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“Testing the origin of primordial perturbation: Use of bi- and tri-spectrum”

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Testing the origin of primordial perturbations

-Use of bi and tri-spectrum-

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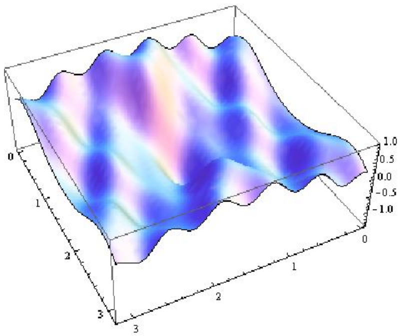
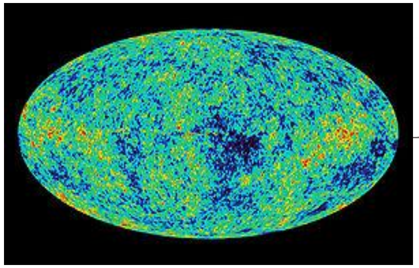
$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$

Primordial fluctuation

Primordial
fluctuation



300,000yrs



(WMAP data)

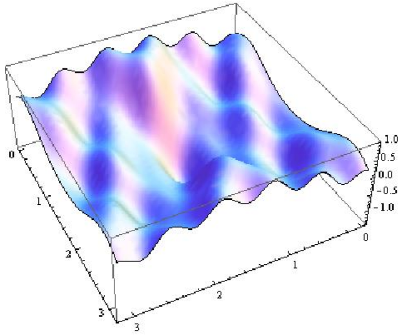
13 billion yrs

- matter fluctuation = fluctuation of spacetime (=curvature perturbation)
- It is a source of the current cosmic structure.
- From observations, we already know it does exist.
- But we don’t know its origin.

Generation of the primordial fluctuation

Basic paradigm

Inflation happened in the early Universe. During the inflationary stage, any light scalar field acquires fluctuations that are eventually stretched to cosmological scales.



$$\delta\phi \simeq \frac{H}{2\pi}$$

When such a scalar field affects the expansion of the Universe, its fluctuating energy density creates the curvature perturbation through the Einstein equations.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \left(T^{1}_{\mu\nu} + T^{2}_{\mu\nu} + \cdots \right)$$

Generation of the primordial fluctuation

- **Minimal scenario**

Inflaton fluctuations create the curvature perturbations.

$$\zeta \simeq \frac{H}{\dot{\phi}} \delta\phi$$

- Almost scale invariant and almost Gaussian.
- Good in the sense that it is simple, economical and consistent with all the observations so far. But this is just an assumption rather than a prediction.

Generation of the primordial fluctuations

- **Non-minimal scenario**

  Anything else

  Over the last decade, many non-minimal scenario have been proposed.

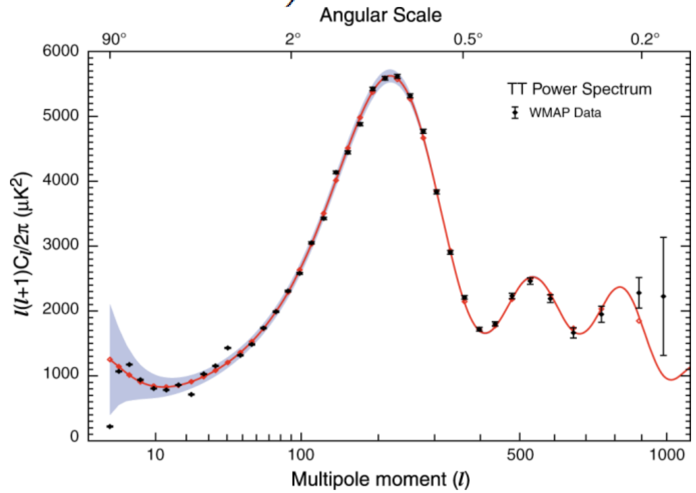
  Non-vacuum state, higher derivative interactions(DBI, Galileon, etc.), self-interactions, features in potential, conversion of isocurvature after inflation (multi-field models, curvaton, modulated reheating, multi-brid, inhomogeneous end of inflation, etc.), ….

Now, there are many models for the generation of the primordial fluctuation. We want to get useful information regarding its nature.

How can we reveal the origin of perturbations?

- Power spectrum

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle = (2\pi)^3 P_\zeta(k_1) \delta(\vec{k}_1 + \vec{k}_2)$$



All the models predict nearly scale invariant spectrum.

$$n_S - 1 = \frac{d \log(k^3 P_\zeta)}{d \log k} = \mathcal{O}(0.01)$$

P is not enough to disentangle the degeneracy.

How can we reveal the origin of perturbations?

Is there another way other than the power spectrum?

3- and 4-point functions could be useful!!

(non-Gaussianity)

Bi-spectrum: 3-point function

Tri-spectrum: 4-point function

- Useful in the sense that they can provide us what cannot be probed by means of the power spectrum.
- Any deviation from the minimal scenario generically leads to the detectable level of non-Gaussianity.
- Potentially observable in the near future.

Various shapes of bi-spectrum

- Local type (Komatsu&Spergel 2001)

$$B(k_1, k_2, k_3) \propto \left(\frac{1}{k_1^3 k_2^3} + 2 \text{ perms.} \right)$$

- Equilateral type (Creminelli et al 2005)

$$B(k_1, k_2, k_3) \propto \left(\frac{3(k_1 + k_2 - k_3)(k_2 + k_3 - k_1)(k_3 + k_1 - k_2)}{k_1^3 k_2^3 k_3^3} \right)$$

- Orthogonal type (Senatore et al 2009)

$$B(k_1, k_2, k_3) \propto \left(-\frac{3}{k_1^3 k_2^3} - \frac{3}{k_2^3 k_3^3} - \frac{3}{k_3^3 k_1^3} - \frac{8}{k_1^2 k_2^2 k_3^2} + \frac{3}{k1 k_2^2 k_3^3} + 5 \text{ perms.} \right)$$

Detection of primordial non-gaussianity and distinction of the scale dependence allows to constrain the models.

