Yousuke Itoh, JGRG 22(2012)111306

"Stacking of cluster profiles"

RESCEU SYMPOSIUM ON

GENERAL RELATIVITY AND GRAVITATION

JGRG 22

November 12-16 2012

Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan

STACKING CLUSTERS

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22nd workshop on General Relativity and Gravitation in Japan, RESCEU symposium, 2012 Sep. 12-16 @ University of Tokyo

In this talk…

1. Introduction
2. Universality of NFW?
3. Introduce the Hough transform as a semi-non-parametric method for the purpose 2.

Introduction

- One big mystery in astronomy: late time accelerated expansion of the universe.

$$\frac{1}{a}\frac{d^2a}{dt^2} > 0$$

- Possible solutions: Modified gravity or new form of matter (or …).

$$G = 8\pi\,(T_{ordinary} + T_{new})$$

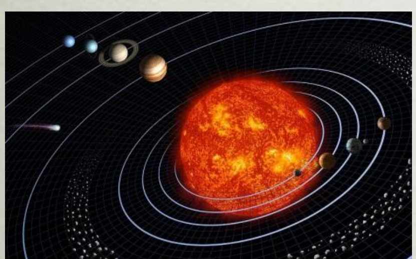
or

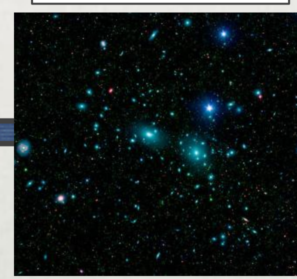
$$G + \delta G = 8\pi\, T_{ordinary}$$

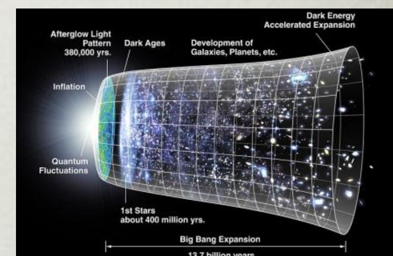
Introduction (cont'd)

- Assume modified gravity. It should coincide with the Newtonian grav. at smaller scale.

Transition







Small scale:
Solar system test.
Newtonian or GR.

Large scale:
Acceleration.
Modified Gravity.

New test of modified gravity

Narikawa & Yamamoto (2012).
Narikawa’s talk Wednesday afternoon for details.

- At the cluster of galaxies scale, scalar degrees of freedom may be apparent through, e.g., gravitational lensing.

$$\kappa \simeq \frac{f_K(\chi_{\rm S}-\chi_{\rm L})f_K(\chi_{\rm L})}{f_K(\chi_{\rm S})}\int_0^{\chi_{\rm S}} d\chi' \left[4\pi G\rho(r') - \frac{\alpha+2\xi}{2}\frac{\triangle}{a_{\rm L}^2}\varphi\right] a_{\rm L}^2$$

Convergence = dimensionless surface mass density

Conventional part. Assuming NFW, gNFW, or Einasto.

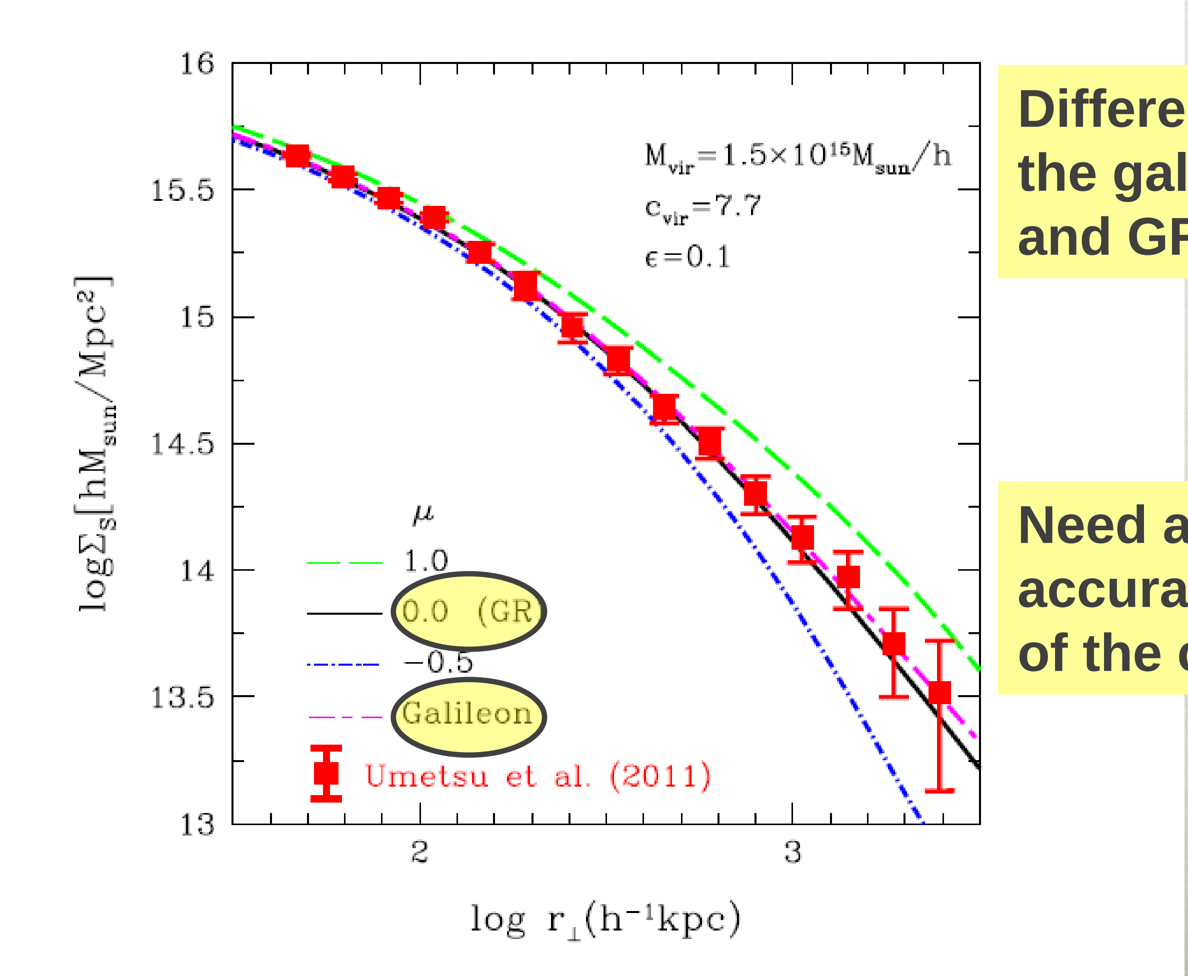
Effect of Scalar dof

New test of modified gravity (cont'd)

Narikawa & Yamamoto (2012).

Narikawa's talk Wednesday afternoon for details.

- In principle, it seems work…, but.



Difference between the galileon gravity and GR is very small.

Need a sufficiently accurate measurement of the cluster profile.

Stacking clusters of galaxies

- **Need a sufficiently accurate measurement of the cluster profile.**
- **Stacking many signals from clusters.**
  - **Averaging out "personalities" of clusters. (asymmetry, clumps, environment, …)**
- **Subtlity:**
  - **Even if the NFW (or other) is universal, it contains scaling radius/density.**

After all, is NFW really universal?

Reminder: NFW

- NFW: Navarro, Frenk & White ApJ (1996).
- ``Universal'' profile of a dark matter halo around a cluster of galaxies.

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

- Two parameters fimily: ρ_s and r_s
- In reality, each one may be deviated from NFW (environment, evolution history, asymmetry…).
- If stacking many $\rho(r)$'s, ...

Toy model for stacking experiment.

- NFW Cluster mass function (M_{ps} = 5x10^{14} solar masses)

$$p(>M) \propto \left(\frac{M}{M_{\rm PS}}\right)^{-1} \exp\left(-\frac{M}{M_{\rm PS}}\right)$$

