

Kent Yagi, JGRG 22(2012)111307



"Black hole solution and binary gravitational waves in dynamical

Chern-Simons gravity"

RESCEU SYMPOSIUM ON

GENERAL RELATIVITY AND GRAVITATION

JGRG 22

November 12-16 2012

Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan





BLACK HOLE SOLUTION AND BINARY GRAVITATIONAL WAVES IN DYNAMICAL CHERN-SIMONS GRAVITY

JGRG22 @ University of Tokyo

November 13th 2012

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Probing alternative theories of gravity using compact binaries



Why modifications of gravity?

(I) **Problems** within GR can be **naturally solved**.





(II) Classical gravitational theory as a **low-energy effective theory** of a more fundamental theory

e.g. superstring theory



Chern-Simons, Gauss-Bonnet,

Scalar-tensor theories

Testing GR

<u>(I) Weak field, non-dynamical regime:</u>

Solar System



Binary Pulsar



(II) Strong field, dynamical regime:

> Gravitational Waves



Dynamical Chern-Simons Gravity

-Standard Model -Superstring Theory -Loop Quantum Gravity -Inflation

$\begin{array}{ll} & \begin{array}{l} \begin{array}{l} \textbf{Dynamical Chern-Simons Gravity} \\ \textbf{-Standard Model} \\ \textbf{-Superstring Theory} \end{array} & \begin{array}{l} \textbf{-Loop Quantum Gravity} \\ \textbf{-Inflation} \\ \end{array} \\ & \begin{array}{l} \textbf{O Particle Construction} \\ \textbf{O Action:} \\ c = G = 1 \\ \kappa_g \equiv (16\pi)^{-1} \end{array} & \begin{array}{l} S \end{array} \equiv \int d^4x \sqrt{-g} \Big\{ \kappa_g R + \frac{\alpha}{4} \vartheta R_{\nu\mu\rho\sigma}^{*} R^{\mu\nu\rho\sigma} \\ -\frac{\beta}{2} [\nabla_{\mu} \vartheta \nabla^{\mu} \vartheta + 2V(\vartheta)] + \mathcal{L}_{mat} \Big\} . \end{array} \end{array}$





$$\begin{array}{l} & \begin{array}{l} \begin{array}{l} \textbf{Dynamical Chern-Simons Gravity} \\ \hline \text{Standard Model} & \text{Loop Quantum Gravity} \\ \hline \text{Superstring Theory} & \text{Inflation} \end{array} \\ & \begin{array}{l} \textbf{Option Constant} \\ \hline \text{Superstring Theory} \\ \hline \text{Superstring Theory} \\ \hline \text{Superstring Theory} \\ \hline \text{Inflation} \end{array} \\ & \begin{array}{l} \textbf{Option Constant} \\ \hline \text{Superstring Theory} \\ \hline \text{S$$

Corrections to GWs from BH Binaries

(I) Dissipative

Scalar & Gravitational Radiation



Modifies the orbital evolution



(II) Conservative

Modified **BH Solution**



Modifies the binding energy



Modifies the binary orbit



Dissipative Corrections in DCS Gravity



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Metric Perturbation:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \alpha'^2 g^{(2)}_{\mu\nu} + \mathcal{O}(\alpha'^4)$$
GR CS

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Expansion in spin:

$$\left[g_{\mu\nu}^{(0)} \right] = g_{\mu\nu}^{(0,0)} + \chi' g_{\mu\nu}^{(1,0)} + \chi'^2 g_{\mu\nu}^{(2,0)} + \mathcal{O}(\chi'^3)$$

$$g^{(m,n)}_{\mu
u} \propto \chi^m \alpha^n$$

Metric Perturbation:

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Metric Perturbation:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \alpha'^2 g^{(2)}_{\mu\nu} + \mathcal{O}(\alpha'^4)$$
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Expansion in spin:

$$\begin{aligned} g_{\mu\nu}^{(0)} &= g_{\mu\nu}^{(0,0)} + \chi' g_{\mu\nu}^{(1,0)} + \chi'^2 g_{\mu\nu}^{(2,0)} + \mathcal{O}(\chi'^3) \\ \alpha'^2 g_{\mu\nu}^{(2)} &= \alpha'^2 g_{\mu\nu}^{(0,2)} + \alpha'^2 \chi' g_{\mu\nu}^{(1,2)} + \alpha'^2 \chi'^2 g_{\mu\nu}^{(2,2)} \\ + \mathcal{O}(\alpha'^2 \chi'^3) \,, \end{aligned}$$

 $g^{(m,n)}_{\mu
u} \propto \chi^m \alpha^n$

Metric Perturbation: $g_{\mu\nu} = \left[g^{(0)}_{\mu\nu}\right] + \left[\alpha'^2 g^{(2)}_{\mu\nu}\right] + \mathcal{O}(\alpha'^4)$ CS GR