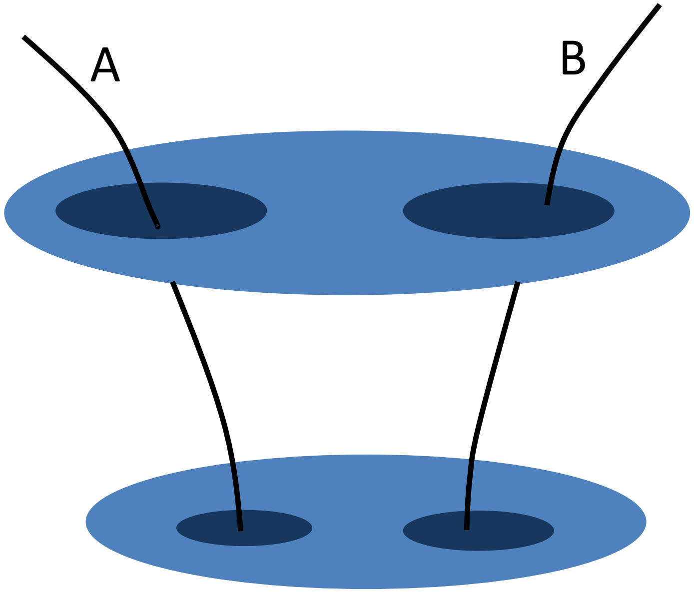
How can we reveal the origin of perturbations?

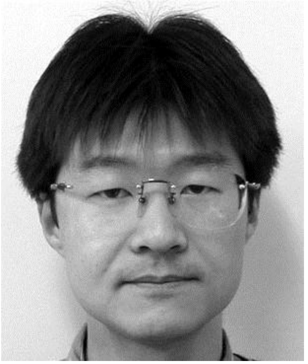
From now on, I will focus on the so-called local type perturbation.

The curvature perturbation is generated from field perturbations on super-horizon scales. (curvaton, modulated reheating, inhomogeneous end of inflation, multi-brid, etc.)

How can we reveal the origin of perturbations?

Separate universe approach (Kodama&Hamazaki 1998, Nambu&Taruya 1998, Wands et al 2000)





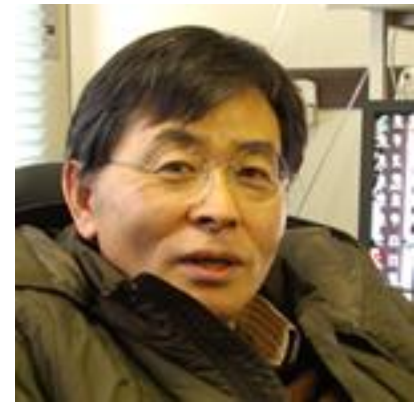
Perturbation is obtained by differentiating the background solutions.

Each Hubble region evolves like FLRW universe with different energy density and pressure.

δN formalism

(e.g. Starobinsky 1985, Salopek&Bond 1980, Sasaki&Stewart 1996, Sasaki&Tanaka 1998, Malik, Lyth&Sasaki 2004, Sugiyama,Komatsu&Futamase 2012)



$$\zeta = \delta N$$

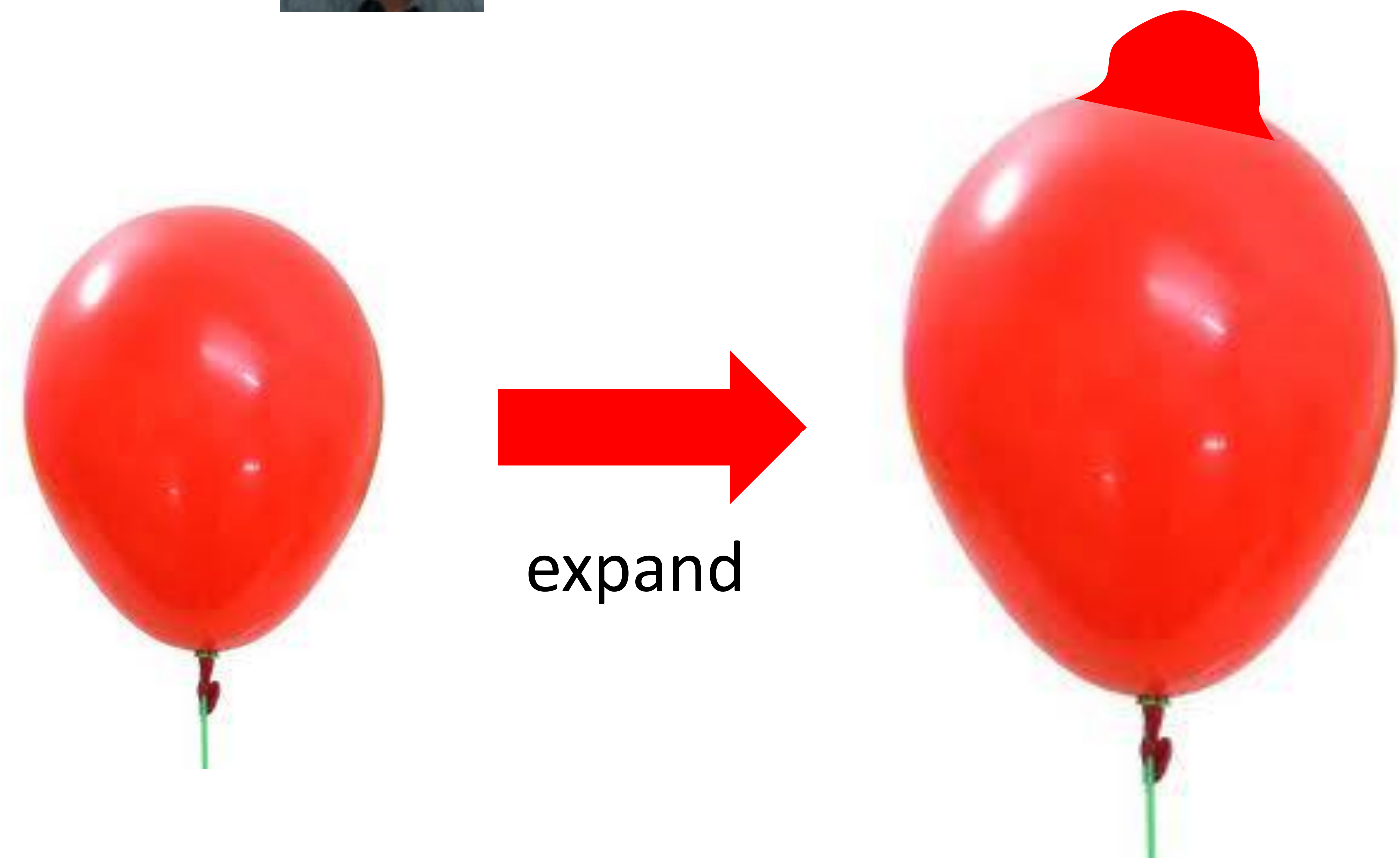
$$\zeta(t,\vec{x}) = \sum_a N_a(t)\delta\phi^a(\vec{x}) + \frac{1}{2}\sum_{ab} N_{ab}(t)\delta\phi^a(\vec{x})\delta\phi^b(\vec{x}) + \cdots$$