- Cluster redshift distribution (z_0 = 0.45)

$$n(z)dz \propto z^2 \exp\left(-\frac{z}{z_0}\right) dz$$

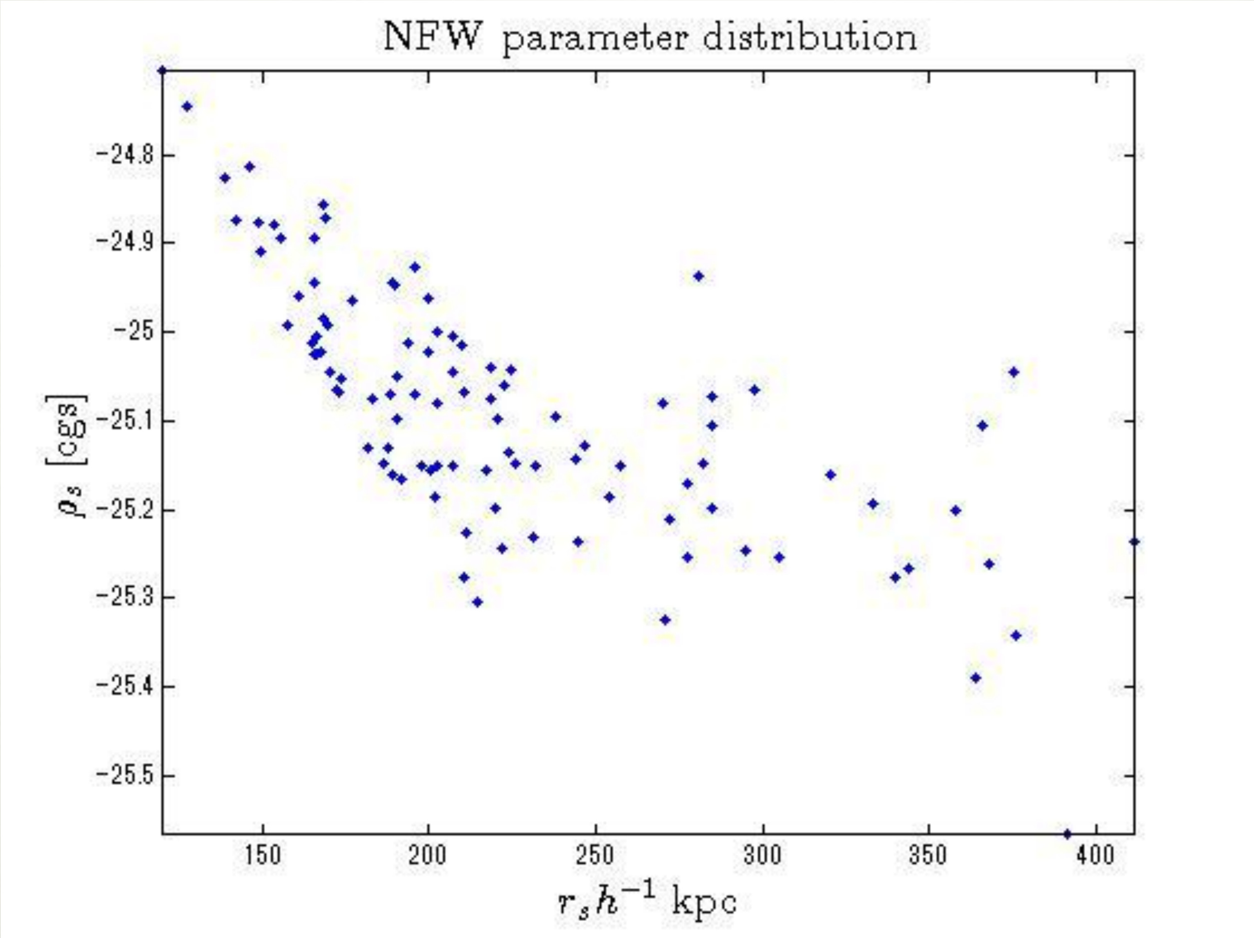
- Concentration parameter (From Duffy 2011 Nbody simulation): $c = \bar{c} + \delta c$

$$\bar{c} = A(M/M_{\rm pivot})^B (1+z)^C$$

$$p(c_{200})dc_{200} = \frac{1}{\sqrt{2\pi\sigma^2_{\log_{10} c_{200}}}} \exp\left(-\frac{(\log_{10} c_{200} - \log_{10} \bar{c}_{200})^2}{2\sigma^2_{\log_{10} c_{200}}}\right) d\ln c_{200}$$

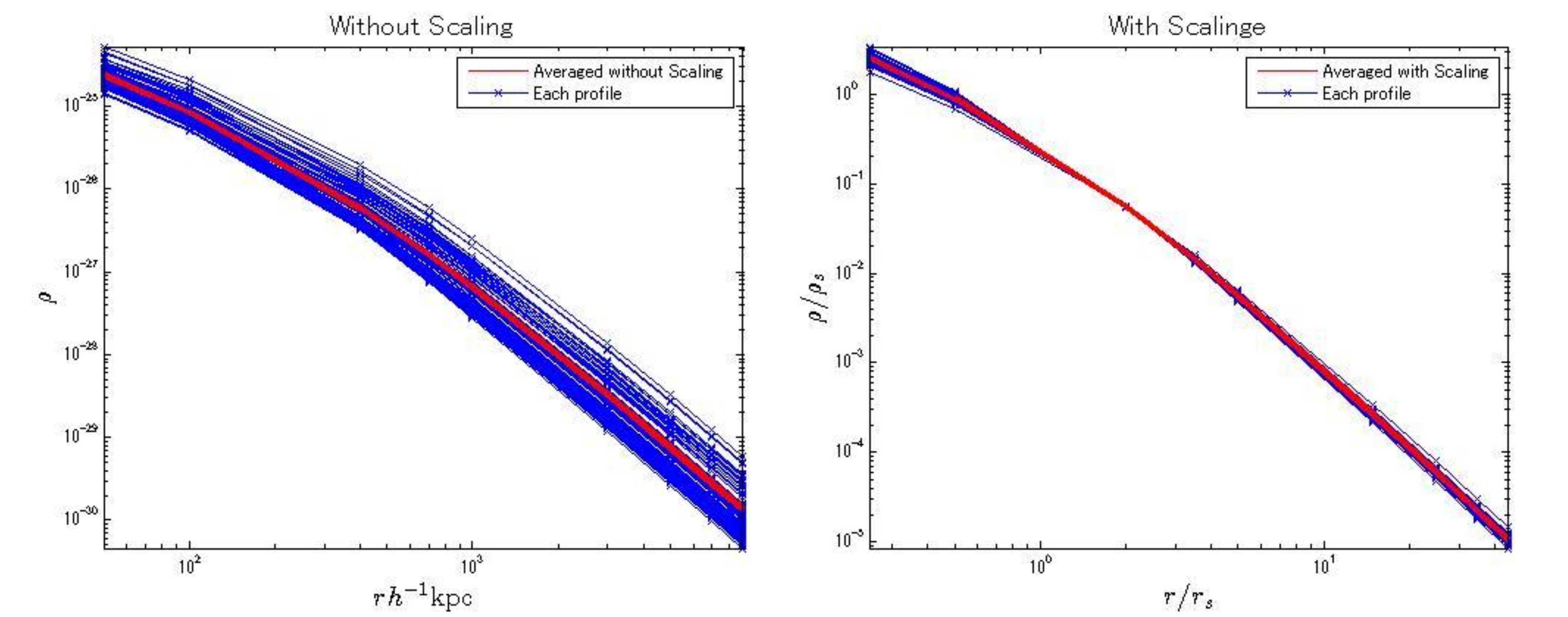
Toy model

Parameters of 100 toy NFW clusters.



CMBCG catalogue contains over 55,000 clusters, though the number of known lensing clusters are much smaller than it at this moment….

With or Without scaling



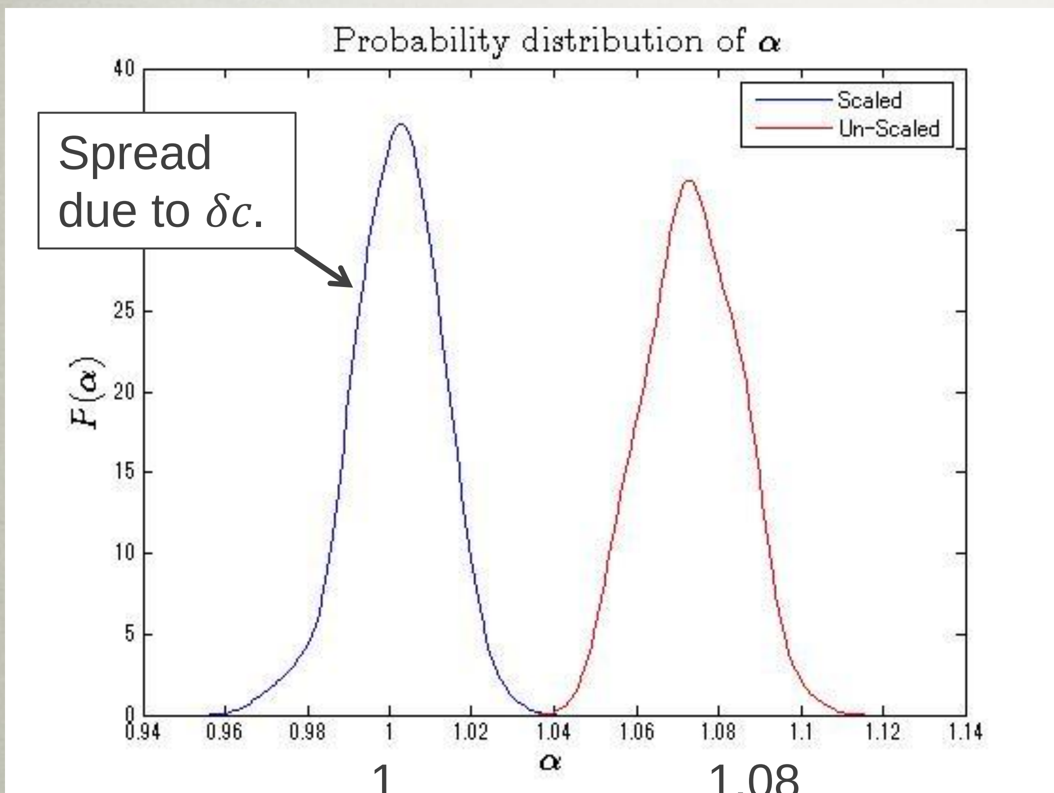
Blue: 100 cluster radial profiles (left: un-scaled, right: scaled)

Red: left: Averated, right: Scaled using true (M,$\bar{c}$) then Averaged.

$$\rho_{w.o\ scaling}(r;\ \{M_i, c_i\}_i) = \frac{1}{N}\sum_i^N \rho\,(r; M_i, c_i)$$

$$\bar{\rho}_{w\ scaling}(x; \{\delta c_i\}_i) = \frac{1}{N}\sum_i^N \frac{\rho(x\, r_{si}; M_i, \overline{c_i})}{\rho_{si}}$$

Estimating α : without noise.



