'$ (Integer)

Energy eigenstates of the horizontal component

Wave function :

$$\psi = A r^{\ell} \exp\left(-\frac{\beta}{2} r^2\right) L_n^{\ell}\left(\beta r^2\right)$$

$$\beta \equiv \sqrt{\frac{q^2 B^2}{4\hbar^2} + \frac{mqB}{\hbar^2}\varpi(R)}, \quad L_n^{\ell} : \text{ associated Laguerre function}$$

Eigen values :

$$E_{nn'} \simeq mU + \hbar\left(\frac{eB}{m} + 2\varpi(R)\right)\left(n + \frac{1}{2}\right) + K_{n'}$$

↑ Non-inertial effects

Order estimates

As $B = 10^8 - 10^{15}[\mathrm{G}],\ M = M_\odot,\ R = 10[\mathrm{km}]$

$$\omega_B(m_e) = \frac{qB}{m_e} \simeq 10^{15} \sim 10^{22}[\mathrm{Hz}],$$

$$\omega_B(m_p) = \frac{qB}{m_p} \simeq 10^{12} \sim 10^{19}[\mathrm{Hz}],$$

$$\omega_\Omega = 2\Omega \simeq 10^3[\mathrm{Hz}], \quad \omega_a = \frac{4GMa}{cR^3} \simeq 10^2[\mathrm{Hz}],$$

Thermal effects : $\Delta\omega_T = k_\mathrm{B} T / \hbar \simeq 10^{17}[\mathrm{Hz}]$

Summary

We investigated quantum systems on a stellar surface. We seriously took account of non-inertial and gravitational effects for the quantum system through which magnetic field penetrates.

We obtained the analytic solution for the Schrodinger equation with relativistic corrections.

Landau levels are modified by the rotational and frame dragging effects.