$$N_a(t) = \frac{\partial N(t)}{\partial \phi_a}, \quad N_{ab}(t) = \frac{\partial^2 N(t)}{\partial\phi_a \partial\phi_b}$$

Intuitive understanding of the δN formalism

Wisdom by T.Tanaka

Curvature = Expansion



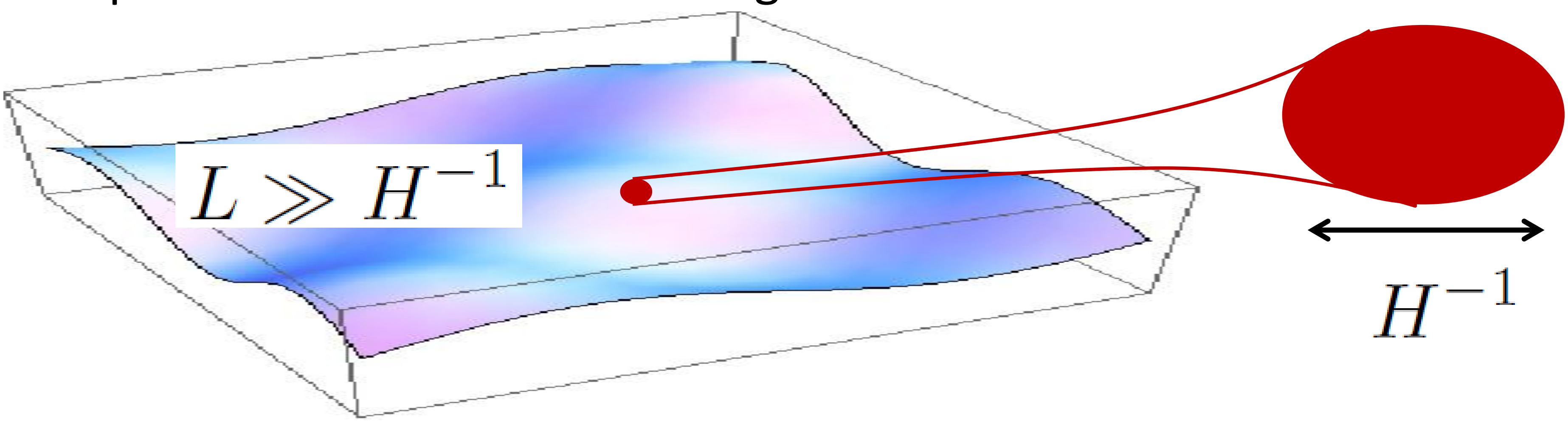
How can we reveal the origin of perturbations?

$$\zeta(t,\vec{x}) = \sum_a N_a(t)\delta\phi^a(\vec{x}) + \frac{1}{2}\sum_{ab} N_{ab}(t)\delta\phi^a(\vec{x})\delta\phi^b(\vec{x}) + \cdots$$

Same position

$\delta\phi^a(\vec{x})$: Gaussian(assumption)

This type of perturbation is realized if the focused scales are super-horizon at the time of its generation.

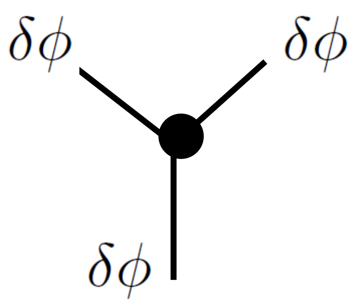


How can we reveal the origin of perturbations?