We estimate the inner power index α using 25 clusters and do it 100 times.

$$\rho(r) = \frac{\rho_s}{(r/r_s)^{\alpha}(1+r/r_s)^{3-\alpha}}$$

Ok, just small difference…

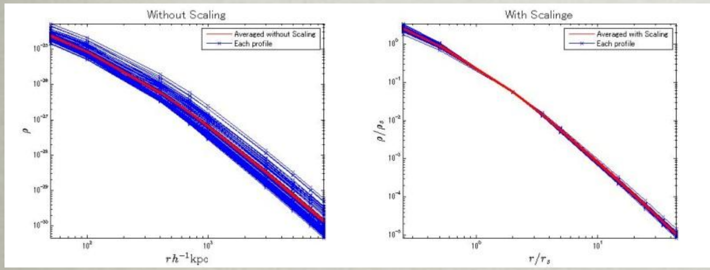
Toy model Including noise…

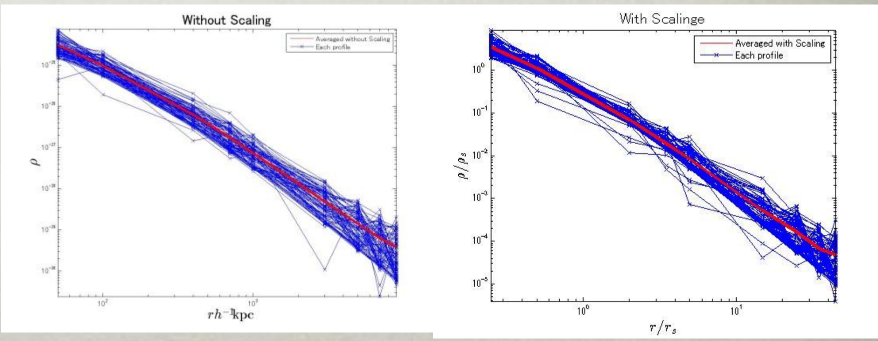
- Adding noise term to the NFW radial profile.

(r_i is the inner edge of the “observed” NFW profile.)

$$
\begin{aligned}
\rho(r) &= \rho_{\mathrm{NFW}}(r) + \delta\rho(r), \\
\delta\rho(r) &= 0.3\rho(r_i)\left(\frac{r_i}{r}\right)^2 \zeta(r), \\
\zeta &\sim N(0,1).
\end{aligned}
$$

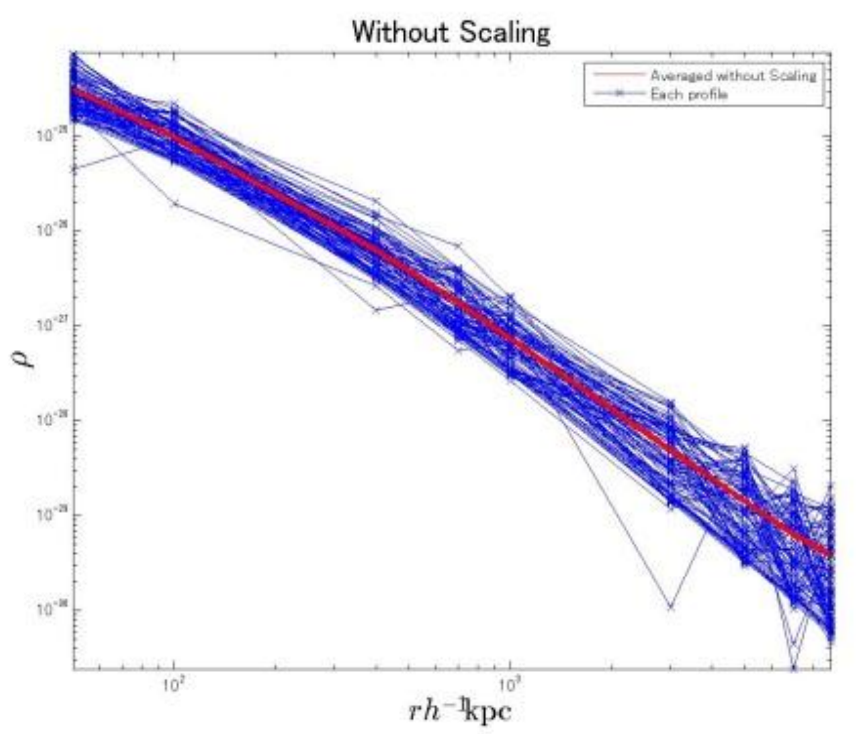
At each radal bin, random noises ζ(r) follow
Gaussian distribution with mean zero and variance 1.