Expansion in spin:

$$\begin{split} g^{(0)}_{\mu\nu} &= g^{(0,0)}_{\mu\nu} + \chi' g^{(1,0)}_{\mu\nu} + \chi'^2 g^{(2,0)}_{\mu\nu} + \mathcal{O}(\chi'^3) \\ \alpha'^2 g^{(2)}_{\mu\nu} &= \alpha'^2 g^{(0,2)}_{\mu\nu} + \alpha'^2 \chi' g^{(1,2)}_{\mu\nu} + \alpha'^2 \chi'^2 g^{(2,2)}_{\mu\nu} \\ &+ \mathcal{O}(\alpha'^2 \chi'^3), \text{ Yunes \& Pretorius,} \\ g^{(m,n)}_{\mu\nu} &\propto \chi^m \alpha^n \end{split}$$

Metric Perturbation:

 $g_{\mu\nu} = \underbrace{g_{\mu\nu}^{(0)}}_{\mathsf{GR}} + \underbrace{\alpha'^2 g_{\mu\nu}^{(2)}}_{\mathsf{CS}} + \mathcal{O}(\alpha'^4)$

Expansion in spin: (0, 0)

$$\begin{aligned} g_{\mu\nu}^{(0)} &= g_{\mu\nu}^{(0,0)} + \chi' g_{\mu\nu}^{(1,0)} + \chi'^2 g_{\mu\nu}^{(2,0)} + \mathcal{O}(\chi'^3) \\ \alpha'^2 g_{\mu\nu}^{(2)} &= \alpha'^2 g_{\mu\nu}^{(0,2)} + \alpha'^2 \chi' g_{\mu\nu}^{(1,2)} + \alpha'^2 \chi'^2 g_{\mu\nu}^{(2,2)} \\ + \mathcal{O}(\alpha'^2 \chi'^3), & \text{Yunes \& Pretorius, New!!} \\ g_{\mu\nu}^{(m,n)} \propto \chi^m \alpha^n \end{aligned}$$

BH Solution at Quadratic Order in Spin Metric Perturbation: $g_{\mu\nu} = g^{(0)}_{\mu\nu} + \alpha'^2 g^{(2)}_{\mu\nu} + \mathcal{O}(\alpha'^4)$ CS GR Expansion in spin: $\left[g_{\mu\nu}^{(0)}\right] = g_{\mu\nu}^{(0,0)} + \chi' g_{\mu\nu}^{(1,0)} + \chi'^2 g_{\mu\nu}^{(2,0)} + \mathcal{O}(\chi'^3)$ $\alpha'^2 g^{(2)}_{\mu\nu} = \alpha'^2 g^{(0,2)}_{\mu\nu} + \alpha'^2 \chi' g^{(1,2)}_{\mu\nu} + \alpha'^2 \chi'^2 g^{(2,2)}_{\mu\nu}$ $+\mathcal{O}(\alpha'^2\chi'^3)$, Yunes & Pretorius, New!! Konno et al. $q_{\mu\nu}^{(m,n)} \propto \chi^m \alpha^n$ Field Eqs.: $G_{\mu\nu}^{(2,2)} \left[g_{\mu\nu}^{(0,0)}, g_{\mu\nu}^{(2,2)} \right] = S_{\mu\nu}^{(2,2)} \left[g_{\mu\nu}^{(0,0)}, g_{\mu\nu}^{(1,0)}, g_{\mu\nu}^{(1,2)}, \vartheta^{(1,1)} \right]$ at order (2,2)

BH Solution at Quadratic Order in Spin Metric Perturbation: $g_{\mu\nu} = g^{(0)}_{\mu\nu} + \alpha'^2 g^{(2)}_{\mu\nu} + \mathcal{O}(\alpha'^4)$ CS GR Expansion in spin: $\left[g_{\mu\nu}^{(0)}\right] = g_{\mu\nu}^{(0,0)} + \chi' g_{\mu\nu}^{(1,0)} + \chi'^2 g_{\mu\nu}^{(2,0)} + \mathcal{O}(\chi'^3)$ $\alpha'^2 g^{(2)}_{\mu\nu} = \alpha'^2 g^{(0,2)}_{\mu\nu} + \alpha'^2 \chi' g^{(1,2)}_{\mu\nu} + \alpha'^2 \chi'^2 g^{(2,2)}_{\mu\nu}$ $+\mathcal{O}(\alpha'^2\chi'^3)$, Yunes & Pretorius, New!! Konno et al. $q_{\mu\nu}^{(m,n)} \propto \chi^m \alpha^n$ Field Eqs.: $G^{(2,2)}_{\mu\nu} \left[g^{(0,0)}_{\mu\nu}, g^{(2,2)}_{\mu\nu} \right] = S^{(2,2)}_{\mu\nu} \left[g^{(0,0)}_{\mu\nu}, g^{(1,0)}_{\mu\nu}, g^{(1,2)}_{\mu\nu}, \vartheta^{(1,1)} \right]$ at order (2,2)**Known functions**