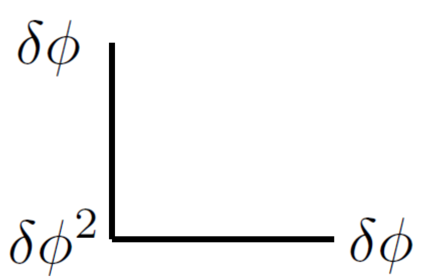
$$\zeta(t,\vec{x}) = \sum_a N_a(t)\delta\phi^a(\vec{x}) + \frac{1}{2}\sum_{ab} N_{ab}(t)\delta\phi^a(\vec{x})\delta\phi^b(\vec{x}) + \cdots$$

Two sources for non-Gaussianity of the curvature perturbation

- Non-gaussianity of field fluctuations



- Non-linear relation between field fluctuations and ζ



Non-Gaussianity(non-linearity) parameters

(Byrnes, Sasaki and Wands, 2006)

Bi-spectrum

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

Tri-spectrum

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = (2\pi)^3 T_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} \underline{f_{\mathrm{NL}}} \left(P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right)$$

$$\begin{aligned} T_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) &= \underline{\tau_{\mathrm{NL}}} \left(P_\zeta(k_{13}) P_\zeta(k_3) P_\zeta(k_4) + 11 \text{ perms} \right) \\ &\quad + \frac{54}{25} \underline{g_{\mathrm{NL}}} \left(P_\zeta(k_2) P_\zeta(k_3) P_\zeta(k_4) + 3 \text{ perms} \right) \end{aligned}$$

Three non-linearity parameters that are observables.

Non-Gaussianity parameters

Current observational bounds on the non-linearity parameters

$$-10 < f_{\rm NL} < 70$$

(Komatsu et al., 2010)

$$-0.6 \times 10^4 < \tau_{\rm NL} < 3.3 \times 10^4$$

$$-7.4 \times 10^5 < g_{\rm NL} < 8.2 \times 10^5$$

(Smidt et al., 2010)

Planck is expected to give us $f_{\rm NL} < \mathcal{O}(1)$ and $\tau_{\rm NL} < 600$.

(Kogo and Komatsu, 2006)

More stringent bound may be obtained by using other cosmological probes. (e.g. Pajer&Zaldarriaga 2012)

Non-Gaussianity parameters

- Standard canonical inflaton fluctuation

$$f_{\rm NL} = \mathcal{O}(\epsilon)$$ (e.g. Maldacena 2002)

- Non-inflaton scenarios

$$f_{\rm NL} = \mathcal{O}(1) \ \text{or} \ \gg \mathcal{O}(1)$$ depending on the model

Bispectrum can be useful to disentangle the degeneracy of models.

What can we say if we also include the trispectrum?

Local-type inequality

$$\frac{6}{5} f_{\rm NL} = \frac{N_a N_b N_{ab}}{(N_c N_c)^2} \qquad \tau_{\rm NL} = \frac{N_a N_b N_{ac} N_{bc}}{(N_d N_d)^3}$$

$$\tau_{\rm NL} \geq \left(\frac{6}{5} f_{\rm NL} \right)^2$$

(TS and M.Yamaguchi, 2008)

This is universal in the sense that it is independent of the underlying models.

Detection of fnl >20 means we surely detect $\tau_{\rm NL}$ as well.

This inequality also suggests a possibility that $f_{\rm NL}$ is small but $\tau_{\rm NL}$ can be very large.

Local-type inequality

More general statement (Smith, Lo Verde&Zaldarriaga 2011)

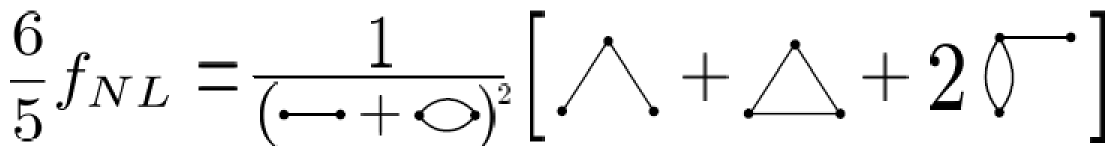
$$f_{NL} = \frac{5}{12} \lim_{k_1 \to 0} \frac{\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle'}{P_\zeta(k_1) P_\zeta(k_2)},$$

$$\tau_{NL} = \frac{1}{4} \lim_{|\mathbf{k}_1 + \mathbf{k}_2| \to 0} \frac{\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle'_c}{P_\zeta(k_1) P_\zeta(k_3) P_\zeta(|\mathbf{k}_1 + \mathbf{k}_2|)}.$$

$$\tau_{\rm NL} \geq \left(\frac{6}{5} f_{\rm NL} \right)^2$$

Local-type inequality

More general statement (Sugiyama 2012)



Inclusion of the loop corrections still leads to

$$\tau_{\rm NL} \geq \left(\frac{6}{5} f_{\rm NL}\right)^2$$

Local-type inequality

- Single source case (e.g. curvaton model)

$$\zeta = N_\phi \delta\phi + \frac{1}{2} N_{\phi\phi} \delta\phi^2 + \cdots$$

$$\frac{6}{5} f_{\rm NL} = \frac{N_a N_b N_{ab}}{(N_c N_c)^2} = \frac{N_{\phi\phi}}{N_\phi^2}$$

$$\tau_{\rm NL} = \frac{N_a N_b N_{ac} N_{bc}}{(N_d N_d)^3} = \frac{N_{\phi\phi}^2}{N_\phi^2}$$

$$\tau_{\rm NL} = \frac{36}{25} f_{\rm NL}^2$$

A consistency relation for the single source case

Local-type inequality

- Multi-source case (for example, inflaton and curvaton)