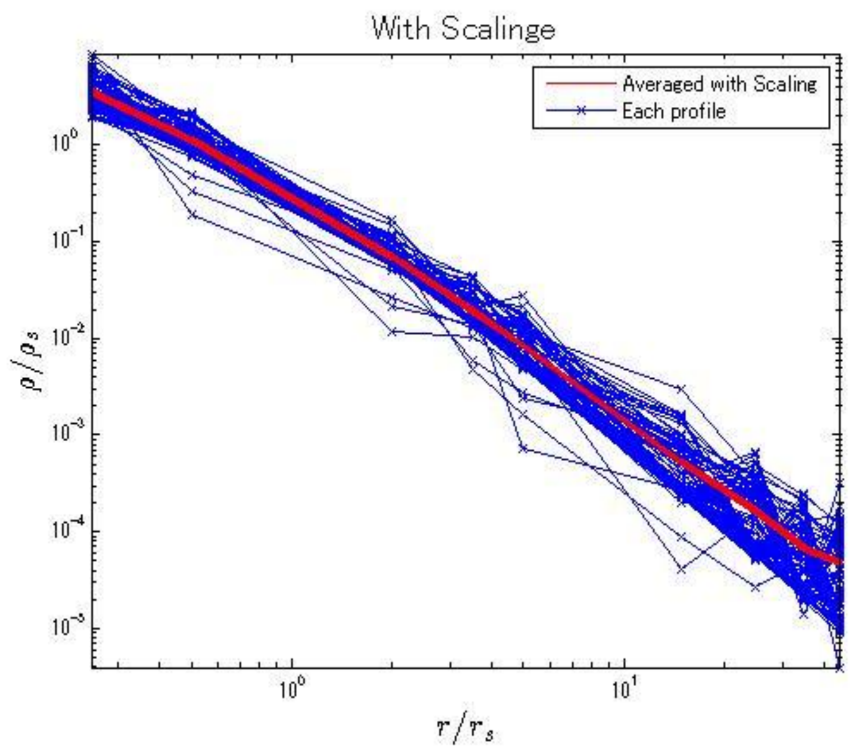


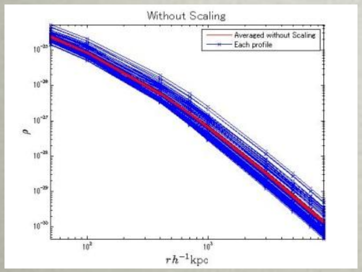


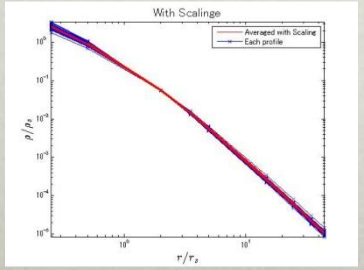
Toy model Including noise…

- Adding noise term to the NFW radial profile

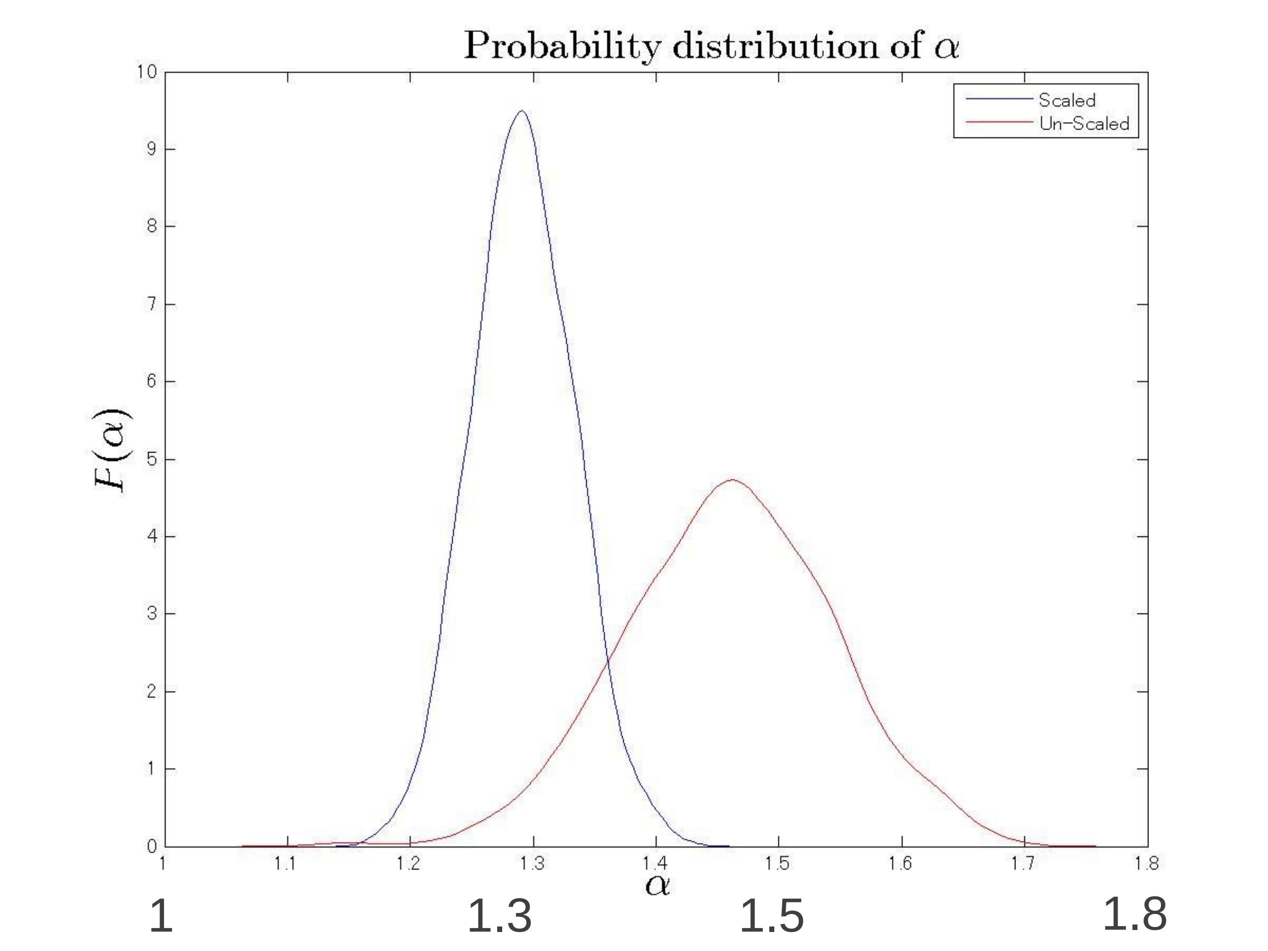








Estimating α : with noise.



From 25 clusters estimate α, do it 1000 times.

Large bias for α from the constraint ρ(r)>0.
Noise is larger at smaller radius.

Concerns

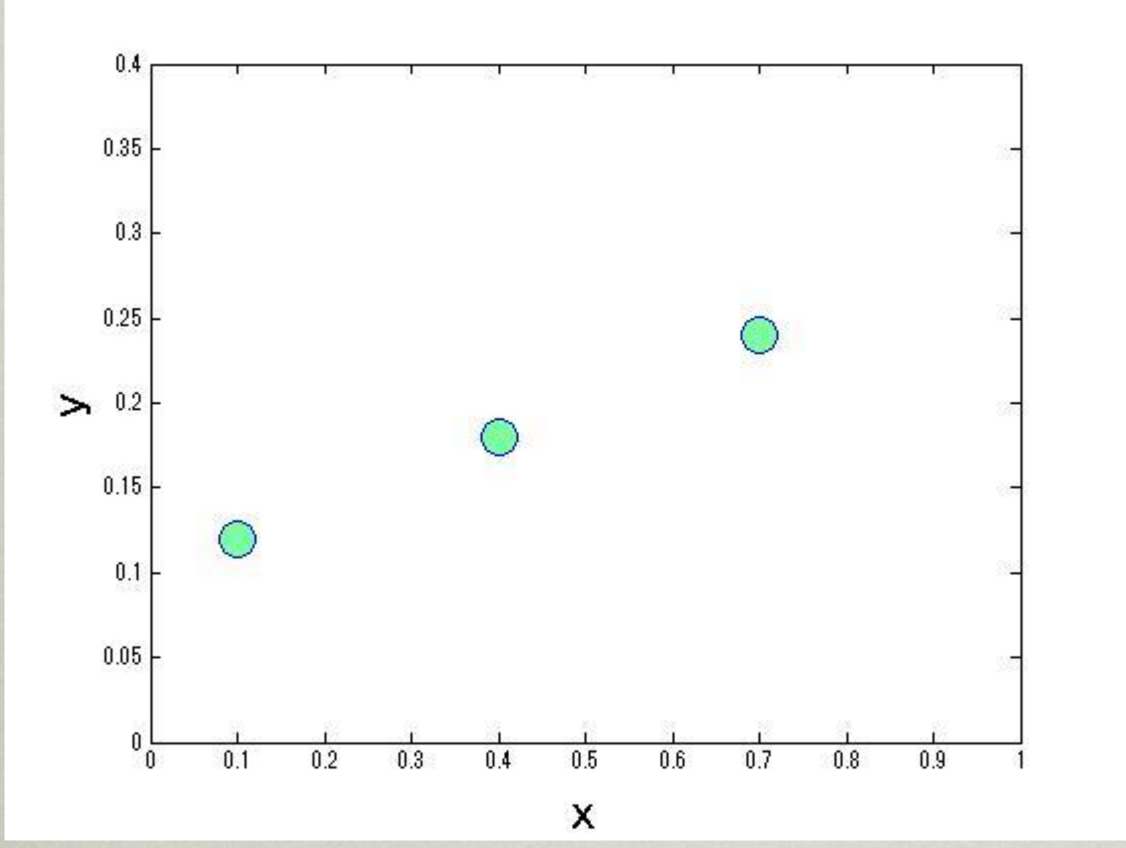
- We do not know a priori the true NFW parameters ρ_s and r_s .
- How can we scale $\rho(r)$ if we do not know scaling parameters ρ_s and r_s ?
- Estimating ρ_s and r_s for each cluster may bias results (e.g., test of modified gravity) if the NFW profile is not "the universal" profile in the first place.

Hough Transform (used in GW community as well)

- Assume a master equation $y = f(x; p_i)$ with M parameters p_i. (For explanation's sake, let's assume a curve for the master equation.)
- Given N>M pair of "data" (x_k, y_k), we obtain N curves in the M-dim param. space.
- If there is no noise, and if the master equation is correct, we get a solution as an intersection of the curves in the parameter space.
- When there is noise, there may be no solution. Yet, there may be a region where many intersections between two curves cluster.

Hough Transform (cont'd)

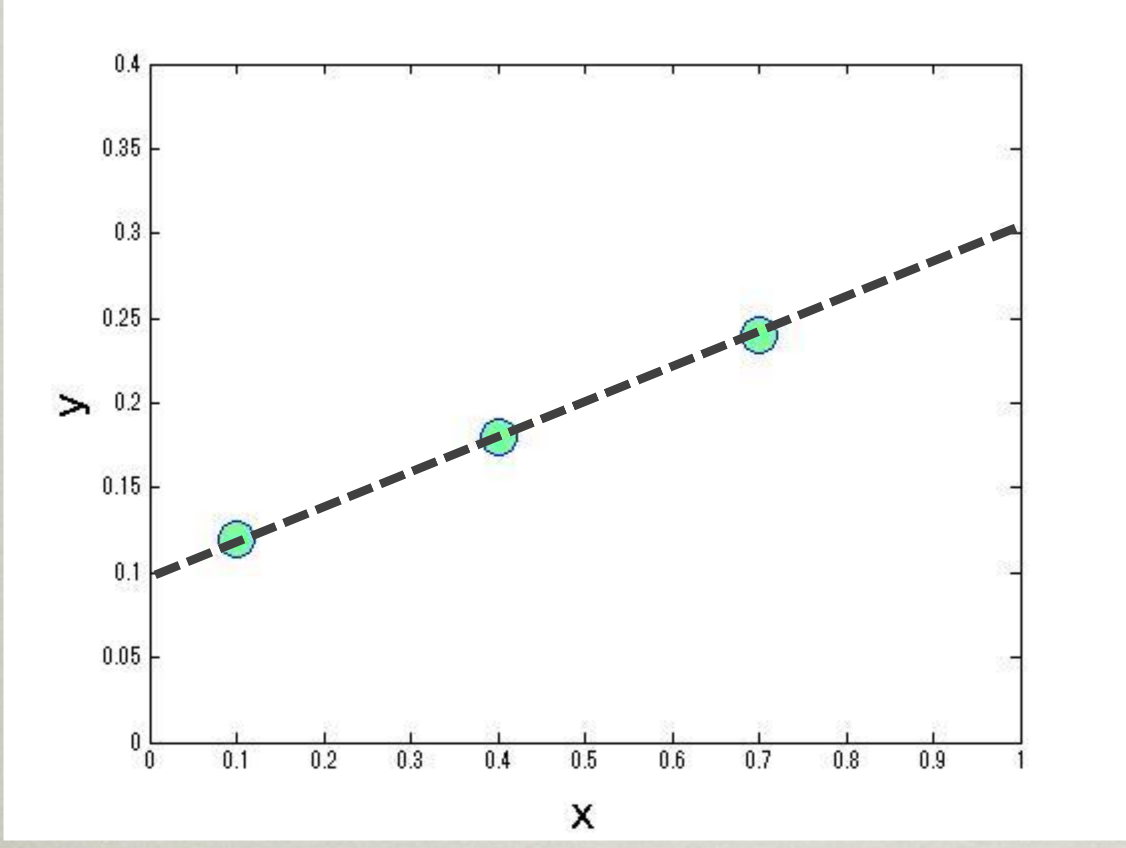
Straight line master eq.