The New Metric [KY+ (2012)]

$$\begin{split} g_{tt}^{cs} &= \zeta \chi^2 \frac{M^3}{r^3} \Biggl[\frac{201}{1792} \left(1 + \frac{M}{r} + \frac{4474}{4221} \frac{M^2}{r^2} - \frac{2060}{469} \frac{M^3}{r^3} + \frac{1500}{469} \frac{M^4}{r^4} - \frac{2140}{201} \frac{M^5}{r^5} + \frac{9256}{201} \frac{M^6}{r^6} - \frac{5376}{67} \frac{M^7}{r^7} \right) (3 \cos^2 \theta - 1) \\ &- \frac{5}{384} \frac{M^2}{r^2} \left(1 + 100 \frac{M}{r} + 194 \frac{M^2}{r^2} + \frac{2220}{7} \frac{M^3}{r^3} - \frac{1512}{5} \frac{M^4}{r^4} \right) \Biggr] + \mathcal{O}(\alpha'^2 \chi'^4) \,, \end{split}$$

$$\begin{aligned} &(41) \\ g_{rr}^{cs} &= \zeta \chi^2 \frac{M^3}{r^3 f(r)^2} \Biggl[\frac{201}{1792} f(r) \left(1 + \frac{1459}{603} \frac{M}{r} + \frac{20000}{4221} \frac{M^2}{r^2} + \frac{51580}{1407} \frac{M^3}{r^3} - \frac{7580}{201} \frac{M^4}{r^4} - \frac{22492}{201} \frac{M^5}{r^4} - \frac{22492}{201} \frac{M^5}{r^5} - \frac{40320}{67} \frac{M^6}{r^6} \right) (3 \cos^2 \theta - 1) \\ &- \frac{25}{384} \frac{M}{r} \left(1 + 3\frac{M}{r} + \frac{322}{5} \frac{M^2}{r^2} + \frac{198}{5} \frac{M^3}{r^3} + \frac{6276}{175} \frac{M^4}{r^4} - \frac{17496}{25} \frac{M^5}{r^5} \right) \Biggr] + \mathcal{O}(\alpha'^2 \chi'^4) \,, \end{aligned} \\ g_{\theta\theta}^{cs} &= \frac{201}{1792} \zeta \chi^2 M^2 \frac{M}{r} \left(1 + \frac{1420}{603} \frac{M}{r} + \frac{18908}{4221} \frac{M^2}{r^2} + \frac{1480}{603} \frac{M^3}{r^3} + \frac{22460}{1407} \frac{M^4}{r^4} + \frac{3848}{201} \frac{M^5}{r^5} + \frac{5376}{67} \frac{M^6}{r^6} \right) (3 \cos^2 \theta - 1) \\ &+ \mathcal{O}(\alpha'^2 \chi'^4) \,, \end{aligned}$$

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 $\begin{array}{l} \underline{\text{GWs from BH Binaries in CS Gravity}}\\ \underline{\text{parameterized post-Einsteinian waveform:}} & \text{Yunes & Pretorius (2009)}\\ \\ \tilde{h}(f) = \tilde{h}_{\text{GR}}(f) \left[1 + \alpha_{\text{ppE}} u^{a_{\text{ppE}}}\right] \exp\left(i\beta_{\text{ppE}} u^{b_{\text{ppE}}}\right)\\ & u \equiv \pi \mathcal{M} f \propto v^{3} \end{array}$

GWs from BH Binaries in CS Gravity
parameterized post-Einsteinian waveform: Yunes & Pretorius (2009)

$$\tilde{h}(f) = \tilde{h}_{GR}(f) \left[1 + \alpha_{ppE} u^{a_{ppE}}\right] \exp \left(i\beta_{ppE} u^{b_{ppE}}\right)$$

 $u \equiv \pi \mathcal{M} f \propto v^{3}$
CS Gravity: $\alpha_{ppE} = 0$ $\beta_{ppE} = C(m_{1,2}, \hat{S}^{i}_{1,2})\zeta\chi^{2}$ $b_{ppE} = -\frac{1}{3}$
2PN correction







 $m_1/m_2 = 2$

Spins are known

Colored Region:



 $m_1/m_2 = 2$

Spins are known

Colored Region:



$$m_1/m_2 = 2$$

Spins are known *a priori*

Colored Region:

 $\Delta \zeta < 1$

The bound roughly corresponds to the BH horizon size.





```
m_1/m_2 = 2
```

Spins are known *a priori*

Colored Region: $\Delta \zeta < 1$

The bound roughly corresponds to the BH horizon size.

Solar system: $\xi^{1/4} < \mathcal{O}(10^8) \text{ km}$

7 orders of magnitude stronger constraint than the solar system bound!!

Results: Other Detectors



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Current constraints from double binary pulsar obs.