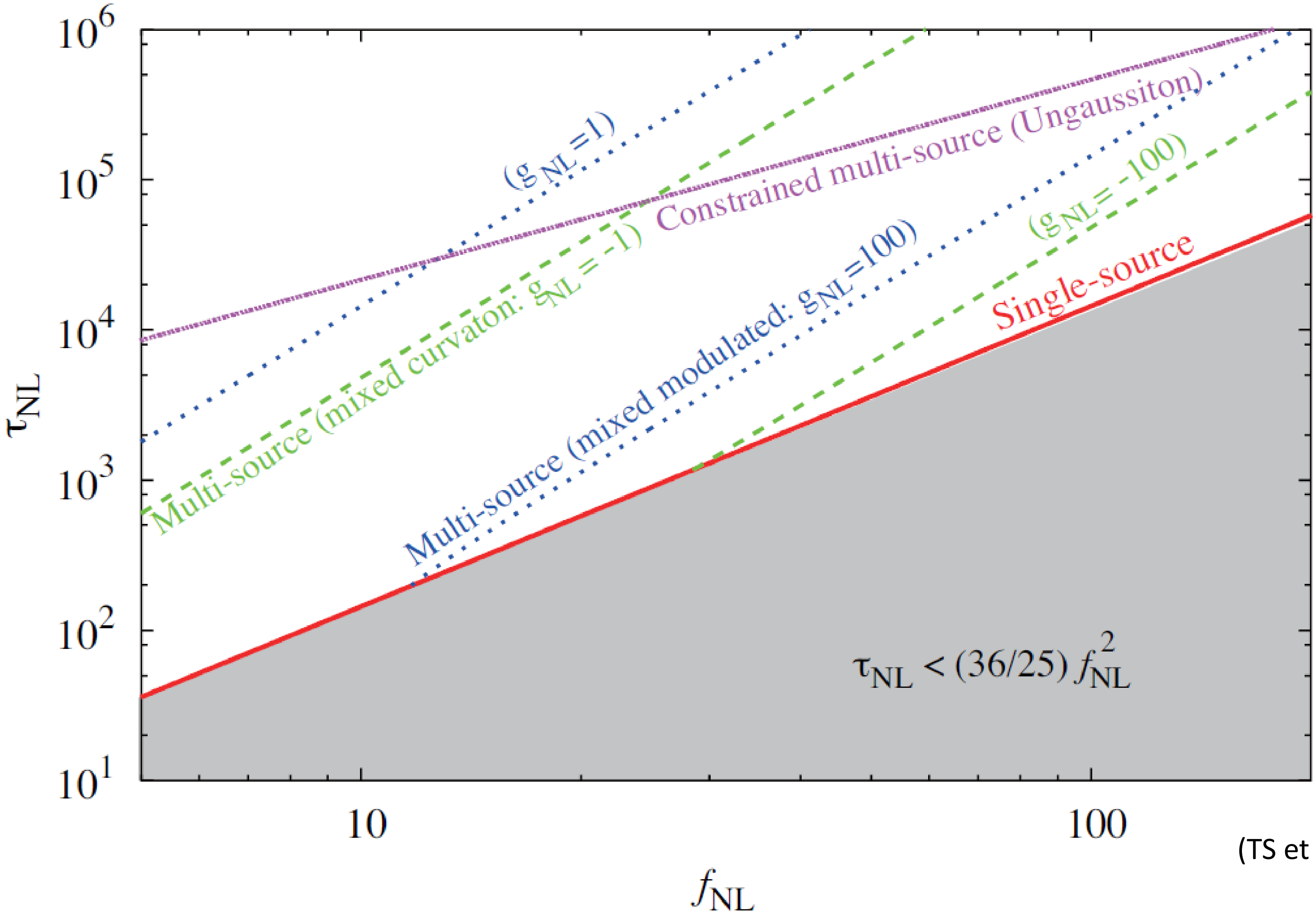
$$\zeta = N_\phi \delta\phi + N_\sigma \delta\sigma + \frac{1}{2} N_{\sigma\sigma} \delta\sigma^2 + \cdots$$ (Ichikawa et al. 2008)

Gaussian Non-Gaussian

$$\Rightarrow \quad \frac{25}{36} \frac{\tau_{\rm NL}}{f_{\rm NL}^2} = 1 + \frac{N_\phi^2}{N_\sigma^2} = 1 + \frac{\mathcal{P}_{\zeta\phi}}{\mathcal{P}_{\zeta\sigma}}$$

If the non-gaussian part is subdominant, the ratio becomes much larger than 1.

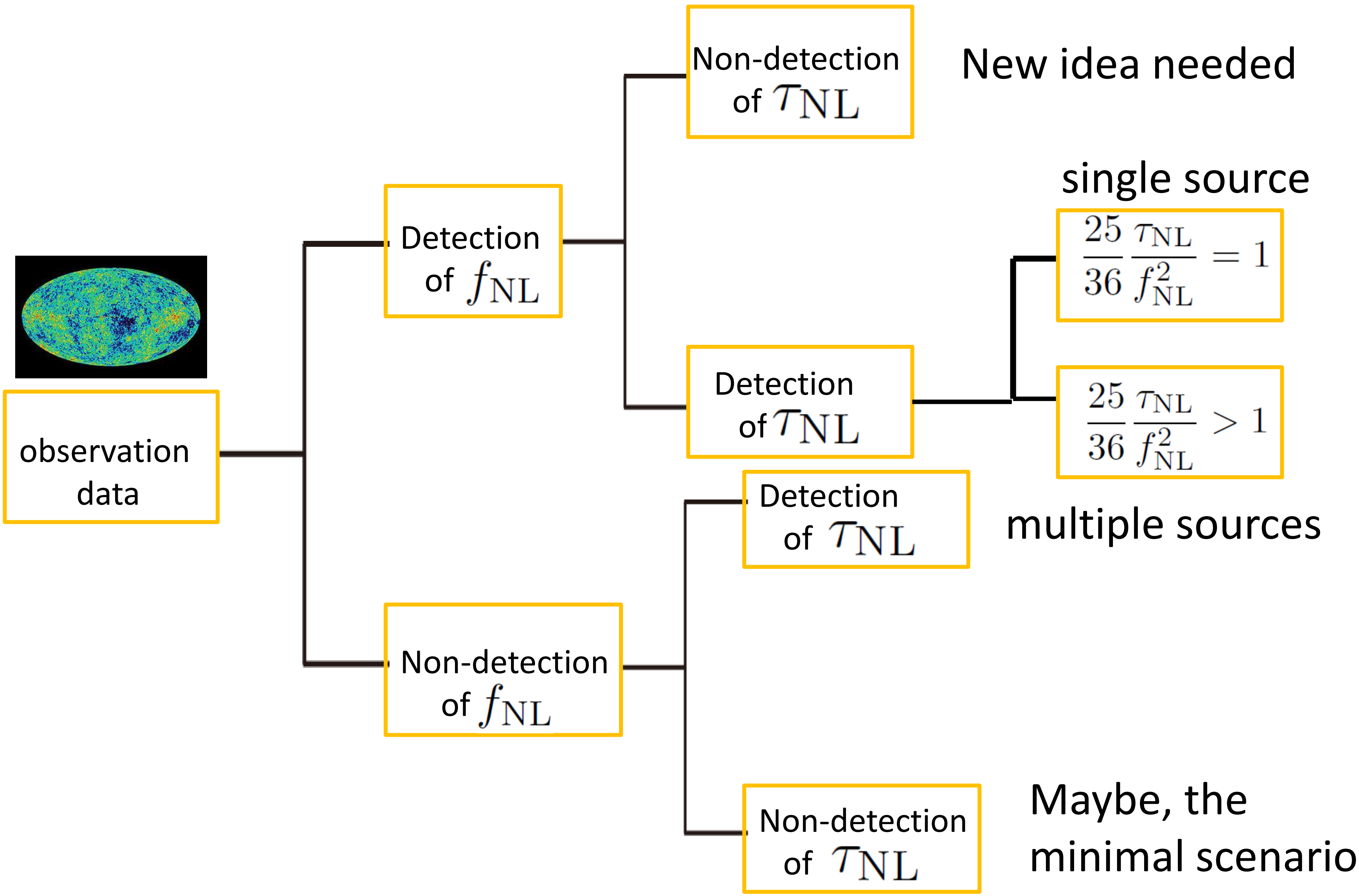
A ratio $\tau_{\rm NL}/f_{\rm NL}^2$ is a good indicator to get information of number of fields that contribute to the curvature perturbation.