Given 3 "data".

Hough Transform (cont'd)

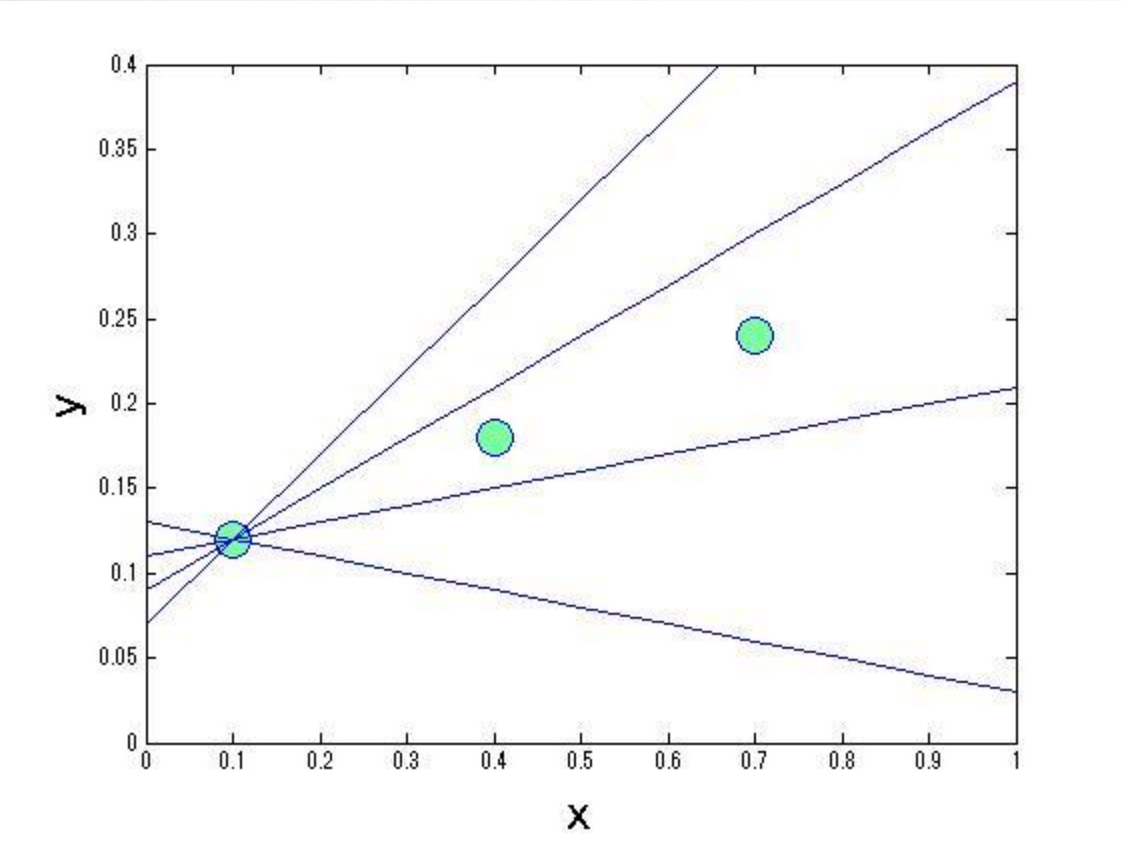
Straight line master eq.



This "data" is actually on $y = 0.2\ x + 0.1$ line.

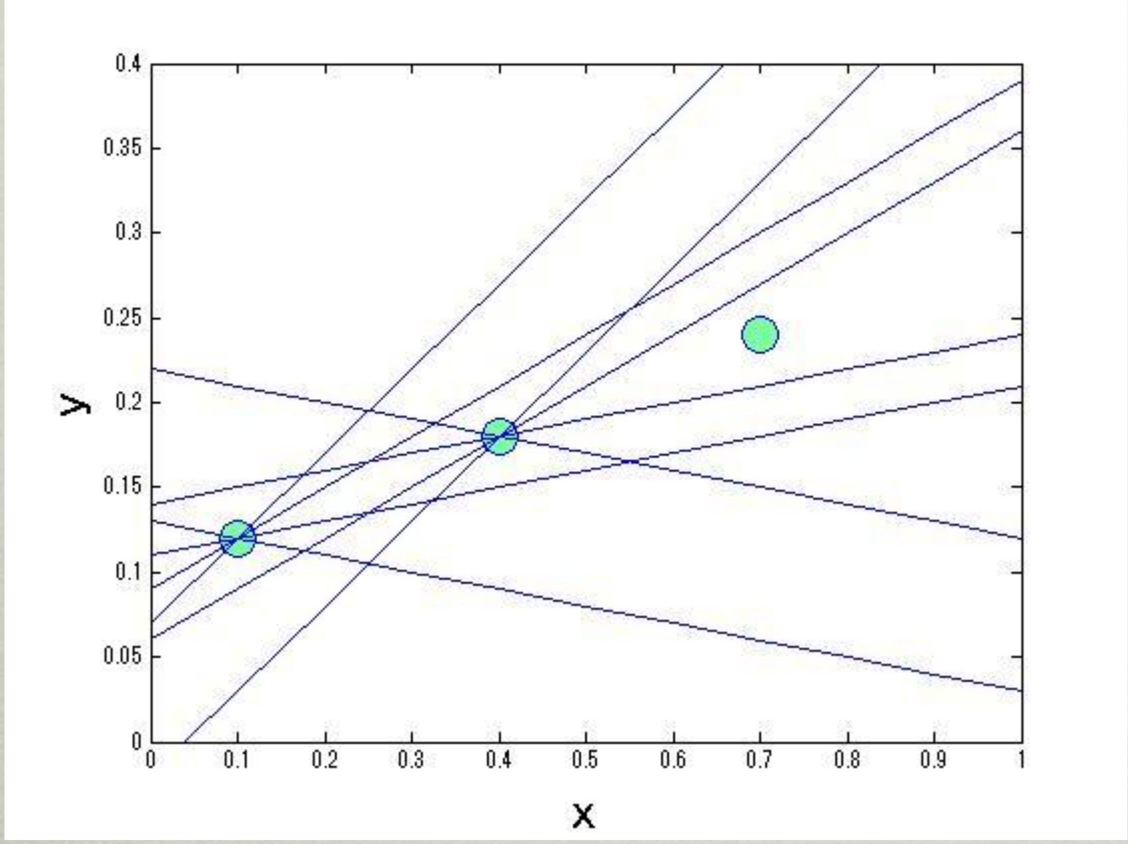
Hough Transform (cont'd)

Straight line master eq.