10^6
10^5
10^4
10^3
10^2
10^1
τ_{NL}
10
100
f_{NL}
$(g_{NL}=1)$
Constrained multi-source (Ungaussiton)
Multi-source (mixed curvaton: $g_{NL}=-1$)
Multi-source (mixed modulated: $g_{NL}=100$)
$(g_{NL}=-100)$
Single-source
$\tau_{NL} < (36/25) f_{NL}^2$

(TS et al. 2010)

Basic strategy to test the origin of perturbation



What about gnl?

$$\frac{54}{25}g_{\rm NL} = \frac{N_{abc}N_aN_bN_c}{(N_dN_d)^3}$$

This is not directly related to fnl and taunl. Thus, it brings another information specific to each model.

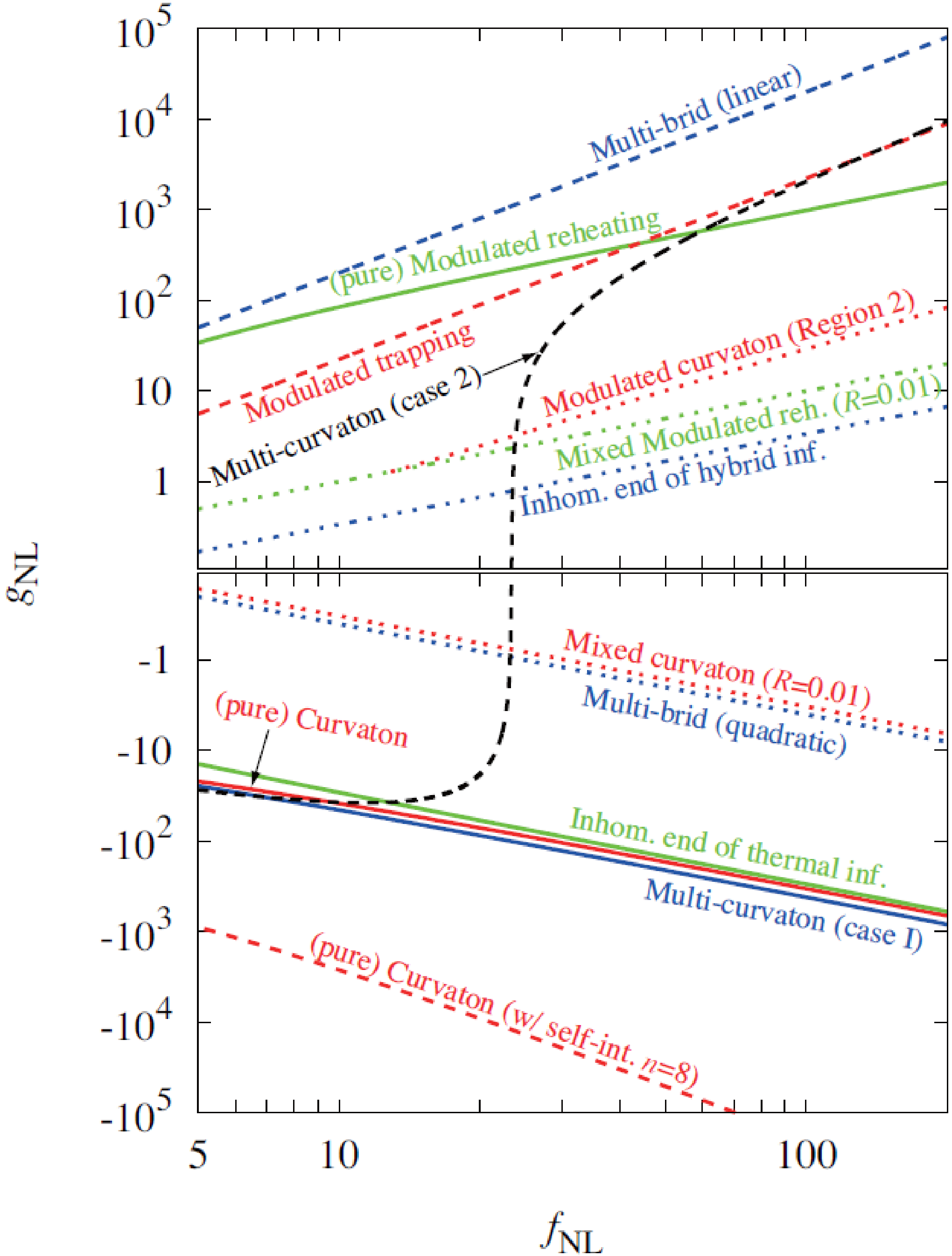
Example: self-interacting curvaton (e.g. Engvist&Nurmi 2005)

$$V(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \lambda m_\sigma^4\left(\frac{\sigma}{m_\sigma}\right)^n$$

$$g_{\rm NL} = -\frac{10}{3}f_{\rm NL} - \frac{575}{108}$$ without self-interaction

$$g_{\rm NL} \simeq \underline{A_{\rm NQ}f_{\rm NL}^2} + B_{\rm NQ}f_{\rm NL} + C_{\rm NQ}$$ with self-interaction

gnl is sesitive to the shape of the field potential.



Multi-brid (linear)
(pure) Modulated reheating
Modulated trapping
Multi-curvaton (case 2)
Modulated curvaton (Region 2)
Mixed Modulated reh. (R=0.01)
Inhom. end of hybrid inf.
Mixed curvaton (R=0.01)
Multi-brid (quadratic)
(pure) Curvaton
Inhom. end of thermal inf.
Multi-curvaton (case I)
(pure) Curvaton (w/ self-int. n=8)
g_{NL}
f_{NL}

What about gnl?

We find gnl is model dependent. But, we can classify the models into three types:

- Suppressed $g_{\rm NL}$ type ($g_{\rm NL} \sim$ [suppression factor] $\times f_{\rm NL}$)
- Linear $g_{\rm NL}$ type ($g_{\rm NL} \sim f_{\rm NL}$)
- Enhanced $g_{\rm NL}$ type ($g_{\rm NL} \sim f_{\rm NL}^n$ with $n > 1$ or $n = 2$ for many models)

Consistency relations among non-linearity parameters

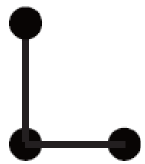
Category	$f_{\rm NL}$–$\tau_{\rm NL}$ relation	Examples and $f_{\rm NL}$–$g_{\rm NL}$ relation
Single-source	$\tau_{\rm NL} = (6 f_{\rm NL}/5)^2$	(pure) curvaton (w/o self-interaction) $[g_{\rm NL} = -(10/3) f_{\rm NL} - (575/108)]^{(a)}$
		(pure) curvaton (w/ self-interaction) $[g_{\rm NL} = A_{\rm NQ} f_{\rm NL}^2 + B_{\rm NQ} f_{\rm NL} + C_{\rm NQ}]^{(b)}$
		(pure) modulated reheating $[g_{\rm NL} = 10 f_{\rm NL} - (50/3)]^{(c)}$
		modulated-curvaton scenario $\left[g_{\rm NL} = 3 r_{\rm dec}^{1/2} f_{\rm NL}^{3/2}\right]^{(d)}$
		Inhomogeneous end of hybrid inflation $[g_{\rm NL} = (10/3)\eta_{\rm cr} f_{\rm NL}]$
		Inhomogeneous end of thermal inflation $[g_{\rm NL} = -(10/3) f_{\rm NL} - (50/27)]^{(e)}$
		Modulated trapping $[g_{\rm NL} = (2/9) f_{\rm NL}^2]^{(f)}$
Multi-source	$\tau_{\rm NL} > (6 f_{\rm NL}/5)^2$	mixed curvaton and inflaton $[g_{\rm NL} = -(10/3)(R/(1+R)) f_{\rm NL} - (575/108)(R/(1+R))^3]^{(g)}$
		mixed modulated and inflaton $[g_{\rm NL} = 10(R/(1+R)) f_{\rm NL} - (50/3)(R/(1+R))^3]^{(h)}$
		mixed modulated trapping and inflaton $[g_{\rm NL} = (2/9)((1+R)/R) f_{\rm NL}^2 = (25/162)\tau_{\rm NL}]^{(i)}$
		multi-curvaton $[g_{\rm NL} = C_{\rm mc} f_{\rm NL}, \ \ g_{\rm NL} = (4/15) f_{\rm NL}^2]^{(j)}$
		Multi-brid inflation (quadratic potential) $[g_{\rm NL} = -(10/3)\eta f_{\rm NL}, \ \ g_{\rm NL} = 2 f_{\rm NL}^2]^{(k)}$
		Multi-brid inflation (linear potential) $[g_{\rm NL} = 2 f_{\rm NL}^2]^{(l)}$
Constrained multi-source	$\tau_{\rm NL} = C f_{\rm NL}^n$	ungaussiton ($C \simeq 10^3$, $n = 4/3$)

(TS et al. 2010)

Different consistency relations for different models.