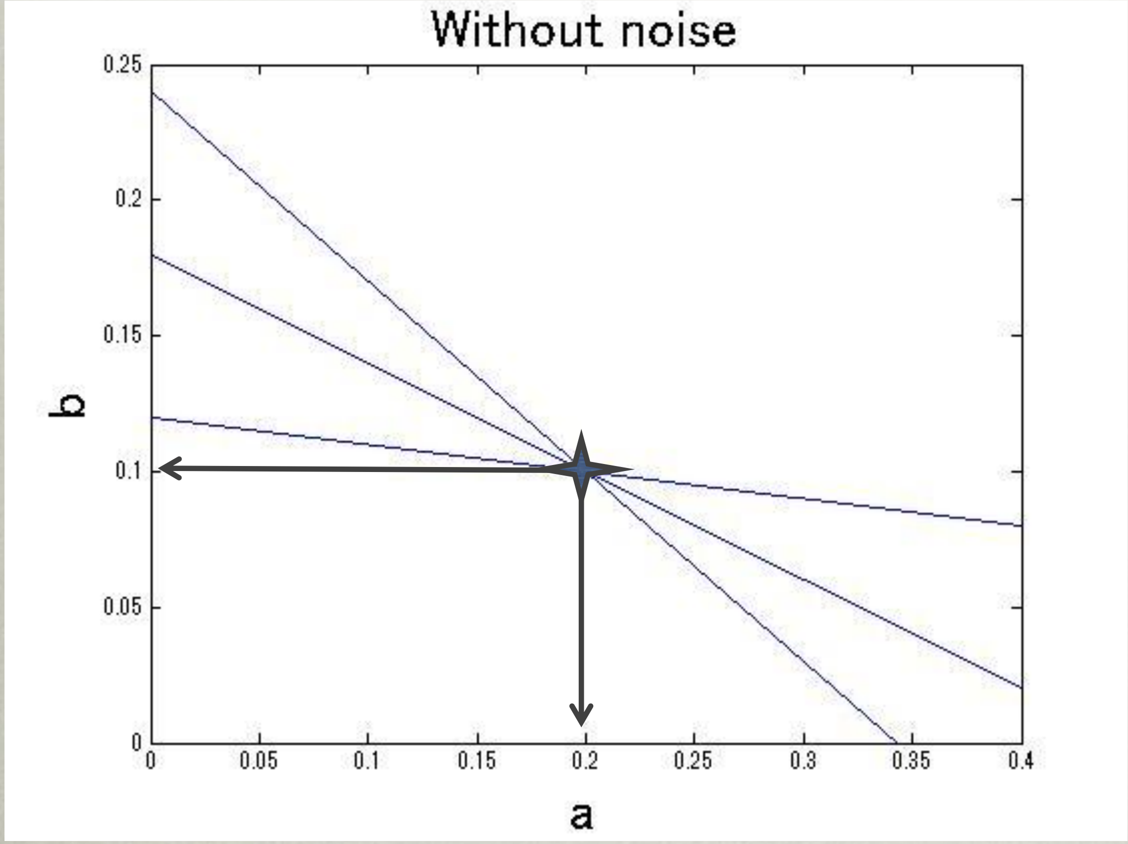
For the rightmost data, plot a master equation (straight line in this example) that pass through the data.

Hough Transform (cont'd)
Straight line master eq.



Do the same for the second data.

Hough Transform (cont'd)
Straight line master eq.



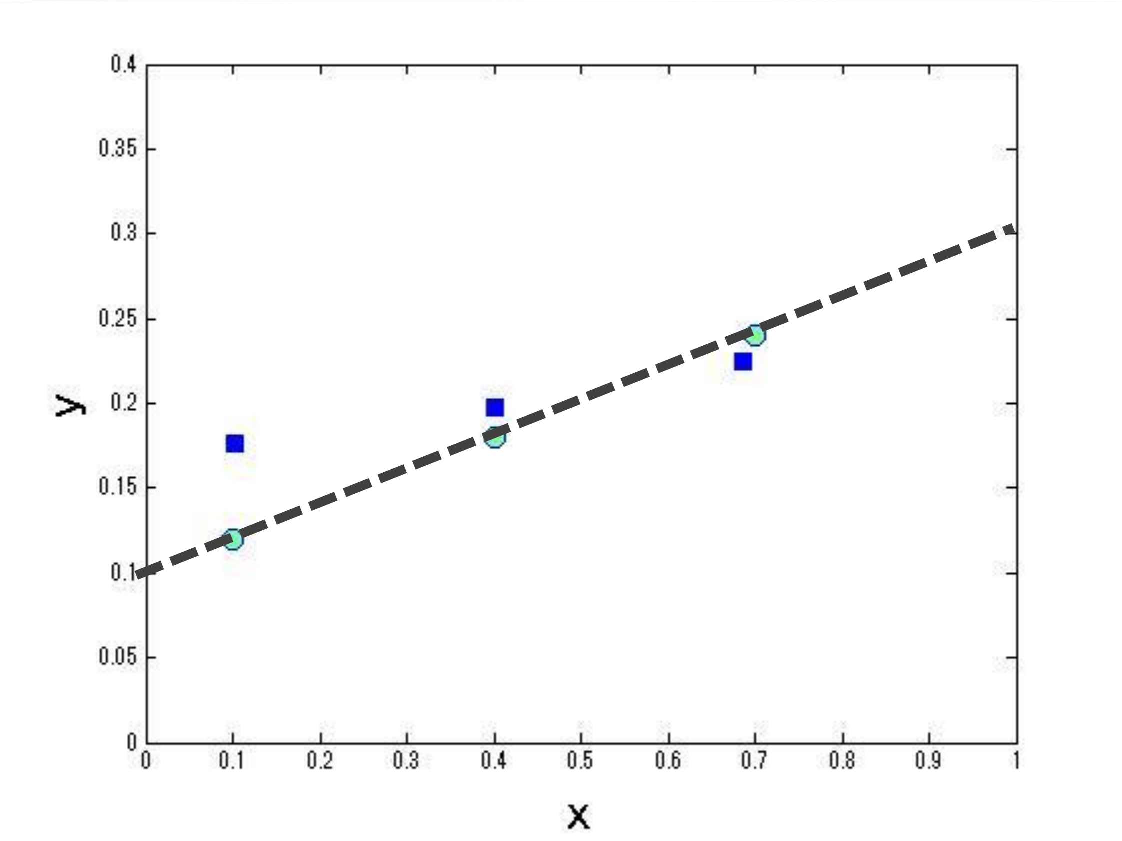
Move on to the parameter space.

If there is no noise, we have "the" solution.

NOTE:
Correct equation is
$y = 0.2\,x + 0.1$

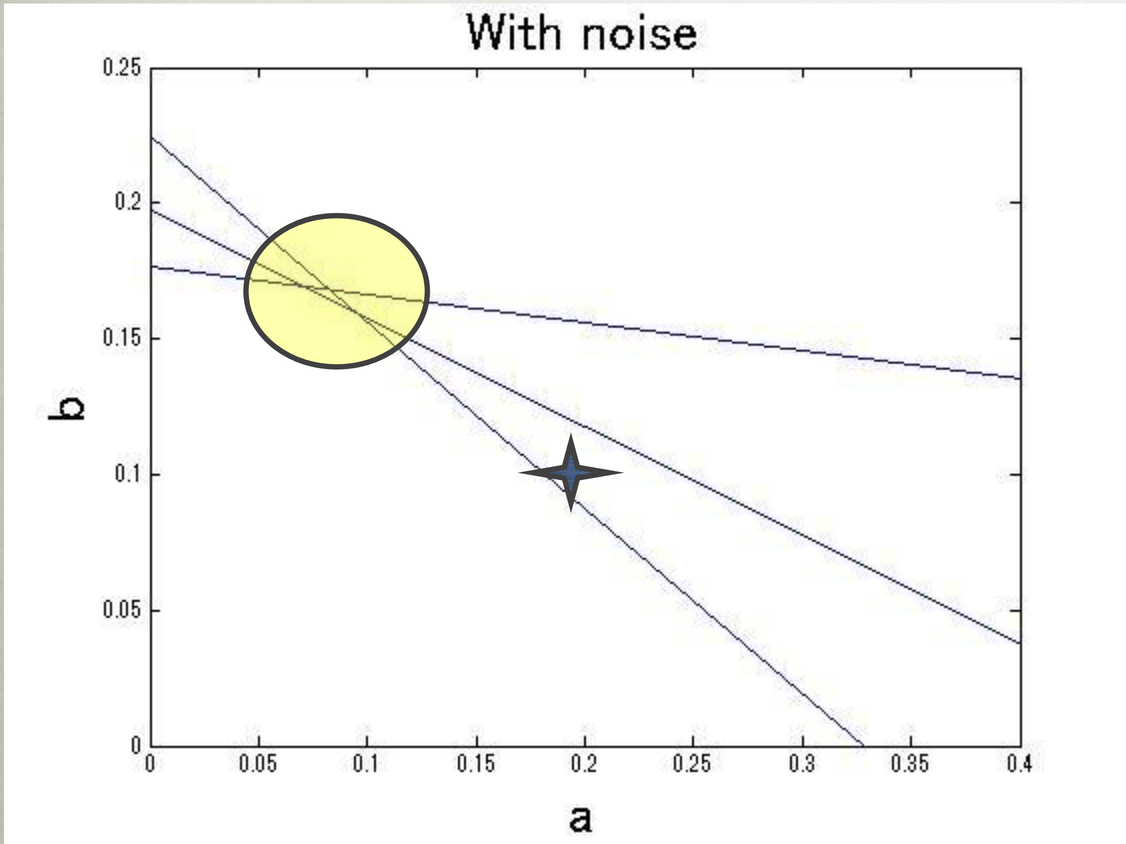
Hough Transform (cont'd)

Straight line master eq.



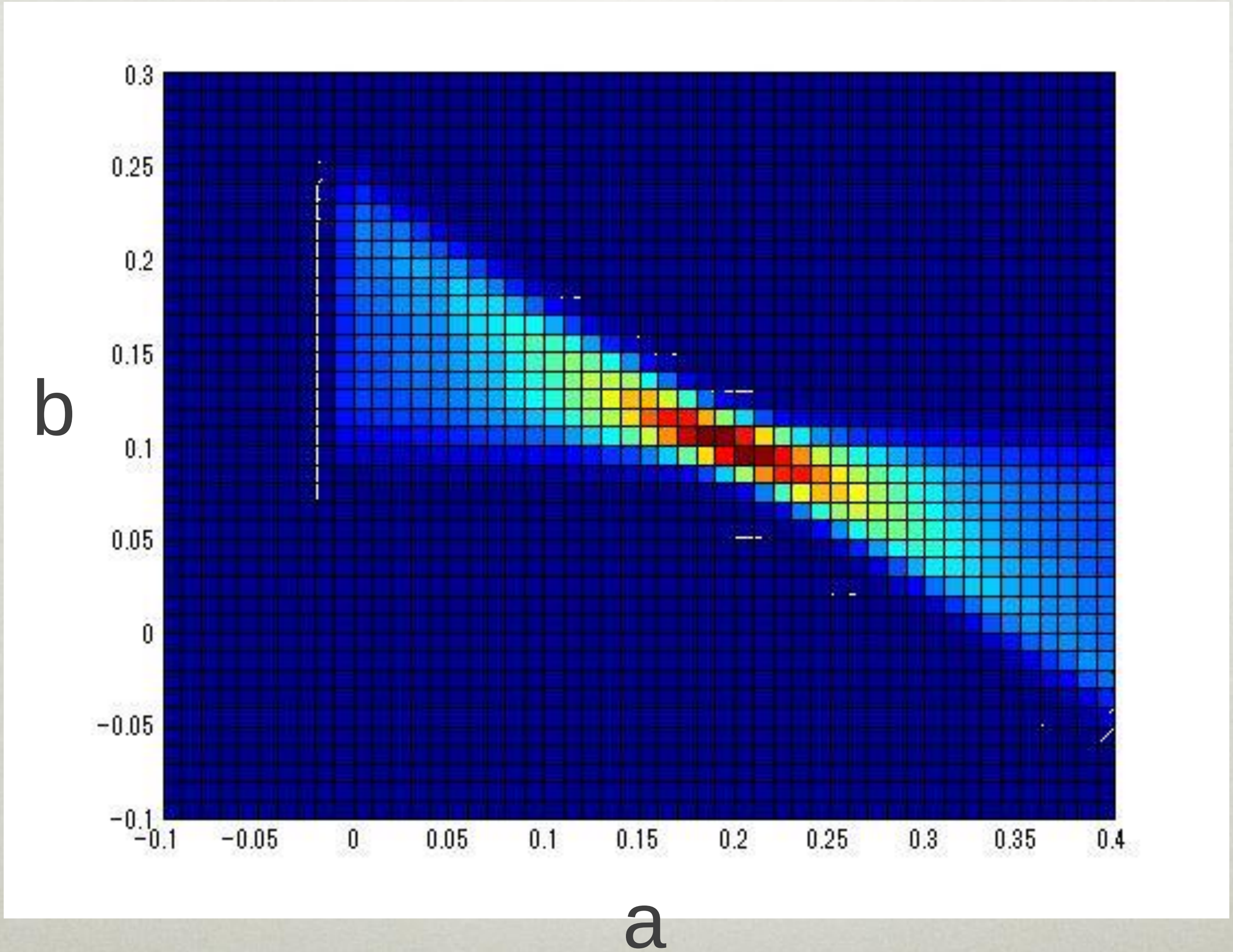
In reality, there is noise.

Hough Transform (cont'd)
Straight line master eq.

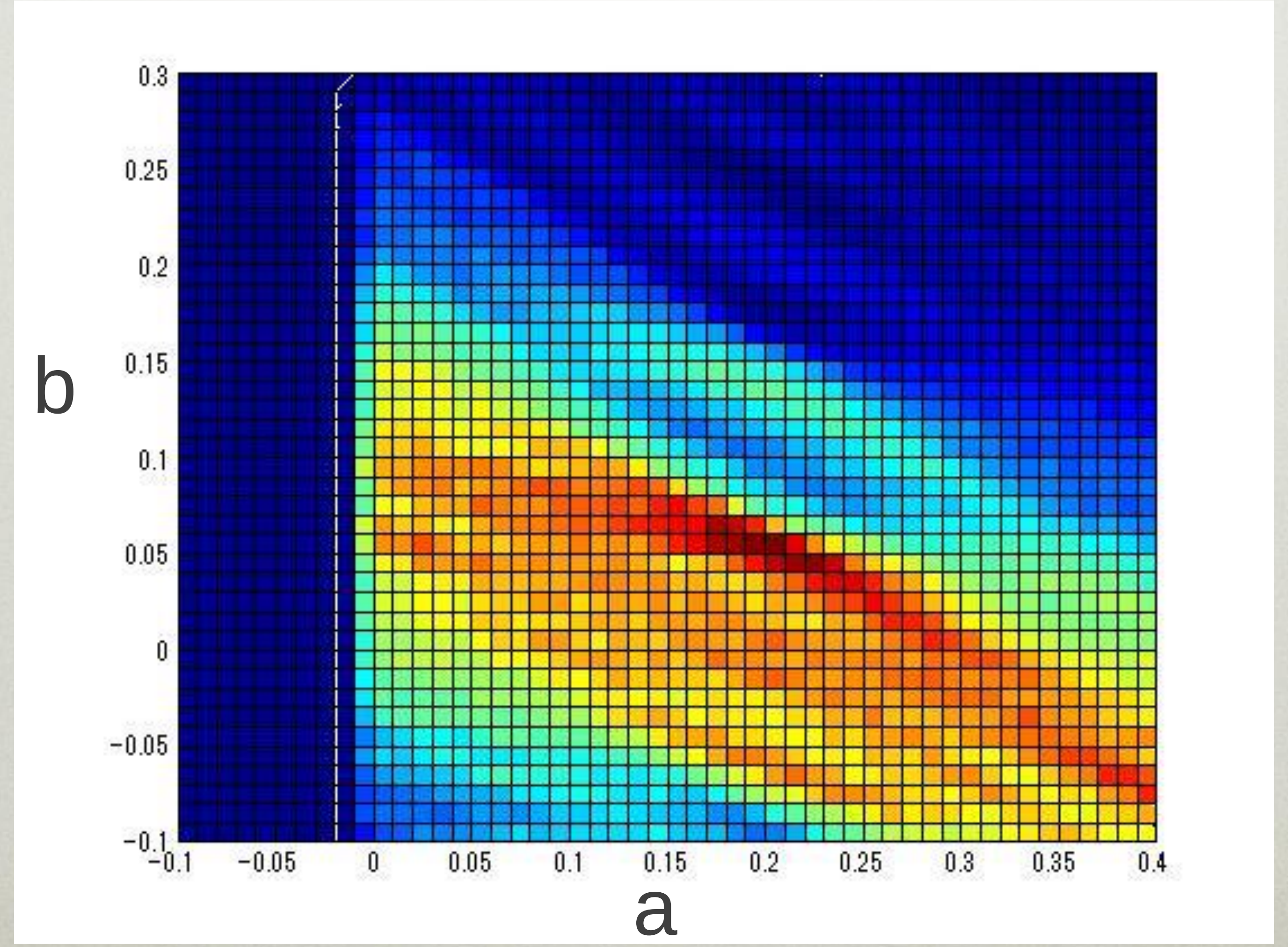


There is no "solution".
Yet the intersections cluster (hopefully) around the true value.