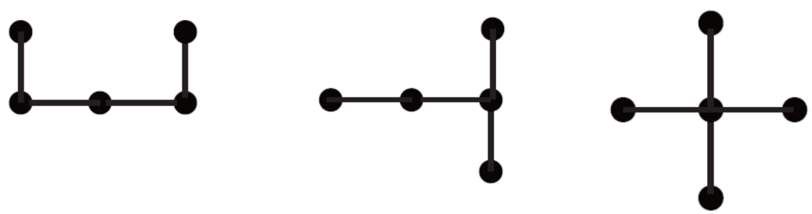
Higher order correlators

3-point function

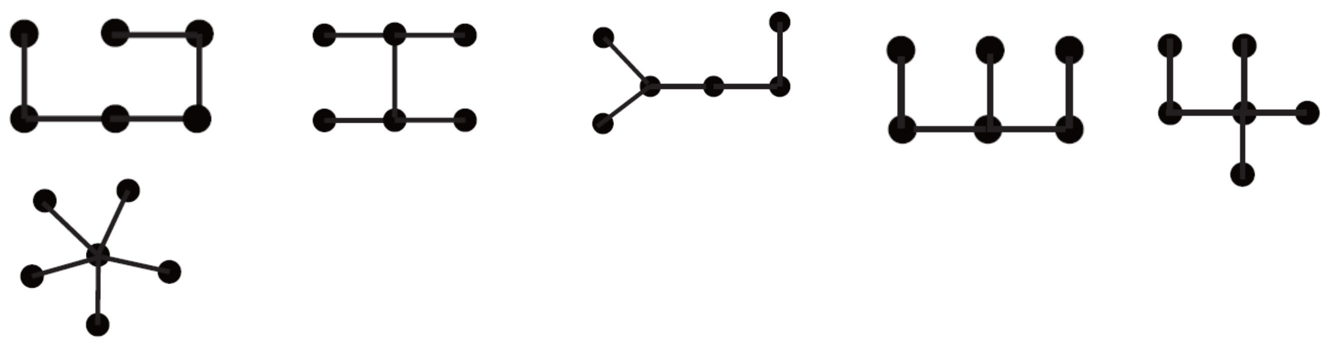


4-point function

5-point function



6-point function



# of Non-linearity parameters=# of graphs with different topologies

General formula of # for arbitrary n is known. (Fry 1984)

The local type inequalities for n-point function (n:even)

General method to derive the local type inequalities

(TS and Yokoyama, 2011)

Example: 6-point function

$$\tau_6^{(1)} \tau_6^{(2)} \geq \left(g_6^{(1)} \right)^2$$

$$\tau_6^{(1)} \tau_4 \geq \left(f_5^{(1)} \right)^2, \qquad \tau_6^{(2)} \tau_4 \geq \left(f_5^{(2)} \right)^2$$

$$\tau_6^{(1)} \geq \tau_4^2, \qquad \tau_6^{(2)} \geq g_4^2$$

Five independent inequalities

$f_{\rm NL}$

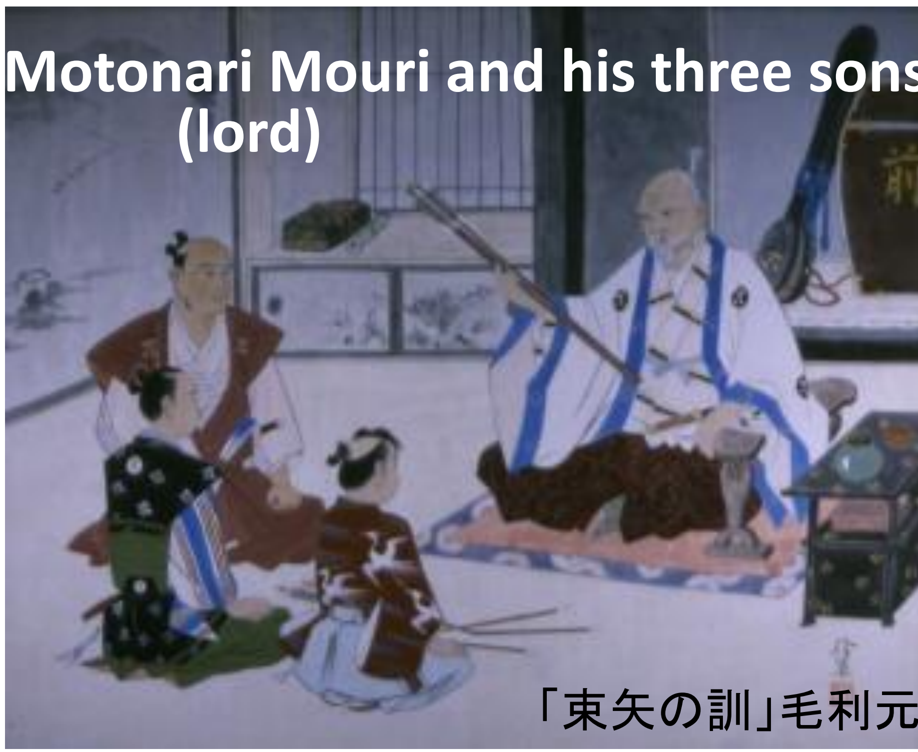
suggests significant contribution of non-inflaton source.

$\tau_{\rm NL}$

clarifies if the perturbation is sourced by a single field or not.

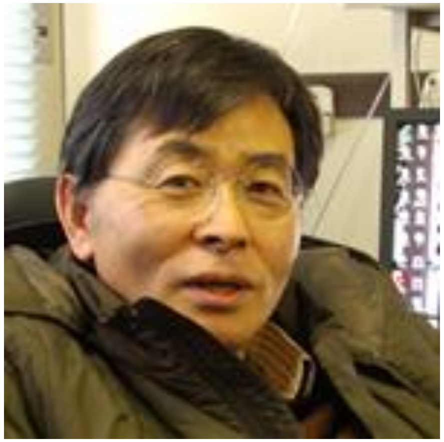
$g_{\rm NL}$

probes interactions(parameters) of the candidate model.



「束矢の訓」毛利元就一代絵（高増径草筆）

But, I do hope this is not the case for the following three persons. Individuals are already awesome for me.



Summary

Test of non-gaussianity can be useful to reveal the origin of primordial perturbations.

We can obtain information about number of fields contributing to the curvature perturbation by using fnl and taunl.

Detection of all the non-linearity parameters greatly helps us constrain the model of the early Universe.

Very soon, Planck results will tell us something.