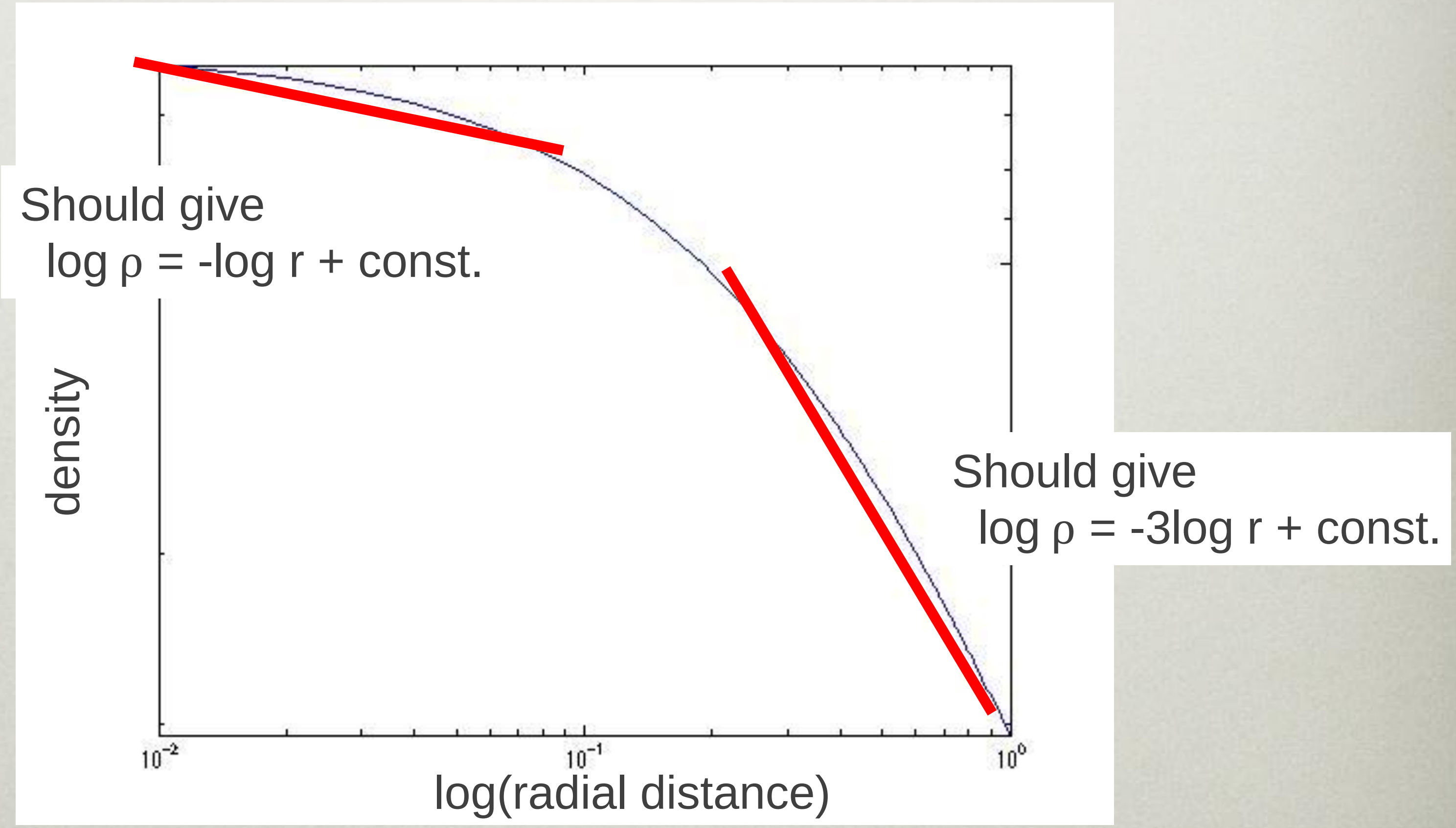
Do it many times….



Even when b is random variable



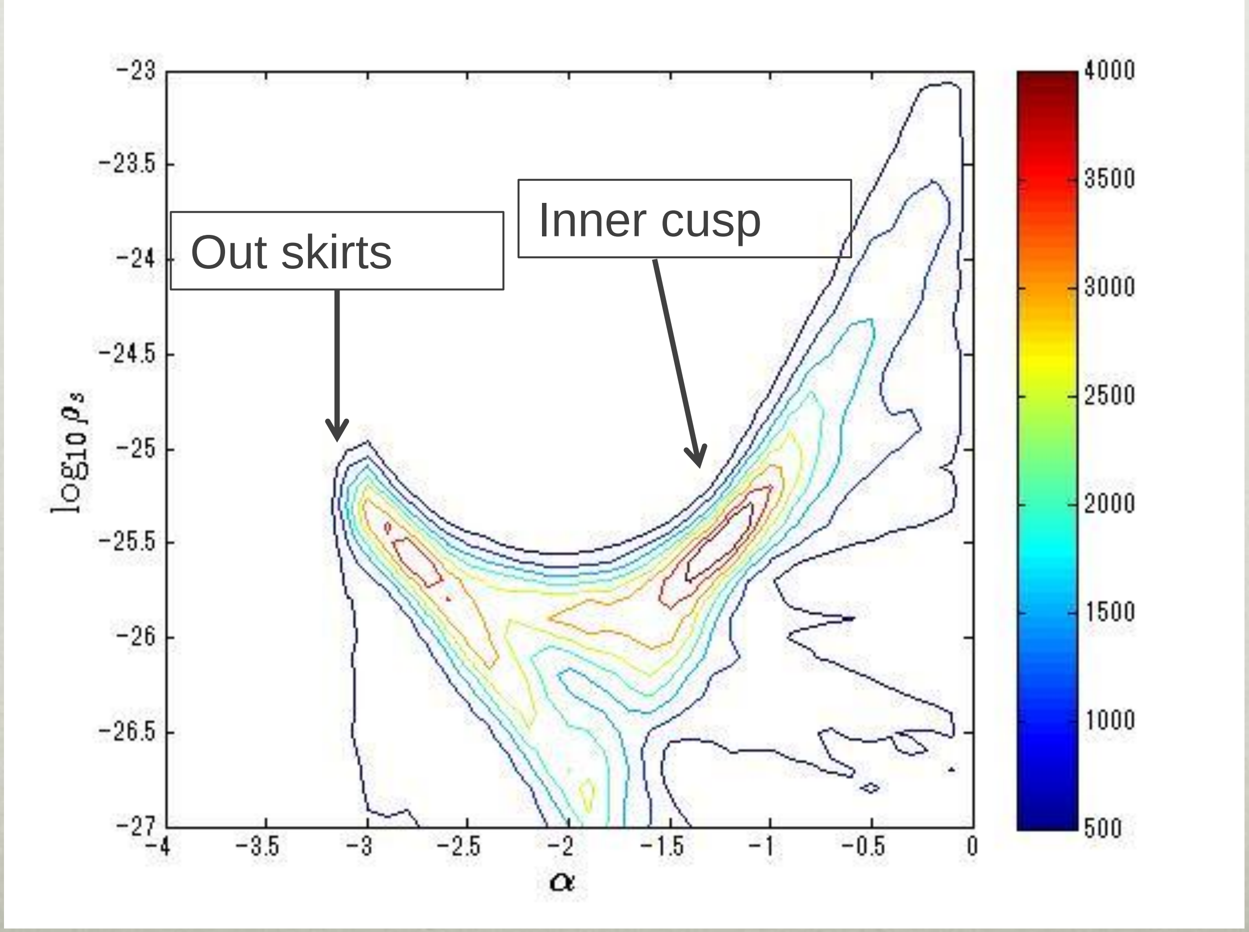
Semi-non-parametric HT on NFW profile.



Master equation: $\log_{10} \rho(r) = a \log_{10} r + b$

Re-examining Toy model using Hough Transform

Fit 1000 $\log_{10}\rho(r)$ toy-data using a master equation $$\log_{10}\rho(r) = a\log_{10} r + b$$



Conclusion: Stacking

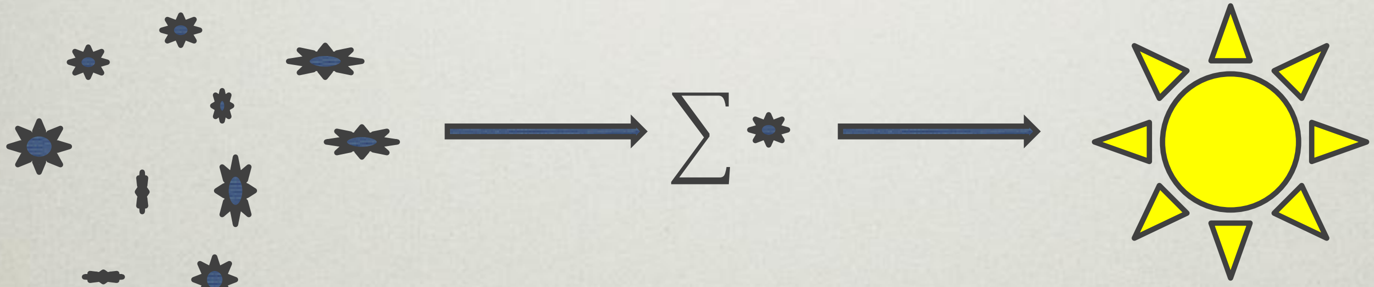
- In the case of finding “the” cluster “universal” profile.
  - Without scaling, stacking gives biased results.
- Introduce Hough transform.
  - It seems useful in many fields including testing power-law indices of the averaged cluster radial profiles.

Appendix

Power of Stacking

Gravitational wave: Cutler & Schutz 2005.

“We show that gravitational waves from collection of the few brightest (in gravitational waves) neutron stars could perhaps be detected before the single brightest source, … ” (Cutler & Schutz PRD 2005).



New test of modified gravity (cont'd)

Narikawa & Yamamoto (2012).

Narikawa's talk Wednesday afternoon for details.

- Assuming Navarro-Frenk-White (NFW), gNFW, or Einasto for the cluster density profile, the lensing potential becomes different from that of GR, due to the scalar DOF.
- At the outer-skirt of the lensing cluster observed, compare the observed tangential shear radial profile with the assumed one expected from the NFW density profile under GR.

New test of modified gravity (cont'd)

Narikawa & Yamamoto (2012).

Narikawa's talk Wednesday afternoon for details.

- Determine two parameters:
  - The strength of the modification to the Newtonian Gravity $G_{\rm eff} = G(1+\mu)$
  - And the length scale smaller than which the Newtonian gravity is recovered.

$$r_V = 13.4\epsilon^{2/3}\left(\frac{M_{\rm vir}}{10^{15}{\rm M}_\odot}\right)^{1/3} h^{-1}{\rm Mpc}.$$